

# Double beta decay NME from deformed QRPA with realistic forces

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# Outline

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- Formalism**
- Results**
- Errors**
- Conclusion and Outlook**

# Formalism

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- **Many-body Methods adopted for the calculations of NME**
  - **Closure without involvement of intermediate states**
    - **IBM, PHFB, DFT, CDFT,.....**
  - **Non-Closure with intermediated states**
    - **Shell Model**
    - **QRPA: realistic forces; Skyrme force;.....**

# Formalism

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**P. Ring et. al., Nuclear Many-Body Problem**

- QRPA is a method used to describe the small amplitude vibrations for open shell nuclei**
- pn-QRPA treats the states of intermediate odd-odd nuclei as iso-vector excitation of even-even ground states**
- Only 1 phonon excitation is considered, no multi-phonon excitations, no phonon-phonon interactions**
- QRPA is constructed on the quasi-particle grounds, particle number is not conserved**

# Formalism

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- Solve the Schroedinger eq. with mean field potential (W.S. Potential)
- Solve the BCS equations to get the quasiparticle, where the residual interactions (G-matrix) are obtained by solving the Bruckner eq. (Overall renormalized paremeters  $g_{\text{pair}}$  are needed)
- Solve the QRPA equations to get the wave functions of the intermediate states ( $g_{\text{ph}}$  and  $g_{\text{pp}}$ )

# Formalism

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- Introduction of deformed QRPA
  - Adiabatic approx. separate the intrinsic and rotation d.f.
  - Quasi-particle constructed on intrinsic frame
- Why deformation:
  - $^{150}\text{Nd}$  lies in the heavily deformed rare earth region
  - This nucleus has the largest phase space factor

# Formalism

Kotila and Iachello, PRC85,034316

Nucleus	$G_{0\nu}^{(0)}$ ( $10^{-15}$ yr $^{-1}$ )	$G_{0\nu}^{(1)}$ ( $10^{-15}$ yr $^{-1}$ )	$Q_{\beta\beta}$ (MeV)
$^{48}\text{Ca}$	24.81	-23.09	4.27226(404)
$^{76}\text{Ge}$	2.363	-1.954	2.03904(16)
$^{82}\text{Se}$	10.16	-9.074	2.99512(201)
$^{96}\text{Zr}$	20.58	-18.67	3.35037(289)
$^{100}\text{Mo}$	15.92	14.25	3.03440(17)
$^{110}\text{Pd}$	4.815	-4.017	2.01785(64)
$^{116}\text{Cd}$	16.70	-14.83	2.81350(13)
$^{124}\text{Sn}$	9.040	-7.765	2.28697(153)
$^{128}\text{Te}$	0.5878	-0.3910	0.86587(131)
$^{130}\text{Te}$	14.22	-12.45	2.52697(23)
$^{136}\text{Xe}$	14.58	12.73	2.45783(37)
$^{148}\text{Nd}$	10.10	-8.506	1.92875(192)
$^{150}\text{Nd}$	63.03	-57.76	3.37138(20)
$^{154}\text{Sm}$	3.015	-2.295	1.21503(125)
$^{160}\text{Gd}$	9.559	-7.932	1.72969(126)
$^{198}\text{Pt}$	7.556	-5.868	1.04717(311)
$^{232}\text{Th}$	13.93	-10.95	0.84215(246)
$^{238}\text{U}$	33.61	28.13	1.14498(125)

- Recent results on phase space factor

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- Recent results on phase space factor

# Formalism

- Nuclear matrix elements for  $2\nu\beta\beta$  under intrinsic frame

$$M_{\text{GT}}^{2\nu} = \sum_{K=0,\pm 1} \sum_{m_i m_f} \frac{\langle 0_f^+ | \bar{\beta}_K^- | K^+, m_f \rangle \langle K^+, m_f | K^+, m_i \rangle \langle K^+, m_i | \beta_K^- | 0_i^+ \rangle}{\bar{\omega}_{K, m_i m_f}}$$

- NME for  $0\nu\beta\beta$

$$M^{0\nu}(K^\pi) = \sum_{m_i, m_f} \langle 0_f^+ | c_p^\dagger c_n | K^\pi m_f \rangle \langle K^\pi m_f | K^\pi m_i \rangle \langle K^\pi m_i | c_{p'}^\dagger c_{n'} | 0_i^+ \rangle$$

$$\times \sum_J \sum_{\substack{\eta_p \eta_{p'} \\ \eta_n \eta_{n'}}} F_{p\eta_p n\eta_n}^{JK} F_{p'\eta_{p'} n'\eta_{n'}}^{JK} \sum_{\mathcal{J}} (-1)^{j_n + j_{p'} + J + \mathcal{J}} \hat{\mathcal{J}} \left\{ \begin{matrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{matrix} \right\} \langle p(1), p'(2); \mathcal{J} \| \mathcal{O}_\ell(1, 2) \| n(1), n'(2); \mathcal{J} \rangle$$

