Overview of and Future of BSM Calculations, $Ov\beta\beta$, WIMPs

J. Engel

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New Physics

Some Basic Questions What underlies the Standard Model? Why is the θ parameter in QCD so small? Why is there more matter than antimatter in our universe? Are neutrinos Majorana?

New Physics

Some Basic Questions		
What underlies the Standard Model?		
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Why is there more matter than antimatter in our universe?		
Are neutrinos Majorana?		

The observation (or continued non-observation) of electric dipole moments and $Ov\beta\beta$ decay will help us address these questions.

Atomic EDMs yield some of the best experimental limits. Heavy diamagnetic atoms are particularly sensitive to new physics within the nucleus.

Many $\beta\beta$ decay candidates are also heavy.

EDMs: How Atoms Can Get Them

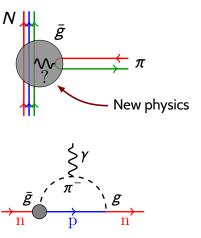
EDMs require CP violation

and

an undiscovered source of *CP* violation is required to explain why there is so much more matter than antimatter.

The source can work its way into nuclei through CP-violating πNN vertices (in chiral EFT)...

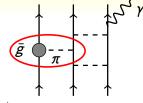
leading, e.g. to a neutron EDM...



EDMs: How Atoms Can Get Them (cont.)

...and to a nuclear EDM from the nucleon EDM or a *T*-violating *NN* interaction:

Note:
$$\mathcal{CP} = \mathcal{T}$$

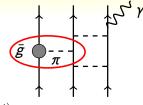


$$V_{PT} \propto \bar{g} \left(\vec{\sigma}_1 \pm \vec{\sigma}_2 \right) \cdot \left(\vec{\nabla}_1 - \vec{\nabla}_2 \right) \frac{\exp\left(-m_\pi |\vec{r}_1 - \vec{r}_2| \right)}{m_\pi |\vec{r}_1 - \vec{r}_2|} + \text{contact term}$$

The \bar{g} 's (isoscalar, isovector and isotensor) depend on source of CP violation.

EDMs: How Atoms Can Get Them (cont.)

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The \bar{g} 's (isoscalar, isovector and isotensor) depend on source of CP violation.

Atoms gets an EDM from nuclei. But electronic shielding replaces nuclear dipole operator with "Schiff operator,"

$$S \approx \sum_{p} r_{p}^{2} z_{p} + \dots ,$$

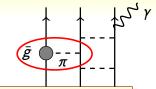
making relevant nuclear quantity the Schiff moment:

$$\langle S \rangle = \sum_{m} \frac{\langle O | S | m \rangle \langle m | V_{PT} | O \rangle}{E_{O} - E_{m}} + c.c.$$

EDMs: How Atoms Can Get Them (cont.)

...and to a nuclear EDM from the nucleon EDM or a *T*-violating *NN* interaction:

Note: CP = T



Job of nuclear-structure theory: compute dependence of $\langle S \rangle$ on the \bar{g} 's (and on the contact term and nucleon EDM).

It's up to QCD to compute the dependence of the \bar{g} vertices on fundamental sources of *CP* violation.

$$S \approx \sum_{p} r_{p}^{2} z_{p} + \dots ,$$

making relevant nuclear quantity the Schiff moment:

$$\langle S \rangle = \sum_{m} \frac{\langle O | S | m \rangle \langle m | V_{PT} | O \rangle}{E_{O} - E_{m}} + c.c.$$

More precisely, because the \bar{g}_i are so small,

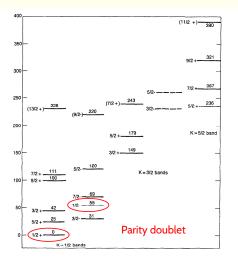
$$\langle S \rangle = \sum_{i} a_{i} g \bar{g}_{i},$$

and we have to calculate the three a_i . These reflect action of both the *S* and V_{PT} operators.

