Effective friction induced by the dynamical Casimir emission and its fluctuations

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OUTLINE

- **Back-reaction problem**

- **Dynamical Casimir effect**

- **Remarks**

$$g_{\mu\nu}(x)$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}(g, \phi)$$

Spacetime

$$\frac{1}{\sqrt{-g(x)}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g(x)}\, g^{\mu\nu}(x) \frac{\partial \hat{\psi}}{\partial x^\nu} \right) = 0$$

Quantum field

Quantum Field Theory in Curved Spacetime Hawking, Parker, Fulling, Birrel, Davies, Wald, Toms, …

$$g_{\mu\nu}(x)$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^c_{\mu\nu}(\phi) + \langle\Psi|\,\hat{T}_{\mu\nu}(\hat{\psi})\,|\Psi\rangle\right)$$

$$\frac{1}{\sqrt{-g(x)}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g(x)}\,g^{\mu\nu}(x)\frac{\partial\hat{\psi}}{\partial x^\nu}\right) = 0$$

Spacetime

Quantum field

Quantum Field Theory in Curved Spacetime   Hawking, Parker, Fulling, Birrel, Davies, Wald, Toms, …

Semi-classical Gravity   Parker *69*, *71*, Zeldovich & Starobinsky *72*, Parker & Fulling *73*, Hu & Parker *78*, Fischetti Hartle & Hu *79*, Hartle & Hu *79*, *80*, Hartle *80*, …

$\langle\Psi|\,\hat{T}_{\mu\nu}(\hat{\psi})\,|\Psi\rangle$ is the meanfield (classical) component of the stress tensor of the quantum field

$$g_{\mu\nu}(x)$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}\left(T^c_{\mu\nu}(\phi) + \langle\Psi|\,\hat{T}_{\mu\nu}(\hat{\psi})\,|\Psi\rangle + T^s_{\mu\nu}(\hat{\psi})\right)$$

Spacetime

$$\frac{1}{\sqrt{-g(x)}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g(x)}\, g^{\mu\nu}(x)\frac{\partial\hat{\psi}}{\partial x^\nu}\right) = 0$$

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$\langle\Psi|\,\hat{T}_{\mu\nu}(\hat{\psi})\,|\Psi\rangle$ is the meanfield (classical) component of the stress tensor

Stochastic Gravity Calzetta & Hu *94*, Hu & Sinha *95*, Hu & Matacz *95*, Hu Roura & Verdauer *04*, …
(See also Hu & Verdaguer, *Liv. Rev. Rel. 08* ; Hu & Verdaguer, *Class. Quant. Grav. 03*)

$T^s_{\mu\nu}(\hat{\psi})$ is a stochastic term representing the (quantum) fluctuations of the stress tensor

Microscopic description

Quantum Gravity

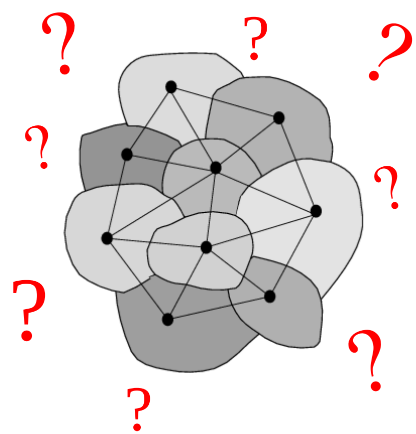


Figure: Adapted from C. Rovelli, *CQG* (2011)

Microscopic description

Analog models of gravity

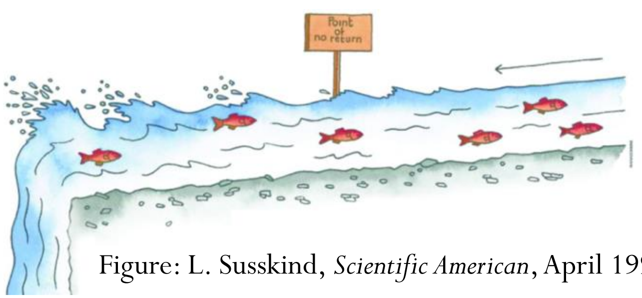


Figure: L. Susskind, *Scientific American*, April 1997

(Effective) curved spacetime

Excitations (phonons)

