Black holes with unusual horizons

Silke Klemm
Dipartimento di Fisica, Università di Milano,
& INFN, Sezione di Milano

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Plan:

-Overview on possible horizon topologies

-Noncompact horizons with finite volume

-Black holes with Bianchi horizons in 5 dimensions

-Multi-centered solutions in AdS

-Final remarks

I) Possible horizon topologies

- Hawking 1972: Event horizon cross sections of four-dimensional asymptotically flat stationary black holes obeying the dominant energy condition are topologically two-spheres

This result extends to outer apparent horizons in black hole spacetimes that are not necessarily stationary (Hawking 1972)

Such restrictive uniqueness theorems do not hold in higher dimensions, the most famous counterexample being the black ring of Emparan and Reall, with horizon topology $\mathrm{S}^2 \times \mathrm{S}^1$ (Emparan/Reall 2001)

But: In arbitrary dimensions, horizon cross sections are of positive Yamabe type, i.e., admit metrics of positive scalar curvature (Galloway/Schoen 2005)

In 4 dimensions, one can have black holes with nonspherical horizons by relaxing some of the assumptions that go into Hawking's theorem.

For instance, in asymptotically AdS space, the horizon of a black hole can be a compact Riemann surface of any genus (Lemos 1994, Mann 1996, Cai/Zhang 1996, Vanzo 1997)

In this case, both the asymptotically flat and dominant energy conditions are violated.

Note: Unless genus = 0, these spacetimes are asymptotically only locally AdS; their global structure is different. This is in contrast to the black rings in 5d, which are asymptotically Minkowski, in spite of their nontrivial horizon topology.

These possibilities do not exhaust the spectrum of potential horizon geometries of asymptotically AdS_4 black holes.

In particular, we will see that there exist black holes whose event horizons are noncompact manifolds with yet finite area (and thus finite entropy), which are topologically spheres with two punctures.

(Gnecchi/Hristov/SK/Toldo/Vaughan, 1311.1795; SK, 1401.3107)

II) Noncompact horizons with finite volume

-consider D=4 Einstein-Maxwell-Lambda system:

Carter-Plebanski solution:

$$ds^2 = -\frac{Q(q)}{p^2+q^2}(d\tau - p^2 d\sigma)^2 + \frac{P(p)}{p^2+q^2}(d\tau + q^2 d\sigma)^2 + (p^2+q^2)\left(\frac{dq^2}{Q(q)} + \frac{dp^2}{P(p)}\right),$$

$$F = \frac{\mathsf{Q}(p^2-q^2)+2\mathsf{P}pq}{(p^2+q^2)^2}dq\wedge(d\tau - p^2 d\sigma) + \frac{\mathsf{P}(p^2-q^2)-2\mathsf{Q}pq}{(p^2+q^2)^2}dp\wedge(d\tau + q^2 d\sigma),$$

w/ the quartic structure functions

$$\begin{aligned} P(p) &= \alpha - \mathsf{P}^2 + 2np - \varepsilon p^2 + (-\Lambda/3)p^4 , \\ Q(q) &= \alpha + \mathsf{Q}^2 - 2mq + \varepsilon q^2 + (-\Lambda/3)q^4 . \end{aligned}$$

P, Q : electric and magnetic charges. In what follows: $\mathsf{P} = 0$

m, n : mass and NUT charge. Take $n = 0$

α and ε: additional non-dynamical parameters

$\Lambda = -3/l^2$: Cosmological constant

Physical discussion:

q radial coordinate, horizon at $q = q_{\mathrm{h}}$, where $Q(q_{\mathrm{h}}) = 0$

-Induced metric on horizon has correct signature iff $P(p) \geq 0$

-Since $n = 0$, $P(p)$ has roots $\pm p_a$, $\pm p_b$, where $0 < p_a < p_b$

-Then $P(p) \geq 0$ for $|p| \leq p_a$ or $|p| \geq p_b$. Consider range $|p| \leq p_a$

(Range $|p| \geq p_b$ leads to different horizon topology)

-Set $p = p_a \cos\theta, \quad 0 \leq \theta \leq \pi$

-Use scaling symmetry

$$p \to \lambda p\,, \quad q \to \lambda q\,, \quad \tau \to \tau/\lambda\,, \quad \sigma \to \sigma/\lambda^3\,,$$

$$\alpha \to \lambda^4 \alpha\,, \quad \mathsf{Q} \to \lambda^2 \mathsf{Q}\,, \quad m \to \lambda^3 m\,, \quad \varepsilon \to \lambda^2 \varepsilon\,,$$

to set $p_b = l$ without loss of generality

-Define rotation parameter j by $p_a^2 = j^2$

Def. also $\tau =: t + \frac{j\phi}{\Xi}\,, \quad \sigma =: \frac{\phi}{j\Xi}\,, \quad \Xi := 1 - \frac{j^2}{l^2}\,, \quad \Delta_\theta := 1 - \frac{j^2}{l^2}\cos^2\theta$

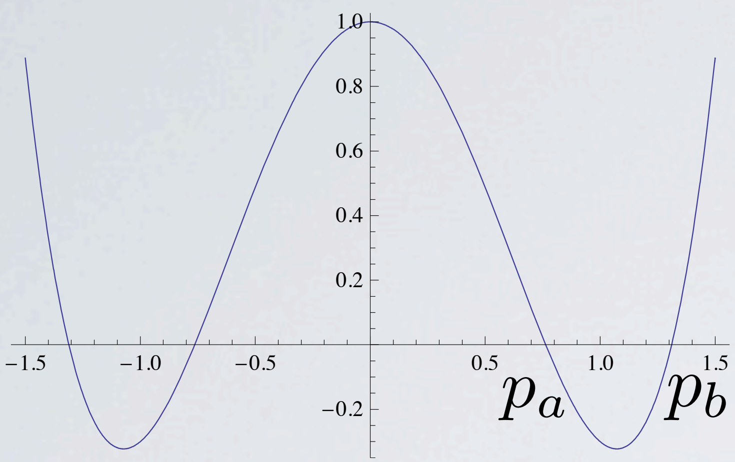
Then:

$$ds^2 = -\frac{Q(q)}{(q^2 + j^2\cos^2\theta)^2}\left[dt + \frac{j\sin^2\theta}{\Xi}d\phi\right]^2 + (q^2 + j^2\cos^2\theta)\left(\frac{dq^2}{Q(q)} + \frac{d\theta^2}{\Delta_\theta}\right)$$
$$+ \frac{\Delta_\theta\sin^2\theta}{q^2 + j^2\cos^2\theta}\left[j dt + \frac{q^2 + j^2}{\Xi}d\phi\right]^2\,,$$

