Pion production within AMD+JAM approach

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Challenges to Transport Theory for Heavy-Ion Collisions
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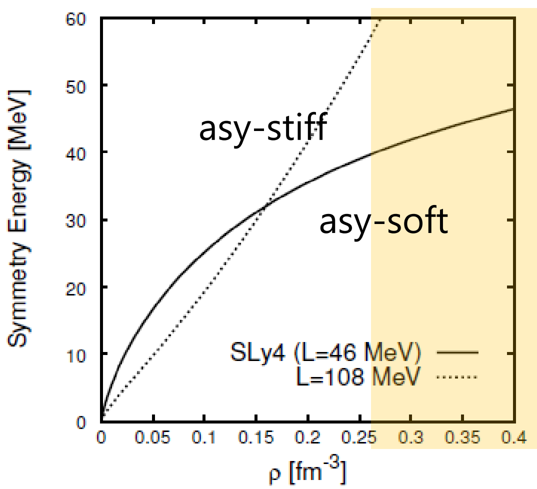
Symmetry energy and Heavy-ion collision

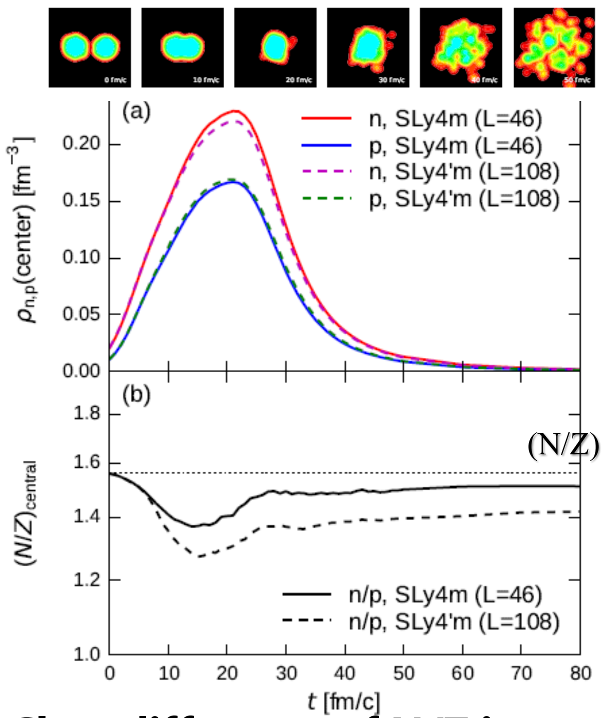
Symmetry energy soft / stiff ⟷ Nucleon N/Z ⟷ Δ Particle Δ^-/Δ^{++} ⟷ Pion π^-/π^+

(Nucleon dynamics) $(NN \leftrightarrow N\Delta)$ $(\Delta \leftrightarrow N\pi)$



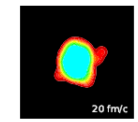
Interest:
High density $\rho \sim 2\rho_0$

^{132}Sn + ^{124}Sn, E/A = 300 MeV, $b \sim 0$



Clear difference of N/Z in high density due to different **S(ρ)**

Formation in NN collisions



π^- production

$$nn \to p\Delta^-$$
$$\Delta^- \to n\pi^-$$

π^+ production

$$pp \to n\Delta^{++}$$
$$\Delta^{++} \to p\pi^+$$

Simple expectation :

$$\left(\frac{\pi^-}{\pi^+}\right) \simeq \frac{5N^2 + NZ}{5Z^2 + NZ} \simeq \left(\frac{N}{Z}\right)^2$$

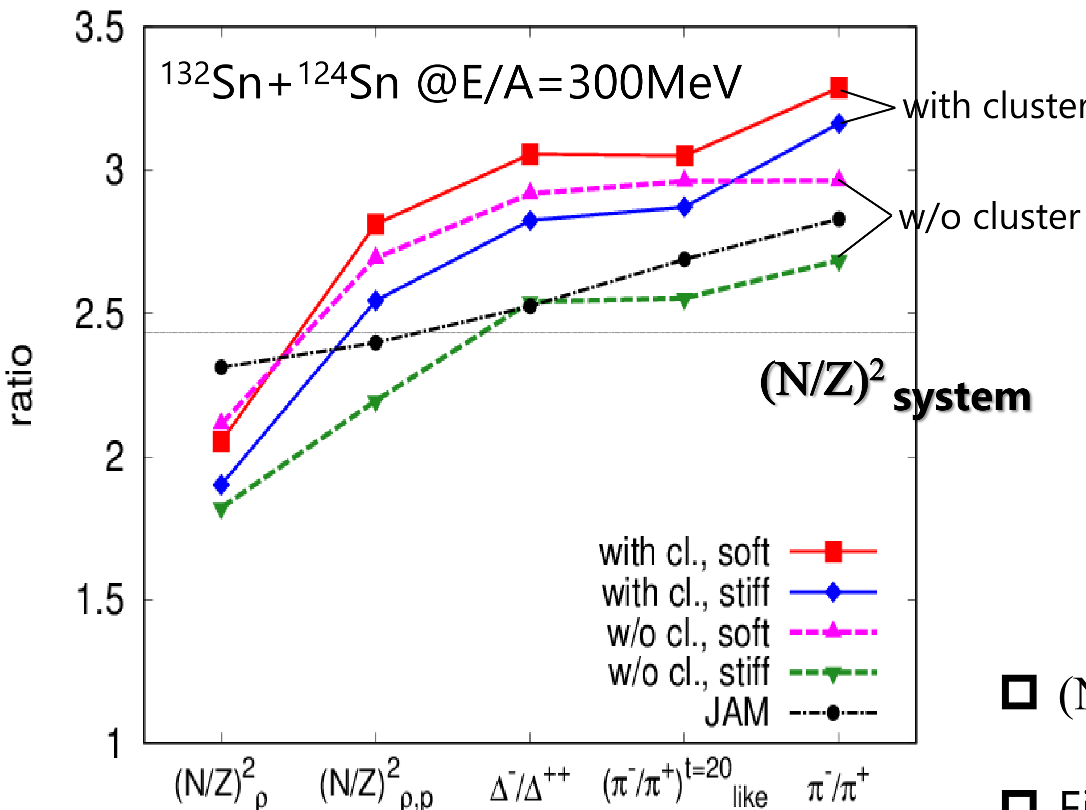
B. A. Li, PRL 88 (2002) 192701

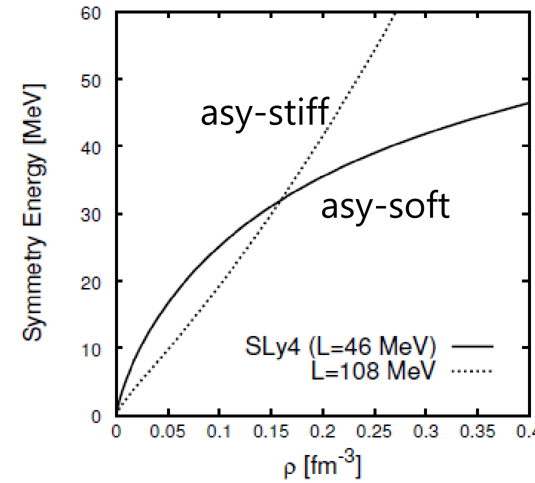
⇒ π^-/π^+ ratio is related to some kind of $(N/Z)^2$ ratio which is supposed to be sensitive to the symmetry energy at high densities.

Pion production based on AMD+JAM calculation

- From nucleons to pion ratios

N. Ikeno, A. Ono, Y. Nara, A. Ohnishi,
PRC93, 044612 (2016); PRC97, 069902(E) (2018)





Effective interaction:

Skyrme force

- $(N/Z)^2_{\rho p} \sim \Delta^-/\Delta^{++} \sim (\pi^-/\pi^+)^{t=20}_{like}$
- Final stage:
  π^-/π^+ is modified from $(\pi^-/\pi^+)^{t=20}_{like}$
  - ✓ Cluster correlation → π^-/π^+ up
  - ✓ S(ρ) effect: 30% weaker

Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$$

$$\frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-) dt}{\int_0^\infty (pp \to n\Delta^{++}) dt}$$

N(t), Z(t) : Numbers of nucleon which satisfy the conditions

Some effects on pion production

- Cluster correlation

  Larger ratios, weaker dependence on E_{sym}

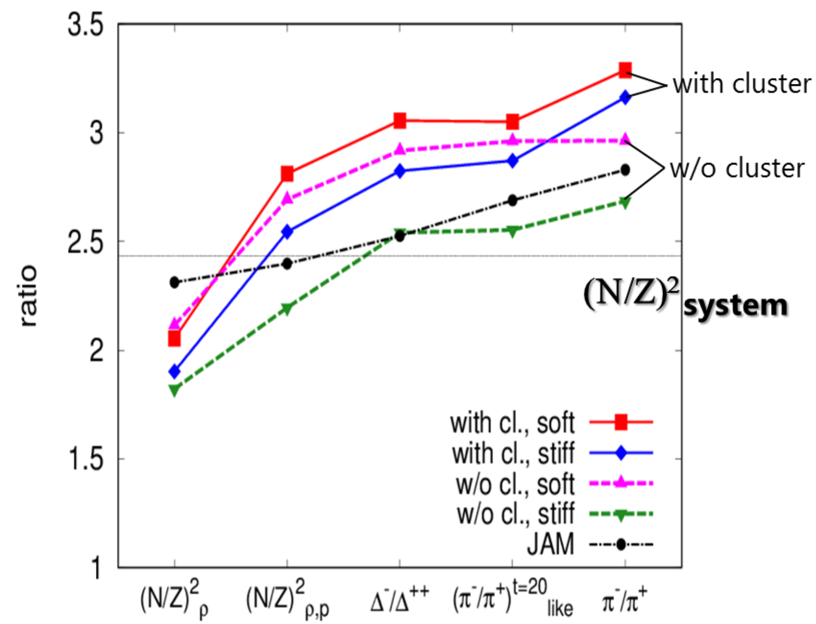
- Pauli-blocking effect

- Pion potential effect

- Delta threshold energy (NN→NΔ)

- Details of transport codes

  Y. X. Zhang *et al.*, PRC97 (2018) 034625,
  A. Ono *et al.*, arXiv1904.02888 (2019)



- Other unexpected effects .... etc.

