Trento, May 21, 2019

Clusters and Correlations in Dense Matter

Gerd Röpke, Rostock

Nuclear Many-Body System

Nucleons (neutron, proton): intrinsic quark-gluon structure, cluster?

degrees of freedom: position – momentum, spin, isospin

single-particle states $|1\rangle = |\mathbf{p}_1, \sigma_1, \tau_1\rangle$

occupation numbers, distribution function

correlation function in a given ensemble $\langle a_1^\dagger(t) a_{1'} \rangle$

spectral function
$$\langle a_1^\dagger(t) a_{1'} \rangle = \int \frac{d\omega}{2\pi} e^{i\omega t} \frac{1}{e^{(\omega-\mu_\tau)/T}+1} A(1\ 1', \omega)$$

self energy
$$A(1,\omega) = \frac{2\mathrm{Im}\Sigma(1,\omega - i0)}{(\omega - E(1) - \mathrm{Re}\Sigma(1,\omega))^2 + (\mathrm{Im}\Sigma(1,\omega - i0))^2}$$

equation of state
$$n_\tau^{\mathrm{tot}}(T,\mu_n,\mu_p) = \frac{1}{\Omega} \sum_{p_1,\sigma_1} \int \frac{d\omega}{2\pi} \frac{1}{e^{(\omega-\mu_\tau)/T}+1} A(1,\omega)$$

Different approximations

medium effects

Ideal Fermi gas:
protons, neutrons,
(electrons, neutrinos,…)

Quasiparticle quantum liquid:
mean-field approximation
BHF, Skyrme, Gogny, RMF

bound state formation

Nuclear statistical equilibrium:
ideal mixture of all bound states
(clusters:) chemical equilibrium

Chemical equilibrium
of quasiparticle clusters:
self-energy and Pauli blocking

continuum contribution

Second virial coefficient:
account of continuum contribution,
scattering phase shifts, Beth-Uhl.Eq.

Generalized Beth-Uhlenbeck formula:
medium modified binding energies,
medium modified scattering phase shifts

chemical & physical picture

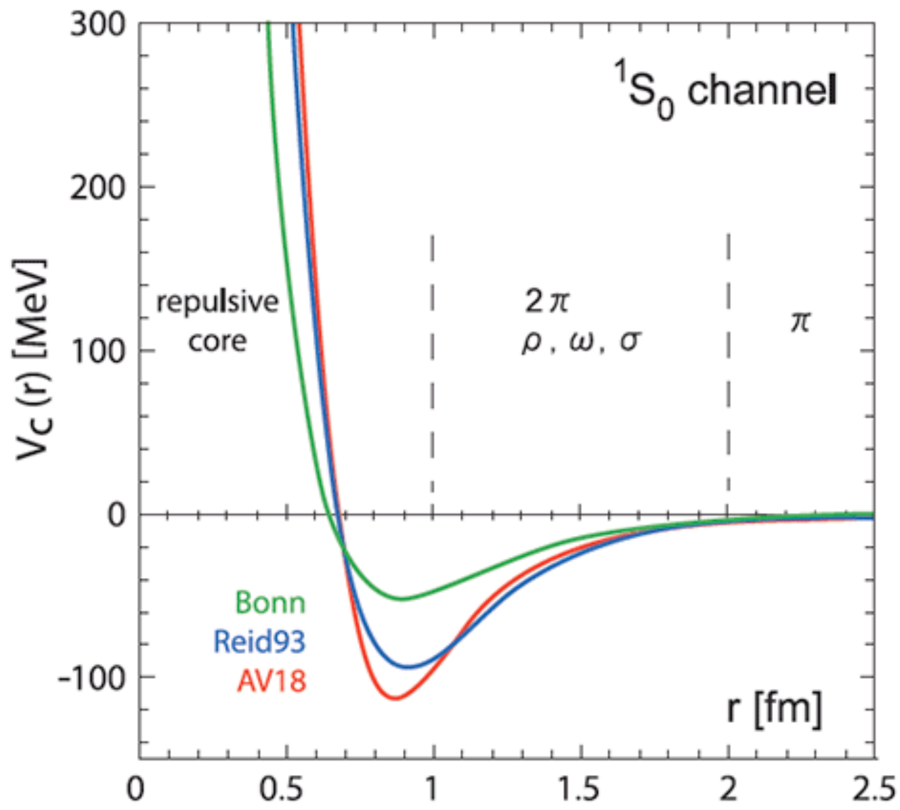
Cluster virial approach:
all bound states (clusters)
scattering phase shifts of all pairs

Correlated medium:
phase space occupation by all bound states
in-medium correlations, quantum condensates

Evaluation of correlation functions

nucleon-nucleon interaction potential ? (atom-atom potential ?)

- non-local, energy-dependent?
  QCS?
- microscopic calculations
  (AMD, FMD)
- single-particle descriptions:
  Thomas-Fermi approximation
  shell model
  density functional theory (DFT)
- correlations, clustering
  low-density nα nuclei,
  α decay: preformation



Cluster: few-nucleon system

Nucleon pair (position, spin, isospin)

$$-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_1}\Psi(\mathbf{r}_1,\mathbf{r}_2)-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_2}\Psi(\mathbf{r}_1,\mathbf{r}_2)+V(\mathbf{r}_1-\mathbf{r}_2)\Psi(\mathbf{r}_1,\mathbf{r}_2)=E\Psi(\mathbf{r}_1,\mathbf{r}_2)$$

Separation ansatz $\Psi(\mathbf{r}_1,\mathbf{r}_2)=\varphi^{\rm intr}(\mathbf{s})\Phi(\mathbf{R})$

$$-\frac{\hbar^2}{m}\nabla^2_{\mathbf{s}}\varphi^{\rm intr}(\mathbf{s})+V(\mathbf{s})\varphi^{\rm intr}(\mathbf{s})=E^{\rm intr}\varphi^{\rm intr}(\mathbf{s})$$

$$-\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}}\Phi(\mathbf{R})=E^{\rm c.m.}\Phi(\mathbf{R})$$

Nucleon quartet $\{n_\uparrow,n_\downarrow,p_\uparrow,p_\downarrow\}$

$$-\sum_i\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_i}\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)+V_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=E\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$$

Separation ansatz $\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=\varphi^{\rm intr}(\mathbf{S},\mathbf{s}_1,\mathbf{s}_2)\Phi(\mathbf{R})$

$$T[\nabla_{\mathbf{s}_j}]\varphi^{\rm intr}(\mathbf{S},\mathbf{s}_1,\mathbf{s}_2)+V(\mathbf{S},\mathbf{s}_1,\mathbf{s}_2)\varphi^{\rm intr}(\mathbf{S},\mathbf{s}_1,\mathbf{s}_2)=E^{\rm intr}\varphi^{\rm intr}(\mathbf{S},\mathbf{s}_1,\mathbf{s}_2)$$

$$-\frac{\hbar^2}{8m}\nabla^2_{\mathbf{R}}\Phi(\mathbf{R})=E^{\rm c.m.}\Phi(\mathbf{R})$$

Composition of nuclear matter

- Composition in equilibrium
  low-density limit: nuclear statistical equilibrium (NSE)

- Cluster decomposition, partial virial coefficients
  bound states
  continuum correlations, scattering states

- Medium modifications, density effects, gRMF, excluded volume, quantum statistical approach

Composition of dense nuclear matter

$$n_p(T,\mu_p,\mu_n) = \frac{1}{V}\sum_{A,\nu,K} Z_A f_A\{E_{A,\nu K} - Z_A\mu_p - (A-Z_A)\mu_n\}$$

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mass number A
charge Z_A
energy $E_{A,\nu,K}$
ν: internal quantum number

$$f_{A(z)} = \frac{1}{\exp(z/T) - (-1)^A}$$

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- Medium effects: correct behavior near saturation
  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

Beth-Uhlenbeck formula

rigorous results at low density: virial expansion

Beth-Uhlenbeck formula

$$
\begin{aligned}
n(T,\mu) &= \frac{1}{V}\sum_p e^{-(E(p)-\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{nP} e^{-(E_{nP}-2\mu)/k_BT} \\
&+ \frac{1}{V}\sum_{\alpha P}\int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_BT}\frac{d}{dE}\delta_\alpha(E) \\
&+ \ldots
\end{aligned}
$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

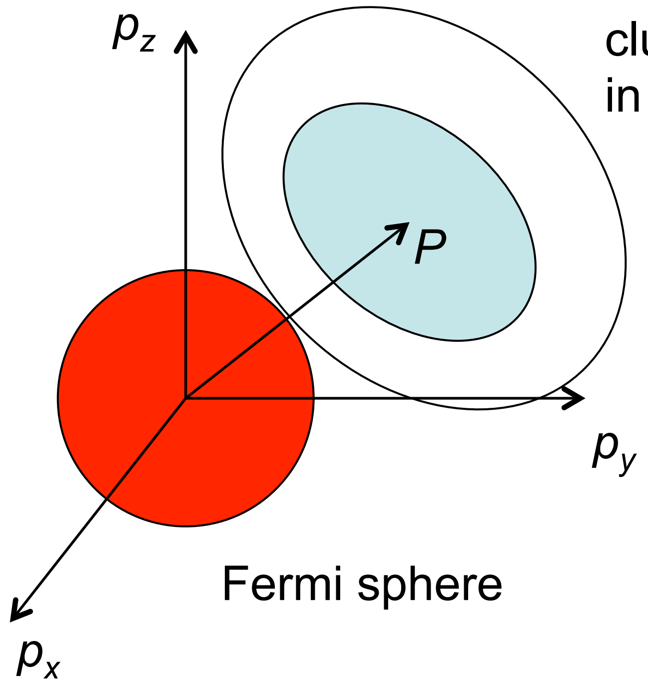
$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Pauli blocking – phase space occupation



cluster wave function (deuteron, alpha,…)
in momentum space

P - center of mass momentum

The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum *P*

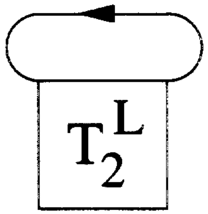
momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Generalized Beth-Uhlenbeck formula

low density limit:

$$G_2^L(12,1'2',i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi^*_{n\mathbf{P}}(12)$$

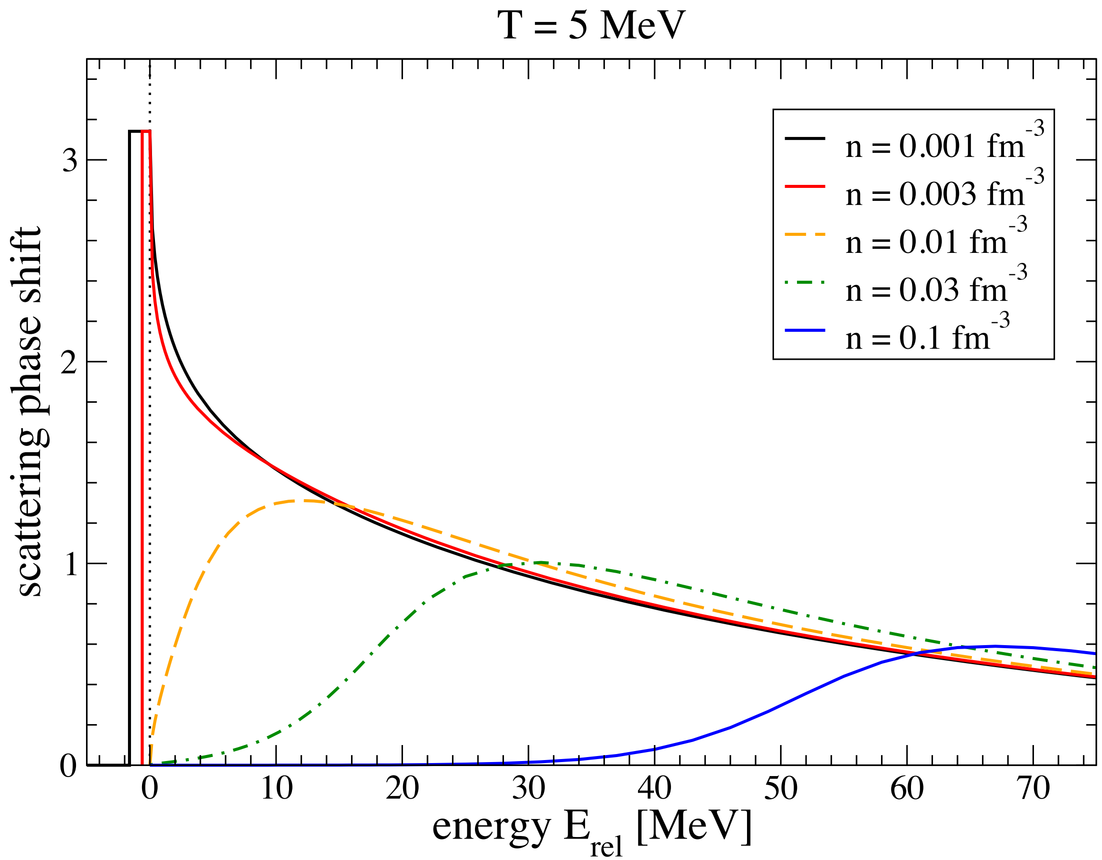
$\Sigma =$ 

$$n(\beta,\mu) = \sum_1 f_1(E^{\rm quasi}(1)) + \sum_{2,n\mathbf{P}}^{\rm bound} g_{12}(E_{n\mathbf{P}})$$
$$+ \sum_{2,n\mathbf{P}} \int_0^\infty \mathrm{d}k\; \delta_{\mathbf{k},\mathbf{p}_1-\mathbf{p}_2} g_{12}(E^{\rm quasi}(1) + E^{\rm quasi}(2)) 2\sin^2\delta_n(k) \frac{1}{\pi}\frac{\mathrm{d}}{\mathrm{d}k}\delta_n(k)$$

- generalized Beth-Uhlenbeck formula
  correct low density/low temperature limit:
  mixture of free particles and bound clusters

