Calculation of Standard Model β spectra

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ECT* Workshop, April 8th 2019

IKS, KU Leuven, Belgium

Trento is enthousiastic for symmetry breaking

ACFI Workshop, Nov 1-3 2018

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Introduction

β decay context: Colliders

Drought at LHC leaves limited number of viable theories standing

FCC is still rather far away, if it comes

β decay context: (B)SM

Standard Model internal consistency test through CKM unitarity

- Superallowed $0^+ \rightarrow 0^+$ decays
- $T = 1/2$ mirror decays
- Neutron decay (see later)

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BSM: Since EFT entered scene directly compare high and low energy:

- Competitive for scalar & tensor currents
- Complementary for right-handed currents

Cirigliano *et al.*, PPNP 71 (2013) 93

β decay horizon

Significant experimental progress, new & improved techniques

- Atomic traps: Jerusalem, TRIUMF, CERN, ANL, TAM, CENPA

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Great for theory!

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Push both in β decay fundamentals and nuclear structure calculations

β decay context: Outside the box

Landscape has changed significantly past 10-20 years

- Neutrino physics: oscillations, reactor anomaly
- Astrophysics: β decay lifetimes, *r*-process
- Big Bang Nucleosynthesis: mass abundances

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Different regimes, overlapping challenges

Introduction

General Hamiltonian

$$\mathcal{H} = \sum_{j=V,A,S,P,T} \langle f| \mathcal{O}_j |i\rangle \langle e| \mathcal{O}_j [C_j + C_j^{'} \gamma_5] |\nu\rangle + h.c.$$

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In Standard Model only V-A $\rightarrow$ where are the **others**?

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Questions:

In Standard Model only V-A $\rightarrow$ where are the **others**?

QCD influences $\rightarrow$ *induced* currents, influenced through **nuclear structure**?

BSM Observables in β decay

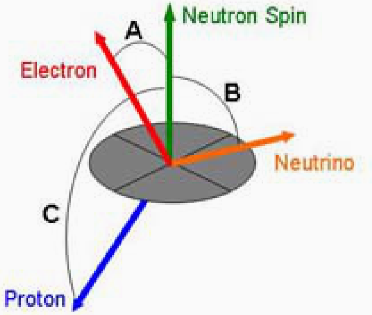
Typical BSM searches through correlations

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} \propto 1 + a_{\beta\nu}\frac{\vec{p_e}\cdot\vec{p_\nu}}{E_e E_\nu} + b_F \frac{m_e}{E_e} + A\frac{\vec{p_e}}{E_e}\langle\vec{I}\rangle + \ldots$$

Measure effective correlations

$$\tilde{X} = \frac{X}{1 + b_F\langle\frac{m_e}{E_e}\rangle}$$

Sensitivity to full spectrum!



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Sensitivity comes from b_F

$$b_F = \pm \frac{1}{1+\rho^2} \left[\mathrm{Re}\left(\frac{C_S + C_S'}{C_V} \right) + \rho^2 \mathrm{Re}\left(\frac{C_T + C_T'}{C_A} \right) \right]$$

because it's linear in coupling constants

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because it's linear in coupling constants

$\rightarrow$ measure β spectrum directly & fit for $1/E_e$

Beta Spectrum Shape

Exploring the Standard Model and Beyond via the allowed β spectrum shape:

$$\frac{dN}{dE_e} \propto 1 + b_{\text{Fierz}} \frac{m_e}{E_e} + b_{WM} E_e$$

b_{Fierz}: Proportional to scalar (Fermi) and tensor (Gamow-Teller) couplings

b_{WM}: Weak Magnetism (main induced current), poorly known for $A > 60$, forbidden decays

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This requires knowledge of the theoretical spectrum shape to $\leq \mathbf{10^{-3}}$ level!

