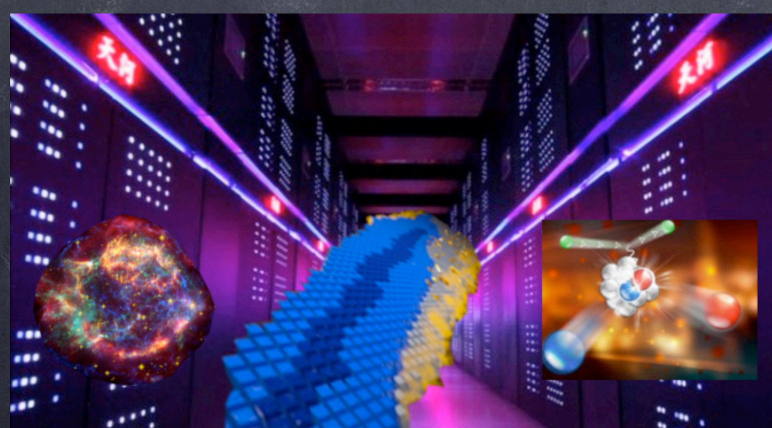
Lattice Gauge Theories

Andrea Shindler

shindler@frib.msu.edu

www.nscl.msu.edu/directory/shindler.html

TALENT School - From quarks and gluons to nuclear forces and structures



ECT* - Trento
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QCD path integral

$$\mathcal{D}[\psi,\overline{\psi}] = \prod_{x\in\Lambda}\prod_{f,\alpha,A} d\psi^{A}_{f\alpha}(x)\, d\overline{\psi}^{A}_{f\alpha}(x)$$

Grassmann numbers

$$\mathcal{D}[U] = \prod_{x\in\Lambda}\prod_{\mu=1}^{4} dU(x,\mu)$$

Haar measure

$$\left\langle \Phi' | \widehat{O}(x) | \Phi \right\rangle = O[\Phi]\delta(\Phi' - \Phi)$$

Pure gauge theory

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, \mathrm{e}^{-S_G[U]} O[U]$$

$$Z = \int \mathcal{D}[U]\, \mathrm{e}^{-S_G[U]}$$

$$S_G[U] = \frac{1}{g^2} \sum_x \sum_{\mu,\nu} P_{\mu\nu}(x)$$

$$\mathcal{D}[U] = \prod_{x\in\Lambda} \prod_{\mu=1}^{4} dU(x,\mu)$$

Haar measure

$$S_G[U] = \frac{\beta}{3} \sum_x \sum_{\mu<\nu} \mathrm{Re}\mathrm{tr}[1 - U(x,x;\Box)]$$

$$\beta = \frac{6}{g^2}$$

Haar measure

$$U(x,\mu) \to U'(x,\mu) = \Omega(x)U(x,\mu)\Omega(x+a\hat{\mu})^\dagger$$

$$S_G[U'] = S_G[U]$$

$$Z = \int \mathcal{D}[U]\ \mathrm{e}^{-S_G[U]} = \int \mathcal{D}[U']\ \mathrm{e}^{-S_G[U']} = \int \mathcal{D}[U']\ \mathrm{e}^{-S_G[U]}$$

$$\mathcal{D}[U] = \mathcal{D}[U'] \qquad dU(x;\mu) = dU'(x;\mu) = d(\Omega(x)U(x;\mu)\Omega(x+a\hat{\mu})^\dagger)$$

Haar measure

Haar measure

$$dU(x;\mu) = dU'(x;\mu) = d(\Omega(x)U(x;\mu)\Omega(x+a\hat{\mu})^\dagger)$$

$$dU = d(UV) = d(VU) \qquad U \in G \quad V \in G \qquad \int dU\ 1 = 1$$

$$\int_{SU(3)} dU\ U_{AB} = 0 \qquad \int_{SU(3)} dU\ U_{AB}U_{CD} = 0$$

$$\int_{SU(3)} dU\ U_{AB}(U^\dagger)_{CD} = \frac{1}{3}\delta_{AD}\delta_{BC}$$

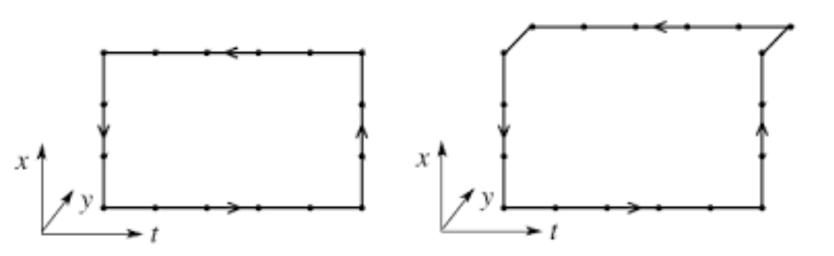
$$\int_{SU(3)} dU\ \mathrm{tr}[VU]\mathrm{tr}[U^\dagger W] = \frac{1}{3}\mathrm{tr}[VW]$$