Most heavy nuclei must be treated in something like DFT for now, leading to uncertainty in the a_i that is large and difficult to estimate.

But other observables can help.

²²⁵Ra: Octupole Physics



Unlike in other nuclei, these two states are the whole story.

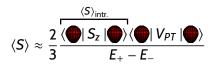


Deformed density

Two members of the parity doublet correspond to the same intrinsic mean-field state:

$$|\frac{1}{2}^{\pm}\rangle\approx\frac{1}{\sqrt{2}}\left(|\textcircled{0}\rangle\pm|\textcircled{0}\rangle\right)$$

and, to good approximation,

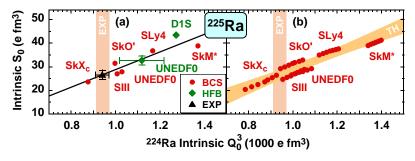


Correlation of $\langle S \rangle_{intr.}$ with Octupole Defm. in ²²⁴Ra



Gaffney et al., 2013

Correlation of $\langle S \rangle_{intr.}$ with Octupole Defm. in ²²⁴Ra

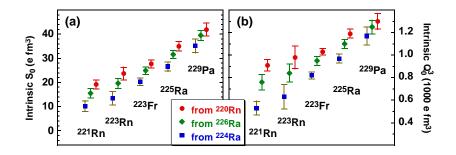


J. Dobaczewski, JE, M. Kortelainen, P. Becker

Correlation with octupole moment of ²²⁵Ra even better.

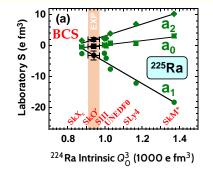
Will be determined at ANL.

Light Actinides More Generally



The error bars represent statistical uncertainty only, but systematic variation is not large.

Implications for Lab Schiff Moment



Looks good, but situation is more complicated when we include octupole moments in other nuclei. The resulting a_i for ²²⁵Ra:

isoscalar	isovector	isotensor
-0.4 - 0.8	-28	2 – 5

Range doesn't include systematic uncertainty.

Reducing Uncertainty of Lab Moments

The problem is that we don't have information about $\langle V_{PT} \rangle$.

Can β decay constrain its matrix element?

 V_{PT} has same space-spin form as two-body axial-charge operator:

$$A_{2b}^{\mathsf{O}} \propto \vec{\tau}_1 \times \vec{\tau}_2 \left(\vec{\sigma}_1 + \vec{\sigma}_2 \right) \cdot \left(\vec{\nabla}_1 - \vec{\nabla}_2 \right) \frac{e^{-m_{\pi} \left| \vec{r}_1 - \vec{r}_2 \right|}}{m_{\pi} \left| \vec{r}_1 - \vec{r}_1 \right|}$$

Because the one-body part,

$$A_{2b}^{0} \propto rac{1}{M} ec{\sigma} \cdot ec{
abla}$$

is suppressed by q/M, the pion-exchange contribution is significant. Also, the effective one-body form of V_{PT} :

$$V_{PT}^{\text{eff}} \propto \vec{\sigma} \cdot \vec{\nabla} \rho$$

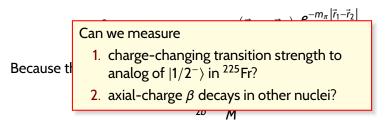
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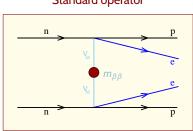


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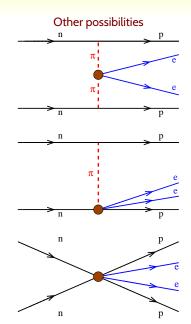
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ho$$

has a similar form.