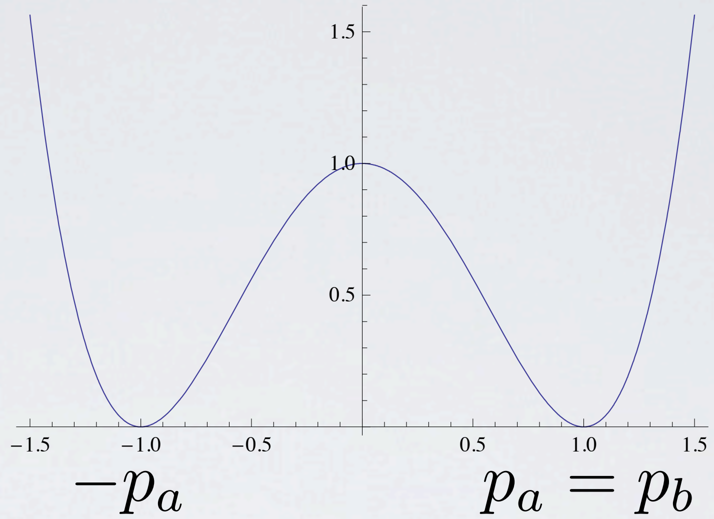
+ some expression for the gauge pot. A

$\Rightarrow$ gives Kerr-Newman-AdS

- Now consider case of coinciding roots of $P(p)$, $p_a = p_b$:



$\rightarrow$



Note: Since above we had $p_a = j$, $p_b = l$, this can be considered the ultraspinning limit $j = l$ of the solution that we had before!

- Can show: Conformal boundary (which is rotating Einstein universe) rotates at the speed of light in this limit

Of course the solution above is singular in this limit

$\Rightarrow$ Have to study this case separately

$\Rightarrow$ Consider region $|p| \leq p_a$ and use scaling symmetry to set $p_a = l$

$\Rightarrow P(p) = \frac{1}{l^2}(p^2 - l^2)^2$

- Shift $\tau \to \tau + l^2\sigma$ to avoid CTCs (we want σ to be compact coordinate)

$\Rightarrow$ Induced metric on horizon $q = q_{\rm h}$ (where $Q(q)$ vanishes):

$$ds_{\rm h}^2 = \frac{P(p)}{q_{\rm h}^2 + p^2}(q_{\rm h}^2 + l^2)^2 d\sigma^2 + \frac{q_{\rm h}^2 + p^2}{P(p)} dp^2$$

$\rightarrow$ Singular for $p = \pm l$ (where $P(p) = 0$)

What happens at these singularities?

Take limit $p \to l$ and define $\rho \equiv l - p$. Then:

$$ds_{\rm h}^2 \to (q_{\rm h}^2 + l^2)\left[\frac{d\rho^2}{4\rho^2} + 4\rho^2 d\sigma^2\right] \qquad \Rightarrow \text{Hyperbolic space } \mathrm{H}^2\,!$$

$\Rightarrow$ For $p \to \pm l$, horizon approaches a space of constant negative curvature

$\Rightarrow$ No true singularity there!

$\Rightarrow$ Noncompact horizon!

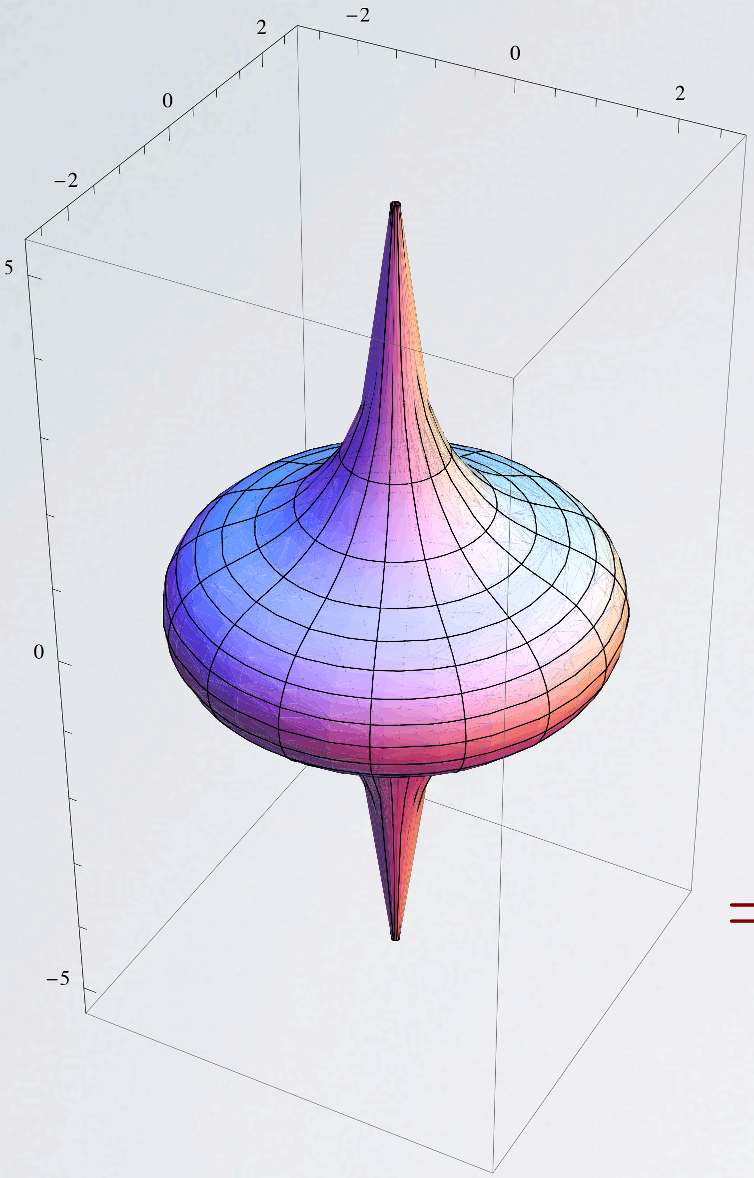
This is a surprise, since one might have expected the limit of coincident roots $p_a = p_b$ to be smooth, and for $p_a \neq p_b$ horizon was topologically S^2

