Four-wave mixing and enhanced analog Hawking effect in a nonlinear optical waveguide

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Scott Robertson

Simulating gravitation and cosmology in condensed matter and optical systems

Trento, 22 July 2019

Analogue Gravity

Unruh (1981):

- Analogy between wave propagation in curved spacetime and in inhomogeneous (moving) media
- Can establish analogue horizon (BH or WH) in media, predicted to emit analogue Hawking radiation
- Testable in the lab?

Necessary ingredients:

- inhomogeneous background that scatters probe waves
- conserved norm, positive or negative (conjugates, $\hat{a}$ and $\hat{a}^\dagger$ in QM)
- dispersion relation allowing mixing of opposite-norm modes

$\Rightarrow$

Anomalous scattering

(vacuum)

Spontaneous emission of entangled pairs

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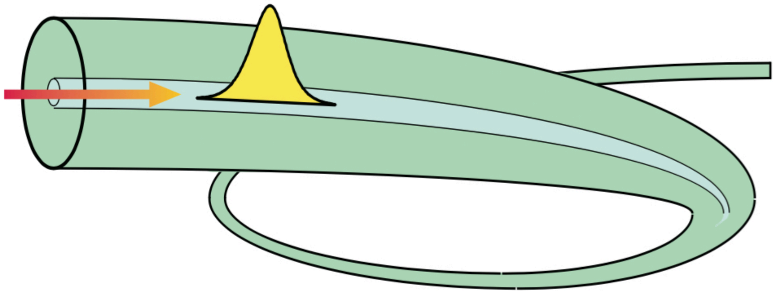
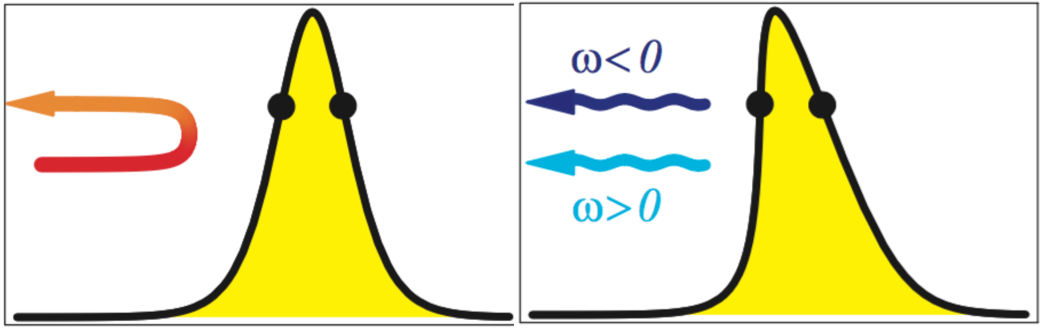
Solitons in waveguides:





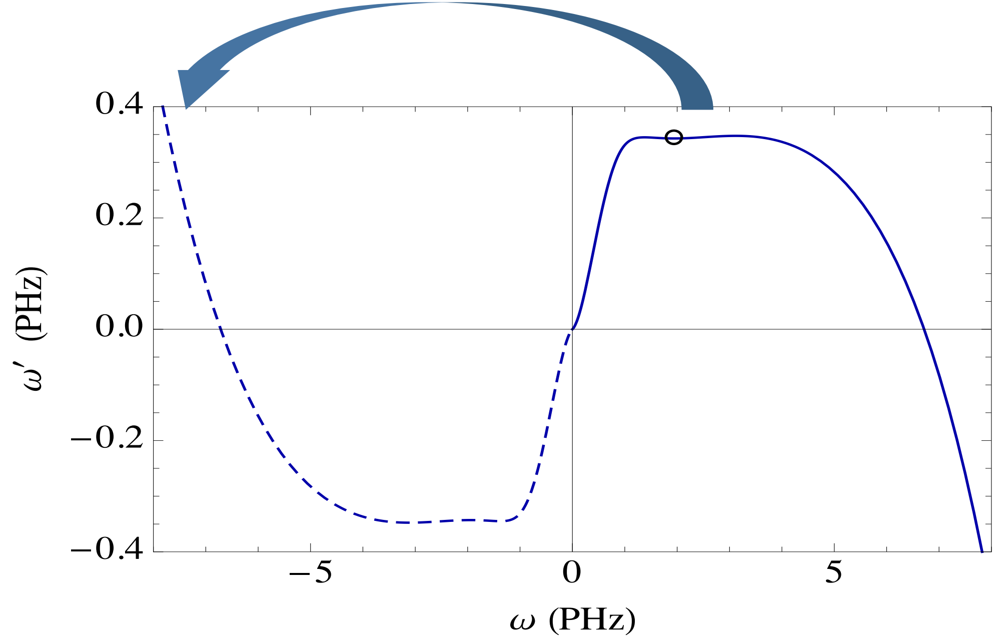
Philbin *et al.*, *Science* **319**, 1367 (2008)

"Hard" v. "Soft" processes

Previous works: "**hard**" photon production

- sign of norm: conjugation of **full field** (inc. carrier)
- frequencies well separated
- (anomalous) scattering coefficients suppressed

$|\beta|^2 \sim 10^{-10}$ (Robertson, **arXiv:1106.1805** (2011))