- Overlaps :

$$\langle K^\pi m_f | K^\pi m_i \rangle = \sum_{l_i l_f} [X_{l_f K^\pi}^{m_f} X_{l_i K^\pi}^{m_i} - Y_{l_f K^\pi}^{m_f} Y_{l_i K^\pi}^{m_i}] \mathcal{R}_{l_f l_i} \langle \text{BCS}_f | \text{BCS}_i \rangle$$

# Formalism

F. Simkovic et. al., PRC60,055502(1999)

□ Induced weak hadron Current

$$J^\mu(\vec{x}) = \sum_{n=1}^A \tau_n^+ [g^{\mu 0} J^0(\vec{q}^2) + g^{\mu k} J_n^k(\vec{q}^2)] \delta(\vec{x} - \vec{r}_n)$$

□ With

$$J^0(\vec{q}^2) = g_V(q^2), \quad \vec{J}_n(\vec{q}^2) = g_M(\vec{q}^2) i \frac{\vec{\sigma}_n \times \vec{q}}{2m_p} + g_A(\vec{q}^2) \vec{\sigma} - g_P(\vec{q}^2) \frac{q \vec{\sigma}_n \cdot \vec{q}}{2m_p}$$

□ Therefore

$$M_{\text{type}}^I = \langle H_{\text{type-F}}^I(r_{12}) + H_{\text{type-GT}}^I(r_{12}) \sigma_{12} + H_{\text{type-T}}^I(r_{12}) S_{12} \rangle$$

□ Where

$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{\mathbf{r}}_{12})(\vec{\sigma}_2 \cdot \hat{\mathbf{r}}_{12}) - \sigma_{12}, \quad \sigma_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

□ And

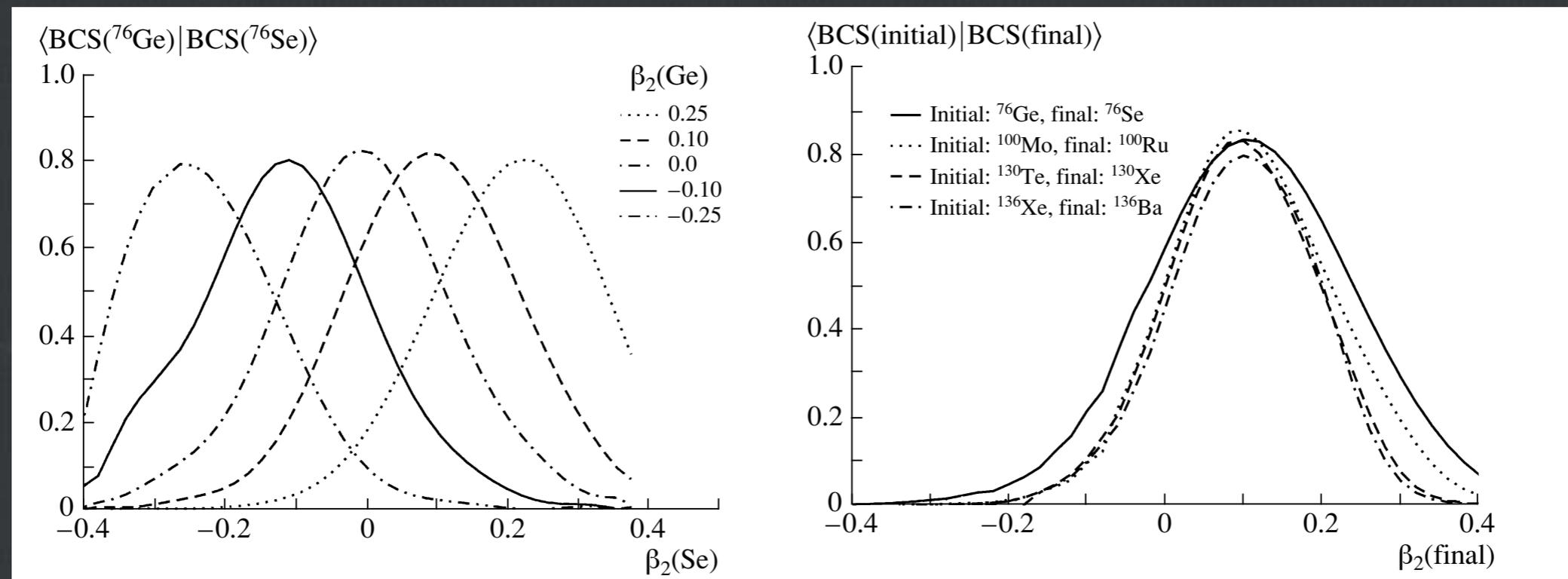
$$H_{\text{type-K}}^{\text{light}}(r_{12}) = \frac{2}{\pi g_A^2} \frac{R}{r_{12}} \int_0^\infty \frac{\sin(qr_{12})}{q + E_J^m - (E_{\text{g.s.}}^i + E_{\text{g.s.}}^f)/2} h_{\text{type-K}}(q^2) dq$$