- Horizon area:

$$A_{\rm h} = \int (q_{\rm h}^2 + l^2) d\sigma dp = 2Ll(q_{\rm h}^2 + l^2) \qquad (\sigma \sim \sigma + L)$$

$\Rightarrow$ Though being noncompact, the event horizon has finite area!

- Embed horizon in $\mathbb{R}^3$ as a surface of revolution:



- Metric on conformal boundary:

$$ds^2_{\text{bdry}} = -d\tau^2 + 2d\tau d\sigma(p^2 - l^2) + l^2\frac{dp^2}{P(p)}$$

- Is conformally flat (Cotton tensor vanishes)

- Near $p = l$: $(\rho \equiv l - p)$

$$ds^2_{\text{bdry}} \to -(d\tau + 2l\rho d\sigma)^2 + l^2\left[\frac{d\rho^2}{4\rho^2} + 4\rho^2 d\sigma^2\right]$$

$\Rightarrow$ AdS_3, written as a Hopf-like fibration over H^2

- Thermodynamics:

Compute M and J as Komar integrals associated to Killing vectors ∂_τ and ∂_σ $\Rightarrow$ Chirality-type condition $M = -J/l^2$

$\Rightarrow$ Suggests that these exotic black holes are described by chiral excitations of a CFT

- In this case more convenient to use $L_0 = (M + J/l^2)/2$, $\tilde{L}_0 = (M - J/l^2)/2$ instead of M, J

  $\Rightarrow$ First law should be

$$TdS = (1 - \Omega l^2) dL_0 + (1 + \Omega l^2) d\tilde{L}_0 - \phi_{\mathrm{el}} d\mathsf{Q}$$

  (Ω: Ang. velocity of horizon, ϕ_{el}: electric potential)

- One finds that 1st law is indeed satisfied with $L_0 = 0$

$\Rightarrow$ These exotic solutions may provide interesting new testgrounds to address questions related to black hole physics or holography

Comments:

The ‘punctures’ cannot be reached by null geodesics emanating from the bulk in a finite affine parameter (Hennigar et al. 1504.07529)

These solutions can be generalized to matter-coupled gauged sugra (Gnecchi et al. 1311.1795) and to higher dimensions (Hennigar et al. 1411.4309, 1504.07529)

Solution can also be obtained from KNAdS by
i) transforming to coordinate system rotating at infinity
ii) boosting this rotation to the speed of light
iii) compactifying the corresponding azimuthal direction

These black holes violate reverse isoperimetric inequality

$\Rightarrow$ Superentropic black holes (1411.4309)

Parenthesis: Λ as a thermodynamic variable and the reverse isoperimetric inequality

Kastor/Ray/Traschen 0904.2765:

Proposed Smarr formula for AdS black holes and associated extended version of first law that accounts for variations in the black hole mass w.r.t. variations in the cosmological constant

(Note: Variable Λ goes back to Brown/Teitelboim 1987/1988 $\Rightarrow$ 4d cosm. const. represents energy density of a 4-form gauge field strength. This idea was first applied to the thermodynamics of AdS black holes (KNAdS) in Caldarelli/Cognola/DK 9908022.)

Note also: In string compactifications the cosm. const. typically is related to the 'radius' of the compactifying manifold. If we allow the size of the extra dimensions to change w/ time, then Λ should also be allowed to vary.

KRT: Obtained a general expression for the quantity $\Theta \equiv 8\pi G \frac{\partial M}{\partial \Lambda}$, that appears in both the first law and the Smarr formula, in terms of surface integrals of the Killing potential ω ($\xi^\nu = \nabla_\mu \omega^{\mu\nu}$)

(This is comparable to knowing that $\frac{\partial M}{\partial A} = \frac{\kappa}{8\pi G}$)

$\Rightarrow$ new term in first law of the form $\frac{\Theta}{8\pi G}\delta\Lambda$

Note: The cosmological constant can be thought of as a perfect fluid stress-energy w/ pressure $P = -\frac{\Lambda}{8\pi G}$

$\Rightarrow$ suggests to interpret Θ as minus a *volume* $\Rightarrow$ have $\frac{\Theta}{8\pi G}\delta\Lambda = V\delta P$

Notice: The interpretation $\Theta = -V$ has an independent motivation:

Express the surface integral for Θ as a volume integral using Gauss

$\Rightarrow$ One finds that $-\Theta$ gives a measure for the volume excluded from the spacetime by the black hole horizon

For static black holes, the first law becomes $\delta M = T\delta S + V\delta P$, which is precisely the variation of the *enthalpy* $H = E + PV$

$\Rightarrow$ The mass of an AdS black hole should be thought of as the enthalpy of spacetime

Cvetič/Gibbons/Kubizňak/Pope 1012.2888: Showed that

$$\left(\frac{(D-1)V}{\mathcal{A}_{D-2}}\right)^{\frac{1}{D-1}} \geq \left(\frac{A}{\mathcal{A}_{D-2}}\right)^{\frac{1}{D-2}} \qquad (1)$$

holds for a large class of black holes in AdS

Here: D = dimension of spacetime

A = horizon area

$\mathcal{A}_{D-2}$ = volume of unit S^{D-2}

(1): 'Reverse isoperimetric inequality'

(If there was a '$\leq$', it would be the usual isoperimetric inequality for Euclidean bounded volumes)

Cvetič et al. conjectured that all black holes satisfy (1).