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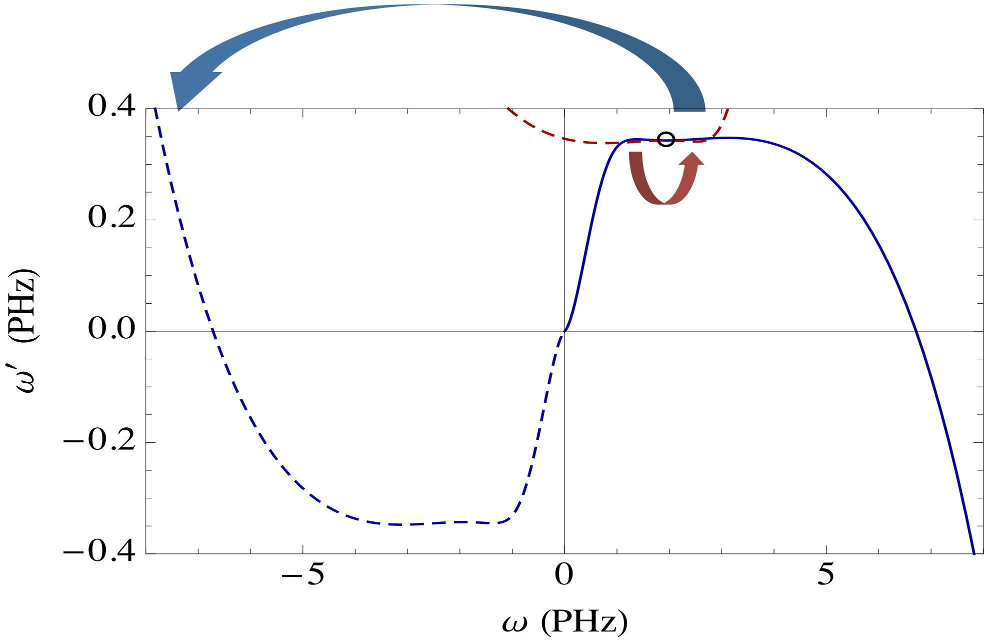

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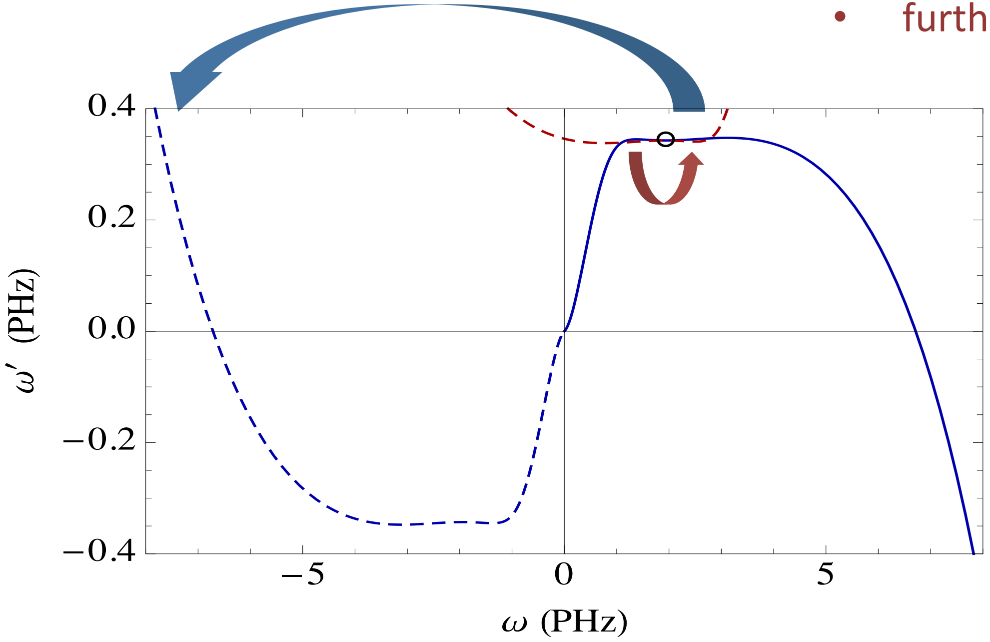
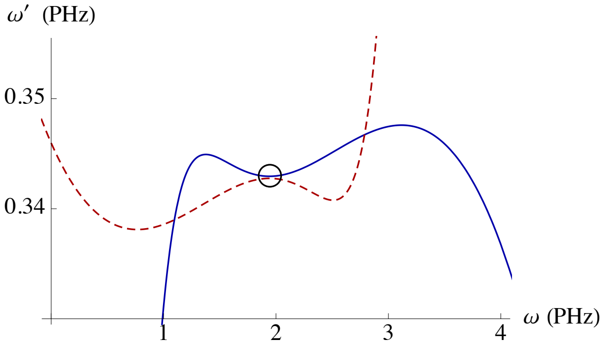
- further enhanced by **phase matching**

$|\beta|^2 \sim 10^{-1}$





Wave equation

Envelope: $E(t,z) = A(t,z)\exp\left(i\beta_0 z - i\omega_0 t\right)$

Retarded time: $\tau = t - \beta_1 z$

Dispersion: $B\left(\Delta\omega\right) = \beta\left(\omega_0 + \Delta\omega\right) - \beta_0 - \beta_1 \Delta\omega$

Nonlinear wave equation

$$\boxed{-i\partial_z A = B\left(i\partial_\tau\right)A + \gamma\left|A\right|^2 A}$$

(neglecting losses, retardation, etc.)

