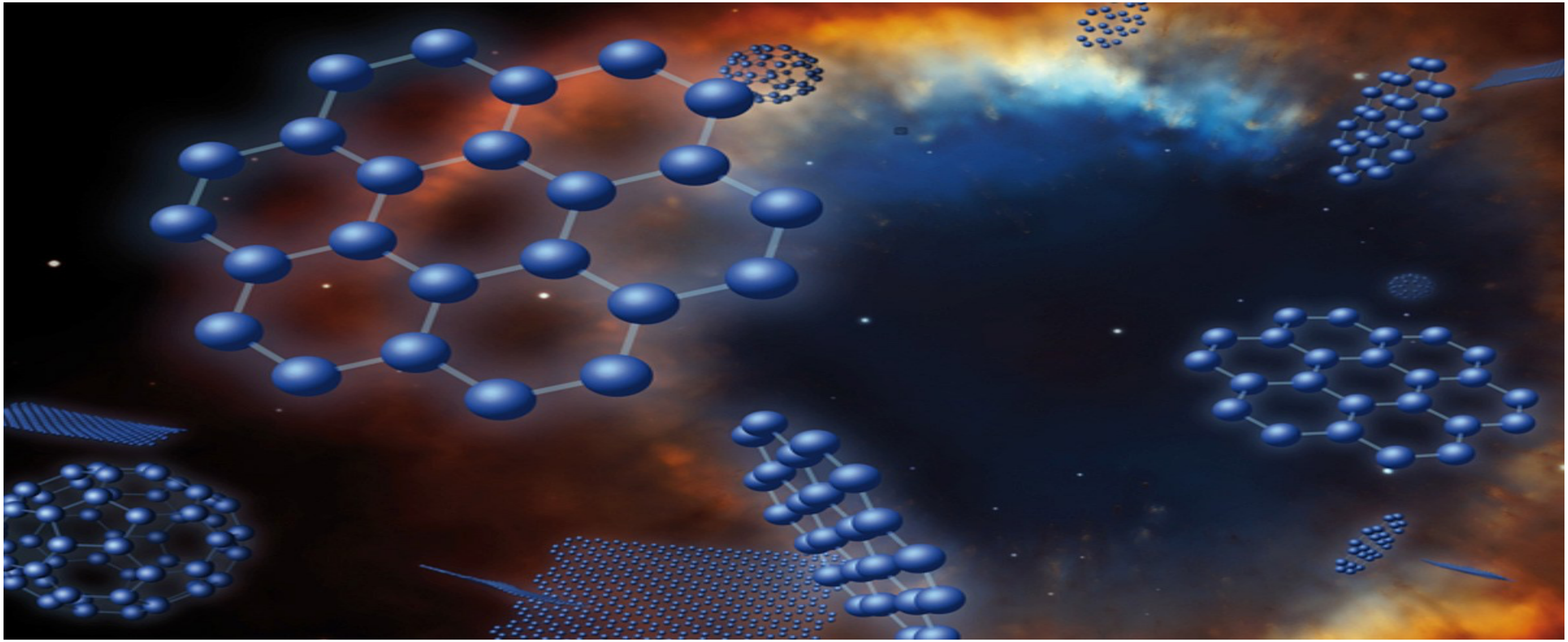
Chern-Simons Graphene Wormholes

(…cosmology in the lab?)

Salvatore Capozziello



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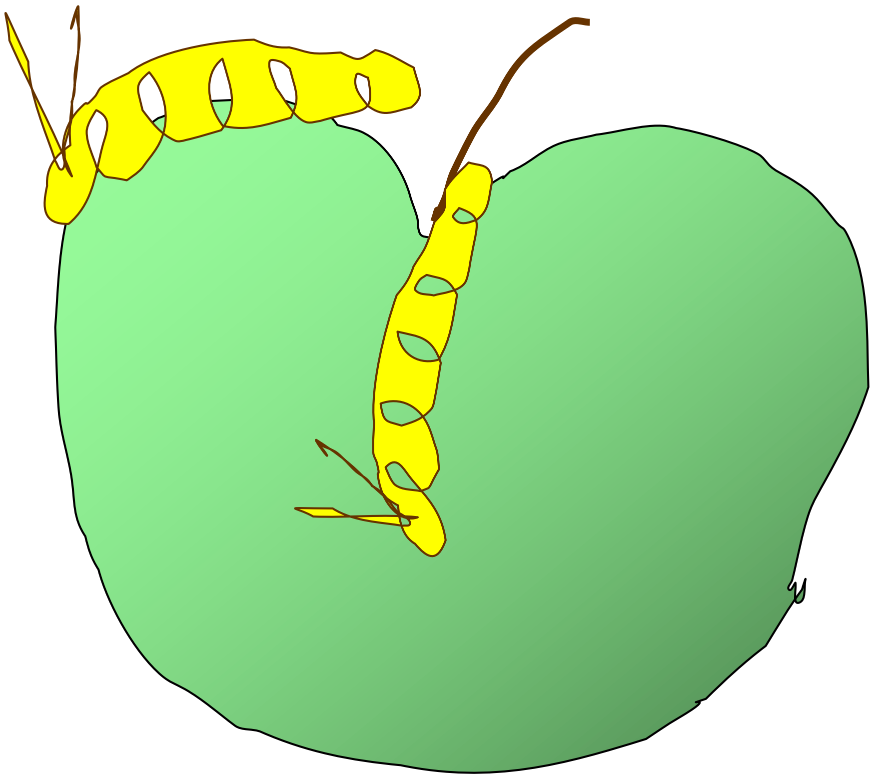
Summary

- Whormholes: some definitions
- Open issues with stable and traversable WHs
- The graphene
- Current densities in graphene
- Geometric defects instead of exotic matter
- The graphene wormhole
- Cosmology in the Lab
- Perpectives

Wormholes

- **The term WH was introduced by J. A. Wheeler in 1957**
- **Already in 1921 by H. Weyl (mass in terms of EM)**
- **The name WH comes from the following obvious picture.**



The worm could take a shortcut to the opposite side of the apple's skin by burrowing through its center, instead of traveling the entire distance around.

Whormole: definitions

- The wormholes are solutions of the Einstein field equations having a non-trivial topological structure linking separate points of spacetime, much like a tunnel with two ends.

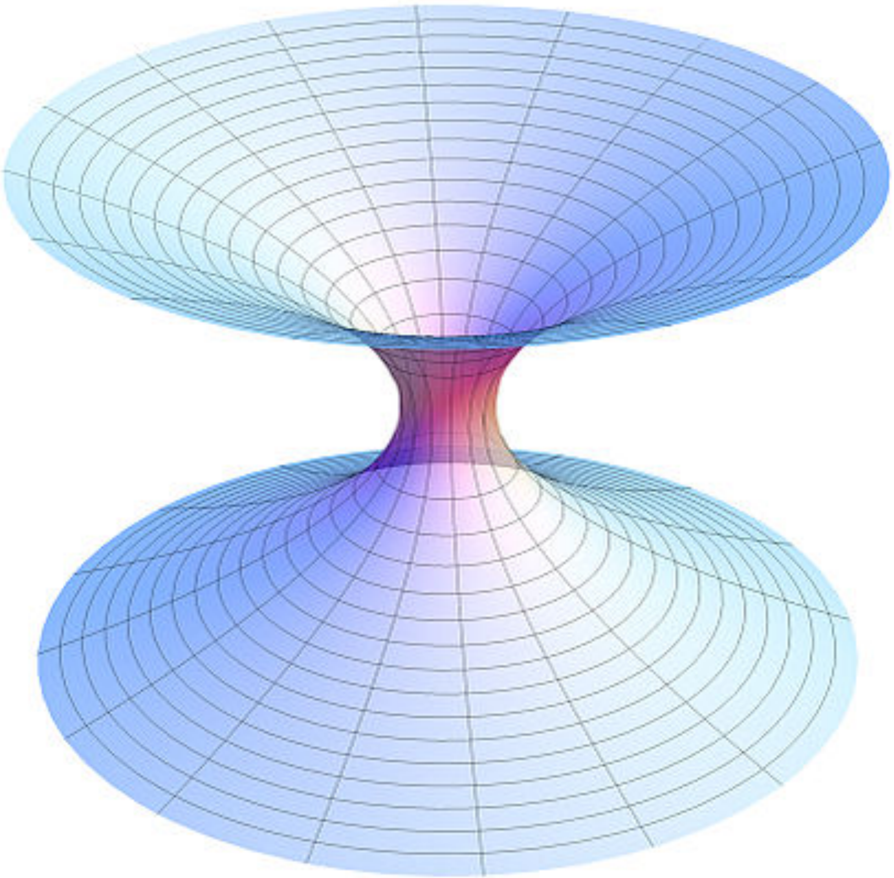
- They are related to space-time topology changes.

- Problems with causality notion.

