# Scattering of co-current surface waves on an analogue black hole

Based on: arXiv:1806.05539 (2018/19)

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*Simulating gravitation and cosmology in condensed matter and optical systems*

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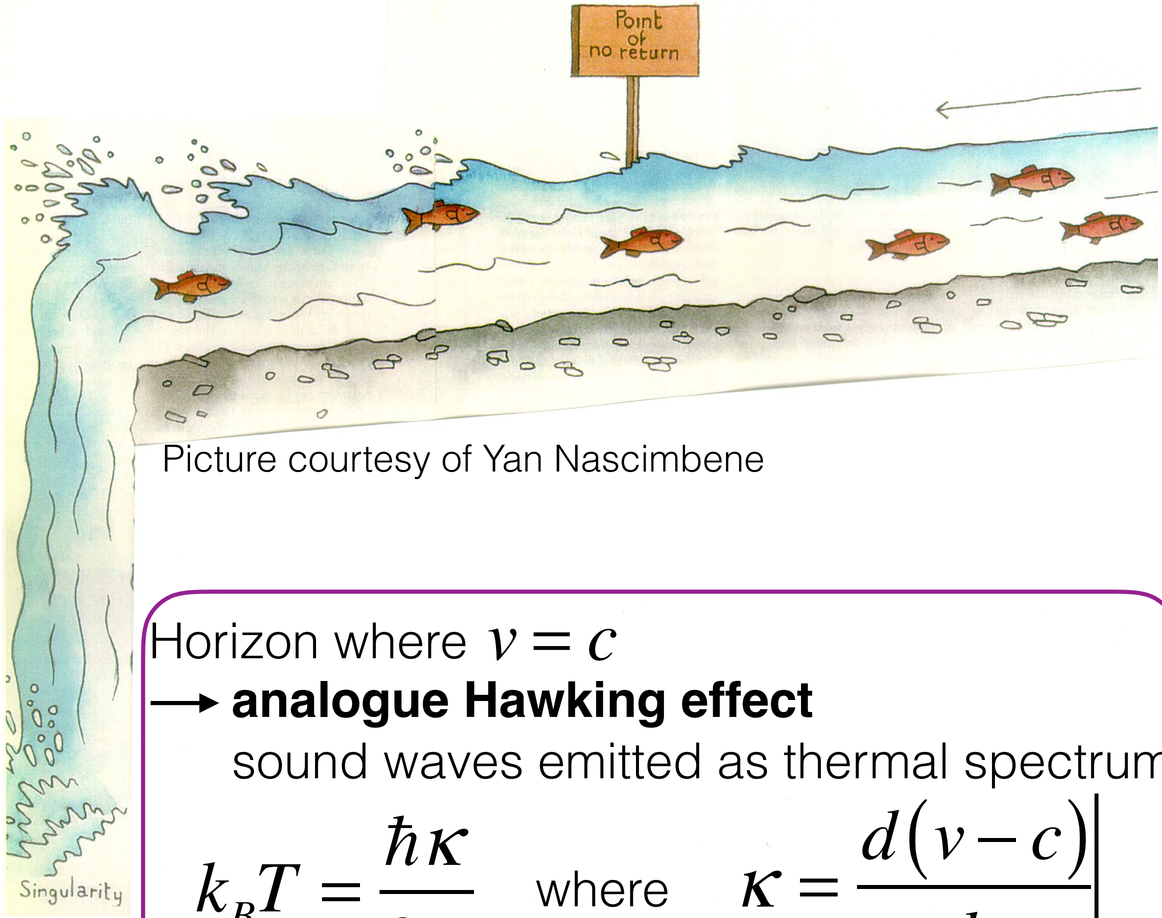
# Analogue black holes

**Unruh (1981)**: on large scales, waves in moving media propagate as if in curved spacetime

$$\phi = \phi_0 + \delta\phi$$

- fixed background $\phi_0$ $\qquad g_{\mu\nu}(\phi_0)\,dx^\mu dx^\nu = c^2 dt^2 - (d\vec{x} - \vec{v}\,dt)^2$
- linear perturbations $\delta\phi$ $\qquad g^{\mu\nu}(\phi_0)\nabla_\mu \nabla_\nu (\delta\phi) = 0$



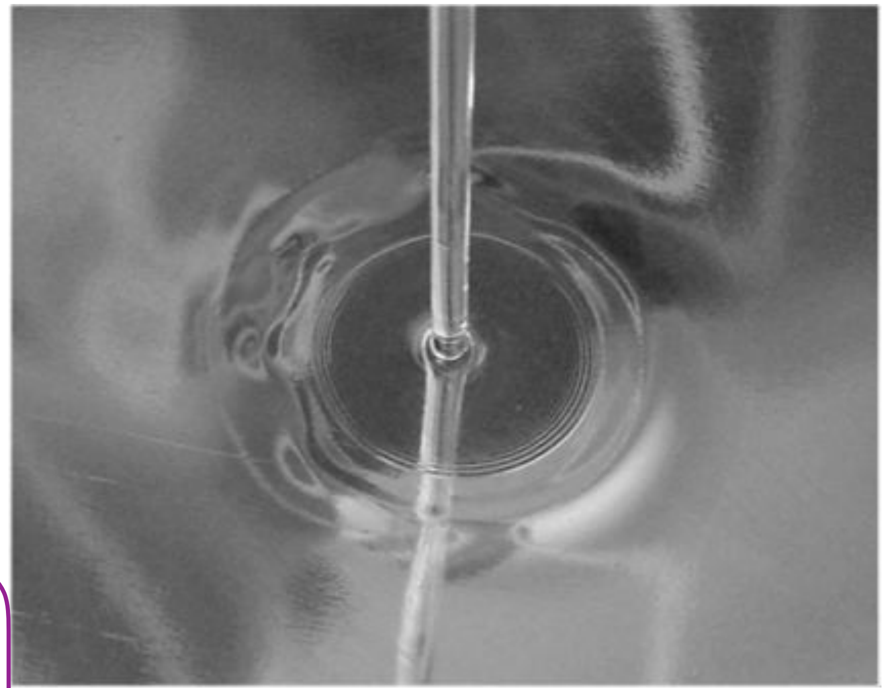

Picture courtesy of Yan Nascimbene

Horizon where $v = c$

$\longrightarrow$ **analogue Hawking effect**

sound waves emitted as thermal spectrum

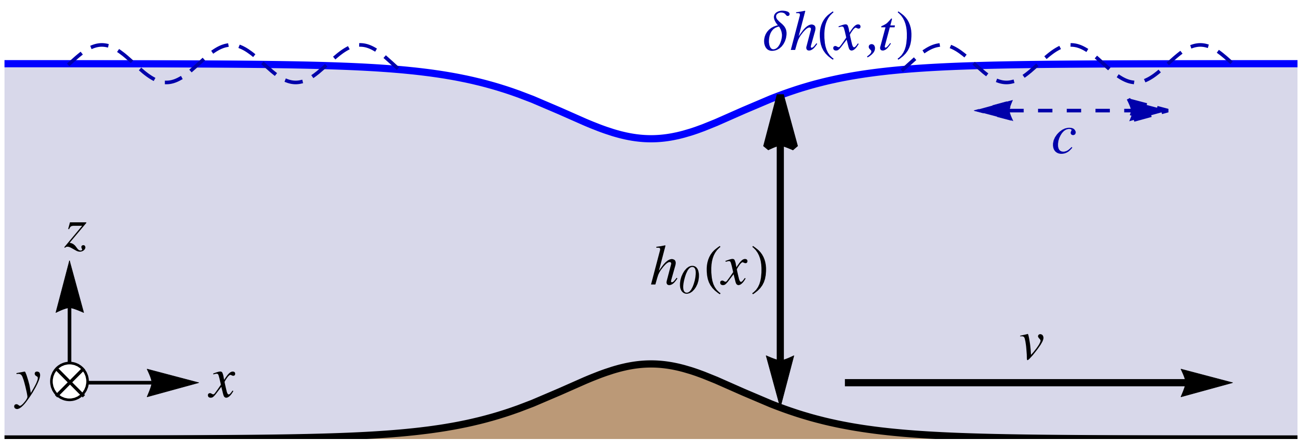
$$k_B T = \frac{\hbar \kappa}{2\pi} \quad \text{where} \quad \kappa = \left.\frac{d(v-c)}{dx}\right|_{\text{hor.}}$$



Picture courtesy of Piotr Pieranski

# The surface wave analogy

(Unruh and Schützhold, 2002)




Flow velocity $v = \dfrac{q}{h}$

Wave speed $c = \sqrt{gh}$

**Total wave speed** $= v \pm c$

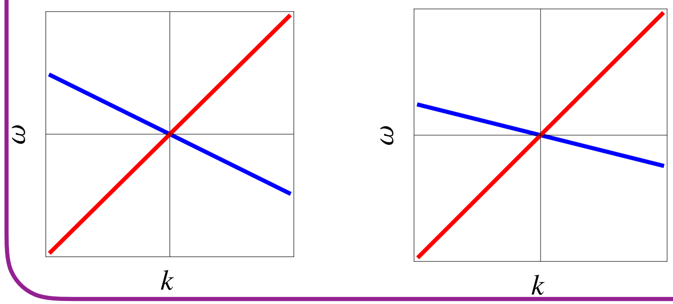
$\delta h$ ⟷ canonical momentum in the (2+1)-d spacetime metric

$$g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = c^{2}\left[c^{2}\,\mathrm{d}t^{2} - \left(\mathrm{d}x - v\,\mathrm{d}t\right)^{2} - \mathrm{d}y^{2}\right]$$