Dynamical Casimir effect

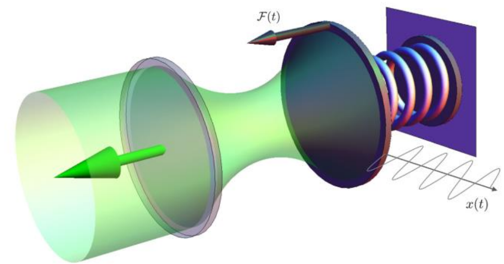
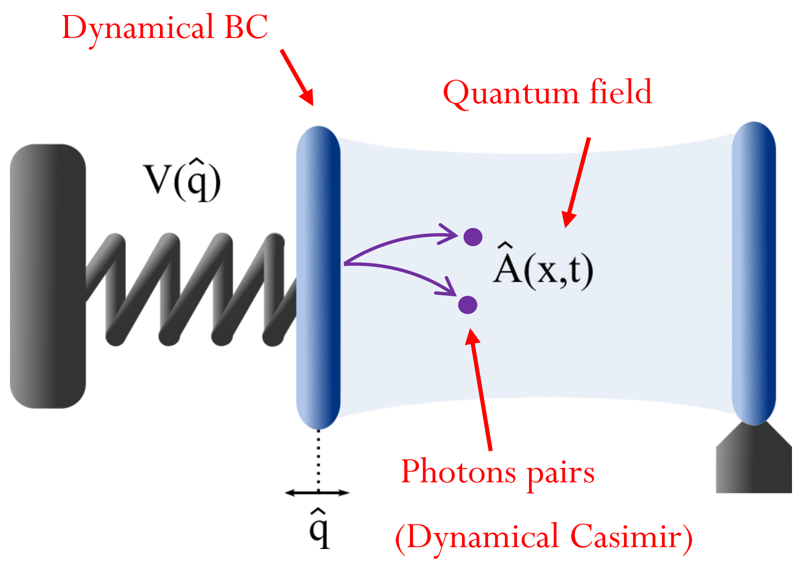


Figure: V. Macrì et al., *PRX* (2018)

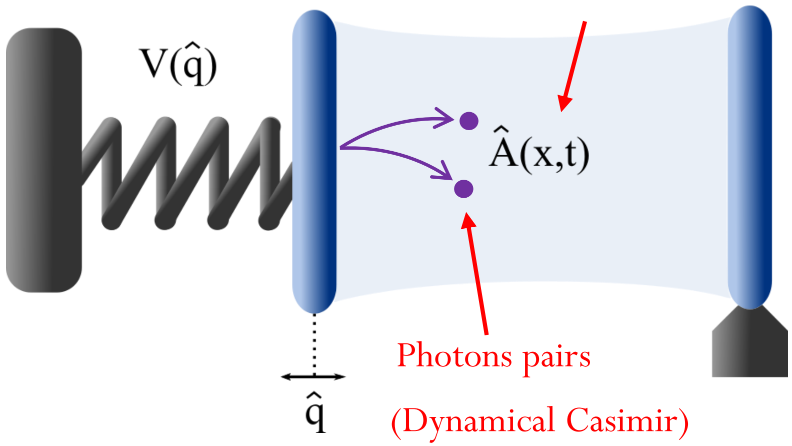
Boundary conditions

Photons

(Dynamical) boundary condition as nontrivial spacetime



(Dynamical) boundary condition as nontrivial spacetime



Mirror

$q(t)$

Field

$$m\ddot{q} = -\frac{\partial V(q)}{\partial q} + \frac{1}{2}\left(\frac{\partial A(x,t)}{\partial t}\right)^2_{x=q(t)}$$

$$\frac{\partial^2 A(x,t)}{\partial x^2} = \frac{\partial^2 A(x,t)}{\partial t^2}$$

$$A(0,t) = A(q(t),t) = 0$$

Radiation pressure

Hamiltonian

Free mirror-field dynamics

$$\hat{H} = \boxed{\hat{H}_0} + \boxed{\hat{H}_{\mathrm{DCE}}} + \boxed{\hat{H}_{\mathrm{drive}}}$$

External drive on the mirror

Radiation pressure interaction

Assumptions

mirror field

- Single cavity mode → $\hat{H}_0 = \hbar\omega_b \hat{b}^\dagger \hat{b} + \hbar\omega_a \hat{a}^\dagger \hat{a}$
- Rotating wave appx. for the mirror-field interaction → $\hat{H}_{\mathrm{DCE}} = \hbar\omega_c \left(\hat{a}^2 \hat{b}^\dagger + \left(\hat{a}^\dagger\right)^2 \hat{b}\right)$
- Coherent drive → $\hat{H}_{\mathrm{drive}} = -\hbar\left(\hat{b}F^*(t) + \hat{b}^\dagger F(t)\right)$