Equality is attained for Schwarzschild-AdS

(Schwarzschild-AdS black holes are 'maximally entropic')

But...

The black holes that have noncompact horizon with finite area always violate the reverse isoperimetric inequality!

$\Rightarrow$ 'Superentropic black holes'

(Hennigar/Mann/Kubizňak 1411.4309)

$\Rightarrow$ Suggests that reverse isoperimetric inequality conjecture might apply only to black holes w/ compact horizon

The proof of this restricted conjecture remains an interesting open problem

III) Black holes with Bianchi horizons in five dimensions

- Start from action $$I = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\,[R-2\Lambda]$$

The eom admit the well-known black hole solutions

$$ds^2 = -\left(k - \frac{m}{r} + \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{k - \frac{m}{r} + \frac{r^2}{\ell^2}} + r^2 d\Sigma_k^2\,,$$

(Birmingham, hep-th/9808032)

where $m =$ mass parameter, $\Lambda = -\frac{6}{\ell^2}$, $k = 0, \pm 1$, and $d\Sigma_k^2$ is the standard metric on S^3 $(k=1)$, H^3 $(k=-1)$ or $\mathbb{E}^3$ $(k=0)$, i.e., on a manifold of constant curvature (homogeneous and isotropic)

- Can we relax this?
  (Homogeneous, but not isotropic horizons)

Three-dimensional homogeneous manifolds were classified (9 Bianchi cosmologies)

The Bianchi types are in correspondence (not one to one) with the eight Thurston model geometries S^3, H^3, $\mathbb{E}^3$, $\mathrm{S}^2 \times \mathbb{R}$, $\mathrm{H}^2 \times \mathbb{R}$, Nil, Sol, $\widetilde{\mathrm{SL}}(2, \mathbb{R})$.

Among these, we are particularly interested in Nil and Sol

Lie group Sol: Semidirect product $\mathbb{R}^2 \rtimes \mathbb{R}$ with the multiplication given by

$$((x, y), z) \cdot ((x', y'), z') = ((x + e^{-z}x', y + e^{z}y'), z + z')$$

Sol-invariant 1-forms: $\omega^1 = e^z dx$, $\omega^2 = e^{-z} dy$, $\omega^3 = dz$

Left-invariant metric on Sol: $e^{2z}dx^2 + e^{-2z}dy^2 + dz^2$

Black hole w/ Sol horizon:

$$ds^2 = -\left(-\frac{2\Lambda}{9}r^2 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{-\frac{2\Lambda}{9}r^2 - \frac{2M}{r}}$$
$$+ \frac{3}{-\Lambda}\left[r^2(e^{2z}dx^2 + e^{-2z}dy^2) + dz^2\right]$$

(Cadeau/Woolgar, gr-qc/0011029)

Note: The horizon can be compactified by dividing by some properly chosen discrete subgroup of Sol, making it a torus bundle over a circle.

Lie group Nil: Consists of all 3×3 upper triangular matrices of the form

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{where} \quad x, y, z \in \mathbb{R}.$$

(Heisenberg group)

Identify $(x, y, z) \in \mathbb{R}^3$ w/ this matrix, giving the multiplication

$$(x, y, z) \cdot (x', y', z') = (x + x', y + y', z + z' + xy')$$

Left-inv. metric on Nil: $dx^2 + dy^2 + (dz - xdy)^2$

Black hole w/ Nil horizon:

$$ds^2 = -\left(-\frac{2\Lambda}{11}r^2 - \frac{2M}{r^{5/3}}\right)dt^2 + \frac{dr^2}{-\frac{2\Lambda}{11}r^2 - \frac{2M}{r^{5/3}}}$$
$$+ r^{4/3}(dx^2 + dy^2) + r^{8/3}\left(dz - \sqrt{\frac{-4\Lambda}{9}}xdy\right)^2$$

(Cadeau/Woolgar, gr-qc/0011029)

Also here, the horizon can be compactified by taking the quotient Nil/Γ, where $\Gamma \subset$ Nil consists of matrices with only integer entries. Nil/Γ is then a circle bundle over a torus.

Can we add charge to these solutions?

Add Maxwell term $\int d^5x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$ and Chern-Simons term $\sim \int F \wedge F \wedge A$ to the above action, to get the bosonic sector of pure $N = 2$, $d = 5$ gauged supergravity.

(We might eventually be interested in supersymmetric solutions)

Let us restrict to the case of Sol horizons

Then we find that the eom are satisfied for the configuration

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + \sqrt{\frac{p^2}{-\Lambda}}(e^{2z}dx^2 + e^{-2z}dy^2) + \frac{r^2}{A}dz^2\,,$$

$$F = p\,dx \wedge dy\,, \qquad V(r) = -\frac{\Lambda}{2}r^2 - 2A\ln\frac{r}{B}$$

A, B: Positive integration constants $\qquad$ p: Magnetic charge

$\rightarrow$ Magnetically charged black hole w/ Sol horizon

(F. Faedo/D. A. Farotti/SK, ‘Black holes in Sol minore’, to appear)

Note: The solution is singular in the limit $p \to 0$, and it is thus disconnected from the Cadeau/Woolgar solution

If the horizon is compactified, the geometry approaches asymptotically for $r \to \infty$ a torus bundle over AdS_3.