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Linearized wave equation

Strong background (e.g. soliton) + weak probe (or vacuum fluctuations)

If $e^{i\delta\beta_0 z}A_0(\tau)$ is a solution, write $A(z,\tau) = e^{i\delta\beta_0 z}\left(A_0(\tau) + \delta A(z,\tau)\right)$

$$\longrightarrow -i\partial_z(\delta A) = \underbrace{\boxed{\left(B\left(i\partial_\tau\right) - \delta\beta_0\right)\delta A}}_{\text{dispersion}} + \underbrace{\boxed{2\gamma\left|A_0\right|^2 \delta A}}_{\text{XPM}} + \underbrace{\boxed{\gamma A_0^2\,\delta A^*}}_{\text{FWM}}$$

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"Doublet" wave equation

$\left(\delta A, \delta A^*\right) \rightarrow w = \left(w_+, w_-\right)$ is a solution of:

Doublets as for phonons in BEC: Leonhardt *et al.*, *PRA* **67**, 033602 (2003)

$$-i\partial_z w = \begin{bmatrix} \boxed{B(i\partial_\tau) - \delta\beta_0} + \boxed{2\gamma|A_0|^2} & \boxed{\gamma A_0^2} \\ \boxed{-\gamma\left(A_0^2\right)^*} & \boxed{-B(-i\partial_\tau) + \delta\beta_0} - \boxed{2\gamma|A_0|^2} \end{bmatrix} w$$

FWM term couples two components of doublet; without it, they remain completely uncoupled

Norm and dispersion relation(s)

Conserved (in *z*) **norm**: $\int \left(\left| w_{+}(z,\tau) \right|^{2} - \left| w_{-}(z,\tau) \right|^{2} \right) \mathrm{d}\tau$

Upper (lower) component of doublet carries positive (negative) norm

Neglecting FWM term eliminates possibility of anomalous scattering

Two components of doublet decouple when $A_0 \to 0$

$\Longrightarrow$ Obey same **dispersion relations** as δA and δA^{*} : $w_{\pm} = e^{iD_{\pm} z - i\Delta\omega\tau}$

$$D_{+} = B(\Delta\omega) - \delta\beta_0$$
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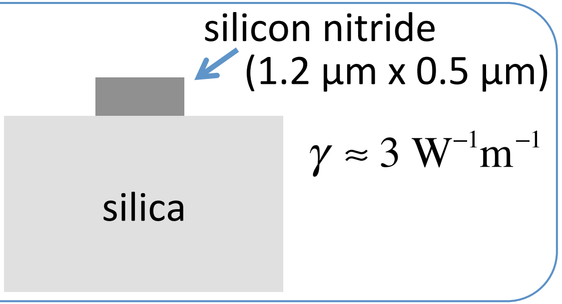
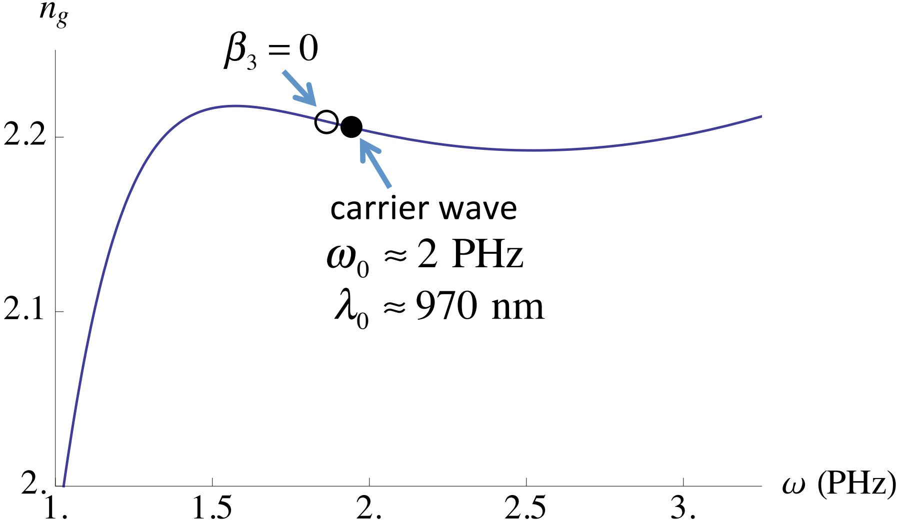


$\gamma \approx 3\ \mathrm{W}^{-1}\mathrm{m}^{-1}$



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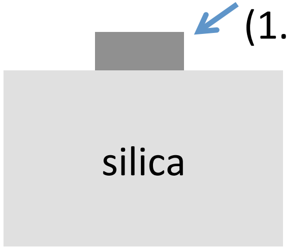

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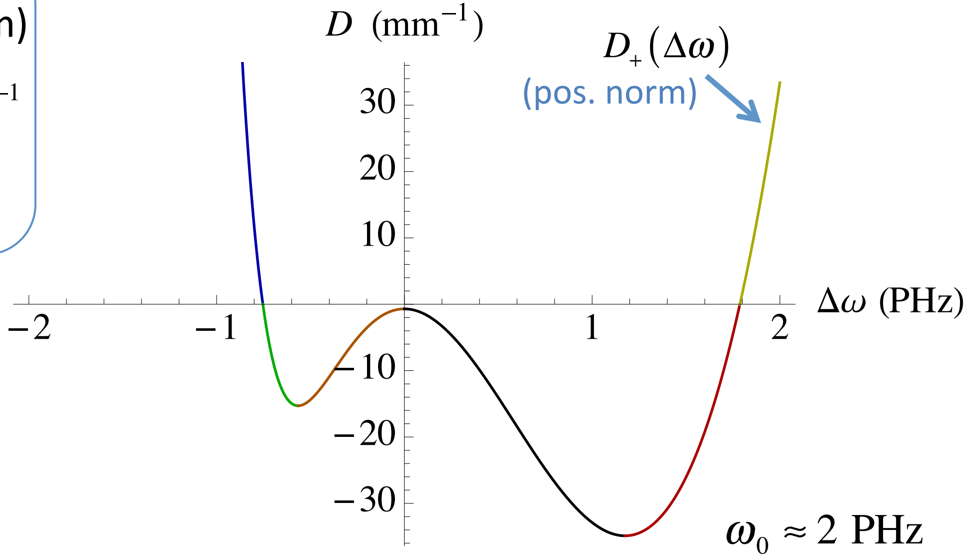