Density matrix

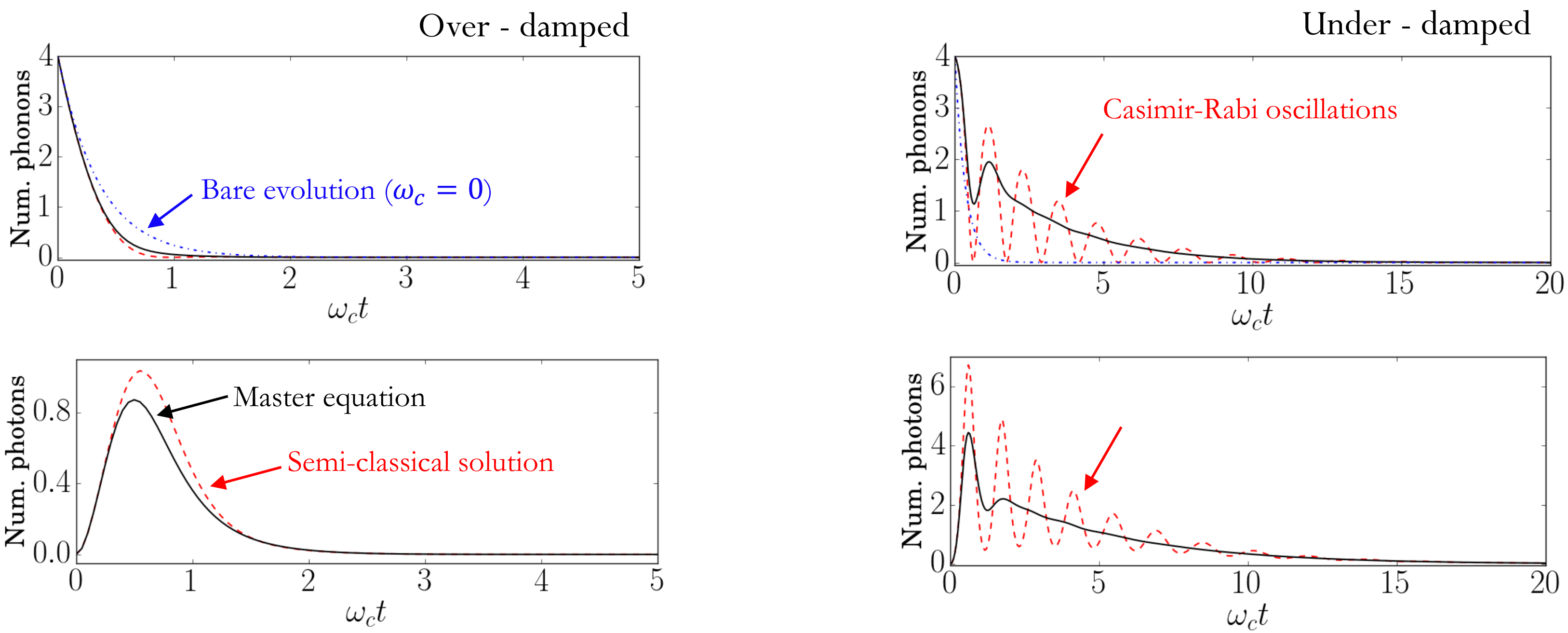
- Master equation (losses) → $\dfrac{d\hat{\rho}}{dt} = \dfrac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right] + \mathcal{L}_{\hat{a}}[\hat{\rho}] + \mathcal{L}_{\hat{b}}[\hat{\rho}]$