Deuteron-like scattering phase shifts

Virial coeff. $\propto e^{-E_d^0/T} - 1 + \frac{1}{\pi T}\int_0^\infty dE\; e^{-E/T}\left\{\delta_c(E) - \frac{1}{2}\sin[2\delta_c(E)]\right\}$



deuteron bound state -2.2 MeV

G. Roepke, J. Phys.: Conf. Series 569, 012031 (2014)
Phys. Part. Nucl. 46, 772 (2015) [arXiv:1408.2654]

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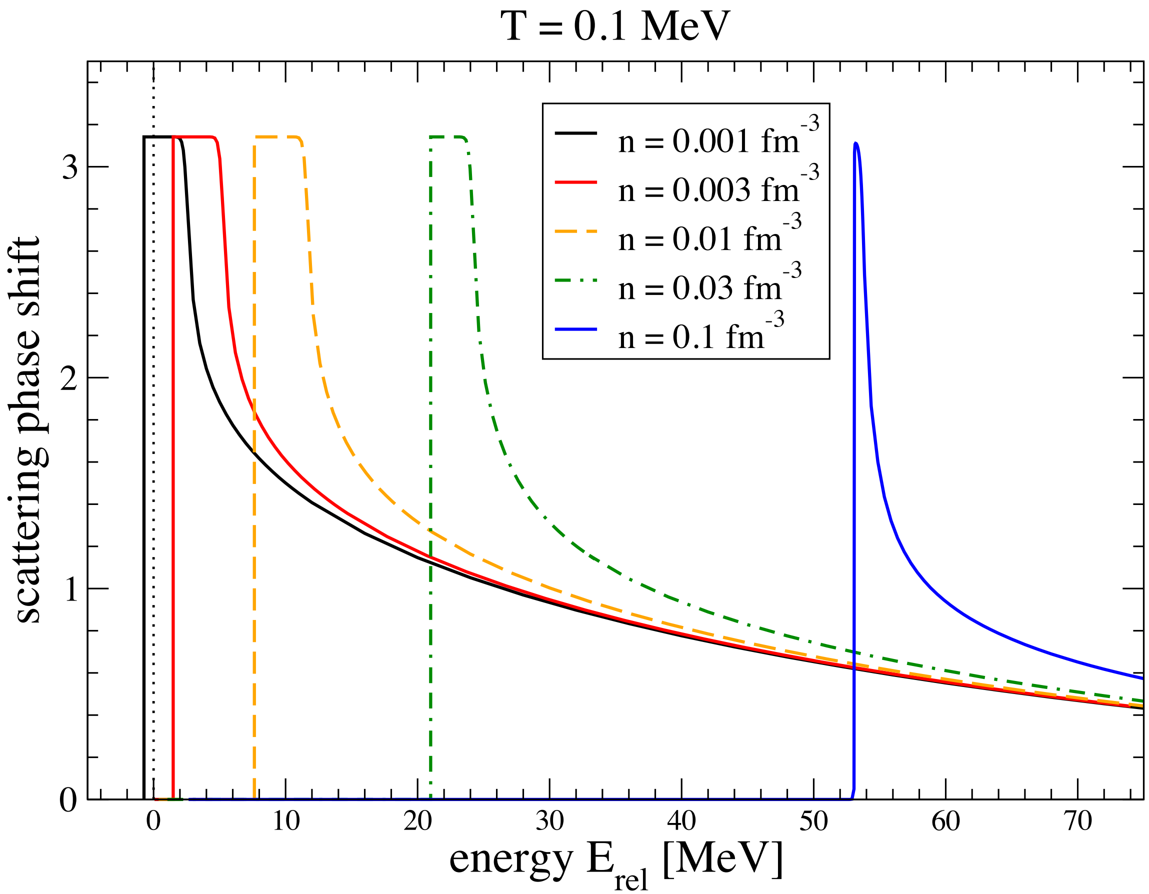

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  self-energy and Pauli blocking shifts of binding energies,
  Coulomb corrections due to screening (Wigner-Seitz, Debye)

EOS: continuum contributions

Partial density of channel A,c at P (for instance, $^3S_1$= d):

$$z_{A,c}^{\rm part}(\mathbf{P};T,\mu_n,\mu_p) = e^{(N\mu_n+Z\mu_p)/T}\left\{\sum_{\nu_c}^{\rm bound} g_{A,\nu_c}\, e^{-E_{A,\nu_c}(\mathbf{P})/T}\,\Theta\left[-E_{A,\nu_c}(\mathbf{P})+E_{A,c}^{\rm cont}(\mathbf{P})\right]+z_{A,c}^{\rm cont}(\mathbf{P})\right\}$$

separation: bound state part – continuum part ?

$$z_c^{\rm part}(\mathbf{P};T,n_B,Y_p) = e^{[N\mu_n+Z\mu_p-NE_n(\mathbf{P}/A;T,n_B,Y_p)-ZE_p(\mathbf{P}/A;T,n_B,Y_p)]/T}$$
$$\times g_c\left\{\left[e^{-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)/T}-1\right]\Theta\left[-E_c^{\rm intr}(\mathbf{P};T,n_B,Y_p)\right]+v_c(\mathbf{P};T,n_B,Y_p)\right\}$$

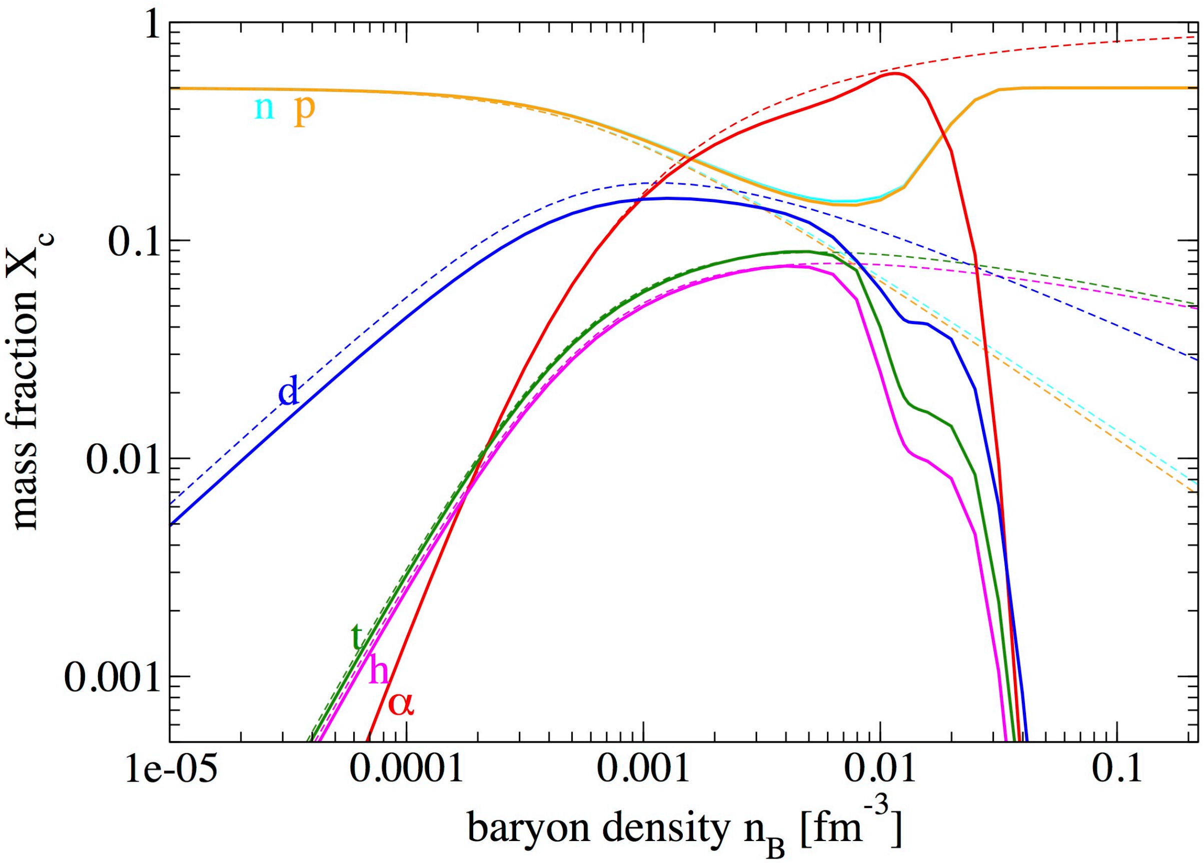
parametrization (d – like):

$$v_c(\mathbf{P}=0;T,n_B,Y_p)\approx\left[1.24+\left(\frac{1}{v_{T_I=0}(T)}-1.24\right)e^{\gamma_c n_B/T}\right]^{-1}.$$

$$v_d^0(T)=v_{T_I=0}^0(T)\approx 0.30857+0.65327\; e^{-0.102424\;T/\mathrm{MeV}}$$

G. Roepke, PRC 92,054001 (2015)

Light Cluster Abundances



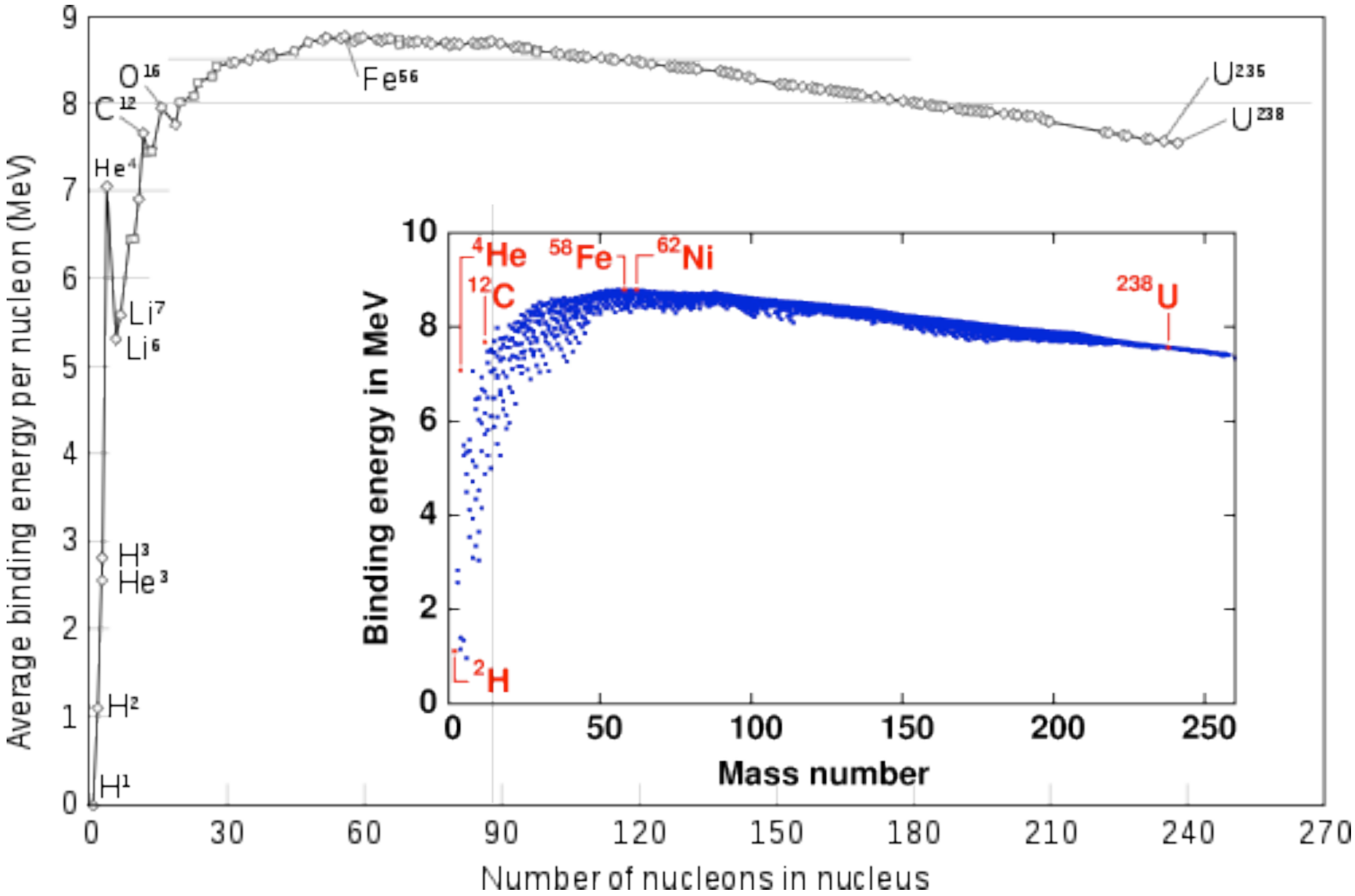
Composition of symmetric matter in dependence on the baryon density n_B, T = 5 MeV.
Quantum statistical calculation (full) compared with NSE (dotted).

G. R., PRC **92**, 054001 (2015)

Inclusion of heavier clusters

- Z > 2: “metals”
- Asymmetric matter, Stellar matter
- “Exotic light clusters” 4 < A < 12
- Light clusters, “exotic” nuclei ^{4}H
- Thermodynamic stability, pasta phases