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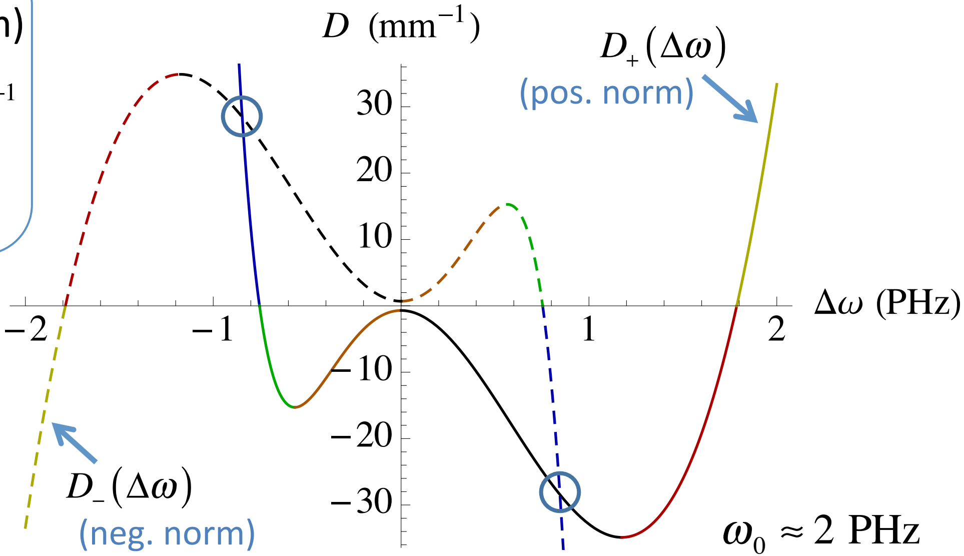


Phase matching: $\omega_{\text{black}} + \omega_{\text{blue}} = 2\omega_0$

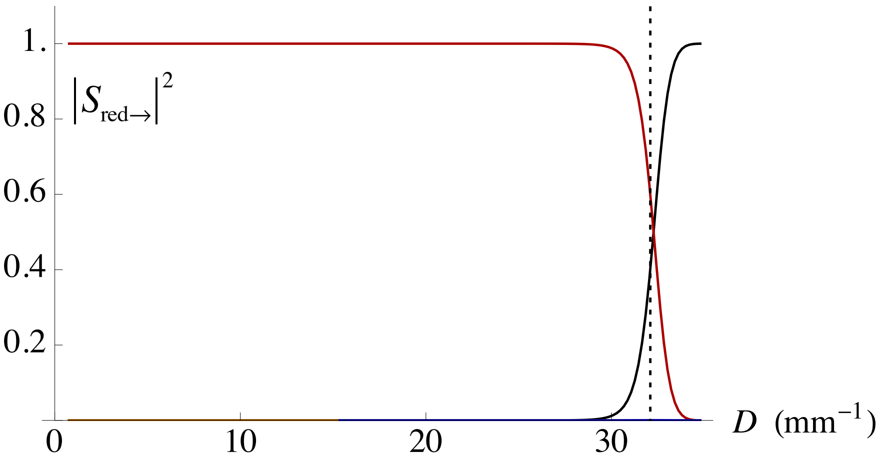
$$\beta_{\text{black}} + \beta_{\text{blue}} = 2(\beta_0 + \delta\beta_0)$$

Requires at least β_4 (otherwise does not occur)

Leads to additional modulation instability for CW

Pitois + Millot, Opt. Comm. 226, 415 (2003)