Beta spectrum shape

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Active participation of QED, QCD & WI $\rightarrow$ Complicated system

Weak Hamiltonian is modified

1. Emitted β particle immersed in Coulomb field: (electroweak) radiative corrections

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Large scale gap to cross

Quark $\rightarrow$ Nucleon $\rightarrow$ Nucleus $\rightarrow$ Atom $\rightarrow$ Molecule

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Whole slew of approximations introduced

Standard Model Calculation: Quark

Starting from the Standard Model $SU(2)_L \times U(1)_Y$ EW sector

$$\mathcal{M} = \frac{g^2}{8} V_{ud} \bar{u}\gamma^\mu(1-\gamma^5)d \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \bar{e}\gamma^\nu(1-\gamma^5)\nu$$

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Since $q \ll M_W$, identify Fermi coupling constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Standard Model Calculation: Nucleon

Moving to the nucleon system, we face

$$\langle p|\bar{u}\gamma^{\mu}(1-\gamma^{5})d|n\rangle$$

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Symmetries to the rescue! CVC & PCAC define new *nucleon* currents

$$V^{\mu}+A^{\mu}\approx g_V(q^2)\gamma^{\mu}(1-\lambda\gamma^{5})$$

where $g_V(q^2)\approx 1$ and λ from the lattice

Great progress from lattice QCD, including scalar & tensor charges

Standard Model Calculation: Nucleon

Strong interaction introduces extra terms into the vertex $\rightarrow$ Construct all Lorentz invariants

$$\langle p|V^\mu|n\rangle = \bar{p}\left[g_V\gamma^\mu + \frac{g_M - g_V}{2M}\sigma^{\mu\nu}q_\nu + i\frac{g_S}{2M}q^\mu\right]n$$
$$\langle p|A^\mu|n\rangle = \bar{p}\left[g_A\gamma^\mu\gamma^5 + \frac{g_T}{2M}\sigma^{\mu\nu}q_\nu\gamma^5 + i\frac{g_P}{2M}q^\mu\gamma^5\right]n$$

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Introduction of recoil ($\sim q/M$) terms

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Introduction of recoil ($\sim q/M$) terms

CVC requires $g_S = 0$ & $g_M = \mu_p^{an} - \mu_n = 4.7$

Standard Model Calculation: Nucleus

Nucleus is spherical system $\rightarrow$ multipole decomposition,
elementary particle

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Nucleus is spherical system $\rightarrow$ multipole decomposition, *elementary particle*

Relativistic generalization in Breit frame

$$\langle f|V^0 + A^0|i\rangle \propto \sum_{LM} (-)^{J_f - M_f} \begin{pmatrix} J_f & L & J_i \\ -M_f & M & M_i \end{pmatrix} (Y_L^M)^* F_L(q^2)$$

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Conservation of angular momentum limits # terms

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Require transformation from form factors to matrix elements

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Immediately faced with several issues:

- Weak current in strongly bound system?

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Here the going gets rough $\rightarrow$ severe approximations

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$\rightarrow$ Impulse approximation, non-interacting nucleons

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- Nucleon-nucleon interaction present in many-body methods

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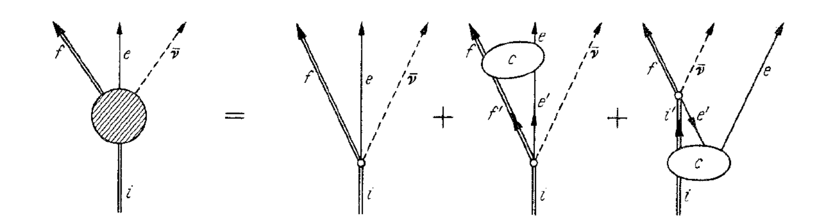
Relativistic nuclear wave functions
$\rightarrow$ Non-relativistic nucleons

- expand operator to $\mathcal{O}(v/c)$
- Incomplete wave function basis, core polarization