Binding energy per nucleon



Half-lives $t_{1/2}$

Radioactive decay of instabile Isotopes

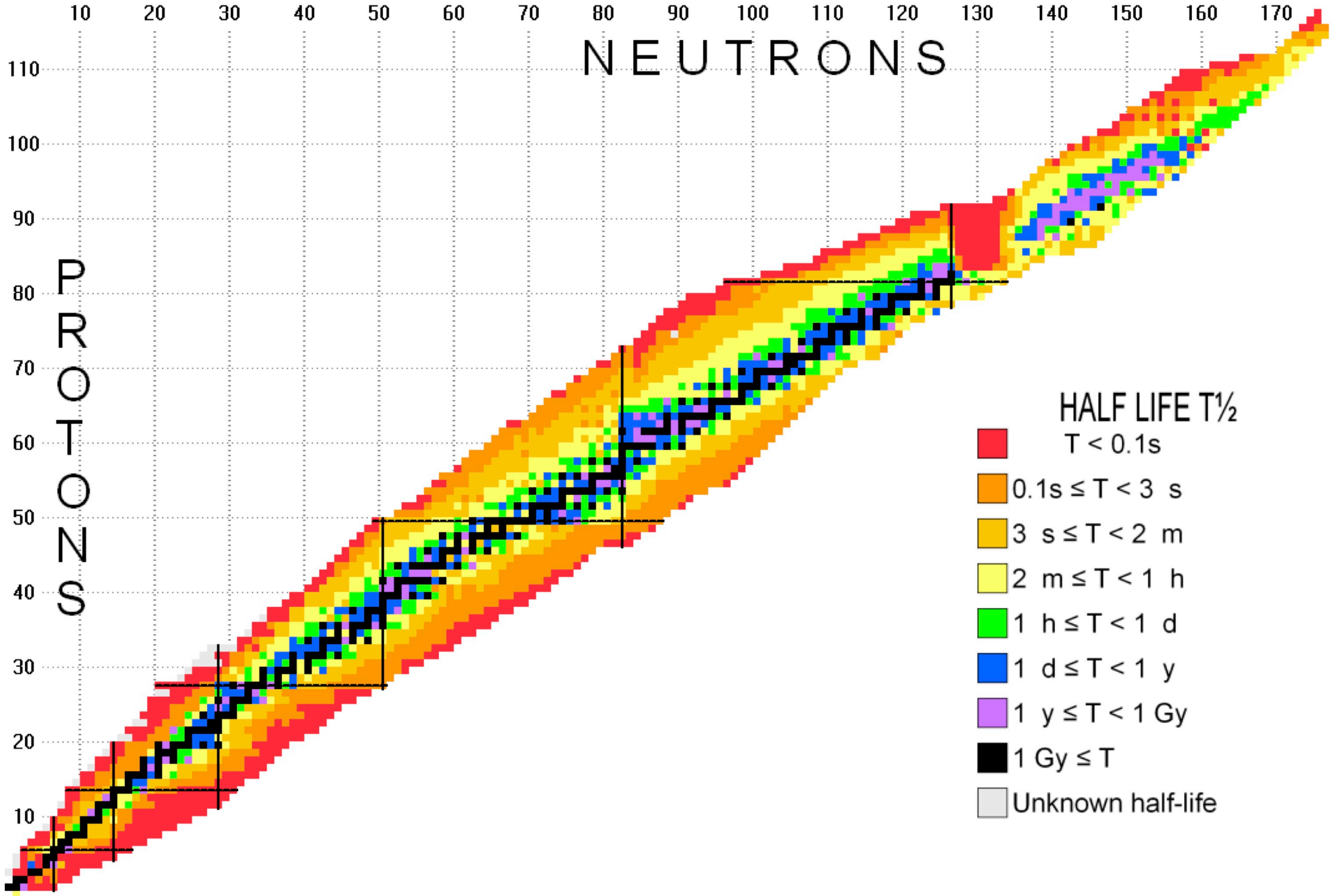
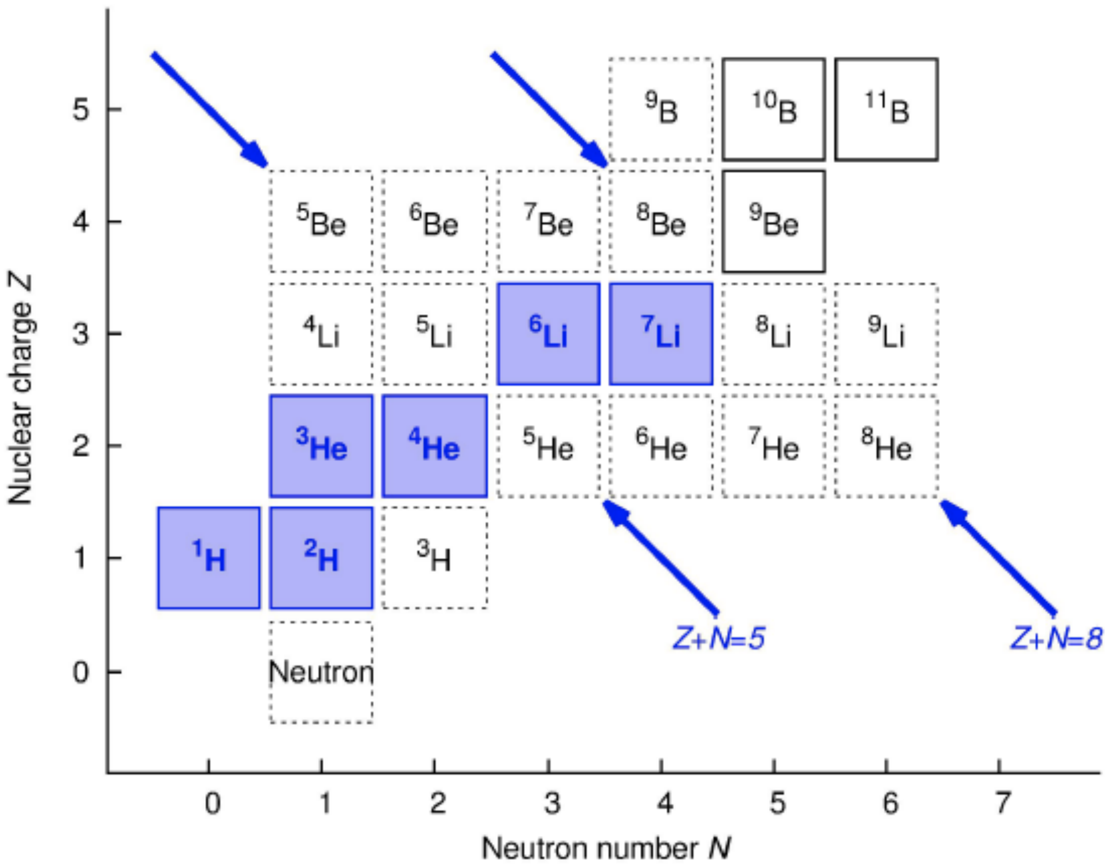


Figure 2: Chart of the nuclides for half-lives (created by NUCLEUS-AMDC).

G. Audi et al. / Nuclear Physics A 729 (2003) 3–128

Big-Bang nucleosynthesis: H, He, Li, __________



Deuterium bottleneck

- Deuterium only stable at sufficiently low temperature

Mass 5 and 8 barriers

- No stable nucleus with mass $Z+N = 5$
- No stable nucleus with mass $Z+N = 8$

Nuclear binding energy

- ^{4}He has the highest binding energy of all stable light nuclei

Electrostatic repulsion

- Probability for capture of a nuclide drops exponentially for increasing Z and sqrt($Z+N$) of the captured nuclide

Courtesy: D. Bemmerer, lecture at Karpacz-2019

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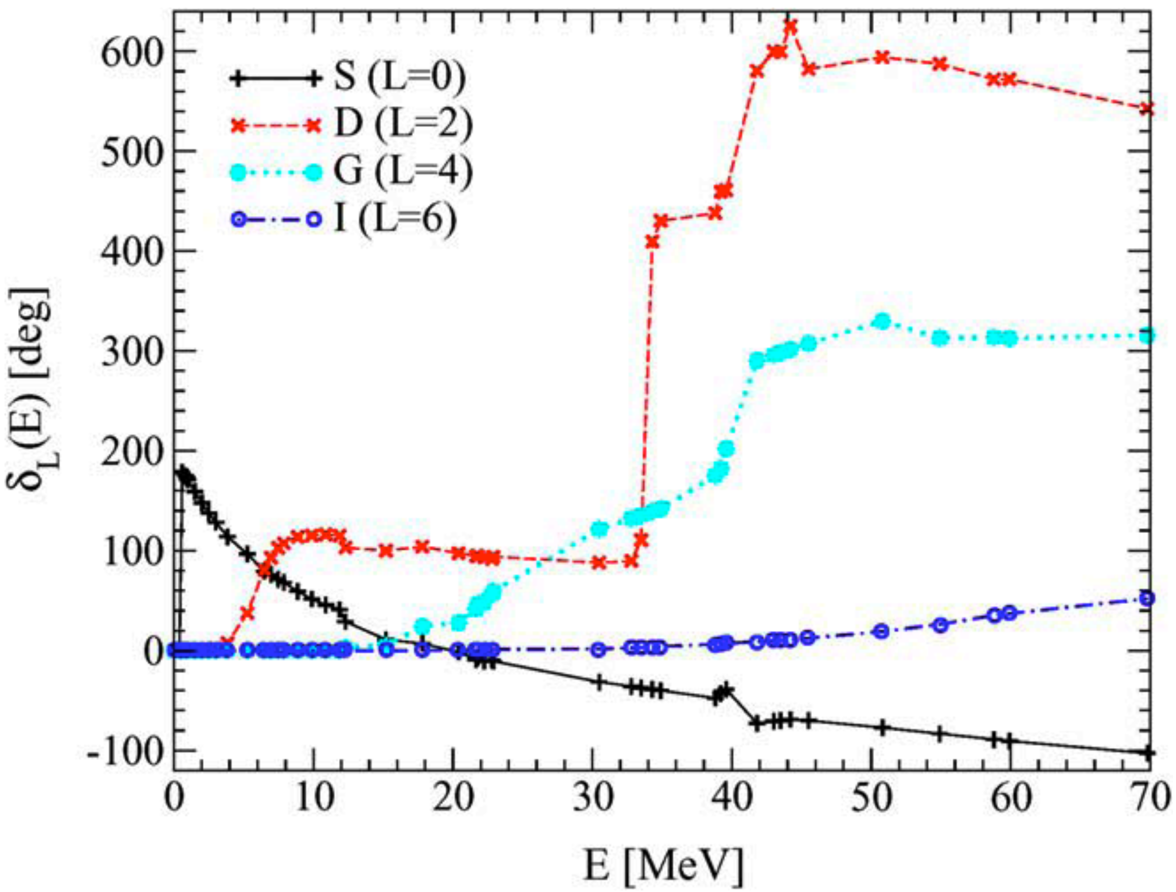
G. Roepke, PRC 92,054001 (2015)

α-α scattering phase shifts



C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

α-n scattering phase shifts

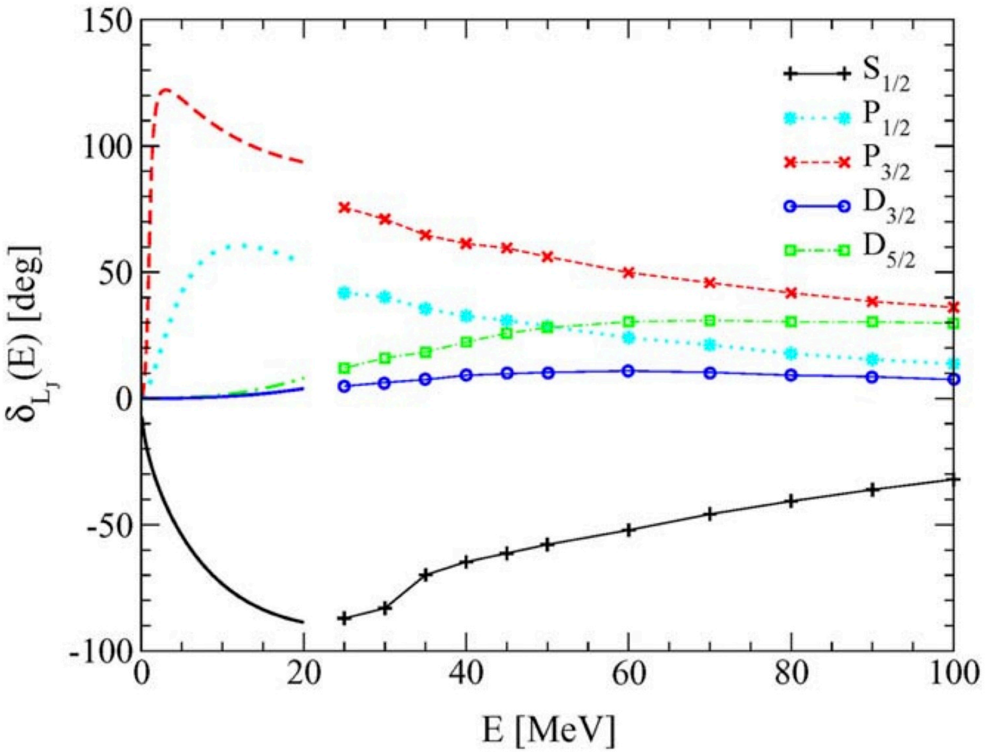
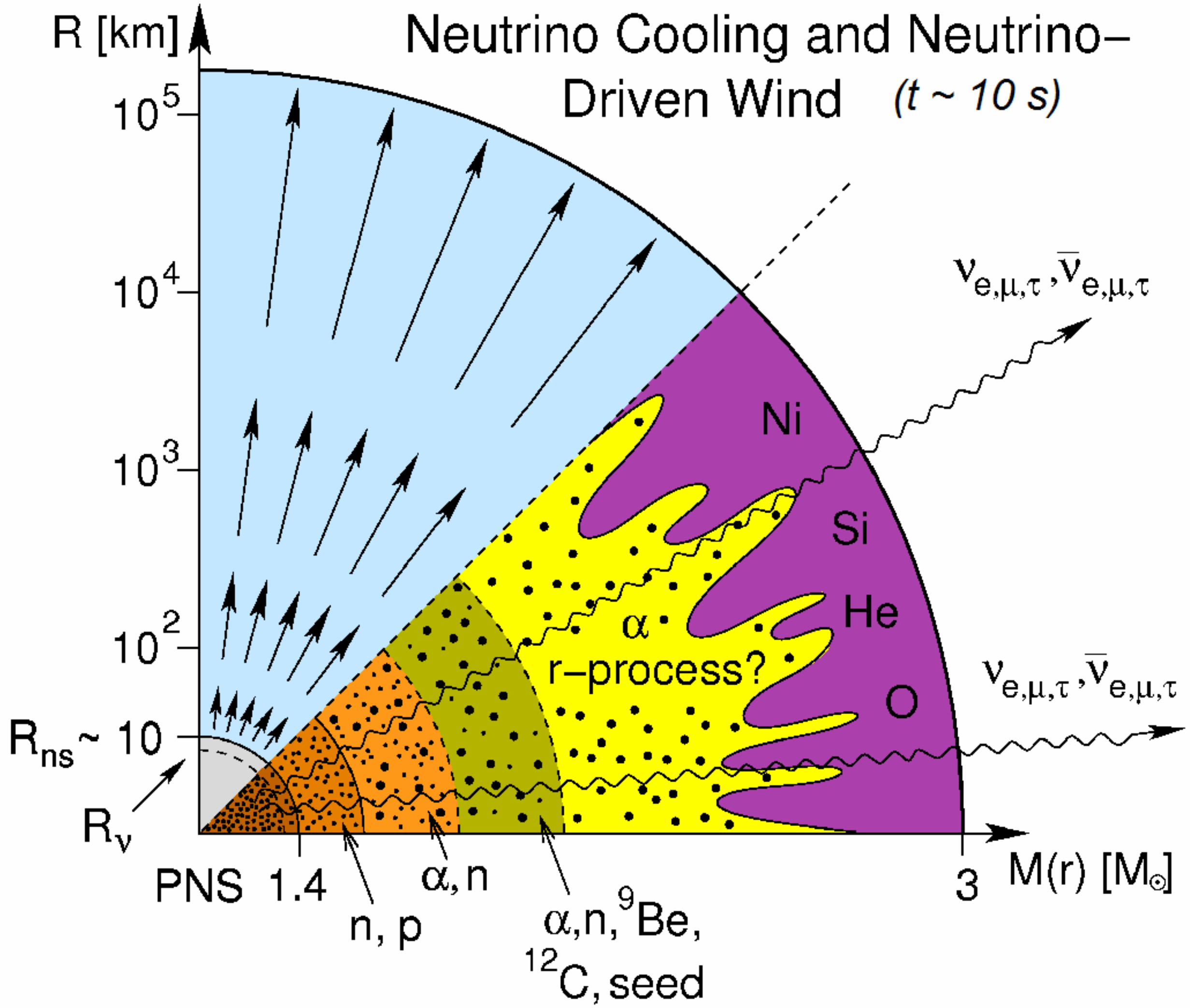


Fig. 2. (Color online.) The phase shifts for elastic neutron-alpha scattering $\delta_{L_J}(E)$ versus laboratory energy E. As discussed in the text, the solid lines are from Arndt and Roper [37] and the symbols are from Amos and Karataglidis [38]. For clarity, we do not show the F-waves included in our results for $b_{\alpha n}$.