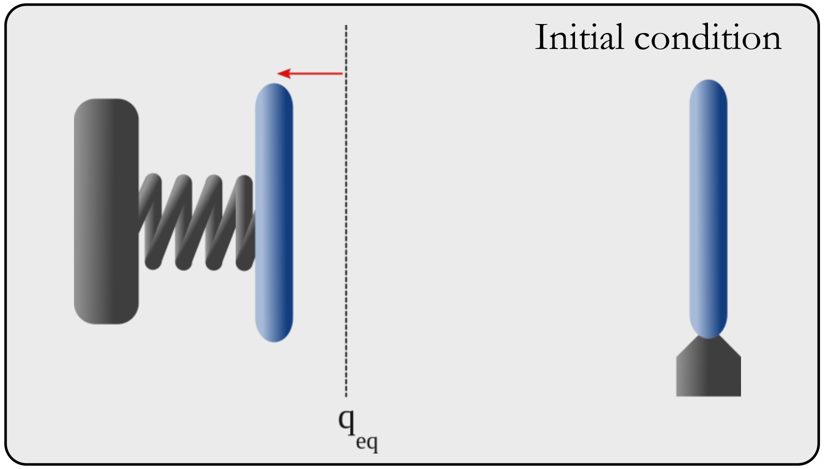
Lindblad super-operators

$$\mathcal{L}_{\hat{o}} \equiv (\gamma_o/2)\left(2\hat{o}\hat{\rho}\hat{o}^\dagger - \hat{o}^\dagger\hat{o}\hat{\rho} - \hat{\rho}\hat{o}^\dagger\hat{o}\right)$$

SB, I. Carusotto, PRA 99 (2019)

- Resonant mirror-field coupling: $\Delta \equiv \omega_b - 2\omega_a = 0$
- Symmetric damping: $\gamma_a = \gamma_b$





SB, I. Carusotto, PRA 99 (2019)

$$b \equiv \langle \hat{b} \rangle : \quad \frac{db}{dt} = -i\omega_c q, \longleftarrow \text{Note that } \gamma_b = 0$$

$$n_a \equiv \langle \hat{n}_a \rangle : \quad \frac{dn_a}{dt} = -\gamma_a n_a - 2i\omega_c \langle q^\dagger b \rangle + 2i\omega_c \langle q b^\dagger \rangle,$$

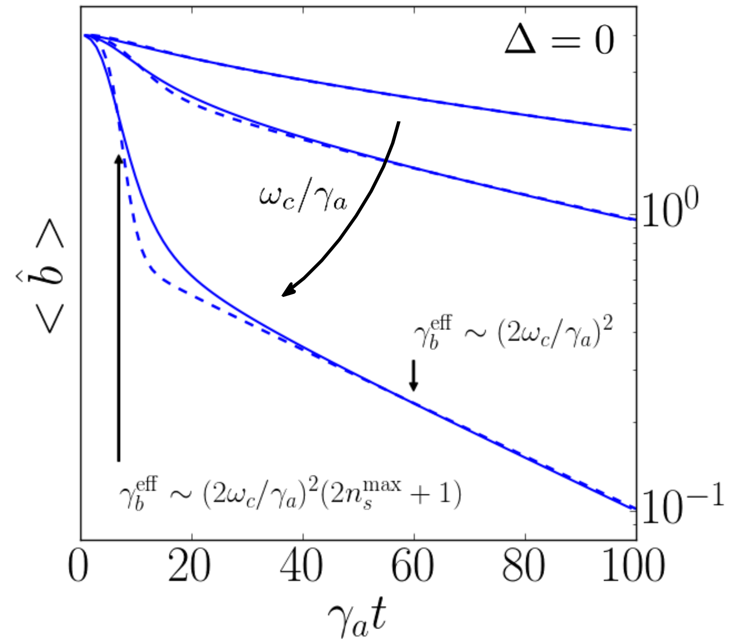
$$q \equiv \langle \hat{a}^2 \rangle : \quad \frac{dq}{dt} = i(\Delta + i\gamma_a) q - 4i\omega_c \langle n_a b \rangle - 2i\omega_c b.$$

- Semi-classical limit: $\omega_c/\gamma_a \ll 1 \longrightarrow \langle \hat{q}\hat{b}^\dagger \rangle \approx q b^* \quad \langle \hat{n}_a \hat{b}^\dagger \rangle \approx n_a b^*$

  Coupled non-linear set of equations

- Born-Oppenheimer : $\tau_b \gg \tau_a$ Uncoupled set of equations

  Mirror time scale Field time scale



Effective mirror dynamics

$$*** \quad \frac{db}{dt} = -\frac{\Gamma_b(n_a)}{2} b \quad ***$$

Effective friction

Lamb shift

$$\Gamma_b(n_a) = \frac{4\omega_c^2 \gamma_a}{\gamma_a^2 + \Delta^2}\left(1 + i\frac{\Delta}{\gamma_a}\right)(2n_{a,SS} + 1)$$

Steady-state number of photons

SB, I. Carusotto, PRA 99 (2019)