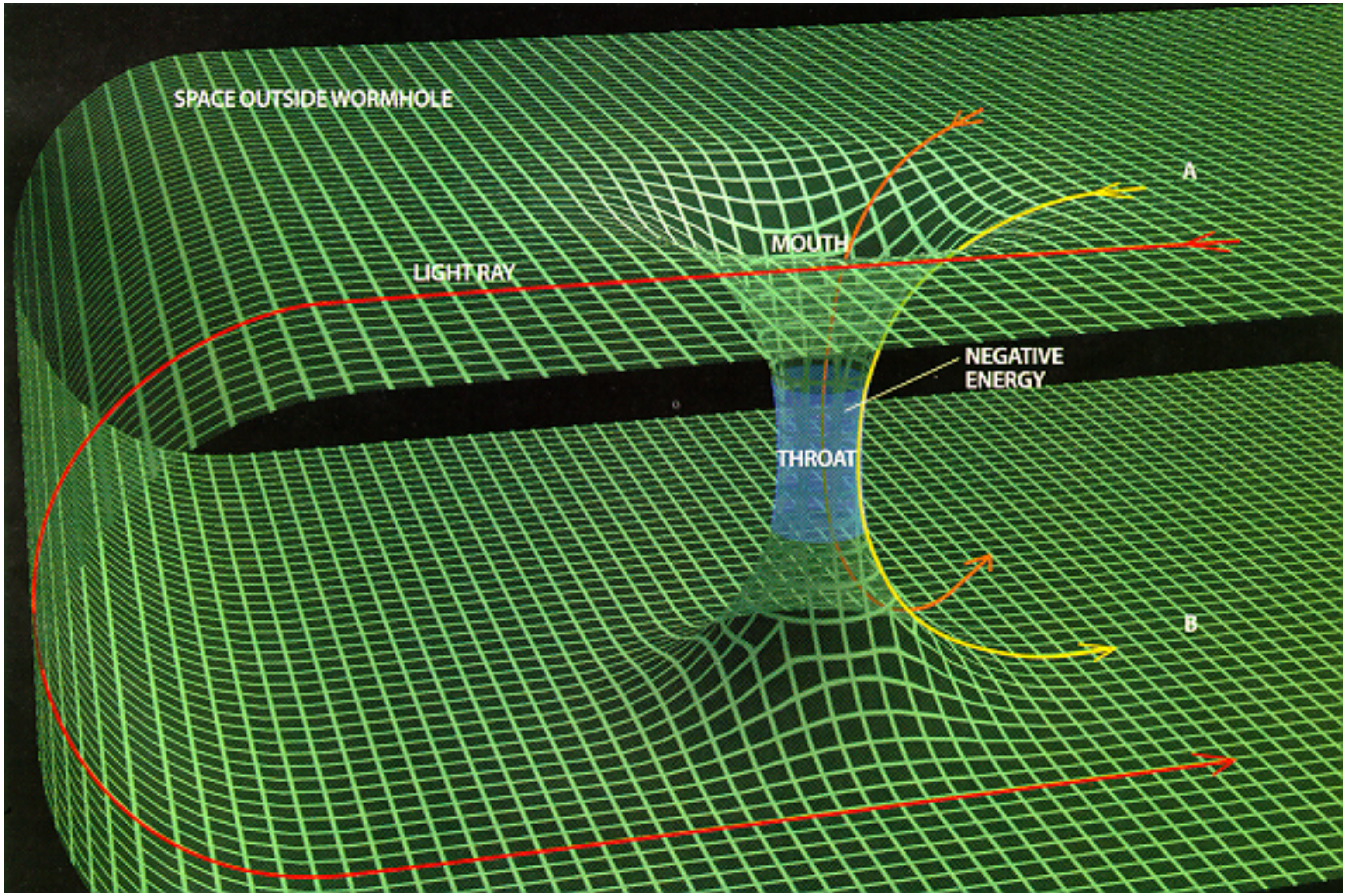
- Cronology Protection Conjecture.

- **Their existence remains hypothetical at astrophysical level.**

Einstein-Rosen bridge (1935)



Two Schwarzschild solutions joined by a throat



The traveler, just as a worm, could take a shortcut to the opposite side of the universe through a topologically nontrivial tunnel. A **negative energy** is requested to give rise to the topology change and the throat.

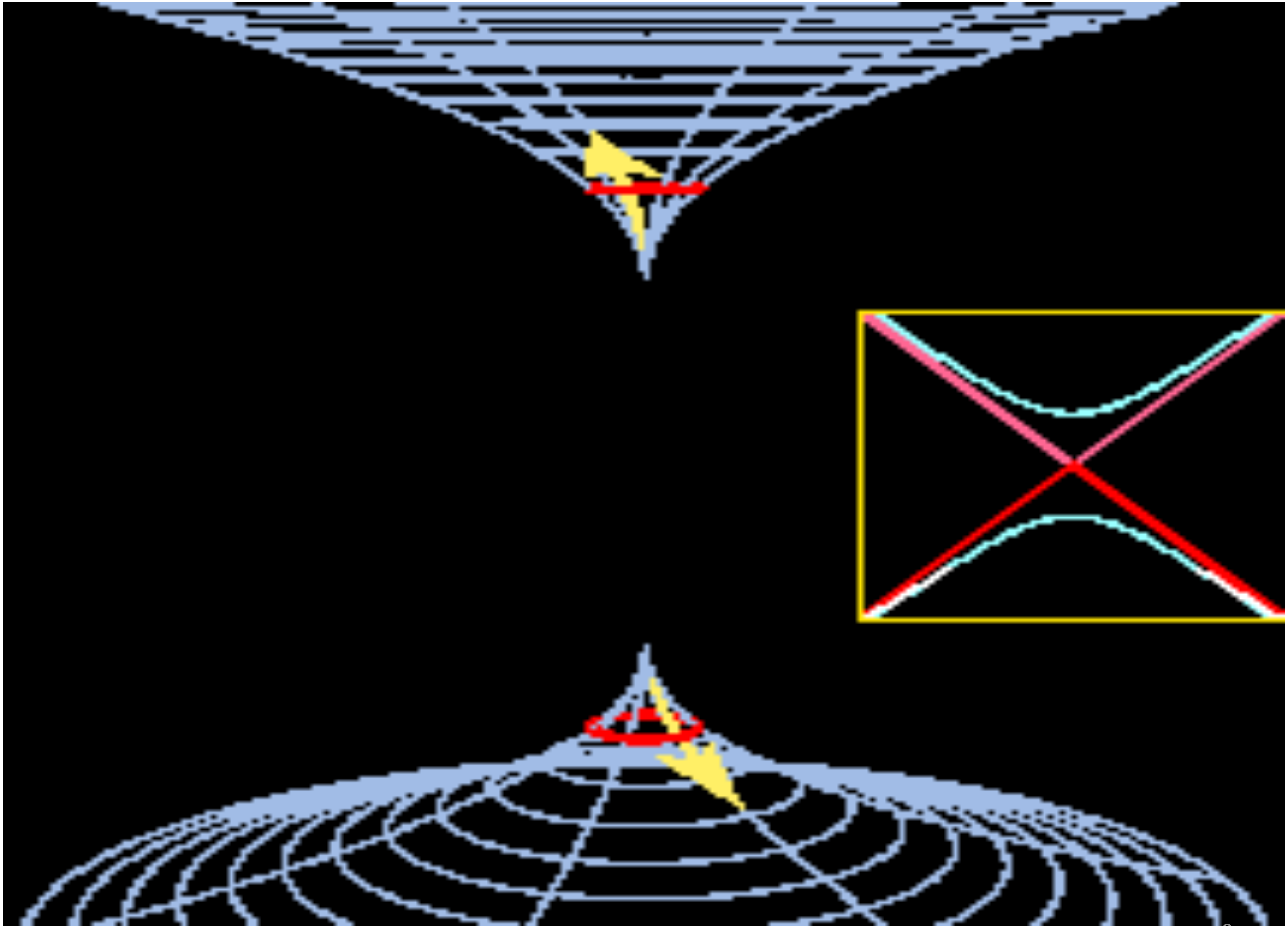
Traversable Wormholes

Morris-Thorne 1988

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r^2}} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

Traversable WHs need **exotic matter** for stability. For standard matter energy conditions are violated!

We need VIOLATIONS or GENERALIZATIONS of Energy Conditions.



- Several WH solutions exist, however standard matter forbids their formation.
- Some kinds of **Dark Matter** could solve the problem but no final detection at fundamental level.
- We need huge energies and masses (at least stellar masses) to give rise to WHs at astrophysical level.

……………………………BUT………………………………..

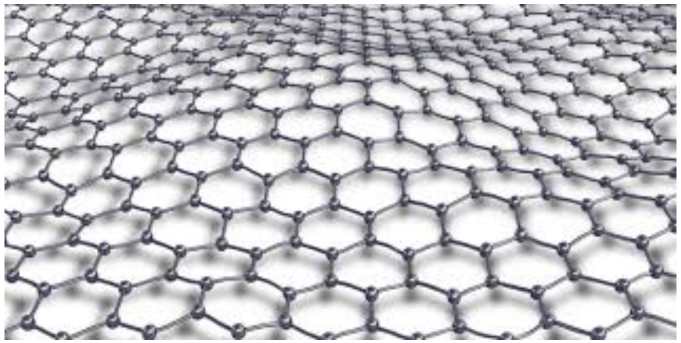
IN NATURE, ALL THAT IS NOT FORBIDDEN,
IT IS NECESSARY!
(E. Fermi)

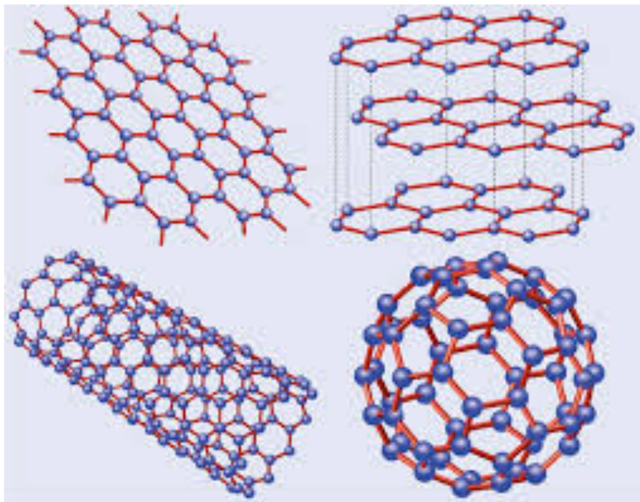
- Gravitational Field Equations DO NOT forbid the formation of WHs!