Standard Model Calculation: Nucleus

Final state interactions

1. Coulomb interaction



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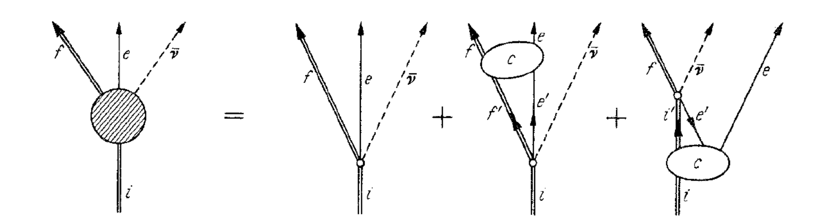
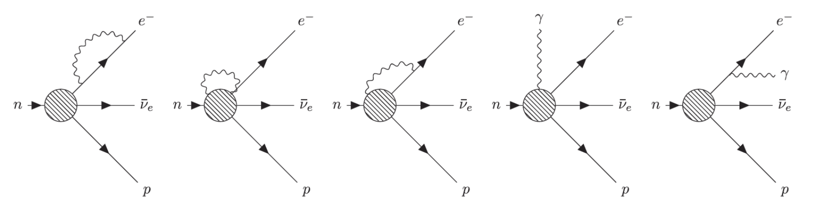


→ Fermi function, *induced Coulomb* terms

Standard Model Calculation: Nucleus

Final state interactions

2. EW Radiative corrections



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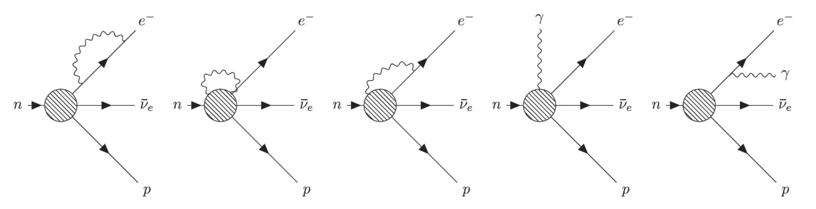

$+$ higher orders, γW boxes: talks by M. Gorshteyn, C. Y. Seng, M. Ramsey-Musolf

Standard Model Calculation: Atom

Must consider total nuclear + atomic Hamiltonian

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Changes

- Available phase space
- Final state interactions
- Opens new decay modes (bound & exchange)

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Must consider total nuclear + atomic Hamiltonian

Changes

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- Final state interactions
- Opens new decay modes (bound & exchange)

Require *atomic* wave functions

- Central & static potential
- Sudden approximation

Standard Model Calculation: Molecule

Similar as atomic system, but changes

- Available phase space
- Molecular excitation, ionization
- Recoil correction & distribution

Standard Model Calculation: Molecule

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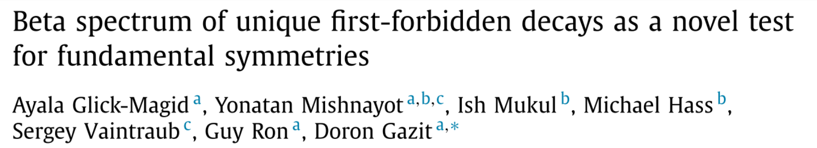
Enter quantum chemistry

- Born-Oppenheimer approximation
- MOLCAO

Current status

Beta Spectrum Shape

Significant effort to rediscover & renew formalisms


Beta spectrum of unique first-forbidden decays as a novel test for fundamental symmetries

Ayala Glick-Magid [a], Yonatan Mishnayot [a,b,c], Ish Mukul [b], Michael Hass [b], Sergey Vaintraub [c], Guy Ron [a], Doron Gazit [a,*]

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PHYSICAL REVIEW C **95**, 024327 (2017)

Spectrum-shape method and the next-to-leading-order terms of the β-decay shape factor

M. Haaranen, J. Kotila, and J. Suhonen
Department of Physics, University of Jyväskylä, P.O. Box 35 (YFL), FI-40014 Jyväskylä, Finland
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Relativistic Theory and Ab Initio Simulations of Electroweak Decay Spectra in Medium-Heavy Nuclei and of Atomic and Molecular Electronic Structure

Tommaso Morresi, Simone Taioli, Stefano Simonucci

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$$\begin{aligned}
N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3}\ & F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ R_N(W,W_0,M) \\
\times\ & Q(Z,W,M)\ R(W,W_0)\ S(Z,W)\ X(Z,W)\ r(Z,W) \\
\times\ & C(Z,W)\ D_C(Z,W,\beta_2)\ D_{\mathrm{FS}}(Z,W,\beta_2) \\
\times\ & pW(W_0-W)^2\ dW
\end{aligned}$$