□

$$H_{\text{type-K}}^{\text{heavy}}(r_{12}) = \frac{1}{m_p m_e} \frac{2}{\pi g_A^2} \frac{R}{r_{12}} \int_0^\infty \sin(qr_{12}) h_{\text{type-K}}(q^2) q dq$$

# Results

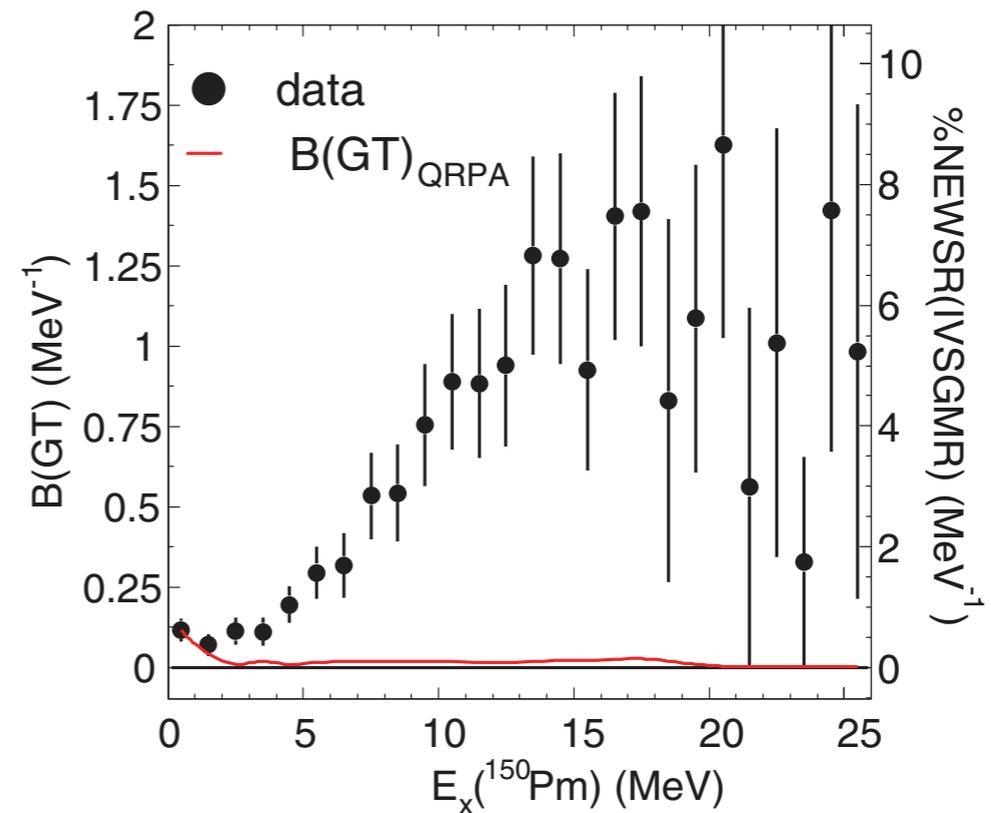
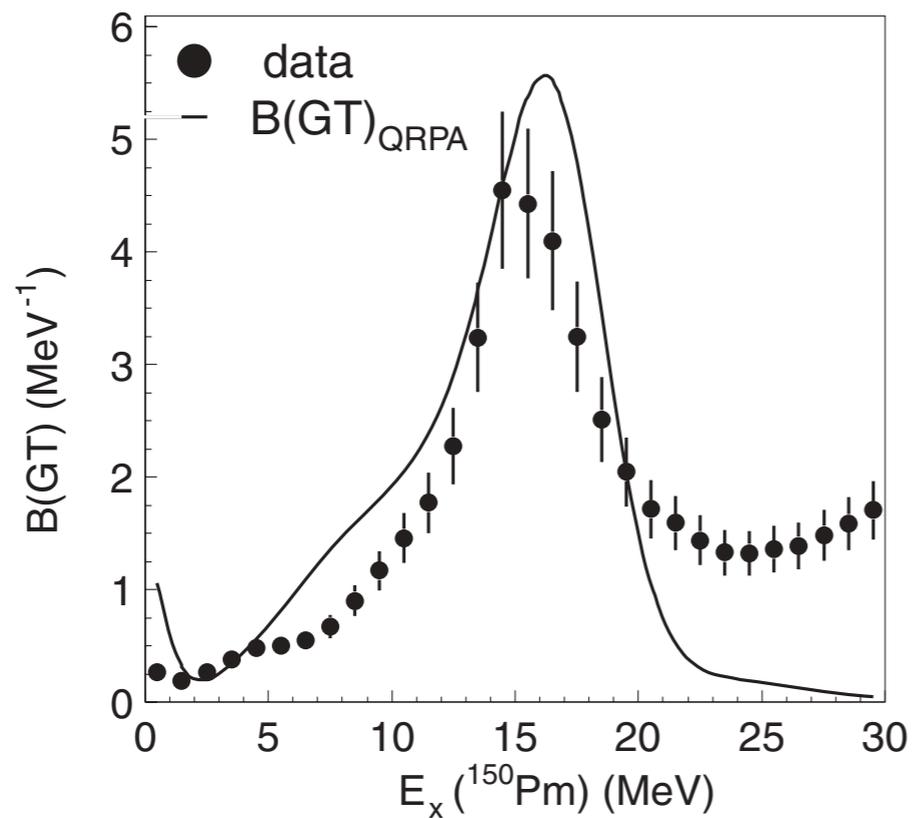
L. Pacerescu et al. Phys. Atom Nucl. 67,1210(2004)