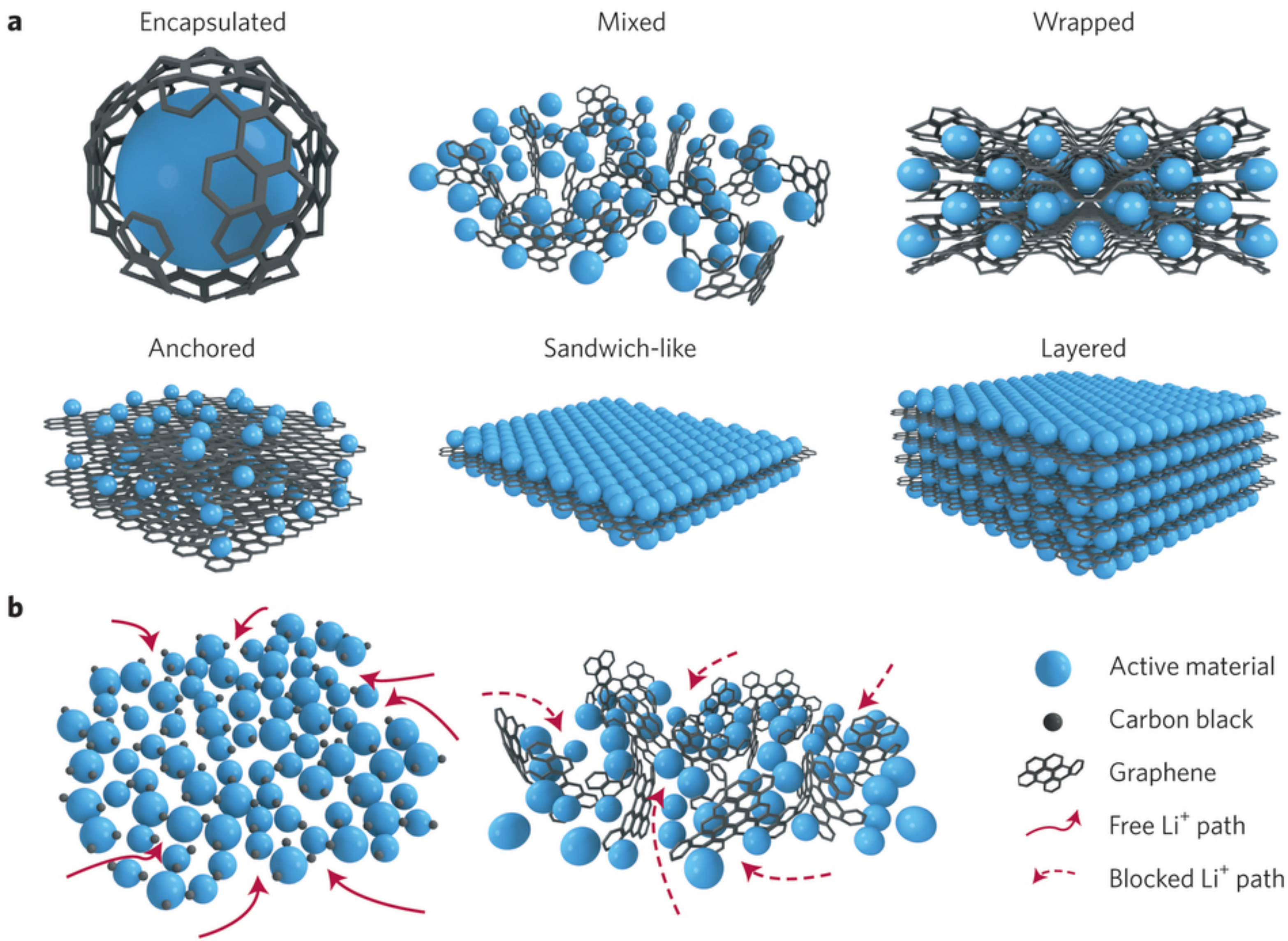
- Are there similar structures in which to generate topology changes as in space-time?
- Is it possible to change the geometry instead of searching for exotic matter?
- Is it possible to find out analogue quantities?

Graphene

Graphene is an allotrope of carbon in the form of a two-dimensional, atomic-scale, hexagonal lattice in which one atom forms each vertex. It is the basic structural element of other allotropes, including graphite, charcoal, carbon nanotubes and fullerenes. It efficiently conducts heat and electricity and is nearly transparent. The material was discovered, isolated, and characterized in 2004 by Andre Geim and Konstantin Novoselov (Nobel Prize 2010).






a
Encapsulated
Mixed
Wrapped
Anchored
Sandwich-like
Layered
b
Active material
Carbon black
Graphene
Free Li+ path
Blocked Li+ path

Working Hypotheses

- Graphene behaves as a **bidimensional spacetime**
- We can apply analogue Einstein field equations to graphene
- Instead of searching for **exotic matter**, we can take into account the graphene **curvature** and its **geometric defects**
- Graphene defects and bonds generate **electric currents**.
- Coupled electrons give rise to **analogue gravitons** that bring information (current = information)
- Equations yield EXACT SOLUTIONS

⇓

Graphene Wormhole


WHAT
DO I
NEED TO
KNOW?

The current density in graphene

Let us introduce the concepts of Mp-branes to extract the current density of free electrons in terms of inequality between curvatures of parallel spins and anti-parallel spins.

Let us consider scalar fields X and the generators of a Lie 3-algebra

$$X^M = X^M_\alpha T^\alpha$$

$$[T^\alpha, T^\beta, T^\gamma] = f^{\alpha\beta\gamma}_\eta T^\eta$$

Nambu-Poisson algebra

$$[X^M, X^N, X^L] = [X^M_\alpha T^\alpha, X^N_\beta T^\beta, X^L_\gamma T^\gamma]$$

Let us consider a 2-form gauge field A_{ab}

$$S_{co-Graphene} = \int d^3x \sum_{n=1}^{U} \beta_n \left(\delta^{a_1,a_2 \ldots a_n}_{b_1 b_2 \ldots . b_n} L^{b_1}_{a_1} \ldots L^{b_n}_{a_n}\right)^{1/2}$$

$$(L)^a_b = \delta^b_a STr\left\{-det\left(P_{abc}[E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{kln}] + \lambda F_{abc}\right) det(Q^i_{j,k})\right\}$$

with

$$E^{\alpha,\beta,\gamma}_{mnl} = G^{\alpha,\beta,\gamma}_{mnl} + B^{\alpha,\beta,\gamma}_{mnl},$$
$$Q^i_{j,k} = \delta^i_{j,k} + i\lambda[X^j_\alpha T^\alpha, X^k_\beta T^\beta, X^{k'}_\gamma T^\gamma] E^{\alpha,\beta,\gamma}_{k'jl}$$
$$F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab}.$$

$$G_{mnl} = g_{mn}\delta^{n'}_{n,l} + \partial_m X^i \partial_{n'} X^i \sum_j (X^j)^2 \delta^{n'}_{n,l} + \frac{1}{2}\langle \partial^b \partial^a X^i, \partial_b \partial_a X^i \rangle$$

The current density in graphene

Applying the above action to graphene, we assume that the scalar fields X*i* play the role of pairs of anti-parallel electrons and the 2-form gauge tensor fields A_{ab} play the role of gravitons which are exchanged between electrons

This assumption is justified in graphene, the three electrons of each atom are paired with three electrons of another atom by exchanging 2-form gauge fields forming spinless pairs which can be treated as scalars.

Moreover, free electrons are represented by ψ, while electrons in each pair are denoted by **ψ**

Hence, with these positions, we can define

$$
\begin{aligned}
&A_{ab} \to \psi_a^U \psi_b^L - \psi_a^L \psi_b^U \\
&X \to \psi_a^U A^{ab} \psi_b^L - \psi_a^L A^{ab} \psi_b^U + \Psi_a^U A^{ab} \Psi_b^L - \Psi_a^L A^{ab} \Psi_b^U + \Psi_a^U A^{ab} \psi_b^L - \psi_a^L A^{ab} \Psi_b^U \\
&\partial_a = \partial_a^U + \partial_a^L \\
&\partial_a^U \psi_a^U = 1, \ \partial_a^L \psi_a^L = 1.
\end{aligned}
$$

The current density in graphene

Using these definitions we can make the splitting

$$\langle F^{abc}, F_{abc} \rangle_{tot} \equiv \langle F^{abc}, F_{abc} \rangle_{Free-Free} + \langle F^{abc}, F_{abc} \rangle_{Free-Bound} + \langle F^{abc}, F_{abc} \rangle_{Bound-Bound}$$

where the subscript

- "Free-Free" indicates the mutual interaction of two free electrons,
- "Free-Bound" denotes the mutual interaction of free and bound electrons,
- "Bound-Bound" denotes the mutual interaction of two bound electrons.

we can calculate the different terms of the above equation in terms of couplings of parallel and anti-parallel spins

Graphene geometry

Analogue Metric

$$A^{ab} = g^{ab} = h^{ab} = h_1^{ab'} \otimes h_2^{b'b} - h_2^{bb'} \otimes h_1^{ab'}$$
$$F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab} = 2(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}) = 2\Gamma_{\mu\nu\lambda}$$

$$\begin{aligned}\langle F^{\rho}{}_{\sigma\lambda}, F^{\lambda}{}_{\mu\nu}\rangle &= \langle [X^\rho, X_\sigma, X_\lambda], [X^\lambda, X_\mu, X_\nu]\rangle \\ &= [X_\nu, [X^\rho, X_\sigma, X_\mu]] - [X_\mu, [X^\rho, X_\sigma, X_\nu]] \\ &\quad + [X^\rho, X_\lambda, X_\nu][X^\lambda, X_\sigma, X_\mu] - [X^\rho, X_\lambda, X_\mu][X^\lambda, X_\sigma, X_\nu] \\ &= \partial_\nu \Gamma^\rho_{\sigma\mu} - \partial_\mu \Gamma^\rho_{\sigma\nu} + \Gamma^\rho_{\lambda\nu}\Gamma^\lambda_{\sigma\mu} - \Gamma^\rho_{\lambda\mu}\Gamma^\lambda_{\sigma\nu} = R^\rho_{\sigma\mu\nu},\end{aligned}$$

Analogue Curvature

$$\langle F_{abc}, F^{bc}_{a'}\rangle = R^{anti-parallel}_{aa'} - R^{parallel}_{aa'}$$

$$\begin{aligned}R_{MN} = R_{aa'} + R_{ia'} + R_{ij'} &= R^{anti-parallel}_{Free-Free} + R^{anti-parallel}_{Free-Bound} + R^{anti-parallel}_{Bound-Bound} \\ &\quad - R^{parallel}_{Free-Free} - R^{parallel}_{Free-Bound} - R^{parallel}_{Bound-Bound}.\end{aligned}$$

Same "**machinery**" of General Relativity

Graphene currents with parallel and anti-parallel electronic spins

$$
\begin{aligned}
\langle F^{abc}, F_{abc}\rangle_{Free-Free} = {} & A^{ab} i\sigma^2_{ij}\partial^i_a\psi^j_b + \sigma^0_{ij}\psi^{\dagger a,i}\psi^j_a - \sigma^1_{ij}\psi^{\dagger a,i}\psi^j_a \\
& + \sigma^0_{i'i}(\psi^{\dagger a,i'} i\sigma^0_{i'j}\sigma^1_{jk}\partial^{a,j}\psi^k_a)(\psi^{\dagger i}_a i\sigma^0_{ij}\sigma^1_{jk}\partial^{a,j}\psi^k_a) \\
& + \sigma^1_{i'i}(\psi^{\dagger a,i'} i\sigma^0_{i'j}\sigma^1_{jk}\partial^{a,j}\psi^k_a)(\psi^{\dagger i}_a i\sigma^0_{ij}\sigma^1_{jk}\partial^{a,j}\psi^k_a) \\
& - \sigma^0_{i'i}(\psi^{\dagger a,i'} i\sigma^1_{i'j}\sigma^1_{jk}\partial^{a,j}\psi^k_a)(\psi^{\dagger i}_a i\sigma^1_{ij}\sigma^1_{jk}\partial^{a,j}\psi^k_a),
\end{aligned}
$$

