Short-range three-nucleon interactions

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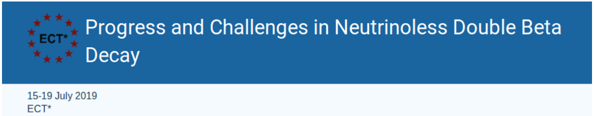
based on L.G., A. Kievsky, M. Viviani and L.E. Marcucci, Phys. Rev. C 99 (2019) 054003



Outline

- Preamble: a simple minded perspective on naturality and power counting
- The subleading three-nucleon contact interaction
- Accurate description of low-energy $N - d$ scattering
- Testing hieararchies from large-N_c and relativistic counting

Choosing the right cutoff

renormalization:

$$\text{Loops}(\Lambda) + \text{LECs}(\Lambda) = \text{observables}$$

- an unnatural cutoff Λ leads to unnatural LECs
- LECs are natural when comparable to loops
- unnatural LECs are subject to fine-tuning problems when fitted to data
- if the theory is to be effective, the cutoff must be natural
- this requires having renormalized the theory, not easy to do non-perturbatively
- renormalization can also be checked *a posteriori*, inspecting the order-by-order convergence

A destabilizing accident

$$B(^2\mathrm{H}) = \overbrace{\langle T \rangle}^{\sim \frac{Q^2}{\Lambda_\mathrm{H}} \sim 20\mathrm{MeV}} + \overbrace{\langle V \rangle}^{\sim \frac{Q^3}{4\pi F_\pi^2} \sim \frac{Q^2}{\Lambda_\mathrm{H}} \sim 20\mathrm{MeV}} \sim 2\mathrm{MeV}$$

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and the NN force enter in the 3N system quite non-linearly

✔ 3NF makes ~ 1 MeV attraction in the ^{3}H, comparable to $\sim$ 2MeV/pair of the 2NF: is this a symptom of such instabilities?

Naïve dimensional analysis

$$\mathcal{L} = \sum_{klm} c_{klm} A \left(\frac{\bar{N}N}{B}\right)^k \left(\frac{\partial^\mu, M_\pi}{C}\right)^l \left(\frac{\pi}{D}\right)^m, \quad c_{klm} \sim 1$$

The scale factors are uniquely fixed by the lowest order Lagrangian

$$\mathcal{L} = \bar{N}(i\partial\!\!\!/ - m_N)N + \frac{1}{2}\partial^\mu \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} - \frac{1}{2}M_\pi^2 \boldsymbol{\pi}^2 - \frac{g_A}{2F_\pi}\bar{N}\gamma^\mu\gamma_5\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau}N + \ldots$$

to be

$$\mathcal{L} = \sum_{klm} c_{klm}\Lambda^2 F_\pi^2 \left(\frac{\bar{N}N}{F_\pi^2\Lambda}\right)^k \left(\frac{\partial^\mu, M_\pi}{\Lambda}\right)^l \left(\frac{\pi}{F_\pi}\right)^m$$

[Georgi, Manohar, Friar]

if a new scale is identified as ϵ, it must come from a further interaction

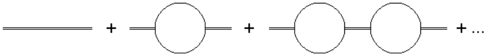
$$\Delta\mathcal{L} = -\frac{D_0}{2}(\bar{N}N)^2, \quad D_0 \sim \frac{4\pi a}{m_N} \sim \frac{4\pi}{m_N\epsilon} \sim \frac{1}{F_\pi\epsilon}$$

$$\Longrightarrow \mathcal{L} = \sum_{klm} c_{klm}\Lambda^2 F_\pi\epsilon \left(\frac{\bar{N}N}{F_\pi\Lambda\epsilon}\right)^k \left(\frac{\partial^\mu, M_\pi}{\Lambda}\right)^l \left(\frac{\pi}{F_\pi}\right)^m$$

Tracking the soft scale

use auxiliary dibaryon fields [Kaplan, Bedaque, Hammer, van Kolck,...]

$$\mathcal{L} = N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) N + \vec{T}^\dagger \left(i\partial_0 + \frac{\nabla^2}{4M} - \Delta_T \right) \cdot \vec{T} - \frac{g_T}{2} \left(\vec{T}^\dagger \cdot N^T \tau_2 \sigma_2 \vec{\sigma} N + \mathrm{h.c} \right) + \ldots$$



$$= \frac{i}{\Delta_T - \frac{M g_T^2}{2\pi} \sqrt{-ME + \frac{\mathbf{p}^2}{4} - i\eta} + \ldots}$$

