Non-linear effects in modulated 1D BEC

Decoherence and out of equilibrium physics,

Observing the *pre-heating* scenario?

Scott Roberston[1], Florent Michel[2], and Renaud Parentani[1]

[1]LPT and LAL, Paris-Sud Orsay,

[2]Durham University.

ECT* workshop, July 2019

Mainly based on **1802.00739**, PRD 98 (2018),
but also on PRD 97 (2018) 065018, PRD 96 (2017) 045012.

QFT in curved space-time

QFT in curved space-time leads to two remarkable predictions

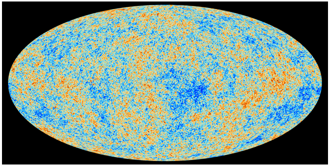
- black holes emit a **steady** thermal flux of quanta
  that are entangled to inside partners of **opposite energy,** Hawking '74.

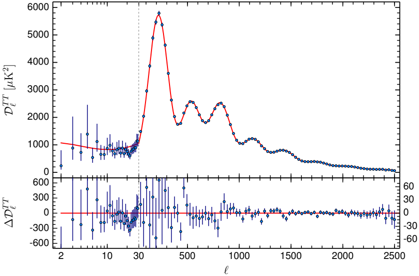
- pairs of quanta **with opposite momenta** $(\vec{k}, -\vec{k})$ are spontaneously produced in cosmological **t-dependent** metrics, Parker-Zeldovich late '60s, *aka DEC (Dynamical Casimir Effect).*

Pair creation in physical cosmology: type I and II.

Type I. During inflation, tensor and scalar fluctuations are **amplified**, and give rise (Starobinski-Mukhanov '80-81)

- to **nearly scale-invariant spectra:** $P_k \propto H^2\, k^\epsilon$ with $\epsilon \ll 1$.
- Sakharov oscillations in the CMB anisotropy spectrum.





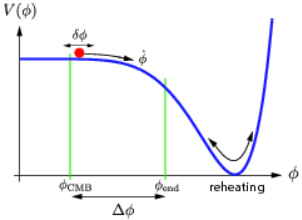
Data from Planck Mission,

Oscillations at fixed time as fct of $k \sim l$.

Pair creation in physical cosmology: type I and II.

Type II. At the end of inflation, resonant modes are **exponentially amplified** by the late oscillations of the inflaton field ϕ during the *"pre-heating"* phase (Kofman, Linde, Starobinski '94):

Frequency of **resonant modes** $\boxed{\omega_{res} \simeq \omega_\phi/2}$



Then, during the *"re-heating"* period

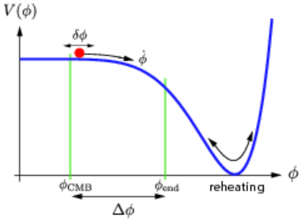
- the exponential growth saturates due to non-linearities,
- there is an energy transfer to non-resonant modes, with $\omega_k \neq \omega_\phi/2$
- matter d.o.f. progressively thermalize giving rise to the radiation dominated era, i.e., to the standard Hot BB scenario.

  see D. Figueroa et al. JCAP (2018) for a recent review.

Pair creation in physical cosmology: type I and II.

Type II. At the end of inflation, resonant modes are **exponentially amplified** by the late oscillations of the inflaton field ϕ during the *"pre-heating"* phase (Kofman, Linde, Starobinski '94):

Frequency of **resonant modes** $\omega_{res} \simeq \omega_\phi/2$



Then, during the *"re-heating"* period

- the exponential growth saturates **due to non-linearities,**
- there is an energy transfer to non-resonant modes, with $\omega_k \neq \omega_\phi/2$
- matter d.o.f. **progressively thermalize** giving rise to the radiation dominated era, i.e., to the **standard Hot BB scenario**.
  see D. Figueroa et al, JCAP (2018) for a recent review.

Comparing types I and II

- When starting from vacuum, both rely on **spontaneous** pair creation.
- As long as non-linearities can be neglected, **i.e., before decoherence**, the bi-partite states $(\vec{k}, -\vec{k})$ display **non-classical features**:
  - **non-separability** (Werner 1989),
  - *violations of some Bell inequalities.*

- However
  - in type I, **non-linearities** (non-Gaussianities) are **very small**: $f_{NL} \sim 1$,
  - whereas **strong non-linearities** and **out of equilibrium physics** play key roles in the (p)re-heating scenario (type II).
  - **In this sense, types I and II radically differ.**

Comparing types I and II

- When starting from vacuum, both rely on **spontaneous** pair creation.
- As long as non-linearities can be neglected, **i.e., before decoherence**, the bi-partite states $(\vec{k}, -\vec{k})$ display **non-classical features**:
  - **non-separability** (Werner 1989),
  - *violations of some Bell inequalities.*
- However
  - in type I, **non-linearities** (non-Gaussianities) are **very small**: $f_{NL} \sim 1$,
  - whereas **strong non-linearities** and **out of equilibrium physics** play key roles in the (p)re-heating scenario (type II).
  - **In this sense, types I and II radically differ.**

Observing these phenomena in the lab

- Following *"Experimental BH evaporation?"* *Unruh PRL (1981)* , experiments aiming at observing cosmological-like effects have been suggested.
- Here we focus on atomic BEC, Garay et al. PRL (2000), Fedichev and Fisher, PRA (2004)
- Since 2007, several experiments have been performed
  - "Observation of Faraday waves in a BEC", Engels et al. PRL 2007
  - "Observation of Sakharov oscillations in a Quenched ..." Hung at al, Science 2103
  - ...
- **Since** $\sim$ 2011,
  - **Critical analysis of reported observations**,
  - **Identification of the relevant mechanisms**,
  - New schemes for observing
    - pre-heating-like mecanism, Zache et al, PRD (2017), Feng et al. Nature Phys (2018), ...
    - re-heating mechanism, Eckel et al. PRX (2018), ...

Observing these phenomena in the lab

- Following *"Experimental BH evaporation?"* *Unruh PRL (1981)* , experiments aiming at observing cosmological-like effects have been suggested.
- Here we focus on atomic BEC, Garay et al. PRL (2000), Fedichev and Fisher, PRA (2004)
- Since 2007, several experiments have been performed
  - "Observation of Faraday waves in a BEC", Engels et al. PRL 2007
  - "Observation of Sakharov oscillations in a Quenched ..." Hung at al, Science 2103
  - ...
- **Since** $\sim$ 2011,
  - **Critical analysis of reported observations**,
  - **Identification of the relevant mechanisms**,
  - New schemes for observing
    - pre-heating-like mecanism, Zache et al, PRD (2017), Feng et al. Nature Phys (2018), ...
    - re-heating mechanism, Eckel et al. PRX (2018), ...

Observing these phenomena in the lab

- Following *"Experimental BH evaporation?"* *Unruh PRL (1981)* , experiments aiming at observing cosmological-like effects have been suggested.
- Here we focus on atomic BEC, Garay et al. PRL (2000), Fedichev and Fisher, PRA (2004)
- Since 2007, several experiments have been performed
  - "Observation of Faraday waves in a BEC", Engels et al. PRL 2007
  - "Observation of Sakharov oscillations in a Quenched ..." Hung at al, Science 2103
  - ...
- **Since** $\sim$ 2011,
  - **Critical analysis of reported observations**,
  - **Identification of the relevant mechanisms**,
  - New schemes for observing
    - pre-heating-like mecanism, Zache et al, PRD (2017), Feng et al. Nature Phys (2018), ...
    - re-heating mechanism, Eckel et al. PRX (2018), ...