This talk: Pion and Pauli-blocking effect within AMD+JAM approach

Pauli-blocking effect

π^- production

$$\underline{nn \to p\Delta^-}$$
$$\Delta^- \to n\pi^-$$

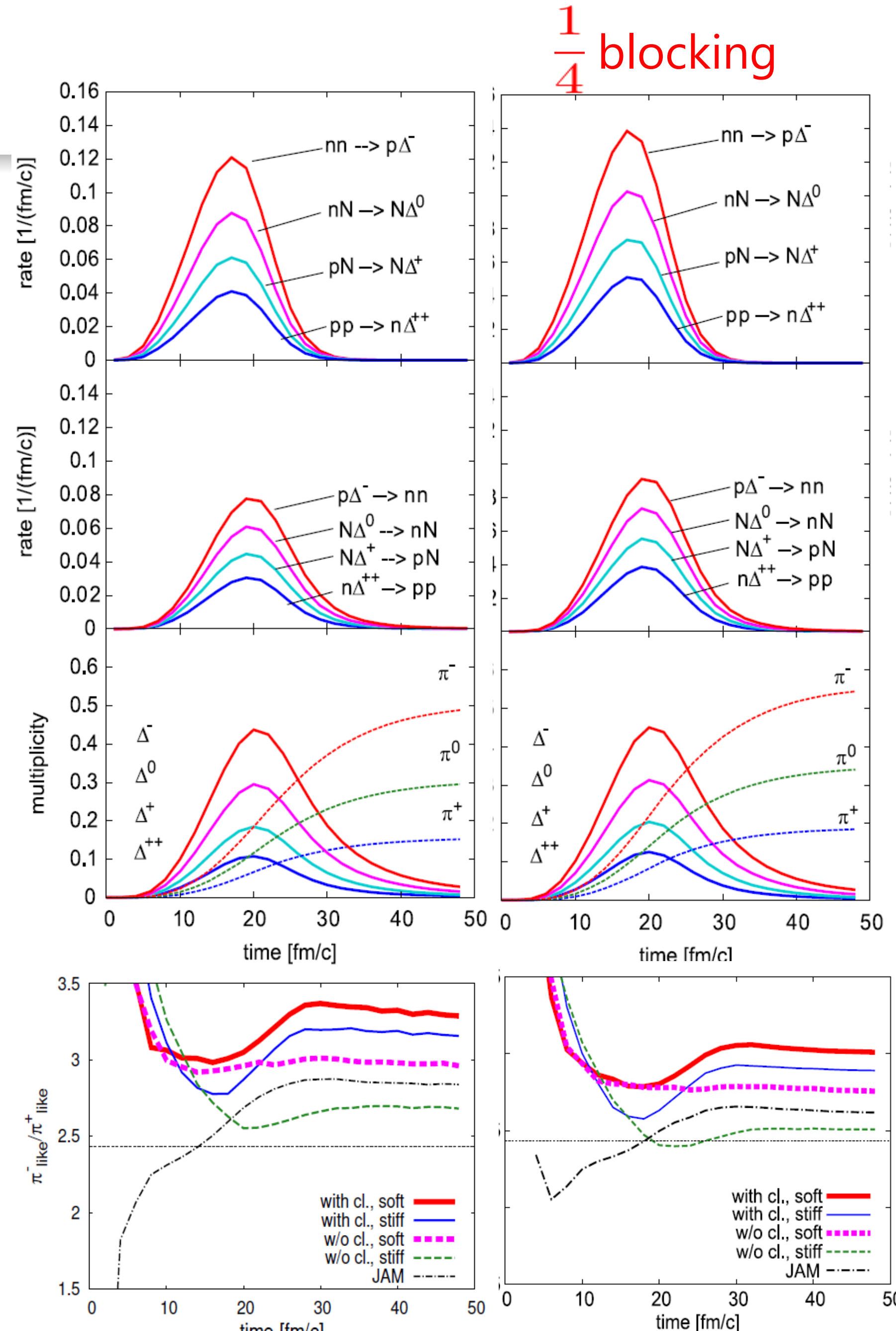
π^+ production

$$pp \to n\Delta^{++}$$
$$\Delta^{++} \to p\pi^+$$

Pauli blocking factor (1-*f*) for the final nucleon

If Pauli blocking is stronger,
Δ and π numbers are smaller

Pauli blocking may play some important role on the pion observables.
⇒ We need to estimate
Pauli blocking factor (1-f) precisely



N. Ikeno, A. Ono, Y. Nara, A. Ohnishi,
PRC93, 044612 (2016); PRC97, 069902(E) (2018)

Transport model

- Coupled equations for $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ ($\alpha = \mathrm{N}, \Delta, \pi$)

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N}{\partial \boldsymbol{p}} = I_N[f_N, f_{\Delta,\pi}]$$

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N, f_{\Delta,\pi}]$$

$I_{\mathrm{N}}[f_{\mathrm{N}}, f_{\Delta,\pi}]$:collision term

$$\left(\begin{array}{ll} \mathrm{N\,N} \to \mathrm{N\,N} & \\ \mathrm{N\,N} \to \mathrm{N}\,\Delta & \\ \mathrm{N}\,\Delta \to \mathrm{N\,N} & \\ \Delta \to \mathrm{N}\,\pi & \\ \mathrm{N}\,\pi \to \Delta & \text{... etc.} \end{array}\right)$$

- **Our model: JAM coupled with AMD**

Perturbative treatment of pion and Δ particle production

$$\boxed{I_N = (NN \to NN) + \lambda \cdot (NN \to N\Delta) + \ldots}$$

- **Nucleon f_{N} : Zeroth order equation**

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N^{(0)}, 0]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{p}} = I_N^{\mathrm{el}}[f_N^{(0)}, 0]$$

Solved by AMD

- **Δ particle f_Δ and pion f_π : First order equation**

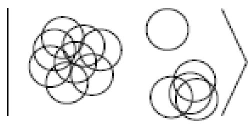
$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]$$

Solved by JAM for given $f_{\mathrm{N}}^{(0)}$

Transport model (AMD)

- **AMD** (Antisymmetrized Molecular Dynamics)
  A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, PTP87 (1992) 1185

  - AMD wave function at a time *t* for an event



$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\boldsymbol{r}_j - \frac{\boldsymbol{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

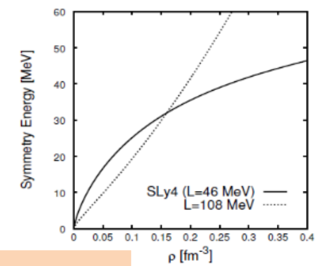
$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Solve the time evolution of the wave packet centroids Z

✓ Effective interaction

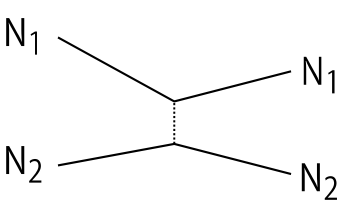


Skyrme force

- Turn on/off **Cluster correlation**

  - Without Cluster

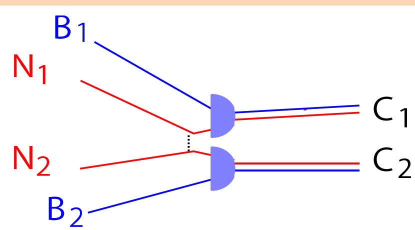
N1 + N2 -> N1 + N2



N1, N2 : Colliding nucleons

  - With Cluster

N1 + B1 + N2 + B2 -> C1 + C2



N1, N2: Colliding nucleons

B1, B2: Spectator nucleons/clusters

C1, C2: N, (2N), (3N), (4N) (up to α cluster)

Transport model (AMD+JAM)

- Nucleon test Particles

$$f^{\alpha}_{\mathrm{AMD}}(\boldsymbol{r},\boldsymbol{p}) = 2^3 \sum_{j\in\alpha}\sum_{k\in\alpha} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2} e^{-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

$$\boldsymbol{R}_{jk} = (\boldsymbol{Z}^*_j + \boldsymbol{Z}_k)/\sqrt{\nu}$$

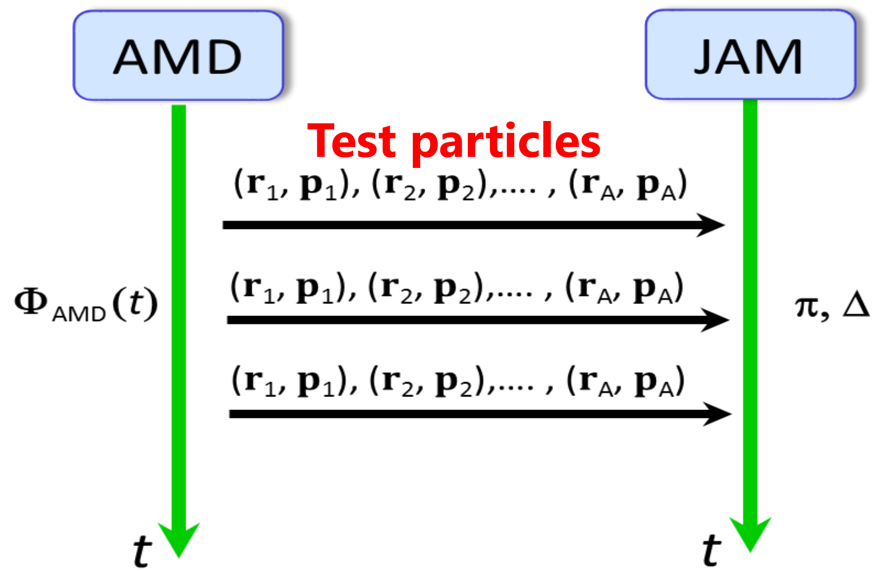
$$\boldsymbol{P}_{jk} = 2i\hbar\sqrt{\nu}(\boldsymbol{Z}^*_j - \boldsymbol{Z}_k)$$

$$B_{jk} = \langle\varphi_j|\varphi_k\rangle$$

$$\alpha = p\uparrow,\ p\downarrow,\ n\uparrow,\ n\downarrow$$

Test particles $(\mathbf{r}_1, \mathbf{p}_1), (\mathbf{r}_2, \mathbf{p}_2), \ldots, (\mathbf{r}_A, \mathbf{p}_A)$

- generated following the Wigner function $f_{\mathrm{AMD}}(\mathbf{r}, \mathbf{p})$
- sent from AMD to JAM at every 2 fm/c with corrections for the conservation of baryon number and charge

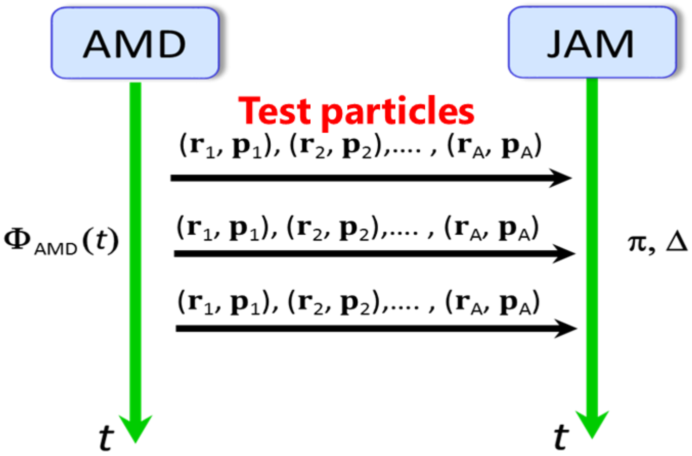


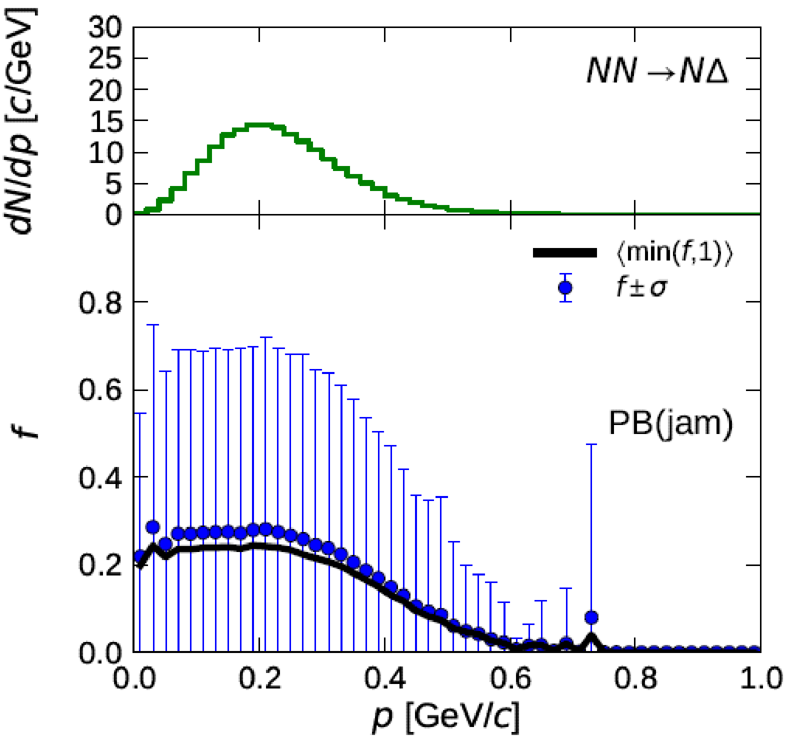
- **JAM** (Jet AA Microscopic transport model)

  Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901

  - Applied to high-energy collisions (1 ～ 158 A GeV)
  - Hadron-Hadron reactions are based on experimental data and the detailed balance.
  - No mean field (default), *s*-wave pion production (NN→NNπ) is turned off. ... etc.

