Exploring a superradiant laser of Ginzburg phonons

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(joint work with J. Marino and I. Carusotto)

Simulating gravitation and cosmology in condensed matter and optical systems

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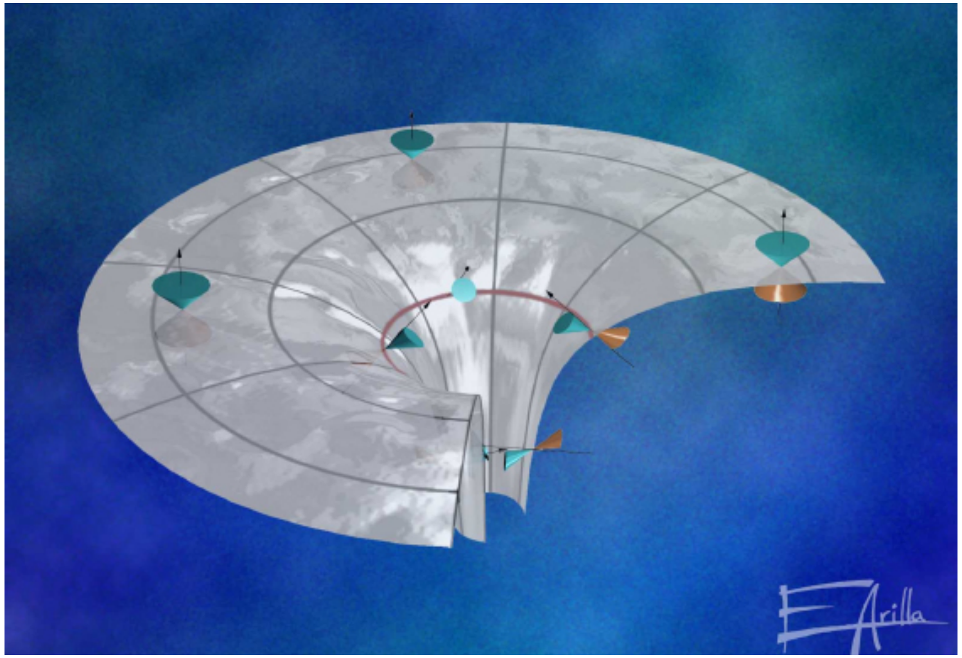
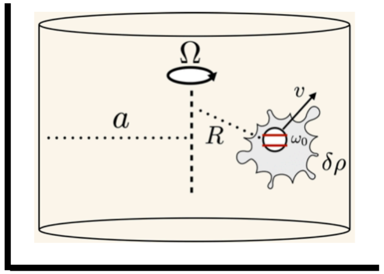
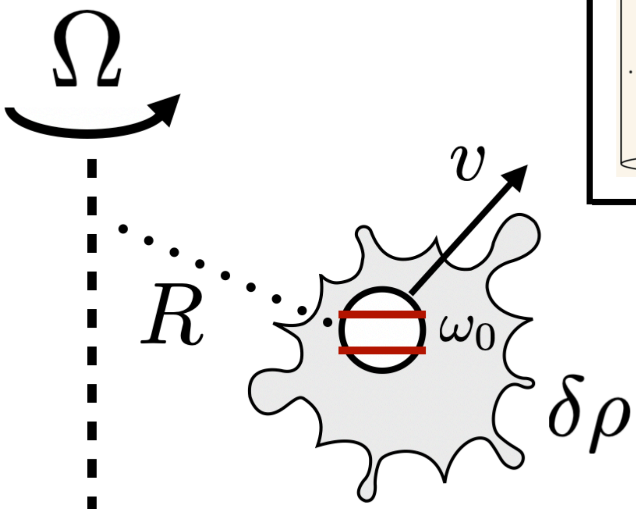
Analogs of classical gravity phenomena with ultra-cold gases represents a growing line of inquiry in modern condensed matter physics.



Extracted from "Analogue Gravity" (Barceló, Liberati, Visser,2011)





Our exploration: Rotating detector embedded in a weakly interacting Bose gas, and we study the amplification of the ground state excitation rate of the detector at supersonic speeds.

Hamiltonian: $$H = \omega_0 \Sigma_z + g_- \Sigma_x \delta\rho(\mathbf{r}) + \int d\mathbf{q}\, \omega_{\mathbf{q}}\, b^\dagger_{\mathbf{q}} b_{\mathbf{q}}$$

Physical situations of interest: Excitation rates for a two-level system (Marino, Recati, Carusotto, 2017) and a simple harmonic oscillator!

Motivation: *Superradiance!* $(\Omega > \omega/n)$

Rotating black hole: Superradiant amplification condition is a consequence of the area theorem. Moreover, its existence requires the presence of an event horizon and a region called ergosphere.