□ BCS overlaps

# Results

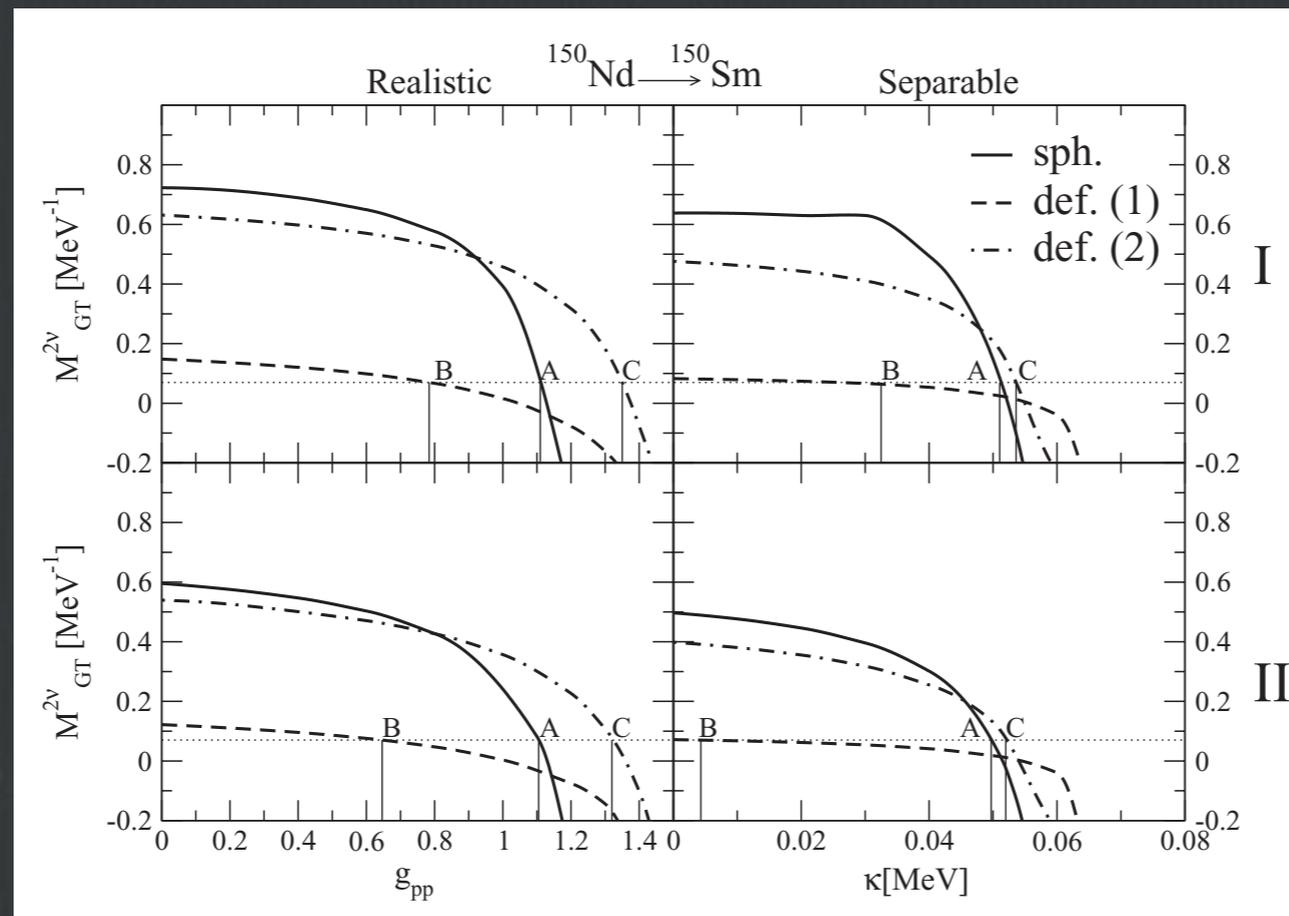
C. J. Guess et al. PRC83,064318(2011)