$\to$ Again unusual asymptotics

- Curvature singularity at $r = 0$, horizon for $V(r) = 0$.

Open questions:

- Solution that contains both Cadeau/Woolgar and ours?
- Generalization to $N = 2$ matter-coupled gauged supergravity
- Electrically charged case?
- Charged black holes w/ Nil horizons?
- Charged (or uncharged) black holes w/ $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ horizons? Note in this context: A supersymmetric near-horizon geometry of a rotating dyonic black hole w/ $\widetilde{\mathrm{SL}}(2,\mathbb{R})$ horizon is known (Gutowski/Reall, hep-th/0401042)

...work in progress

Faedo/Farotti/SK (to appear):

- There are no static BPS black holes with Sol horizons and purely electric or purely magnetic field strengths in $N = 2$ gauged sugra coupled to abelian vector multiplets
- There are no static $\mathrm{AdS}_2 \times \mathrm{Sol}$ or $\mathrm{AdS}_2 \times \mathrm{Nil}$ attractors (susy or not) with purely electric field strengths in $N = 2$ gauged sugra coupled to abelian vector multiplets
- Magnetic (non-BPS) $\mathrm{AdS}_2 \times \mathrm{Sol}$ attractors exist. The horizon values of the scalar fields follow from extremization of an effective potential that can be computed explicitely. The value of this effective potential at the extremum gives the Bekenstein-Hawking entropy

IV) Multi-centered black holes in AdS

Start from charged generalization of McVittie solution (Reissner-Nordström immersed in FLRW, Shah/Vaidya 1968):

$$ds^2 = -\frac{\left[1-(M^2-Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^2}{\left[1+M\frac{\sqrt{1+kr^2}}{a\,r}+(M^2-Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^2}dt^2$$

$$+4\,a^2\left[1+M\frac{\sqrt{1+kr^2}}{a\,r}+(M^2-Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^2\frac{dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2}{(1+kr^2)^2}\,,$$

$$F=\frac{Q}{ar^2}\frac{1}{\sqrt{1+kr^2}}\frac{\left[1-(M^2-Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]}{\left[1+M\frac{\sqrt{1+kr^2}}{a\,r}+(M^2-Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^2}dr\wedge dt\,. \qquad (a=a(t))$$

$M=Q=0$: FLRW universe

$a=$ const., $k=0$: Reissner-Nordström (in isotropic coordinates)

- Solves Einstein-Maxwell equations

$$G_{\mu\nu}=8\pi T_{\mu\nu}\,, \qquad \nabla_\nu F^{\mu\nu}=4\pi J^\mu$$

where
$$T_{\mu\nu} = \frac{1}{4\pi}\left[F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\lambda}F^{\rho\lambda}\right] + \rho u_\mu u_\nu + p(u_\mu u_\nu + g_{\mu\nu})\,,$$

(Maxwell + perfect fluid)
$$J^\mu = \sigma u^\mu$$

- Pressure, energy density, charge density and 4-velocity of fluid:

$$8\pi p = -\,2\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\frac{\left[1 + M\frac{\sqrt{1+kr^2}}{a\,r} + (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]}{\left[1 - (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]} - 3\frac{\dot{a}^2}{a^2}$$
$$-k\left\{a^2\left[1 + M\frac{\sqrt{1+kr^2}}{a\,r} + (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^2\left[1 - (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]\right\}^{-1}\,,$$

$$8\pi\rho = 3\frac{\dot{a}^2}{a^2} + \frac{3k}{2a^2}\left[1 + M\frac{\sqrt{1+kr^2}}{a\,r} + (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^{-3}\left[2 + M\frac{\sqrt{1+kr^2}}{a\,r}\right]\,,$$

$$4\pi\sigma = -\,\frac{3}{4}\frac{kQ}{a^3}\frac{\sqrt{1+kr^2}}{r}\left[1 + M\frac{\sqrt{1+kr^2}}{a\,r} + (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}\right]^{-3}\,,$$

$$u = \frac{1 - (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}}{1 + M\frac{\sqrt{1+kr^2}}{a\,r} + (M^2 - Q^2)\frac{1+kr^2}{4\,a^2\,r^2}}\,dt\,.$$

$\Rightarrow$ For $k \neq 0$, the fluid must be charged!

- Note also: p, ρ, σ are inhomogeneous (pressure gradient prevents surrounding matter from accreting into black hole)
- Now consider 'extremal' case $M = Q$: (and def. $r = \frac{1}{\sqrt{k}} \tan \frac{\sqrt{k}\psi}{2}$)

$$
\begin{aligned}
ds^2 =& -\left[1 + M \frac{\sqrt{k}}{a\,\sin(\sqrt{k}\,\psi/2)}\right]^{-2} dt^2 \\
&+ a^2 \left[1 + M \frac{\sqrt{k}}{a\,\sin(\sqrt{k}\,\psi/2)}\right]^{2} \left[d\psi^2 + \frac{\sin^2(\sqrt{k}\psi)}{k}\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \\
F =& d\left[\left(1 + M \frac{\sqrt{k}}{a\,\sin(\sqrt{k}\,\psi/2)}\right)^{-1} dt\right],
\end{aligned}
$$

(plus some expressions for p, ρ, σ)

$\Rightarrow$ Solution completely determined by function $H = \frac{M\sqrt{k}}{\sin(\sqrt{k}\,\psi/2)}$

For $k = 0$: $H \propto \frac{M}{\psi}$, harmonic function on flat base space $d\psi^2 + \psi^2 d\Omega^2$

$\Rightarrow$ For $a =$ const., $k = 0$ usual recipe to construct extremal black holes in terms of harmonic functions

And for $k \neq 0$?

$H = \frac{M\sqrt{k}}{\sin(\sqrt{k}\,\psi/2)}$ satisfies *conformal Laplace equation* on base space

E^3, S^3 or H^3, $\qquad \nabla^2 H = \frac{1}{8} R H \qquad$ ($R = 6k$ scalar curvature)

One easily checks: For any function H satisfying the conf. Laplace equ., the resulting fields still solve the Einstein-Maxwell eqns.!