In the present case, $R \gtrsim c/\Omega$ is the critical threshold.

Upshots:

- Mechanism of excitation by superradiant Bogolyubov quanta has a close connection to the process of emission of supersonic Ginzburg radiation for the cylindrically confined system.
- We find conditions for the onset of dynamical instabilities, and find prospects of lasing of superradiant Bogolyubov quanta!

- Density fluctuations (Bogoliubov theory, unbounded space):

$$\delta\rho(t,\mathbf{r}) = \sqrt{\rho_0}\,\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}dk\int_0^{\infty}dq q(u_K+v_K)\Big(\phi_{qnk}b_{qnk}(t)+\phi^*_{qnk}b^\dagger_{qnk}(t)\Big)$$

$$\begin{aligned}\phi_{qnk} &= J_n(qr)e^{in\theta}e^{ikz}\\ b_{qnk}(t) &= b_{qnk}(0)e^{-i\omega_K t}\end{aligned}$$

$$\begin{aligned}(u_K+v_K)^2 &= E_K/\omega_K\\ E_K &= K^2/2m\\ \omega_K &= cK\sqrt{1+\left(\frac{K\xi}{2}\right)^2}\\ K^2 &= q^2+k^2\end{aligned}$$

- Density fluctuations (Bogoliubov theory, confined system, rotating frame):

$$\delta\rho(t,\mathbf{r}) = \sqrt{\rho_0}\,\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\sum_{\nu=1}^{\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}dk(u_{n\nu k}+v_{n\nu k})\Big(\bar\psi_{n\nu k}\bar b_{n\nu k}(t)+\bar\psi^*_{n\nu k}\bar b^\dagger_{n\nu k}(t)\Big)$$

$$\begin{aligned}\bar\psi_{n\nu k} &= J_n\left(\frac{\xi_{n\nu}r}{a}\right)e^{in\bar\theta}e^{ikz}\\ \bar b_{n\nu k}(t) &= \bar b_{n\nu k}(0)e^{-i\bar\omega_{n\nu k}t}\end{aligned}$$

$$\begin{aligned}(u_{n\nu k}+v_{n\nu k})^2 &= \frac{2}{a^2[J_{n+1}(\xi_{n\nu})]^2}\frac{E_{n\nu k}}{(\bar\omega_{n\nu k}+n\Omega)}\\ E_{n\nu k} &= K^2_{n\nu k}/2m\\ \bar\omega_{n\nu k} &= \omega_{n\nu k}-n\Omega\\ \omega_{n\nu k} &= cK_{n\nu k}\sqrt{1+\left(\frac{K_{n\nu k}\,\xi}{2}\right)^2}\\ K^2_{n\nu k} &= \xi^2_{n\nu}/a^2+k^2\end{aligned}$$