Elastic scattering coefficients



➢ total transmission → total reflection

[see, e.g., Philbin *et al.*, *Science* **319**, 1367 (2008);
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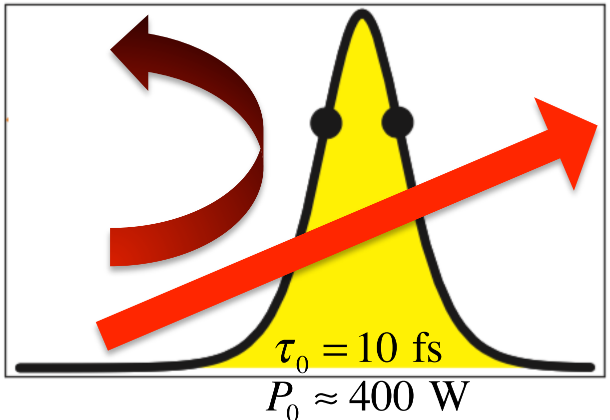
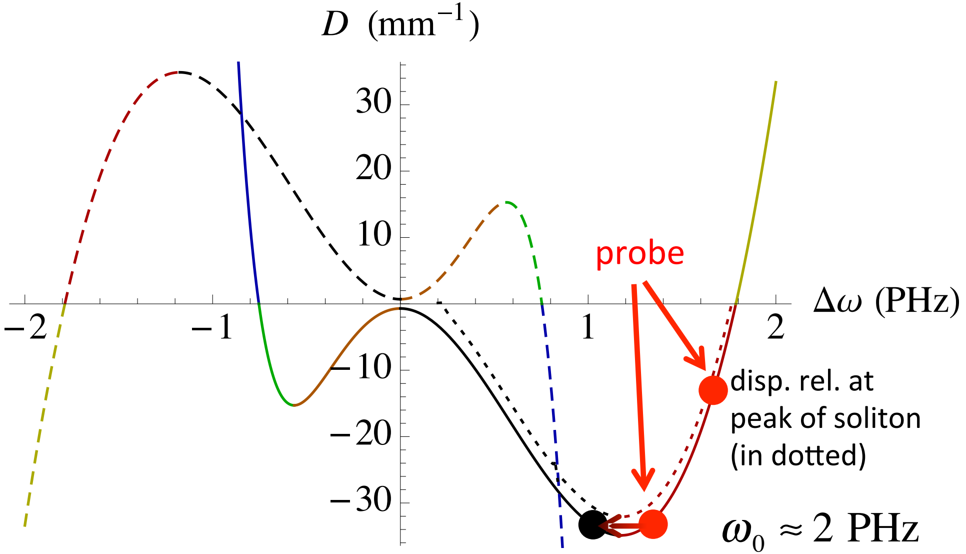
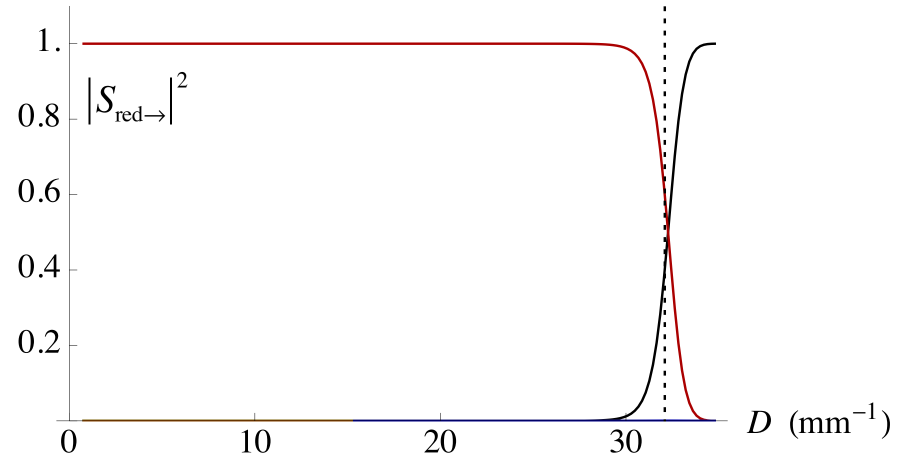
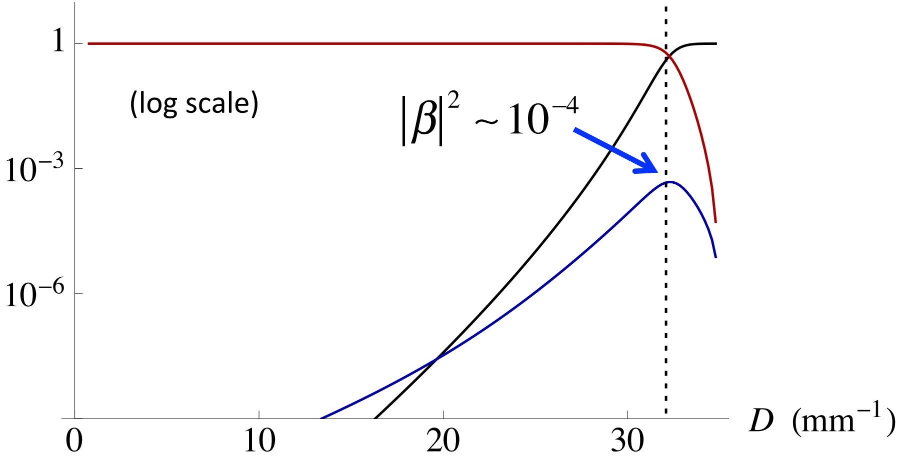