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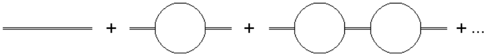

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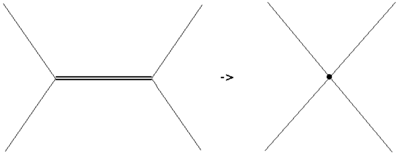

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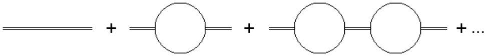

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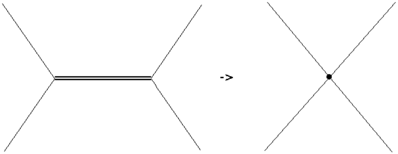

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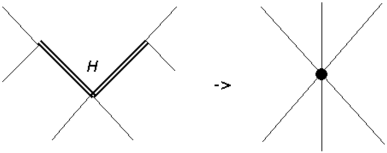

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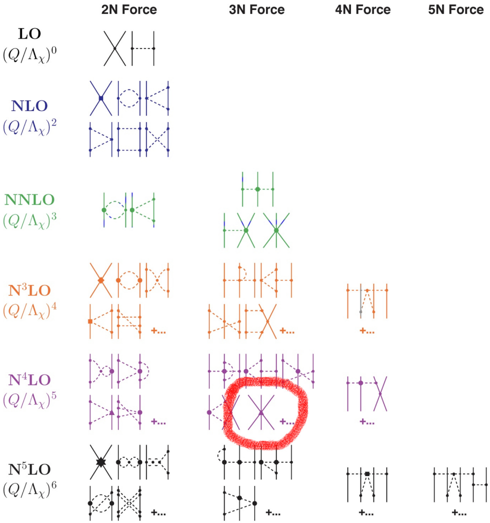
The subleading contact TNI



(picture by Machleidt)

Motivation

- accurate NN potentials developed up to N4LO and N5LO
- N3LO $3N$ forces awaiting revision (see Evgeny's talk)
- well-known discrepancies (cfr. A_y puzzle) [LENPIC, EPJA(2014)]

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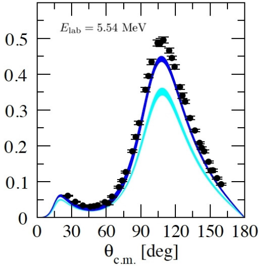
  [Viviani et al. PRL111 (2013) 172302]



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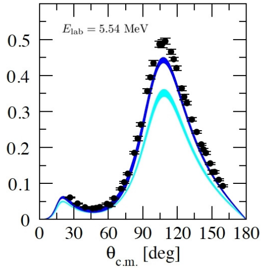

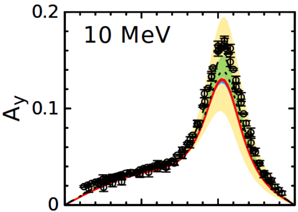

- For Nd, possibly affected by large uncertainty [LENPIC, PRC93 (2016) 044002], [Epelbaum et al. 1907.03608]

Minimal subleading contact TNI

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\begin{aligned}
V = \sum_{i\neq j\neq k} & (E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k)(\mathbf{L}\cdot\mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
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Spin-orbit terms suitable for the A_y puzzle [Kievsky PRC60 (1999) 034001]

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Numerical implementation

The N-d scattering wave function is written as

$$\Psi_{LSJJ_z} = \Psi_C + \Psi_A$$

with Ψ_C expanded in the HH basis

$$|\Psi_C\rangle = \sum_\mu c_\mu |\Phi_\mu\rangle$$

and Ψ_A describing the asymptotic relative motion

$$\Psi_A \sim \Omega^R_{LS}(k,r) + \sum_{L'S'} R_{LS,L'S'}(k)\Omega^I_{L'S'}(k,r)$$

with the unknown c_μ and R-matrix elements (related to the S-matrix) to be determined so that the Kohn functional is stationary

$$[R_{LS,L'S'}] = R_{LS,L'S'} - \langle \Psi_C + \Psi_A | H - E | \Psi_C + \Psi_A \rangle$$