Chris Westbrook's experiments, PRL 2012

- Inspired by "Density correlations and DCE in modulated atomic BEC", I. Carusotto *et al.* (2010), his team performed **two** experiments aiming at observing the correlations among pair produced phonons with opposite k.

- Although they looked for the 2-mode $(k, -k)$ entanglement, they observed *rather weak correlations*, **well below their expectations**.
  (C. Westbrook private communication, fall 2013)

- This **interesting remark** motivated us to reconsider **DCE in 1D quasi-condensates**:
  - *Quantum entanglement due to modulated DCE*, PRA (2014)
  - *Controlling and observing nonseparability of phonons created in time-dependent 1D BEC*, PRD96 (2017)
  - *Assessing degrees of entanglement of phonon states in BEC through the measurement of* **commuting** *observables*, PRD97 (2018) {applied also to Jeff's BH experiments (2015)}
  - *Nonlinearities induced by parametric resonance in effectively 1D atomic Bose condensates*, PRD98 (2018)

Chris Westbrook's experiments, PRL 2012

- Inspired by "Density correlations and DCE in modulated atomic BEC", I. Carusotto *et al.* (2010), his team performed **two** experiments aiming at observing the correlations among pair produced phonons with opposite k.

- Although they looked for the 2-mode $(k, -k)$ entanglement, they observed *rather weak correlations,* **well below their expectations**.
  (C. Westbrook private communication, fall 2013)

- This **interesting remark** motivated us to reconsider **DCE in 1D quasi-condensates**:
  - *Quantum entanglement due to modulated DCE,* PRA (2014)
  - *Controlling and observing nonseparability of phonons created in time-dependent 1D BEC,* PRD96 (2017)
  - *Assessing degrees of entanglement of phonon states in BEC through the measurement of* **commuting** *observables,* PRD97 (2018) {applied also to Jeff's BH experiments (2015)}
  - *Nonlinearities induced by parametric resonance in effectively 1D atomic Bose condensates,* PRD98 (2018)

Chris Westbrook's experiments, PRL 2012

- Inspired by "Density correlations and DCE in modulated atomic BEC", I. Carusotto *et al.* (2010), his team performed **two** experiments aiming at observing the correlations among pair produced phonons with opposite k.

- Although they looked for the 2-mode $(k, -k)$ entanglement, they observed *rather weak correlations,* **well below their expectations**.
  (C. Westbrook private communication, fall 2013)

- This **interesting remark** motivated us to reconsider **DCE in 1D quasi-condensates**:

  - *Quantum entanglement due to modulated DCE,* PRA (2014)
  - *Controlling and observing nonseparability of phonons created in time-dependent 1D BEC,* PRD96 (2017)
  - *Assessing degrees of entanglement of phonon states in BEC through the measurement of* **commuting** *observables,* PRD97 (2018) (applied also to Jeff's BH experiments (2015))
  - *Nonlinearities induced by parametric resonance in effectively 1D atomic Bose condensates,* PRD98 (2018)

Chris Westbrook's experiments, PRL 2012

- Inspired by "Density correlations and DCE in modulated atomic BEC", I. Carusotto *et al.* (2010), his team performed **two** experiments aiming at observing the correlations among pair produced phonons with opposite k.

- Although they looked for the 2-mode $(k, -k)$ entanglement, they observed *rather weak correlations,* **well below their expectations**. *(C. Westbrook private communication, fall 2013)*

- This **interesting remark** motivated us to reconsider **DCE in 1D quasi-condensates**:
  - *Quantum entanglement due to modulated DCE,* PRA (2014)
  - *Controlling and observing nonseparability of phonons created in time-dependent 1D BEC,* PRD96 (2017)
  - *Assessing degrees of entanglement of phonon states in BEC through the measurement of* ***commuting*** *observables,* PRD97 (2018) {applied also to Jeff's BH experiments (2015)}
  - *Nonlinearities induced by parametric resonance in effectively 1D atomic Bose condensates,* PRD98 (2018)

Elongated quasi-condensates

- Have been studied theoretically (in the mean field + linear perturbations)
  Zambera (1998), Stringari-Menotti (2002), Dalfovo-Tozzo (2004), Gerbier (2004), Kagan (2006), ...
- and experimentally:
  - Dispersion relations, Steinhauer-Dalfovo *et al* (2003),
  - Density-density correlation fct, several papers of Steinhauer *et al* (2013), ...
- **Out of equilibrium physics** is less investigated (as far as I know)

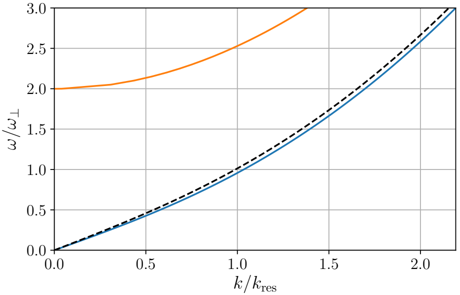
Elongated quasi-condensates

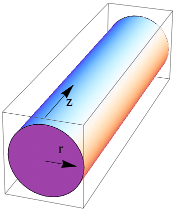
- Have been studied theoretically (in the mean field + linear perturbations)
  Zambera (1998), Stringari-Menotti (2002), Dalfovo-Tozzo (2004), Gerbier (2004), Kagan (2006), ...
- and experimentally:
  - Dispersion relations, Steinhauer-Dalfovo *et al* (2003),
  - Density-density correlation fct, several papers of Steinhauer *et al* (2013), ...
- **Out of equilibrium physics** is less investigated (as far as I know)

Elongated quasi-condensates

- **Longitudinal excitations (in z)** with $\boxed{\omega \ll \omega_\perp}$ are effective 1D phonons, where $\omega_\perp = \hbar/(m\, a_\perp^2)$ is the **radial trapping freq.** and $a_\perp$ the cloud radius.

- Dispersion relations of cyl. symmetric lowest and "breathing" modes
  for $n_1 a_s = 0.6$, in the so-called "quasi-condensate 1D regime", see next slide

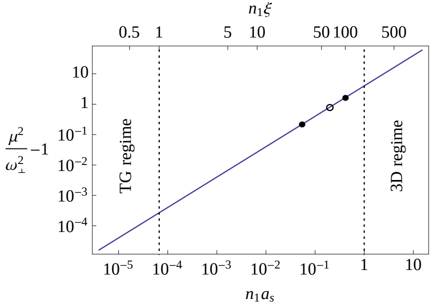




The dotted line is the approximate law obtained using the factorization
$\Phi = \Phi_0(r) \times (\sqrt{n_1} + \delta\phi(z,t))$.

Elongated quasi-condensates

- Have a rather rich phase space characterized by several scales: $a_\perp$, L, a_s, ξ, ...



Chemical potential $\mu/\omega_\perp$ as a fct of $n_1 a_s$ **(bottom)** and $n_1\xi$ (top), for $a_s/a_\perp = 5.5\,10^{-3}$.

empty circle is Chris' 2012,

filled circles are sup. and subsonic Jeff's 2016

- Have been used in many analog gravity experiments:
  - Engels et al. for studying "Faraday" waves (PRL 2007)
    Abstract: "intriguing nonlinear dynamics ... are observed"
  - Westbrook et al. for studying $k, -k$ correlations (PRL 2012).
    very close settings as the above
  - Jeff' experiments of the *BH-laser* (Nat. Phys. 2014), for *entanglement* (Nat. Phys. 2016), and *thermality* (Nature. 2019) of analog BH radiation

Elongated cyl. symm. quasi-condensates

The Bose gaz is described by

$$\hat{\Phi} = \Phi_0 + \delta\hat{\Phi} \,, \tag{1}$$

where Φ_0 is the mean field condensate and $\delta\hat{\Phi}$ describes perturbations.