LH *et al.*, Rev. Mod. Phys. 90 (2018) 015008

LH, Severijns, Comp. Phys. Comm. 10.1016/j.cpc.2019.02.012

Order of magnitude estimates

Nuclear structure sensitivity in shape factor

$$C(Z,W) \sim 1 \pm \frac{4}{3}\frac{W}{M_N}\frac{\boldsymbol{b}}{Ac} \pm \frac{4\sqrt{2}}{21}\alpha Z W R \boldsymbol{\Lambda} - \frac{1}{3WMc}(\pm 2\boldsymbol{b} + \boldsymbol{d})$$

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Fill in typical numbers to obtain

Matrix element	Name	Slope (% MeV^{-1})
b	Weak Magnetism	0.5
d	Induced Tensor	0.1
Λ	Induced Pseudoscalar	0.1

Weak magnetism is generally more stable than others

$\rightarrow$ essential to get this right

Nuclear matrix elements

Overview

$$b = A(g_M \mathcal{M}_{GT} + g_V \mathcal{M}_L)$$

$$d = A(g_A \mathcal{M}_{\sigma L} + g_T \mathcal{M}_{GT})$$

$$\Lambda \propto \mathcal{M}_{1y}$$

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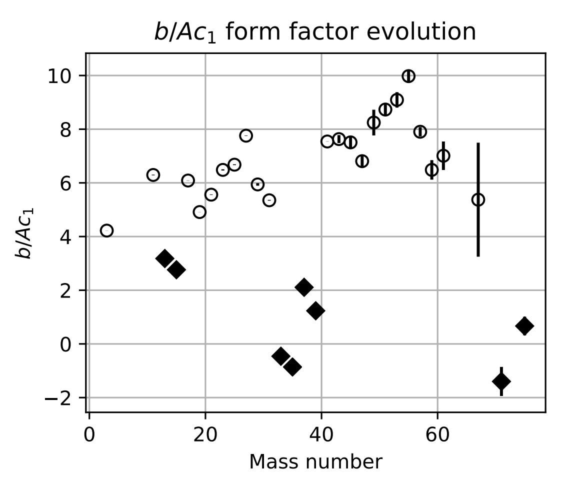
where

$$\mathcal{M}_L = \langle\beta|| \sum \tau_i \vec{l}_i ||\alpha\rangle$$
$$\mathcal{M}_{\sigma L} = \langle\beta|| \sum \tau_i i\vec{\sigma}_i \times \vec{l}_l ||\alpha\rangle$$
$$\mathcal{M}_{1y} \propto \langle\beta|| \sum \tau_i r^2 C_{121}^{nn\prime k} \sigma_{i,n} Y_2^{n\prime}(\hat{r}_i) ||\alpha\rangle$$

Weak magnetism

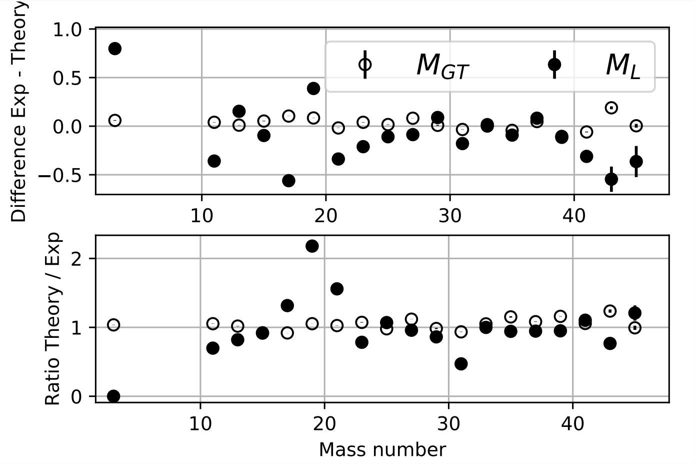
Mirror nuclei have CVC-determined WM



open: $l+1/2$, closed: $l-1/2$

Weak magnetism

How does shell model perform right now?