$\Rightarrow$ Use this to construct multi-centered solutions!

Note: For $k = 0$, $a = \exp(\sqrt{\Lambda} t/3)$ (De Sitter), this was used by Kastor and Traschen in '92 to construct multi-centered black holes in dS, comoving with the cosmic expansion. Here we saw that one can generalize this to arbitrary FLRW and any k.

- A multi-centered solution is $\quad H = \frac{1}{\sqrt{2}} \left[1 + k\vec{x}^{\,2}\right]^{1/2} \sum_{I=1}^{N} \frac{Q_I}{|\vec{x} - \vec{x}_I|}\,,$

obtained by using conformal invariance,

$$\tilde{\nabla}^2 \tilde{H} = \frac{1}{8} \tilde{R} \tilde{H}\,, \qquad \tilde{g}_{ij} = \Omega^2 g_{ij}\,, \qquad \tilde{H} = \Omega^{-1/2} H\,,$$

$$\tilde{g}_{ij} dx^i dx^j = \frac{4 d\vec{x}^{\,2}}{\left[1 + k\vec{x}^{\,2}\right]^2}\,.$$

- To construct multi-centered solutions in AdS:

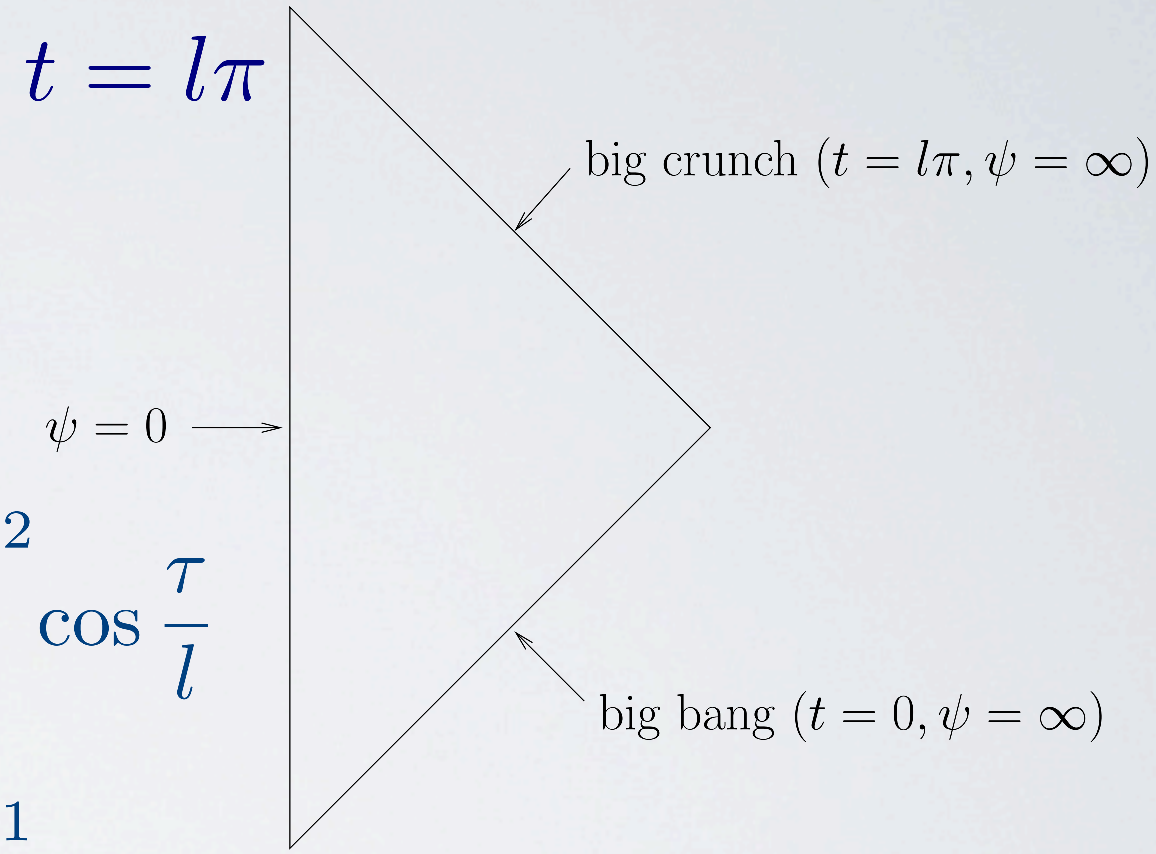
Write AdS in FLRW coordinates,

$$ds^2 = -dt^2 + l^2 \sin^2\frac{t}{l}\left(d\psi^2 + \sinh^2\psi d\Omega^2\right) \quad (0 < t/l < \pi,\ \Lambda = -3/l^2)$$

$\Rightarrow$ Big bang in $t = 0$, big crunch in $t = l\pi$

Of course coordinate artefacts:

Global coordinates $\tau, \hat{r}$



$$\hat{r} = l \sin\frac{t}{l}\sinh\psi\,, \qquad \cos\frac{t}{l} = \left(1 + \frac{\hat{r}^2}{l^2}\right)^{1/2}\cos\frac{\tau}{l}$$

$$\Rightarrow ds^2 = -\left(1 + \frac{\hat{r}^2}{l^2}\right)d\tau^2 + \left(1 + \frac{\hat{r}^2}{l^2}\right)^{-1}d\hat{r}^2 + \hat{r}^2 d\Omega^2$$

$\Rightarrow$ Can extend beyond big bang/big crunch singularities

Note: Only one point $(\tau/l = \pi/2)$ of conformal boundary $\hat{r} \to \infty$ visible in FLRW coordinates

- Consider single-centered asymptotically AdS case:

(Is not Reissner-Nordström-AdS, but highly dynamical)