imposing the Kohn functional to be stationary leads to a *linear* system

$$\sum_{L''S''} R_{LS,L''S''} X_{L'S',L''S''} = Y_{LS,L'S'}$$

with the matrices

$$X_{LS,L'S'} = \langle \Omega^I_{LS} + \Psi^I_C | H - E | \Omega^I_{L'S'} \rangle \quad Y_{LS,L'S'} = -\langle \Omega^R_{LS} + \Psi^R_C | H - E | \Omega^I_{L'S'} \rangle$$

and the $\Psi^{R/I}_C$ solutions of

$$\sum_{\mu'} c_\mu \langle \Phi_\mu | H - E | \Phi_{\mu'} \rangle = -D^{R/I}_{LS}(\mu)$$

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and the $\Psi_C^{R/I}$ solutions of

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with

$$D^{R/I}_{LS}(\mu) = \langle \Phi_\mu | H - E | \Omega^{R/I}_{LS} \rangle$$

11 set of matrices are calculated once for all, and only linear systems are solved for each choice of E_i's

Fit strategy

- we ask whether the subleading contact interaction has enough flexibility to solve the existing puzzles in low-energy $N-d$ scattering
- to start, we consider this interaction as a remainder to the phenomenological AV18+UIX
- we have 11 LECs, $E = \frac{c_E}{F_\pi^4 \Lambda}$ (LO) and $E_{i=1,\ldots,10} = \frac{e_i^{NN}}{F_\pi^4 \Lambda^3}$ (NLO) to be fitted to $B(^3H)$, $^2a_{nd}$, $^4a_{nd}$ and accurate $p-d$ scattering data at 3 MeV proton energy (~ 300 data), for different values of Λ
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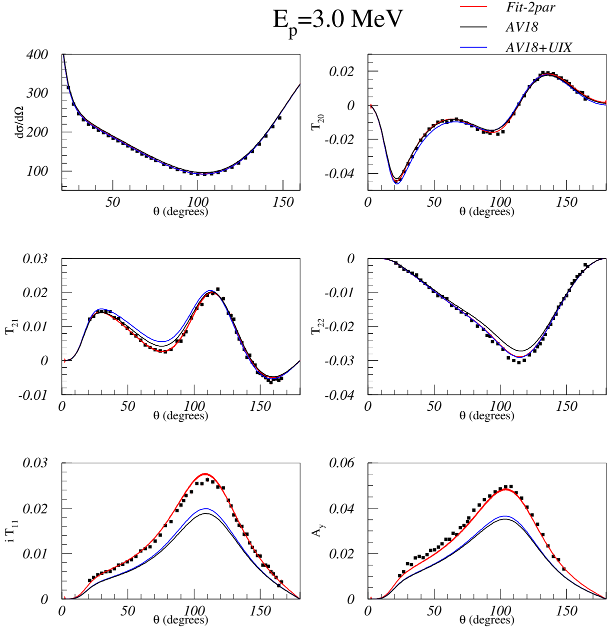
- data are mostly sensitive to the tensor and spin-orbit operators

Λ (MeV)	200	300	400	500
χ^2/d.o.f.	2.0	2.0	2.1	2.1
e_0	-0.074	-0.037	0.053	0.451
e_5	-0.212	-0.248	-0.403	-0.799
e_7	1.104	1.195	1.686	2.598
$\langle \mathrm{AV18} \rangle$ (MeV)	-7.353	-7.373	-7.394	-7.343
$\langle \mathrm{UIX} \rangle$ (MeV)	-1.118	-1.095	-1.058	-1.031
$\langle V^{(0)} \rangle$ (MeV)	-0.057	-0.069	0.125	0.841
$\langle E_5 O_5 \rangle$ (MeV)	-0.032	-0.182	-0.609	-1.553
$\langle E_7 O_7 \rangle$ (MeV)	0.079	0.237	0.454	0.605



E_p=3.0 MeV
Fit-2par
AV18
AV18+UIX
dσ/dΩ
T_{20}
T_{21}
T_{22}
i T_{11}
A_y
θ (degrees)

Isospin projection

- $N - d$ scattering only gives access to the $T = 1/2$ component of 3NF
- we can project each operator on isospin channels

$$o_i = P^{(1)}(o_i) + P^{(3)}(o_i) \equiv P_{1/2} o_i P_{1/2} + P_{3/2} o_i P_{3/2}$$

$$P_{1/2} = \frac{1}{2} - \frac{1}{6}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3), \quad P_{1/2} + P_{3/2} = 1$$

- the projected operators can again be expressed in the initial 10-operator basis, using the Fierz identities
- at the end we find 9 independent operators among the 10 $P^{(1)}(o_i)$

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$$o_{3/2} = 3o_1 - 3o_3 - o_4 - 3o_5 - o_6 - 36o_7 - 12o_8 - 9o_9 - 3o_{10}$$

(up to cutoff effects ...)