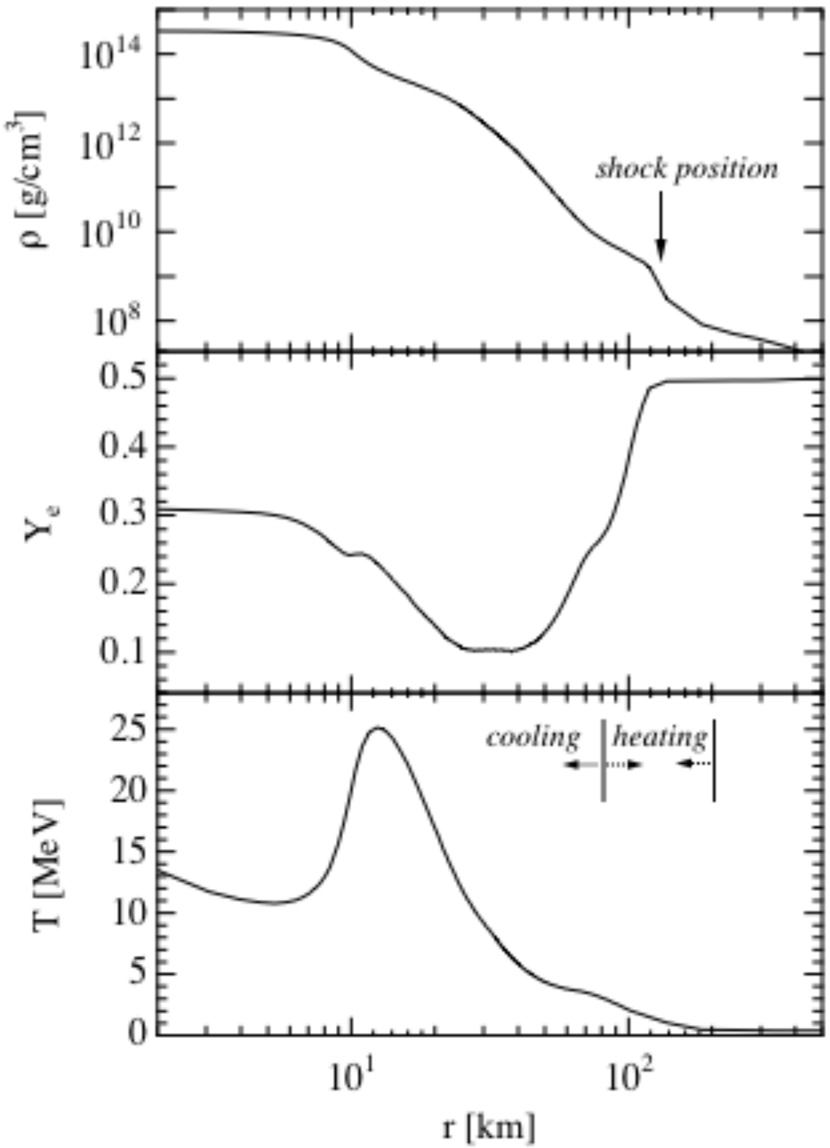
C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Supernova explosion



T.Janka

Core-collapse supernovae



Density.

electron fraction, and

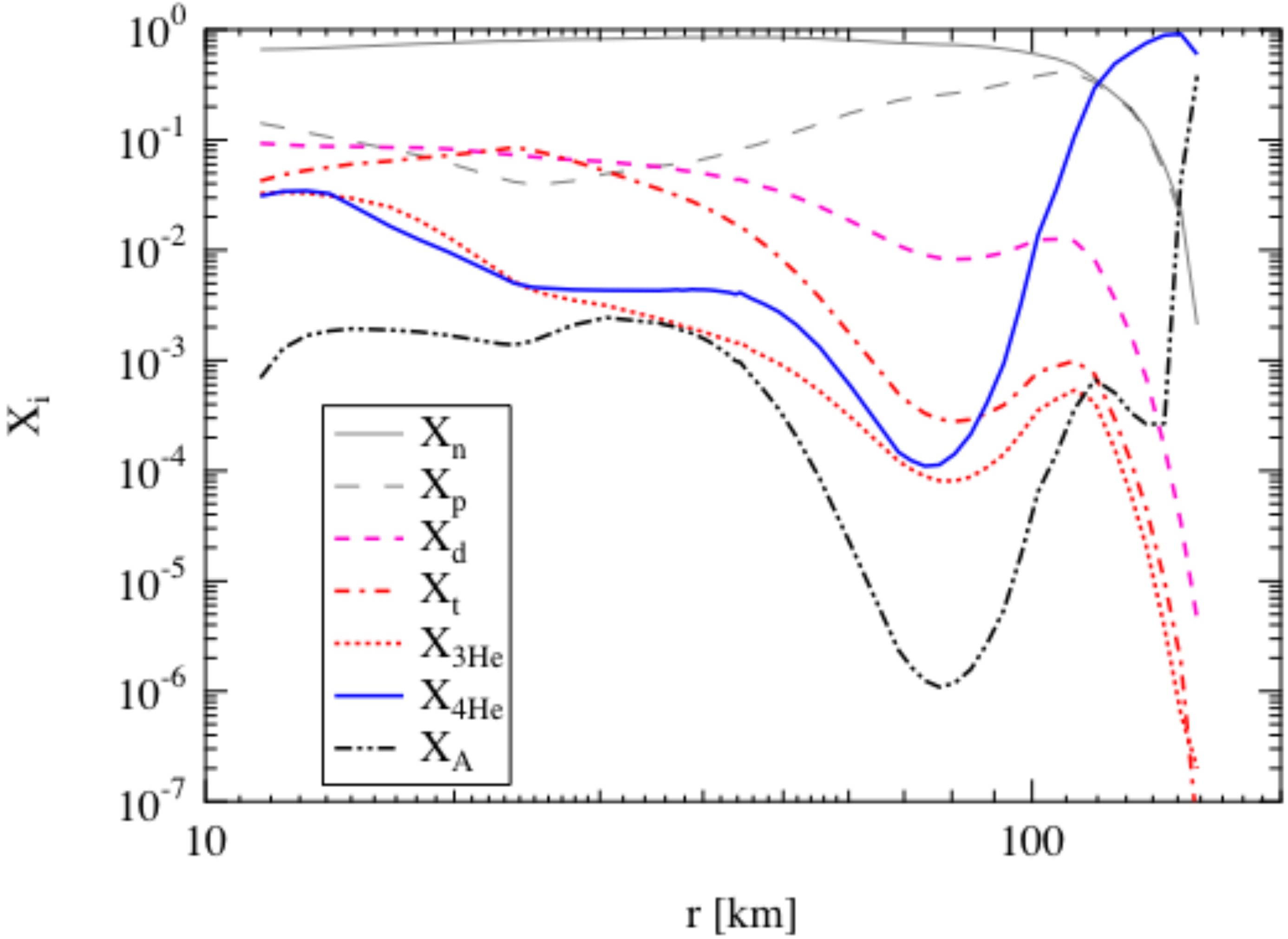
temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core



Mass fraction X of light clusters for a post-bounce supernova core

K.Sumiyoshi, G. R., PRC **77**, 055804 (2008)

Asymmetric nuclear light clusters in supernova matter

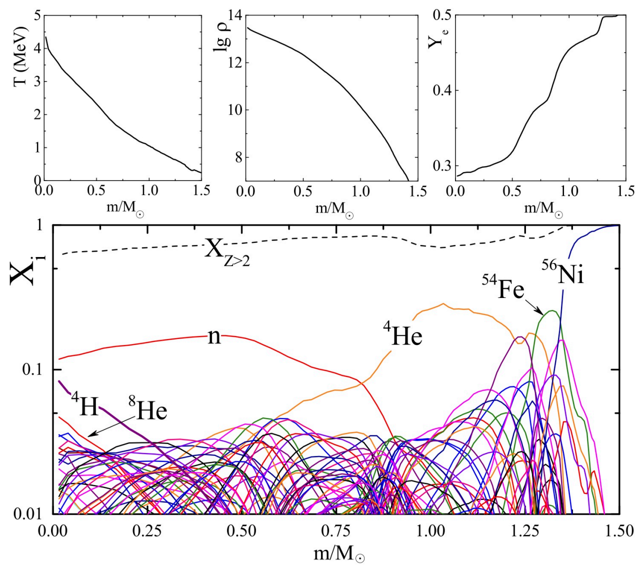


Figure 1. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $g \cdot cm^{-3}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions of of nuclei X_i as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of elements with $Z > 2$. EoS is pure NSE.

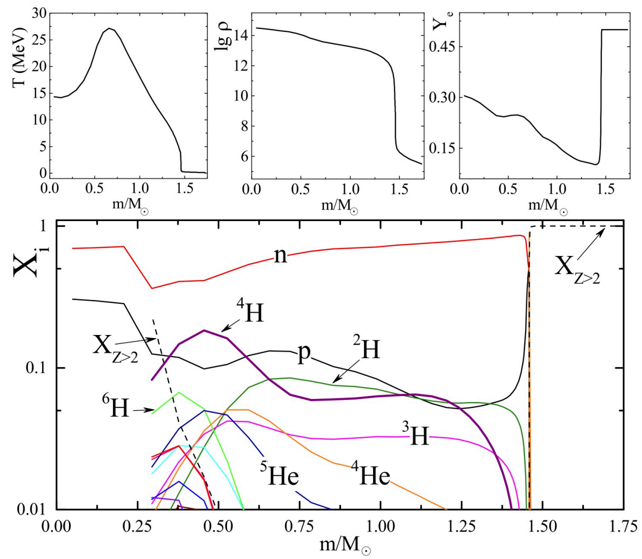


Figure 7. Upper three panels, from left ro right: temperature T (in MeV), log of density ρ (in $g \cdot cm^{-3}$) and electron fraction Y_e as a functions of mass coordinate m. Lower panel: mass fractions X_i of of hydrogen and helium isotopes as a function of m. The black dashed line marked $X_{Z>2}$ shows the total mass fraction of all rest nuclei. Stellar profile corresponds to 200 ms after bounce approximately, calculations according to modified HS EoS.

A. V. Yudin, M. Hempel, S. I. Blinnikov, D. K. Nadyozhin, I. V. Panov,
Monthly Notices of the Royal Astronomical Society **483**, 5426 (2019)

In-medium isospin-triplet phase shift

$$^4\mathrm{H} \rightleftharpoons {}^3\mathrm{H} + n$$

$$\Delta E = +1.601 \text{ MeV}$$

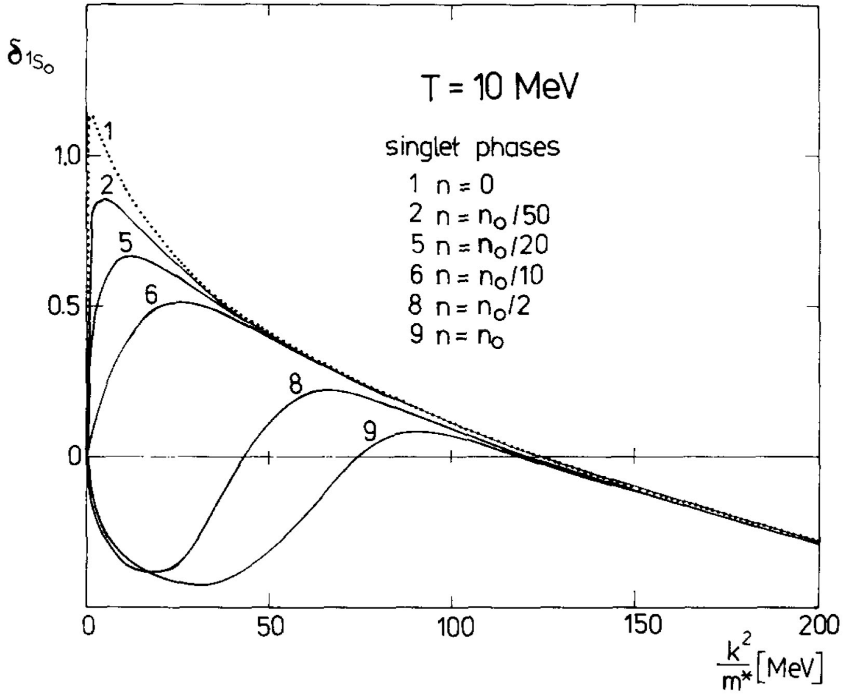


FIG. 4. The singlet scattering phase shifts as a function of the relative energy at given temperature $T = 10$ MeV and several density values of the medium for total momentum $K = 0$. The dotted line gives the phase shift for the free scattering.

Light cluster production at NICA★

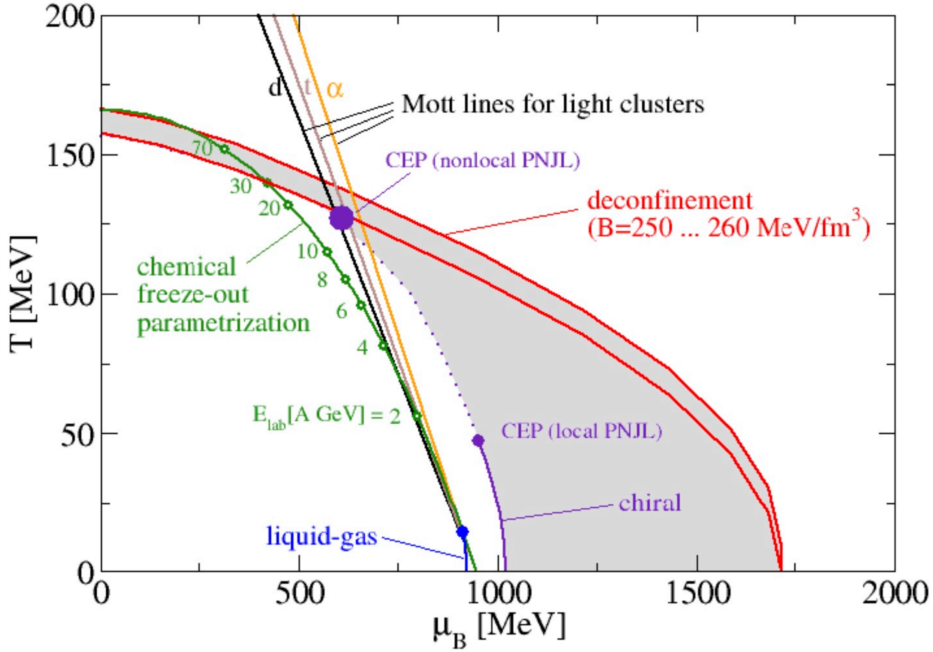
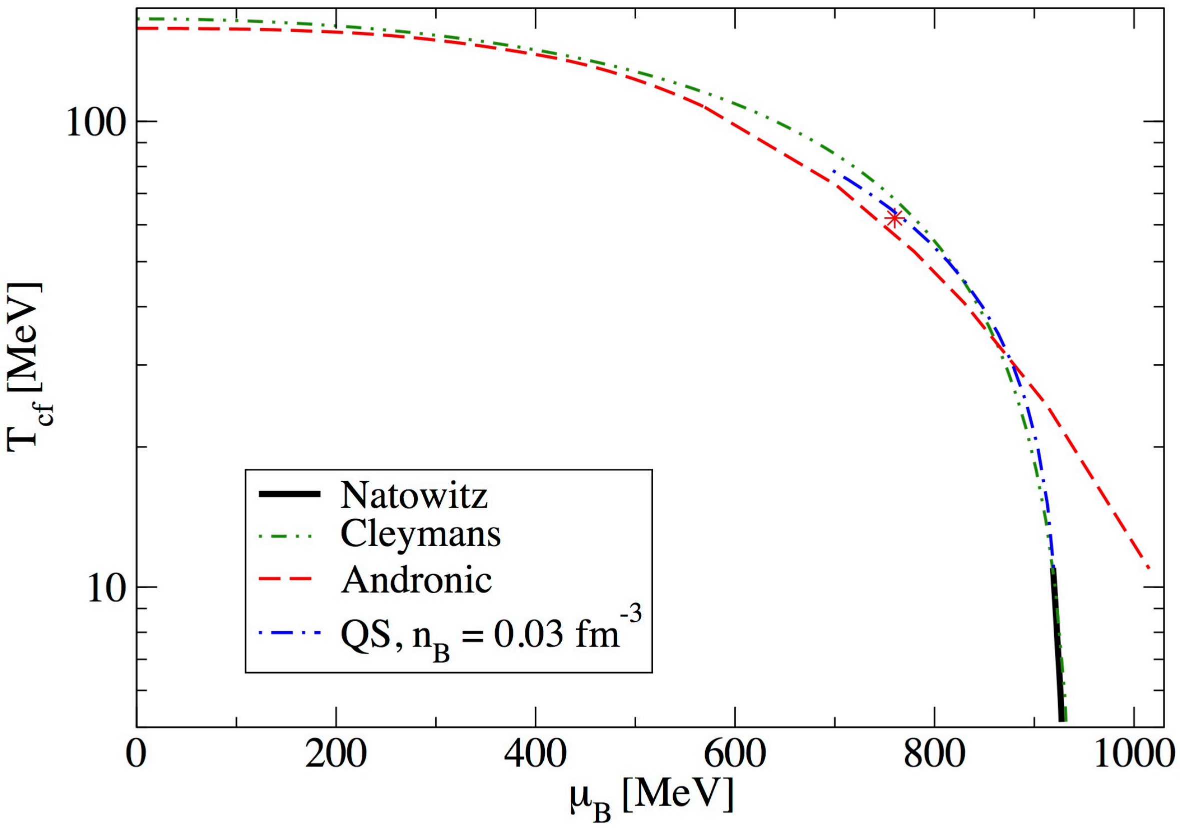