Methods for Pauli-blocking factor *f*





- **Do Pauli blocking within JAM**

  (Natural prescription in AMD+JAM)

  $\mathrm{NN} \rightarrow \mathrm{N}\Delta,\ \Delta \rightarrow \mathrm{N}\pi$ etc.

$$f^{\tau}_{\mathrm{jam}}(\boldsymbol{r},\boldsymbol{p}) = \frac{2^3}{2}\sum_{j\in\tau} e^{-(\boldsymbol{r}-\boldsymbol{r}_j)^2/2L} e^{-2L(\boldsymbol{p}-\boldsymbol{p}_j)^2/\hbar^2}$$

$\tau = p, n$ L=2.0 fm^2

Pauli blocking factor $1 - f_{\mathrm{jam}}(\mathbf{r}_i, \mathbf{p}'_i)$ calculated for Test particles $\{(\mathbf{r}_j, \mathbf{p}_j);\ , j=1,2,\ \dots\ ,A\}$

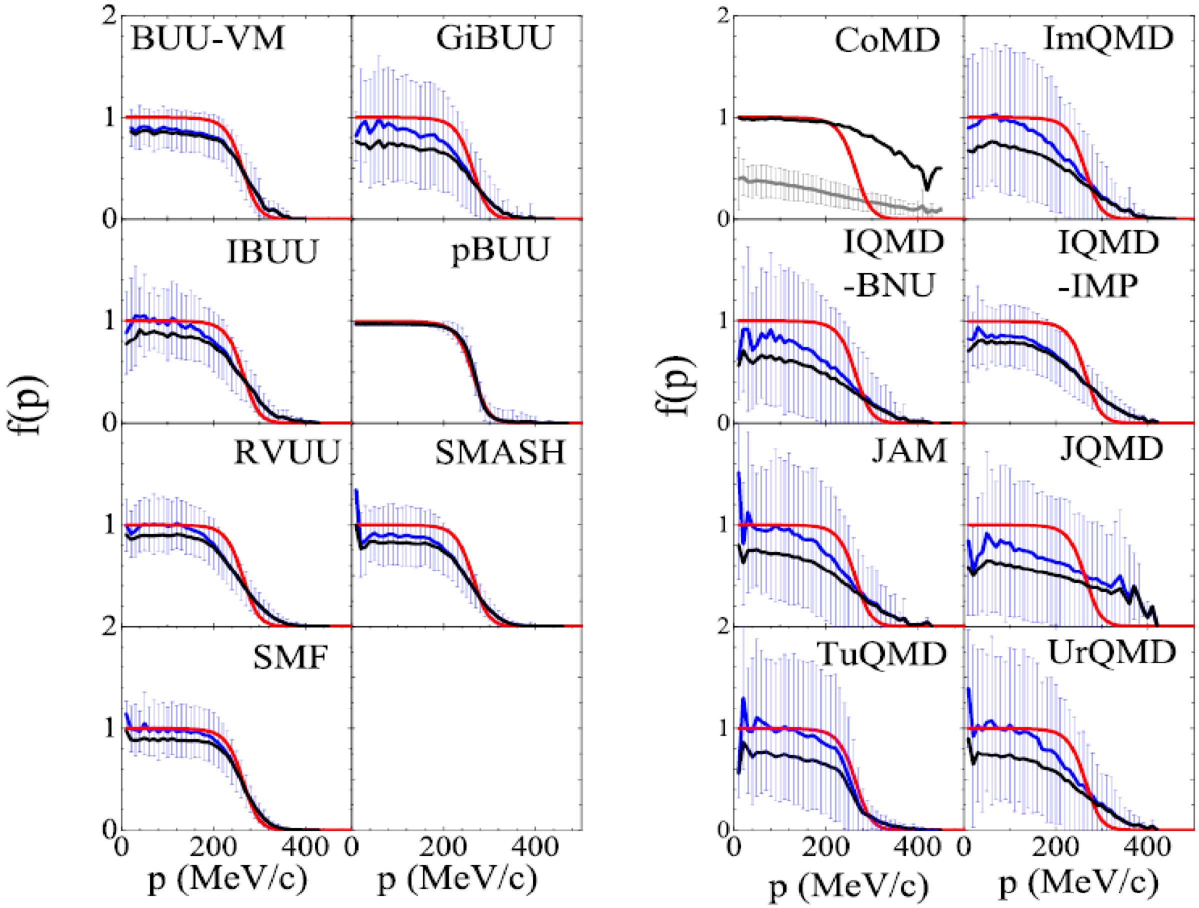
- ✓ f_{jam} sometimes becomes larger than 1 because of fluctuation

  $\langle \min(f,1)\rangle \le \langle f \rangle$

- ✓ f_{jam} is more broadly distributed than test particles because of smearing

Pauli blocking in Box HW1

Y. X. Zhang et al.,
PRC97 (2018) 034625



Fermi distribution

$$f = \frac{1}{1 + e^{(E-\mu)/T}}$$

Particularly in QMD codes

- ✓ **<*f*>** does not reproduce the original Fermi distribution because of smearing
- ✓ ***f*** sometimes becomes larger than 1 because of fluctuation

In HIC, Pauli blocking may be weaker because of low values of $\langle f \rangle$ due to the high temperature.

But we should still check this problem

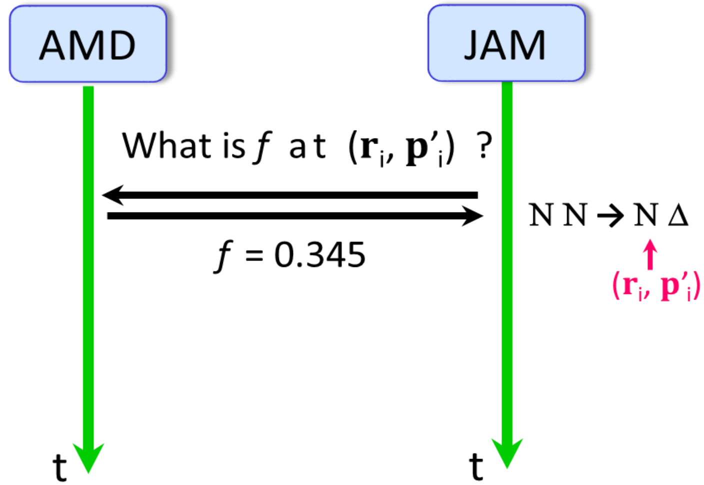
Methods for Pauli-blocking factor *f*

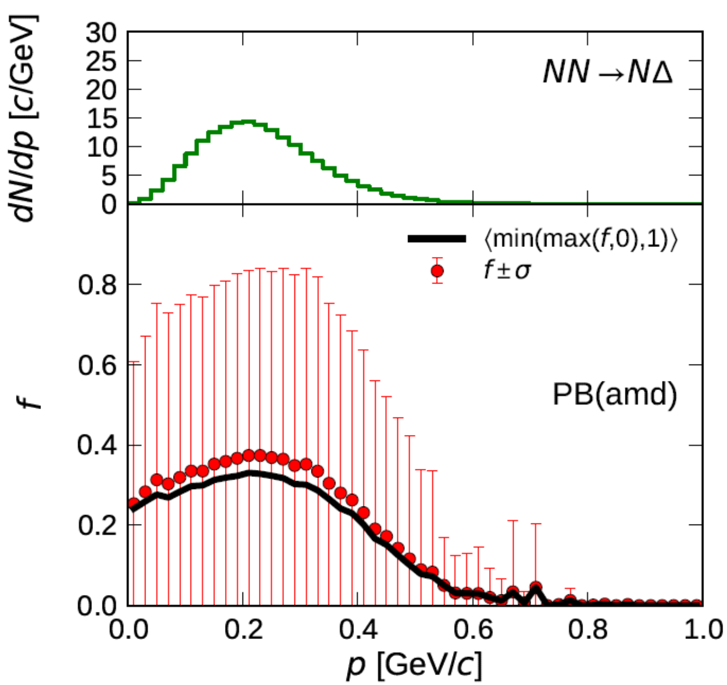
- **Use *f* of AMD for Pauli blocking**

Wigner function calculated for the AMD wave function, for τ = neutron or proton, is

$$f^{\tau}_{\mathrm{amd}}(\boldsymbol{r},\boldsymbol{p}) = \frac{2^3}{2}\sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2} e^{-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

Pauli-blocking factor $1 - f_{\mathrm{amd}}(\mathbf{r}_i, \mathbf{p}'_i)$ for the final phase-space point $(\mathbf{r}_i, \mathbf{p}'_i)$.





- Communication between AMD and JAM
- AMD accepts a question from JAM, calculates f_{amd}, and answers it to JAM