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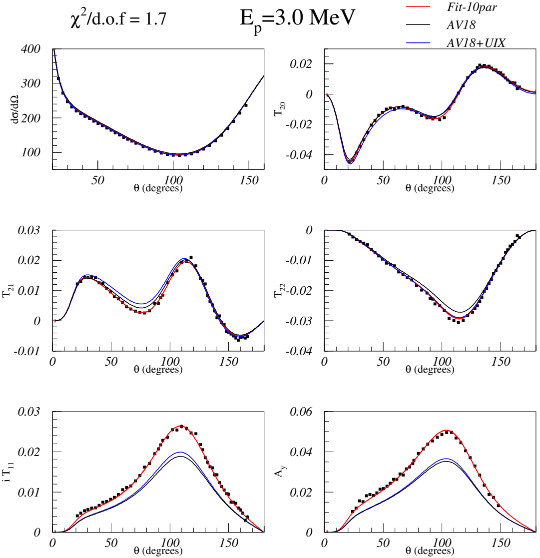
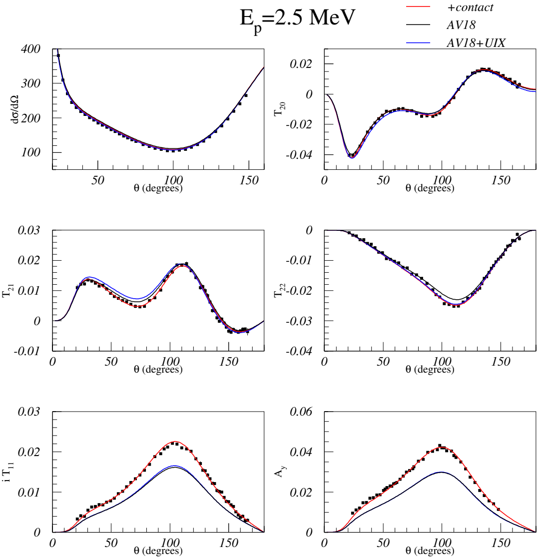
- we can exclude 1 LEC from the fits and absorb its effect in the remaining LECS

10-parameter fits



Predictions at lower energies



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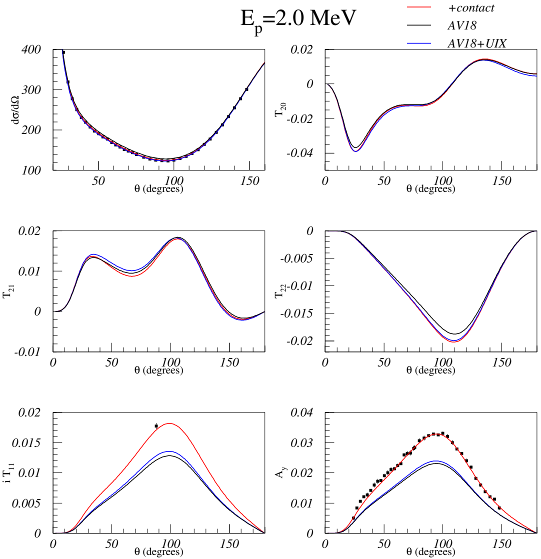