The mean field Φ_0 obeys the **3D GPE ($\hbar = 1$):**

$$\boxed{\mathbf{i}\partial_t \Phi_0 = -\frac{1}{2m}\nabla^2\Phi_0 + V(\vec{x})\,\Phi_0 + g\,|\Phi_0|^2\,\Phi_0} \tag{2}$$

Assuming a cylindrically symmetric harmonic potential:

$$V(\vec{x}) = \frac{m}{2}\left(\omega_\perp^2 r^2 + \omega_\parallel^2 z^2\right), \tag{3}$$

$\omega_\parallel^2 \ll \omega_\perp^2$, gives an elongated cigar with $a_\parallel \gg a_\perp$, where $a_i \doteq 1/(m\omega_i)^{1/2}$.

In our analysis, we put $\omega_\parallel = 0$ and work with homogeneous cyl. symm. condensates of size $L \gg a_\perp$, typically $L/a_\perp \sim 100$.

Longitudinal excitations with momentum k

Linear longitudinal (node-less) pert. are (approx.) described by a 1D field $\hat{\phi}(t, z)$

$$\hat{\Phi} = \Phi_0(t, r) \left(\sqrt{n_1} + \hat{\phi}(t, z) \right) , \tag{4}$$

where $\rho_0(t, r) = n_1 \times |\Phi_0(t, r)|^2$ is the background density of mean radius $a_\perp$.

The linearized 3D GPE then gives the **1D-BdG equation**

$$\boxed{ i\, \partial_t \hat{\phi} = -\frac{1}{2m} \partial_z^2 \hat{\phi} + n_1\, g_1 \left(\hat{\phi} + \hat{\phi}^\dagger \right) } \tag{5}$$

n_1 is the linear atom density, and $g_1 \doteq g/2\pi a_\perp^2$ the effective 1D coupling
(which becomes t-dependent when varying $\omega_\perp$ or g).

Exploiting **homogeneity**, work with Fourier components $\hat{\phi}_k(t)$

$$\hat{\phi}(t, z) = \sum_{k \in 2\pi\mathbb{Z}/L} \hat{\phi}_k(t) \frac{e^{ikz}}{\sqrt{L}} , \tag{6}$$

where $\hat{\phi}_k$ destroys an **atom** carrying longit. momentum k.

Phonon dispersion relation

The 1D-BdG eq. for the **atomic operators** $\hat{\phi}_k,\ \hat{\phi}^\dagger_{-k}$ reads

$$i\,\partial_t \begin{bmatrix} \hat{\phi}_k \\ \hat{\phi}^\dagger_{-k} \end{bmatrix} = \begin{bmatrix} \Omega_k & g_1 n_1 \\ -g_1 n_1 & -\Omega_k \end{bmatrix} \begin{bmatrix} \hat{\phi}_k \\ \hat{\phi}^\dagger_{-k} \end{bmatrix}, \tag{7}$$

where $\Omega_k \equiv k^2/2m + g_1 n_1$.

They are related to **phonon operators** $\hat{\varphi}_k,\ \hat{\varphi}^\dagger_{-k}$ by a Bogoliubov transformation

$$\begin{bmatrix} \hat{\phi}_k \\ \hat{\phi}^\dagger_{-k} \end{bmatrix} \equiv \begin{bmatrix} u_k & v_k \\ v_k & u_k \end{bmatrix} \begin{bmatrix} \hat{\varphi}_k \\ \hat{\varphi}^\dagger_{-k} \end{bmatrix}, \tag{8}$$

where u_k, v_k obey $|u_k| - |v_k|^2 = 1$.

The phonic dispersion relation is (approx.)

$$\boxed{\omega_k^2 = c^2\,k^2 + \left(\frac{k^2}{2m}\right)^2}, \tag{9}$$

where $c \equiv (g_1 n_1/m)^{1/2}$ is the eff. 1D speed of sound (on the lowest phonic branch).

Probing *in situ* the phonon state

- Whereas the FT of the 1-body correlation $G_1(x, x') = \langle \hat{\phi}^\dagger(t, z)\, \hat{\phi}(t, z') \rangle$ gives the **atomic** content: $\boxed{G_1(k \neq 0, t) = n_k^{at}(t)}$: number of **depleted atoms,**
- the FT of the 2-body (dens-dens) correlation $G_2(x, x') = \langle \hat{\rho}(t, z)\, \hat{\rho}(t, z') \rangle$ where $\hat{\rho} = |\hat{\phi}|^2$, gives its **phonic** content (when fluctuations are small enough).
- When $c^2 = \text{cst}$, i.e., before/after a "quench", phonon modes are $\hat{\varphi}_k = \hat{b}_k\, e^{-i\omega t}$,
- in any homogeneous (isotr.) state, the FT $G_{2,k}(t)$ has the form

$$\boxed{G_{2,k}(t) = (u_k + v_k)^2 \left(2n_k^{ph} + 1 + 2\,\mathrm{Re}\left[c_k^{ph}\, e^{-2i\omega_k t} \right] \right)}, \quad (10)$$

where

$$\boxed{n_k^{ph} = \mathrm{Tr}[\hat{\rho}_T\, \hat{b}_k^\dagger\, \hat{b}_k]} \quad (11)$$

is the **mean occ. number of $\pm k$-phonons**, and the norm of

$$\boxed{c_k^{ph} = \mathrm{Tr}[\hat{\rho}_T\, \hat{b}_k\, \hat{b}_{-k}]} \quad (12)$$

gives the **strength of the correlations** between k and $-k$.

Probing *in situ* the phonon state

- Whereas the FT of the 1-body correlation $G_1(x, x') = \langle \hat{\phi}^\dagger(t, z)\, \hat{\phi}(t, z') \rangle$ gives the **atomic** content: $\boxed{G_1(k \neq 0, t) = n_k^{at}(t)}$: number of **depleted atoms,**

- the FT of the 2-body (dens-dens) correlation $G_2(x, x') = \langle \hat{\rho}(t, z)\, \hat{\rho}(t, z') \rangle$ where $\hat{\rho} = |\hat{\phi}|^2$, gives its **phonic** content (when fluctuations are small enough).

- When c^2 = cst, i.e., before/after a "quench", phonon modes are $\hat{\varphi}_k = \hat{b}_k\, e^{-i\omega t}$,

- in any homogeneous (isotr.) state, the FT $G_{2,k}(t)$ has the form

$$\boxed{G_{2,k}(t) = (u_k + v_k)^2 \left(2 n_k^{ph} + 1 + 2\,\mathrm{Re}\left[c_k^{ph}\, e^{-2i\omega_k t}\right]\right)}, \qquad (10)$$

where

$$\boxed{n_k^{ph} \equiv \mathrm{Tr}[\hat{\rho}_T\ \hat{b}_k^\dagger\, \hat{b}_k]} \qquad (11)$$

is the **mean occ. number of $\pm k$-phonons**, and the norm of

$$\boxed{c_k^{ph} \equiv \mathrm{Tr}[\hat{\rho}_T\ \hat{b}_k\, \hat{b}_{-k}]} \qquad (12)$$

gives the **strength of the correlations** between k and $-k$.