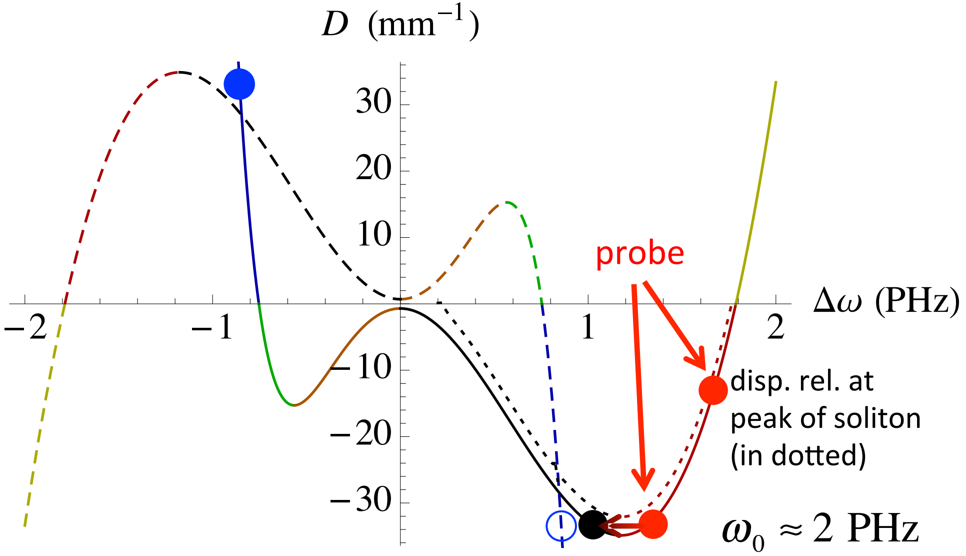


Anomalous scattering coefficients

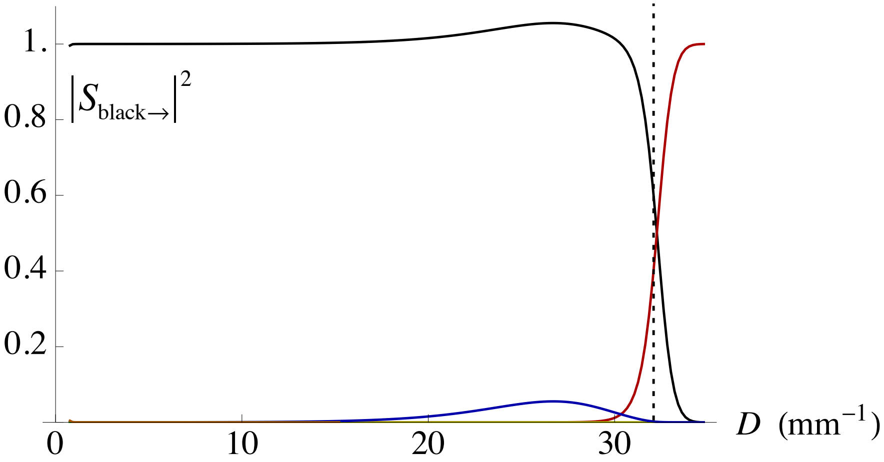




- total transmission → total reflection
  [see, e.g., Philbin *et al.*, *Science* **319**, 1367 (2008); Ciret *et al.*, *Opt. Express* **24**, 114 (2016)]
- small anomalous coefficient much larger than in "hard" scattering
- behaves as in other media, e.g., water in flume
  [Robertson *et al.*, *PRD* **93**, 124060 (2016)]

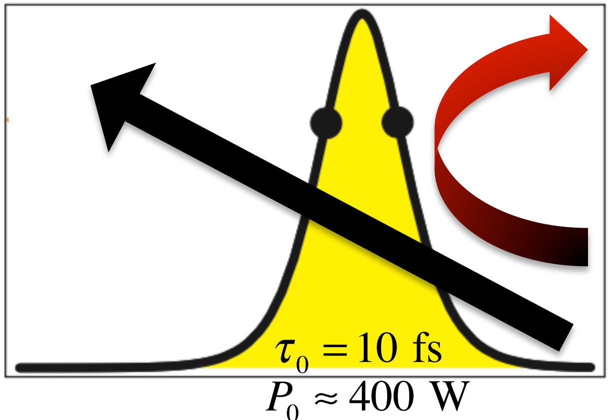


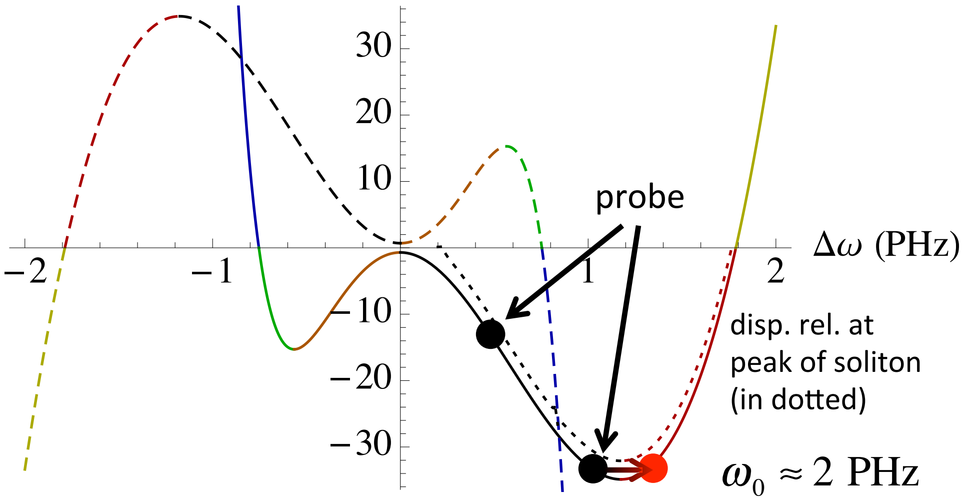
Enhanced anomalous scattering coefficients



- total transmission $\rightarrow$ total reflection (as before)

D (mm^{-1})