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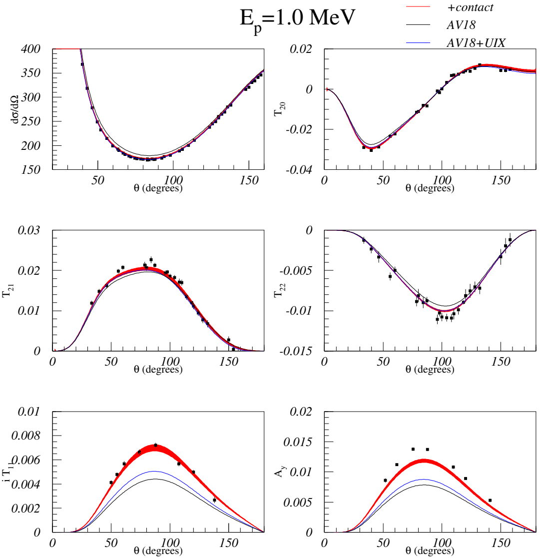
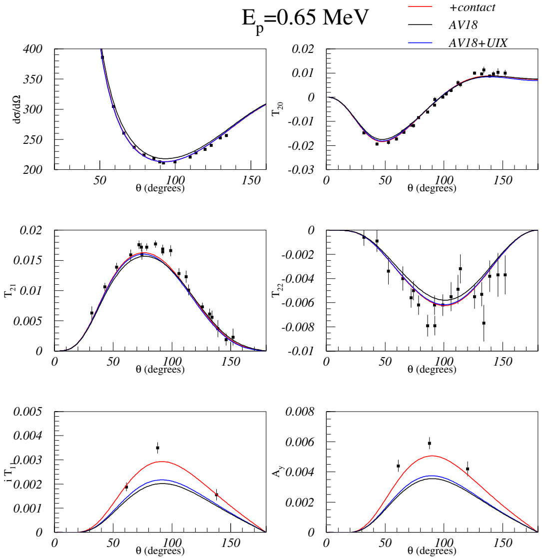


Predictions at lower energies



Insight from the large-N_c limit

- initially proposed by 't Hooft in 1974, to define a *weak coupling* limit of QCD, $g^2 N_c$ =const giving rise to substantial simplifications over QCD, but with similar physical properties
- a topological expansion emerges in which only *planar diagrams* survive, and no dynamical quark loops
- extended to baryons by Witten in 1979
- a spin-flavour symmetry appears, in which e.g. N and Δ belong to the same SU(4) multiplet

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- as a result, one finds e.g.

$$\mathbf{1} \sim \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim O(N_c)$$

while

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \sim \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim O(1/N_c)$$

Large-N_c and Pauli principle

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- in an effective theory one obtains that amplitude from

$$\mathcal{L} = c_1 N^\dagger N N^\dagger N + c_2 N^\dagger \sigma_i N N^\dagger \sigma_i N + c_3 N^\dagger \tau^a N N^\dagger \tau^a N + c_4 N^\dagger \sigma_i \tau^a N N^\dagger \sigma_i \tau^a N \equiv \sum_i c_i o_i$$

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$$\mathcal{L} = (c_1 - 2c_3 - 3c_4) N^\dagger N N^\dagger N + (c_2 - c_3) N^\dagger \sigma_i N N^\dagger \sigma_i N$$

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reobtaining the well-established fact that $C_S >> C_T$

3NF and large-N_c

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[D.R.Phillips and C.Schat, PRC88 (2013) 034002]

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& -E_5 N^\dagger \sigma^i N N^\dagger \sigma^i \tau^a N N^\dagger \tau^a N - E_6 \epsilon^{ijk} \epsilon^{abc} N^\dagger \sigma^i \tau^a N N^\dagger \sigma^j \tau^b N N^\dagger \sigma^k \tau^c N
\end{aligned}
$$

- only E_1, E_4 and E_6 are $O(N_c)$

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- operators with different scaling properties in $1/N_c$ get mixed

Large-N_c constraints on subleading 3N contact interaction

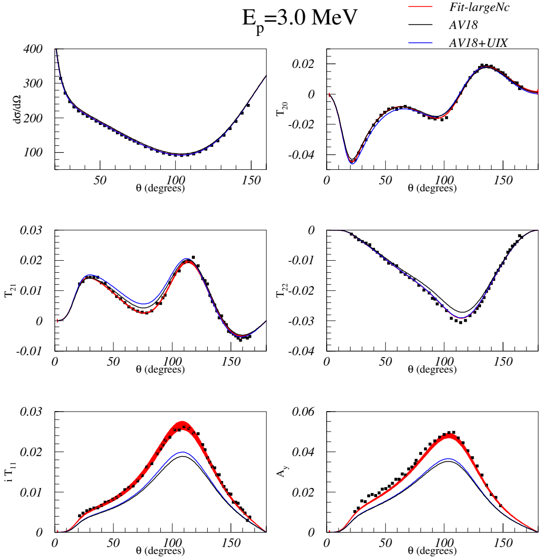
- applying Phillips and Schat counting to our redundant operators we get 13 leading structures
- using Fierz identities we find 4 vanishing LECs in the large-N_c limit

$$E_2 = E_3 = E_5 = E_9 = 0$$

thus reducing the number of subleading LECs to 6

Testing the large-N_c hierarchy

$E_{2,3,5,9} = 0$
$\Lambda = 200 - 500$ MeV
χ^2/d.o.f.~ 2



Subleading contact terms from “relativistic counting”