$$
\begin{aligned}
\langle \partial^b\partial^a X^i, \partial_b\partial_a X^i\rangle = {} & \varepsilon^{abc}\varepsilon^{ade}(\partial_b\partial_c X^i_\alpha)(\partial_e\partial_d X^i_\beta) = \\
& \Psi^{\dagger a,U}\langle F_{abc}, F^{a'bc}\rangle\Psi^L_{a'} + \Psi^{\dagger a,L}\langle F_{abc}, F^{a'bc}\rangle\Psi^U_{a'} - \Psi^{\dagger a,U}\langle F_{abc}, F^{a'bc}\rangle\Psi^U_{a'} \\
& - \Psi^{\dagger a,L}\langle F_{abc}, F^{a'bc}\rangle\Psi^L_{a'} + \Psi^{\dagger a,L}\Psi^{\dagger d,U}\partial_d\partial^{d'}\langle F_{abc}, F^{a'bc}\rangle\Psi^L_{a'}\Psi^U_{d'} \\
& - \Psi^{\dagger a,L}\Psi^{\dagger d,U}\partial_d\langle F_{abc}, F^{a'bc}\rangle\Psi^L_{a'} - \Psi^{\dagger a,U}\Psi^{\dagger d,L}\partial_d\langle F_{abc}, F^{a'bc}\rangle\Psi^U_{a'} \\
& + \psi^{\dagger i,U}\langle F_{ijk}, F^{i'jk}\rangle\psi^L_{i'} + \psi^{\dagger i,L}\langle F_{ijk}, F^{i'jk}\rangle\psi^U_{i'} - \psi^{\dagger i,U}\langle F_{ijk}, F^{i'jk}\rangle\psi^U_{i'} \\
& - \psi^{\dagger i,L}\langle F_{ijk}, F^{i'jk}\rangle\psi^L_{i'} + \psi^{\dagger i,L}\psi^{\dagger m,U}\partial_m\partial^{m'}\langle F_{ijk}, F^{i'jk}\rangle\psi^L_{i'}\psi^U_{m'} \\
& - \psi^{\dagger i,L}\psi^{\dagger m,U}\partial_m\langle F_{ijk}, F^{i'jk}\rangle\psi^L_{i'} - \psi^{\dagger i,U}\psi^{\dagger m,L}\partial_m\langle F_{ijk}, F^{i'jk}\rangle\psi^U_{i'} \\
& + \Psi^{\dagger a,U}\langle F_{abc}, F^{i'bc}\rangle\psi^L_{i'} + \Psi^{\dagger a,L}\langle F_{abc}, F^{i'bc}\rangle\psi^U_{i'} - \Psi^{\dagger a,U}\langle F_{abc}, F^{i'bc}\rangle\psi^U_{i'} \\
& - \Psi^{\dagger a,L}\langle F_{abc}, F^{i'bc}\rangle\psi^L_{i'} + \Psi^{\dagger a,L}\Psi^{\dagger d,U}\partial_d\partial^{i'}\langle F_{abc}, F^{j'bc}\rangle\psi^L_{j'}\psi^U_{i'} \\
& - \Psi^{\dagger a,L}\Psi^{\dagger d,U}\partial_d\langle F_{abc}, F^{i'bc}\rangle\psi^L_{i'} - \Psi^{\dagger a,U}\Psi^{\dagger d,L}\partial_d\langle F_{abc}, F^{i'bc}\rangle\psi^U_{i'}. \qquad (A.2)
\end{aligned}
$$

Differences with respect to the Einstein Gravity

Two kinds of effective gravity emerge:

- One is generated by parallel spins, the other by antiparallel spins.

- **Three types of curvatures**:

- Interaction of free electrons.
- Interactions of free-bound electrons
- Interactions of bound-bound electrons.

- Antiparallel electrons give positive curvature, parallel electrons give negative curvature.
- We can have **ATTRACTIVE and REPULSIVE GRAVITY**

This feature gives **stable** and **traversable** WHs!

Chern-Simons Effective Gravity

$$
\begin{aligned}
E_{system} &= \int d^4x\rho \\
&= \int d^4x\sqrt{-g}\Big\{ -(1-m_g^2)\Big[(R^{parallel}_{Free-Free})^2 + (R^{anti-parallel}_{Free-Free})^2 + (R^{parallel}_{Free-Bound})^2 \\
&\quad +(R^{anti-parallel}_{Free-Bound})^2 + (R^{parallel}_{Bound-Bound})^2 + (R^{anti-parallel}_{Bound-Bound})^2 \\
&\quad +(R^{parallel}_{Free-Free}R^{anti-parallel}_{Free-Free})\,\partial^2(R^{parallel}_{Free-Free} + R^{anti-parallel}_{Free-Free}) \\
&\quad +(R^{parallel}_{Free-Bound}R^{anti-parallel}_{Free-Bound})\,\partial^2(R^{parallel}_{Free-Bound} + R^{anti-parallel}_{Free-Bound}) \\
&\quad +(R^{parallel}_{Bound-Bound}R^{anti-parallel}_{Bound-Bound})\,\partial^2(R^{parallel}_{Bound-Bound} + R^{anti-parallel}_{Bound-Bound})\Big] \\
&\quad +m_g^2\lambda^2\delta^{\mu_1\nu_1}_{\rho_1\sigma_1}\Big[R^{anti-parallel,\rho_1\sigma_1}_{Free-Free,\mu_1\nu_1} + R^{anti-parallel,\rho_1\sigma_1}_{Bound-Bound,\mu_1\nu_1} + R^{anti-parallel,\rho_1\sigma_1}_{Free-Bound,\mu_1\nu_1} \\
&\quad -R^{parallel,\rho_1\sigma_1}_{Free-Free,\mu_1\nu_1} + R^{parallel,\rho_1\sigma_1}_{Bound-Bound,\mu_1\nu_1} + R^{parallel,\rho_1\sigma_1}_{Free-Bound,\mu_1\nu_1}\Big]\Big\},
\end{aligned}
$$

$$m_g^2 = (\lambda)^2\det([X^j_\alpha T^\alpha, X^k_\beta T^\beta, X^{k'}_\gamma T^\gamma])$$

Effective "graviton" mass.

see e.g. S. Alexander, N. Yunes
Chern-Simons Modified General Relativity,
Phys. Rept. 480 (2009) 1-55

Defects and *f(R)* gravity

We can show that defects produce a particular form of modifired gravity.

The graphene action can be modelled as an *f(R)* function:

$$f(R) = \Big\{ 2(1 - m_g^2) \Big[(R^{parallel}_{Free-Free})^2 + (R^{parallel}_{Free-Bound})^2 + (R^{parallel}_{Bound-Bound})^2 + (R^{parallel}_{Free-Free} R^{anti-parallel}_{Free-Free}) \, \partial^2 (R^{parallel}_{Free-Free}) + (R^{parallel}_{Free-Bound} R^{anti-parallel}_{Free-Bound}) \, \partial^2 (R^{parallel}_{Free-Bound}) + (R^{parallel}_{Bound-Bound} R^{anti-parallel}_{Bound-Bound}) \, \partial^2 (R^{parallel}_{Bound-Bound}) \Big] \Big\}^N ,$$

N is the number of atoms in graphene.

Defects and *f(R)* gravity

- Gravity produced by hexagonal molecules is different from gravity produced by pentagonal or heptagonal molecules.
- The wormhole is achieved for $f(R)=R+\alpha R^2+\dots$ by regular hexagonal molecules. R^2 plays the role of "exotic matter" and the wormhole is stable. No violation of energy conditions (S.C. et al. Phys. Rev. D 91 (2015) 124019, S.C. et al. Phys. Lett.B 730 (2014) 280).
- Geometrical defects give rise to currents.
- Graphene becomes a superconductor by improving the number of defects.
- Gravitons have a similar role to the Cooper pairs.
- Graphene wormhole behaves like a cosmological solution!
- Int.Jou. Mod. Phys.D 26 (2017) 1750094

$f(R)$ gravity

The general gravitational action is $$\mathcal{A} = \int d^4x\sqrt{-g}[f(R) + \mathcal{X}\mathcal{L}_m],$$

S. Capozziello and M. De Laurentis, Phys. Rep. 509, 167 (2011).

The field equations, in metric formalism

$$f'(R)R_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box_g f'(R) = \frac{\mathcal{X}}{2}T_{\mu\nu},$$

and the trace $$3\Box f'(R) + f'(R)R - 2f(R) = \frac{\mathcal{X}}{2}T,$$

In particular, the Lagrangian $\mathcal{L} = R + \alpha R^2 + 2\chi\mathcal{L}_M$ is useful in cosmology

A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).

The trace $$6\alpha\Box R - R = \chi T$$ ⇒ $$\Box R - \frac{1}{6\alpha}(R + \chi T) = 0.$$

Effective scalar mass related to curvature $$-\frac{1}{6\alpha} = \omega^2 = m^2.$$

In terms of graphene's couplings

Combining electrons

$$\Psi^{\dagger a,L}\psi_a^U = \Psi^{\dagger a,U}\psi_a^L = \Psi^{\dagger a,L}\Psi_a^U = \Psi^{\dagger a,U}\Psi_a^L = \psi^{\dagger a,U}\psi_a^L = \psi^{\dagger a,L}\psi_a^U = l_1$$

$$\Psi^{\dagger a,U}\psi_a^U = \Psi^{\dagger a,L}\psi_a^L = \Psi^{\dagger a,U}\Psi_a^U = \Psi^{\dagger a,L}\Psi_a^L = \psi^{\dagger a,U}\psi_a^U = \psi^{\dagger a,L}\psi_a^L = l_2,$$

- l_1 coupling for anti-parallell spin electrons
- l_2 coupling for parallel spin electrons

combining the couplings we get

Repulsion $$R^{parallel}_{Free-Free} = R^{parallel}_{Free-Bound} = R^{parallel}_{Bound-Bound} \approx l_2 - l_2'$$

Attraction $$R^{anti-parallel}_{Free-Free} = R^{anti-parallel}_{Free-Bound} = R^{anti-parallel}_{Bound-Bound} \approx l_1 + l_1'.$$

Current dynamics on graphene

Action for an atom in the graphene

$$S_{co-atom} \approx V \int d\cos\theta\, \Sigma_{n=1}^{p} \Big\{ 6m_g^2\lambda^2 \left[l_1 - l_2 - l_1' + l_2' + (l_1')^2 - (l_2')^2\right] -3(1-m_g^2)\left[2l_1^2 + 2l_2^2 + 2(l_1')^2 + 2(l_2')^2 + l_1^2 l_2^2 (l_1^2 + l_2^2)''\right] \Big\}^{\frac{1}{2}}$$

- l_2 is the coupling for parallel spins, l_1 is the coupling for antiparallel spins
- V is the atomic volume
- The couplings depend on the angle θ between two electrons in a graphene atom.