Far away from the black hole, p, ρ approach values given by a $\Lambda < 0$ and charge density $\sigma \to 0$ $\Rightarrow$ 'asymptotically AdS'

Metric:

$$ds^2 = -\frac{g^2}{f^2}dt^2 + a^2 f^2 \left(d\psi^2 + \frac{\sin^2(\sqrt{k}\psi)}{k} d\Omega^2 \right) ,$$

$$f = 1 + \frac{\sqrt{k}M}{a \sin(\sqrt{k}\psi/2)} + k\frac{M^2 - Q^2}{4\, a^2 \sin^2(\sqrt{k}\psi/2)} , \qquad g = 1 - k\frac{M^2 - Q^2}{4\, a^2 \sin^2(\sqrt{k}\psi/2)} .$$

- Curvature singularities: $2a \sinh(\psi/2) = \sqrt{M^2 - Q^2}$

For $M = Q$: $t = 0$, $t = l\pi$ or $\psi = 0$

- Trapping horizons (Hayward '93):

Compute expansions of outgoing and ingoing radial null geodesics:

$$\theta_+ \equiv -2m^{(\mu}\bar{m}^{\nu)}\nabla_\mu l_\nu , \qquad \theta_- \equiv -2m^{(\mu}\bar{m}^{\nu)}\nabla_\mu n_\nu$$

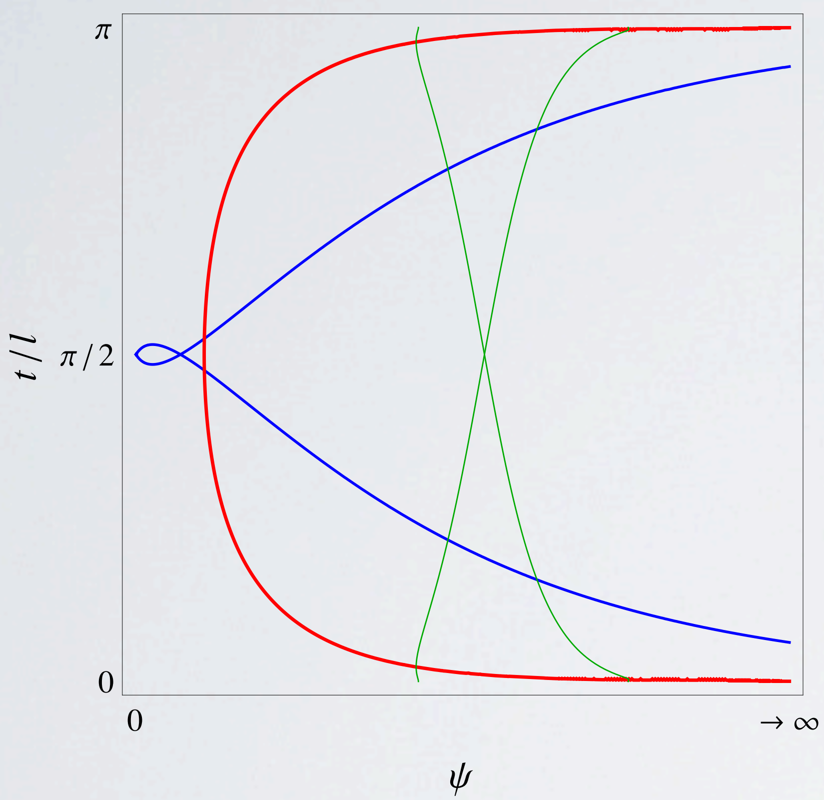
(l, m, n: Newman-Penrose null tetrad)

Marginal surfaces: Spacelike 2-surfaces on which $\theta_+ = 0$ ($\theta_- = 0$)

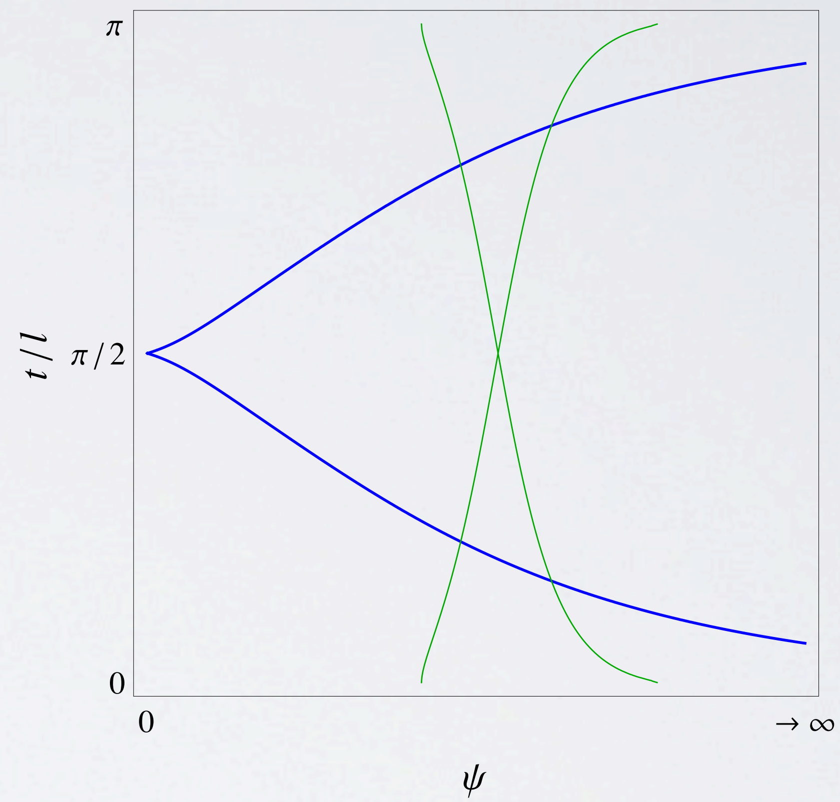
Trapping horizons: Defined as closure of 3-surfaces foliated by marginal surfaces