Assessing *in situ* non-separability

Theorem: The expression,

$$G_{2,k}(t) = (u_k + v_k)^2 \left(1 + 2n_k + 2\,\mathrm{Re}\{c_k\, e^{-2i\omega_k t}\}\right) \ ,$$

guarantees that the **observation** of the inequality *(e.g. at some given time t after a quench)*

$$\boxed{G_{2,k}(t) < G_{2,k}^{\mathrm{vac}} = (u_k + v_k)^2 \equiv S_k^0} \tag{13}$$

is **sufficient** to assess that the (phonic) 2-mode state $(k, -k)$ is **non-separable**,
see ...?, X. Busch-RP (2013), ...

Indeed inequality (13) can be satisfied only if

$$\boxed{\Delta_k \doteq n_k - |c_k| < 0} . \tag{14}$$

which is a **sufficient condition** for non-sep. and equivalent to **Perez-Horodecki** criterion.

NB. $G_{2,k}(t)$ acts as an **interferometer** with a phase shift $\boxed{\delta\phi = \arg(c_k) - 2\omega_k t}$.
When minima obey Eq. (13), the "dark port" reveals a **sub-fluctuating mode**,

Assessing *in situ* non-separability

Theorem: The expression,

$$G_{2,k}(t) = (u_k + v_k)^2 \left(1 + 2n_k + 2\,\mathrm{Re}\{c_k\, e^{-2i\omega_k t}\}\right) ,$$

guarantees that the **observation** of the inequality *(e.g. at some given time t after a quench)*

$$\boxed{G_{2,k}(t) < G^{\mathrm{vac}}_{2,k} = (u_k + v_k)^2 \equiv S^0_k} \qquad (13)$$

is **sufficient** to assess that the (phonic) 2-mode state $(k, -k)$ is **non-separable**,
see ...?, X. Busch-RP (2013), ...

Indeed inequality (13) can be satisfied only if

$$\boxed{\Delta_k \doteq n_k - |c_k| < 0}. \qquad (14)$$

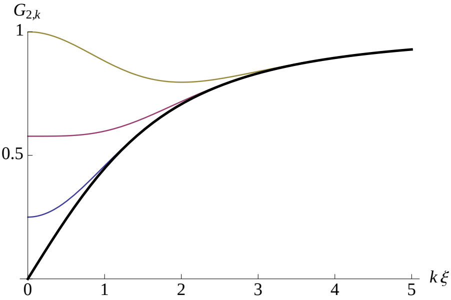
which is a **sufficient condition** for non-sep. and equivalent to **Perez-Horodecki** criterion.

NB. $G_{2,k}(t)$ acts as an **interferometer** with a phase shift $\boxed{\delta\phi = \arg(c_k) - 2\omega_k t}$.
When minima obey Eq. (13), the "dark port" reveals a **sub-fluctuating mode**,

Behavior of $G_{2,k}$ before a quench

Before a quench, starting with a stationary **incoherent thermal state**, $c_k \equiv 0$ (hence no $k, -k$ correl.), and n_k is Planckian.

For 4 different temperatures, $G_{2,k}$ as a function of k is

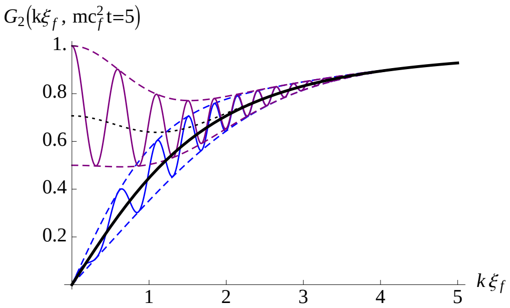


Temperature: $T/mc^2 = 0$ **(thick black)**, 0.25 (blue), $1/\sqrt{3}$ (purple) and 1 (yellow).

The low-k intercept gives the temperature in units of mc^2,

see *in situ* observations of J. Steinhauer et al, PRL 2013

Observing Sakharov oscillations after a quench

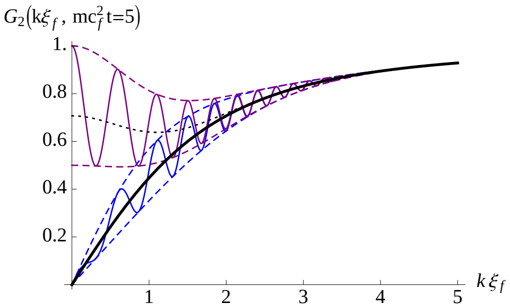


$G_{2,k}(t_f)$ as a fct of k, following a **quench** $c_{\rm fin}^2/c_{\rm in}^2 = 2$ **of rate** $\boxed{H \equiv \dot{c}/c \simeq 3c_f/\xi_f}$
by a lapse of time $t_f = 5\xi_f/c_f$ for $T = 0$ and $T = mc_{\rm in}^2$, (see Hung et al. Nature, 2013).

The jump is 'sudden' (non-adiab.) for freq. k-phonons with $\boxed{\omega_k \lesssim H}$.

The Sakharov oscill. have a larger amplit. for $T = mc_{\rm in}^2$ due to stimulated amplific.,
yet, the $(k, -k)$ 2-mode states are **less entangled than that at** $T = 0$.

Observing Sakharov oscillations after a quench



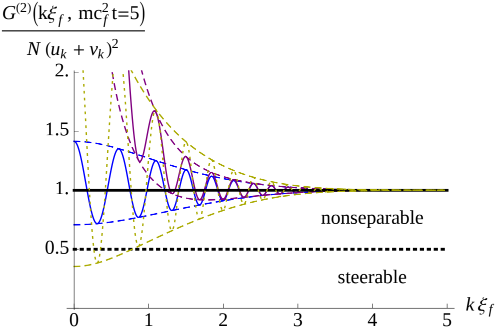
$G_{2,k}(t_f)$ as a fct of k, following a **quench** $c_{\rm fin}^2/c_{\rm in}^2 = 2$ **of rate** $H \equiv \dot{c}/c \simeq 3c_f/\xi_f$

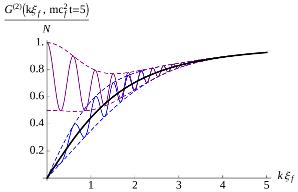
by a lapse of time $t_f = 5\xi_{\rm f}/c_{\rm f}$ for $T = 0$ and $T = mc_{\rm in}^2$, (see Hung et al. Nature, 2013).

The jump is 'sudden' (non-adiab.) for freq. k-phonons with $\omega_k \lesssim H$.

The Sakharov oscill. have a larger amplit. for $T = mc_{\rm in}^2$ due to stimulated amplific., yet, the $(k, -k)$ 2-mode states are **less entangled than that at** $T = 0$.

Assessing *in situ* non-separability





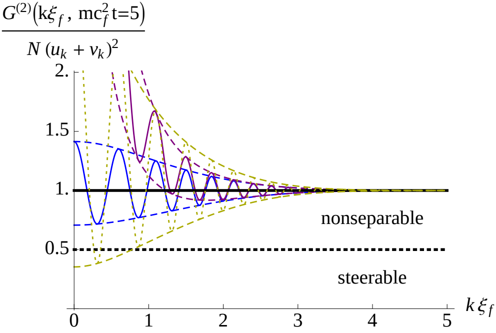
$G_{2,k}$ of the former plot

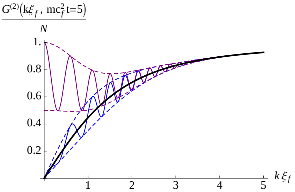
Ratio $\boxed{G_{2,k}/G_{2,k}^{\rm vac}}$ as a fct. of $k\xi_f$, following the quench

where $G_{2,k}^{\rm vac} \equiv S_k^0 = (u_k + v_k)^2$ is the final **zero-temp. "structure factor"**.