'Easy' matrix elements only accurate to 10-20%

Weak magnetism

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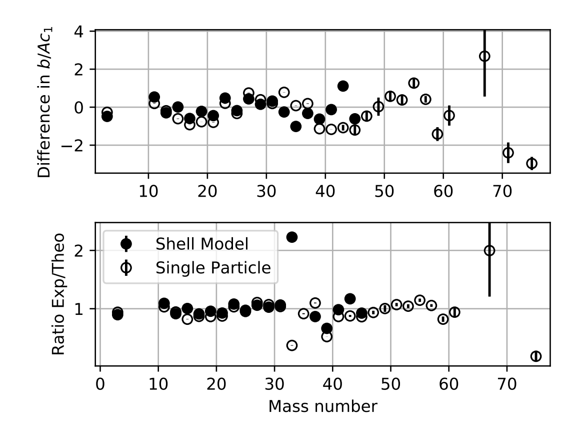

$\Delta b/Ac = 1 \rightarrow 0.1\%\ \mathrm{MeV}^{-1}$

Induced tensor

Still large discrepancies for d/Ac

PHYSICAL REVIEW C **95**, 035501 (2017)

2_1^+ to 3_1^+ γ width in ^{22}Na and second class currents

S. Triambak,[1,2,*] L. Phuthu,[1] A. García,[3] G. C. Harper,[3] J. N. Orce,[1] D. A. Short,[3] S. P. R. Steininger,[3] A. Diaz Varela,[4] R. Dunlop,[4] D. S. Jamieson,[4] W. A. Richter,[1] G. C. Ball,[5] P. E. Garrett,[4] C. E. Svensson,[4] and C. Wrede[3,6]

$$21(6) \geq d/Ac \geq 3(6)$$

Factor 7 differences depending on shell model results → killer!

Induced tensor

Immediate response last workshop (Alex Brown)

3+ to 2+	USDB	USDA	USD
M(s-tau) (c_1)	0.042	0.012	0.027
M(l-tau) (part of b)	-1.07	-1.00	-1.00
M(d-tau)	0.062	0.081	0.066

Relative phases look robust but s-tau is not very uncertain so we should look at b/d (not b/c and d/c)

Alex Brown, ND2013, NYC, March 4, 2013

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General lesson: when looking at allowed transitions, make sure $\mathcal{M}_{GT}$ is large and stable

Neutron V_{ud} calculation

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Neutron is extremely well-studied system, ideal system for V_{ud}

$$|V_{ud}|^2 \tau_n \left(f_V + 3 f_A \lambda^2\right) = \frac{2\pi^3}{G_F^2 m_e^5 g_V^2} \frac{1}{1 + RC}$$

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From β decay perspective, need 3 things

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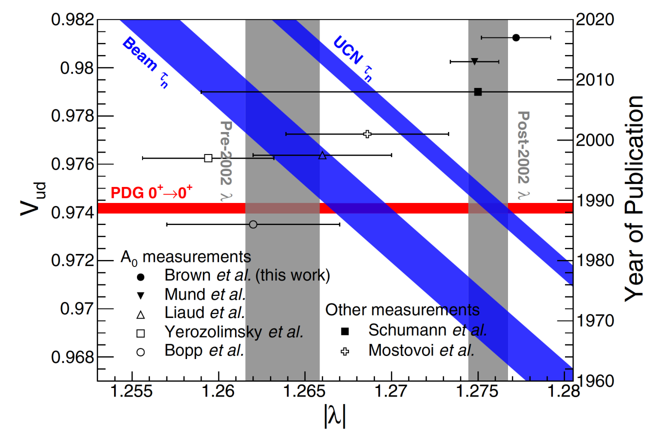
Clearly, all *trivial* things