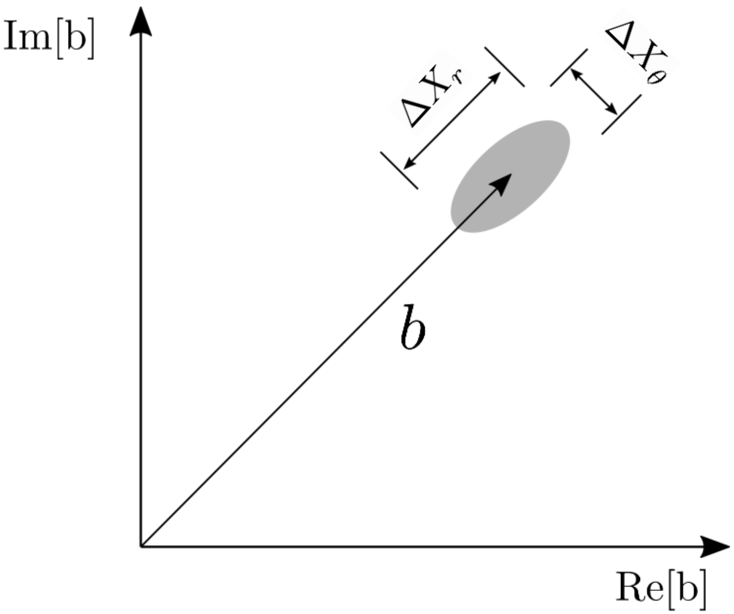
At the **meanfield** level, the back-reaction appears as an **effective friction** acting on the mirror, due to the particle creation by DCE

What are the **strictly quantum** features of this friction? ⟶ Go beyond meanfield

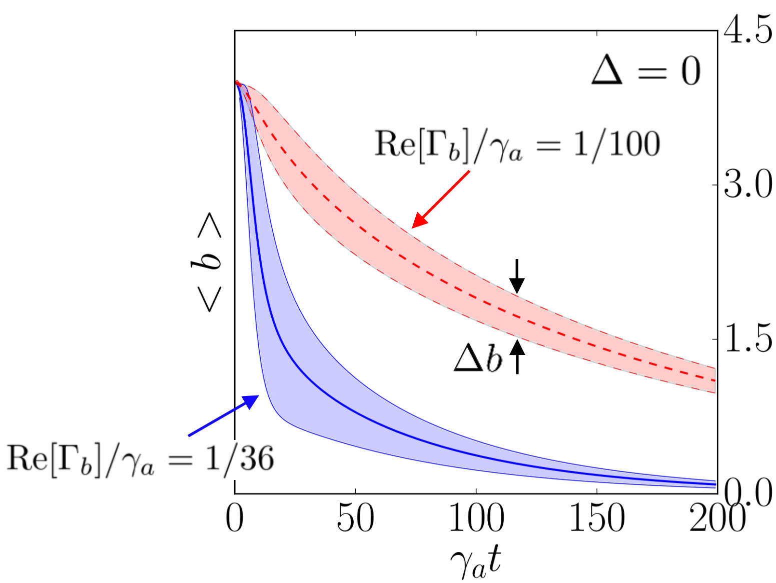
Fluctuation of the mirror (boundary condition)