Fig. 1. Phase diagram of dense nuclear matter in the plane of temperature T and baryochemical potential μ_B. The diagram includes Mott lines for the dissociation of light nuclear clusters, extrapolated also to the deconfinement region. For details, see text.

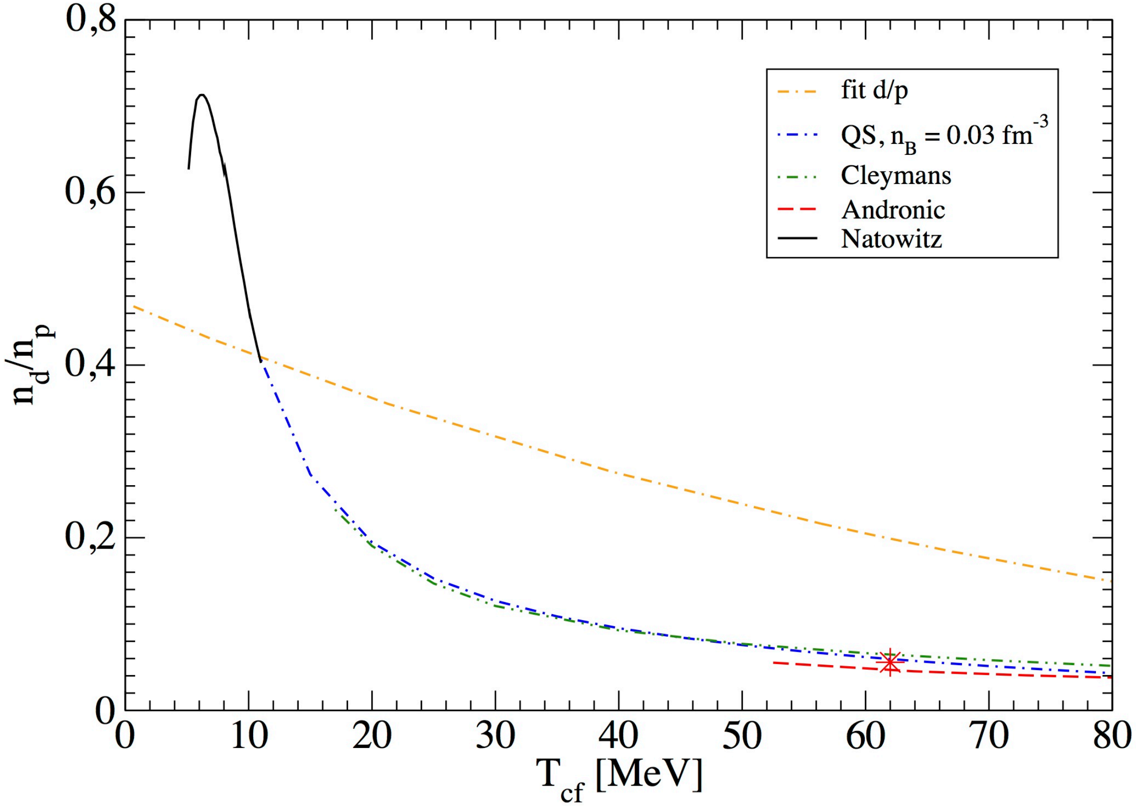
N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A **52**, 244 (2016)

Freeze-out in HIC



$$\frac{T_{cf}^{\text{Cleymans}}}{\text{GeV}} = 0.166 - 0.139\left(\frac{\mu_B}{\text{GeV}}\right)^2 - 0.053\left(\frac{\mu_B}{\text{GeV}}\right)^4$$

Deuteron to proton fraction at freeze-out temperature



Fit d/p $$d/p = 0.8[\sqrt{s_{NN}}/\mathrm{GeV}]^{-1.55} + 0.0036$$

Feckova Z., Steinheimer J., Tomasik B., Bleicher M.
Phys. Rev. C . 92, 064908 (2015)

Light cluster production at NICA★

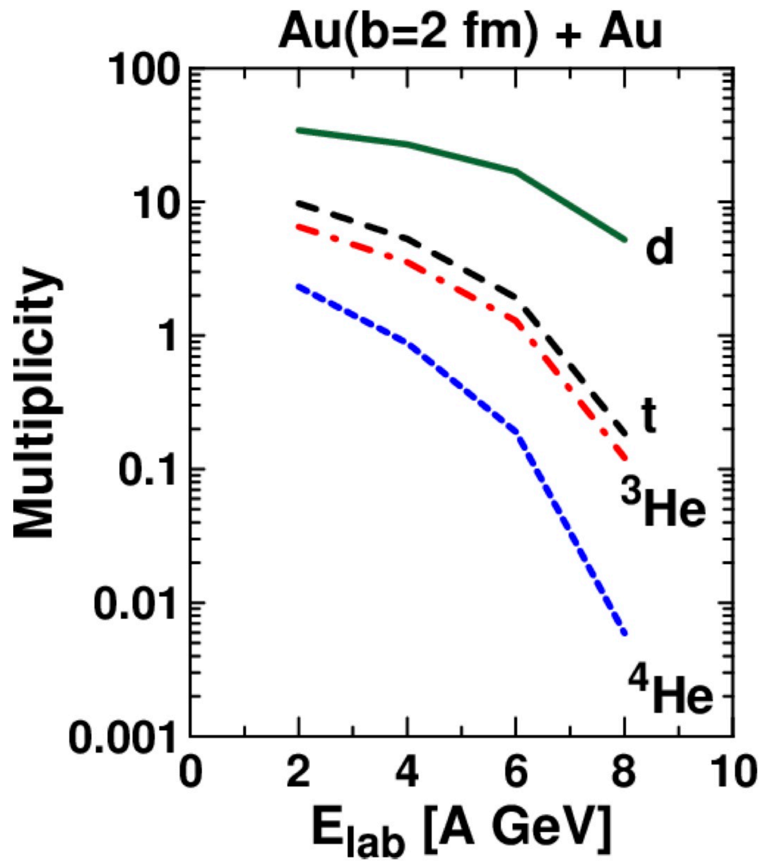


Fig. 5. Multiplicities of light clusters in central Au + Au collisions in the NICA energy range (calculated for an energy scan with $E_{\rm lab} = 2, 4, 6, 8\,A\,\mathrm{GeV}$). Results from a 3-fluid hydrodynamics description with cluster coalescence [22].

N.-U. Bastian, P. Batyuk, D. Blaschke, P. Danielewicz, Yu.B. Ivanov, Iu. Karpenko, G. Ropke, O. Rogachevsky, and H.H. Wolter, Eur. Phys. J. A (2016) 52: 244

Formation of light clusters in heavy ion reactions, transport codes

PHYSICAL REVIEW C, VOLUME 63, 034605

Medium corrections in the formation of light charged particles in heavy ion reactions

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Wigner distribution

cluster mean-field potential

$$\partial_t f_X + \{\mathcal{U}_X, f_X\} = \mathcal{K}_X^{\text{gain}}\{f_N, f_d, f_t, \ldots\}(1 \pm f_X) - \mathcal{K}_X^{\text{loss}}\{f_N, f_d, f_t, \ldots\} f_X,$$

$$X = N, d, t, \ldots .$$

loss rate

in-medium
breakup transition operator

$$\mathcal{K}_d^{\text{loss}}(P,t) = \int d^3k \int d^3k_1\, d^3k_2\, d^3k_3 |\langle k_1 k_2 k_3 | U_0 | kP\rangle|^2_{dN\to pnN} \times f_N(k_1,t) f_N(k_2,t) f_N(k_3,t) f_N(k,t) + \cdots . \qquad (3)$$

breakup cross section

$$\sigma^0_{\text{bu}}(E) = \frac{1}{|v_d - v_N|}\frac{1}{3!}\int d^3k_1\, d^3k_2\, d^3k_3 |\langle kP | U_0 | k_1 k_2 k_3\rangle|^2 \times 2\pi\delta(E' - E)(2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3), \qquad (4)$$

Mott effect, in-medium cross section

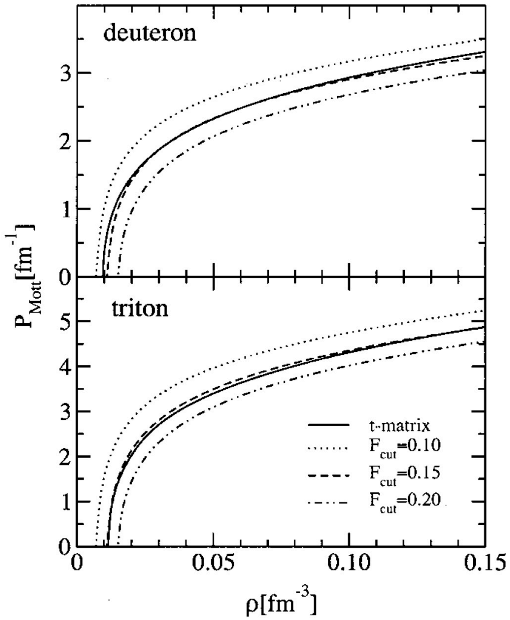


FIG. 1. Deuteron and triton Mott momenta P_{Mott} shown as a function of density ρ at fixed temperature of $T=10$ MeV. The solid line represents results of the t matrix approach. The dashed, dotted, and dashed-dotted lines represent the deuteron Mott momenta from the parametrization given in Eq. (24) for three different cutoff values F_{cut}.

$$\int d^3 q \; f\left(\mathbf{q}+\frac{\mathbf{P}_{\mathrm{c.m.}}}{2}\right) \; |\phi(\mathbf{q})|^2 \leq F_{\mathrm{cut}}$$

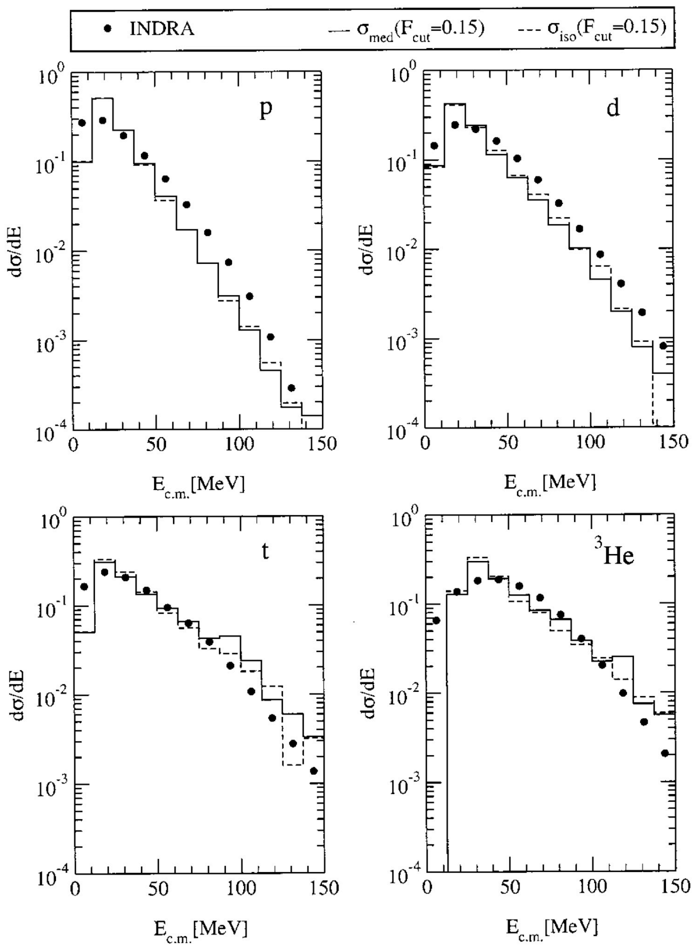
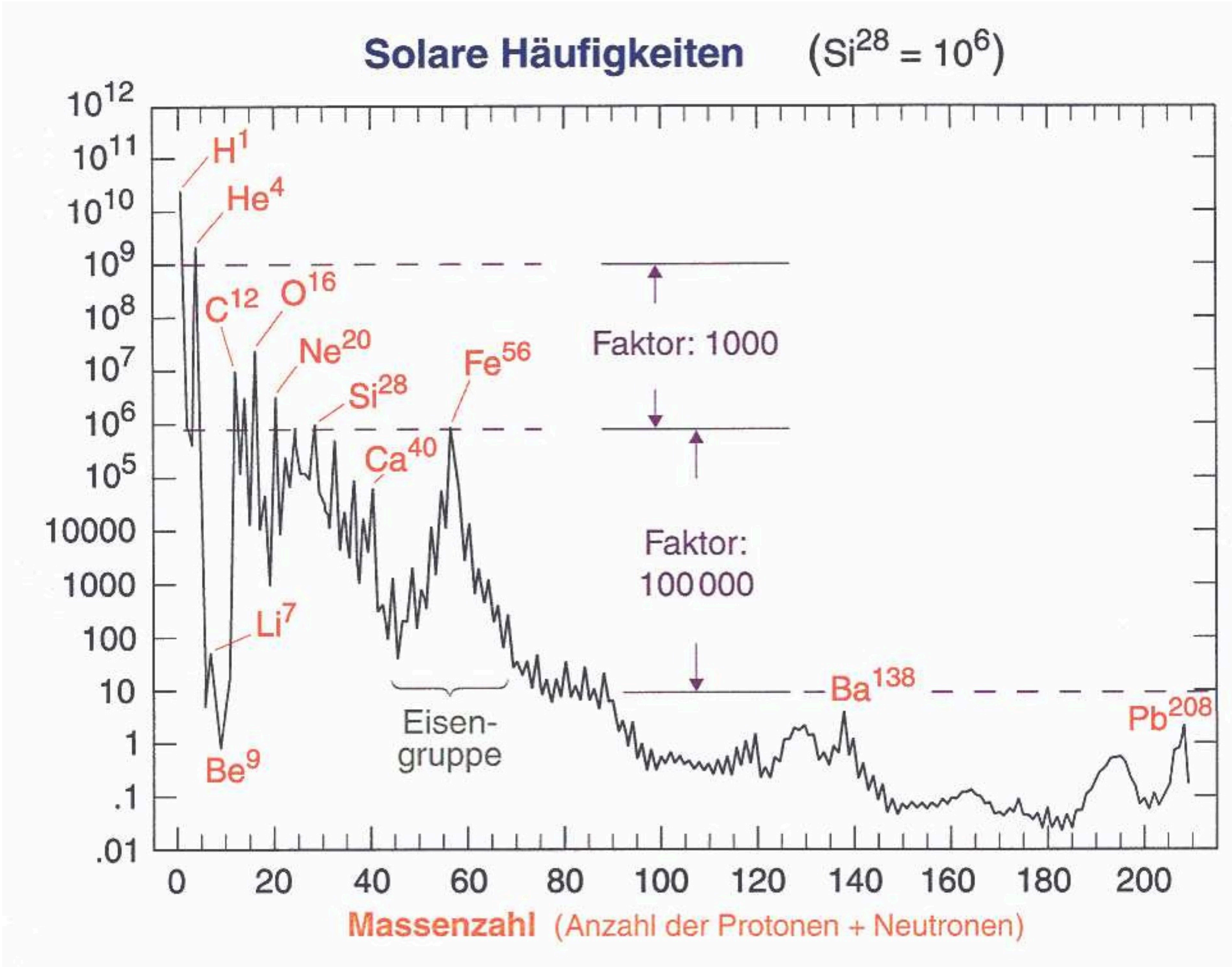