Graphene Equations

Equations of motions of electrons in graphene:

$$\Big\{m_g^2 l_1' \left[\lambda^2 - (1+2l_1^2 l_2^2)\right] \{6m_g^2\lambda^2[l_1 - l_2 - l_1' + l_2' + (l_1')^2 - (l_2')^2]$$
$$-3(1-m_g^2)\left[2l_1^2 + 2l_2^2 + 2(l_1')^2 + 2(l_2')^2 + l_1^2 l_2^2 (l_1^2 + l_2^2)''\right]\}^{-\frac{1}{2}}\Big\}' =$$
$$\{(1-m_g^2) l_1 \left[1 + 3l_2^2(l_1^2 + l_2^2)''\right] + m_g^2\lambda^2\}\,\{6m_g^2\lambda^2[l_1 - l_2 - l_1' + l_2' + (l_1')^2 - (l_2')^2]$$
$$-3(1-m_g^2)[2l_1^2 + 2l_2^2 + 2(l_1')^2 + 2(l_2')^2 + l_1^2 l_2^2(l_1^2 + l_2^2)'']\}^{-\frac{1}{2}},$$

$$\Big\{l_2' \left[(1-m_g^2)(1-2l_1^2 l_2^2) - m_g^2\lambda^2\right] \{6m_g^2\lambda^2[l_1 - l_2 - l_1' + l_2' + (l_1')^2 - (l_2')^2]$$
$$-3(1-m_g^2)[2l_1^2 + 2l_2^2 + 2(l_1')^2 + 2(l_2')^2 + l_1^2 l_2^2(l_1^2 + l_2^2)'']\}^{-\frac{1}{2}}\Big\}' =$$
$$\{(1-m_g^2) l_2 [1 - 3l_1^2(l_1^2 + l_2^2)''] - m_g^2\lambda^2\}\,\{6m_g^2\lambda^2[l_1 - l_2 - l_1' + l_2' + (l_1')^2 - (l_2')^2]$$
$$-3(1-m_g^2)[2l_1^2 + 2l_2^2 + 2(l_1')^2 + 2(l_2')^2 + l_1^2 l_2^2(l_1^2 + l_2^2)'']\}^{-\frac{1}{2}}.$$

solutions

$$l_1 \approx \cos(\theta_1)$$
$$l_2 \approx \cos(\theta_2) = (1-m_g^2)\cos(\theta_1) - m_g^2\lambda^2 \sin(\theta_1).$$

..and then

$$\Psi = \psi \approx \sqrt{\cos(\theta)}.$$

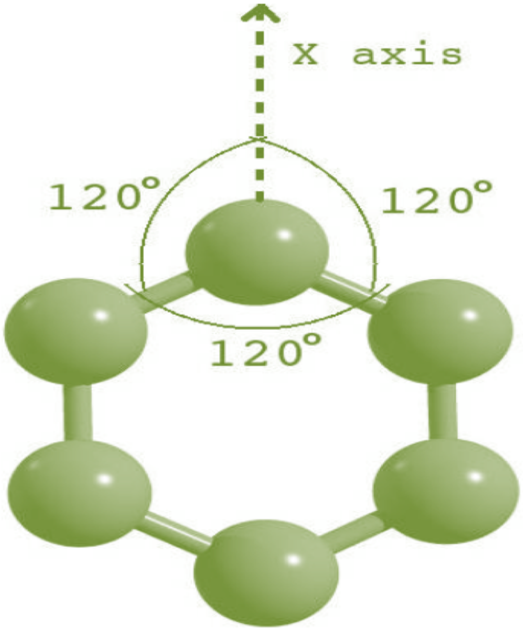
…we can evaluate curvature and current densities

Graphene without defects (hexagonal)

$$R^{anti-parallel}_{Free/Bound-Free/Bound} = l_1^{1-1} + l_1^{1-2} + l_1^{1-3} + (l_1')^{1-1} + (l_1')^{1-2} + (l_1')^{1-3} = 0$$
$$R^{parallel}_{Free/Bound-Free/Bound} = l_2^{1-1} + l_2^{1-2} + l_2^{1-3} - (l_2')^{1-1} - (l_2')^{1-2} - (l_2')^{1-3} = 0,$$

$$J \approx 0.$$

For graphene without defects, the **current density is zero**. Electrons do not move in any direction.

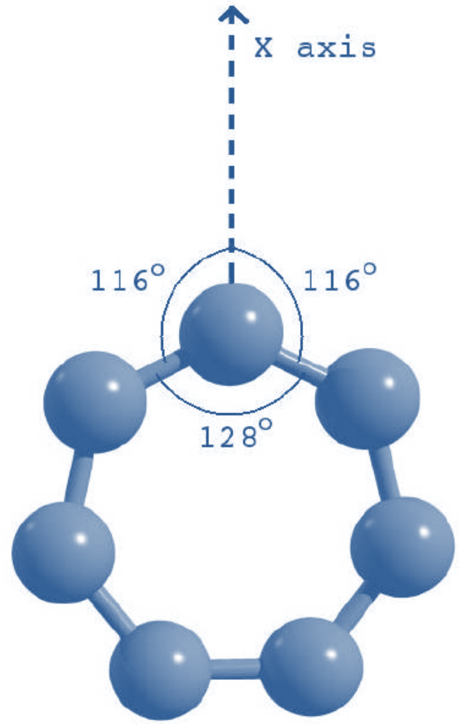


Graphene with heptagonal defects

$$R^{anti-parallel}_{Free/Bound-Free/Bound} = l_1^{1-1} + l_1^{1-2} + l_1^{1-3} + (l_1')^{1-1} + (l_1')^{1-2} + (l_1')^{1-3} = 0.132$$
$$R^{parallel}_{Free/Bound-Free/Bound} = l_2^{1-1} + l_2^{1-2} + l_2^{1-3} - (l_2')^{1-1} - (l_2')^{1-2} - (l_2')^{1-3} = -0.048.$$

$$J \approx 0.458.$$

Electrons are repelled by neighbor molecules and move along the X-axis (the curvature produced by parallel spins is larger than the curvature produced by anti-parallel spins and therefore a negative force is applied to electrons and they move in opposite directions with respect to the molecule). **The current density is positive.**

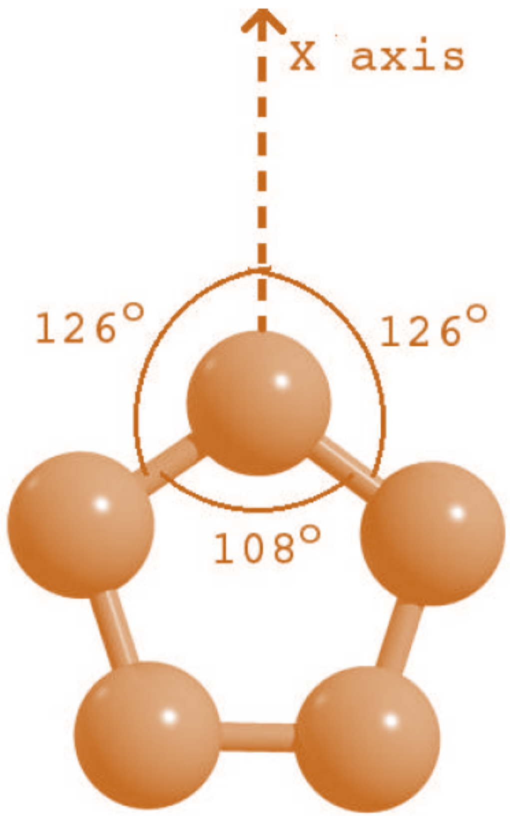


Graphene with pentagonal defects

$$R^{anti-parallel}_{Free/Bound-Free/Bound} = l_1^{1-1} + l_1^{1-2} + l_1^{1-3} + (l_1')^{1-1} + (l_1')^{1-2} + (l_1')^{1-3} = -0.174$$
$$R^{parallel}_{Free/Bound-Free/Bound} = l_2^{1-1} + l_2^{1-2} + l_2^{1-3} - (l_2')^{1-1} - (l_2')^{1-2} - (l_2')^{1-3} = 0.063.$$

$$J \approx -0.539.$$

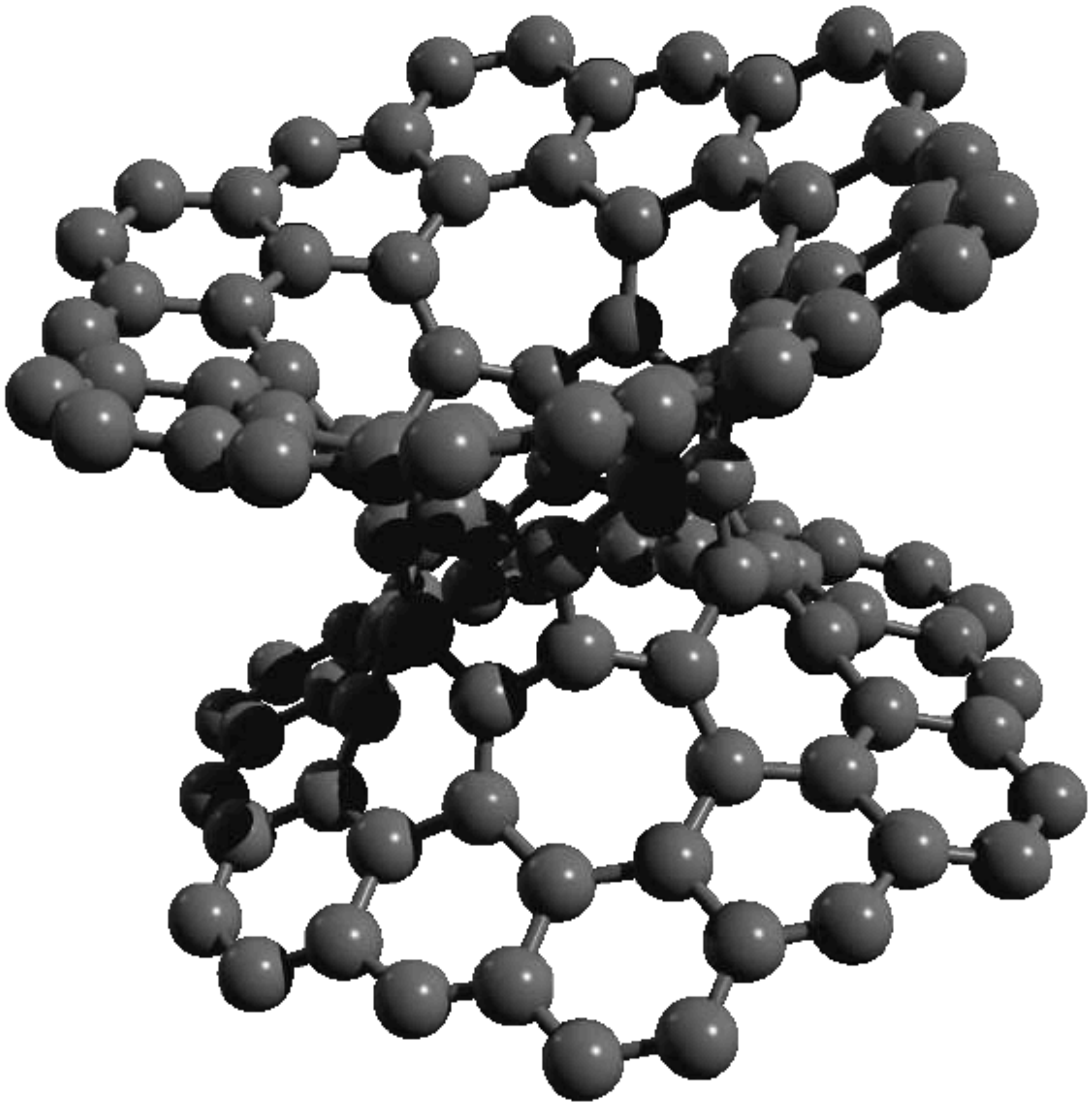
The negative value of the current density implies that the electrons are absorbed by pentagonal defects and move along the negative X-axis, i.e. this type of defects induces a force to the free electrons and leads them to move towards the molecule. **The current density is negative.**