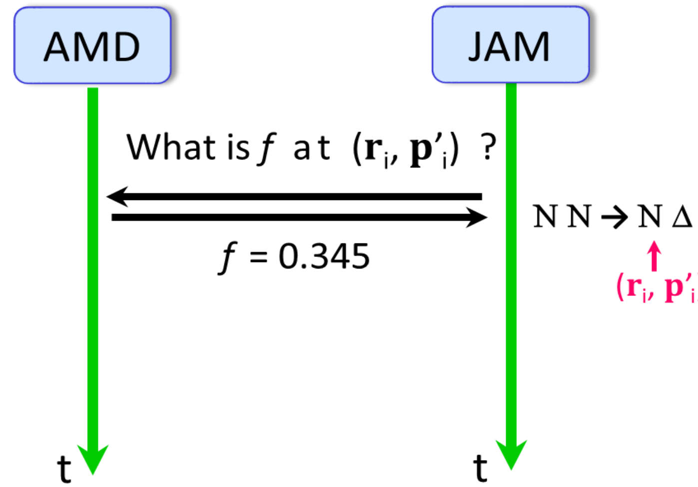
Methods for Pauli-blocking factor f

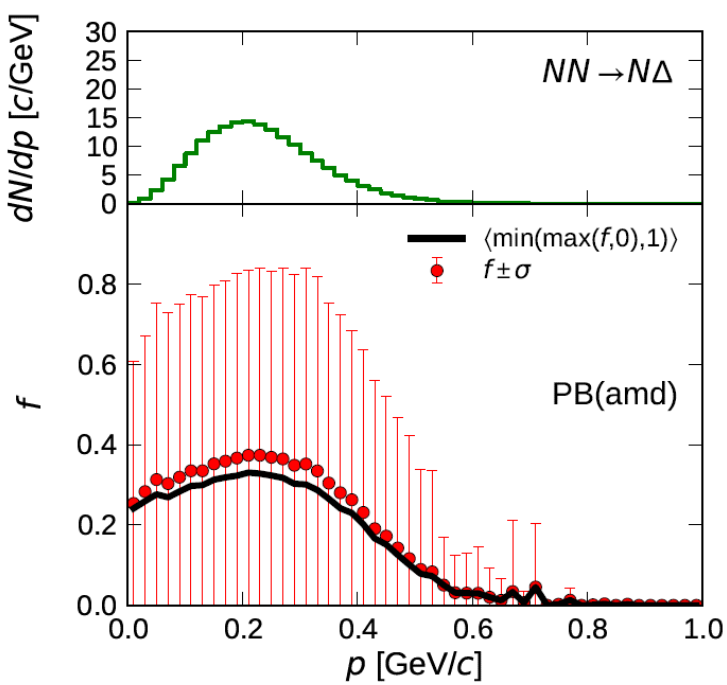
- **Use f of AMD for Pauli blocking**

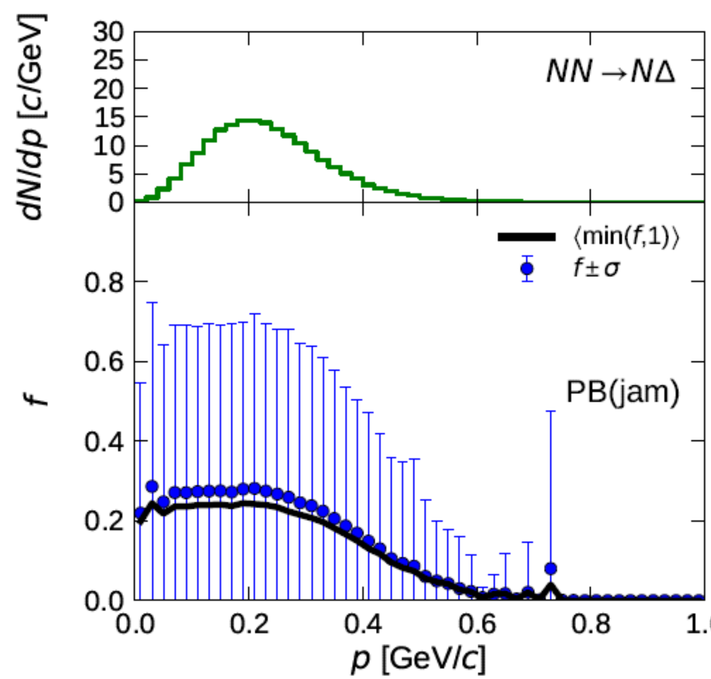
Wigner function calculated for the AMD wave function, for τ = neutron or proton, is

$$f_{\mathrm{amd}}^{\tau}(\boldsymbol{r},\boldsymbol{p}) = \frac{2^3}{2}\sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2} e^{-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk} B_{kj}^{-1}$$

Pauli-blocking factor 1 - $f_{\mathrm{amd}}(\mathbf{r}_i, \mathbf{p}'_i)$ for the final phase-space point $(\mathbf{r}_i, \mathbf{p}'_i)$.

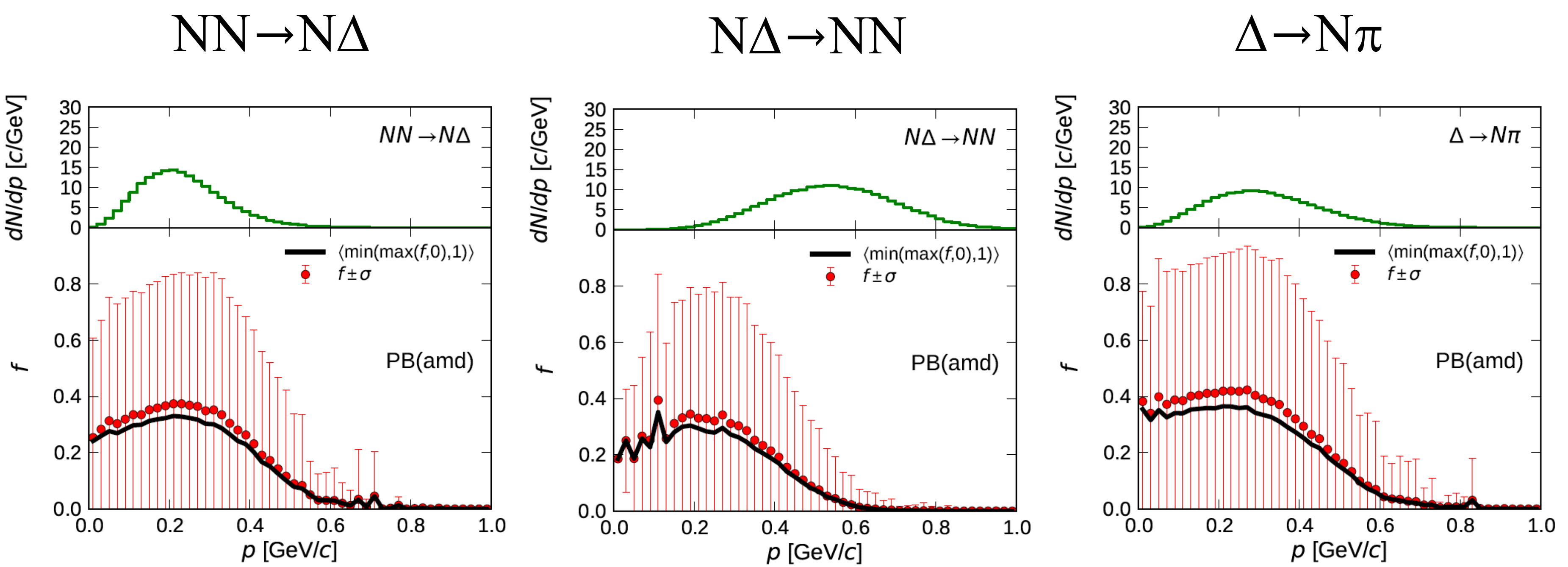






Pauli-blocking probability f_{amd}

$^{132}Sn+^{124}Sn$@E/A=270 MeV



Pauli blocking is important for NN→NΔ and Δ→Nπ
because the final momentum is relatively low

In NΔ→NN reaction, Pauli blocking may not be so important

Calculated system and parameters/options

- ^{132}Sn + ^{124}Sn @E/A=270 MeV 0<b<1 fm

- 5 options: Pauli blocking procedures (PB)

1. PB($\frac{1}{4}$ jam) : Pauli blocking factor f_{jam} is artificially reduced by factor 4

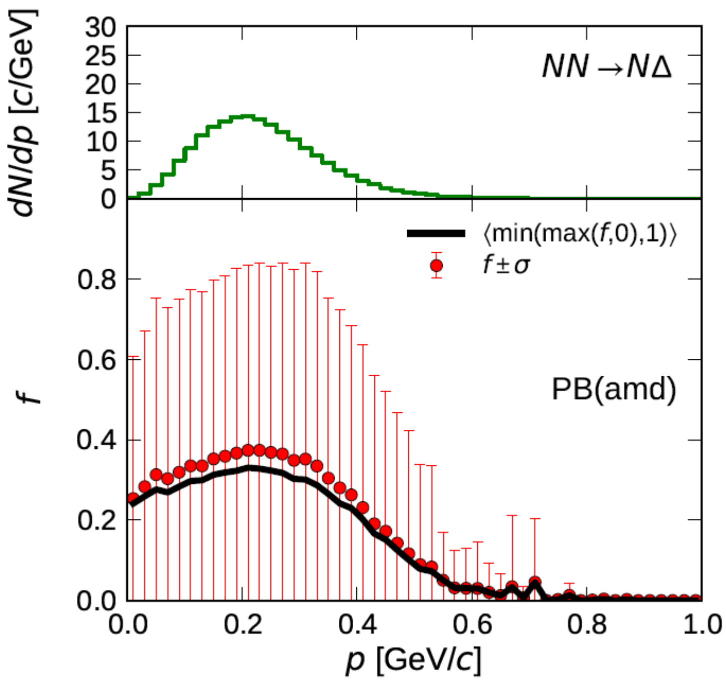
2. **PB(jam)** : Use f_{jam}, Do Pauli blocking within JAM

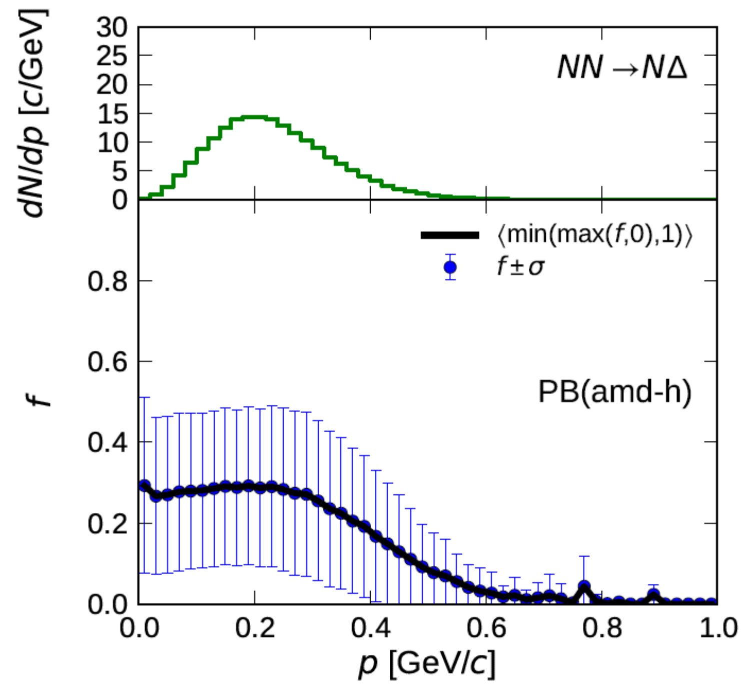
3. PB(amd, jam) : Use f_{amd} (Wigner function of AMD) only for $NN \leftrightarrow N\Delta$
   Use f_{jam} for $\Delta \rightarrow N\pi$

4. **PB(amd)** : Use f_{amd} (Wigner function of AMD)
   both for $NN \leftrightarrow N\Delta$ and $\Delta \rightarrow N\pi$

5. PB(amd-h) : Use Husimi function of AMD

Wigner function and Husimi function





blocking probability = 1 ($f > 1$)

= f ($0 \leq f \leq 1$)

= 0 ($f < 0$)

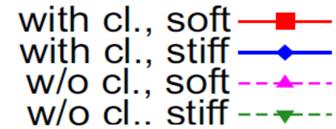
$$f^{\tau}_{\mathrm{amd}}(\boldsymbol{r},\boldsymbol{p}) = \frac{2^3}{2}\sum_{j\in\tau}\sum_{k\in\tau} e^{-2\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2} e^{-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/2\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

Husimi function is guaranteed to be $0 \leq f \leq 1$

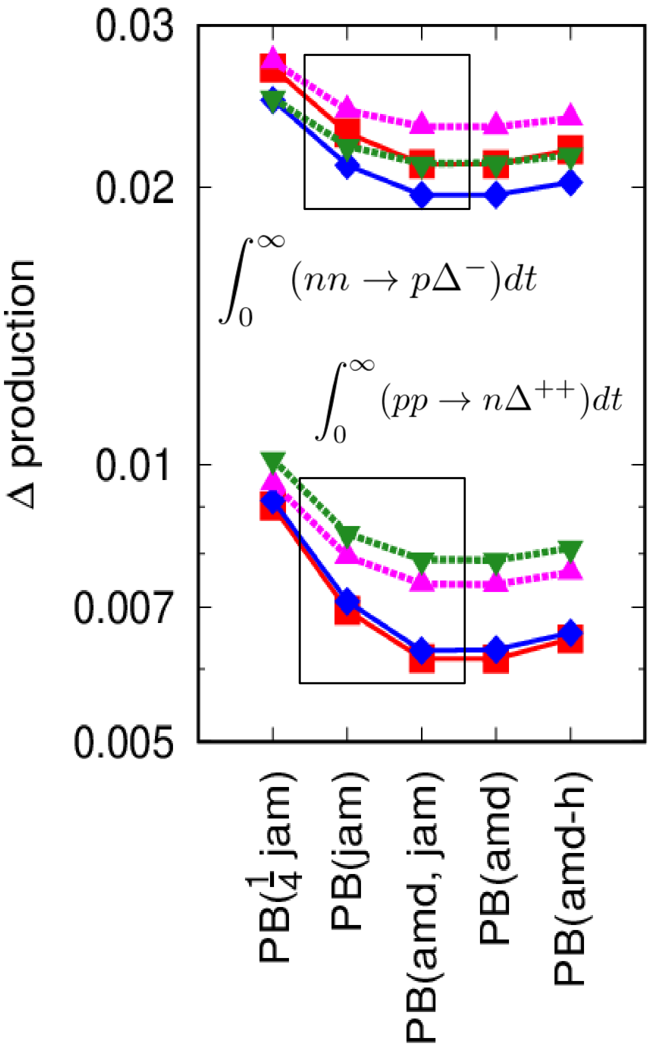
But the Pauli blocking in the Boltzmann equation should be calculated with the Winger function, not the Husimi function.