Whenever $\boxed{G_{2,k}(t)/G_{2,k}^{\rm vac} < 1}$, the bi-partite state $k, -k$ is **non-separable**.

Assessing *in situ* non-separability





$G_{2,k}$ of the former plot

Ratio $\boxed{G_{2,k}/G^{\rm vac}_{2,k}}$ as a fct. of $k\xi_f$, following the quench

where $G^{\rm vac}_{2,k} \equiv S^0_k = (u_k + v_k)^2$ is the final **zero-temp. "structure factor"**.

Whenever $\boxed{G_{2,k}(t)/G^{\rm vac}_{2,k} < 1}$, the bi-partite state $k, -k$ is **non-separable**.

Whenever $\boxed{G_{2,k}(t)/S_k < 1/2}$, the state is necessarily **'steerable'** (Schrodinger '35).

The yellow case corresponds to $c^2_{\rm fin}/c^2_{\rm in} = 8$ (stronger quench); see C. Klempt *et al*, Nature Comm. (2015)

Assessing non-separability, coda.

- Assessing non-separability of the phonon state can be achieved by observing *in situ* $\boxed{G_{2,k}(t) < G_{2,k}^{vac}\ ,}$ at a given time.

- The corresponding observation after TOF was Chris' aim.
  It is more complicate than *in situ* because the condensate-interferometer is lost.

Assessing non-separability, coda.

- Assessing non-separability of the phonon state can be achieved by observing *in situ* $\boxed{G_{2,k}(t) < G_{2,k}^{vac}\,,}$ at a given time.
- The corresponding observation after TOF was Chris' aim.
  It is more complicate than *in situ* because the condensate-interferometer is lost.
- Remarks:
  - **Non-commuting** measurements are **not** necessary.
  - *The observation of* $\boxed{G_{2,k}(t) < G_{2,k}^{vac}\,,}$ *should be* **simpler** *than that reported by J. Steinhauer in a supersonic flow in 2015.*

Westbrook's second 2012 experiment

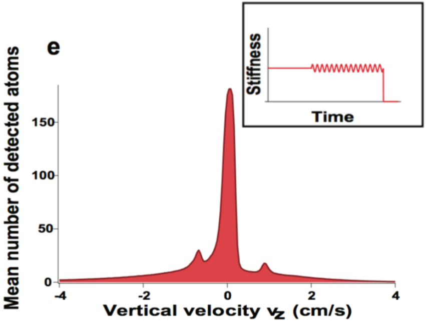
Westbrook's second 2012 experiment

To isolate **resonant modes**, they **periodically modulated** $\omega_\perp$:

$$\omega_\perp(t)/\omega_\perp^0 = 1 + A \sin(\omega_{\rm mod}\, t)$$

with $A = 0.05$, $N_{oscil.} \sim 10$, and with $\boxed{\omega_{\rm mod} = 0.7\,\omega_\perp}$ in order to excite only node-free soft modes, and be **adiabatic wrt cloud oscillations**.

They observed **(after TOF)** the **final atomic distribution** n_k^{at}

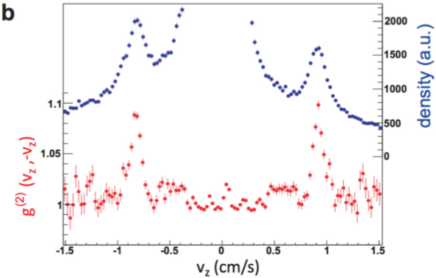


As expected,

- the large $k \simeq 0$ peak of condensed atoms,
- the two peaks of resonant modes $(k, -k)$ with $\omega_k = \omega_m/2$.

Not expected is the asymmetry $k, -k$ which is probably due to the detection device.

Cross-correlation $g^{(2)}_{\rm cross}(k)$ and entanglement



Cross-correlation

$$g^{(2)}_{\rm cross}(k) \equiv g^{(2)}(k,-k)$$

where the **atomic** 2-body fct is

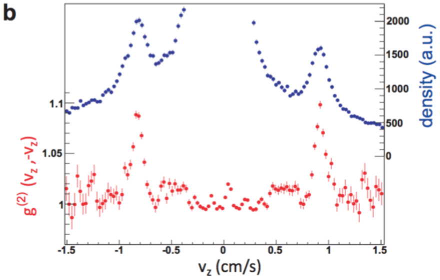
$$g^{(2)}(k,k') \equiv \frac{\langle \hat{a}^\dagger_k \hat{a}^\dagger_{k'} \hat{a}_k \hat{a}_{k'} \rangle}{\langle \hat{a}^\dagger_k \hat{a}_k \rangle \, \langle \hat{a}^\dagger_{k'} \hat{a}_{k'} \rangle}$$

- One gets (Assum. Gaussianity)

$$g^{(2)}_{\rm cross}(k) = \frac{n^{at}_k n^{at}_{-k} + |c^{at}_k|^2}{n^{at}_k \; n^{at}_{-k}}.$$

- **Non-separability** requires $g^{(2)}_{\rm cross}(k) > 2$.
- **They measured** $g^{(2)}_{\rm cross}(k) \lesssim 1.1$, hence very weak correlations **well below BdG predictions**.
- This could be (partially) due to a **weak phonon dissipation** $\Gamma_k/\omega_k \sim 0.01$, X. Busch, S. Robertson, RP (2014)

Cross-correlation $g^{(2)}_{\rm cross}(k)$ and entanglement



Cross-correlation

$$g^{(2)}_{\rm cross}(k) \equiv g^{(2)}(k,-k)$$

where the **atomic** 2-body fct is

$$g^{(2)}(k,k') \equiv \frac{\langle \hat a_k^\dagger \, \hat a_{k'}^\dagger \hat a_k \hat a_{k'}\rangle}{\langle \hat a_k^\dagger \, \hat a_k\rangle \, \langle \hat a_{k'}^\dagger \hat a_{k'}\rangle}$$

- One gets (Assum. Gaussianity)

$$g^{(2)}_{\rm cross}(k) = \frac{n_k^{at} n_{-k}^{at} + |c_k^{at}|^2}{n_k^{at} \; n_{-k}^{at}}.$$

- **Non-separability** requires $g^{(2)}_{\rm cross}(k) > 2$.

- **They measured** $g^{(2)}_{\rm cross}(k) \lesssim 1.1$,

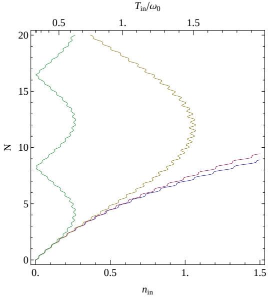
  hence very weak correlations

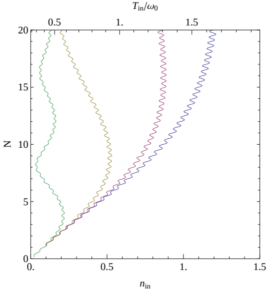
  well below BdG predictions.

- This could be (partially) due to
  a **weak phonon dissipation** $\Gamma_k/\omega_k \sim 0.01$,
  X. Busch, S. Robertson, RP (2014)

Theory: adding some weak dissipation

Effect of **weak dissipation** $\Gamma/\omega_0 = 0.01$ on non-separability.