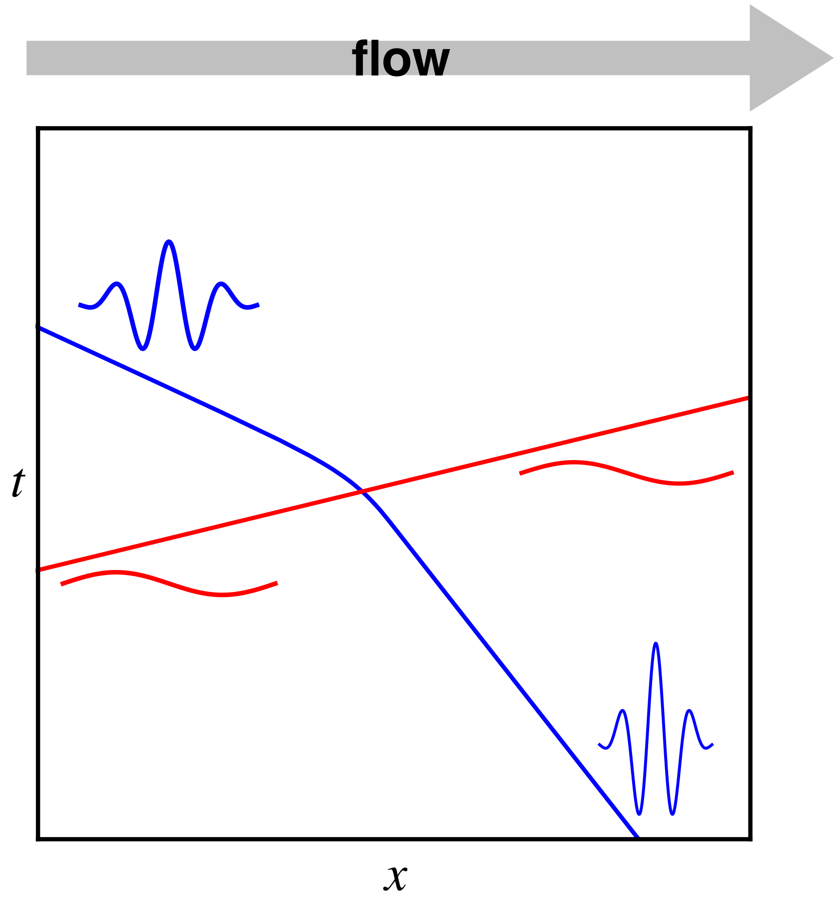
*"conformal factor"*, multiplies simpler metric
generates effective potential *$V_{eff}$*
**allows mixing between co- and counter-propagating waves**

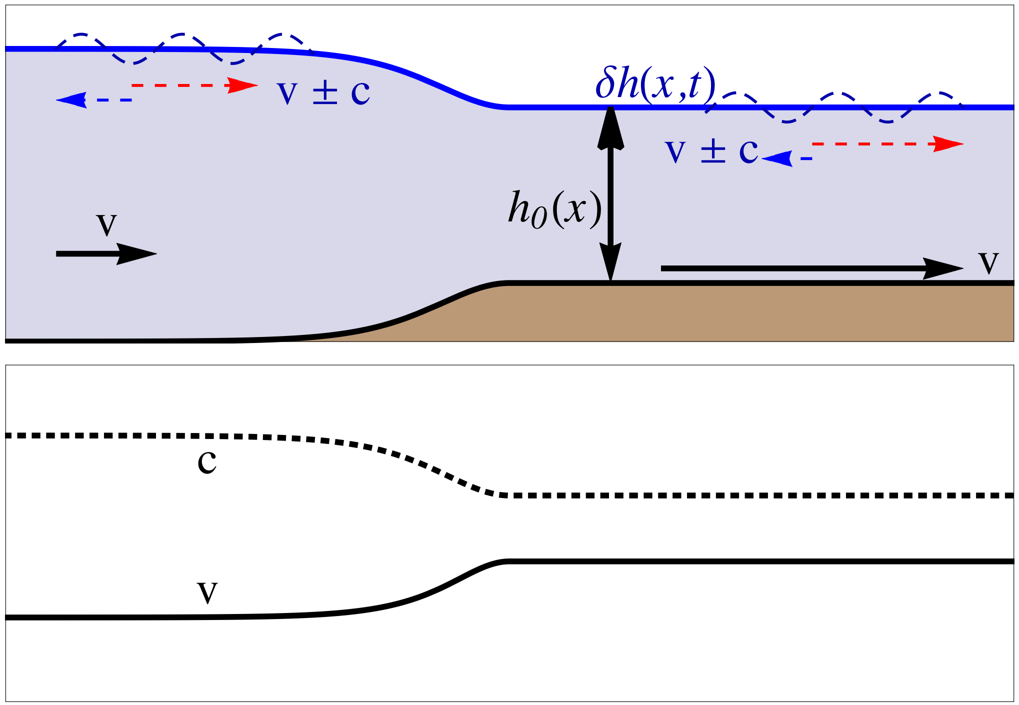
# Wave scattering (subcritical flows)

Waves are solutions of dispersion relation: $\omega - vk = \pm ck$ (Doppler shift)




- co-propagating waves
- counter-propagating waves



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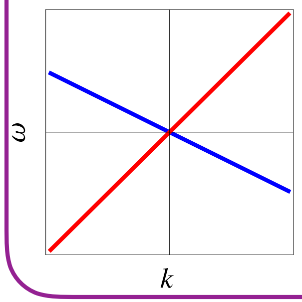
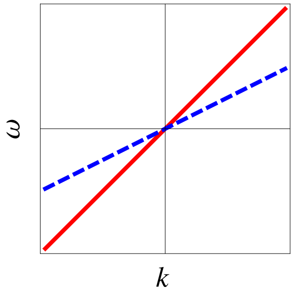
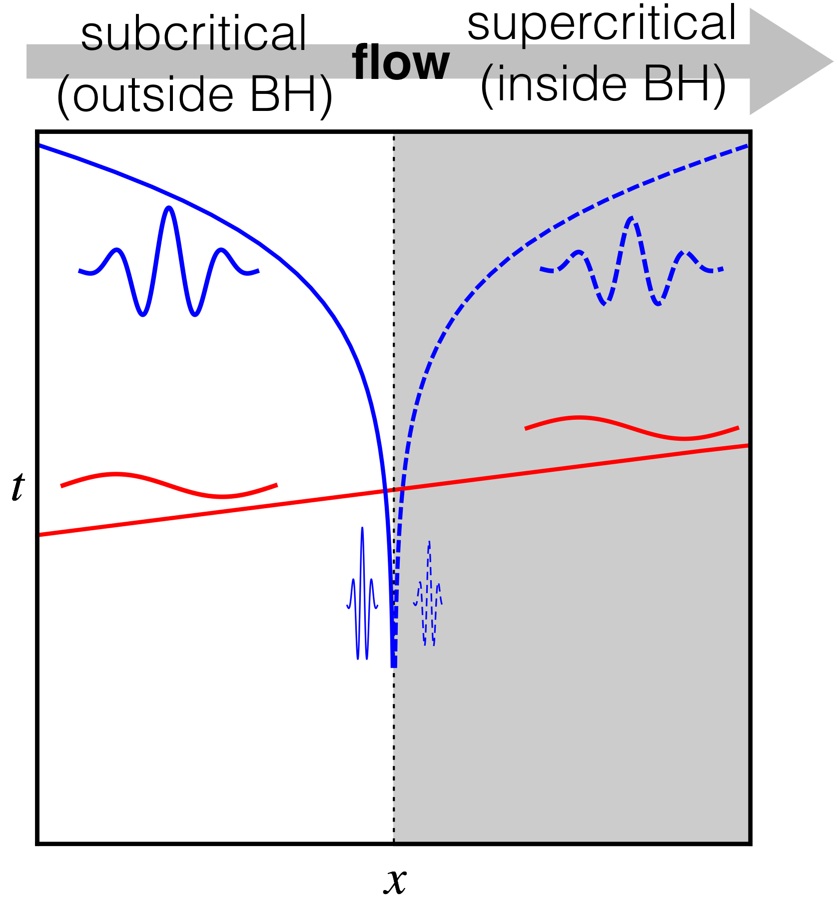
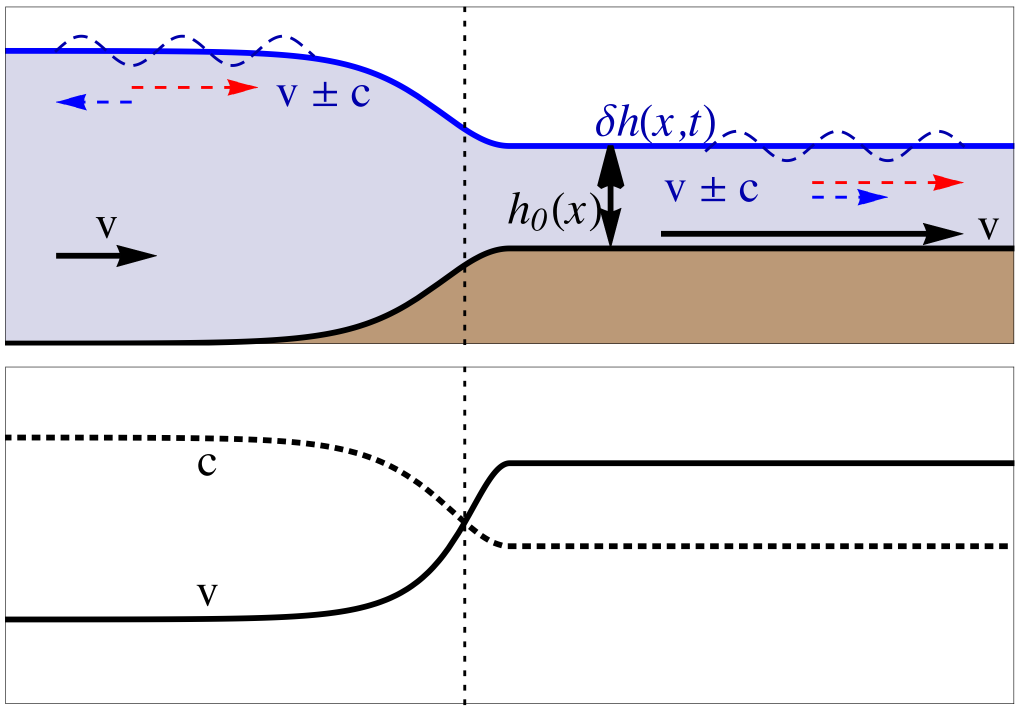



- co-propagating waves
  — experience nothing special at horizon
- counter-propagating waves
  — “split” into **two** separate modes



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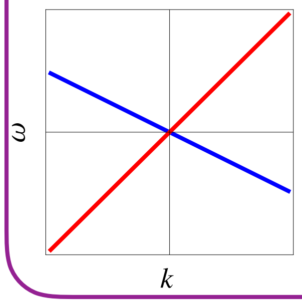