The graphene wormhole

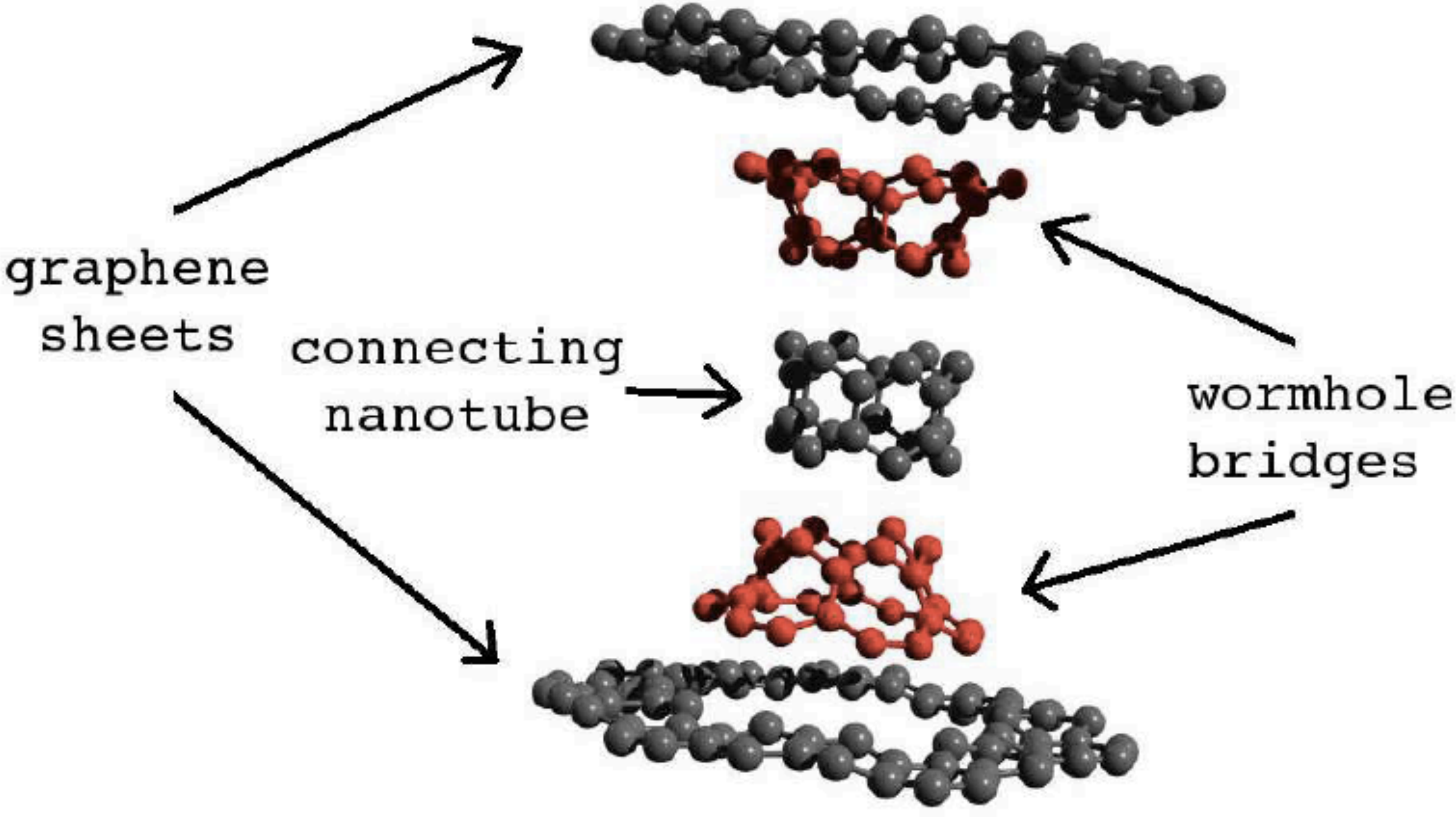
Such an object is created when the structure of 2 plain graphene sheets is connected and stabilized by the presence of heptagonal and pentagonal defects

The structure in the picture is produced by the combination of 12 defects.



This is an **exact solution!**

The graphene wormhole



The current density in a graphene wormhole

- The number of defects in the graphene wormhole can differ from 2 to 12 defects, i.e. from 1 to 6 defects at each side. The process can be iterative with n+6

$$J \approx 6 \cdot 0.458 = 2.748.$$

- The upper and lower sheets have not the geometry of the plain graphene, and the corresponding current density changes, as well as the current density, close to the middle of the connecting nanotube.

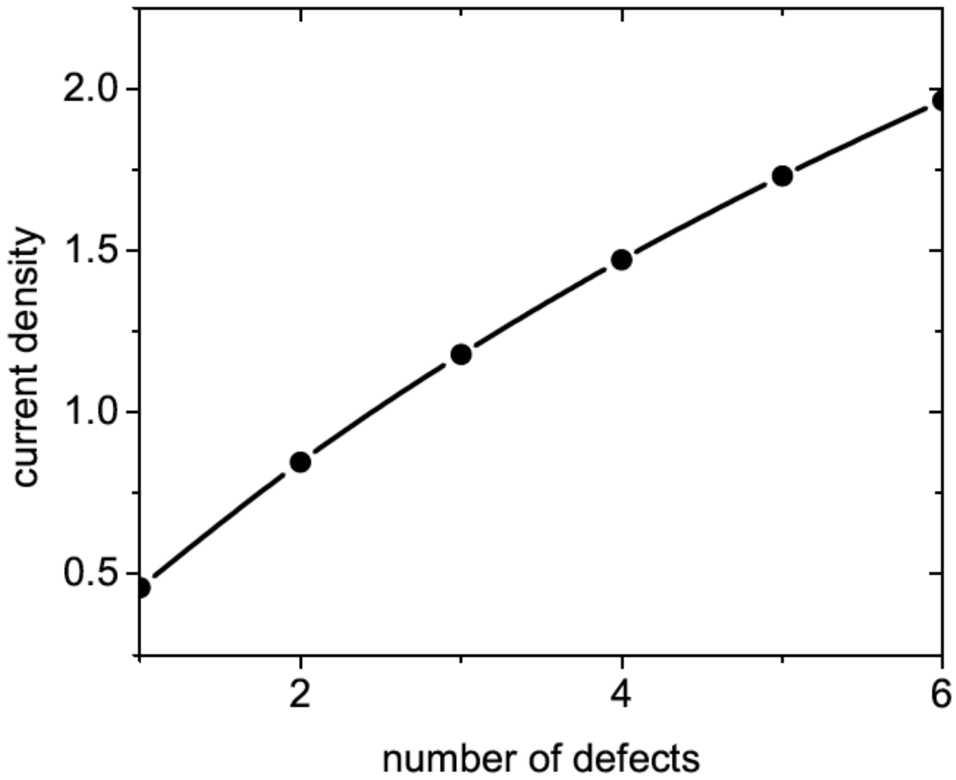
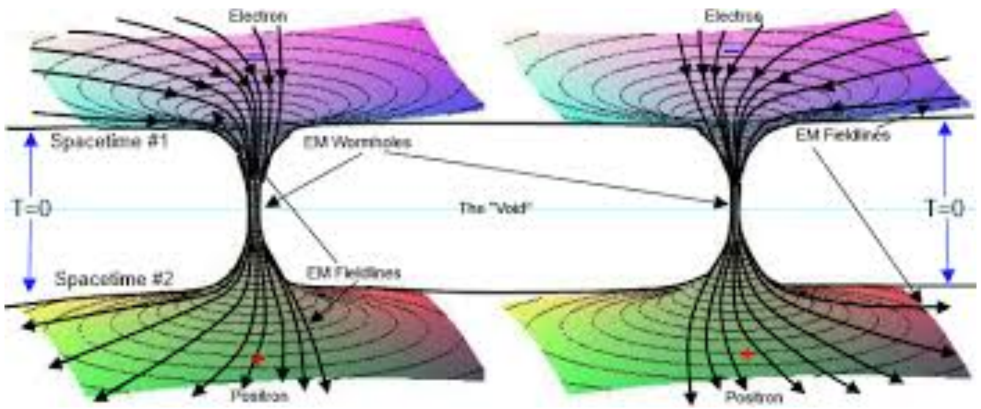


Table of Analogies

Graphene sheet	Space-time sheet
Graphene wormhole	Space-time wormhole
R^2 curvature terms	Exotic matter
Current densities	Information, flux of particles
Electronic circuits	2 anti-parallel doped WHs
Circulating currents	Close Time Curves
Cooper pairs	Gravitons



Cosmology meets superconductivity in the LAB

New Perspectives…..