Non-separability threshold $\Delta_k = 0$ in the plane (n^{in}, N_{osc}):

Left **without dissipation, resonant modes become non-separable** $\forall n^{in}$

Right **with dissipation, for $n^{in} \gtrsim 1.2$ non-sep. is not reached even for $N_{osc} \to \infty$.**

This reduction could play a crucial role in Chris' 2^d experiment

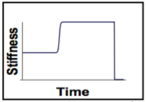
More work is needed to compute the decoherence rate Γ in 1D-BEC at finite T,

work in progress

The first experiment (the 'sudden' case)

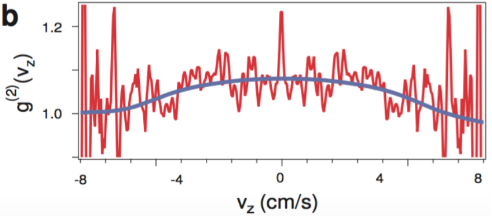
The first experiment (the 'sudden' case)

- They suddenly increased $\omega_\perp$: $\boxed{\omega^2_{\perp fin}/\omega^2_{\perp in} = 2}$,



- They waited 30ms before opening the trap.

  - The **observed cross correlation** $g^{(2)}_{\rm cross}(k)$ is:



- One sees: $\boxed{\textit{smoothed } g^{(2)}_{\rm cross}(k) \lesssim 1.1}$, hence

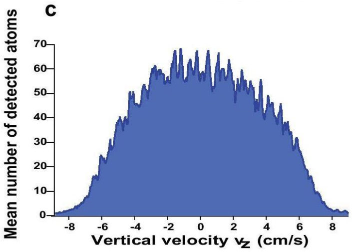
very weak $(k, -k)$ **correlations** over a **wide range:** $|k|\xi \lesssim 3$.

Comparison with theory

- These observations were unexpected/unexplained.
- they are in contradiction with BdG predictions (as we shall see)
- We conjectured in PRD 2017 that they observed
  - the early stage of thermalization **after pre-heating**.
  - that pre-heating (exponential growth) ended after $\sim$ 5ms (i.e. $1/6$ of 30ms)
- This claim is supported by numerical simulations see below

Reconsidering the data

- These observations were unexpected/unexplained.
- they are in contradiction with BdG predictions.
- We conjectured in PRD 2017 that they observed
  - the early stage of thermalization **after pre-heating**.
  - that pre-heating (exponential growth) ended after $\sim$ 5ms (i.e. 1/6 of 30ms)
- This claim is supported by numerical simulations see below
- is also supported by their **measurement of n_k^{at} versus k**

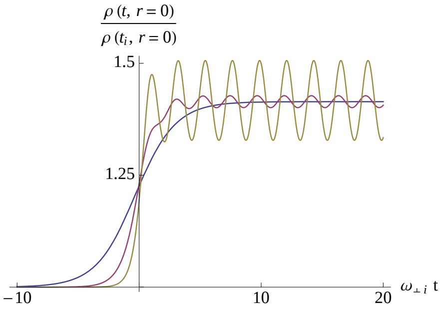


- **no central peak** of atoms near $k = 0$.
- their condensate has been **burned out**:
- they have observed a **hot gas of atoms**, *analogous* to a **radiation dominated era.**

Explanation: varying $\omega_\perp$ suddenly

Their change was **sudden**: $\omega_\perp^2(t)/\omega_{\perp in}^2 \sim A + B \tanh(Ht)$ with $\boxed{H/\omega_\perp \sim 1}$.

It is thus **non-adiabatic w.r.t. the radial cloud dynamic** and induces **large oscillations** of the **"breathing"** mode.



The density $\rho(t, r=0)/\rho_0$ for various $H/\omega_{\perp in}$.

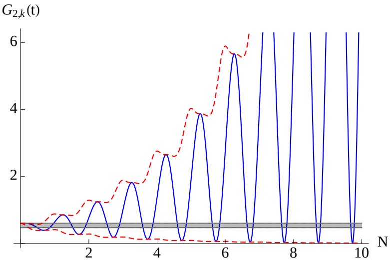
The yellow curve with $H/\omega_\perp = 1$ matches the exp.

The radial density oscillates with a frequency $= 2\omega_\perp$.

see Kagan *et al.* PRA 1996)

Hence one expects an **exponential amplification** of resonant modes with $\boxed{\omega_{res} = \omega_\perp}$.

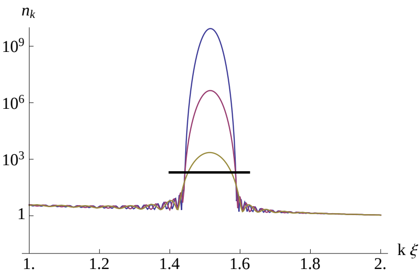
Predictions from BdG PRD 2017



Exponential growth of **resonant modes** with $\omega_{k_{res}} = \omega_\perp$.

They obey a Mathieu eq. driven by the coherent oscill. of the cloud.

NB. In the experiment $N_{\text{oscillations}} \sim 60$.



Phonon nb. $n_{k_{res}}$ for $N_{\text{osc.}} = 15, 30$, and 60, using experimental parameters.

Black line indicates when BdG is no longer valid: for $N_{\text{osc.}} \sim 10$, i.e. $\sim$ 5ms.

One must work beyond (mean field + BdG).

→ **Out of equilibrium physics**

Non-linear dynamics: 1. Longitudinal effects

Keeping the factorization ansatz:

$$\Psi(r, \theta, z, t) = \psi(r, t) \times \phi(z, t)\,,$$

longitudinal perturbations now obey a **non-linear 1D GPE**

$$\boxed{i\,\hbar\,\partial_t \phi = -\frac{\hbar^2}{2m}\,\partial_z^2 \phi + g_1(t)\left(|\phi|^2 - n_1\right)\phi}\,, \qquad (15)$$

where $\boxed{g_1(t) = g/2\pi\sigma(t)^2}$ as in BdG.

Eq. (15) describes a self-interacting 1D field propagating in an oscillatory background described by $\sigma(t)$.

2. Backreaction on radial oscillations

- Since the system is isolated, $\boxed{E_{Total} = E_{\rm rad}(t) + E_{\rm longit}(t)}$ is conserved.
- Hence the increase of $E_{\rm longit}(t)$ causes a reduction of the amplitude of radial (*inflaton*) oscillations.
- The **modified EoM of** $\sigma(t)$ is $\boxed{m\ddot{\sigma} = -\partial_\sigma V_{\rm eff}}$,

  where the effective radial potential is $V_{\rm eff}(\sigma, t) = V_0(\sigma) + V_{\rm longit.}(\sigma, t)$.
- We then self-consistently solved the coupled eqs. for $\phi(t, z)$ and $\sigma(t)$.
- NB. The **interesting effects are found in the longitudinal 1D physics** and not in the (expected) decrease of radial oscillations.

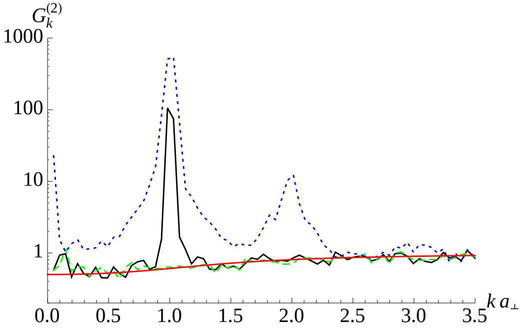
Initial conditions and TWA

The initial conditions are

- for the **radial profile** $\rho^{in}(r)$:
  - at $t < 0$, $\rho^{in}(r)$ describes the cloud at rest in the $\omega_{\perp in}$-trap,
  - at $t = 0^+$, the radial oscillations caused by the jump $\omega^2_{\perp fin}/\omega^2_{\perp in} = 2.$
- for the **longitudinal profile** $\phi^{in}(z)$
  - a homogeneous 1D condensate, and
  - a (homogeneous) phonon bath with initial temperature $T_{\rm in}$.
  - the latter is described a Gaussian set of random realizations, typically 100

    using the Truncated Wigner Approximation (TWA)

Ensemble averaged results: Early times, all k



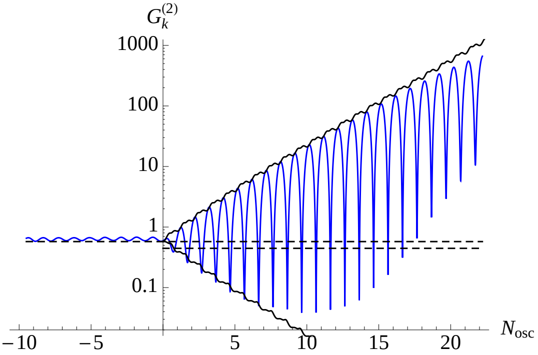
In situ $G_k^{(2)}$ after 0, 14, and 28 oscillations (in a log scale).