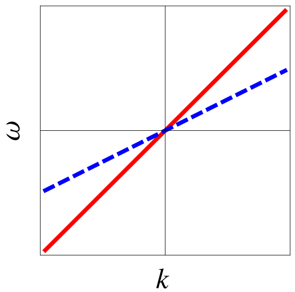


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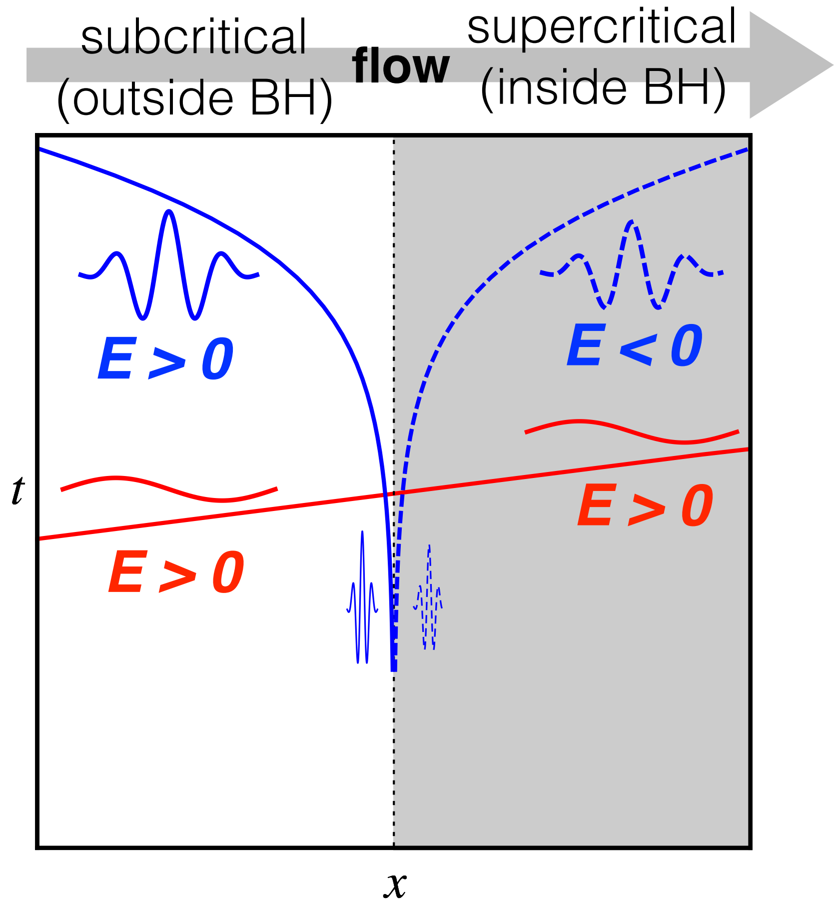


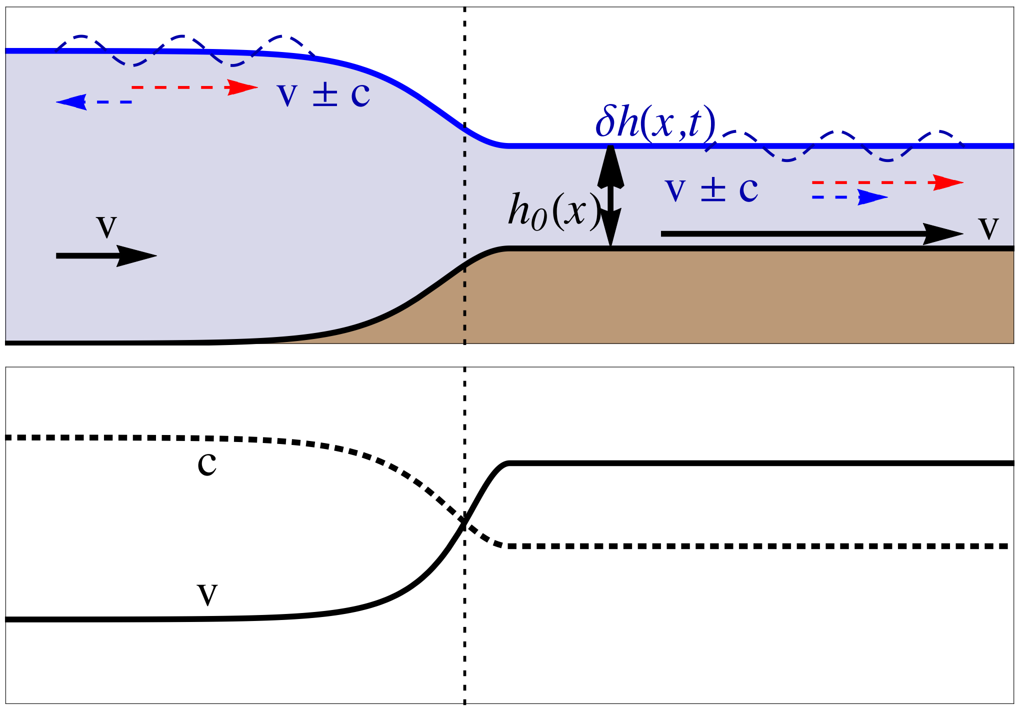

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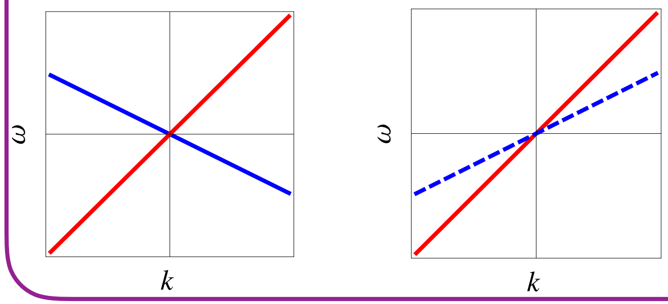


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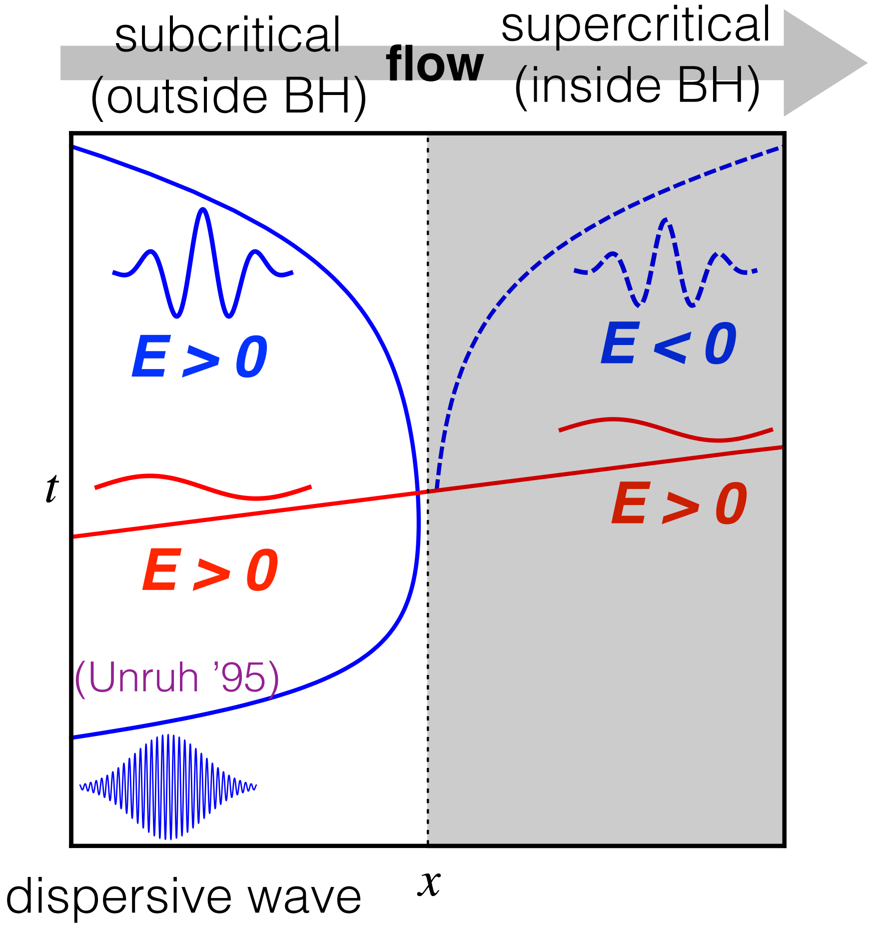
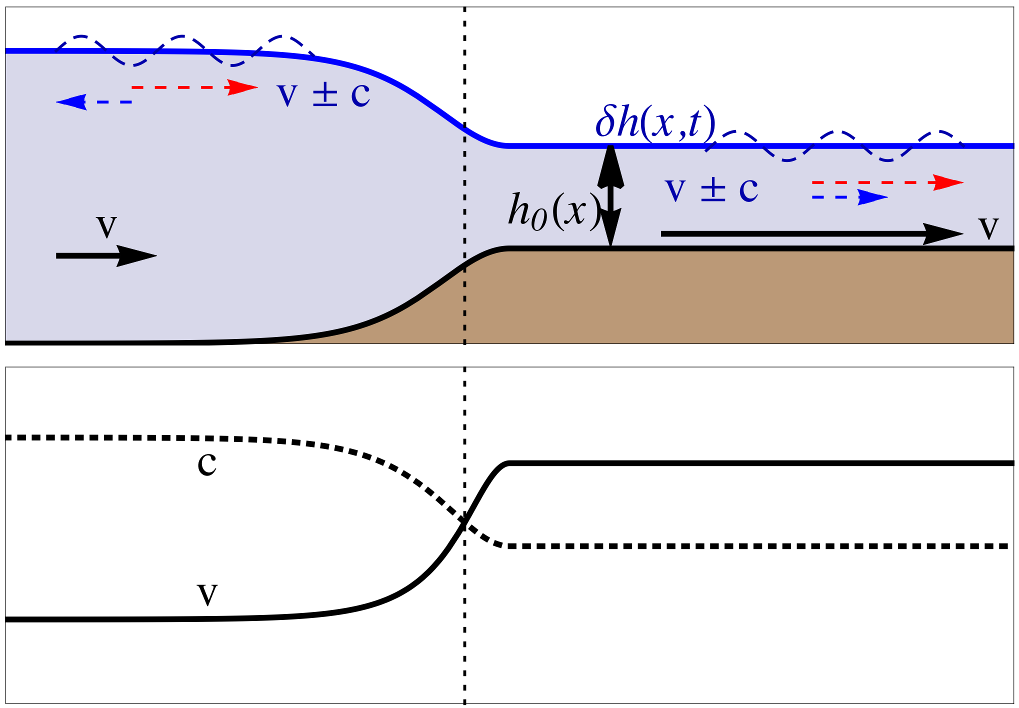