Open Issues

- Wormhole solutions can be reproduced in Lab?
- Yes…by graphene structures!
- It is possible to achieve stable configurations
- Current densities represent **information** flowing into the wormhole troath
- Main role is played by the defects (pentagonal, eptagonal etc…) that determine the intensity and the direction of current densities

The goals:

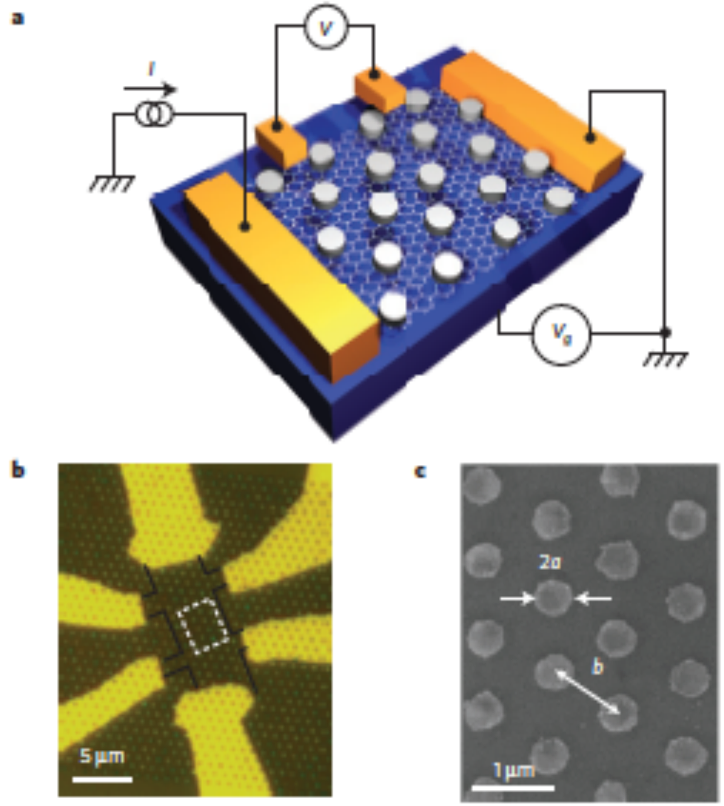
- **Achieving an analogue wormhole by graphene**
- **Stabilize it and controlling the system**
- **Give exact estimation of currents**
- **Standardize the procedure at industrial level**

How generate the supporting structure…
some examples in literature

ARTICLES
PUBLISHED ONLINE: 30 MARCH 2014 | DOI: 10.1038/NPHYS2929
nature physics

Collapse of superconductivity in a hybrid tin-graphene Josephson junction array

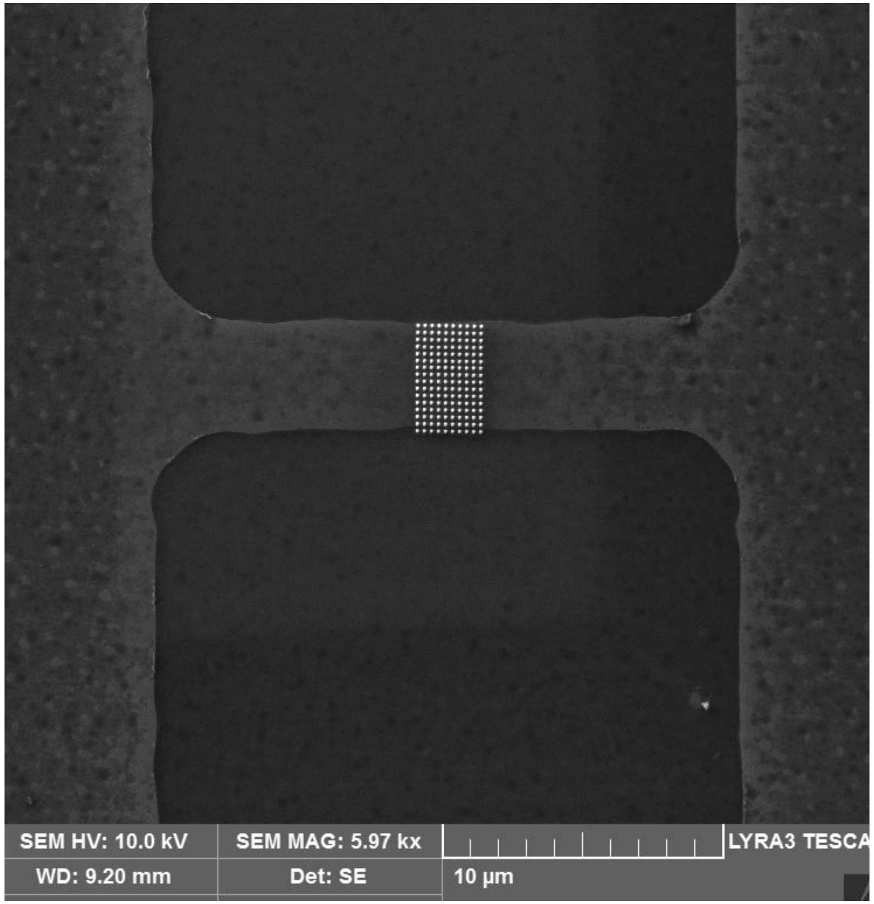
Zheng Han[1,2], Adrien Allain[1,2], Hadi Arjmandi-Tash[1,2], Konstantin Tikhonov[3,4], Mikhail Feigel'man[3,5], Benjamin Sacépé[1,2] and Vincent Bouchiat[1,2]*

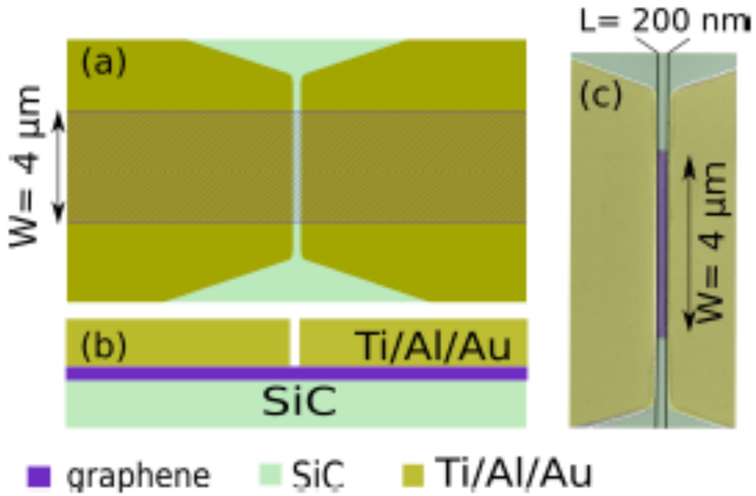


Methods

We used CVD-grown monolayer graphene sheets transferred onto 285 nm oxidized silicon wafer as a 2D diffusive metal. As shown in Fig. 1, the sample was patterned by standard e-beam lithography into a Hall bar geometry (central square area 6 µm^2) and contacted with normal leads (Ti/Au bilayers, 5 nm/50 nm thick). The entire graphene surface was then decorated in a second lithography step by an array of 50-nm-thick Sn discs.

Our DEMONSTRATOR
(the WH as a Josephson junction)





PHYSICAL REVIEW B **94**, 054525 (2016)

Josephson effect in graphene

Al-graphene-Al

Collaboration between Napoli and Chalmers (Sweden).
We used an already available technology in order to get the graphene suspension

LETTERS

Vol 446 | 1 March 2007 | doi:10.1038/nature05545

The structure of suspended graphene sheets

Jannik C. Meyer[1], A. K. Geim[2], M. I. Katsnelson[3], K. S. Novoselov[2], T. J. Booth[2] & S. Roth[1]

Selected for a Viewpoint in *Physics*

PRL **101,** 096802 (2008) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2008

Temperature-Dependent Transport in Suspended Graphene

K. I. Bolotin,[1] K. J. Sikes,[2] J. Hone,[3] H. L. Stormer,[1,2,4] and P. Kim[1]

[1]*Department of Physics, Columbia University, New York, New York 10027, USA*

[2]*Department of Applied Physics, Columbia University, New York, New York 10027, USA*

[3]*Department of Mechanical Engineering, Columbia University, New York, New York 10027, USA*

'4, USA

endent for $5 < T <$

on and a mobility

reases with increas-

. At $T = 240$ K the

rge neutral point we

ent with a density

PRL **105,** 256806 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Local Compressibility Measurements of Correlated States in Suspended Bilayer Graphene

J. Martin, B. E. Feldman, R. T. Weitz, M. T. Allen, and A. Yacoby

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 8 September 2010; published 15 December 2010)

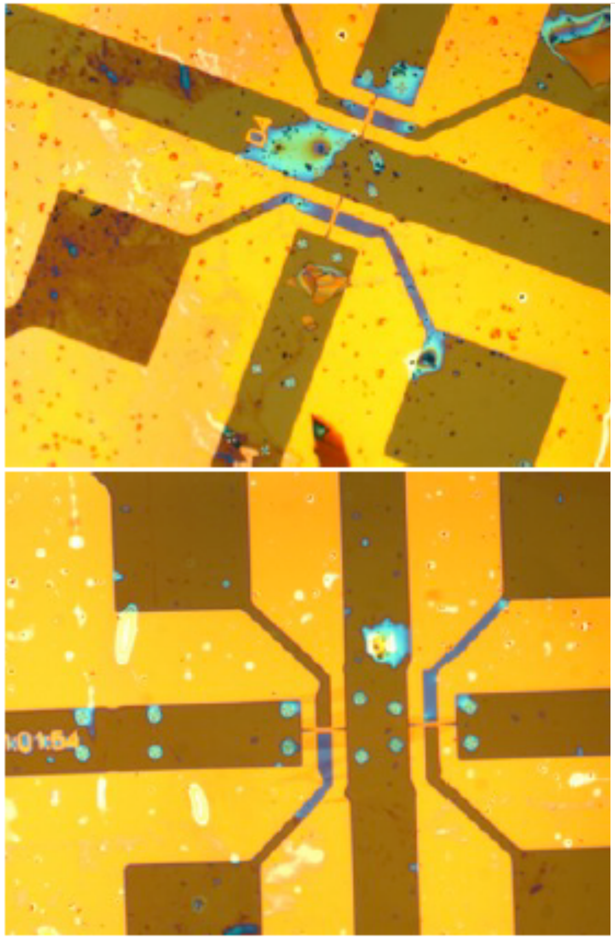
nature materials LETTERS

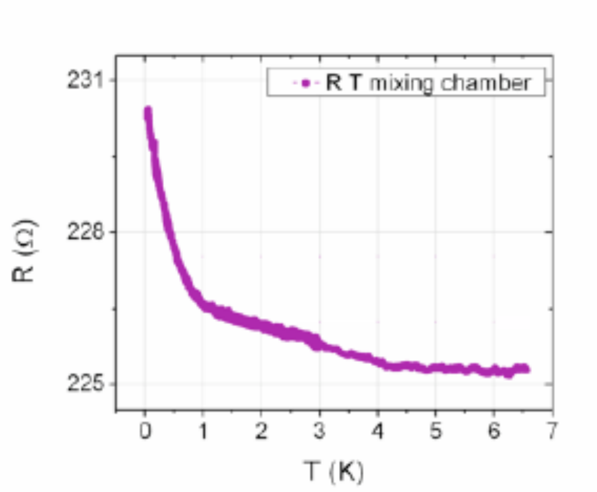
PUBLISHED ONLINE: 1 JULY 2012 | DOI: 10.1038/NMAT3370