A new power-counting scheme for the derivation of relativistic chiral nucleon-nucleon interactions

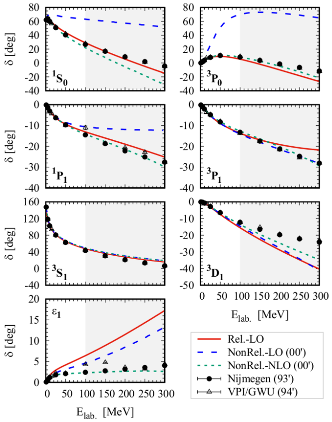
Xiu-Lei Ren,[1] Kai-Wen Li,[2] Li-Sheng Geng,[2,3,*] Bingwei Long,[4] Peter Ring,[5,1] and Jie Meng[1,2,†]
[1]*State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China*
[2]*School of Physics and Nuclear Energy Engineering & International Research Center for Nuclei and Particles in the Cosmos, Beihang University, Beijing 100191, China*
[3]*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*
[4]*Center for Theoretical Physics, Department of Physics, Sichuan University, Chengdu, Sichuan 610064, China*
[5]*Physik Department, Technische Universität München, D-85748 Garching, Germany*
(Dated: February 6, 2017)

Motivated by the successes of relativistic theories in studies of atomic/molecular and nuclear systems and the strong need for a covariant chiral force in relativistic nuclear structure studies, we develop a new covariant scheme to construct the nucleon-nucleon interaction in the framework of chiral effective field theory. The chiral interaction is formulated up to leading order with a covariant power counting and a Lorentz invariant chiral Lagrangian. We find that the covariant scheme induces all the six invariant spin operators needed to describe the nuclear force, which are also helpful to achieve cutoff independence for certain partial waves. A detailed investigation of the partial wave potentials shows a better description of the scattering phase shifts with low angular momenta than the leading order Weinberg approach. Particularly, the description of the $^1S_0$, $^3P_0$, and $^1P_1$ partial waves is similar to that of the next-to-leading order Weinberg approach. Our study shows that the relativistic framework presents a more efficient formulation of the chiral nuclear force.

PACS numbers: 13.75.Cs,21.30.-x

$$
\begin{aligned}
\mathcal{L}^{(0)}_{NN} &= \frac{1}{2}\Big[C_S(\bar\Psi\Psi)(\bar\Psi\Psi) + C_A(\bar\Psi\gamma_5\Psi)(\bar\Psi\gamma_5\Psi) \\
&+ C_V(\bar\Psi\gamma_\mu\Psi)(\bar\Psi\gamma^\mu\Psi) + C_{AV}(\bar\Psi\gamma_\mu\gamma_5\Psi)(\bar\Psi\gamma^\mu\gamma_5\Psi) \\
&+ C_T(\bar\Psi\sigma_{\mu\nu}\Psi)(\bar\Psi\sigma^{\mu\nu}\Psi)\Big]\,,
\end{aligned}
$$

“relativistic corrections are in the data”