# Key surface wave experiments in AG

**All** previous experiments (in 1 dimension) have been in **purely subcritical flows**

Rousseaux et al., *New J. Phys.* **10**, 053015 (2008)
Weinfurtner et al., *Phys. Rev. Lett.* **106**, 021302 (2011)
Euvé et al., *Phys. Rev. Lett.* **117**, 121301 (2016)

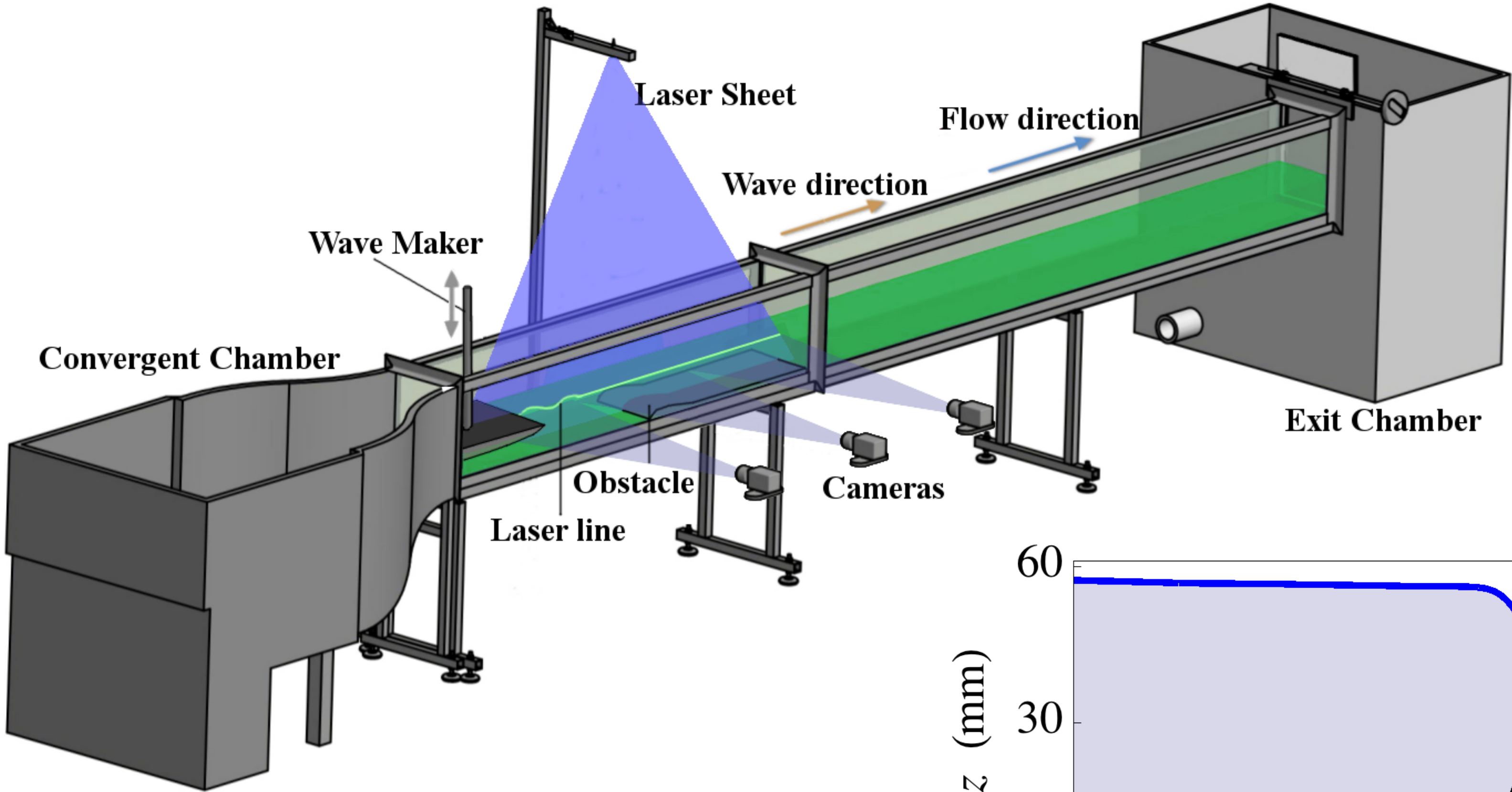
→ **No horizon** in the effective metric

→ **No thermality** in the sense of Hawking/Unruh

(Nontrivial scattering allowed thanks to **dispersion**)

(Michel and Parentani, *Phys. Rev. D* **90**, 044033 (2014))
(Robertson, Michel and Parentani, *Phys. Rev. D* **93** 124060 (2016))

