MULTIPOLE OPERATORS FOR RECOIL CORRECTIONS & TENSOR INTERACTIONS

ECT* 2019

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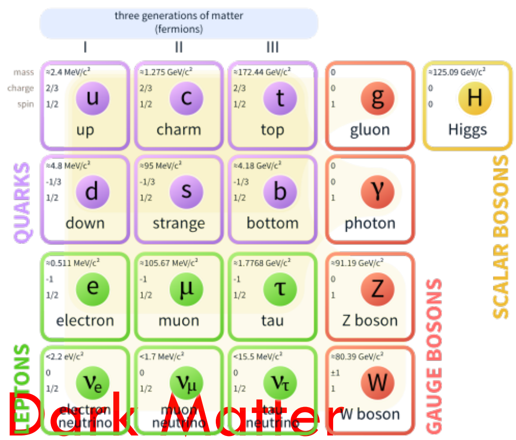
To “translate” the needed operators to the “language” of the nuclear structure calculations

- Introduction
- Multipole Expansion
- Effect of BSM operators on the cross section
  - Emphasis on tensor contributions & operators
- Unique first forbidden decays and their prospect as BSM probes
- Recoil corrections & Operators

Beyond Standard Model (BSM)

- **Elementary particles**



- Dark Matter

- **Fundamental forces**
  - Electromagnetic
  - Strong
  - Weak
    - The Neutrino has mass, even though according to the SM it should not

Ongoing experimental searches:

- Dark Matter Direct detection
- BSM signatures in precision nuclear β-decay experiments

Purpose: Provide the nuclear theory needed to analyze both fields of BSM experimental searches (DM & exotic "weak" interactions)

BSM Signatures @Weak Interaction

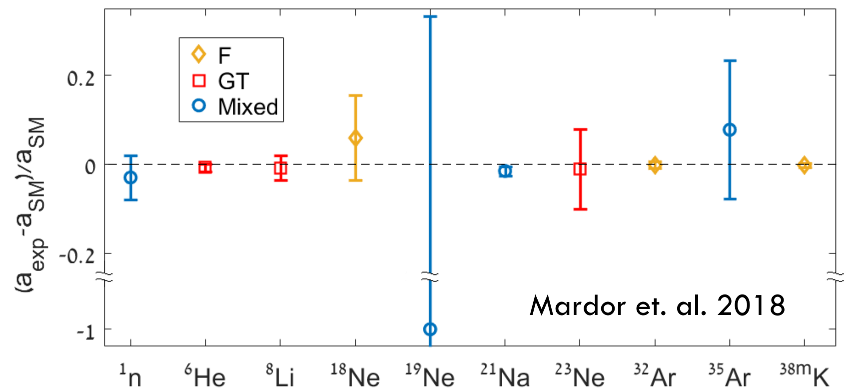
- A-priori:
  - Scalar
  - Pseudoscalar
  - ✔ Vector
  - ✔ Axial vector
  - Tensor
- Experimental: "V-A"
  - Nuclear β-decays
- The SM is incomplete

$$\widehat{\mathcal{H}}_{\mathrm{W}} \sim C_{sym} \int d^3\vec{x}\, \hat{j}\,(\vec{x})\, \hat{\mathcal{J}}\,(\vec{x})$$

Coupling constant

Probe "current" of known symmetry

Nuclear "current" of the same symmetry



>> Ongoing searches for Scalar & Tensor couplings in precision nuclear β-decay experiments

The smoking gun we are looking for

$$\text{Beta Decay Rate } (allowed) \propto 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_{Fierz}\frac{m_e}{\epsilon}$$

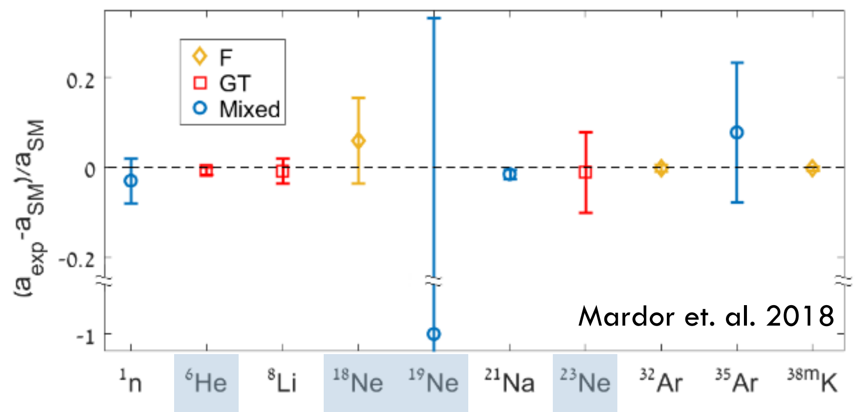
$\vec{\beta}, m_e, \epsilon$ - electron's momentum, mass, energy.
$\hat{\nu}$ - neutrino's momentum

β-ν angular correlation:

- Fermi: $a_{\beta\nu}^{0+} \approx 1 - \frac{|C_S|^2+|C_S'|^2}{|C_V|^2}$
- GT: $a_{\beta\nu}^{1+} \approx -\frac{1}{3}\left(1 - \frac{|C_T|^2+|C_T'|^2}{|C_A|^2}\right)$

Fierz term:

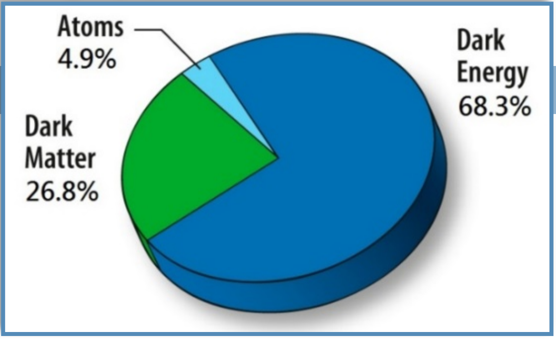
- Fermi: $b^{0+} = 2\,\frac{C_S+C_S'}{C_V}$
- GT: $b^{1+} = 2\,\frac{C_T+C_T'}{C_A}$



C_V, C_A - Vector, Axial vector coupling constants, known
C_S, C_S' - Scalar coupling constants
C_T, C_T' - Tensor coupling constants

>> Required: BSM predictions vs. SM

Dark Matter Direct Detection

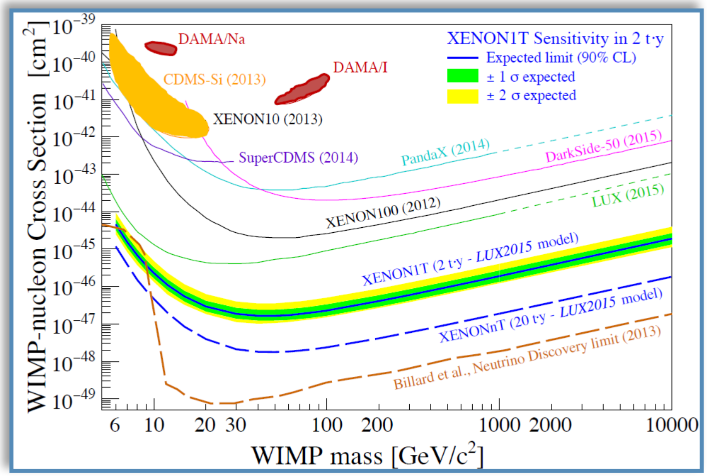


Promising candidates – **WIMPs:**
Weakly-**I**nteracting **M**assive **P**articles

- Challenge – Direct detection:
  - Measuring WIMP scattering off nuclei on detectors
    - Nuclear matrix elements & structure factors
  - Detection capabilities: $q \sim 100\ MeV/c$
  - In search for different types of couplings with standard material:
    - Scalar, Pseudoscalar, Vector, Axial and Tensor

q - momentum transfer

>> Previous works ignored the Tensor symmetry coupling



N. Anand, A. Liam Fitzpatrick, W. C. Haxton, Phys. Rev. C 89, 065501 (2014)
A. Liam Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, Y. Xu, J. Cosmol. Astropart. Phys, 2013(02):004, 2013.

Multipole Expansion

Nuclear β-decay Formalism

Differential distribution:

$$\frac{d^5\omega_{\beta^\mp}}{d\epsilon \frac{d\Omega_k}{4\pi}\frac{d\Omega_\nu}{4\pi}} = \frac{2G^2}{\pi^2}\frac{1}{2J_i+1}\left(Q-\epsilon\right)^2 k\epsilon F^\pm\left(Z_f,\epsilon\right) C_{\text{corrections}} \cdot \Theta\left(q, \vec{\beta}\cdot\hat{\nu}\right)$$

$\vec{k},\epsilon$ - electron's momentum, energy, $\vec{\beta} \equiv \frac{\vec{k}}{\epsilon}$, $\hat{\nu}$ - neutrino's momentum, $F^\mp$ - Fermi function, Z_f - daughter nucleus's charge, J_i - mother nucleus's angular momentum, Q - Q-value

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$$L_0(Z,W)U(Z,W)D_{\mathrm{FS}}(Z,W,\beta_2)R(W,W_0)R_N(W,W_0,M)Q(Z,W)S(Z,W)X(Z,W)r(Z,W)C(Z,W)D_C(Z,W,\beta_2)$$

$\vec{k},\epsilon$ - electron's momentum, energy, $\vec{\beta}\equiv\frac{\vec{k}}{\epsilon}$, $\hat{\nu}$ - neutrino's momentum, $F^{\mp}$ - Fermi function, Z_f - daughter nucleus's charge, J_i - mother nucleus's angular momentum, Q - Q-value