Relativistic counting applied to contact TNI

There are 25 $C-$, $P-$ and $T-$ relativistic invariant operators

$$
\begin{aligned}
o_{1,2} &= (\bar\psi\psi)_1(\bar\psi\psi)_2(\bar\psi\psi)_3[\mathbf{1}, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2] \\
o_{3,4,5} &= (\bar\psi\psi)_1(\bar\psi\gamma_5\psi)_2(\bar\psi\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{6,7,8} &= (\bar\psi\psi)_1(\bar\psi\gamma^\mu\psi)_2(\bar\psi\gamma_\mu\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{9,10,11} &= (\bar\psi\psi)_1(\bar\psi\gamma^\mu\gamma_5\psi)_2(\bar\psi\gamma_\mu\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{12,13,14} &= (\bar\psi\psi)_1(\bar\psi\sigma^{\mu\nu}\psi)_2(\bar\psi\sigma_{\mu\nu}\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{15} &= (\bar\psi\gamma_5\psi)_1(\bar\psi\gamma^\mu\psi)_2(\bar\psi\gamma_\mu\gamma_5\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{16} &= (\bar\psi\sigma^{\mu\nu}\psi)_1(\bar\psi\gamma_\mu\psi)_2(\bar\psi\gamma_\nu\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{17} &= (\bar\psi\sigma^{\mu\nu}\psi)_1(\bar\psi\gamma_\mu\gamma_5\psi)_2(\bar\psi\gamma_\nu\gamma_5\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{18} &= (\bar\psi\sigma^{\mu\nu}\psi)_1(\bar\psi\sigma_{\mu\alpha}\psi)_2(\bar\psi\sigma^{\alpha}_{\nu}\psi)_3[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3] \\
o_{19,20,21} &= (\bar\psi\gamma_5\psi)_1(\bar\psi\sigma^{\mu\nu}\psi)_2(\bar\psi\sigma_{\mu\nu}\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3)] \\
o_{22,23,24,25} &= (\bar\psi\gamma_\mu\psi)_1(\bar\psi\gamma_\nu\gamma_5\psi)_2(\bar\psi\sigma^{\mu\nu}\gamma_5\psi)_3[\mathbf{1}, \boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3, \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2+\boldsymbol{\tau}_3), \boldsymbol{\tau}_1\cdot(\boldsymbol{\tau}_2-\boldsymbol{\tau}_3)]
\end{aligned}
$$

After deriving all sort of Fierz identities like

$$(\sigma^{\mu\alpha})[\sigma_\alpha{}^\nu] - \mu \leftrightarrow \nu = i(\sigma^{\mu\nu}][) - i(][\sigma^{\mu\nu}) + i(\sigma^{\mu\nu}\gamma_5][\gamma_5) - i(\gamma_5][\sigma^{\mu\nu}\gamma_5)$$

using the 3×25 linear relations we are left with 5 operators

$$o_1, \quad o_3, \quad o_6, \quad o_9, \quad o_{12}$$

$\Longrightarrow$ test the relativistic counting by including only 5 combinations of the 10 LECs

Testing the *relativistic counting*

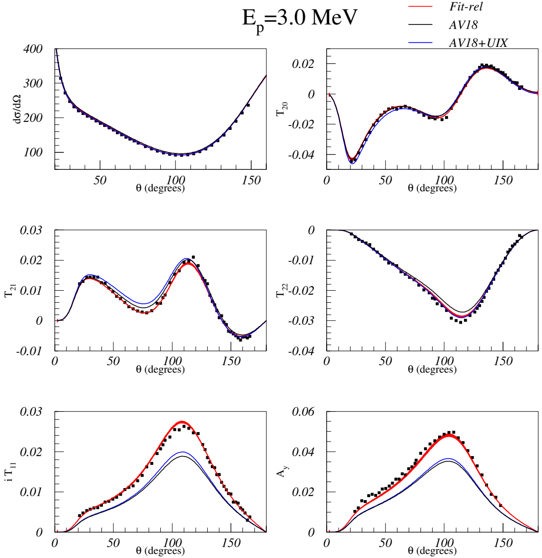
A LO contact 3NF depending on relativistic, derivativeless 6 fermion operators

✔ only 5 operators, like in NN relativistic contact operators

✔ reduced to 4 by the isospin projection

✔ "*natural*" explanation for the size of spin-orbit terms

$\Lambda = 200 - 500$ MeV



Summary and conclusions

- We have assessed, in a hybrid approach, the capability of the N4LO contact interaction to solve long-standing problems in low-energy $N - d$ scattering
- It would be much more desirable from the ChEFT perspective if the revised (parameter-free) N3LO $3N$ force achieved the same result
- Further studies are needed to test the derived interaction in an extended energy domain
- It will also be interesting to investigate its impact in the spectrum of medium-light nuclei.
- We have derived and tested two possible hierarchies among the subleading contact LECs, based on the large-N_c limit and on a recently proposed "relativistic power counting", that are reasonably respected.
- Work to embed the derived interaction in a consistent pionless potential is in progresss