FIG. 5. Renormalized light charged light particle spectra in the center of mass system for the reaction ^{129}Xe+^{119}Sn at 50 MeV/nucleon. The filled circles represent the data of the INDRA Collaboration [21]. The solid line shows the calculations with the in-medium Nd reaction rates, while the dashed line shows a calculation using the isolated Nd breakup cross section; both with $F_{\mathrm{cut}}=0.15$.

C. Kuhrts et al., PRC **63**,034605 (2001)

Element abundances in the Sun



Mass number A

Summary

• Quantum statistical approach: light clusters with in-medium quasiparticle energies. The Pauli blockiing is strongly depending on temperature T. Mott effect: bound states merge with the continuum

• The influence of continuum correlations (clusters) at increasing densities requires detailed investigations.

- Continuum correlations contribute to the symmetry energy (density dependent virial coefficients).
- The blocking of bound states is modified because of correlations in the medium (α matter).

• The „exotic“ light clusters can be taken into acount, but double accounting must be avoided. Medium effects are important.

• Relevant for HIC (freeze-out, transport theory) and astrophysics (supernova explosions: larger clusters (A>4), pasta structures)

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H. Wolter
for collaboration

to you
for attention

D.G.

Equilibrium correlations and transport codes

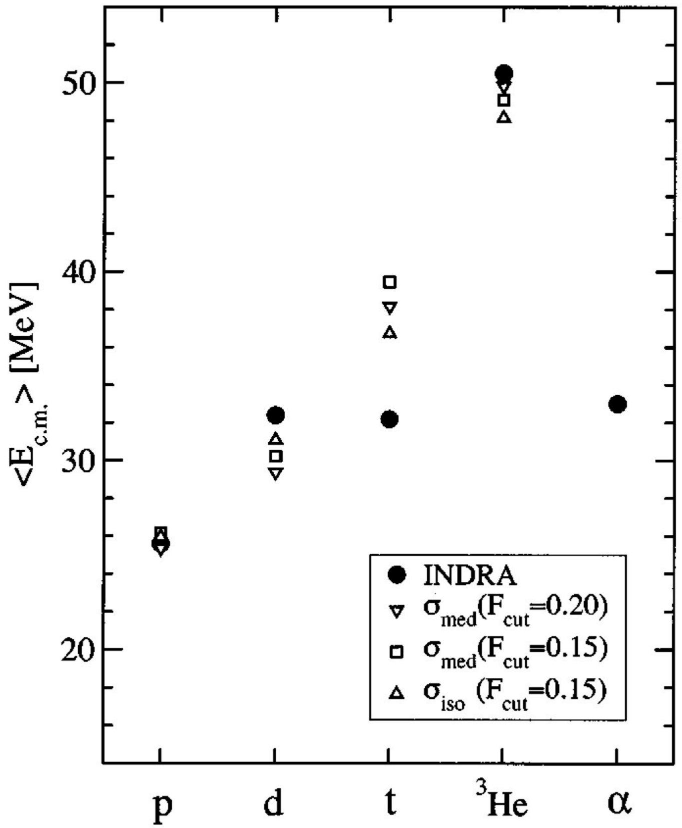


FIG. 6. Mean transverse energy of light charged fragments in the angular range of $-0.5 \leqslant \cos\theta_{c.m.} \leqslant 0.5$.

C. Kuhrts, PRC **63**,034605 (2001)

Important: Mott effect

Minor effects:
in medium cross sections

Missing: inclusion of alphas

Correlated continuum,
correlated medium

Freeze-out and local
thermodynamic equilibrium

single-particle quantum kinetic
equations and correlations

Equilibrium solution?

Clusters in an external potential

c. o. m. coordinate R, relative coordinates s_j $\quad \Psi(\mathbf{R},\mathbf{s}_j)=\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})\,\Phi(\mathbf{R})$

normalization $\quad \int dR\,|\Phi(\mathbf{R})|^2=1 \qquad \int ds_j|\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})|^2=1$

Wave equation for the c.o.m. motion

$$-\frac{\hbar^2}{2Am}\nabla_R^2\Phi(\mathbf{R})-\frac{\hbar^2}{Am}\int ds_j\varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})[\nabla_R\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})][\nabla_R\Phi(\mathbf{R})]$$
$$-\frac{\hbar^2}{2Am}\int ds_j\varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})[\nabla_R^2\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})]\Phi(\mathbf{R})+\int dR'\,W(\mathbf{R},\mathbf{R}')\Phi(\mathbf{R}')=E\,\Phi(\mathbf{R})$$

c.o.m. effective potential

$$W(\mathbf{R},\mathbf{R}')=\int ds_j\,ds'_j\,\varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})\left[T[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}'_j)+V(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)\right]\varphi^{\rm intr}(\mathbf{s}'_j,\mathbf{R}')$$

Wave equation for the intrinsic motion

$$-\frac{\hbar^2}{Am}\Phi^*(\mathbf{R})[\nabla_R\Phi(\mathbf{R})][\nabla_R\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})]-\frac{\hbar^2}{2Am}|\Phi(\mathbf{R})|^2\nabla_R^2\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$
$$+\int dR'\,ds'_j\,\Phi^*(\mathbf{R})\left[T[\nabla_{s_j}]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}'_j)+V(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}'_j)\right]\Phi(\mathbf{R}')\varphi^{\rm intr}(\mathbf{s}'_j,\mathbf{R}')=F(\mathbf{R})\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$

G. Roepke et al., PRC **90**, 034304 (2014)

Effective wave equation for the deuteron in matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$=E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Thouless criterion

$$E_d(T,\mu)=2\mu$$

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

BEC-BCS crossover:
Alm et al.,1993

Few-particle Schrödinger equation in a dense medium

4-particle Schrödinger equation with medium effects

$$\left(\left[E^{HF}(p_1)+E^{HF}(p_2)+E^{HF}(p_3)+E^{HF}(p_4)\right]\right)\Psi_{n,P}(p_1,p_2,p_3,p_4)$$
$$+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{n,P}(p_1',p_2',p_3,p_4)$$
$$+\{permutations\}$$
$$=E_{n,P}\Psi_{n,P}(p_1,p_2,p_3,p_4)$$

Thouless criterion
for quantum condensate:

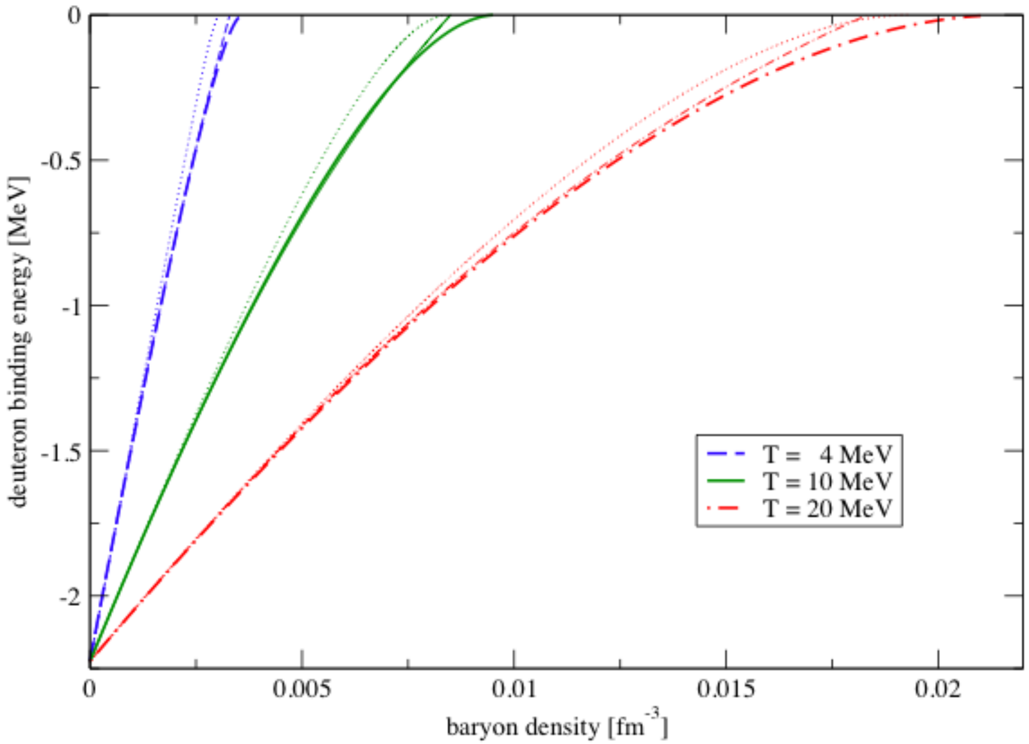
$E_{n,P=0}(T,\mu) = 4\mu$

Shift of the deuteron bound state energy

Dependence on nucleon density, various temperatures,
zero center of mass momentum

thin lines:

fit formula



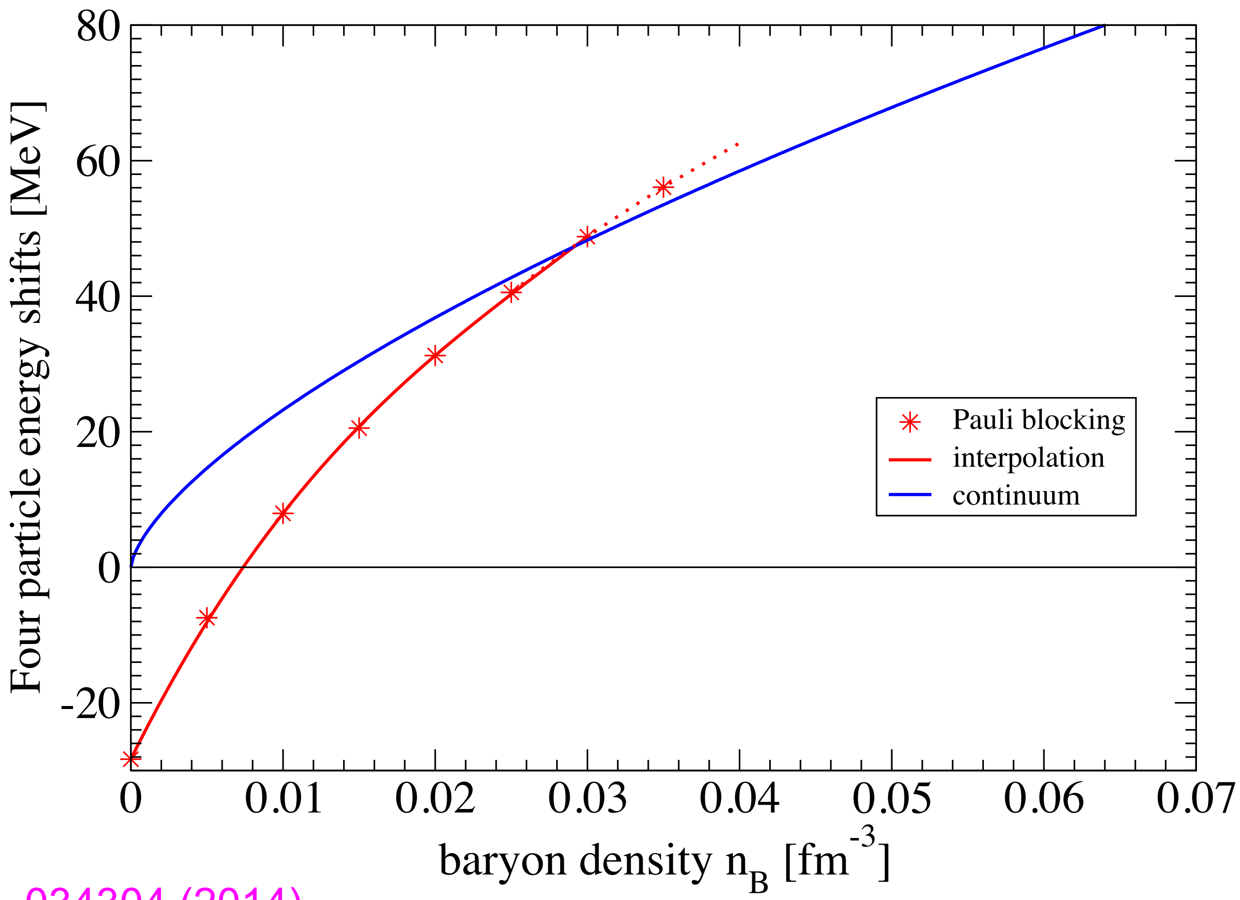
G.R., Nucl. Phys. A **867**, 66 (2011)