Wilson loop

$$L[U] = \mathrm{tr}\left[\prod_{(x,\mu)\in\mathcal{L}} U(x,\mu)\right]$$

Closed loop

$$S(\underline{x},\underline{y},t) = \prod_{(\underline{z},j)\in\mathcal{C}_{\underline{x},\underline{y}}} U(\underline{z},t;j)$$



$$T(\underline{x},t) = \prod_{x_4=0}^{t-a} U(\underline{x},x_4;4)$$

$$W_{\mathcal{L}} = \mathrm{tr}\left[S(\underline{x},\underline{y},t)T(\underline{y},t)^\dagger S(\underline{x},\underline{y},0)^\dagger T(\underline{x},t)\right]$$

Wilson loop: physical interpretation

$$A_4(x) = 0 \qquad U(x;4) = 1$$ **Temporal gauge**

$$\langle W_{\mathcal{L}} \rangle = \langle W_{\mathcal{L}} \rangle_{\rm temp} = \left\langle {\rm tr}\left[S(\underline{x},\underline{y},t) S(\underline{x},\underline{y},0)^\dagger \right] \right\rangle_{\rm temp}$$

$$\left\langle {\rm tr}\left[S(\underline{x},\underline{y},t) S(\underline{x},\underline{y},0)^\dagger \right] \right\rangle_{\rm temp} = \sum_n \left\langle 0 | \widehat{S}(\underline{x},\underline{y}) | n \right\rangle \left\langle n | \widehat{S}(\underline{x},\underline{y})^\dagger | 0 \right\rangle {\rm e}^{-E_n t}$$

$$E_0 = V(r) \qquad r = a|\underline{x} - \underline{y}| \qquad \langle W_{\mathcal{L}} \rangle \propto {\rm e}^{-V(r)t} + \ldots$$

Wilson loop: physical interpretation

$$Q(\underline{x}, \underline{y})_{ab} = \psi_a(\underline{x})\overline{\psi}_b(\underline{y})$$

$$Q(\underline{x}, \underline{y})_{ab} \to \Omega(\underline{x})_{aa'} Q(\underline{x}, \underline{y})_{a'b'} \Omega(\underline{y})^\dagger_{b'b}$$

$$S(\underline{x}, \underline{y}, t)_{ab} \to \Omega_{aa'} S(\underline{x}, \underline{y}, t)_{a'b'} \Omega_{b'b}$$

Static quark potential

$$V(r) = A + \frac{B}{r} + \sigma r$$

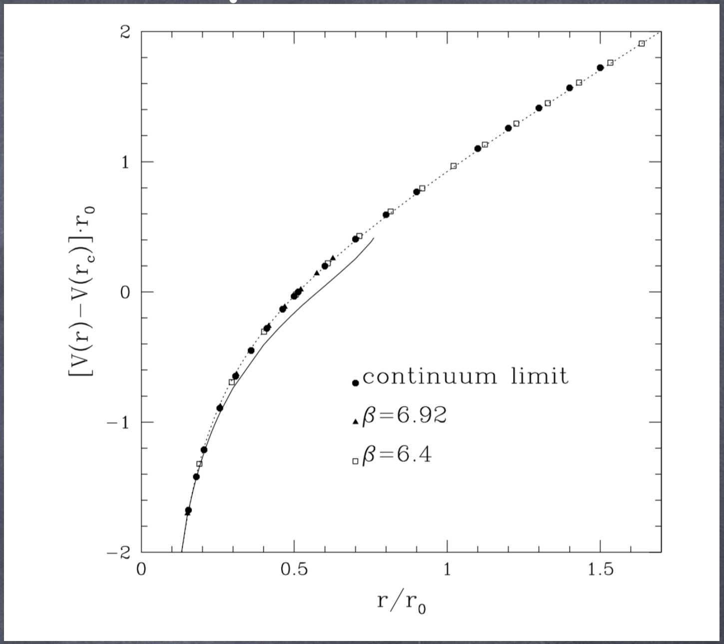
$$\Downarrow$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