# The current Poitiers setup



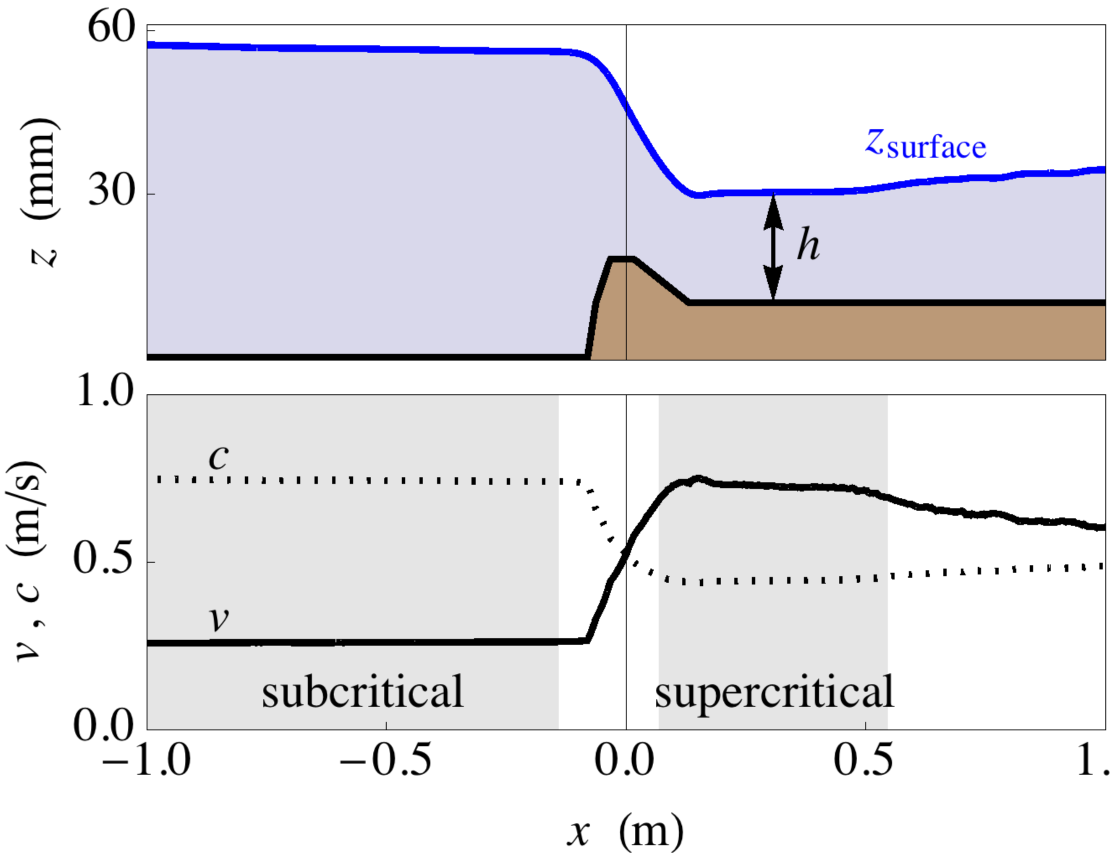

$$v = \frac{q}{h} \qquad c = \sqrt{gh}$$

**Transcritical**: Horizon where $v = c$

*Hawking prediction*

"Surface gravity" $\kappa = \partial_x(v - c) = 4.56\ \mathrm{Hz}$

BH temperature $T = \dfrac{\hbar\kappa}{2\pi k_B} = 5.55\ \mathrm{pK}$



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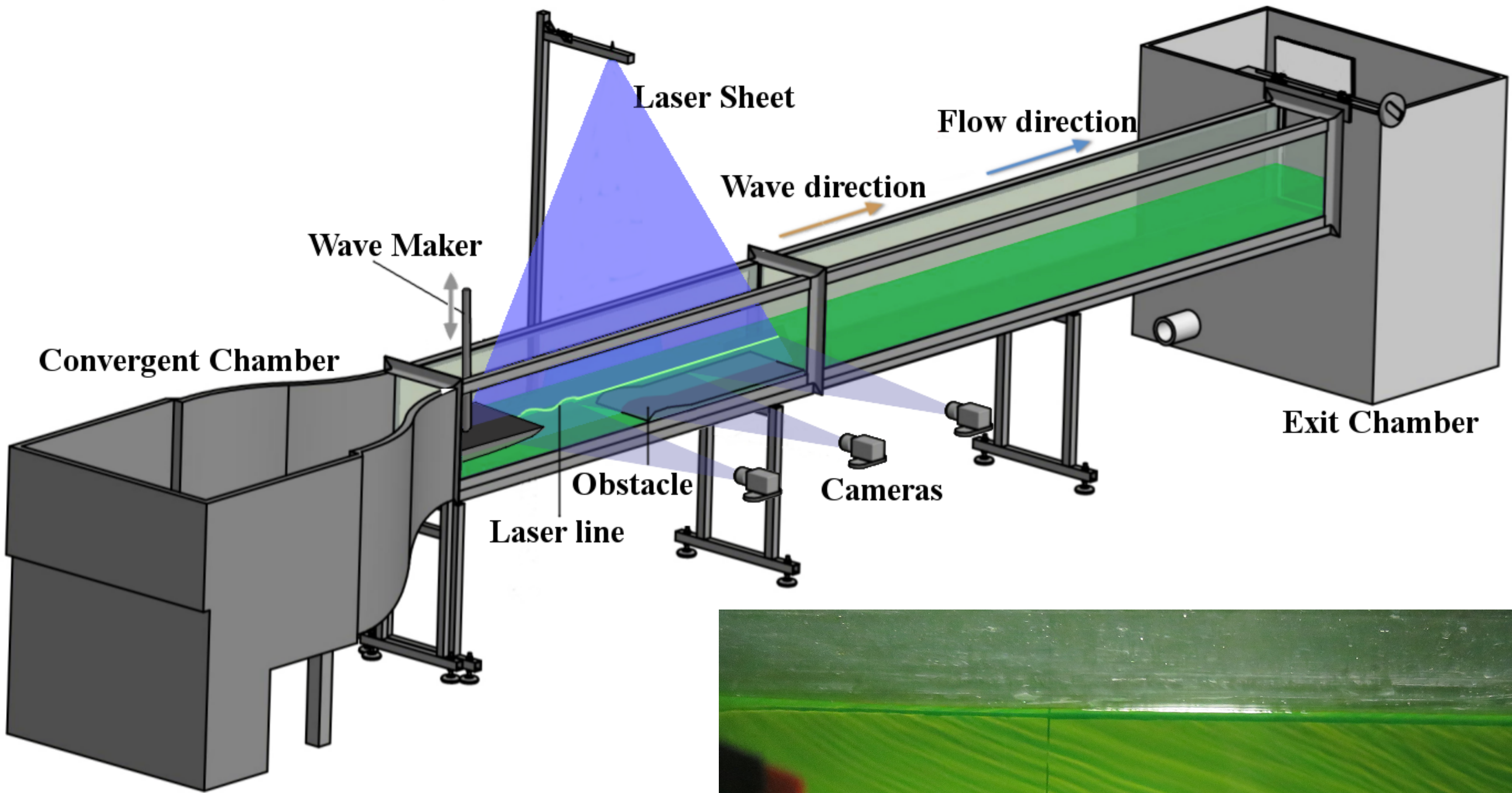
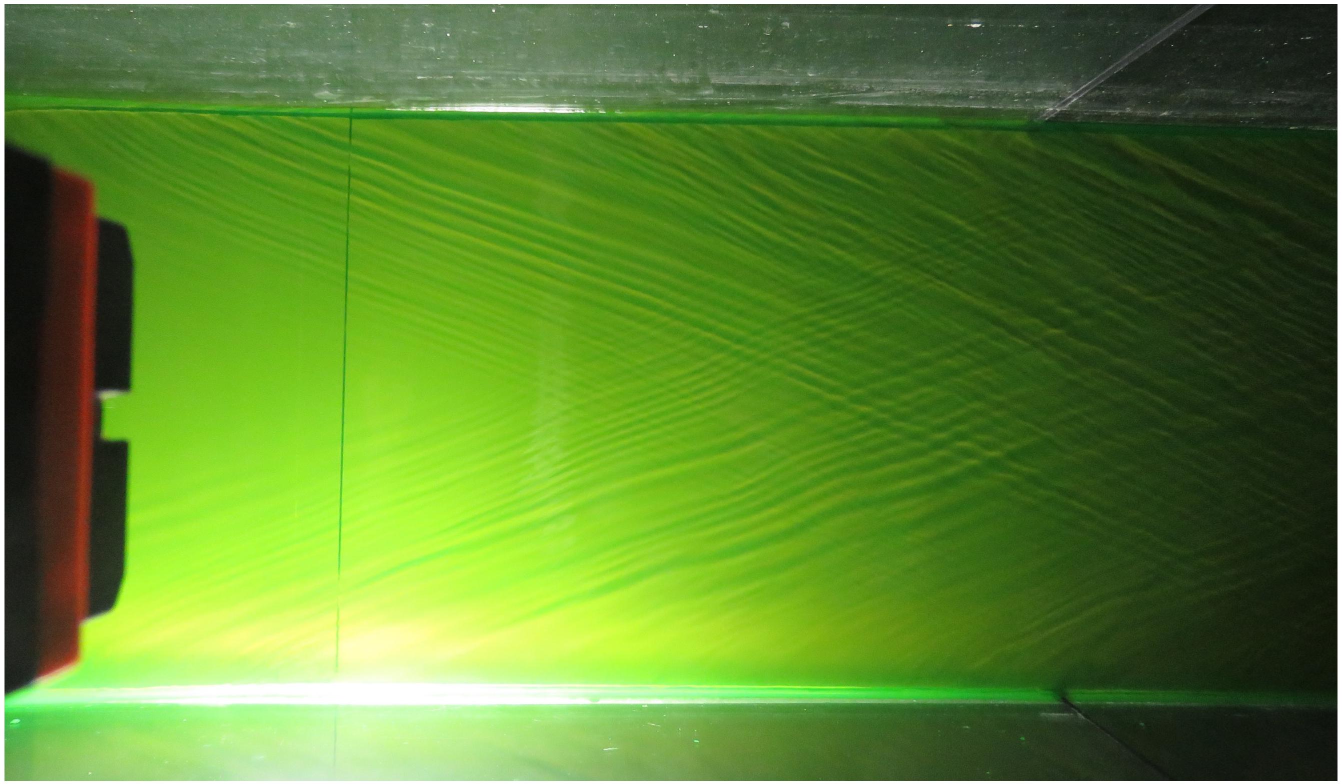
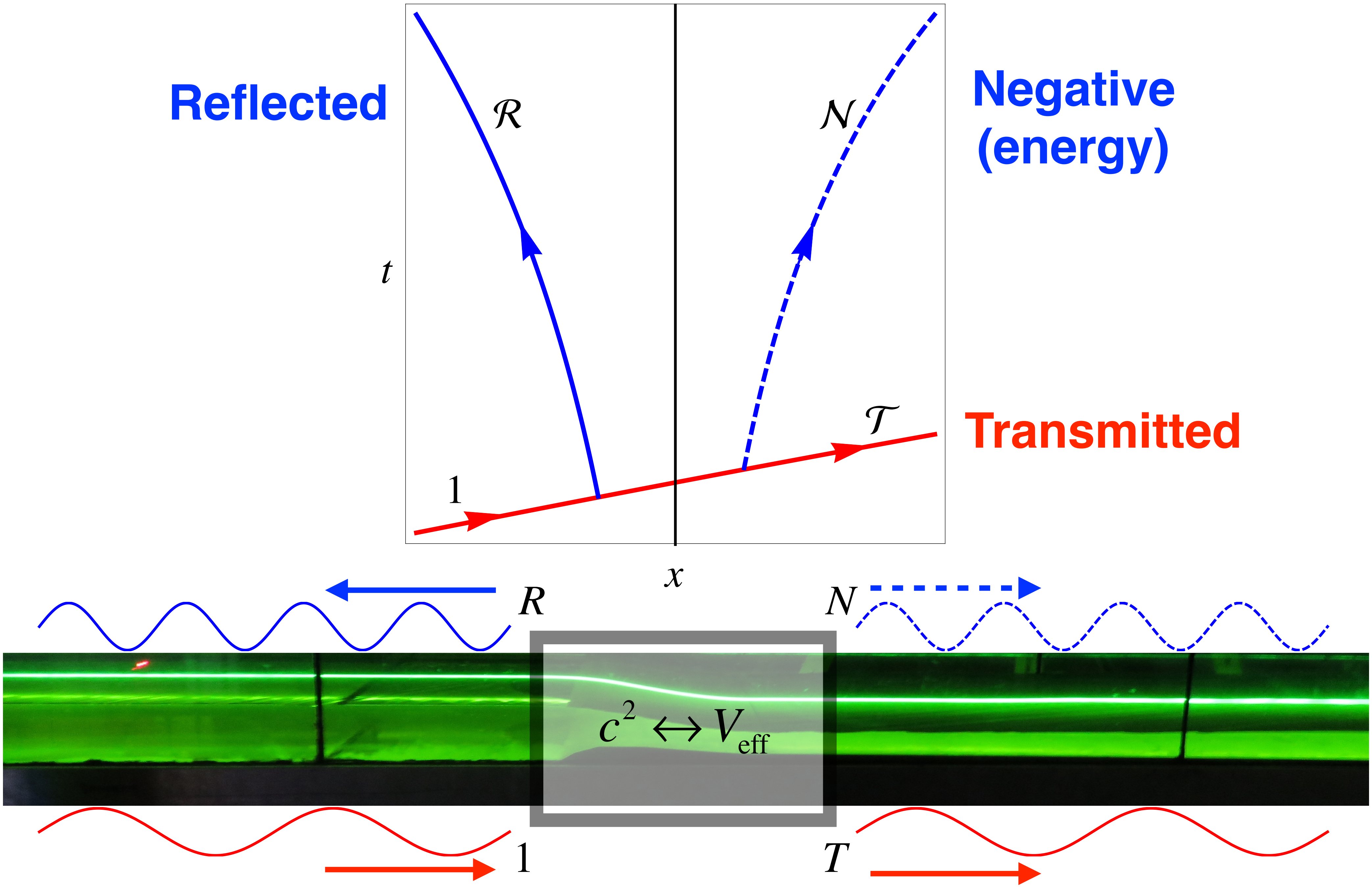