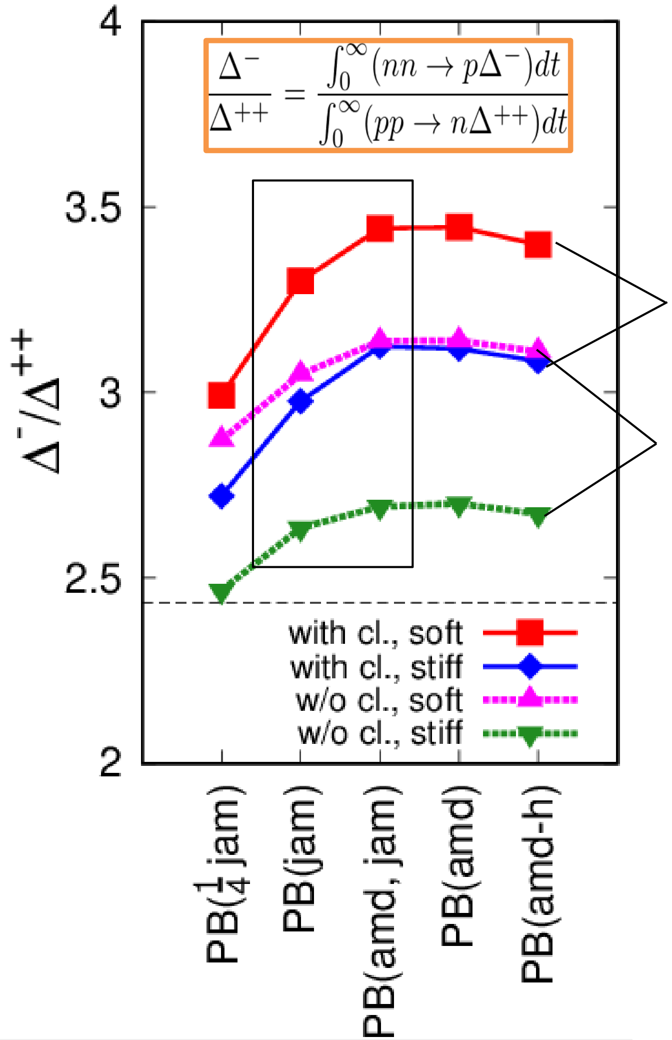
$$f^{\tau}_{\mathrm{amd\text{-}h}}(\boldsymbol{r},\boldsymbol{p}) = \iint \frac{d\boldsymbol{r}'d\boldsymbol{p}'}{(\pi\hbar)^3} e^{-2\nu(\boldsymbol{r}-\boldsymbol{r}')^2} e^{-(\boldsymbol{p}-\boldsymbol{p}')^2/2\hbar^2\nu} f^{\tau}_{\mathrm{amd}}(\boldsymbol{r}',\boldsymbol{p}')$$
$$= \frac{1}{2}\sum_{j\in\tau}\sum_{k\in\tau} e^{-\nu(\boldsymbol{r}-\boldsymbol{R}_{jk})^2} e^{-(\boldsymbol{p}-\boldsymbol{P}_{jk})^2/4\hbar^2\nu} B_{jk}B^{-1}_{kj}$$

Pauli-blocking effect for NN→NΔ



Comparison of PB(jam) and PB(amd, jam)





n-rich system
-> final neutron is blocked more strongly
$pp \to n\Delta^{++}$

with cluster

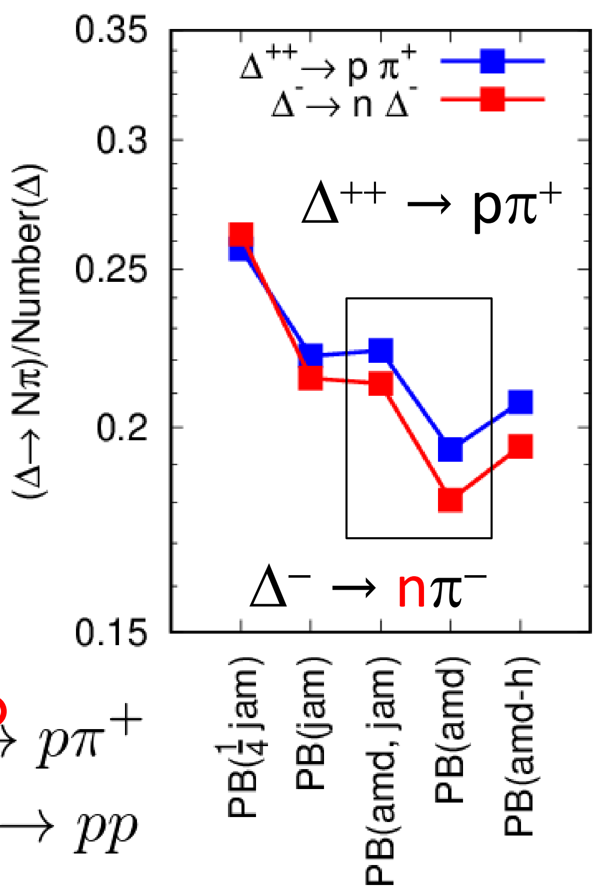
w/o cluster

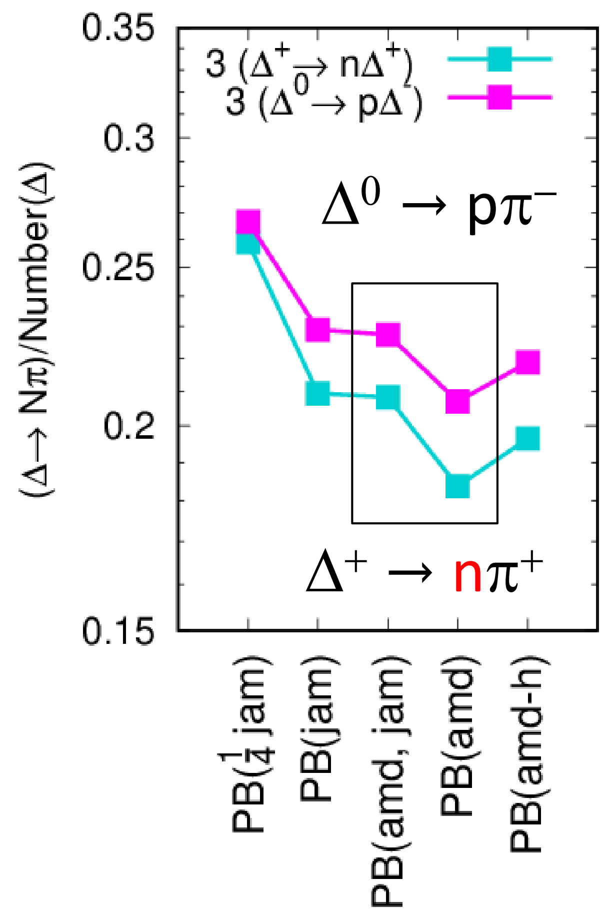
Pauli-blocking effect is stronger for the production of Δ^{++} than Δ^-

In particular when cluster correlation is switched on.

Pauli-blocking effect for Δ→Nπ $\int_0^\infty (\Delta \to N\pi)dt / \int \Delta(t)dt$

Comparison of PB(amd, jam) and PB(amd)