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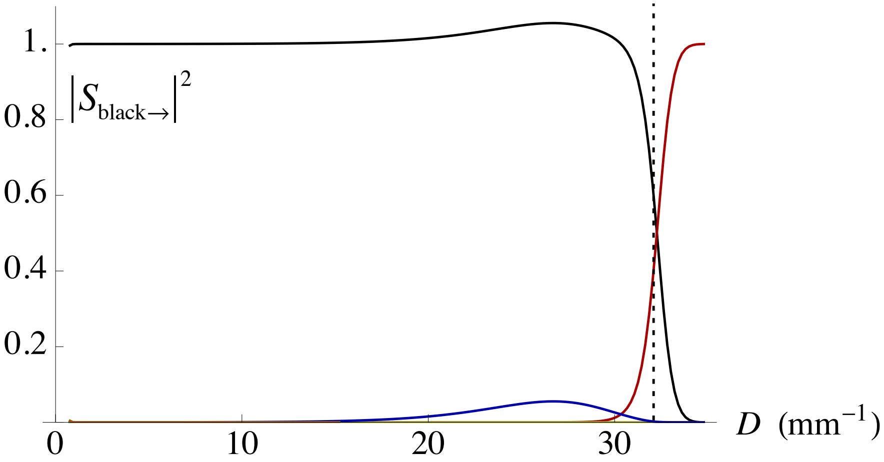
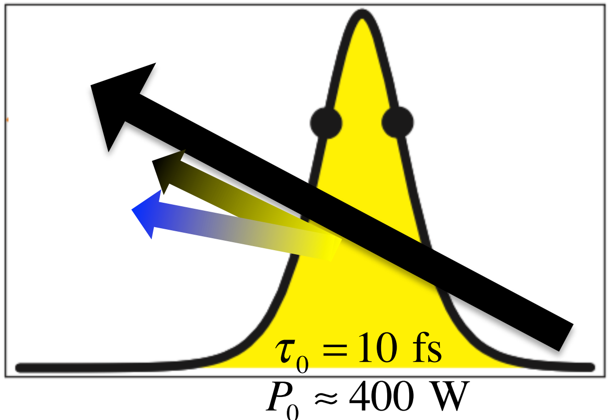
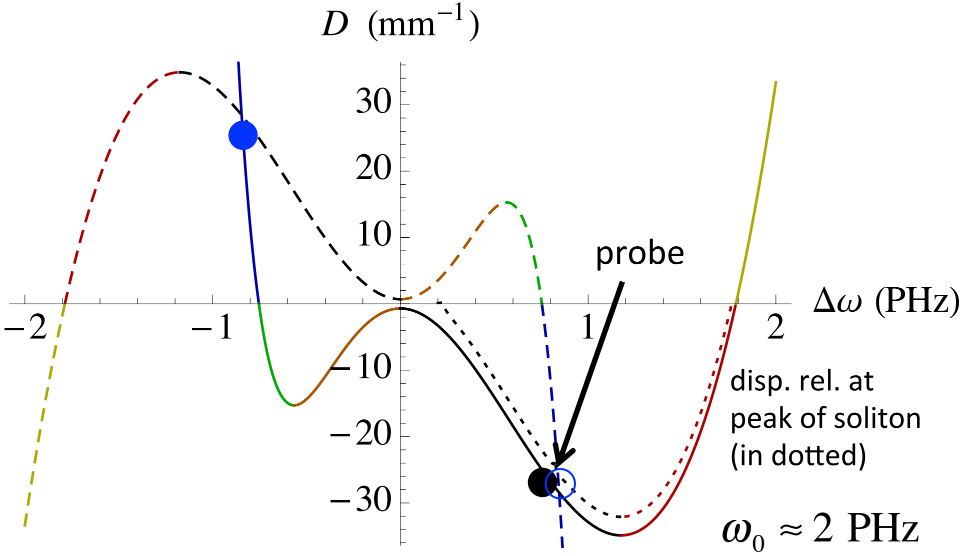


- total transmission → total reflection (as before)
- strong anomalous mixing in vicinity of PM
  - boosted by modulation instability
  - extraction of blue/black pairs from soliton





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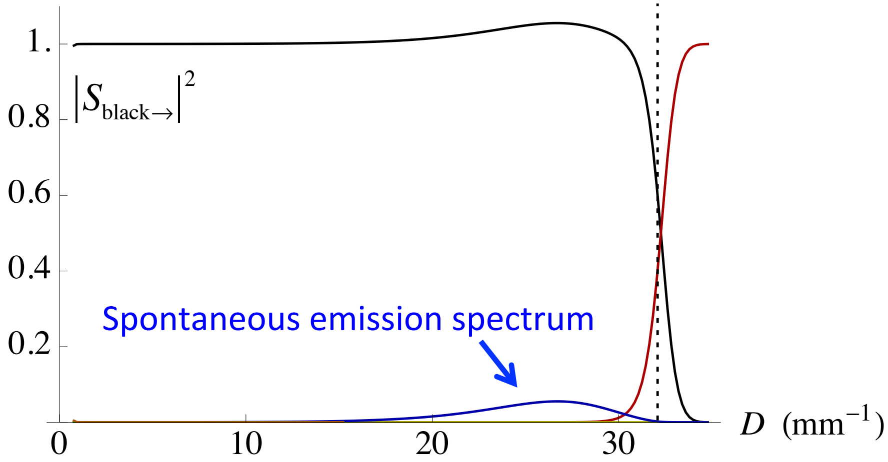