Item	Effect	Formula	Magnitude
1	Phase space factor [a]	$pW(W_0-W)^2$	Unity or larger
2	Traditional Fermi function	F_0	
3	Finite size of the nucleus	L_0	10^{-1}-10^{-2}
4	Radiative corrections	R	
5	Shape factor	C	
6	Atomic exchange	X	
7	Atomic mismatch	r	
8	Atomic screening	S	10^{-3}-10^{-4}
9	Shake-up	See item 7	
10	Shake-off	See item 7	
11	Isovector correction	C_I	
12	Recoil Coulomb correction	Q	
13	Diffuse nuclear surface	U	
14	Nuclear deformation	D_{FS} & D_C	
15	Recoiling nucleus	R_N	
16	Molecular screening	ΔS_{Mol}	
17	Molecular exchange	Case by case	
18	Bound state β decay	Γ_b/Γ_c	Smaller than $1\cdot 10^{-4}$
19	Neutrino mass	Negligible	

p, W - electron's momentum (k), energy (ϵ)
W_0 - Q-value (maximal electron's energy)

L. Hayen, N. Severijns, K. Bodek, D. Rozpedzik, X. Mougeot. Rev. Mod. Phys., 90:015008 (2018)

Nuclear β-decay Formalism

$$\widehat{\mathcal{H}}_W \sim C_T \int d^3\vec{x}\,\hat{j}\,(\vec{x})\,\hat{\mathcal{J}}\,(\vec{x})$$

Probe current

Nuclear current

Differential distribution:

$$\frac{d^5\omega_{\beta^{\mp}}}{d\epsilon\frac{d\Omega_k}{4\pi}\frac{d\Omega_\nu}{4\pi}} = \frac{2G^2}{\pi^2}\frac{1}{2J_i+1}\left(Q-\epsilon\right)^2 k\epsilon F^{\pm}\left(Z_f,\epsilon\right)C_{\text{corrections}}\cdot\Theta\left(q,\vec{\beta}\cdot\hat{\nu}\right)$$

$\vec{k},\epsilon$ - electron's momentum, energy, $\vec{\beta}\equiv\frac{\vec{k}}{\epsilon}$, $\hat{\nu}$ - neutrino's momentum, $F^{\mp}$ - Fermi function, Z_f - daughter nucleus's charge, J_i - mother nucleus's angular momentum, Q - Q-value

$$L_0(Z,W)U(Z,W)D_{FS}(Z,W,\beta_2)R(W,W_0)R_N(W,W_0,M)Q(Z,W)S(Z,W)X(Z,W)r(Z,W)C(Z,W)D_C(Z,W,\beta_2)$$

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Nuclear β-decay Formalism

$$\widehat{\mathcal{H}}_{\mathrm{W}} \sim C_T \int d^3\vec{x}\,\hat{j}\left(\vec{x}\right)\hat{\mathcal{J}}\left(\vec{x}\right)$$

Probe current

Nuclear current

Differential distribution:

$$\frac{d^5\omega_{\beta\mp}}{d\epsilon\frac{d\Omega_k}{4\pi}\frac{d\Omega_\nu}{4\pi}} = \frac{2G^2}{\pi^2}\frac{1}{2J_i+1}\left(Q-\epsilon\right)^2 k\epsilon F^{\pm}\left(Z_f,\epsilon\right)C_{\mathrm{corrections}}\cdot \Theta\left(q,\vec{\beta}\cdot\hat{\nu}\right)$$

$\vec{k},\epsilon$ - electron's momentum, energy, $\vec{\beta}\equiv\frac{\vec{k}}{\epsilon}$, $\hat{\nu}$ - neutrino's momentum, $F^{\mp}$ - Fermi function, Z_f - daughter nucleus's charge, J_i - mother nucleus's angular momentum, Q - Q-value

Multipole Expansion

SM:

$$\Theta\left(q,\vec{\beta}\cdot\hat{\nu}\right) = \sum_{J=1}^{\infty}\left[\left(1-(\hat{\nu}\cdot\hat{q})\left(\vec{\beta}\cdot\hat{q}\right)\right)\left(\left|\langle\|\hat{E}_J\|\rangle\right|^2+\left|\langle\|\hat{M}_J\|\rangle\right|^2\right)\pm\hat{q}\left(\hat{\nu}-\vec{\beta}\right)2\mathfrak{Re}\left(\langle\|\hat{E}_J\|\rangle\langle\|\hat{M}_J\|\rangle^*\right)\right]+$$
$$+\sum_{J=0}^{\infty}\left[\left(1-\hat{\nu}\cdot\vec{\beta}+2\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\right)\left|\langle\|\hat{L}_J\|\rangle\right|^2+\left(1+\hat{\nu}\cdot\vec{\beta}\right)\left|\langle\|\hat{C}_J\|\rangle\right|^2-\hat{q}\left(\hat{\nu}+\vec{\beta}\right)2\mathfrak{Re}\left(\langle\|\hat{L}_J\|\rangle\langle\|\hat{C}_J\|\rangle^*\right)\right],$$

- $\langle\|\hat{O}_J\|\rangle$ – **Reduced matrix element** of a rank J spherical tensor operator $\hat{O}_J$, between the daughter and mother wave functions

Nuclear β-decay Formalism

$$\widehat{\mathcal{H}}_{\mathrm{W}} \sim C_T \int d^3\vec{x}\,\underbrace{\hat{j}\left(\vec{x}\right)}_{\text{Probe current}}\,\underbrace{\hat{\mathcal{J}}\left(\vec{x}\right)}_{\text{Nuclear current}}$$

Differential distribution:

$$\frac{d^5\omega_{\beta\mp}}{d\epsilon\frac{d\Omega_k}{4\pi}\frac{d\Omega_\nu}{4\pi}} = \frac{2G^2}{\pi^2}\frac{1}{2J_i+1}\left(Q-\epsilon\right)^2 k\epsilon F^{\pm}\left(Z_f,\epsilon\right)C_{\mathrm{corrections}}\cdot \Theta\left(q,\vec{\beta}\cdot\hat{\nu}\right)$$

$\vec{k},\epsilon$ - electron's momentum, energy, $\vec{\beta}\equiv\frac{\vec{k}}{\epsilon}$, $\hat{\nu}$ - neutrino's momentum, $F^{\mp}$ - Fermi function, Z_f - daughter nucleus's charge, J_i - mother nucleus's angular momentum, Q - Q-value

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- $\langle\|\hat{O}_J\|\rangle$ – **Reduced matrix element** of a rank J spherical tensor operator $\hat{O}_J$, between the daughter and mother wave functions
- **Multipole operators:**

$$\begin{aligned}
\hat{C}_J(q) &\equiv \int d^3r j_J(qr) Y_J(\hat{r}) J_0(\vec{r}) \\
\hat{L}_J(q) &\equiv \frac{i}{q}\int d^3r \left[\vec{\nabla}\left(j_J(qr)Y_J(\hat{r})\right)\right]\cdot\vec{J}(\vec{r}) \\
\hat{E}_J(q) &\equiv \frac{1}{q}\int d^3r\left[\vec{\nabla}\times\left(j_J(qr)\vec{Y}_{JJ1}(\hat{r})\right)\right]\cdot\vec{J}(\vec{r}) \\
\hat{M}_J(q) &\equiv \int d^3r j_J(qr)\vec{Y}_{JJ1}(\hat{r})\cdot\vec{J}(\vec{r}),
\end{aligned}$$

Nuclear β-decay Formalism

$$\widehat{\mathcal{H}}_{\mathrm{W}} \sim C_T \int d^3\vec{x}\,\hat{j}\left(\vec{x}\right)\hat{\mathcal{J}}\left(\vec{x}\right)$$

Probe current

Nuclear current

Differentia

$$\frac{d^5\omega_{\beta\mp}}{d\epsilon\frac{d\Omega_k}{4\pi}\frac{d\Omega_\nu}{4\pi}} = \frac{4}{\pi} \cdots \left(q,\vec{\beta}\cdot\hat{\nu}\right)$$

$$\Delta J^{\pi}\ :\quad J_i^{\pi_i} \to J_f^{\pi_f}$$

$$|J_i - J_f| \le \Delta J \le J_i + J_f$$

$$\Delta\pi = \pi_i - \pi_f$$

…ectron's momentum, energy, $\vec{\beta} \equiv \frac{\vec{k}}{\epsilon}$, …ino's momentum, $F^{\mp}$ - Fermi function, …ughter nucleus's charge, J_i - mother angular momentum, Q - Q-value

Multipole

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$$+\sum_{J=0}^{\infty}\left[\left(1-\hat{\nu}\cdot\vec{\beta}+2(\hat{\nu}\cdot\hat{q})\left(\vec{\beta}\cdot\hat{q}\right)\right)\left|\langle\|\hat{L}_J\|\rangle\right|^2 + \left(1+\hat{\nu}\cdot\vec{\beta}\right)\left|\langle\|\hat{C}_J\|\rangle\right|^2 - \hat{q}\left(\hat{\nu}+\vec{\beta}\right)2\mathfrak{Re}\left(\langle\|\hat{L}_J\|\rangle\langle\|\hat{C}_J\|\rangle^*\right)\right],$$

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$$\hat{M}_J(q) \equiv \int d^3r\, j_J(qr)\,\vec{Y}_{JJ1}(\hat{r})\cdot\vec{J}(\vec{r}),$$