- At $N = 0$, the thermal $G_k^{(2)}$ with $T = mc^2/2$. (In red: BdG, in dashed green: TWA outcome)

- At $N = 14$, **only** a single narrow peak with $\boxed{\omega_{k_{res}} = \omega_\perp}$, **as in BdG.**

- At $N = 28$, **saturation of the growth** and several broadened peaks
due to non-linearities neglected in BdG

First deviations wrt to BdG for resonant modes



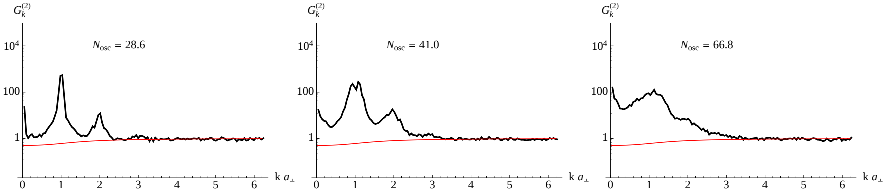
In situ $G^{(2)}_{k=k_{res}}(t)$ as function of N_{osc}, in black the corresp. BdG envelops

For $N \gtrsim 7$, first visible deviations wrt to BdG

The first non-trivial effect is **decoherence** of the $k, -k$ pairs,
i.e. a **loss of non-separability**, here near $N_{osc} = 17$.

NB. In Chris' experim. it should occur earlier.

Behavior of $G_k^{(2)}(t)$ at late times, all k.



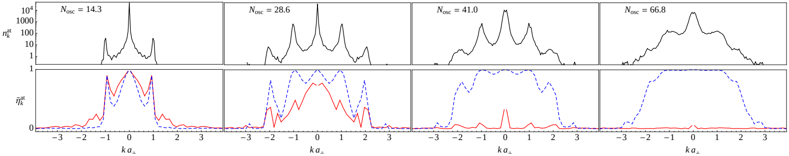
Behavior of $G_k^{(2)}$ for $N = 28.6, 41.0, 66.8$, in a log scale.

The **broadening of the peaks** at $k = 0$, k_{res} and $2k_{res}$ is manifest.

This is (probably) due to exchanges of many soft phonons, work in progress.

They reveal a trend towards thermal equilibrium at high $T \sim 100mc^2$, indicated by the late value of $G_{k\to 0}^{(2)}$.

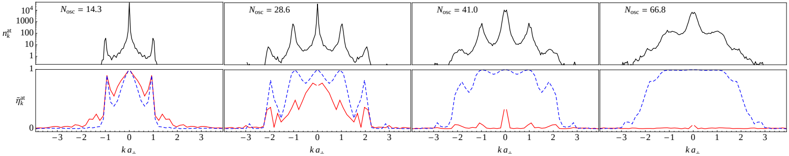
Atomic number and correlations, all k, late times



In **black**, the Log_{10} of n_k^{at} for $N = 14, 26, 41, 67$. ($N_{atoms} = 4.5\, 10^4$)

Harmonics in k_{res} are clearly visible, as the smoothing out of the **atomic** spectrum, including the reduction of the number of **condensed atoms** at $k = 0$.

Atomic number and correlations, all k, late times



In **black**, the Log_{10} of n_k^{at} for $N = 14, 26, 41, 67$. ($N_{atoms} = 4.5\,10^4$)

Harmonics in k_{res} are clearly visible, as the smoothing out of the **atomic** spectrum, including the reduction of the number of **condensed atoms** at $k = 0$.

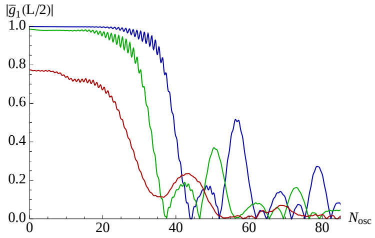
In red, the **atomic** coherence level $\eta_k \in [0, 1]$, where $\eta_k \equiv \frac{|c_k|^2}{(n_k+1/2)^2}$

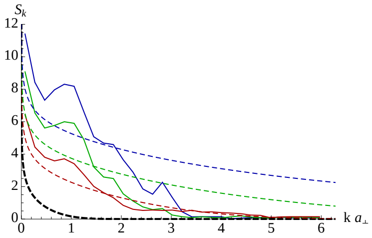
In dashed blue, the **non-separability threshold** $\eta_{k,\text{threshold}} \equiv \frac{|n_k|^2}{(n_k+1/2)^2}$

The early left plot shows atomic non-separability, since $\eta_k > \eta_{k,\text{threshold}}$

The **last plot** shows a loss of (classical) coherence, **in agreement with Chris' observations**

Spatial coherence and entropy





Left: **atomic 1-body** correlation function $g_1(L/2) \equiv \langle \hat{\phi}^\dagger(0)\hat{\phi}(L/2)\rangle$ as a fction of N_{osc} for 3 values of $a_s/a_\perp$: $1.7 \times (10^{-2}, 10^{-3}$, and $10^{-4})$

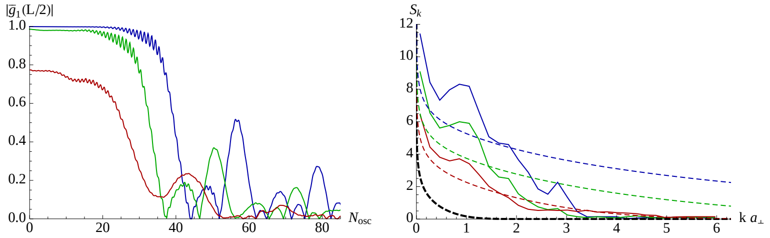
NB. $g_1(L/2) = 1$, i.e. BdG, is reached in the limit $a_s/a_\perp \to 0$, $n_1 a_s = 0.6$ fixed.

One sees a sudden drop signaling a **loss of coherence of the order parameter**, as 'expected' for *hot quasi-cond.* whose coher. length is $r_0(T)/\xi = 2(n_1\xi)\,(mc^2/T)$.

Right, after 67 oscillations, **entropy** $S_{cov}(k)$ (i.e., based on the knowledge of the cov. matrix) in dashed its value in a thermal state having the same energy.

NB. For the lowest value of $a_s/a_\perp$ (blue), high k modes have not yet thermalized. Hence slow process, as found in the cosmological (p)re-heating.

Spatial coherence and entropy



Left: **atomic 1-body** correlation function $g_1(L/2) \equiv \langle \hat{\phi}^\dagger(0)\hat{\phi}(L/2)\rangle$ as a fction of N_{osc} for 3 values of $a_s/a_\perp$: $1.7 \times (10^{-2}, 10^{-3}$, and $10^{-4})$

NB. $g_1(L/2) = 1$, i.e. BdG, is reached in the limit $a_s/a_\perp \to 0$, $n_1 a_s = 0.6$ fixed.