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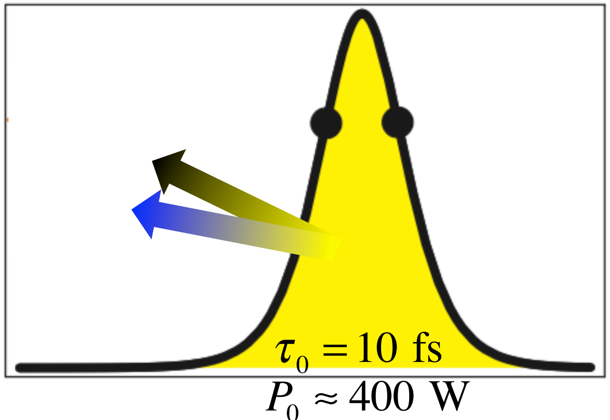
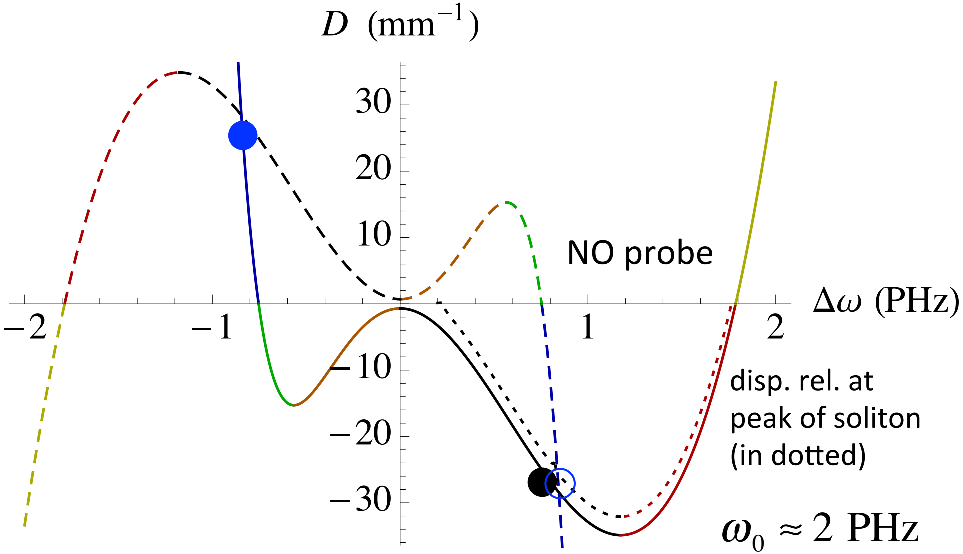
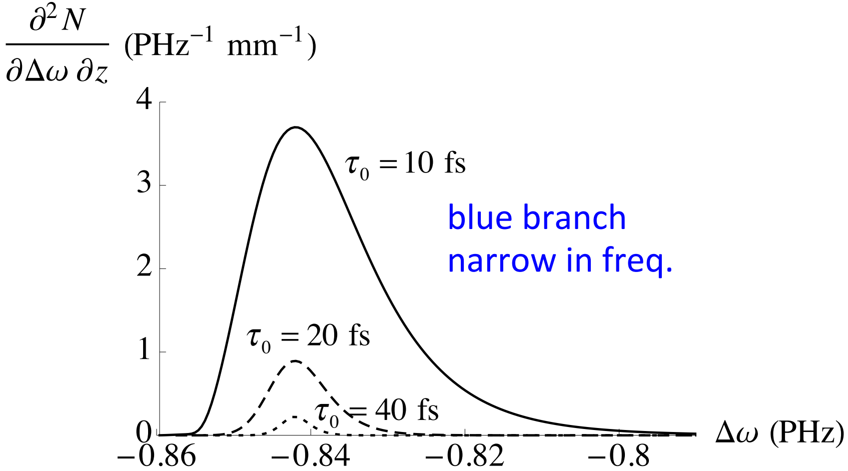
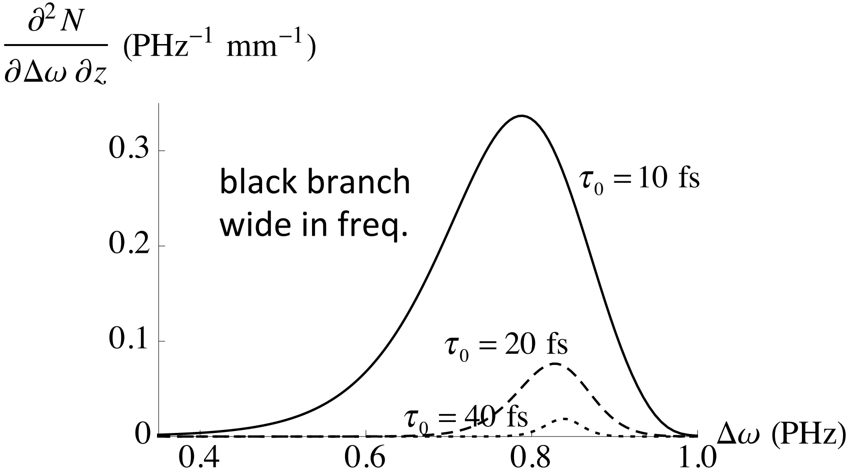




Spontaneous emission





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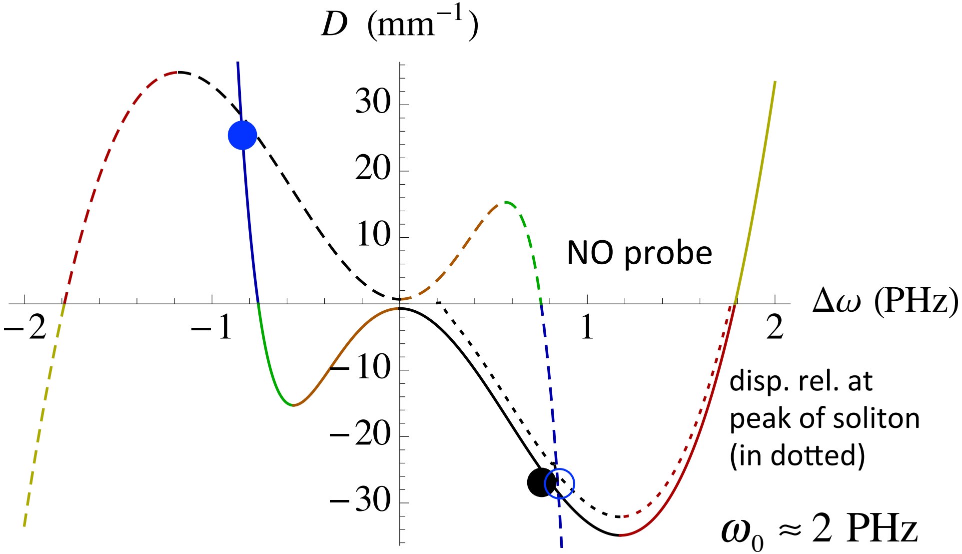
  - extraction of blue/black pairs from soliton
  - spontaneous emission in vacuum state
- integral over $\Delta\omega$ gives total emission rate $\sim \tau_0^{-3}$

For $\tau_0 = 10$ fs, rate $\approx 1/\mathrm{cm}$



Conclusions and outlook

Key conclusions:

- “Soft” pair production also exists as an analogue of Hawking radiation, and is orders of magnitude more efficient than “hard” pair production
- Phase matching occurs through crossing of positive- and negative-norm dispersion relations, leading to a greatly enhanced anomalous scattering / spontaneous emission

Current/Future work:

- Describe “soft” and “hard” processes with single wave equation
- Measurement