Nuclear charge & currents

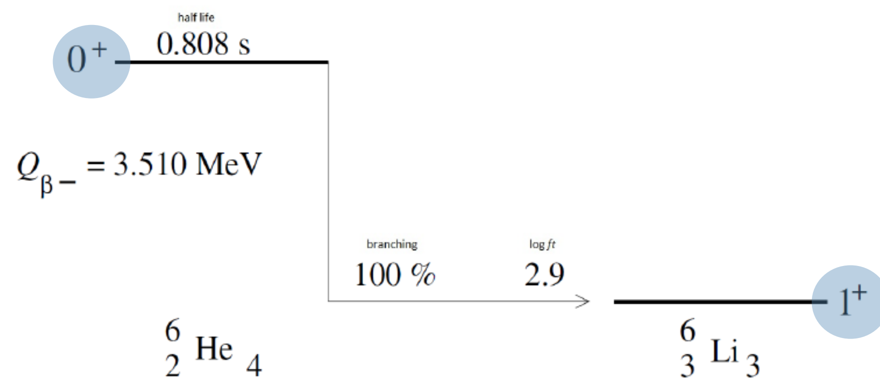
Production @ SARAF

$^{6}He \rightarrow {}^{6}Li$

Also at University of Washington Seattle, GANIL

$\Delta J^{\pi} = 1^{+}$: Pure GT

Corrections (qr)



I. Mardor et al., SARAF: Overview, research programs and future plans. Eur. Phys. J. A, 54(5):91, 2018.
A. Knecht et al., Precision measurement of the ^{6}He half-life and the weak axial current in nuclei. Phys. Rev. C, 86(3), 9 2012.

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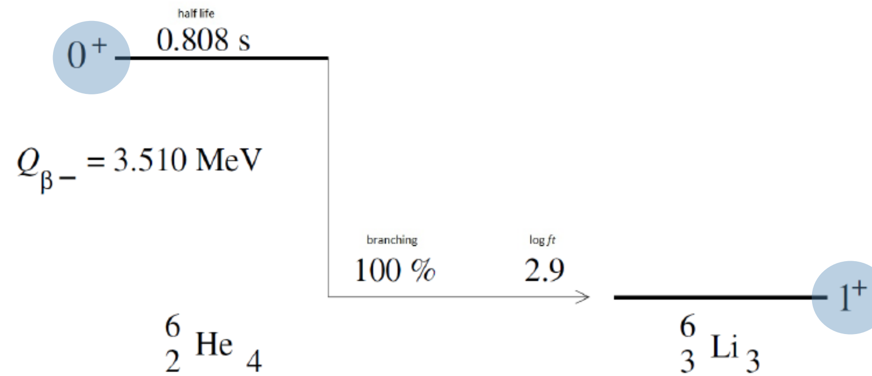
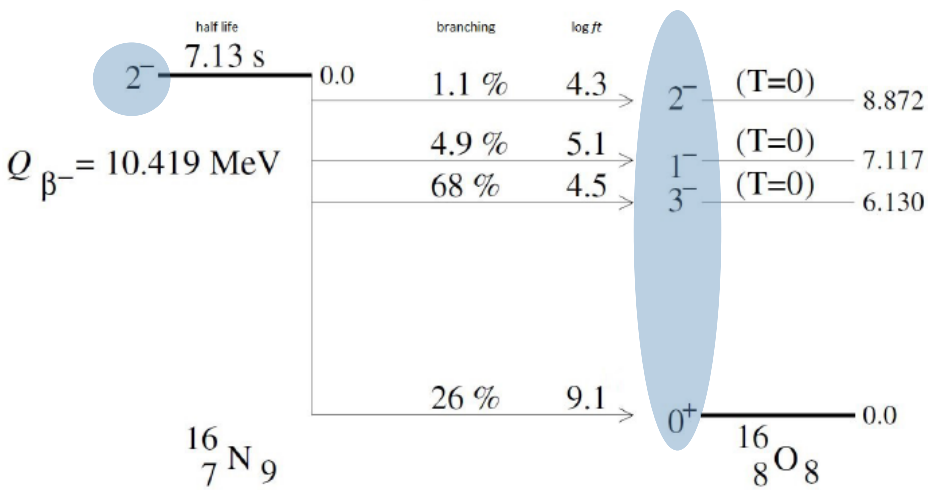


^{16}N → ^{16}O



GT (Fermi)

GT

$\Delta J^{\pi} = 2^{+}, 3^{+}$ corrections

1st forbidden unique ($\Delta J^{\pi} = 2^{-}$)

I. Mardor et al., SARAF: Overview, research programs and future plans. Eur. Phys. J. A, 54(5):91, 2018.
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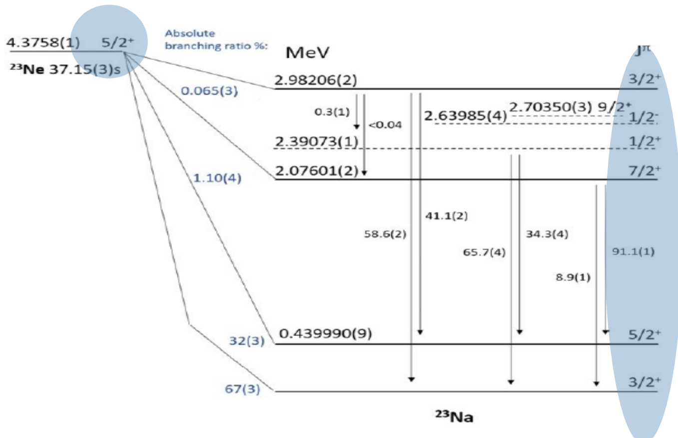

GT

GT

GT (Fermi)

GT

$^{23}Ne \rightarrow {}^{23}Na$

$\Delta J^{\pi} = 2^{+}, 3^{+}$

corrections

Ohayon, Chocron, Hirsh, Glick-Magid, Mishnayot, Mukul, Rahangdale, Vaintraub, Heber, Gazit, Ron, Weak interaction studies at SARAF. Hyperfine Interact. 239: 57 (2018).

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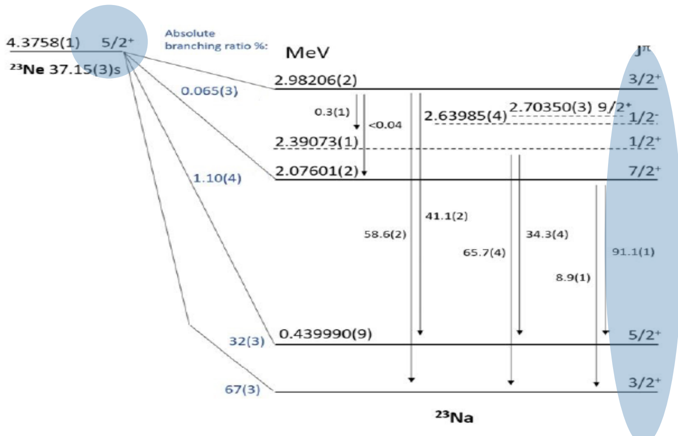


GT

GT

GT (Fermi)

GT

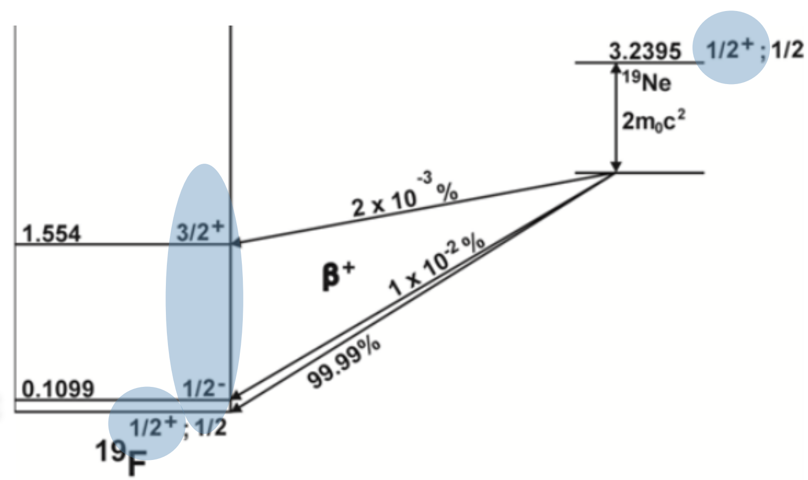
$^{23}Ne \rightarrow {}^{23}Na$

$\Delta J^{\pi} = 2^{+}, 3^{+}$ corrections

$^{19}Ne \rightarrow {}^{19}F$

Ground state corrections originate only from Fermi & GT higher orders

But the second exited state is very close, including the magnetic moment, who dirties the SM calculations



1st forbidden ($\Delta J^{\pi} = 0^{-}, 1^{-}$)

Fermi, GT

Ohayon, Chocron, Hirsh, Glick-Magid, Mishnayot, Mukul, Rahangdale, Vaintraub, Heber, Gazit, Ron, Weak interaction studies at SARAF. Hyperfine Interact. 239: 57 (2018).