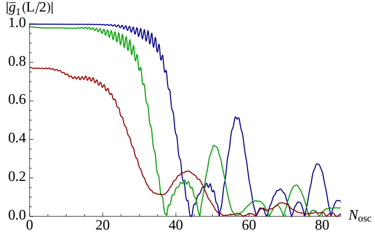
One sees a sudden drop signaling a **loss of coherence of the order parameter**, as 'expected' for *hot quasi-cond.* whose coher. length is $r_0(T)/\xi = 2(n_1\xi)\,(mc^2/T)$.

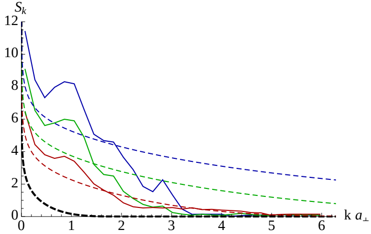
Right, after 67 oscillations, **entropy** $S_{\rm cov}(k)$ (i.e., based on the knowledge of the cov. matrix)

in dashed its value in a thermal state having the same energy.

NB. For the lowest value of $a_s/a_\perp$ (blue), high k modes have not yet thermalized. Hence slow process, as found in the cosmological (p)re-heating.

Spatial coherence and entropy





Left: **atomic 1-body** correlation function $g_1(L/2) \equiv \langle \hat{\phi}^\dagger(0)\hat{\phi}(L/2)\rangle$ as a fction of N_{osc} for 3 values of $a_s/a_\perp$: $1.7 \times (10^{-2}, 10^{-3}$, and $10^{-4})$

NB. $g_1(L/2) = 1$, i.e. BdG, is reached in the limit $a_s/a_\perp \to 0$, $n_1 a_s = 0.6$ fixed.

One sees a sudden drop signaling a **loss of coherence of the order parameter**, as 'expected' for *hot quasi-cond.* whose coher. length is $r_0(T)/\xi = 2(n_1\xi)(mc^2/T)$.

Right, after 67 oscillations, **entropy** $S_{\rm cov}(k)$ (i.e., based on the knowledge of the cov. matrix) in dashed its value in a thermal state having the same energy.

NB. For the lowest value of $a_s/a_\perp$ (blue), high k modes have not yet thermalized. Hence slow process, as found in the cosmological (p)re-heating.

Conclusions

- In both Chris' experim., the entanglement is much below that predicted by BdG.
- in the 2^d modulated one with $\omega_{mod} \sim (2\omega_\perp)/3$, this could be due to
  - the finite life time of phonons in 1D quasi-condensate,
  - the efficiency of their detector.

  For future: Non-separability should be most visible after $\lesssim 5$ oscillations

- In the 1^{st} experim. the sudden change of $\omega_\perp$ causes large oscillations, which exponentially amplify resonant modes with $\omega = \omega_\perp$.

  Non-linearities cause first
  - a loss of 2-mode entanglement,

    this is a **small deviation** wrt to BdG

  then,
  - a saturation of the exponential growth,
  - broadening of resonant and harmonics peaks,
  - significant increase of entropy S_{cov}.

    these are all manifestations of **out of equilibrium physics**

  Finally, signs of slow thermalization.

  This sequence of events seems generic properties of the (p)re-heating scenario

  The whole sequence should be observable in future experiments both *in situ* and after TOF.

Conclusions

- In both Chris' experim., the entanglement is much below that predicted by BdG.
- in the 2^d modulated one with $\omega_{mod} \sim (2\omega_\perp)/3$, this could be due to
  - the finite life time of phonons in 1D quasi-condensate,
  - the efficiency of their detector.

  For future: Non-separability should be most visible after $\lesssim 5$ *oscillations*

- In the 1^{st} experim. the sudden change of $\omega_\perp$ causes large oscillations, which exponentially amplify resonant modes with $\omega = \omega_\perp$.

  Non-linearities cause first
  - a loss of 2-mode entanglement,

    this is a **small deviation** wrt to BdG

  then,
  - a saturation of the exponential growth,
  - broadening of resonant and harmonics peaks,
  - significant increase of entropy $S_{\rm cov}$.

    these are all manifestations of **out of equilibrium physics**

  Finally, signs of slow thermalization.

  This sequence of events seems generic properties of the (p)re-heating scenario

  The whole sequence should be observable in future experiments both *in situ* and after TOF.

Conclusions

- In both Chris' experim., the entanglement is much below that predicted by BdG.

- in the 2^d modulated one with $\omega_{mod} \sim (2\omega_\perp)/3$, this could be due to
  - the finite life time of phonons in 1D quasi-condensate,
  - the efficiency of their detector.

  For future: Non-separability should be most visible after $\lesssim$ 5 oscillations

- In the 1^{st} experim. the sudden change of $\omega_\perp$ causes large oscillations, which exponentially amplify resonant modes with $\omega = \omega_\perp$.

  Non-linearities cause first
  - a loss of 2-mode entanglement,
    this is a **small deviation** wrt to BdG

  then,
  - a saturation of the exponential growth,
  - broadening of resonant and harmonics peaks,
  - significant increase of entropy S_{cov}.
    these are all manifestations of **out of equilibrium physics**

  Finally, signs of slow thermalization.

  This sequence of events seems generic properties of the (p)re-heating scenario

  The whole sequence should be observable in future experiments both *in situ* and after TOF.

Conclusions

- In both Chris' experim., the entanglement is much below that predicted by BdG.

- in the 2^d modulated one with $\omega_{mod} \sim (2\omega_\perp)/3$, this could be due to
  - the finite life time of phonons in 1D quasi-condensate,
  - the efficiency of their detector.

  For future: Non-separability should be most visible after $\lesssim 5$ oscillations

- In the 1^{st} experim. the sudden change of $\omega_\perp$ causes large oscillations, which exponentially amplify resonant modes with $\omega = \omega_\perp$.

  Non-linearities cause first
  - a loss of 2-mode entanglement,

    this is a **small deviation** wrt to BdG

  then,
  - a saturation of the exponential growth,
  - broadening of resonant and harmonics peaks,
  - significant increase of entropy $S_{\rm cov}$.

    these are all manifestations of **out of equilibrium physics**

  Finally, signs of slow thermalization.

  This sequence of events seems generic properties of the (p)re-heating scenario

  The whole sequence should be observable in future experiments both *in situ* and after TOF.

Conclusions

- In both Chris' experim., the entanglement is much below that predicted by BdG.

- in the 2^d modulated one with $\omega_{mod} \sim (2\omega_\perp)/3$, this could be due to
  - the finite life time of phonons in 1D quasi-condensate,
  - the efficiency of their detector.

  For future: Non-separability should be most visible after $\lesssim 5$ oscillations

- In the 1^{st} experim. the sudden change of $\omega_\perp$ causes large oscillations, which exponentially amplify resonant modes with $\omega = \omega_\perp$.

  Non-linearities cause first
  - a loss of 2-mode entanglement,

    this is a **small deviation** wrt to BdG

  then,
  - a saturation of the exponential growth,
  - broadening of resonant and harmonics peaks,
  - significant increase of entropy $S_{\rm cov}$.

    these are all manifestations of **out of equilibrium physics**

  Finally, signs of slow thermalization.

  This sequence of events seems generic properties of the (p)re-heating scenario

  The whole sequence should be observable in future experiments both *in situ* and after TOF.