$\Delta^{++} \to p\pi^+$

$\Delta^{++} n \to pp$

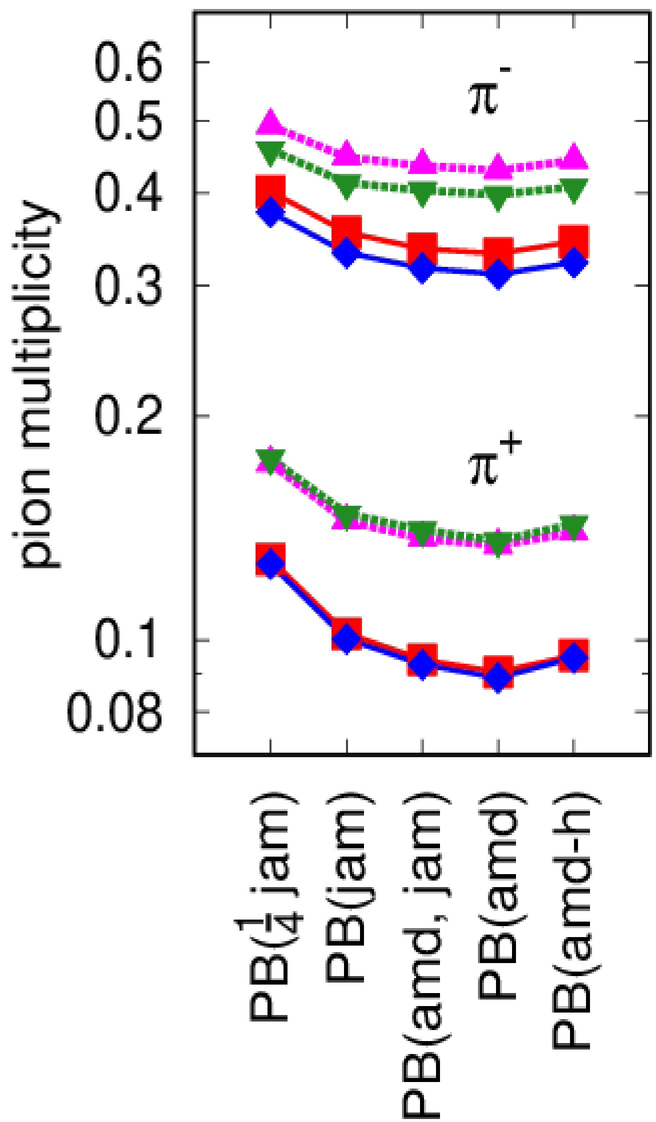
$\Delta^0 \to n\pi^0$

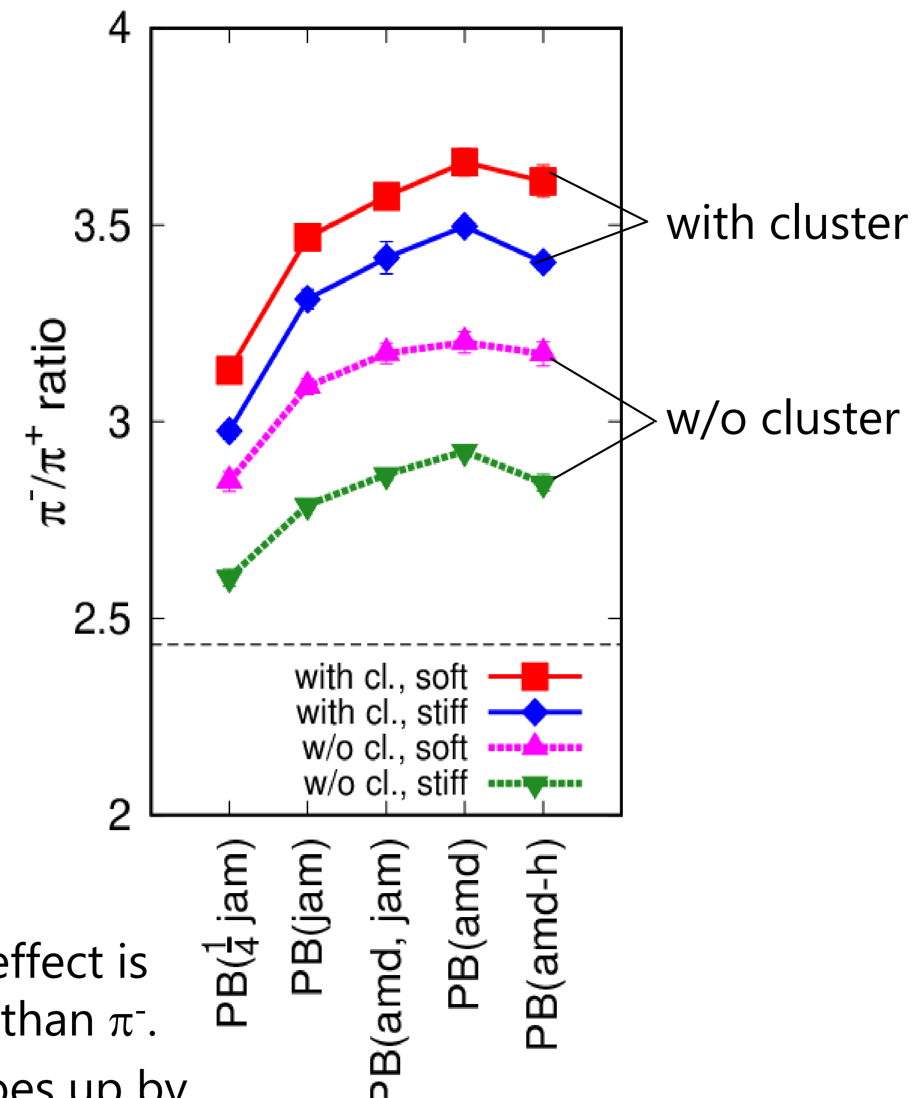
$\Delta^0 \to p\pi^-$

$\Delta^0 N \to NN$

- Final neutron is blocked more strongly than a proton
- Strong Pauli blocking in $\Delta^0 \to n\pi^0$ tends to increase $\Delta^0 \to p\pi^-$

Final pion @ E/A=270 MeV





Pauli-blocking effect is stronger for π^+ than π^-.

-> Pion ratio goes up by Pauli-blocking effect

Summary

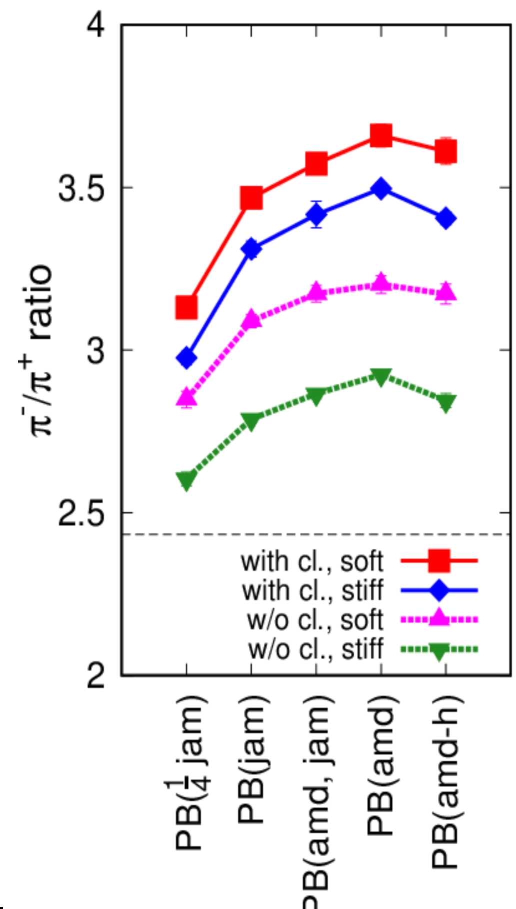
High-ρ symmetry energy and Pion production on HIC

- Improved Pauli blocking procedure

  for NN<−>NΔ, Δ −>Nπ
  - Strength of blocking probability

    PB(amd) > PB(amd-h)> PB(jam)

- Pauli-blocking effect for pion production
  - Pauli-blocking effect is stronger for π^+ (Δ^{++}) than π^- (Δ^-) in n-rich system
  - π^-/π^+ and Δ^-/Δ^{++} ratios go up

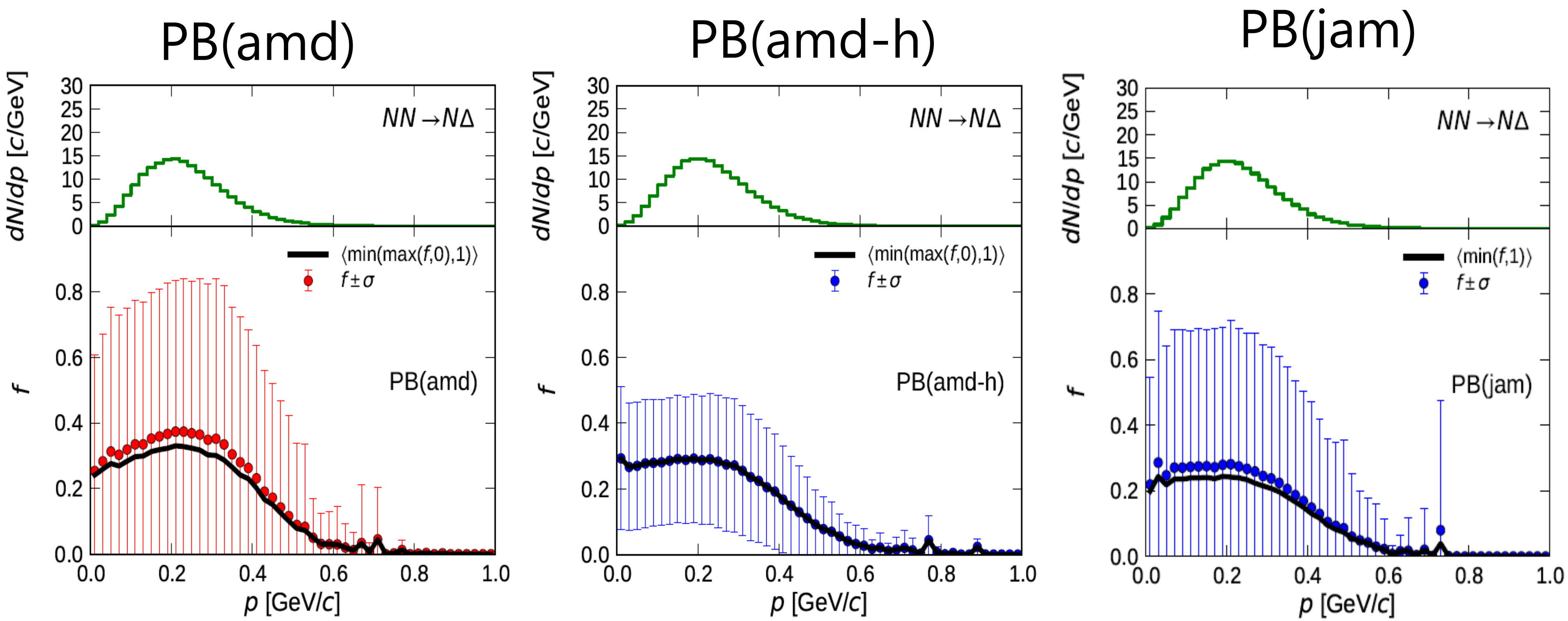


Future work:

We need to study other treatments for pion observables

- Pion potential, Δ resonance production threshold …

blocking probability for NN→NΔ



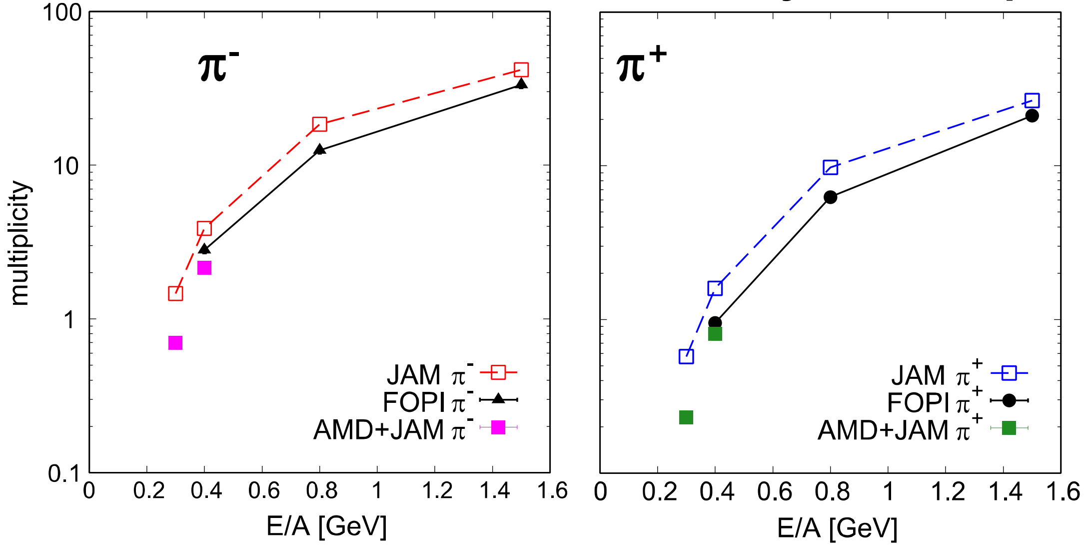
Strength of blocking probability: PB(amd) > PB(amd-h)> PB(jam)

Pion Calcutions in central Au+Au collisions

by transport model (AMD + JAM)

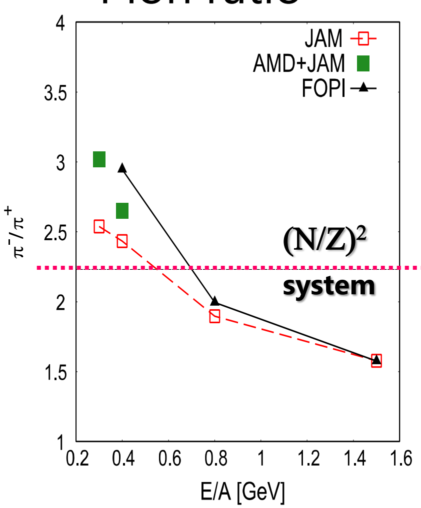
with cluster (asy-soft)