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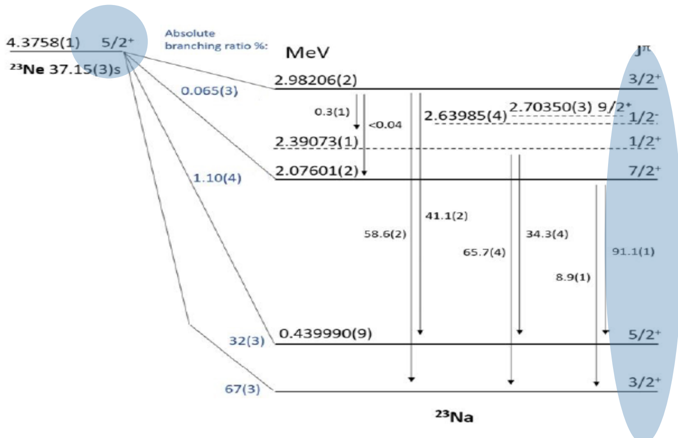

GT

GT

GT (Fermi)

GT

^{23}Ne → ^{23}Na

$\Delta J^{\pi} = 2^{+}, 3^{+}$

corrections

DM Direct detectors

^{19}Ne → ^{19}F

Ground state corrections originate only from Fermi & GT higher orders

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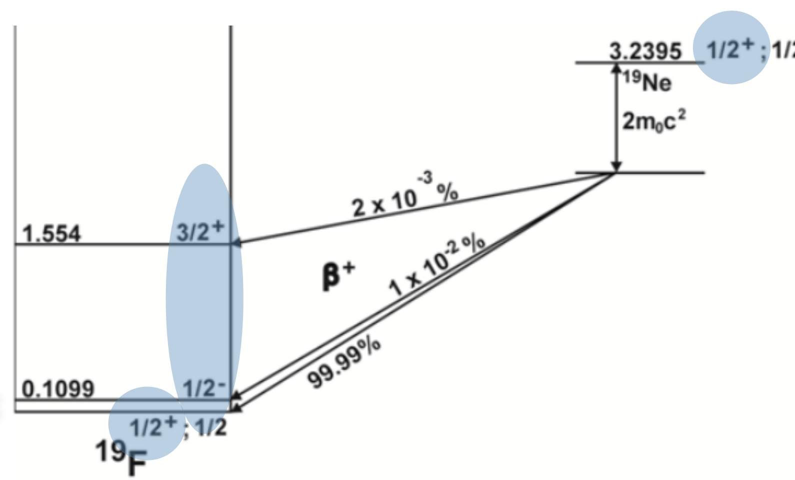

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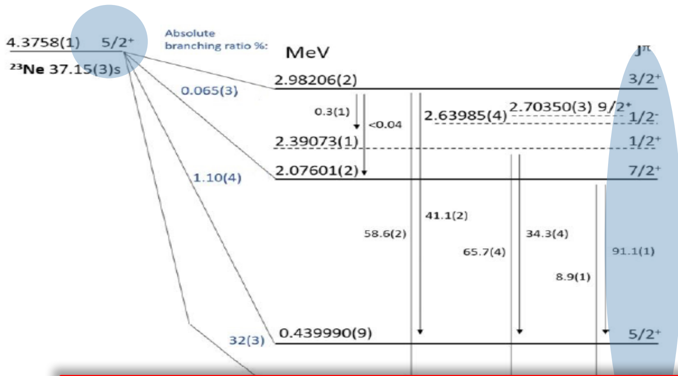


GT

GT

GT (Fermi)

^{23}Ne → ^{23}Na

$\Delta J^{\pi} = 2^{+}, 3^{+}$
corrections

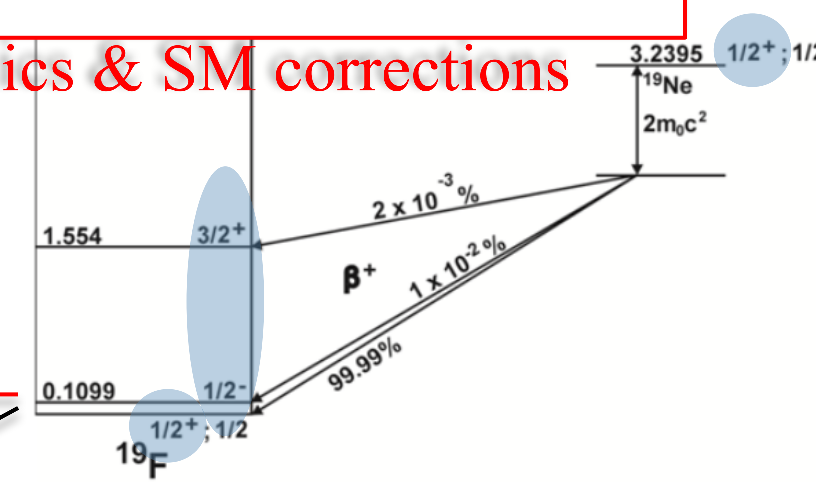
Required: theoretical predictions for all those measurements,
including BSM physics & SM corrections

^{19}Ne → ^{19}F

Ground state corrections originate only from Fermi & GT higher orders
But the second exited state is very close, including the magnetic moment, who dirties the SM calculations

1st forbidden ($\Delta J^{\pi} = 0^{-}, 1^{-}$)

Fermi, GT



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Tensor Contribution & Operators

Precise Tensor analysis

- Since we are looking at a lot of nuclei, the BSM contribution would be different
- There are works about Scalar & Pseudoscalar, but there is yet no accurate analysis for tensor interactions.
- There are terms in the literature for small momentum transfers, but not for all transitions
- There is a need for general terms also for non beta decay BSM searches

Tensor Multipole Expansion

$$\widehat{\mathcal{H}}_{\mathrm{W}} \sim C_T \int d^3\vec{x}\, \hat{j}\left(\vec{x}\right) \hat{\mathcal{J}}\left(\vec{x}\right)$$

Probe current

Nuclear current

General expression of the Tensor coupling for non-vanishing momentum transfer q:

$$d\omega^T = \frac{4}{\pi^2} k\epsilon \left(W_0-\epsilon\right)^2 d\epsilon \frac{d\Omega_k}{4\pi}\frac{d\Omega_\nu}{4\pi}\frac{1}{2J_i+1}\left\{\sum_{J=0}^{\infty}\left(1+\hat{\nu}\cdot\vec{\beta}-2\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\right)\cdot\right.$$

$$\cdot\left[\left(\left|C_T\right|^2+\left|C_T'\right|^2\right)\left(\left|\left\langle\left\|\hat{L}_J^{(1)}\right\|\right\rangle\right|^2+2\left|\left\langle\left\|\hat{L}_J^{0\cdot}\right\|\right\rangle\right|^2\right)+2\mathfrak{Re}\left(\sqrt{2}\left(C_TC_T'^*+C_T'C_T^*\right)\left\langle\left\|\hat{L}_J^{(1)}\right\|\right\rangle\left\langle\left\|\hat{L}_J^{0\cdot}\right\|\right\rangle^*\right)\right]$$

$$+\sum_{J=1}^{\infty}\left[\left(1+\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\right)\left[\left(\left|C_T\right|^2+\left|C_T'\right|^2\right)\left(\left|\left\langle\left\|\hat{E}_J^{(1)}\right\|\right\rangle\right|^2+\left|\left\langle\left\|\hat{M}_J^{(1)}\right\|\right\rangle\right|^2+2\left(\left|\left\langle\left\|\hat{E}_J^{0\cdot}\right\|\right\rangle\right|^2+\left|\left\langle\left\|\hat{M}_J^{0\cdot}\right\|\right\rangle\right|^2\right)\right)\right.\right.$$

$$\left.+2\mathfrak{Re}\left(\sqrt{2}\left(C_TC_T'^*+C_T'C_T^*\right)\left(\left\langle\left\|\hat{E}_J^{(1)}\right\|\right\rangle\left\langle\left\|\hat{E}_J^{0\cdot}\right\|\right\rangle^*+\left\langle\left\|\hat{M}_J^{(1)}\right\|\right\rangle\left\langle\left\|\hat{M}_J^{0\cdot}\right\|\right\rangle^*\right)\right)\right]$$

$$+\hat{q}\cdot\left(\hat{\nu}+\vec{\beta}\right)2\mathfrak{Re}\left[\left(C_TC_T'^*+C_T'C_T^*\right)\left(\left\langle\left\|\hat{E}_J^{(1)}\right\|\right\rangle\left\langle\left\|\hat{M}_J^{(1)}\right\|\right\rangle^*+2\left\langle\left\|\hat{E}_J^{0\cdot}\right\|\right\rangle\left\langle\left\|\hat{M}_J^{0\cdot}\right\|\right\rangle^*\right)\right.$$

$$\left.\left.\left.+\sqrt{2}\left(\left|C_T\right|^2+\left|C_T'\right|^2\right)\left(\left\langle\left\|\hat{E}_J^{(1)}\right\|\right\rangle\left\langle\left\|\hat{M}_J^{0\cdot}\right\|\right\rangle^*+\left\langle\left\|\hat{M}_J^{(1)}\right\|\right\rangle\left\langle\left\|\hat{E}_J^{0\cdot}\right\|\right\rangle^*\right)\right]\right]\right\}$$