Review of $O_{\nu\beta\beta}$ Decay

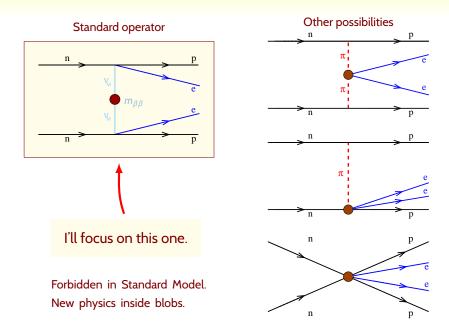


Standard operator

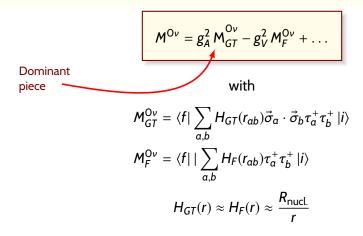


Forbidden in Standard Model. New physics inside blobs.

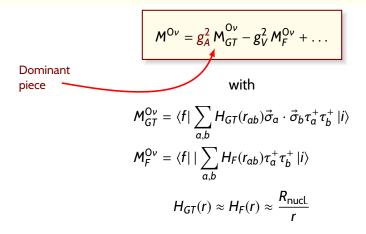
Review of $O_{\nu\beta\beta}$ Decay



Nuclear Matrix Element (Simplified)



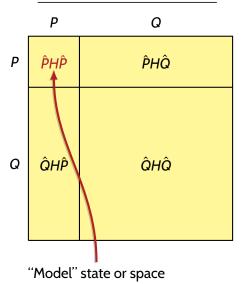
Nuclear Matrix Element (Simplified)



Also:

$$M_{2\nu} = g_A^2 \sum_m \frac{\langle f | \sum_a \vec{\sigma}_a \tau_a^+ | m \rangle \cdot \langle m | \sum_b \vec{\sigma}_b \tau_b^+ | i \rangle}{E_m - \frac{E_f + E_i}{2}}$$

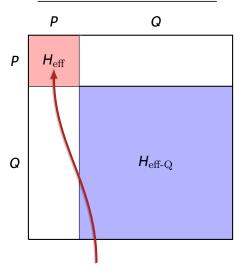
Partition of Full Hilbert Space



P = states we care about Q = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with $H_{\rm eff}$ in P reproducing most important eigenvalues.

Partition of Full Hilbert Space

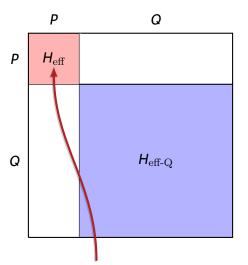


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Now includes more.

Partition of Full Hilbert Space



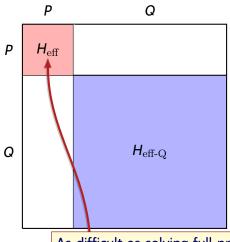
P = states we care about *Q* = the rest

Task: Find unitary transformation to make H block-diagonal in P and Q, with $H_{\rm eff}$ in P reproducing most important eigenvalues.

For transition operator \hat{M} , must apply same transformation to get $\hat{M}_{\rm eff}$.

Now includes more.

Partition of Full Hilbert Space



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Task: Find unitary transformation to make H block-diagonal in P and Q, with $H_{\rm eff}$ in P reproducing most important eigenvalues.

For transition operator \hat{M} , must apply same transformation to get $\hat{M}_{\rm eff}$.

As difficult as solving full problem. But *N*-body effective Nov operators with N > 2 or 3 can be treated approximately.

Coupled Clusters

Wave function ansatz:

$$\begin{split} \Psi \rangle &= e^{\hat{T}} | \text{Slater det.} \rangle \\ &= \exp \left(t_{ij}^1 \alpha_i^{\dagger} \alpha_j + t_{ijkl}^2 \alpha_i^{\dagger} \alpha_j^{\dagger} \alpha_k \alpha_l + \dots \right) | \text{Slater det.} \rangle \end{split}$$

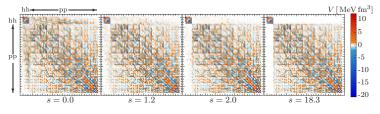
Then using a similarity transform:

$$\hat{H} \longrightarrow e^{-\hat{T}} \hat{H} e^{\hat{T}},$$

means that you work with a Slater determinant rather than the fully correlated state when building excitations.