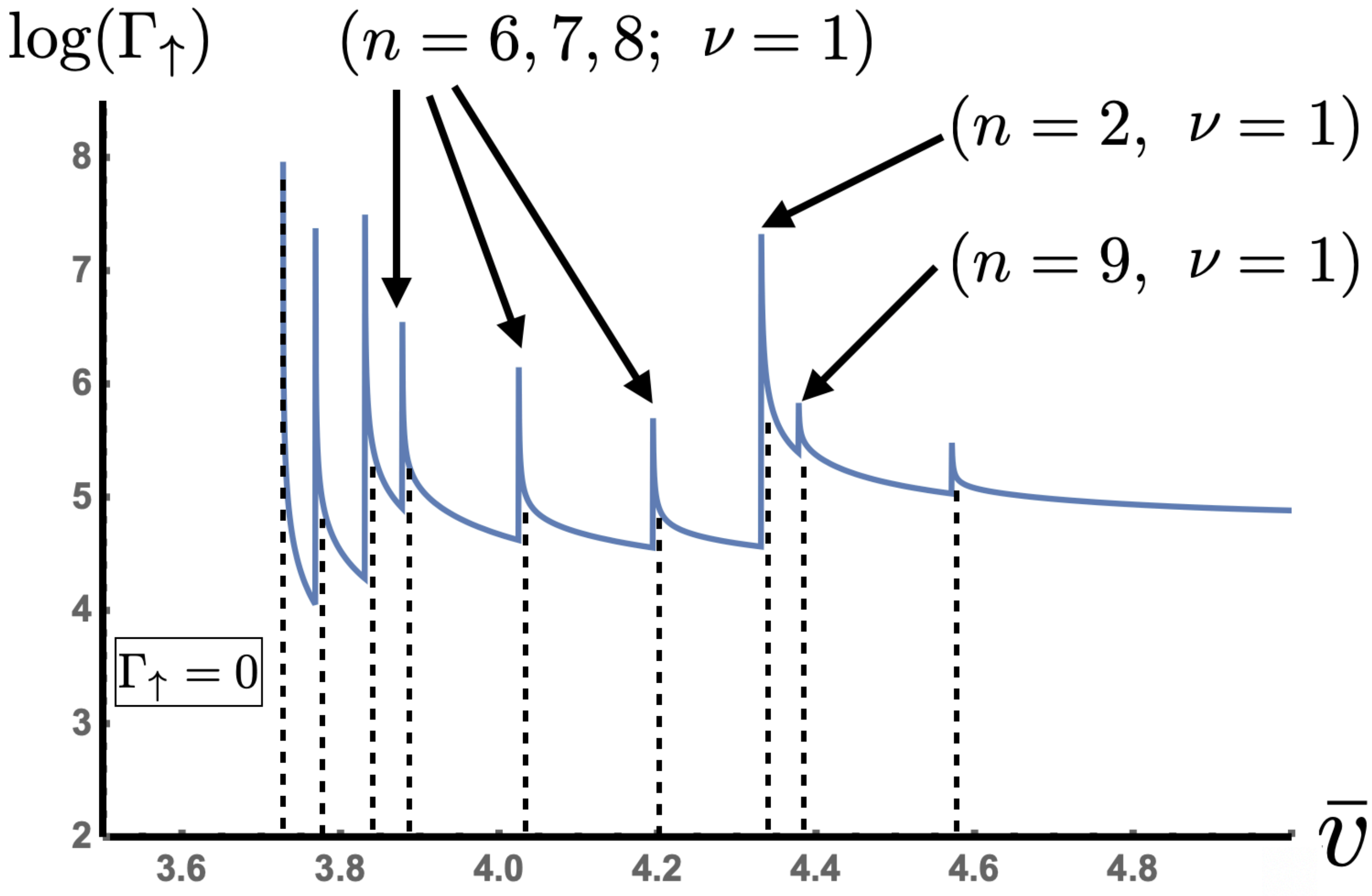
- Unbounded space: Excitation rate is non-vanishing only if

$$\boxed{n\Omega > kc\sqrt{1+\left(\frac{k\xi}{2}\right)^2} + \omega_0}$$

- Confined system: A necessary condition to excite modes involves cylinder radii larger than a characteristic length set by the sound speed and the angular frequency, in close connection to the rotational superradiant condition:

$$\boxed{n\Omega > \frac{c\,\xi_{n\nu}}{a}\sqrt{1+\left(\frac{\xi\xi_{n\nu}}{2a}\right)^2} + \omega_0 \Longrightarrow \Omega a > c}$$

- Superradiant instabilities: Laser of Ginzburg phonons!



The dimensionless excitation rate, $\Gamma_\uparrow$, evaluated for $\bar{y} = \xi/a = 0.4$, $\bar{R} = R/a = 0.6$ and $W = 0.3$, as function of the rescaled radius $\bar{v} = (\Omega a)/c$. For this specific instance, the rate drops to zero for $\bar{v} \lesssim 3.78$. The peaks represent resonant contributions associated to the excitation of the eigenenergies of the modes of the cavity. In figure we indicate the resonance channels that open upon increasing $\bar{v}$; the first three peaks are respectively labelled by the quantum numbers $(n = 4, 5, 3; \nu = 1)$.

- Rotational superradiance can be at the origin of dynamical instabilities.

- In order to demonstrate that, one could first confine the system on a two-dimensional plane, i.e. we assume that excitations along the z-direction are gapped. We also treat the impurity as a single bosonic mode.

- In the frame co-moving with the detector, the Hamiltonian can be written as:

$$H = \omega_0 \left(a^\dagger a + \frac{1}{2} \right) + \sum_{n=-\infty}^{\infty} \sum_{\nu=1}^{\infty} \left[\bar{\omega}_{n\nu} \left(\bar{b}^\dagger_{n\nu} \bar{b}_{n\nu} + \frac{1}{2} \right) + \lambda_{n\nu} (a^\dagger + a) \left(\bar{b}^\dagger_{n\nu} + \bar{b}_{n\nu} \right) \right]$$

Outlook

- Excitation rate is non-vanishing whenever a generalized superradiant condition is satisfied. This superradiant condition can be therefore viewed as the counterpart of Ginzburg emission from a linearly moving impurity in a cold gas.

- A cylindrical mirror confining a circularly moving detector coupled to a BEC constitutes a convenient avenue to achieve self-amplified radiation and lasing, in the spirit of rotational superradiance.

- Connection between our results and 'black-hole bombs' in analogue models?