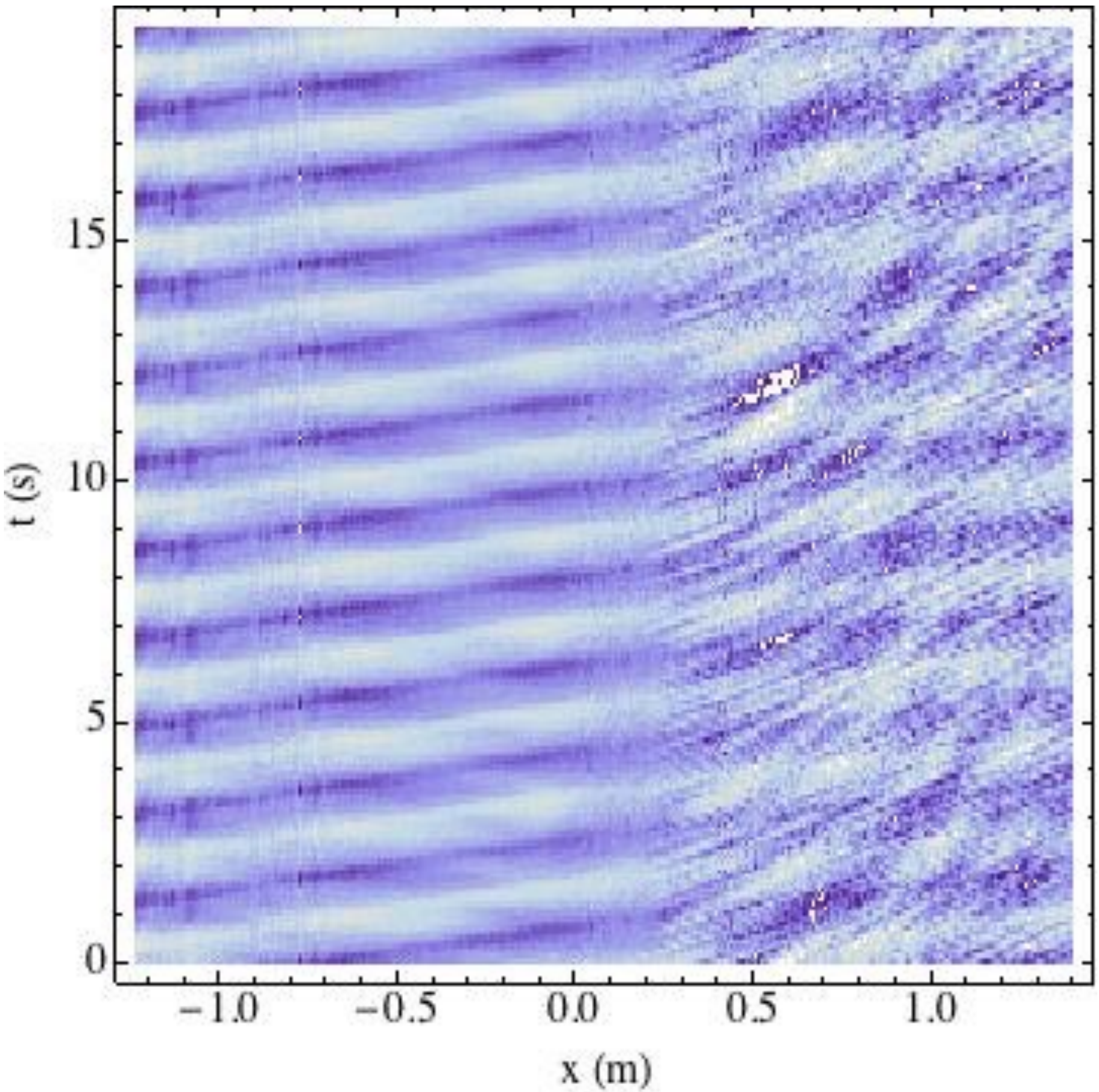
Wake pattern on downstream (supercritical) side