In-Medium Similarity Renormalization Group

Flow equation for effective Hamiltonian. Gradually decouples shell-model space.

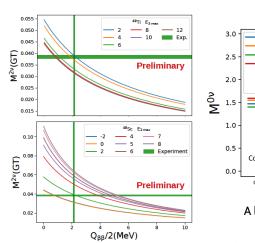


Hergert et al.

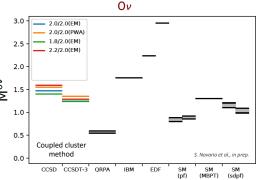
Trick is to keep all 1- and 2-body terms in *H* at each step after normal ordering (approximate treatment of 3-, 4-...terms).

If model space contains just a single state, approach yields ground-state energy. If it is larger, result is effective interaction and operators.

$\beta\beta$ Decay in ⁴⁸Ca with Coupled Clusters



2ν



A little larger than shell-model result.

From G. Hagen

Small Fly in the Ointment



must be supplemented, at same order in chiral EFT, by short-range operator (representing high-energy v exchange):

Coefficient of this term is unknown.



Two-Body Axial Current and Connection with β Decay

β Decay (simplified) with electron lines omitted

Leading order in χ EFT:



Usual β-decay current. Finite-momentum corrections at next order.

plus a contact

Two-Body Axial Current and Connection with β Decay

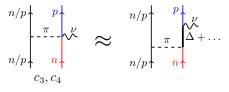
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Higher order:



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Two-Body Axial Current and Connection with β Decay

β Decay (simplified) with electron lines omitted

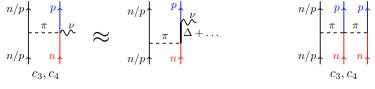
Leading order in χ EFT:



Usual β-decay current. Finite-momentum corrections at next order.

Higher order:

Coefficients same as in three-body interaction:



plus a contact

Product of Currents

In first quantization, let

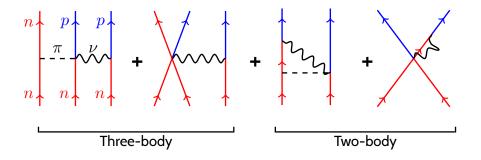
$$\sum_{i} \hat{O}_{i}^{1b} = 1\text{-body operator in } J^{+}$$
$$\sum_{ij} \hat{O}_{ij}^{2b} = 2\text{-body operator in } J^{+}$$

$$J^{+}(\vec{q})J^{+}(-\vec{q}) = \sum_{ij} \hat{O}_{i}^{1b}\hat{O}_{j}^{1b} + \sum_{ijk} \left(\hat{O}_{ij}^{2b}\hat{O}_{k}^{1b} + \hat{O}_{i}^{1b}\hat{O}_{jk}^{2b} \right) + 4\text{-body}$$
$$+ \sum_{ij} \left(\hat{O}_{ij}^{2b} [\hat{O}_{i}^{1b} + \hat{O}_{j}^{1b}] + [\hat{O}_{i}^{1b} + \hat{O}_{j}^{1b}]\hat{O}_{ij}^{2b} \right)$$

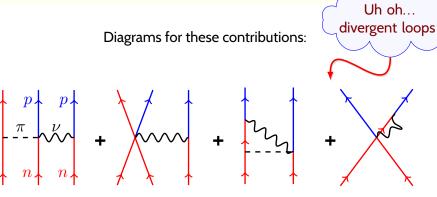
2-body op.