□ Validation of the theory

# Results

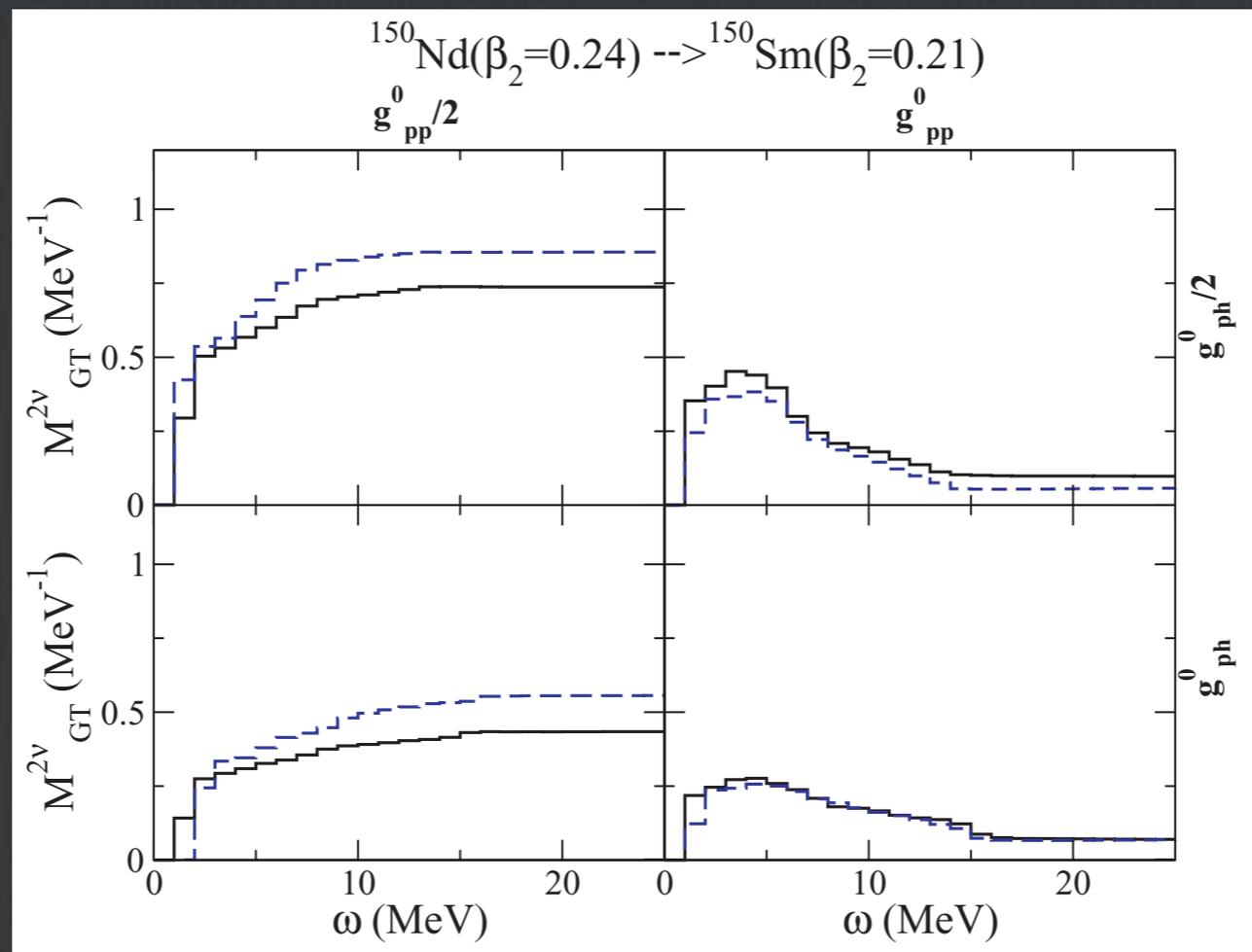
M.S.Yousef et. al. PRC79,014314(2009)