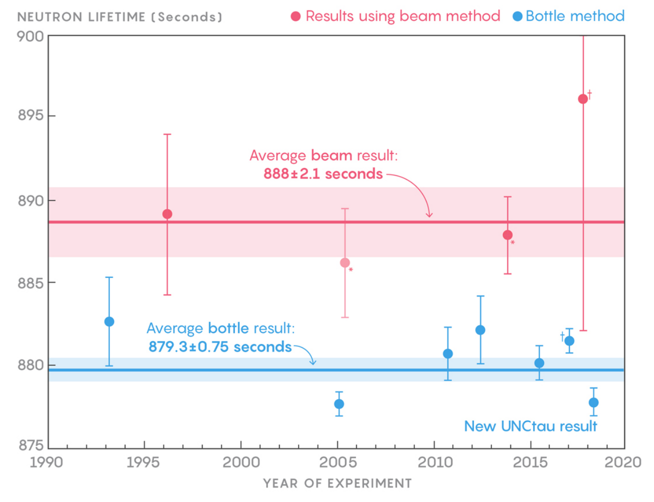
Major decades-long community efforts



UCNA, Phys Rev C 97 (2018) 035505

Neutron V_{ud} calculation

Major decades-long community efforts



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One particular case appears forgotten, however...

Neutron V_{ud} calculation

Recap:

$$\langle p|V^\mu|n\rangle = \bar{p}\left[g_V\gamma^\mu + \frac{g_M - g_V}{2M}\sigma^{\mu\nu}q_\nu + i\frac{g_S}{2M}q^\mu\right]n$$

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gives rise to spectrum shape contribution

$$\left(\frac{\mathrm{d}N}{\mathrm{d}W_e}\right)^{\text{wm}} \propto \frac{4}{3M}\frac{g_M}{g_A\mathcal{M}_{GT}}p_e W_e(W_0 - W_e)^2 \times \left(W_e - \frac{W_0}{2} - \frac{m_e^2}{2W_e}\right)$$

represents vector-axial vector spacelike cross term

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Except...

Weinberg, Phys Rev 115 (1959) 481

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There is one more thing: Coulomb corrections on weak magnetism gives non-negligible terms $\mathcal{O}(\alpha Z/MR)$ besides expected $\mathcal{O}(\alpha Z(q/M)qR)$

$$\frac{f_A}{f_V} = 1 + \frac{4}{5}\frac{\alpha Z}{MR}\frac{g_M}{g_A} = 1.0040(2)$$

Plot twist!

Wilkinson Nucl Phys A 377 (1982) 474; Bottino *et al.* Phys Rev C 9 (1974) 2052;

Holstein Phys Rev C 10 (1974) 1215

Neutron V_{ud} calculation

Using $\Delta_R = 0.02467(22), \quad \lambda = 1.27510(66)$

$$|V_{ud}|^2 \tau_n \left(1 + 3\frac{f_A}{f_V}\lambda^2\right) = 4903.5(1.1)\, s$$

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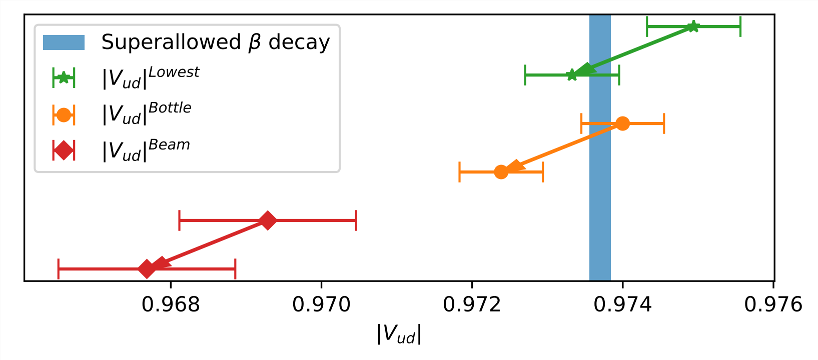