Inclusion of Two-Body Currents

Diagrams for these contributions:



Inclusion of Two-Body Currents



Three-body

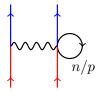
n

Two-body

Prior Work on Effects in Heavy Systems

Javier, Doron, Achim: Symmetric Nuclear Matter

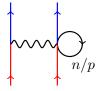
Normal ordered two-body current, to get effective one-body current. Corresponds to:



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Normal ordered two-body current, to get effective one-body current. Corresponds to:



In nuclear matter:

$$g_{A} \longrightarrow g_{A} - g_{A} \frac{\rho}{F_{\pi}^{2}} \left[\frac{c_{d}}{g_{A} \wedge} + \frac{2c_{3}}{3} \frac{q^{2}}{q^{2} + 4m_{\pi}^{2}} + I(\rho, P) \left(\frac{2c_{4} - 3_{3}}{3} + \frac{1}{6m} \right) \right]$$

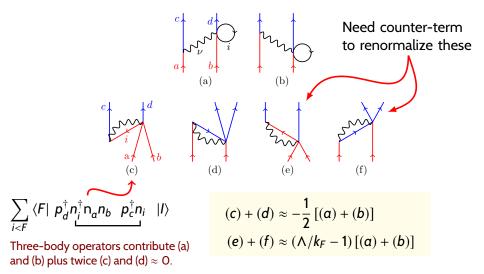
 $I(\rho, P) \approx 2/3$ at nuclear density, with weak dependence on P.

 $O_{\nu\beta\beta}$ decay quenched by about 30%, somewhat less than $2\nu\beta\beta$ decay because of q dependence of effective g_A .

More Complete Nuclear Matter Calculation

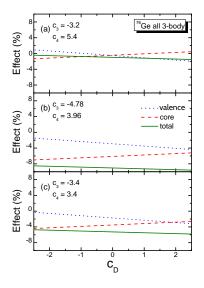
With Simplest Operator: g_A at one-body vertex, c_D at two-body vertex

Goldstone (Time-Ordered) Diagrams



⁷⁶Ge in Shell Model

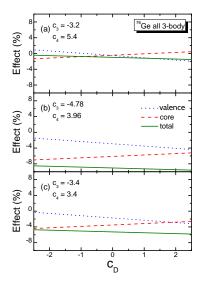
Three-body operators



Approximate ⁷⁶Ge wave function in *fp* shell, inert core underneath.

⁷⁶Ge in Shell Model

Three-body operators



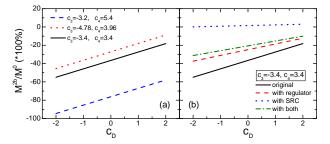
Approximate ⁷⁶Ge wave function in *fp* shell, inert core underneath.

Takeaway: Effects of threebody operators are <u>small</u>.

Two-Body Operators

With Nucleon Form Factors

Right side includes usual modifications.

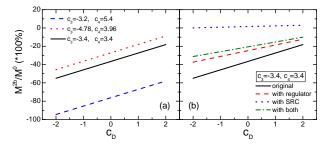


Almost entire contribution from c_D and short-range parts of c_3 , c_4 .

Two-Body Operators

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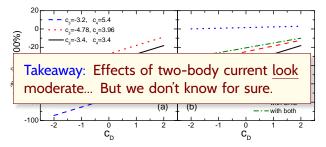
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Need counter term, just like in leading order. Help!

Two-Body Operators

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So, to Sum Up...

1. Schiff moments, for now, must be calculated in DFT, which makes drastic and uncontrolled approximations. Other observables can help constrain calculations.

Can β -decay rates do that?

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Can β -decay rates do that?

 Application of chiral EFT to Ovββ decay implies short-range contribution to neutrino exchange with unknown coefficient. A similar issue hampers our ability to fully examine effects of the two-body current in Ovββ decay.

The part for which we do know coefficients seems to quench very little, however.



\begin{Acknowledgments} Thanks!

 $\ensuremath{\mathsf{Acknowledgments}}\$

 $\ensuremath{\mathsf{Talk}}\$