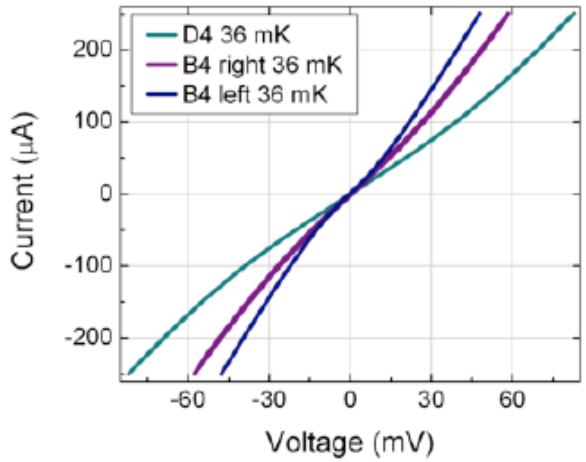
The nature of strength enhancement and weakening by pentagon–heptagon defects in graphene

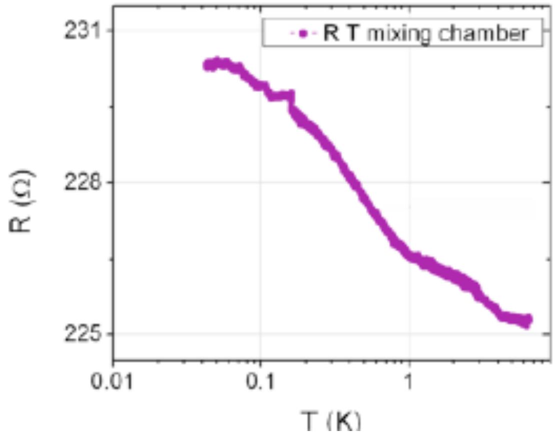
Yujie Wei[1]*, Jiangtao Wu[1], Hanqing Yin[1], Xinghua Shi[1], Ronggui Yang[2]* and Mildred Dresselhaus[3]

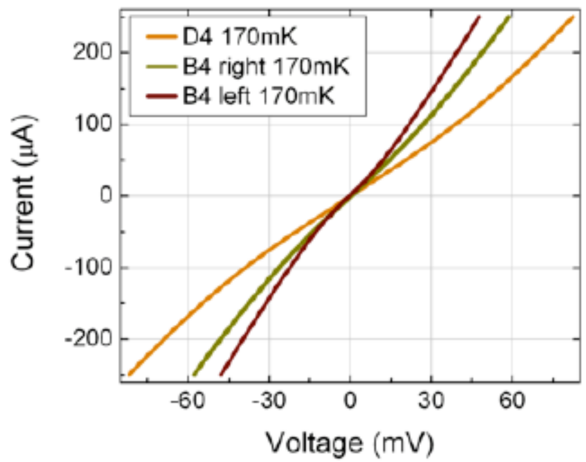
Expertise in Napoli









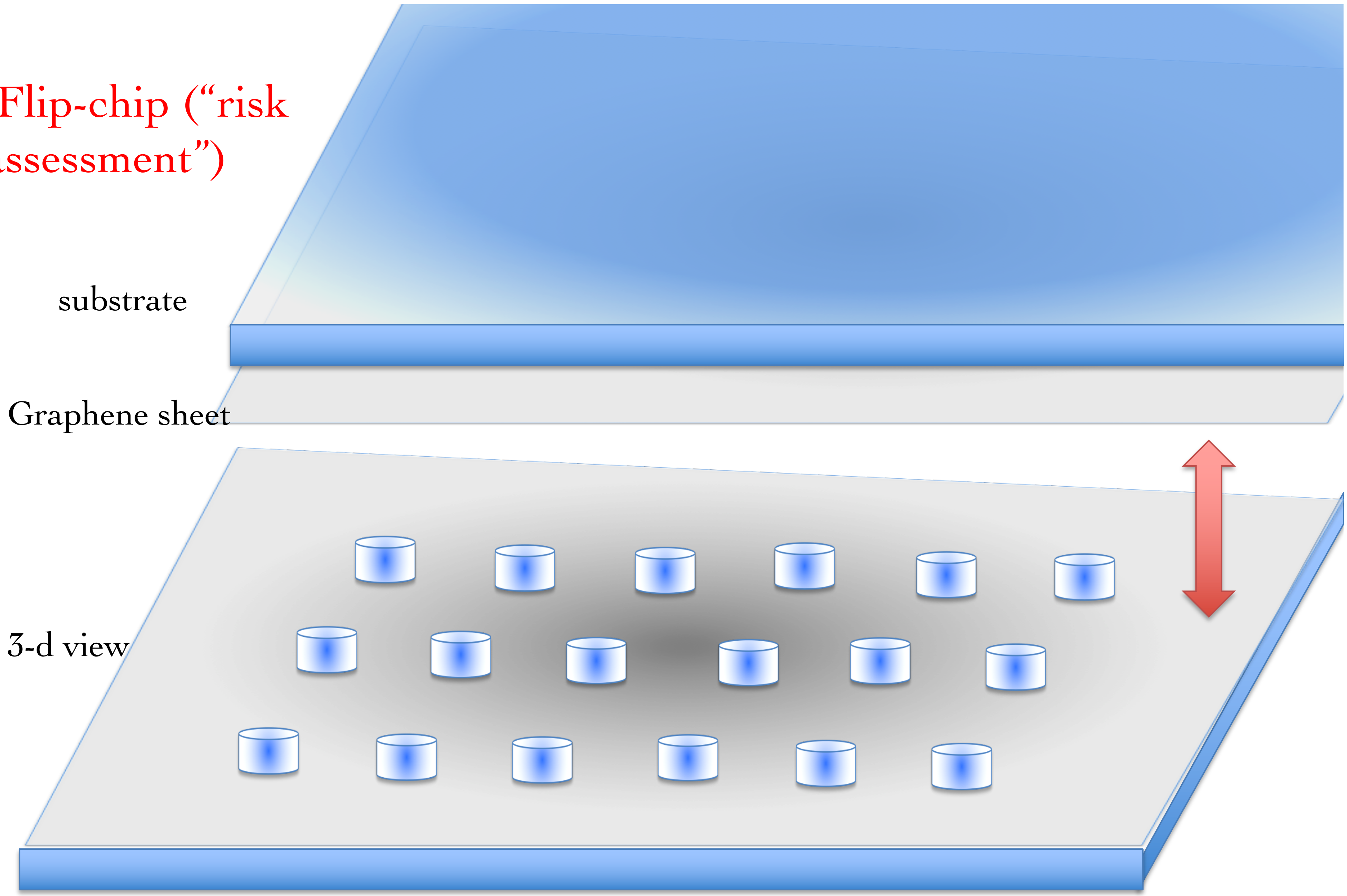


Graphene suspended on YBCO

Collaboration between Napoli and Chalmers

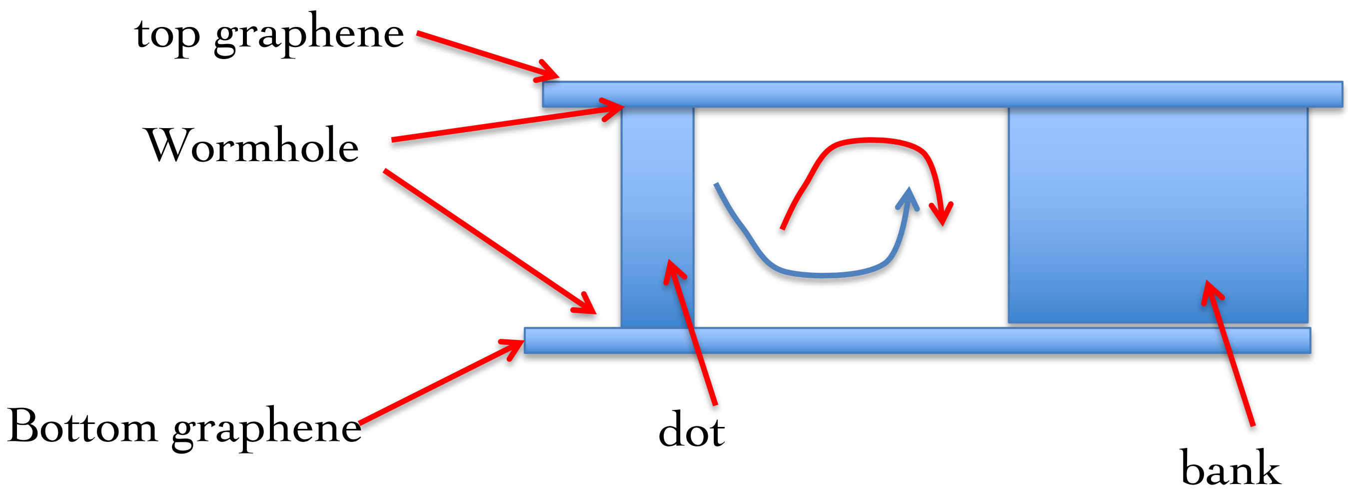
graphene

Flip-chip ("risk assessment")



If we need a certain quality of graphene, which cannot necessarily be achieved in the suspended configuration, we can use the chip-flip technique. Two substrates, each with a suitable graphene layer, and one of them with nanodots, are clamped togheter.

The "industrial" idea is to measure and control the flow of current within the WH and to produce a superconducting device



This is the pattern of a wormhole device that could be used to produce circular currents depending on the doping of graphene sheets. Advantages: **zero resistance, very small currents** (nanoampere).

"I do not know what it is for, but someone, in the future, can put a fee on it "
(M. Faraday)

In summary

- The graphene-WH is an **analogue gravity system.**

- **Graphene** plays the same role of **space** and then we can build up a relativistic theory

- Graphene **WHs** as **EXACT SOLUTIONS** of **Field Equations**

- We do not need exotic matter to stabilize it but only **geometry.**

- An **"observer"** on the WH throat can send and receive signals (**electric current**).

- Negative or positive **curvature** give rise, respectively, to attractive or repulsive forces.
- **Other possible solutions can be achieved from other Chern-Simons effective gravitational theories**

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