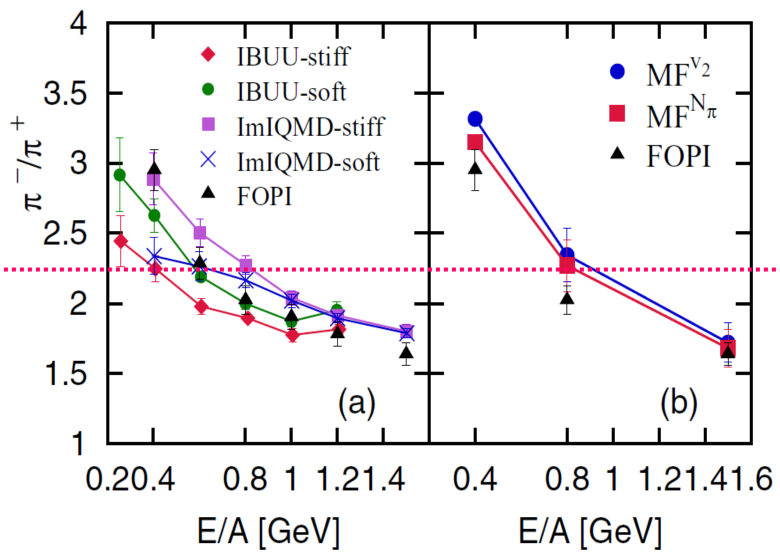
- Pion multiplicity



- ✓ Our calculation almost reproduces the experimental data reasonably well
- ✓ Pion ratios are also larger than $(N/Z)^2_{\text{system}}$

- Pion ratio

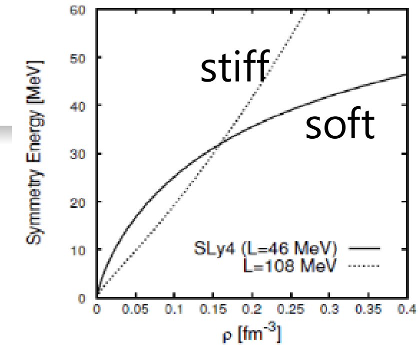




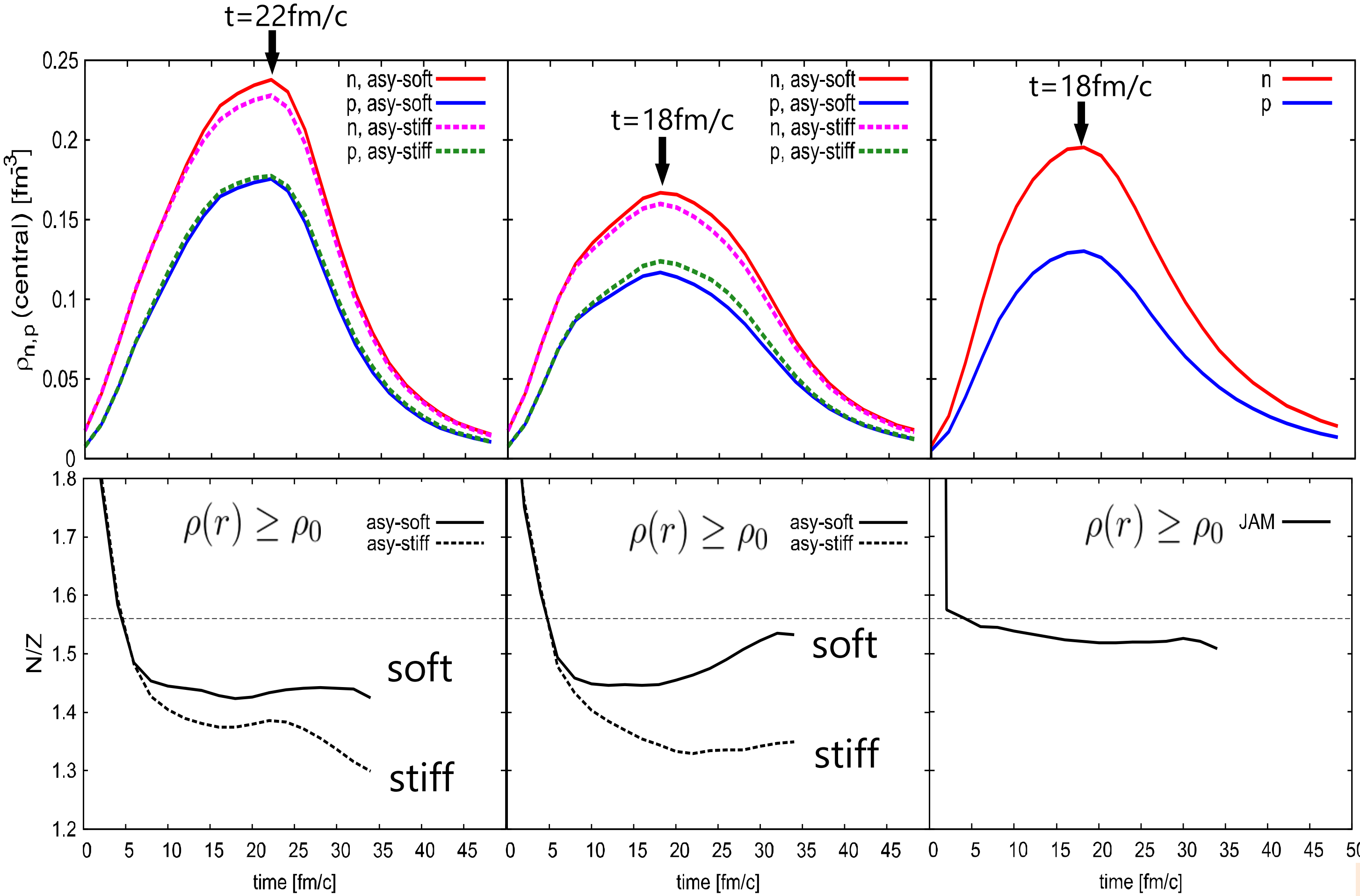
J. Hong and P. Danielewicz , PRC90 (2014) 024605

Exp. Data: Reisdorf *et al.*, NPA 848 (2010) 366

Dynamics of neutrons and protons



- with cluster
- without cluster
- JAM



Effective interaction:
Skyrme force

$^{132}Sn + ^{124}Sn$ Collision
@E/A=300 MeV

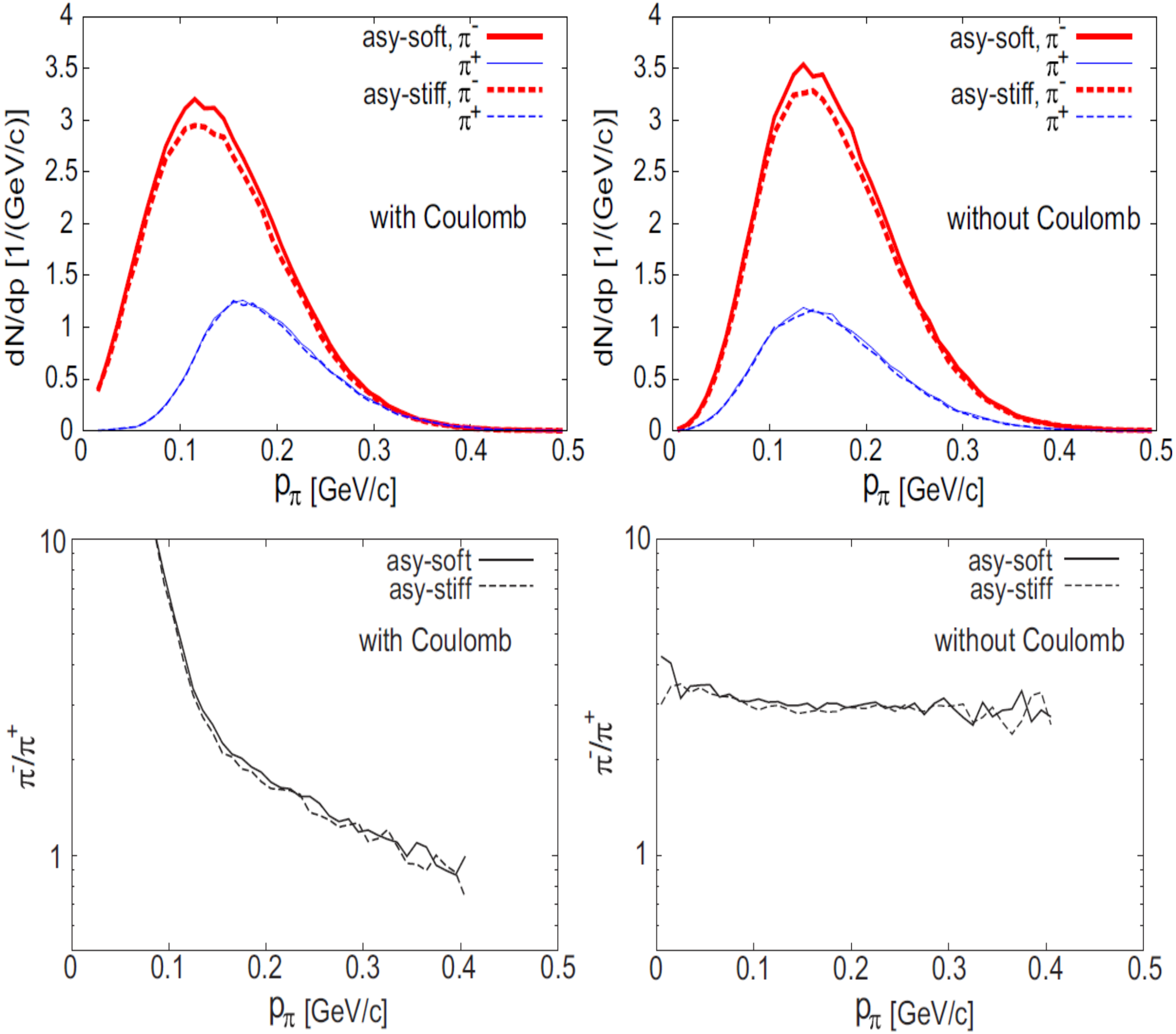
- ✓ Density maximum is different for cases with or without cluster
- ✓ Clear difference of N/Z ratio due to different symmetry energy
- ✓ Especially symmetry energy effect is weaker if there is cluster correlation

Pion spectra

$^{132}Sn + ^{124}Sn$ Collision
@E/A=300 MeV

AMD + JAM with cluster (asy-soft)

- With Coulomb
- Without Coulomb



- Coulomb effect:
  Acceleration of π^+
  Deceleration of π^-

→ Changes of pion spectra

	π^-	π^+	π^-/π^+
with Coulomb	0.577	0.192	**3.01(1)**
w/o Coulomb	0.582	0.193	**3.02(1)**

→ Coulomb effect has almost no effect on the pion multiplicities and the pion ratio.

Potential for Δ and pion

In JAM, reaction thresholds are the same as in free space.

(The production and absorption reactions for Δ and pions occur in the JAM calculation as in the free space)

Nucleons feel potential in the AMD calculation.