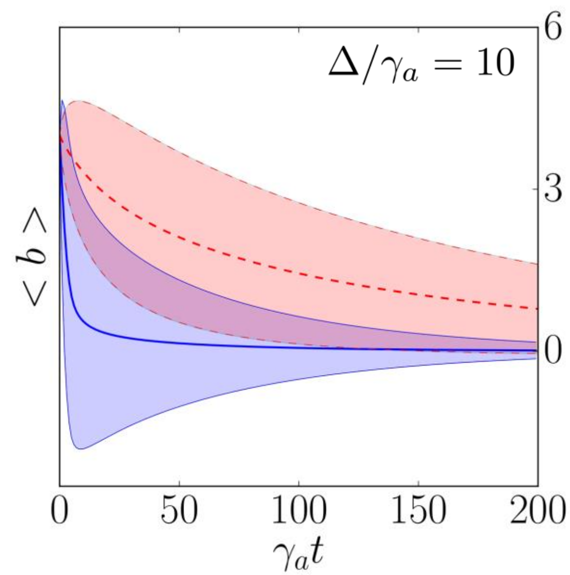
⬍

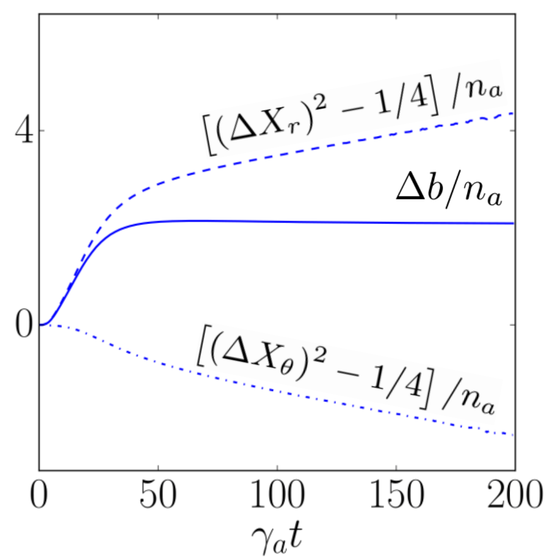
Fluctuation of spacetime (bulk)