Coupling constants

Leptonic traces

Tensor Multipole operators (q)

$$\hat{O}_J^T\left(q\right) \approx -\frac{i}{\sqrt{2}}\frac{g_T+2i\omega g_T^{(3)}}{g_A-\frac{\omega}{2m_N}\tilde{g}_{T(A)}}\hat{O}_J^A\left(q\right) \qquad \hat{O}\in\left\{\hat{L},\hat{E},\hat{M}\right\}$$

$$\hat{L}_J^{T'}\left(q\right) \approx -\frac{1}{\sqrt{2}}\frac{q}{m_N}\sum_{j=1}^{A}\left\{\left(2m_Ng_T^{(1)}+ig_T\right)M_J\left(q\vec{r}_j\right)+g_T\left[\left(\frac{1}{q}\vec{\nabla}M_J\left(q\vec{r}_j\right)\right)\times\vec{\sigma}_j\right]\cdot\frac{1}{q}\vec{\nabla}\right\}\tau_j^{\pm}$$

$$\hat{E}_J^{T'}\left(q\right) \approx \frac{1}{\sqrt{2}}\frac{q}{m_N}\sum_{j=1}^{A}\left\{ig_T\left[\left(\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}\left(q\vec{r}_j\right)\right)\times\vec{\sigma}_j\right]\cdot\frac{1}{q}\vec{\nabla}+\left(\frac{i}{2}g_T+\frac{\omega}{2}g_T^{(1)}-2m_Ng_T^{(3)}\right)\vec{\sigma}_j\cdot\vec{M}_{JJ1}\left(q\vec{r}_j\right)\right\}\tau_j^{\pm}$$

$$\hat{M}_J^{T'}\left(q\right) \approx \frac{1}{\sqrt{2}}\frac{q}{m_N}\sum_{j=1}^{A}\left\{ig_T\left[\vec{M}_{JJ1}\left(q\vec{r}_j\right)\times\vec{\sigma}_j\right]\cdot\frac{1}{q}\vec{\nabla}+\left(\frac{i}{2}g_T+\frac{\omega}{2}g_T^{(1)}-2m_Ng_T^{(3)}\right)\vec{\sigma}_j\cdot\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}\left(q\vec{r}_j\right)\right]\right\}\tau_j^{\pm},$$

- Paves the way to calculating Tensor interactions at:
  - Forbidden β-decays (q dependence)
  - Other BSM searches, e.g. WIMPs:
    - obtaining the non-relativistic reduction, one gets additional operators that do not appear in Haxton's reduction.

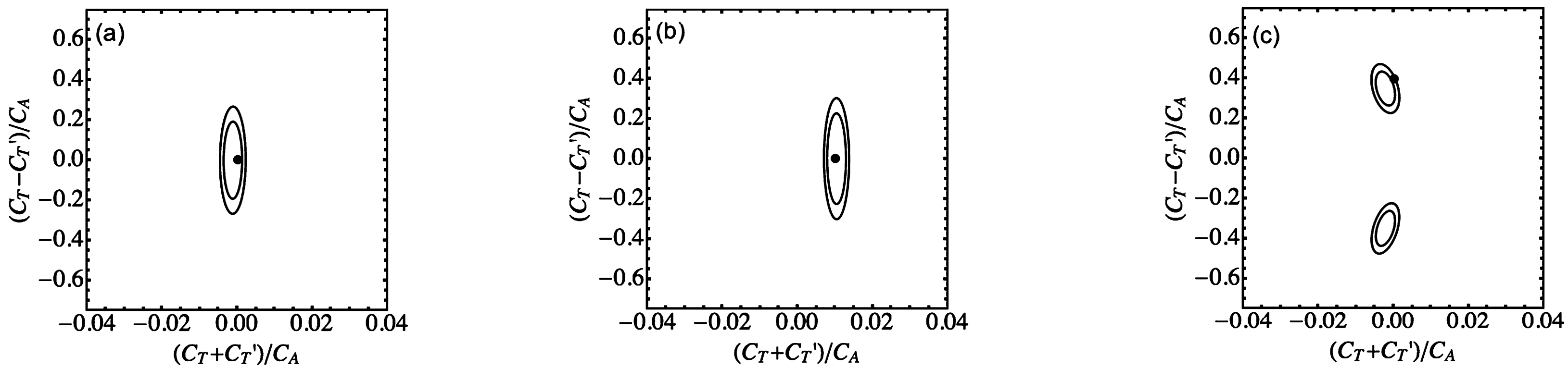
Unique First-Forbidden ($\Delta J^{\pi} = 2^{-}$)

$^{16}N \rightarrow {}^{16}O$

$$\text{Decay Rate } (2^-) \propto 1 + b\frac{m_e}{\epsilon} + a\left[1 - \left(\hat{\beta}\cdot\hat{\nu}\right)^2\right]$$

β energy spectrum is sensitive to both a & b

- Allows simultaneous extraction of tensor coupling to both right & left neutrinos
- Increases the accuracy level



$C_T = C_T' = 0$ $\quad$ $C_T/C_A = C_T'/C_A = 0.005$ $\quad$ $C_T/C_A = -C_T'/C_A = 0.2$

Formalism is nice, but applications are nicer...

A. Glick-Magid, Y. Mishnayot, I. Mukul, M. Hass, S. Vaintraub, G. Ron, D. Gazit, Phys. Lett. B767, 285 (2017)

25

Recoil Corrections & Operators

A consistent expansion in powers of q

For a specific ΔJ^{π} (axial-dominant current transition)

$$\Theta\left(q,\vec{\beta}\cdot\hat{\nu}\right)=\frac{\left|C_A\right|^2+\left|C_A'\right|^2}{2}\frac{2J+1}{J}\left[1+\delta_1^{J^\pi}-\frac{J}{2J+1}\hat{\nu}\cdot\vec{\beta}\left(1+\delta_{\beta\nu}^{J^\pi}\right)+\frac{J-1}{2J+1}\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\left(1+\delta_{proj}^{J^\pi}\right)\right]\left|\left\langle\left\|\hat{L}_J^A\right\|\right\rangle\right|^2$$

Recoil corrections:

$$\begin{aligned}
\delta_1^{J^\pi} &= 2\Re\mathfrak{e}\left\{-\frac{J}{2J+1}\frac{\nu+\frac{k^2}{\epsilon}}{q}\frac{\left\langle\left\|\hat{C}_J^A\right\|\right\rangle}{\left\langle\left\|\hat{L}_J^A\right\|\right\rangle}\pm\frac{\sqrt{J\left(J+1\right)}}{2J+1}\frac{\nu-\frac{k^2}{\epsilon}}{q}\frac{C_AC_V^*+C_A'C_V'^*}{\left|C_A\right|^2+\left|C_A'\right|^2}\frac{\left\langle\left\|\hat{M}_J^V\right\|\right\rangle}{\left\langle\left\|\hat{L}_J^A\right\|\right\rangle}\right.\\
&\qquad\left.-i\sqrt{\frac{J+1}{2J+1}}\frac{\left\langle\left\|\hat{E}_J^{A(1)}\right\|\right\rangle}{\left\langle\left\|\hat{L}_J^A\right\|\right\rangle}\right\}\\
\delta_{\beta\nu}^{J^\pi} &= 2\Re\mathfrak{e}\left\{\frac{\epsilon+\nu}{q}\frac{\left\langle\left\|\hat{C}_J^A\right\|\right\rangle}{\left\langle\left\|\hat{L}_J^A\right\|\right\rangle}\mp\sqrt{\frac{J+1}{J}}\frac{\epsilon-\nu}{q}\frac{C_AC_V^*+C_A'C_V'^*}{\left|C_A\right|^2+\left|C_A'\right|^2}\frac{\left\langle\left\|\hat{M}_J^V\right\|\right\rangle}{\left\langle\left\|\hat{L}_J^A\right\|\right\rangle}\right\}\\
\delta_{proj}^{J^\pi} &= 2\Re\mathfrak{e}\left\{i\frac{\sqrt{\left(J+1\right)\left(2J+1\right)}}{J-1}\frac{\left\langle\left\|\hat{E}_J^{A(1)}\right\|\right\rangle}{\left\langle\left\|\hat{L}_J^A\right\|\right\rangle}\right\}.
\end{aligned}$$

$$\begin{aligned}
\hat{L}_J^A &\propto \frac{(qr)^{J-1}}{(2J-1)!!}\\
\hat{L}_J^{A(1)},\hat{E}_J^{A(1)} &\propto \frac{(qr)^{J+1}}{(2J+1)!!}\\
\hat{C}_J^A,\hat{M}_J^V &\propto \frac{(qr)^{J}}{(2J+1)!!}
\end{aligned}$$

One-body Multipole Operators

SM + Second class currents

Vector

$$\hat{C}^V_J(q) \approx \left(g_V + \frac{\omega}{2m_N}\tilde{g}_S\right)\sum_{j=1}^{A}\tau_j^{\pm}M_J(q\vec{r}_j)$$

$$\hat{L}^V_J(q) \approx \frac{q}{\omega}\hat{C}^V_J(q)$$

$$\hat{E}^V_J(q) \approx \frac{q}{m_N}\sum_{j=1}^{A}\tau_j^{\pm}\left[-ig_V\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\frac{1}{q}\vec{\nabla} + \frac{g_V + \tilde{g}_{T(V)}}{2}\vec{M}_{JJ1}(q\vec{r}_j)\cdot\vec{\sigma}\right]$$

$$\hat{M}^V_J(q) \approx -\frac{iq}{m_N}\sum_{j=1}^{A}\tau_j^{\pm}\left[g_V\vec{M}_{JJ1}(q\vec{r}_j)\cdot\frac{1}{q}\vec{\nabla} + i\frac{g_V + \tilde{g}_{T(V)}}{2}\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\vec{\sigma}\right]$$

$$M_J(q\vec{r}) \equiv j_J(qr)\,Y_J(\hat{r})$$

$$\vec{M}_{JL1}(q\vec{r}) \equiv j_L(qr)\,\vec{Y}_{JL1}(\hat{r})$$

Axial

$$\hat{C}^A_J(q) \approx -\frac{iq}{m_N}\sum_{j=1}^{A}\tau_j^{\pm}\left[g_A M_J(q\vec{r}_j)\,\vec{\sigma}\cdot\frac{1}{q}\vec{\nabla} + \frac{1}{2}\left(g_A + \tilde{g}_{T(A)} - \frac{\omega}{2m_N}\tilde{g}_P\right)\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r})\right]\cdot\vec{\sigma}\right] - \frac{\omega}{2m_N}\tilde{g}_P\Big)\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r})\right]\cdot\vec{\sigma}\Big]$$

$$\hat{L}^A_J(q) \approx i\left(g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right)\sum_{j=1}^{A}\tau_j^{\pm}\left[\frac{1}{q}\vec{\nabla}M_J(q\vec{r}_j)\right]\cdot\vec{\sigma}$$

$$\hat{E}^A_J(q) \approx \left(g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right)\sum_{j=1}^{A}\tau_j^{\pm}\left[\frac{1}{q}\vec{\nabla}\times\vec{M}_{JJ1}(q\vec{r}_j)\right]\cdot\vec{\sigma}$$

$$\hat{M}^A_J(q) \approx \left(g_A - \frac{\omega}{2m_N}\tilde{g}_{T(A)}\right)\sum_{j=1}^{A}\tau_j^{\pm}\vec{M}_{JJ1}(q\vec{r}_j)\cdot\vec{\sigma} \qquad (5)$$