Therefore AMD+JAM assumes

$$\mathrm{NN} \leftrightarrow \mathrm{N}\Delta \qquad \Delta \leftrightarrow \mathrm{N}\pi$$

$$U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} = U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}, \qquad U^{(\Delta)}_{\tau_1} = U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4} \qquad \text{for } \tau_1(+\tau_2) = \tau_3 + \tau_4$$

This is equivalent to the choice in the pBUU calculation
c.f. Hong and Danielewicz, PRC 90 (2014) 024605

$$\begin{aligned}
v_{asy}(\Delta^-) &= 2v_{asy}(n) - v_{asy}(p) = 3v_{asy}(n), \\
v_{asy}(\Delta^0) &= v_{asy}(n), \\
v_{asy}(\Delta^+) &= v_{asy}(p) = -v_{asy}(n), \\
v_{asy}(\Delta^{++}) &= 2v_{asy}(p) - v_{asy}(n) = -3v_{asy}(n).
\end{aligned}$$

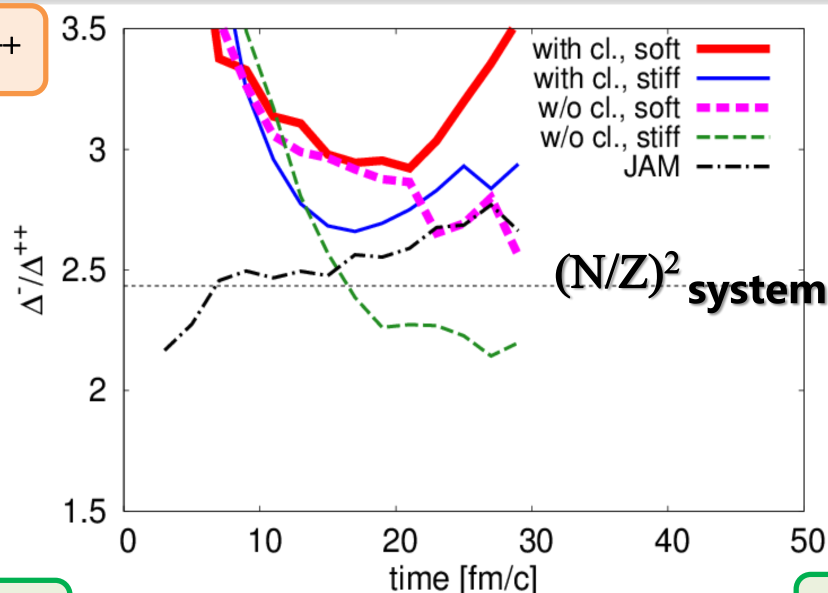
* Different choice,
cf. Bao-An Li

$$\begin{aligned}
v_{asy}(\Delta^-) &= v_{asy}(n), \\
v_{asy}(\Delta^0) &= \frac{2}{3}v_{asy}(n) + \frac{1}{3}v_{asy}(p) = \frac{1}{3}v_{asy}(n), \\
v_{asy}(\Delta^+) &= \frac{1}{3}v_{asy}(n) + \frac{2}{3}v_{asy}(p) = -\frac{1}{3}v_{asy}(n), \\
v_{asy}(\Delta^{++}) &= v_{asy}(p) = -v_{asy}(n).
\end{aligned}$$

Relation between N/Z and Δ^-/Δ^{++}

Nucleon
N/Z
?
Δ resonance
Δ^-/Δ^{++}

Δ^-/Δ^{++}

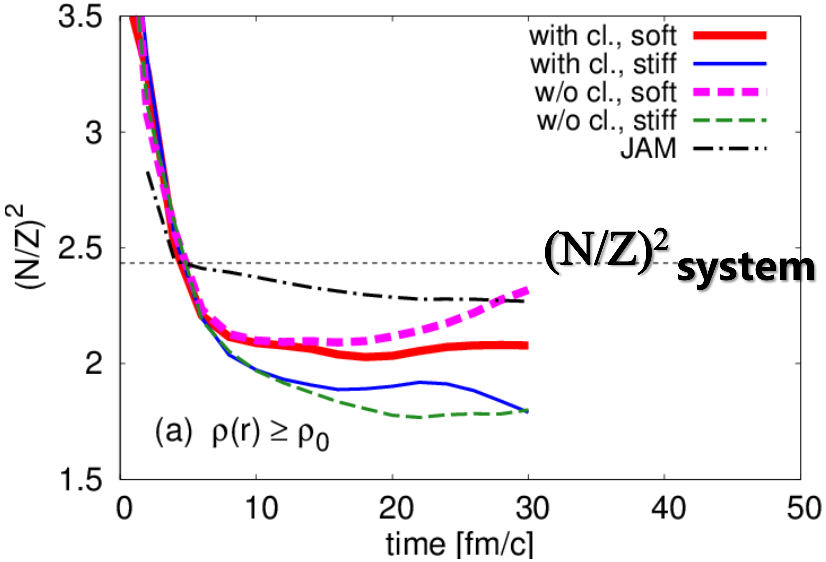


Simple expectation: $\Delta^-/\Delta^{++} \sim (N/Z)^2$

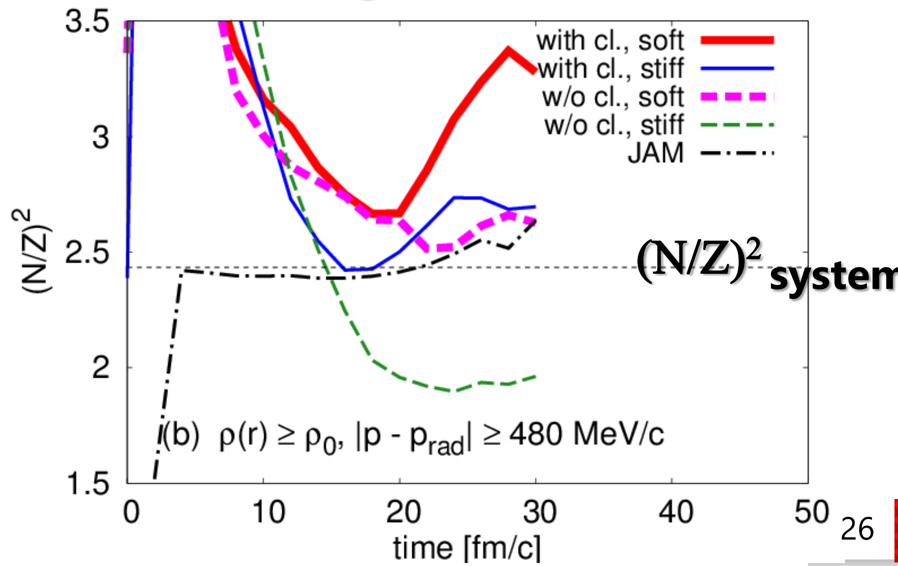
$$\frac{\Delta^-}{\Delta^{++}} = \frac{\mathrm{Rate}(nn \to p\Delta^-)}{\mathrm{Rate}(pp \to n\Delta^{++})}$$

(The collective radial momentum $\boldsymbol{p}_{\mathrm{rad}}$ is subtracted)

$(N/Z)^2_\rho$ Nucleons in the sphere $\rho(r) \geqq \rho_0$ centered at CM.

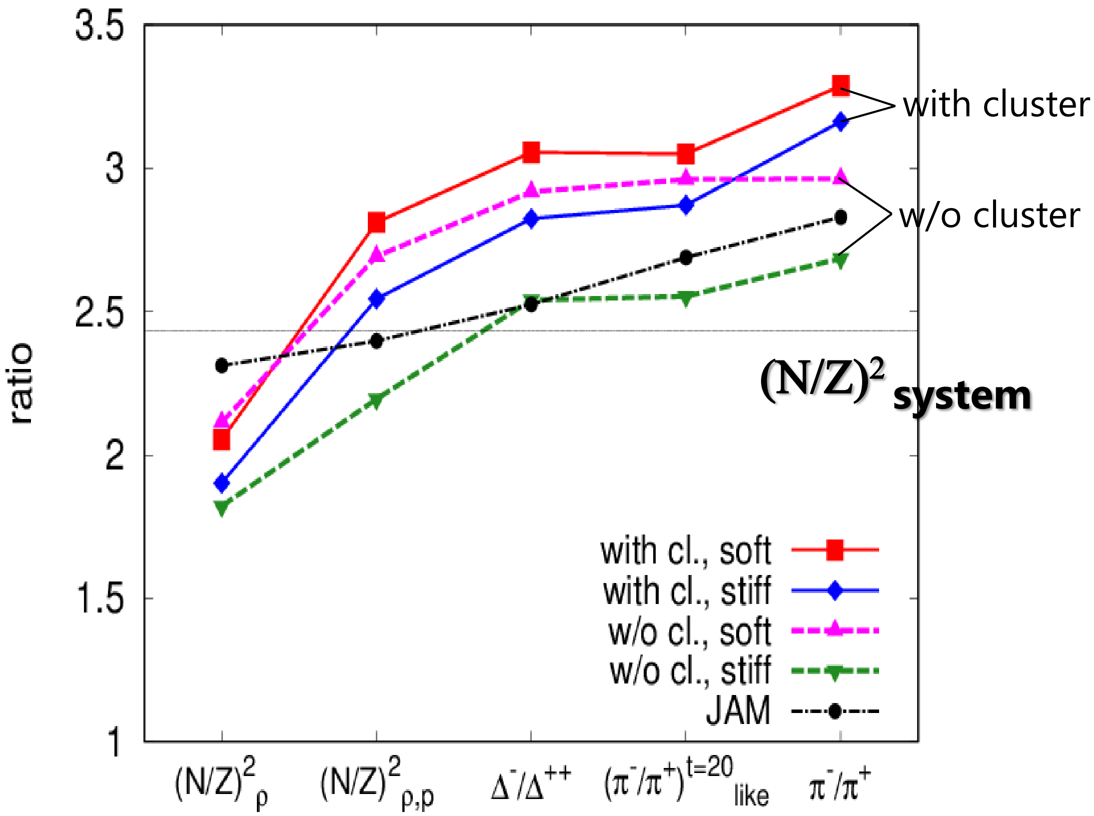


$(N/Z)^2_{\rho,\,p}$ Nucleons in the sphere $\rho(r) > \rho_0$ with **high momentum**



Final π^-/π^+ ratio

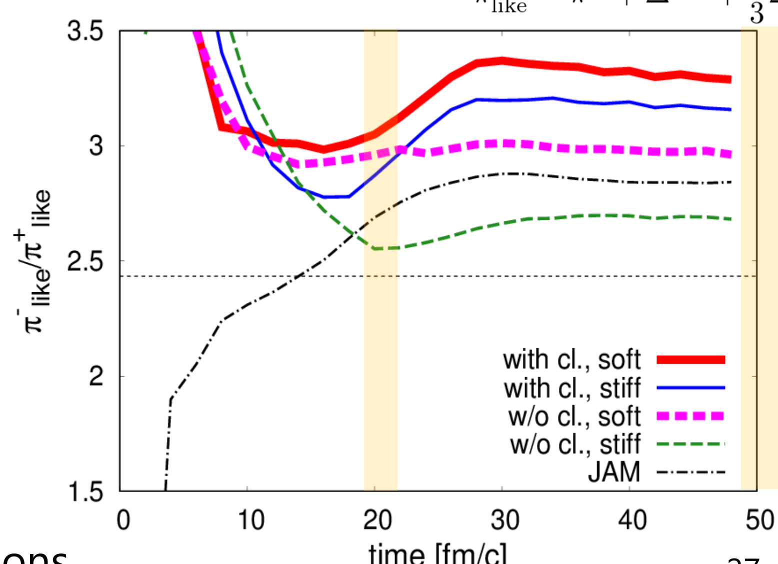
- From nucleons to pion ratios



- $\Delta^-/\Delta^{++} \sim (\pi^-/\pi^+)^{t=20}_{\text{like}}$
- Final stage:
  π^-/π^+ is modified from $(\pi^-/\pi^+)^{t=20}_{\text{like}}$
  - ✓ Cluster correlation → π^-/π^+ up
  - ✓ $S(\rho)$ effect: 30% weaker

$$\pi^-_{\text{like}} = \pi^- + \Delta^- + \frac{1}{3}\Delta^0$$

$$\pi^+_{\text{like}} = \pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+$$



Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$$

$$\frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-)dt}{\int_0^\infty (pp \to n\Delta^{++})dt}$$

N(t), Z(t) : Numbers of nucleon which satisfy the conditions