Setting the scale with the static potential

$$a = \frac{(aM)_{\rm latt}}{M_{\rm phys}}$$

$$\langle W_C \rangle = C \mathrm{e}^{-V(r)t} + \cdots$$



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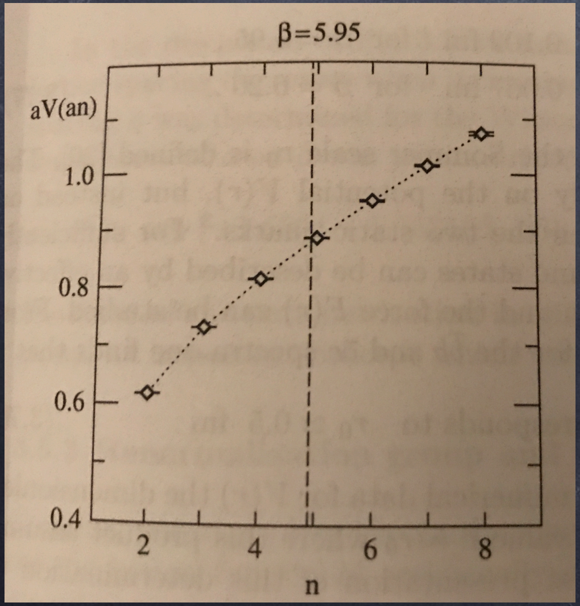
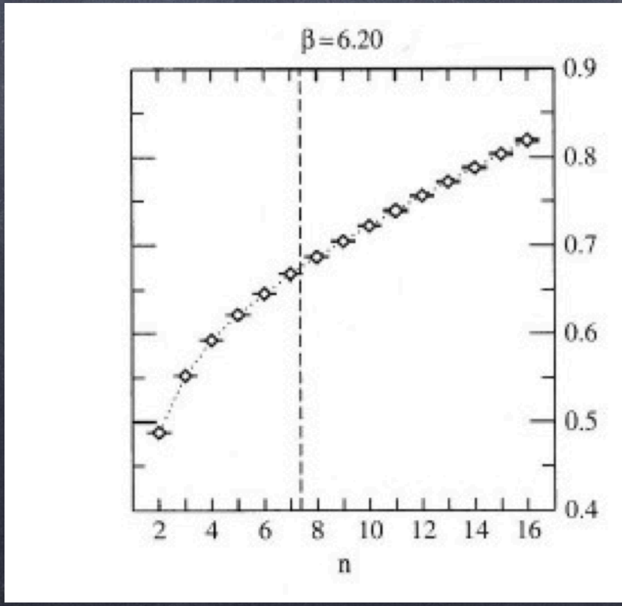




Setting the scale with the static potential

$$F(r) = \frac{dV(r)}{dr}$$

$$F(r_0)r_0^2 = 1.65 \qquad r_0 = 0.5\text{fm}$$

$$F(r) = \frac{dV(r)}{dr} = -\frac{B}{r^2} + \sigma$$

$$F(r_0)r_0^2 = -B + \sigma r_0^2 = 1.65$$

$$\frac{r_0}{a} = \sqrt{\frac{1.65 + B}{a^2\sigma}}$$

$$aV(x) = A \cdot a + B\frac{a}{r} + \sigma a^2 \frac{r}{a}$$

Step 1

Step 2

$$a = \frac{0.5}{r_0/a}\text{fm}$$

RG and running coupling

$$\lim_{a\to 0} P(g(a),a) = P_C$$

$$\frac{dP(g(a),a)}{d\ln a} = 0$$

$$\left(\frac{\partial}{\partial \ln a} - \beta(g)\frac{\partial}{\partial g}\right) = 0$$

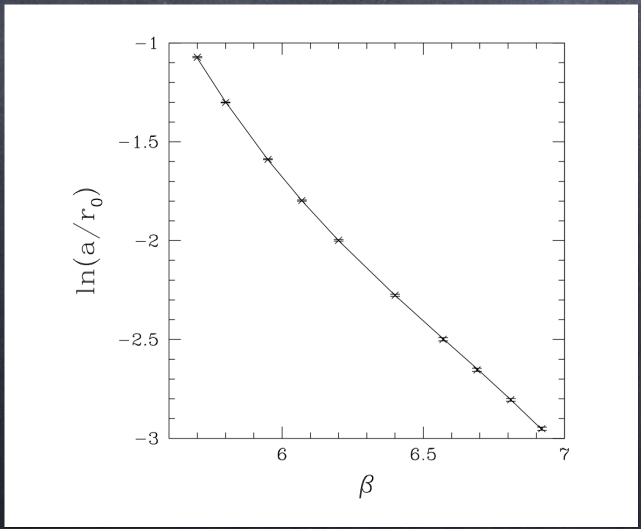
$$\beta(g) = -\frac{\partial g}{\partial \ln a}$$

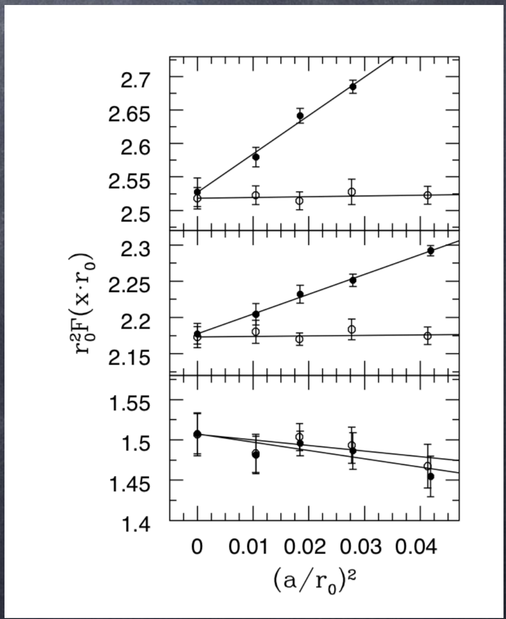
$$\beta(g) = -\beta_0 g^3 - \beta_1 g^5 + O(g^7)$$

RG and running coupling

$$\beta_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

$$a(g) = \frac{1}{\Lambda_L}(\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{2\beta_0 g^2}\right)(1+O(g^2))$$





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