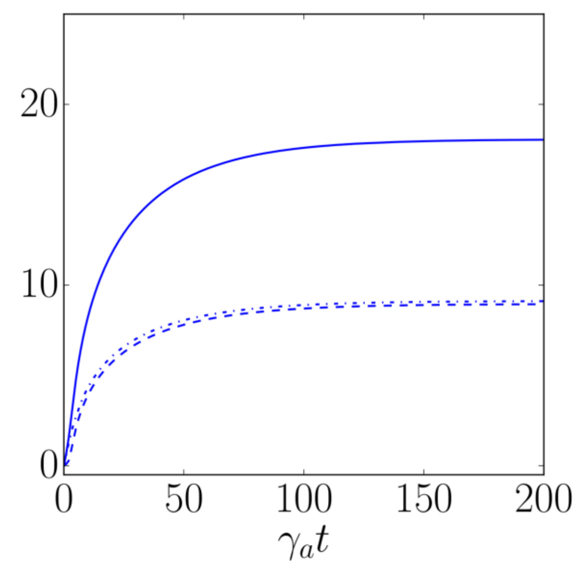


BACK-REACTION FROM DCE — QUANTUM FLUCTUTATIONS



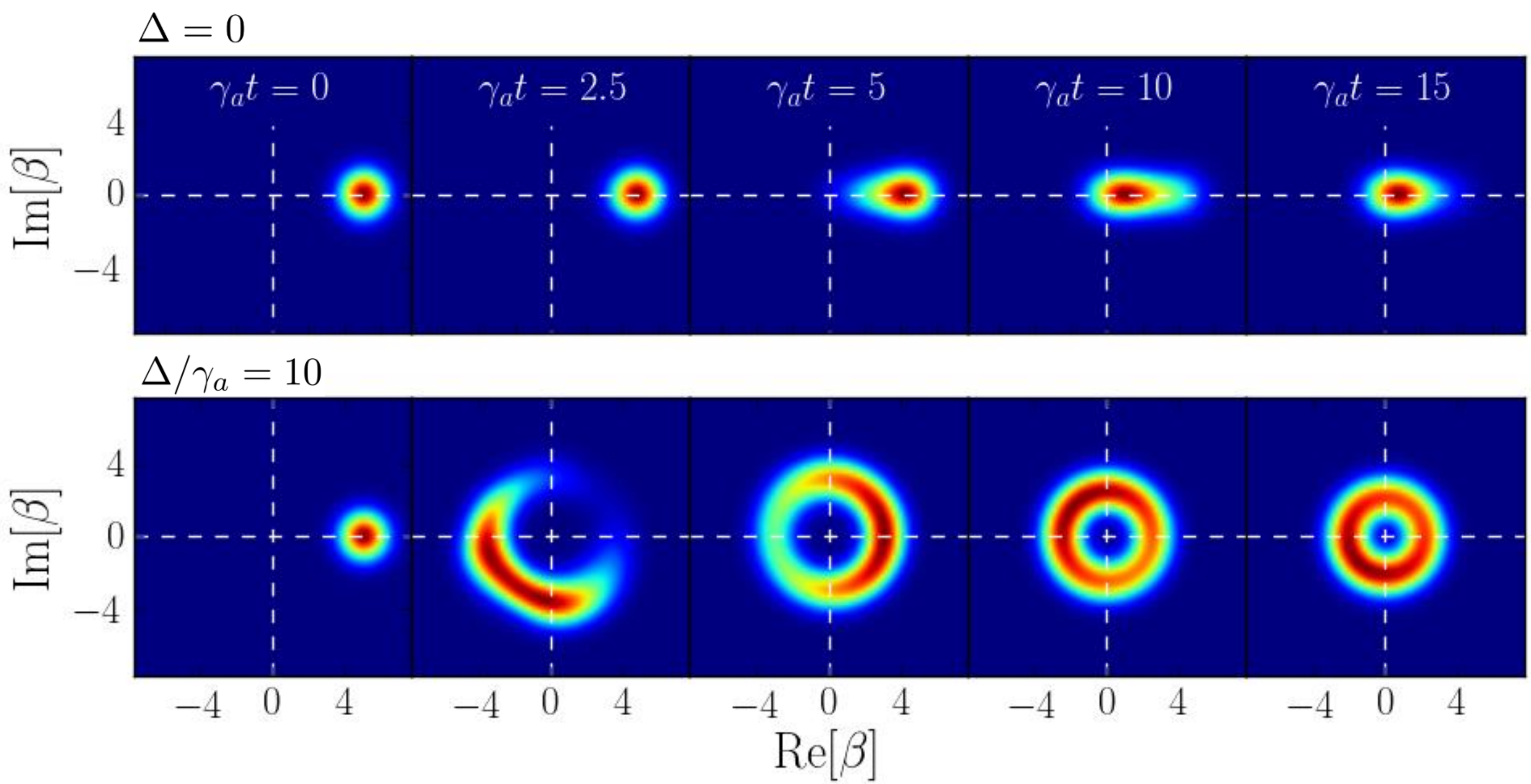




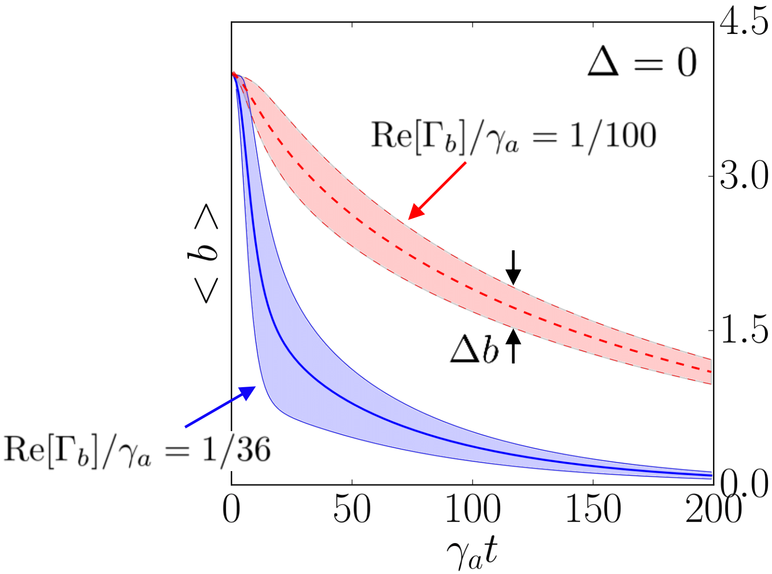


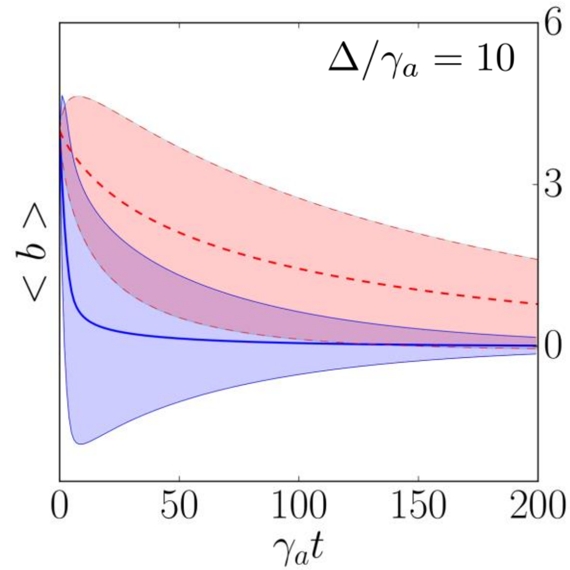
SB, I. Carusotto, in preparation

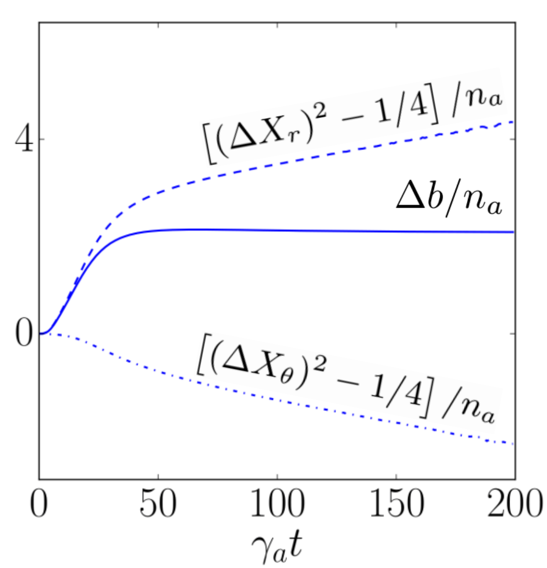
$$Q(\beta) \equiv \frac{1}{\pi} \langle\beta|\,\hat{\rho}\,|\beta\rangle$$

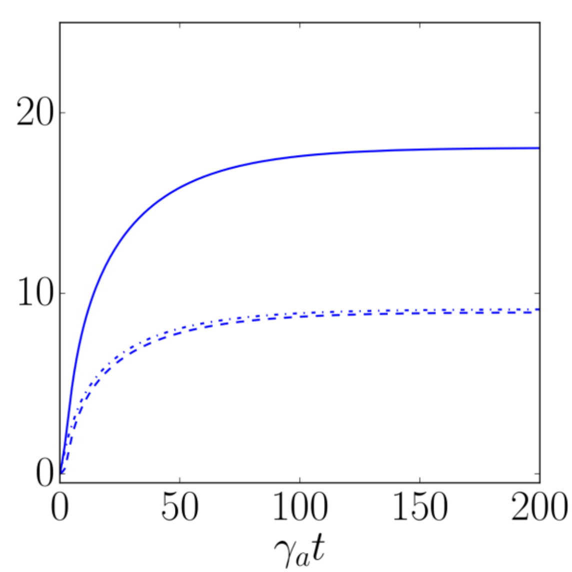


SB, I. Carusotto, in preparation