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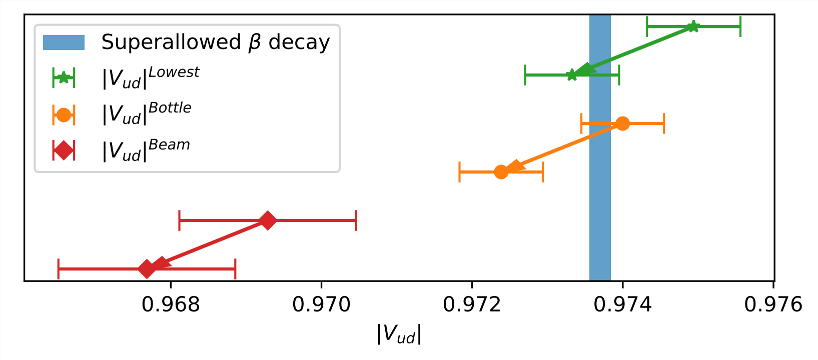
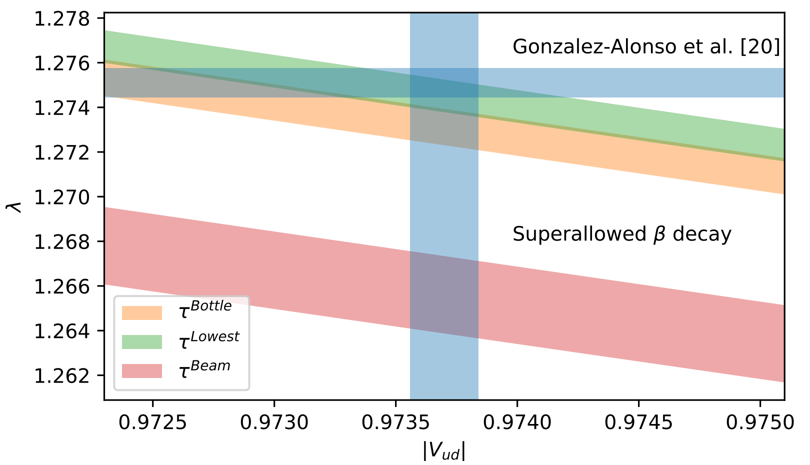


Yikes!

Seng *et al.*, PRL 121 (2019) 241804; PPNP 104 (2019) 165

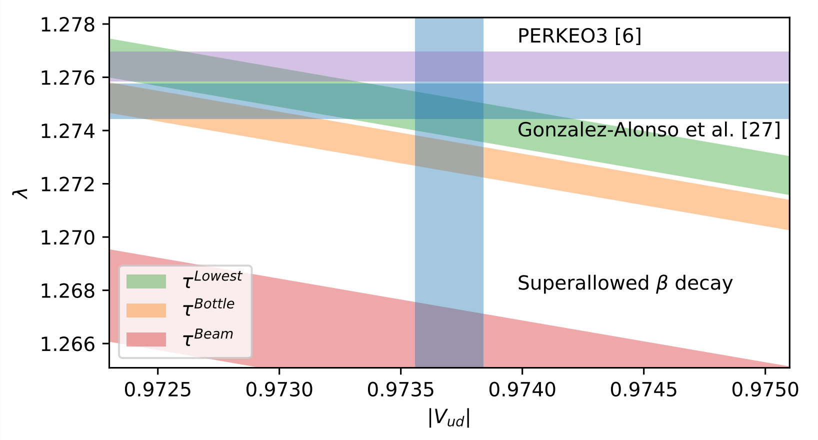
Neutron V_{ud} calculation

Using the new f_A/f_V



Neutron V_{ud} calculation

Using the new f_A/f_V, including latest PERKEO3



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Assume superallowed V_{ud}, predict 'Standard Model' τ_n

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Also impacts Big Bang Nucleosynthesis, helium mass abundance

$$\frac{\Delta Y_p}{Y_p} \approx 0.73 \frac{\Delta \tau_n}{\tau_n}.$$

Shift in neutron lifetime leads to

$$\Delta Y_p = -5.1 \cdot 10^{-4}$$

3 σ shift!

Challenges

Challenges

At $\mathcal{O}(10^{-3})$, nuclear structure is main culprit

- Nuclear matrix elements only precise to 10-20%
- Generally: large meson exchange corrections on induced currents
- Isospin multiplet decays are way to go: WM from CVC, induced tensor $= 0$
- Major ab initio efforts underway

Challenges

At $\leq \mathcal{O}(10^{-4})$, *everything* breaks

Challenges

At $\leq \mathcal{O}(10^{-4})$, *everything* breaks , but not in the same place!

- Low energy: Atomic & Molecular effects (exchange)
- Endpoint: Final state interactions, excitations
- Radiative corrections: higher order, model dependence
- Low Z: recoil corrections to matrix elements
- High Z: everything electromagnetic

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Exciting (unnerving?) developments happening in β decay fundamentals