One finds the following:

$M > Q$:



$M = Q$:



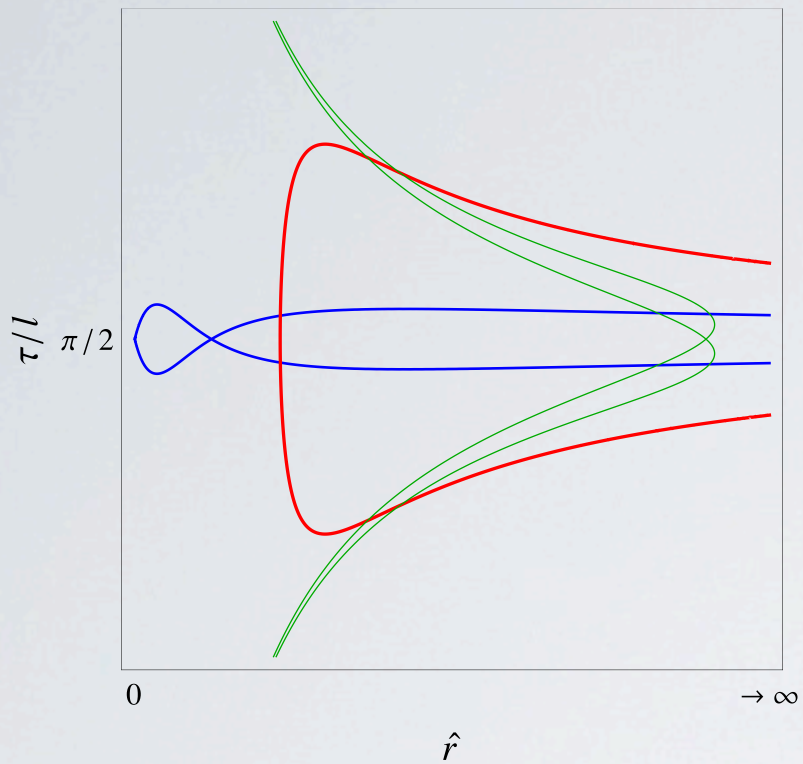
Red: Curvature singularities (coincide w/ axes for $M = Q$)

Blue: Trapping horizons

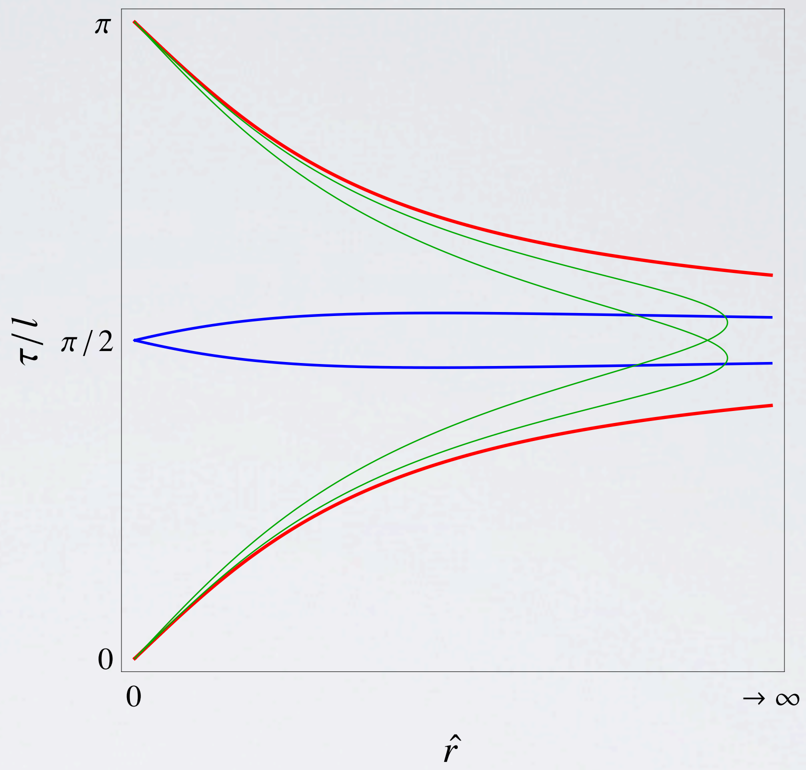
(Green: Pair of radial null geodesics intersecting in $t = l\pi/2$)

In global coordinates $\tau, \hat{r}$:

$M > Q$:



$M = Q$:



$\Rightarrow$ Spurious big bang/big crunch singularities that appear when one writes AdS in FLRW coordinates, become real once such a dynamical black hole is present.

$\Rightarrow$ Only one point of the conformal boundary of AdS survives.

$\Rightarrow$ $\exists$ AdS/CFT interpretation in the usual sense?

IV) Final Remarks

- Black holes with unusual horizons:

  Noncompact manifolds w/ finite volume

  - Chiral excitations
  - violate reverse isoperimetric inequality $\rightarrow$ 'superentropic'
  - can be generalized to higher dimensions and to presence of matter

  Open questions:

  Stability issues?

  Field theory interpretation?

  Is this the end of the story?
  Or can we have still more possibilities for the horizon geometry/topology in presence of a scalar potential?

- Multi-centered solutions in AdS:

  Constructed from solutions of *conformal Laplace equation*

  Deeper reason for appearance of conformal symmetry?

  Why superposition principle? (Neither true nor fake susy)

  Can we mimic the perfect fluid with scalar fields, and embed solution in some model of matter-coupled $N = 2$ supergravity?

  Note: Since charge density σ of cosmic fluid is nonvanishing for $k \neq 0$, these scalars have to be charged under a U(1) gauge field.

  In such a scenario, the cosmological expansion would be driven by the scalars while rolling down their potential.

  (Cf. e.g. black holes constructed by Gibbons/Maeda in 0912.2809)