SB, I. Carusotto, in preparation

Single cavity mode $\longrightarrow$ Multi-mode

Optomechanics:

Decay of mechanical motion into photons

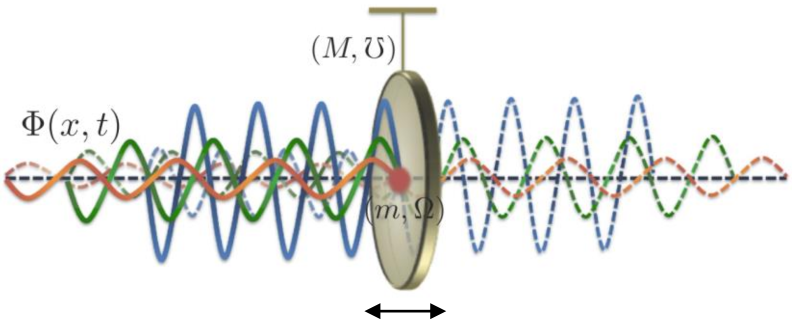


Figure: K. Sinha et al., PRA 2015

Pre-heating:

Decay of the **inflaton** field into matter particles

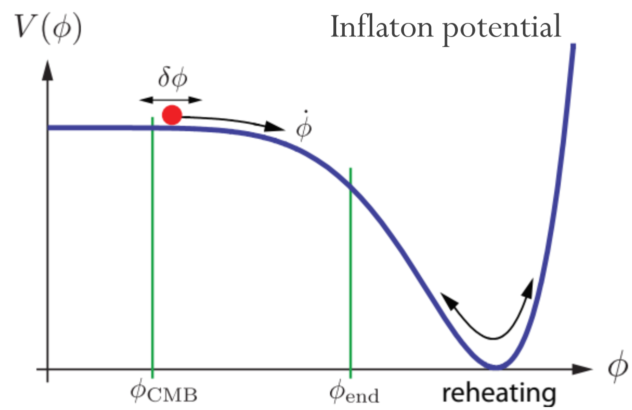


Figure: D. Baumann, The Physics of Inflation

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Thank You!