Four-nucleon energies at finite density

Solution of the in-medium wave equation, T = 0

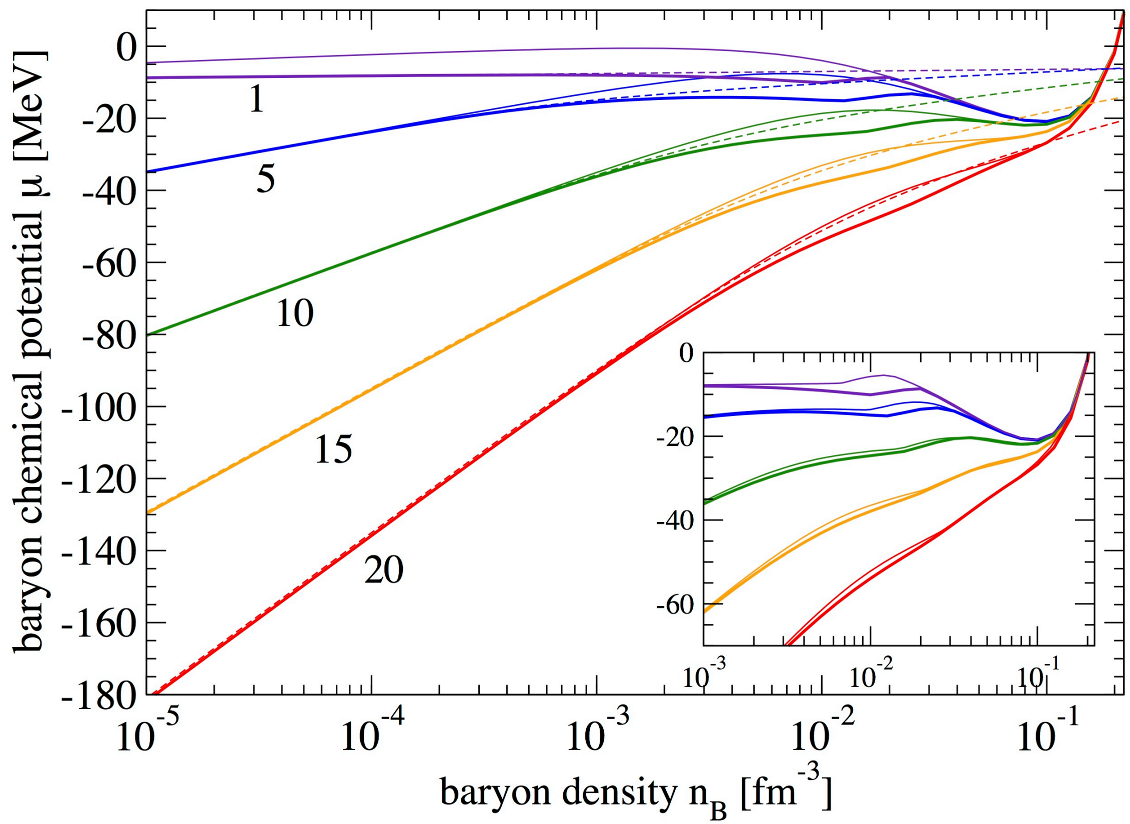
4 free nucleons
at the Fermi energy
(continuum)

bound state
(α particle)
with Pauli blocking



G. Roepke et al., PRC **90**, 034304 (2014)

Equation of state: chemical potential



Chemical potential for symmetric matter. T=1, 5, 10, 15, 20 MeV.
QS calculation compared with RMF (thin) and NSE (dashed).
Insert: QS calculation without continuum correlations (thin lines).

3. Inhomogeneous matter: nuclei

Correlations are important in the low density region ($n_B < n_{sat}/5 = 0.03$ fm^{-3}): excited nuclei (Hoyle – like), surface of heavy nuclei, neck emission, etc.

Quartetting wave-function approach:
mixed gradient terms $[\nabla_R \varphi^{\rm intr}(\mathbf{s}_j, \mathbf{R})]$ are neglected,
Pauli blocking must be taken into account.

$$[E_4 - \hat{h}_1 - \hat{h}_2 - \hat{h}_3 - \hat{h}_4]\Psi(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4) = \int d^3\mathbf{r}'_1\, d^3\mathbf{r}'_2 \langle \mathbf{r}_1\mathbf{r}_2 | B\, V_{N-N} | \mathbf{r}'_1\mathbf{r}'_2\rangle \Psi(\mathbf{r}'_1\mathbf{r}'_2\mathbf{r}_3\mathbf{r}_4)$$

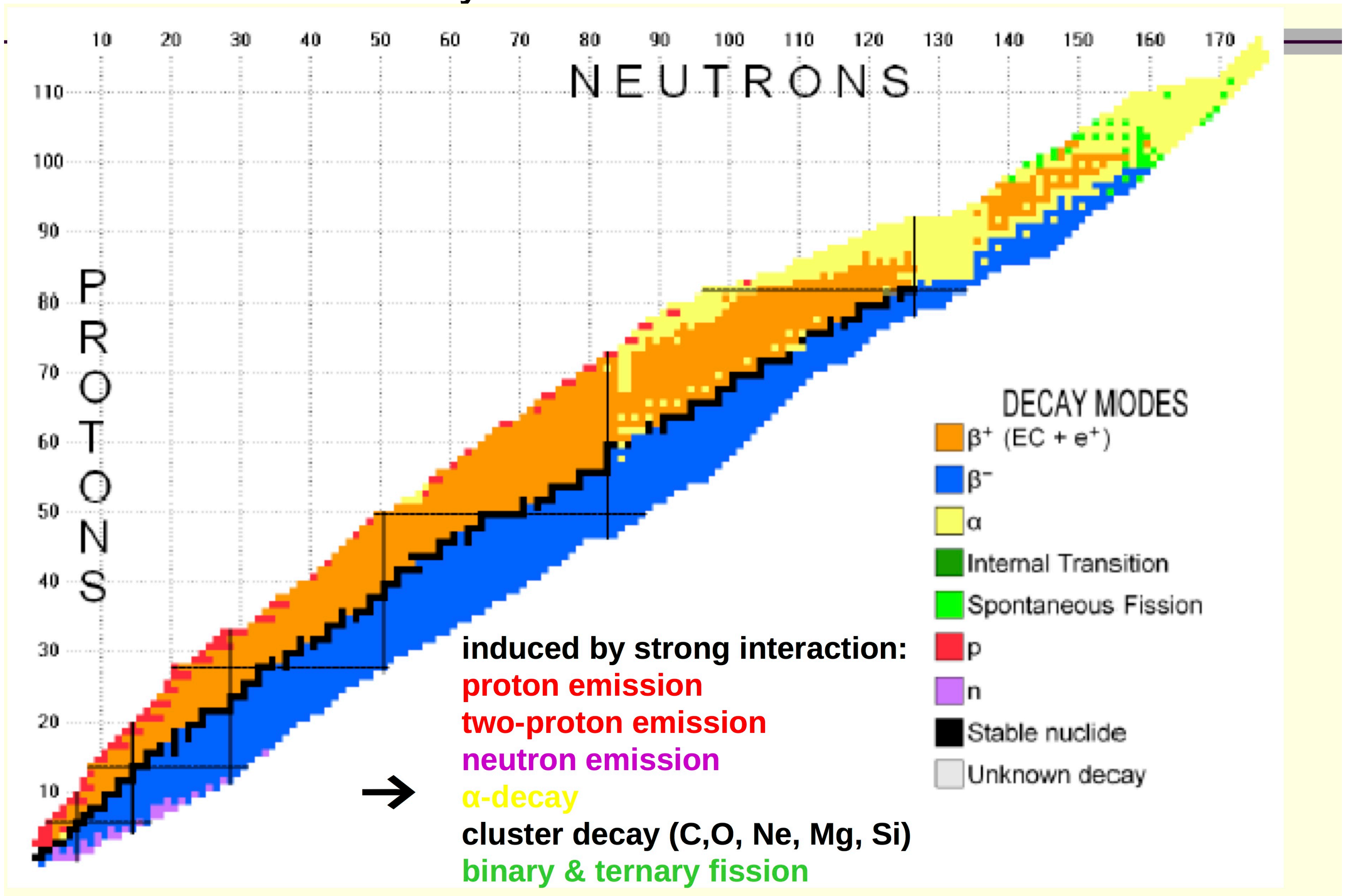
$$+ \int d^3\mathbf{r}'_1\, d^3\mathbf{r}'_3 \langle \mathbf{r}_1\mathbf{r}_3 | B\, V_{N-N} | \mathbf{r}'_1\mathbf{r}'_3\rangle \Psi(\mathbf{r}'_1\mathbf{r}_2\mathbf{r}'_3\mathbf{r}_4) + \text{four further permutations.} \quad (8)$$

Pauli blocking

$$\hat{h} = \frac{\hbar^2 p^2}{2m} + [1 - \sum_i^{\rm occ.} |n\rangle\langle n|] V^{\rm mf}(r) \qquad B(1,2) = [1 - f_1(\hat{h}_1) - f_2(\hat{h}_2)]$$

α decay of heavy nuclei

Decay modes of nuclei



Landau Fermi liquid

Strongly degenerate Fermi system: excitations near the Fermi energy, well-defined quasiparticles

Inverse of compressibility, T=0

$$K = n \frac{\partial \mu}{\partial n}\Big|_T = \frac{p_F^2}{3E_F}(1 + f_0)$$

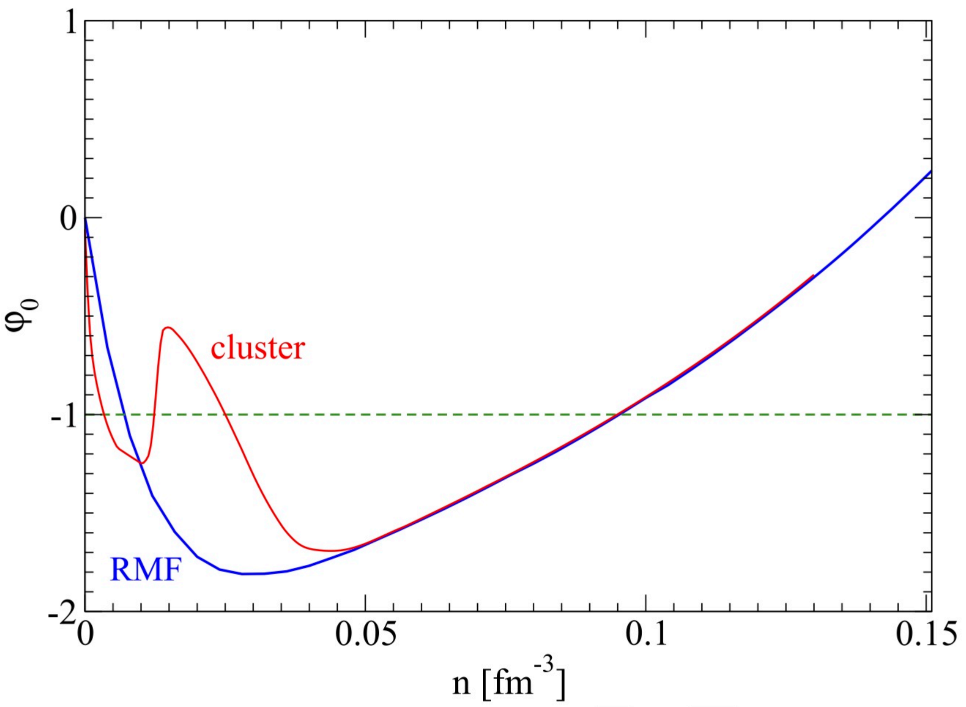
Landau-Migdal parameter f_0 (T=0)

General case, correlations, finite T

$$K(T,n) = K^{(0)}(T,n)[1 + \varphi_0(T,n)]$$

fluctuation-dissipation theorem

response function $\chi(q, \omega)$

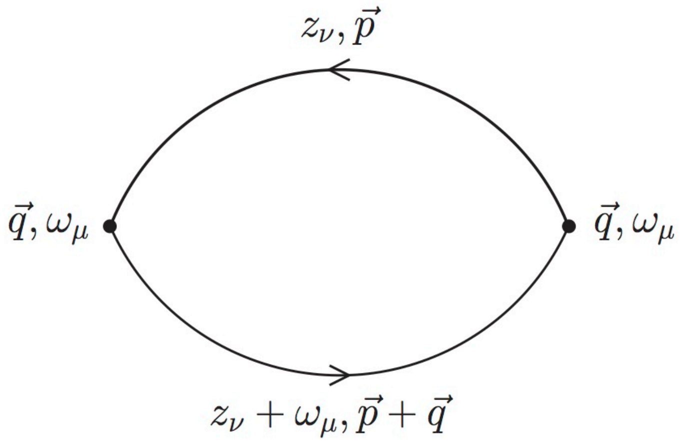


G. Ropke, D.N. Voskresensky, I.A. Kryukov, D. Blaschke, Nucl. Phys. A 970, 224 (2018)

Noninteracting Fermi-gas

polarization loop

$$L(\vec{q},\omega_\mu) \approx \Pi_1^0(\vec{q},\omega_\mu) \;=$$



$$L_0(\mathbf{q}, z) = g_\nu \int \frac{d^3p}{(2\pi)^3} \frac{f_{\mathbf{p}}^0 - \ldots}{z + \epsilon_{\mathbf{p}}^0 - \ldots}$$

Dynamical structure factor

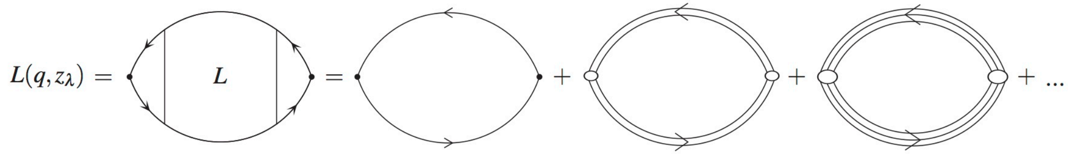
$$S_0(\mathbf{q},\omega) = \frac{1}{e^{\beta\omega} - 1} g_\nu \int \frac{d^3p}{(2\pi)^3} \left(f_{\mathbf{p}}^0 - f_{\mathbf{p}+\mathbf{q}}^0\right) \delta(\omega + \epsilon_{\mathbf{p}}^0 - \epsilon_{\mathbf{p}+\mathbf{q}}^0)$$