Nuclear Corrections ($q \neq 0$)

$^6He \to {}^6Li$

$^{16}N \to {}^{16}O$

Ne isotopes

Allowed decays ($\Delta J^\pi = 0^+, 1^+$)

$$\Theta\left(q, \vec{\beta}\cdot\hat{\nu}\right) = \frac{|C_V|^2 + |C_V'|^2}{2}\left[\left(1+\frac{\omega^2}{q^2}\right) + \hat{\nu}\cdot\vec{\beta}\left(1-\frac{\omega^2}{q^2}\right) - 2\frac{\omega}{q}\hat{q}\cdot\left(\hat{\nu}+\vec{\beta}\right) + 2\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\right]\left|\left\langle\left\|\hat{C}_0^V\right\|\right\rangle\right|^2$$

Fermi

$$+\frac{|C_A|^2 + |C_A'|^2}{2} 3\left[1+\delta_1^{1^+} - \frac{1}{3}\hat{\nu}\cdot\vec{\beta}\left(1+\delta_{\beta\nu}^{1^+}\right)\right]\left|\left\langle\left\|\hat{L}_1^A\right\|\right\rangle\right|^2$$

Gamow-Teller

Recoil corrections:

$$\delta_1^{1^+} = 2\Re\mathfrak{e}\left\{-\frac{1}{3}\frac{\nu+\frac{k^2}{\epsilon}}{q}\frac{\left\langle\left\|\hat{C}_1^A\right\|\right\rangle}{\left\langle\left\|\hat{L}_1^A\right\|\right\rangle} \pm \frac{\sqrt{2}}{3}\frac{\nu-\frac{k^2}{\epsilon}}{q}\frac{C_A C_V^* + C_A' C_V'^*}{|C_A|^2+|C_A'|^2}\frac{\left\langle\left\|\hat{M}_1^V\right\|\right\rangle}{\left\langle\left\|\hat{L}_1^A\right\|\right\rangle} - i\sqrt{\frac{2}{3}}\frac{\left\langle\left\|\hat{E}_1^{A(1)}\right\|\right\rangle}{\left\langle\left\|\hat{L}_1^A\right\|\right\rangle}\right\}$$

$$\delta_{\beta\nu}^{1^+} = 2\Re\mathfrak{e}\left\{\frac{\epsilon+\nu}{q}\frac{\left\langle\left\|\hat{C}_1^A\right\|\right\rangle}{\left\langle\left\|\hat{L}_1^A\right\|\right\rangle} \mp \sqrt{2}\frac{\epsilon-\nu}{q}\frac{C_A C_V^* + C_A' C_V'^*}{|C_A|^2+|C_A'|^2}\frac{\left\langle\left\|\hat{M}_1^V\right\|\right\rangle}{\left\langle\left\|\hat{L}_1^A\right\|\right\rangle}\right\}$$

$C_0^V, L_1^A \propto 1$

$C_1^A, M_1^V \propto qR$

higher corrections $\propto (qR)^2$

1st Forbidden Corrections

$$
\begin{aligned}
\Theta\left(q,\vec{\beta}\cdot\hat{\nu}\right) = {} & \frac{|C_A|^2+|C'_A|^2}{2}\Big\{\left(1+\hat{\nu}\cdot\vec{\beta}\right)\left|\langle\|\hat{C}_0^A\|\rangle\right|^2+\left(1-\hat{\nu}\cdot\vec{\beta}+2\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\right)\left|\langle\|\hat{L}_0^A\|\rangle\right|^2 - \\
& -\hat{q}\left(\hat{\nu}+\vec{\beta}\right)2\mathfrak{Re}\left(\langle\|\hat{L}_0^A\|\rangle\langle\|\hat{C}_0^A\|\rangle^*\right)+\left[1-\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\right]\left|\langle\|\hat{M}_1^A\|\rangle\right|^2+ \\
& +\frac{5}{2}\left[1+\delta_1^{2^-}-\frac{2}{5}\hat{\nu}\cdot\vec{\beta}\left(1+\delta_{\beta\nu}^{2^-}\right)+\frac{1}{5}\left(\hat{\nu}\cdot\hat{q}\right)\left(\vec{\beta}\cdot\hat{q}\right)\left(1+\delta_{proj}^{2^-}\right)\right]\left|\langle\|\hat{L}_2^A\|\rangle\right|^2\Big\}+ \\
& +\frac{|C_V|^2+|C'_V|^2}{2}\left\{\left[1+3\frac{\omega^2}{q^2}-2\frac{\omega\left(\nu+\frac{k^2}{\epsilon}\right)}{q^2}+\left(1-\frac{\omega^2}{q^2}-2\frac{\omega\left(\epsilon+\nu\right)}{q^2}\right)\hat{\nu}\cdot\vec{\beta}\right]\left|\langle\|\hat{C}_1^V\|\rangle\right|^2\right. \\
& \left.\pm\frac{\omega}{q}\left(\frac{\nu-\frac{k^2}{\epsilon}}{q}+\frac{\epsilon-\nu}{q}\hat{\nu}\cdot\vec{\beta}\right)2\sqrt{2}\mathfrak{Re}\left(\frac{C_VC_A^*+C'_VC'^*_A}{|C_V|^2+|C'_V|^2}\langle\|\hat{C}_1^V\|\rangle\langle\|\hat{M}_1^A\|\rangle^*\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
\delta_1^{2^-} &= -2\mathfrak{Re}\left\{i\sqrt{\frac{3}{5}}\frac{\langle\|\hat{E}_2^{A(1)}\|\rangle}{\langle\|\hat{L}_2^A\|\rangle}+\right. \\
&\quad \left.+\frac{1}{5}\frac{\nu+\frac{k^2}{\epsilon}}{q}\frac{\langle\|\hat{C}_2^A\|\rangle}{\langle\|\hat{L}_2^A\|\rangle}\mp\frac{\sqrt{6}}{5}\frac{\nu-\frac{k^2}{\epsilon}}{q}\frac{C_AC_V^*+C'_AC'^*_V}{|C_A|^2+|C'_A|^2}\frac{\langle\|\hat{M}_2^V\|\rangle}{\langle\|\hat{L}_2^A\|\rangle}\right\} \\
\delta_{\beta\nu}^{2^-} &= 2\mathfrak{Re}\left\{\frac{\epsilon+\nu}{q}\frac{\langle\|\hat{C}_2^A\|\rangle}{\langle\|\hat{L}_2^A\|\rangle}\mp\sqrt{\frac{3}{2}}\frac{\epsilon-\nu}{q}\frac{C_AC_V^*+C'_AC'^*_V}{|C_A|^2+|C'_A|^2}\frac{\langle\|\hat{M}_2^V\|\rangle}{\langle\|\hat{L}_2^A\|\rangle}\right\} \\
\delta_{proj}^{2^-} &= 2\mathfrak{Re}\left\{i\sqrt{15}\frac{\langle\|\hat{E}_2^{A(1)}\|\rangle}{\langle\|\hat{L}_2^A\|\rangle}\right\}.
\end{aligned}
$$

$$\hat{C}_1^V \propto \frac{q}{\omega}\frac{P}{2m_N} \qquad \hat{C}_0^A \propto \frac{P}{2m_N}$$

$$\hat{L}_0^A,\hat{M}_1^A \propto \frac{qr}{3} \text{ and } \hat{L}_2^A \propto \frac{qr}{15}$$

$$\hat{C}_1^{V(1)} \propto \frac{q}{\omega}\left(\frac{P}{2m_N}\right)^3;\ \hat{C}_0^{A(1)},\hat{L}_1^{V(1)},\hat{E}_1^{V(1)} \propto \left(\frac{P}{2m_N}\right)^3$$

$$\hat{L}_0^{A(1)},\hat{M}_1^{A(1)} \propto \frac{qr}{3}\left(\frac{P}{2m_N}\right)^2 \text{ and } \hat{L}_2^{A(1)},\hat{E}_2^{A(1)} \propto \frac{qr}{15}\left(\frac{P}{2m_N}\right)^2$$

$$\hat{C}_2^A,\hat{M}_2^V \propto \frac{(qr)^2}{15}\frac{P}{2m_N}$$

30

Tensor WINPs Interactions

WIMPs Scattering Formalism

Non-Relativistic Nuclear Reduction

Contact interaction Lagrangian between a spin-$^{1}/_{2}$ WIMP χ & a nucleon N:

$$L_{int} \sim \bar{\chi} O_{\chi} \chi \bar{N} O_{N} N \approx \sum_{i=1}^{16} c_i O_i \bar{\chi} \chi \bar{N} N$$