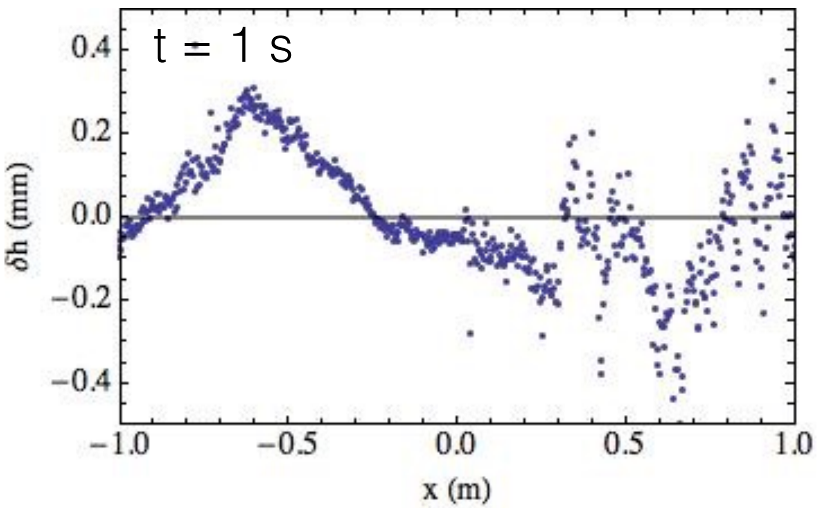
# Scattering of incident probe

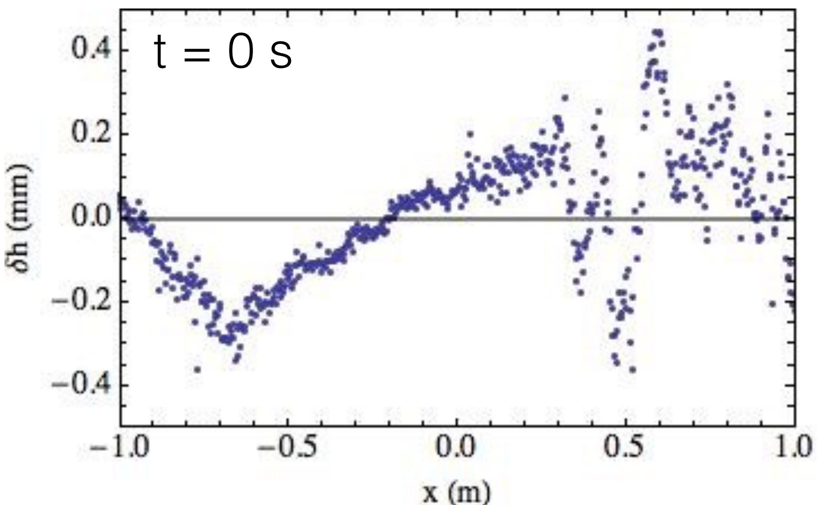


# Example data

$$A_{\mathrm{wm}} = 0.25\,\mathrm{mm}$$
$$\omega = 3.46\,\mathrm{Hz}$$

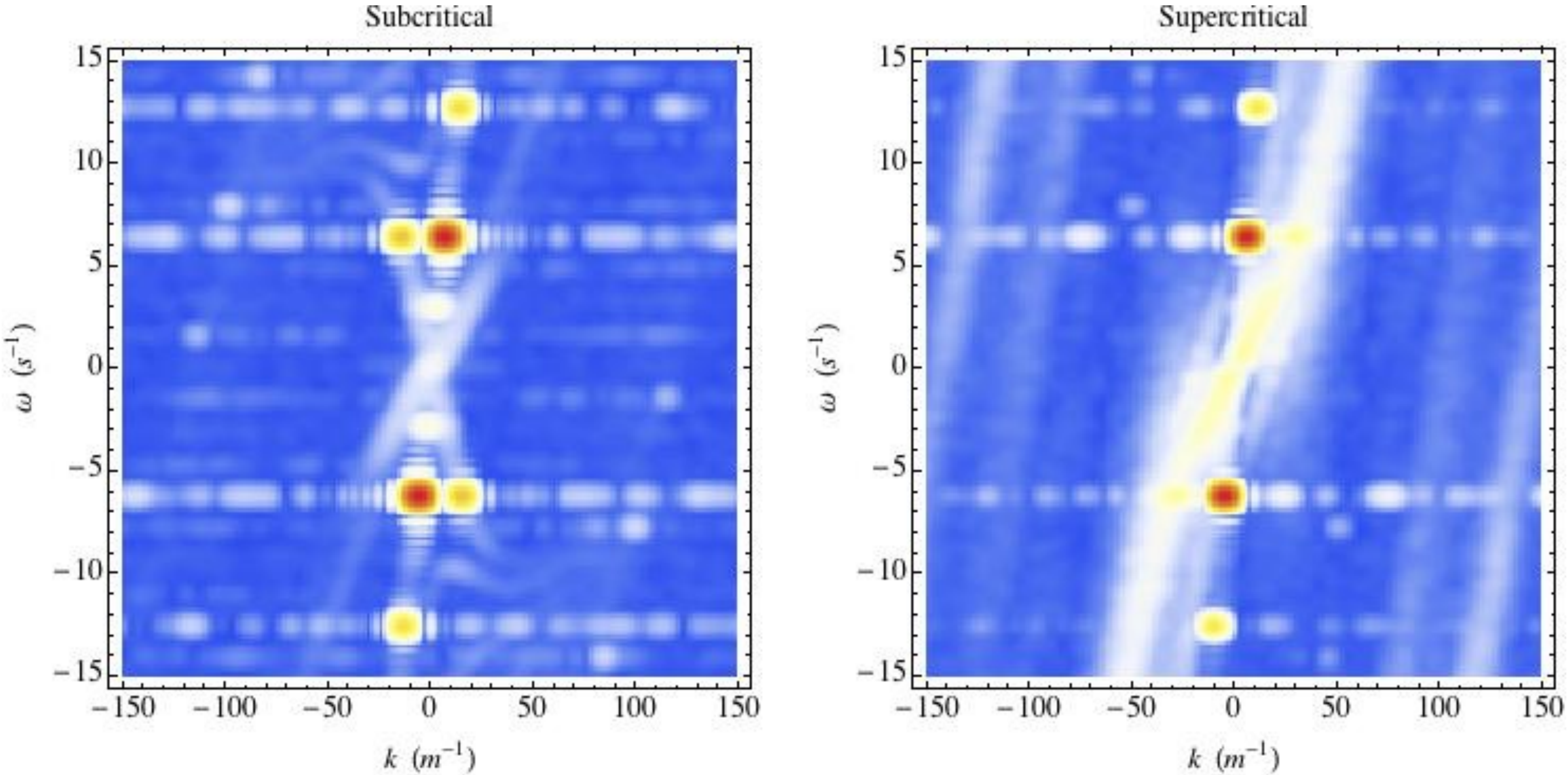


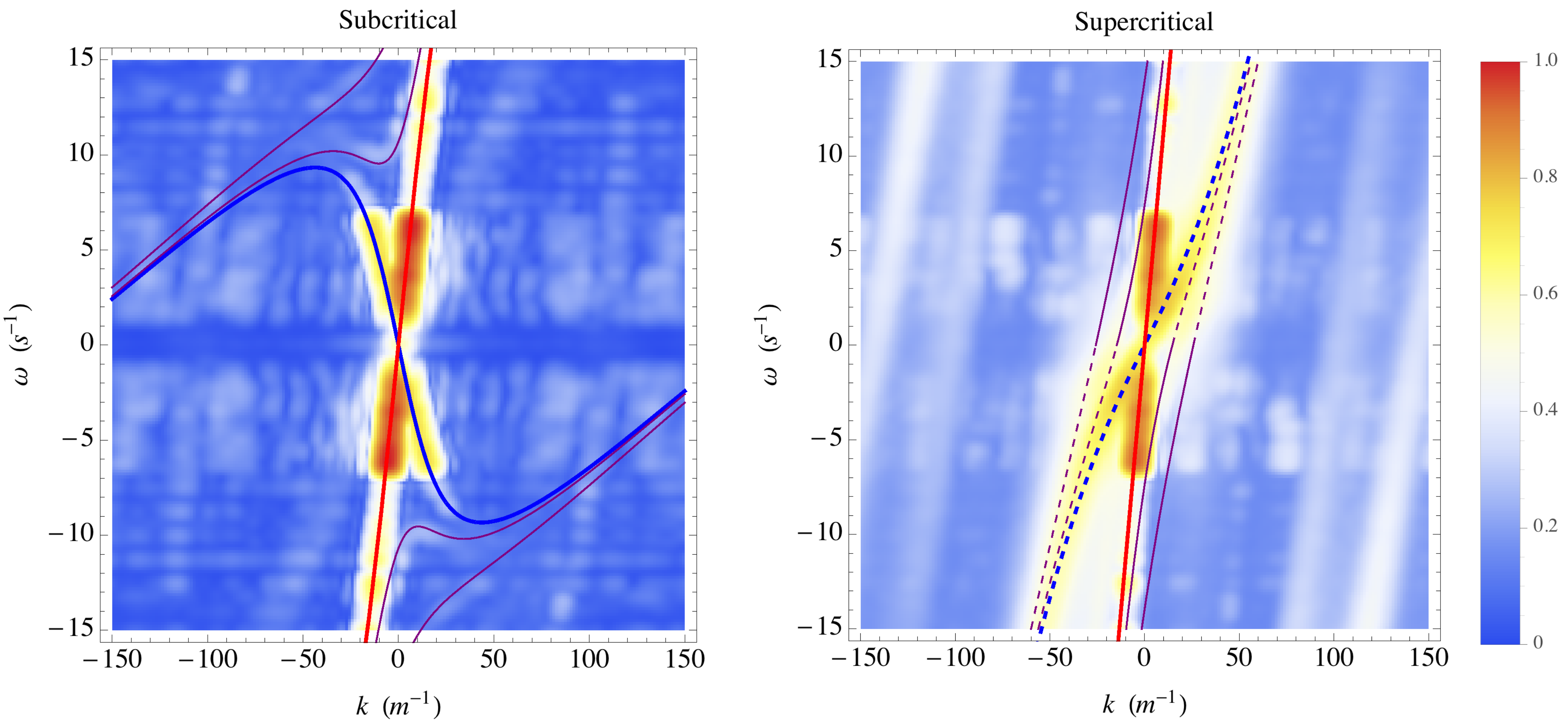
# Dispersion relation

Fourier transforming data in both space and time, at a single frequency ($\omega = 2\pi$ Hz)



# Dispersion relation

Fourier transforming data in both space and time, then combining all frequencies




Full theoretical dispersion relation: $\omega - vk = \pm\sqrt{gk\tanh(hk)}$

- Observations lie close to theoretical dispersion relations
- Close to the linear (non-dispersive) regime, so **metric description valid**
- Counter-propagating branch has flipped sign, indicating **transcriticality** and (indirectly) the **existence of negative-energy waves**

# Fitting the waveforms

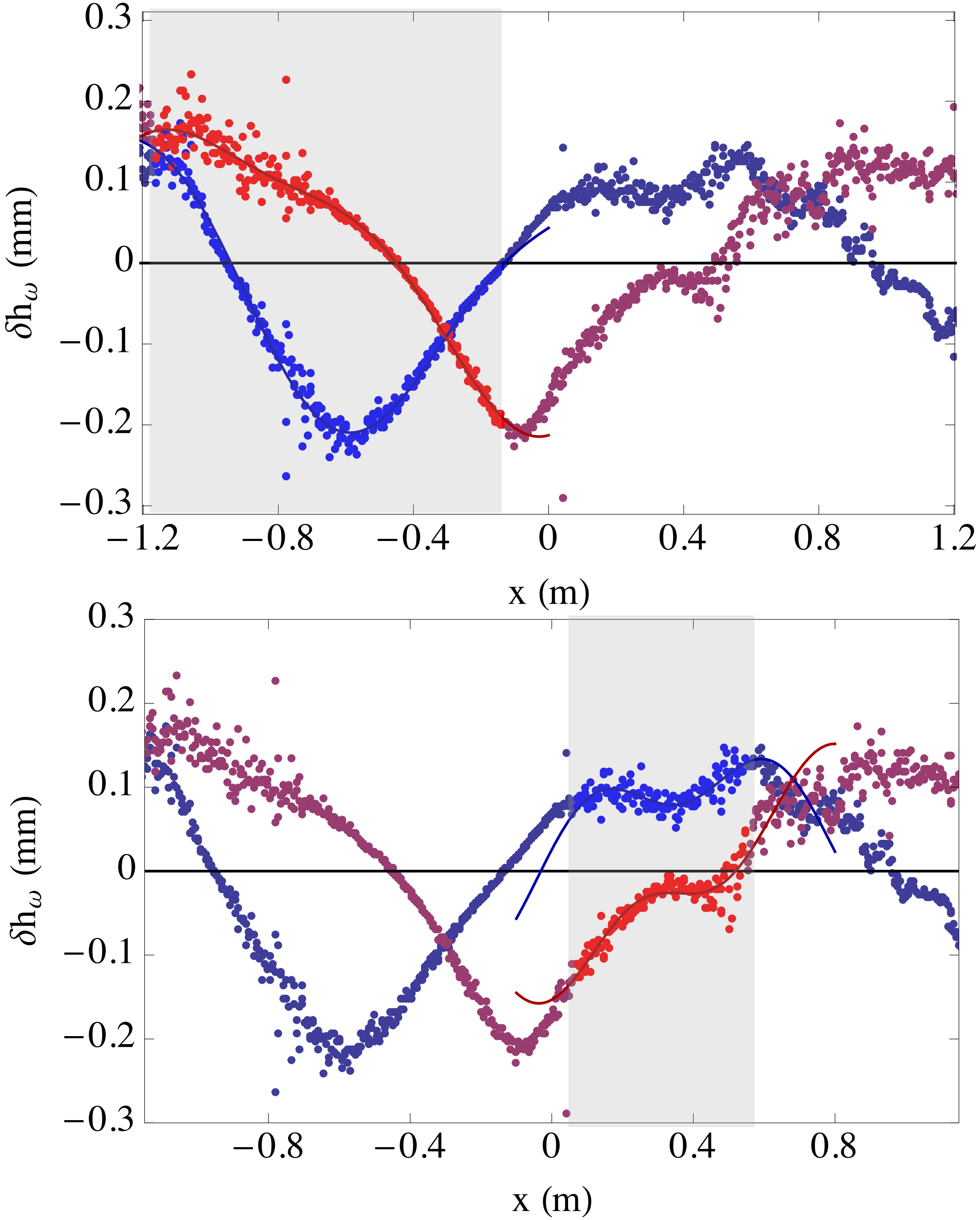
$A_{\mathrm{wm}} = 0.25\,\mathrm{mm}$

$\omega = 3.46\,\mathrm{Hz}$

FT in time at wave maker frequency, yielding real and imaginary parts of

$$\delta h_{\omega}(x)$$

Fit to sum of two plane waves, separately in sub- and supercritical regions



# Dispersion relation

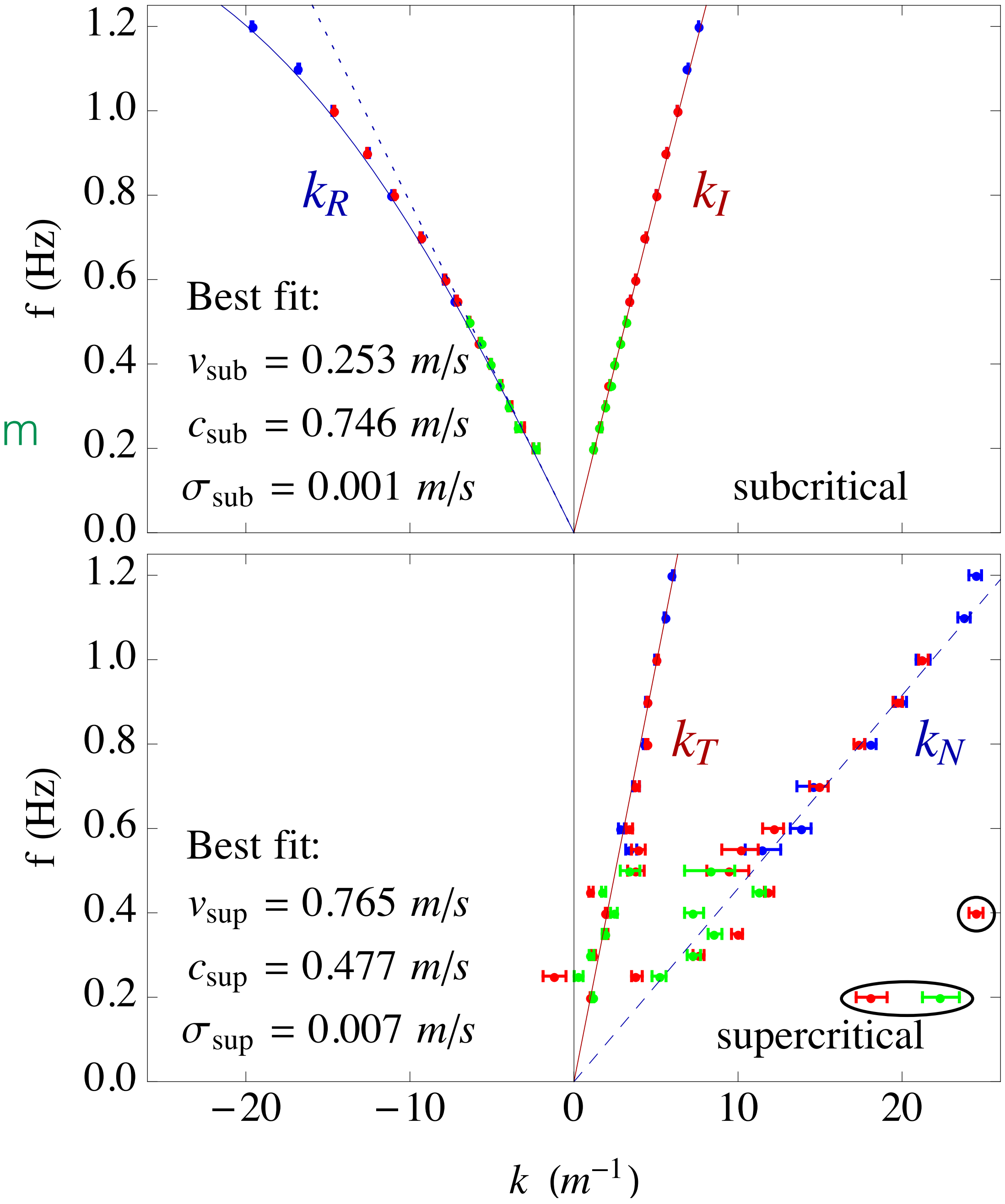
***Latest results***

Found by FT'ing in time
then fitting to sum of two plane waves

Different colours represent different wave maker amplitudes: 0.25 mm, 0.5 mm, 1 mm