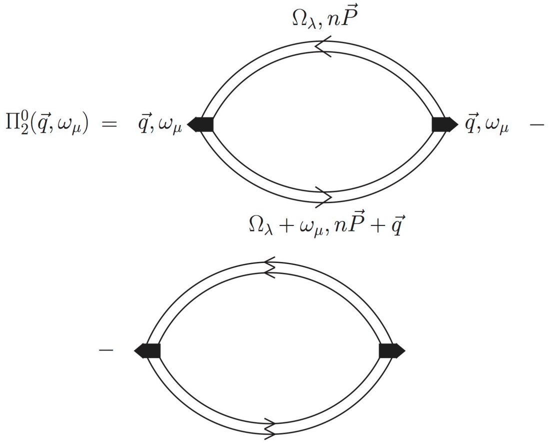
isothermal compressibility

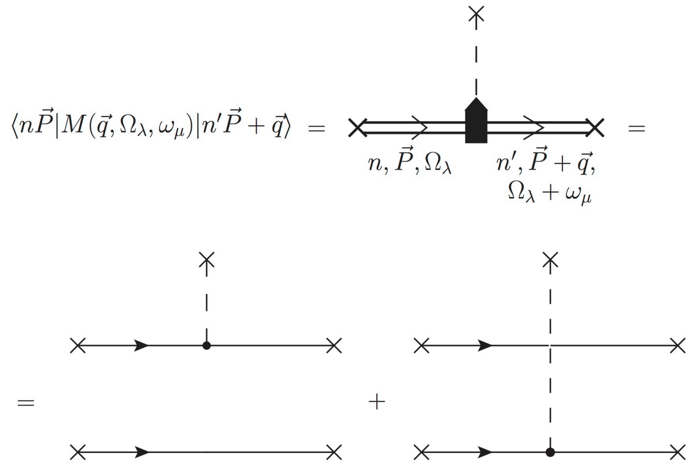
$$\kappa_{\mathrm{iso}}^{(0)}(T,\mu) = \frac{\beta}{n_B^2} g_\nu \int \frac{d^3p}{(2\pi)^3} f_p^0 (1 - f_p^0)$$

$$n_B^{(0)}(\beta,\mu) = \frac{1}{\Omega_0} \sum_p \frac{1}{e^{\beta(\epsilon_p^0 - \mu)} + 1} = \frac{1}{\Omega_0} \sum_p f_p^0 = \frac{N}{\Omega_0}$$

Cluster decomposition of the polarization function

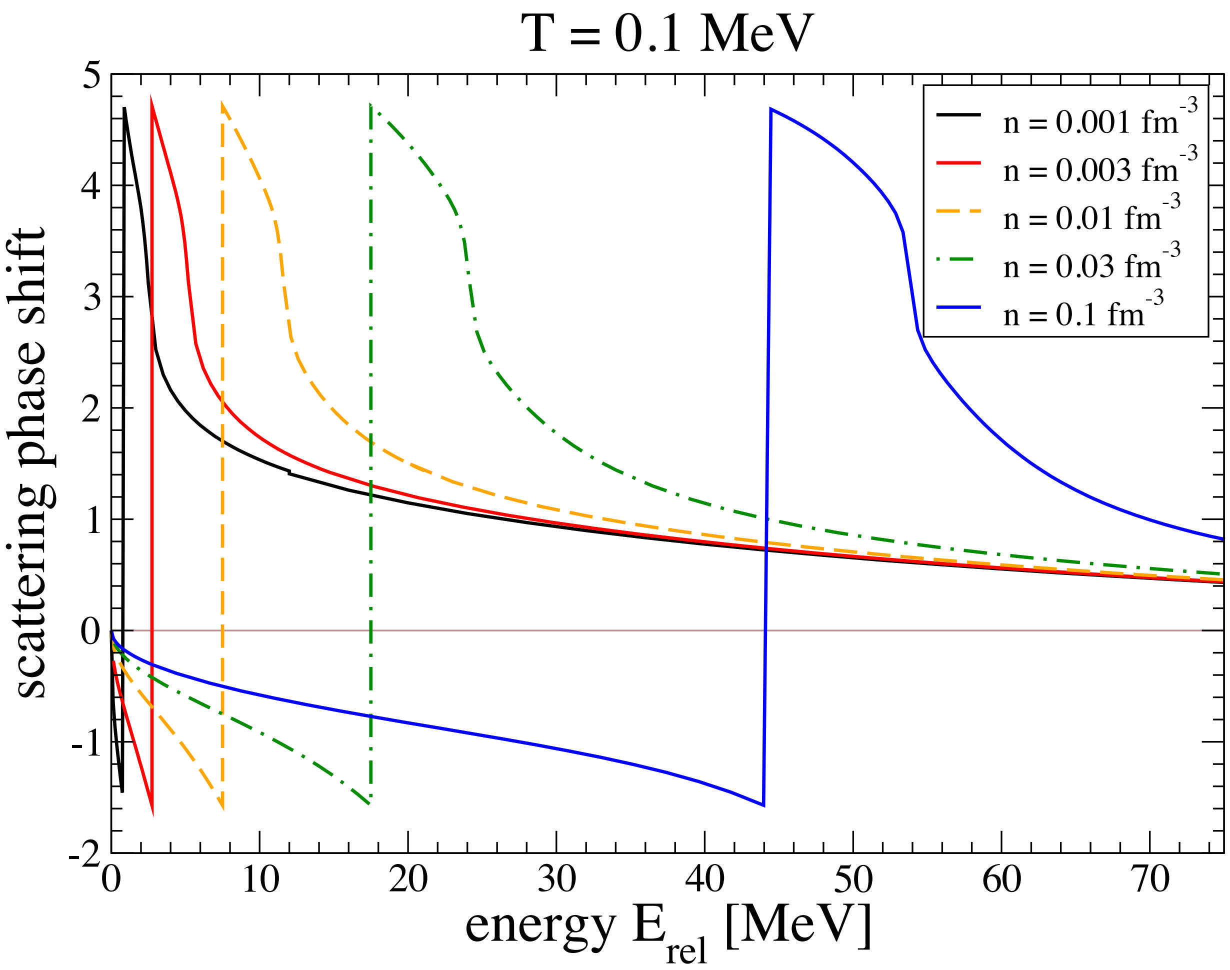




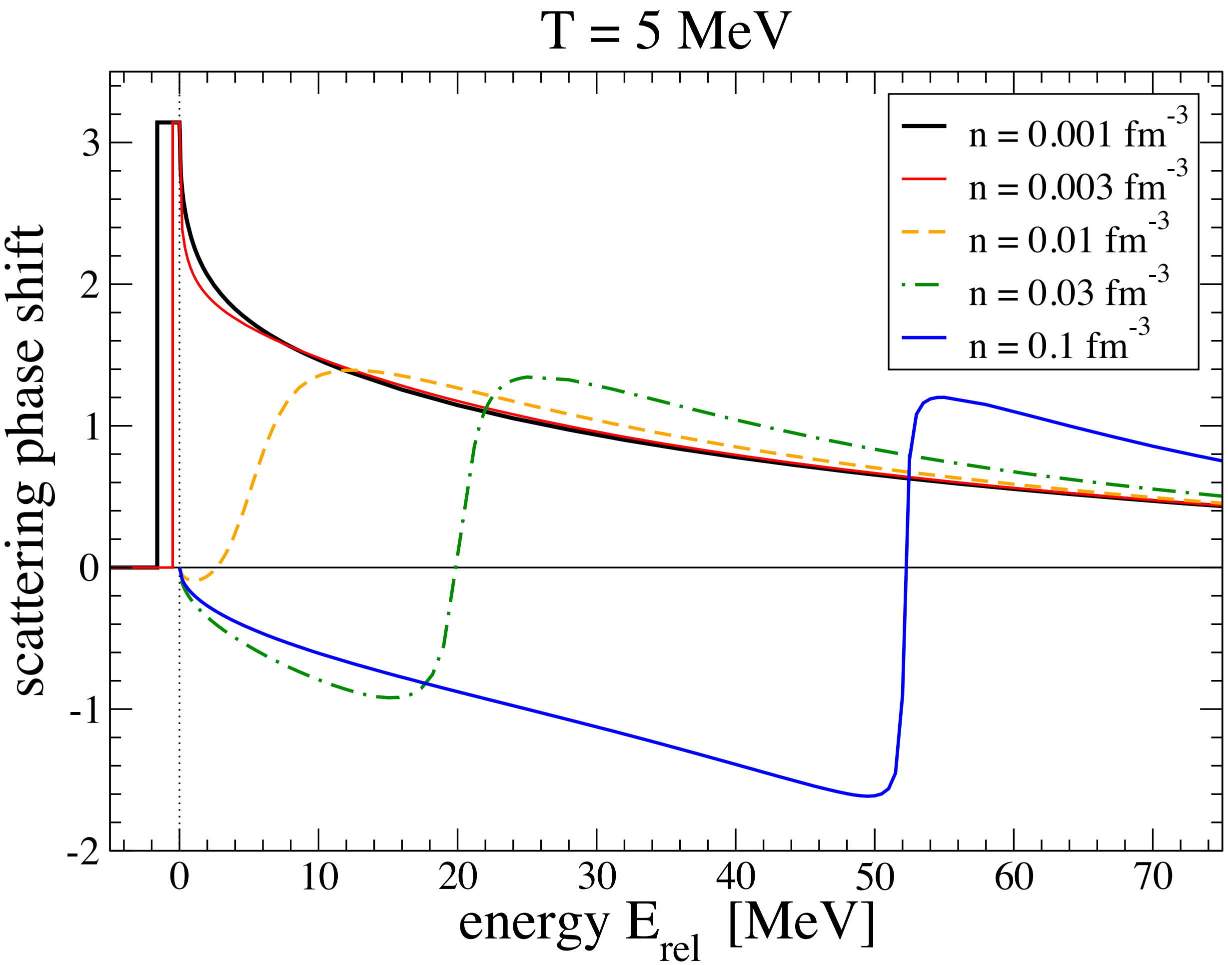


$$M_{\nu\nu'}(\mathbf{q}) = \langle \nu, \mathbf{P}|M(\mathbf{q}, z_\lambda, z_\mu)|\nu', \mathbf{P} + \mathbf{q}\rangle = \sum_{\mathbf{p}_1, \mathbf{p}_2} \psi^*_{\nu,\mathbf{P}}(p_1, p_2)[\psi_{\nu',\mathbf{P}+\mathbf{q}}(\mathbf{p}_1 + \mathbf{q}, \mathbf{p}_2) + \psi_{\nu',\mathbf{P}+\mathbf{q}}(\mathbf{p}_1, \mathbf{p}_2 + \mathbf{q})]$$

$$\kappa_{\rm iso}^{\rm (BU)}(T, \mu_n, \mu_p) = \frac{\beta}{\Omega_0 n_B^2} \left\{ \sum_{\mathbf{p}} f_p^0(1 - f_p^0) + \sum_{\alpha,\mathbf{P}} \int_{-\infty}^{\infty} \frac{dE}{\pi} f_2\left(E + \frac{P^2}{4m}\right) \left[1 + f_2\left(E + \frac{P^2}{4m}\right)\right] D_{\alpha,\mathbf{P}}(E) \right\}$$



T = 0.1 MeV
scattering phase shift
energy E_{rel} [MeV]
n = 0.001 fm^{-3}
n = 0.003 fm^{-3}
n = 0.01 fm^{-3}
n = 0.03 fm^{-3}
n = 0.1 fm^{-3}
5
4
3
2
1
0
-1
-2
0
10
20
30
40
50
60
70



T = 5 MeV
n = 0.001 fm^-3
n = 0.003 fm^-3
n = 0.01 fm^-3
n = 0.03 fm^-3
n = 0.1 fm^-3
scattering phase shift
energy E_rel [MeV]
3
2
1
0
-1
-2
0
10
20
30
40
50
60
70

Cluster: few-nucleon system

Nucleon pair (position, spin, isospin)

$$-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_1}\Psi(\mathbf{r}_1,\mathbf{r}_2)-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_2}\Psi(\mathbf{r}_1,\mathbf{r}_2)+V(\mathbf{r}_1-\mathbf{r}_2)\Psi(\mathbf{r}_1,\mathbf{r}_2)=E\Psi(\mathbf{r}_1,\mathbf{r}_2)$$

Translational invariance: Jacobi coordinates $\mathbf{R}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2}$ $\mathbf{s}=\mathbf{r}_2-\mathbf{r}_1$

Separation ansatz $\Psi(\mathbf{r}_1,\mathbf{r}_2)=\varphi^{\mathrm{intr}}(\mathbf{s})\Phi(\mathbf{R})$

$$-\frac{\hbar^2}{m}\nabla^2_{\mathbf{s}}\varphi^{\mathrm{intr}}(\mathbf{s})+V(\mathbf{s})\varphi^{\mathrm{intr}}(\mathbf{s})=E^{\mathrm{intr}}\varphi^{\mathrm{intr}}(\mathbf{s})$$

$$-\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}}\Phi(\mathbf{R})=E^{\mathrm{c.m.}}\Phi(\mathbf{R})$$

Cluster: few-nucleon system

Nucleon pair (position, spin, isospin)

$$-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_1}\Psi(\mathbf{r}_1,\mathbf{r}_2)-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_2}\Psi(\mathbf{r}_1,\mathbf{r}_2)+V(\mathbf{r}_1-\mathbf{r}_2)\Psi(\mathbf{r}_1,\mathbf{r}_2)=E\Psi(\mathbf{r}_1,\mathbf{r}_2)$$

Translational invariance: Jacobi coordinates $\mathbf{R}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2}$ $\mathbf{s}=\mathbf{r}_2-\mathbf{r}_1$

Separation ansatz $\Psi(\mathbf{r}_1,\mathbf{r}_2)=\varphi^{\mathrm{intr}}(\mathbf{s})\Phi(\mathbf{R})$

$$-\frac{\hbar^2}{m}\nabla^2_{\mathbf{s}}\varphi^{\mathrm{intr}}(\mathbf{s})+V(\mathbf{s})\varphi^{\mathrm{intr}}(\mathbf{s})=E^{\mathrm{intr}}\varphi^{\mathrm{intr}}(\mathbf{s}) \qquad -\frac{\hbar^2}{4m}\nabla^2_{\mathbf{R}}\Phi(\mathbf{R})=E^{\mathrm{c.m.}}\Phi(\mathbf{R})$$

Nucleon quartet $\{n_\uparrow,n_\downarrow,p_\uparrow,p_\downarrow\}$

$$-\sum_i\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}_i}\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)+V_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=E\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)$$

$$V_4(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=V(\mathbf{r}_1-\mathbf{r}_2)+V(\mathbf{r}_1-\mathbf{r}_3)+V(\mathbf{r}_1-\mathbf{r}_4)+V(\mathbf{r}_2-\mathbf{r}_3)+V(\mathbf{r}_2-\mathbf{r}_4)+V(\mathbf{r}_3-\mathbf{r}_4)$$

Jacobi-Moshinsky coordinates

$$\begin{aligned}
\mathbf{r}_{n,\uparrow}&=\mathbf{R}+\mathbf{S}/2+\mathbf{s}/2, & \mathbf{p}_{n,\uparrow}&=\mathbf{P}/4+\mathbf{Q}/2+\mathbf{q},\\
\mathbf{r}_{n,\downarrow}&=\mathbf{R}+\mathbf{S}/2-\mathbf{s}/2, & \mathbf{p}_{n,\downarrow}&=\mathbf{P}/4+\mathbf{Q}/2-\mathbf{q},\\
\mathbf{r}_{p,\uparrow}&=\mathbf{R}-\mathbf{S}/2+\mathbf{s}'/2, & \mathbf{p}_{p,\uparrow}&=\mathbf{P}/4-\mathbf{Q}/2+\mathbf{q}',\\
\mathbf{r}_{p,\downarrow}&=\mathbf{R}-\mathbf{S}/2-\mathbf{s}'/2, & \mathbf{p}_{p,\downarrow}&=\mathbf{P}/4-\mathbf{Q}/2-\mathbf{q}'.
\end{aligned}$$