(Leading order in the particles' velocities $\frac{p}{m_\chi}$, $\frac{k}{m_N}$)

$\{O_i\}_{i=1}^{16}$ – 16 non-relativistic operators appropriate to use with Pauli spinors, built of 4 three-vectors:

- $\frac{i\vec{q}}{m_N}$ ($\vec{q}$ - momentum transfer)
- $\vec{v}^{\perp} \equiv \frac{\vec{P}}{2m_\chi} - \frac{\vec{K}}{2m_N}$
- $\vec{S}_\chi, \vec{S}_N$ - particle spins

j	$\mathcal{L}^j_{\text{int}}$	Nonrelativistic Reduction	$\sum_i c_i \mathcal{O}_i$
1	$\bar{\chi}\chi\bar{N}N$	$1_\chi 1_N$	$\mathcal{O}_1$
2	$i\bar{\chi}\chi\bar{N}\gamma^5 N$	$i\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$\mathcal{O}_{10}$
3	$i\bar{\chi}\gamma^5\chi\bar{N}N$	$-i\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$
4	$\bar{\chi}\gamma^5\chi\bar{N}\gamma^5 N$	$-\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$
5	$\frac{P^\mu}{m_M}\bar{\chi}\chi\frac{K_\mu}{m_M}\bar{N}N$	$4\frac{m_\chi m_N}{m_M^2}1_\chi 1_N$	$4\frac{m_\chi m_N}{m_M^2}\mathcal{O}_1$
6	$\frac{P^\mu}{m_M}\bar{\chi}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$-\frac{m_\chi}{m_N}\frac{\vec{q}^2}{m_M^2}1_\chi 1_N - 4i\frac{m_\chi}{m_M}\vec{v}^\perp\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)$	$-\frac{m_\chi}{m_N}\frac{\vec{q}^2}{m_M^2}\mathcal{O}_1 + 4\frac{m_\chi m_N}{m_M^2}\mathcal{O}_3$
7	$\frac{P^\mu}{m_M}\bar{\chi}\chi\bar{N}\gamma_\mu\gamma^5 N$	$-4\frac{m_\chi}{m_M}\vec{v}^\perp\cdot\vec{S}_N$	$-4\frac{m_\chi}{m_M}\mathcal{O}_7$
8	$i\frac{P^\mu}{m_M}\bar{\chi}\chi\frac{K_\mu}{m_M}\bar{N}\gamma^5 N$	$4i\frac{m_\chi}{m_M}\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$4\frac{m_\chi m_N}{m_M^2}\mathcal{O}_{10}$
9	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\frac{K_\mu}{m_M}\bar{N}N$	$\frac{m_N}{m_\chi}\frac{\vec{q}^2}{m_M^2}1_\chi 1_N + 4i\frac{m_N}{m_M}\vec{v}^\perp\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_\chi\right)$	$\frac{m_N}{m_\chi}\frac{\vec{q}^2}{m_M^2}\mathcal{O}_1 - 4\frac{m_N^2}{m_M^2}\mathcal{O}_5$
10	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4\left(\frac{\vec{q}}{m_M}\times\vec{S}_\chi\right)\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)$	$4\left(\frac{\vec{q}^2}{m_M^2}\mathcal{O}_4 - \frac{m_N^2}{m_M^2}\mathcal{O}_6\right)$
11	$\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar{N}\gamma^\mu\gamma^5 N$	$-4i\left(\frac{\vec{q}}{m_M}\times\vec{S}_\chi\right)\cdot\vec{S}_N$	$-4\frac{m_N}{m_M}\mathcal{O}_9$
12	$i\bar{\chi}i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\frac{K_\mu}{m_M}\bar{N}\gamma^5 N$	$\left[i\frac{\vec{q}^2}{m_\chi m_M} - 4\vec{v}^\perp\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_\chi\right)\right]\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$\frac{m_N}{m_\chi}\frac{\vec{q}^2}{m_M^2}\mathcal{O}_{10} + 4\frac{\vec{q}^2}{m_M^2}\mathcal{O}_{12} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$
13	$\bar{\chi}\gamma^\mu\gamma^5\chi\frac{K_\mu}{m_M}\bar{N}N$	$4\frac{m_N}{m_M}\vec{v}^\perp\cdot\vec{S}_\chi$	$4\frac{m_N}{m_M}\mathcal{O}_8$
14	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$-4i\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)$	$4\frac{m_N}{m_M}\mathcal{O}_9$
15	$\bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma^\mu\gamma^5 N$	$-4\vec{S}_\chi\cdot\vec{S}_N$	$-4\mathcal{O}_4$
16	$i\bar{\chi}\gamma^\mu\gamma^5\chi\frac{K^\mu}{m_M}\bar{N}\gamma^5 N$	$4i\vec{v}^\perp\cdot\vec{S}_\chi\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$4\frac{m_N}{m_M}\mathcal{O}_{13}$
17	$i\frac{P^\mu}{m_M}\bar{\chi}\gamma^5\chi\frac{K_\mu}{m_M}\bar{N}N$	$-4i\frac{m_N}{m_M}\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi$	$-4\frac{m_N^2}{m_M^2}\mathcal{O}_{11}$
18	$i\frac{P^\mu}{m_M}\bar{\chi}\gamma^5\chi\bar{N}i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\left[i\frac{\vec{q}^2}{m_N m_M} - 4\vec{v}^\perp\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)\right]$	$\frac{\vec{q}^2}{m_M^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$
19	$i\frac{P^\mu}{m_M}\bar{\chi}\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$	$4i\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\vec{v}_\perp\cdot\vec{S}_N$	$4\frac{m_N}{m_M}\mathcal{O}_{14}$
20	$\frac{P^\mu}{m_M}\bar{\chi}\gamma^5\chi\frac{K_\mu}{m_M}\bar{N}\gamma^5 N$	$-4\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\frac{\vec{q}}{m_M}\cdot\vec{S}_N$	$-4\frac{m_N^2}{m_M^2}\mathcal{O}_6$

A. L. Fitzpatrick, W. Haxton et al., J. Cosmol. Astropart. Phys, 2013(02):004, 2013

Doesn't take into account Tensor couplings between WIMPs and nucleons

WIMPs Tensor Interactions

Using the same tensor current decomposition we used for achieving the tensor multipole expansion, we were able to obtain, for the first time, the non-relativistic reduction for the tensor coupling:

j	$\mathcal{L}^j_{\text{int}}$	Nonrelativistic Reduction	$\Sigma_i c_i O_i$
21	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{N}\sigma_{\mu\nu}N$	$8\left(\vec{S}_\chi\cdot\vec{S}_N\right)$	$8O_4$
22	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{N}\left(\frac{q_\mu}{m_M}\gamma_\nu-\frac{q_\nu}{m_M}\gamma_\mu\right)N$	$i\frac{\vec{q}^2}{m_\chi m_M}1_\chi 1_N-4i\frac{\vec{q}^2}{m_M m_N}\left(\vec{S}_\chi\cdot\vec{S}_N\right)+$ $+4i\left(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\right)\left(\frac{\vec{q}}{m_N}\cdot\vec{S}_N\right)+2\left(\frac{\vec{P}}{m_\chi}+\frac{\vec{K}}{m_N}\right)\cdot\left(\vec{S}_\chi\times\frac{\vec{q}}{m_M}\right)$	$i\frac{\vec{q}^2}{m_M m_\chi}O_1-4i\frac{\vec{q}^2}{m_M m_N}O_4+$ $+4i\frac{m_N}{m_M}O_6+4i\left(O_3-2O_{17}\right)$
23	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{N}\left(\frac{q^\mu}{m_M}\frac{K^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{K^\mu}{m_M}\right)N$	$4\frac{\vec{K}}{m_M}\cdot\left(\vec{S}_\chi\times\frac{\vec{q}}{m_M}\right)$	$8i\frac{m_N^2}{m_M^2}\left(O_3-O_{17}\right)$
24	$\bar{\chi}\sigma^{\mu\nu}\chi\bar{N}\left(\gamma_\mu\frac{\not{q}}{m_M}\gamma_\nu-\gamma_\nu\frac{\not{q}}{m_M}\gamma_\mu\right)N$	$16i\frac{q_0}{m_M}\left(\vec{S}_\chi\cdot\vec{S}_N\right)$	$16i\frac{q_0}{m_M}O_4$
25	$\bar{\chi}\left(\frac{q^\mu}{m_M}\gamma^\nu-\frac{q^\nu}{m_M}\gamma^\mu\right)\chi\bar{N}\sigma_{\mu\nu}N$	$i\frac{\vec{q}^2}{m_M m_N}1_\chi 1_N-4i\frac{\vec{q}^2}{m_M m_\chi}\left(\vec{S}_\chi\cdot\vec{S}_N\right)+$ $+4i\left(\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi\right)\left(\frac{\vec{q}}{m_M}\cdot\vec{S}_N\right)-2\left(\frac{\vec{P}}{m_\chi}+\frac{\vec{K}}{m_N}\right)\cdot\left(\vec{S}_N\times\frac{\vec{q}}{m_M}\right)$	$i\frac{\vec{q}^2}{m_M m_N}O_1-4i\frac{\vec{q}^2}{m_M m_\chi}O_4+$ $+4i\frac{m_N^2}{m_M m_\chi}O_6-4i\left(O_5-2O_{18}\right)$
26	$\bar{\chi}\left(\frac{q^\mu}{m_M}\gamma^\nu-\frac{q^\nu}{m_M}\gamma^\mu\right)\chi\bar{N}\left(\frac{q_\mu}{m_M}\gamma_\nu-\frac{q_\nu}{m_M}\gamma_\mu\right)N$	$2\frac{\vec{q}^2}{m_M^2}1_\chi 1_N$	$2\frac{\vec{q}^2}{m_M^2}O_1$
27	$\bar{\chi}\left(\frac{q^\mu}{m_M}\gamma^\nu-\frac{q^\nu}{m_M}\gamma^\mu\right)\chi\bar{N}\left(\frac{q^\mu}{m_M}\frac{K^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{K^\mu}{m_M}\right)N$	$2\frac{m_N}{m_M}\frac{\vec{q}^2}{m_M^2}1_\chi 1_N-2\frac{q_0}{m_M}\left(\frac{\vec{q}}{m_M}\cdot\frac{\vec{K}}{m_M}\right)$	$2\frac{m_N}{m_M}\frac{\vec{q}^2}{m_M^2}O_1-2\frac{q_0}{m_M}\frac{m_N^2}{m_M^2}O_{19}$
28	$\bar{\chi}\left(\frac{q^\mu}{m_M}\gamma^\nu-\frac{q^\nu}{m_M}\gamma^\mu\right)\chi\bar{N}\left(\gamma_\mu\frac{\not{q}}{m_M}\gamma_\nu-\gamma_\nu\frac{\not{q}}{m_M}\gamma_\mu\right)N$	$16\frac{q_0}{m_M}\left[\frac{\vec{q}^2}{m_M m_\chi}\left(\vec{S}_\chi\cdot\vec{S}_N\right)-\right.$ $\left.-\left(\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi\right)\left(\frac{\vec{q}}{m_M}\cdot\vec{S}_N\right)-\frac{i}{2}\frac{\vec{P}}{m_\chi}\cdot\left(\vec{S}_N\times\frac{\vec{q}}{m_M}\right)\right]$	$16\frac{q_0}{m_M}\left[\frac{\vec{q}^2}{m_M m_\chi}O_4-\right.$ $\left.-\frac{m_N^2}{m_M m_\chi}O_6-\frac{m_N}{m_M}O_{18}\right]$
29	$\bar{\chi}\left(\frac{q^\mu}{m_M}\frac{P^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{P^\mu}{m_M}\right)\chi\bar{N}\sigma_{\mu\nu}N$	$-4\frac{\vec{P}}{m_M}\cdot\left(\vec{S}_N\times\frac{\vec{q}}{m_M}\right)$	$8i\frac{m_N m_\chi}{m_M^2}O_{18}$
30	$\bar{\chi}\left(\frac{q^\mu}{m_M}\frac{P^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{P^\mu}{m_M}\right)\chi\bar{N}\left(\frac{q_\mu}{m_M}\gamma_\nu-\frac{q_\nu}{m_M}\gamma_\mu\right)N$	$2\frac{m_\chi}{m_M}\frac{\vec{q}^2}{m_M^2}1_\chi 1_N+2\frac{q_0}{m_M}\left(\frac{\vec{q}}{m_M}\cdot\frac{\vec{P}}{m_M}\right)$	$2\frac{m_\chi}{m_M}\frac{\vec{q}^2}{m_M^2}O_1+2\frac{q_0}{m_M}\frac{m_N m_\chi}{m_M^2}O_{19}$
31	$\bar{\chi}\left(\frac{q^\mu}{m_M}\frac{P^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{P^\mu}{m_M}\right)\chi\bar{N}\left(\frac{q^\mu}{m_M}\frac{K^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{K^\mu}{m_M}\right)N$	$2\frac{m_\chi m_N}{m_M^2}\frac{\vec{q}^2}{m_M^2}1_\chi 1_N-2\frac{q_0^2}{m_M^2}\left(\frac{\vec{P}}{m_M}\cdot\frac{\vec{K}}{m_M}\right)$	$2\frac{m_\chi m_N}{m_M^2}\frac{\vec{q}^2}{m_M^2}O_1-2\frac{q_0^2}{m_M^2}\frac{m_N m_\chi}{m_M^2}O_{20}$
32	$\bar{\chi}\left(\frac{q^\mu}{m_M}\frac{P^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{P^\mu}{m_M}\right)\chi\bar{N}\left(\gamma_\mu\frac{\not{q}}{m_M}\gamma_\nu-\gamma_\nu\frac{\not{q}}{m_M}\gamma_\mu\right)N$	$-16i\frac{q_0}{m_M}\frac{\vec{P}}{m_M}\cdot\left(\vec{S}_N\times\frac{\vec{q}}{m_M}\right)$	$-32\frac{q_0}{m_M}\frac{m_N m_\chi}{m_M^2}O_{18}$
33	$\bar{\chi}\left(\gamma^\mu\frac{\not{q}}{m_M}\gamma^\nu-\gamma^\nu\frac{\not{q}}{m_M}\gamma^\mu\right)\chi\bar{N}\sigma_{\mu\nu}N$	$16i\frac{q_0}{m_M}\left(\vec{S}_\chi\cdot\vec{S}_N\right)$	$16i\frac{q_0}{m_M}O_4$
34	$\bar{\chi}\left(\gamma^\mu\frac{\not{q}}{m_M}\gamma^\nu-\gamma^\nu\frac{\not{q}}{m_M}\gamma^\mu\right)\chi\bar{N}\left(\frac{q_\mu}{m_M}\gamma_\nu-\frac{q_\nu}{m_M}\gamma_\mu\right)N$	$16\frac{q_0}{m_M}\left[\frac{\vec{q}^2}{m_M m_N}\left(\vec{S}_\chi\cdot\vec{S}_N\right)-\right.$ $\left.-\left(\frac{\vec{q}}{m_M}\cdot\vec{S}_\chi\right)\left(\frac{\vec{q}}{m_N}\cdot\vec{S}_N\right)+\frac{i}{2}\frac{\vec{K}}{m_N}\cdot\left(\vec{S}_\chi\times\frac{\vec{q}}{m_M}\right)\right]$	$16\frac{q_0}{m_M}\left[\frac{\vec{q}^2}{m_M m_N}O_4-\right.$ $\left.-\frac{m_N}{m_M}O_6-\frac{m_N}{m_M}\left(O_3-O_{17}\right)\right]$
35	$\bar{\chi}\left(\gamma^\mu\frac{\not{q}}{m_M}\gamma^\nu-\gamma^\nu\frac{\not{q}}{m_M}\gamma^\mu\right)\chi\bar{N}\left(\frac{q^\mu}{m_M}\frac{K^\nu}{m_M}-\frac{q^\nu}{m_M}\frac{K^\mu}{m_M}\right)N$	$16i\frac{q_0}{m_M}\frac{\vec{K}}{m_M}\cdot\left(\vec{S}_\chi\times\frac{\vec{q}}{m_M}\right)$	$-32\frac{q_0}{m_M}\frac{m_N^2}{m_M^2}\left(O_3-O_{17}\right)$
36	$\bar{\chi}\left(\gamma^\mu\frac{\not{q}}{m_M}\gamma^\nu-\gamma^\nu\frac{\not{q}}{m_M}\gamma^\mu\right)\chi\bar{N}\left(\gamma_\mu\frac{\not{q}}{m_M}\gamma_\nu-\gamma_\nu\frac{\not{q}}{m_M}\gamma_\mu\right)N$	$32\left(\vec{S}_\chi\cdot\frac{\vec{q}}{m_M}\right)\left(\vec{S}_N\cdot\frac{\vec{q}}{m_M}\right)-32\left(\frac{q_0^2}{m_M^2}+\frac{\vec{q}^2}{m_M^2}\right)\left(\vec{S}_\chi\cdot\vec{S}_N\right)$	$32\frac{m_N^2}{m_M^2}O_6-32\left(\frac{q_0^2}{m_M^2}+\frac{\vec{q}^2}{m_M^2}\right)O_4$

Another 4 non-relativistic operators, built of 1 more three-vector ($\vec{P}$)

$$O_{17}=i\vec{S}_N\cdot\left(\frac{\vec{q}}{m_N}\times\frac{\vec{P}}{2m_\chi}\right)$$

$$O_{18}=i\vec{S}_\chi\cdot\left(\frac{\vec{q}}{m_N}\times\frac{\vec{P}}{2m_\chi}\right)$$

$$O_{19}=\frac{\vec{q}}{m_N}\cdot\frac{\vec{P}}{m_\chi}=\frac{\vec{q}}{m_N}\cdot\frac{\vec{K}}{m_N}$$

$$O_{20}=\frac{\vec{P}}{m_\chi}\cdot\frac{\vec{K}}{m_N}=\frac{\vec{P}^2}{m_\chi^2}-2\vec{v}^\perp\cdot\frac{\vec{P}}{m_\chi}$$

$$L_{int}\approx\sum_{i=1}^{20}c_i O_i\bar{\chi}\chi\bar{N}N$$

New!

Summary

- We have Accurate expressions for SM & BSM (including tensor)
- We have formalism for corrections resulted from nuclear structure, in the "language" of nuclear structure calculations
  - Ability to provide single nucleon matrix elements
- The general expression allows us to pinpoint different types of beta decays that it's interesting to measure, because of their different sensitivity to BSM signatures
  - E.g. unique first forbidden decay @ ^{16}N
  - Allows us to find more interesting decay transitions and nuclei
- The general expression also allows us to analyze other BSM experiments - direct detection WIMPs searches