Allows fitting of *v* and *c*

- very close to values inferred from depth in subcritical region
- small difference in supercritical region (likely due to presence of vorticity)



# Scattering coefficients

(i.e., ratios of wave amplitudes
to amplitude of incident wave)

***Latest results***

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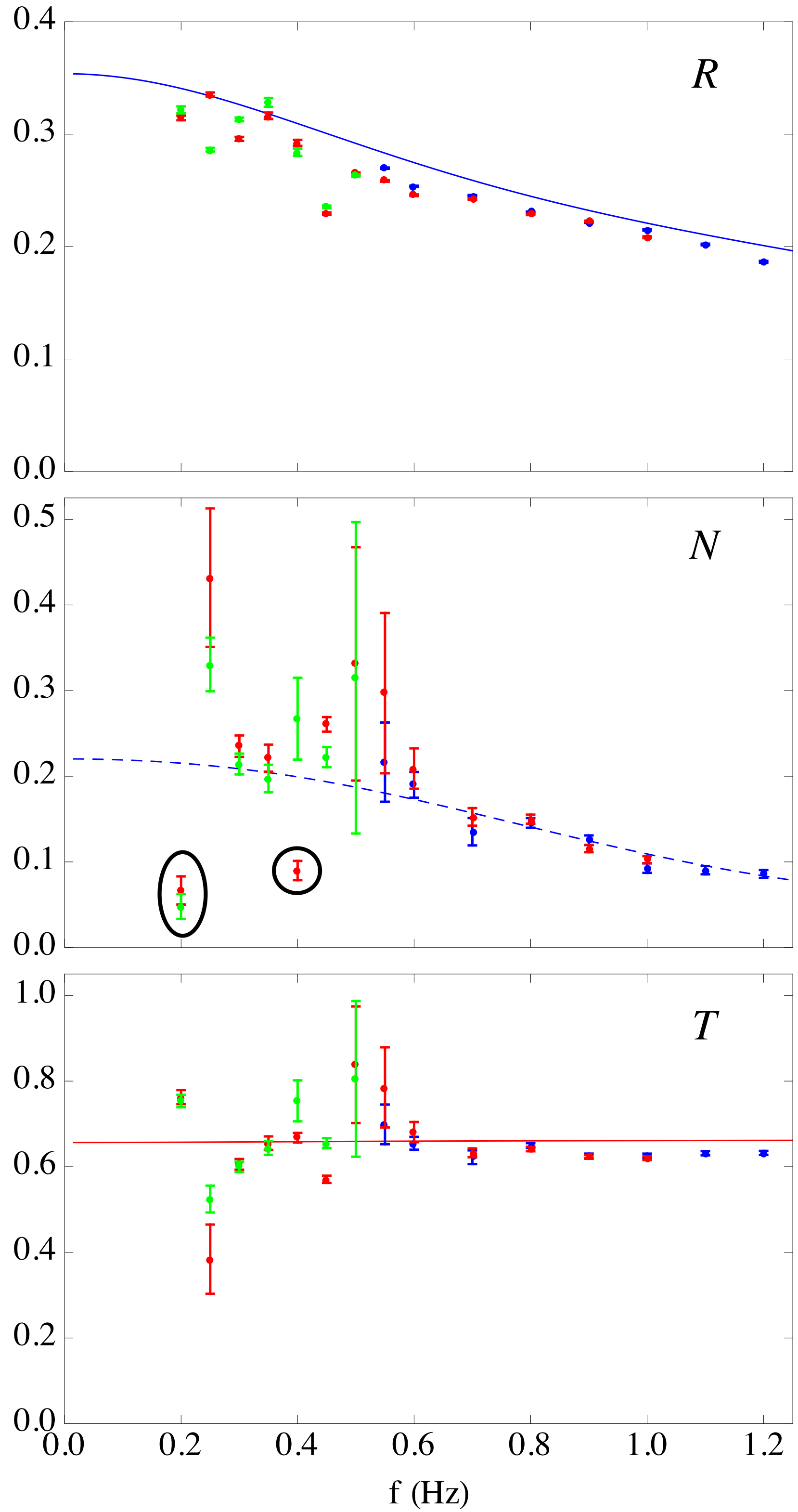
- Reasonable agreement with theory, especially at high frequencies

- Considerable scatter and errors at low freq., especially in supercritical region



# Scattering coefficients

**(normalized)**

## ***Latest results***

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then fitting to sum of two plane waves

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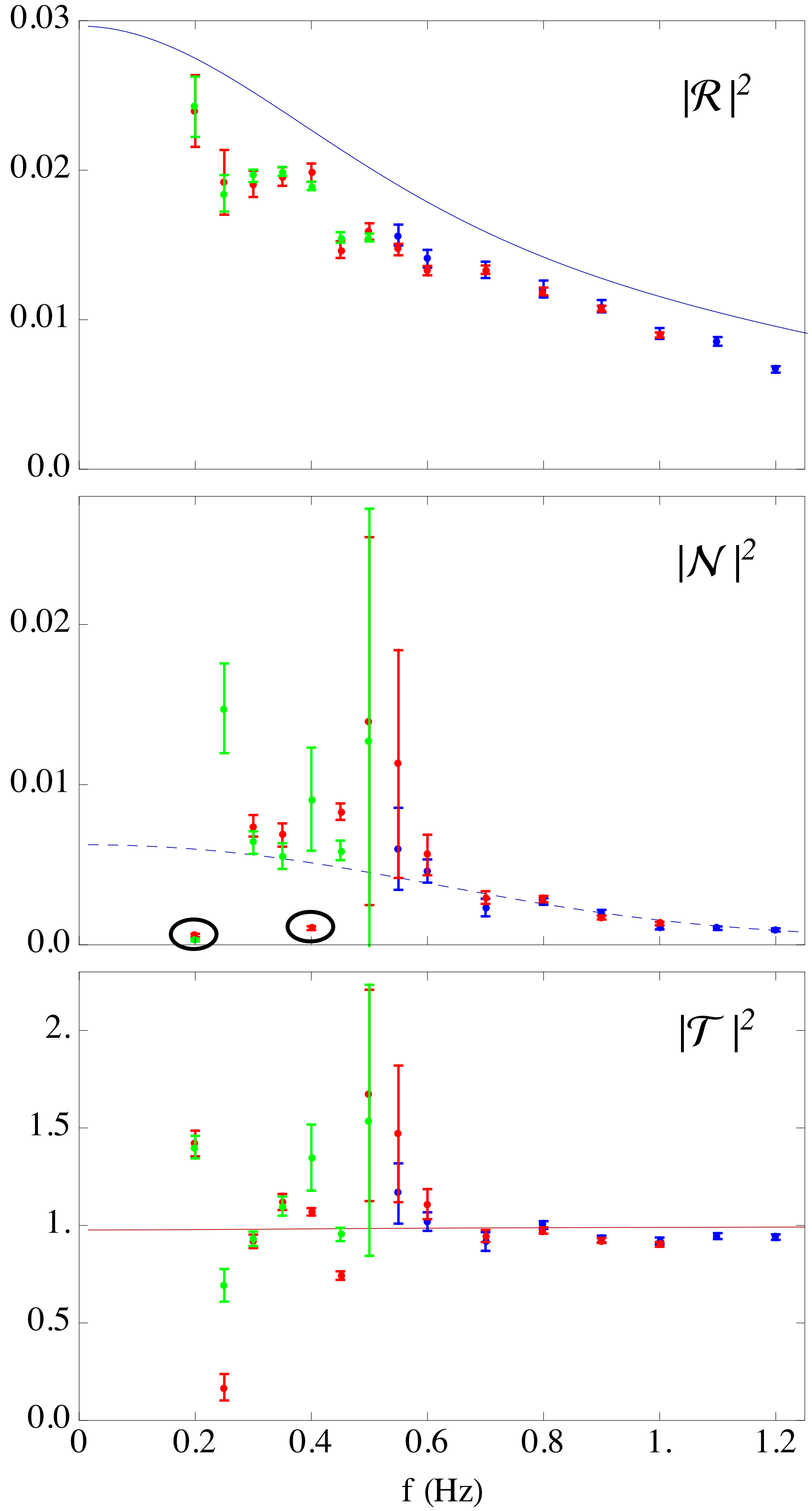
- Reasonable agreement with theory, especially at high frequencies
- Considerable scatter and errors at low freq., especially in supercritical region
- Unable to verify unitarity:

$$|\mathscr{R}|^2 - |\mathscr{N}|^2 + |\mathscr{T}|^2 = 1$$



# Summary

**Transcritical black-hole flow** realized (a first in water wave Analogue Gravity), probed by scattering of incident co-current waves off effective potential

Have access to **dispersion relation** and **scattering coefficients**

Results reasonably close to predictions of **effective metric** description

*But what next…?*

# Future prospects

- Stimulation of Hawking radiation
  - exciting **incident dispersive modes** is technologically challenging

- “Unstimulated” scenario (i.e. without wave maker)
  - incident modes provided by **noise** already present

- Stimulated Hawking radiation in a time-dependent **black hole formation** (*à la* Hawking ’74-75)

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**Thank you for your attention!**