□ Dependence of NME for  $2\nu\beta\beta$  on residual interactions

# Results

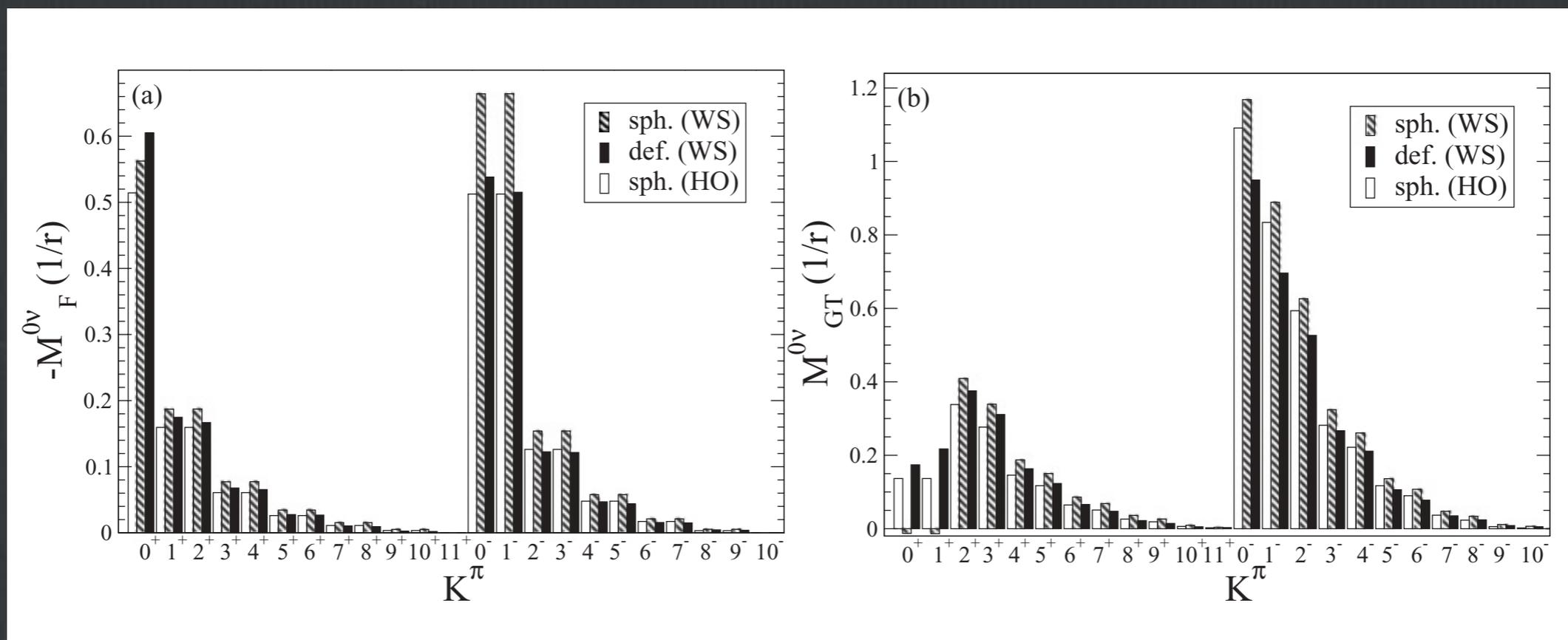
DLF et al. PRC81,037303(2010)



□ Lowlying states dominance

# Results

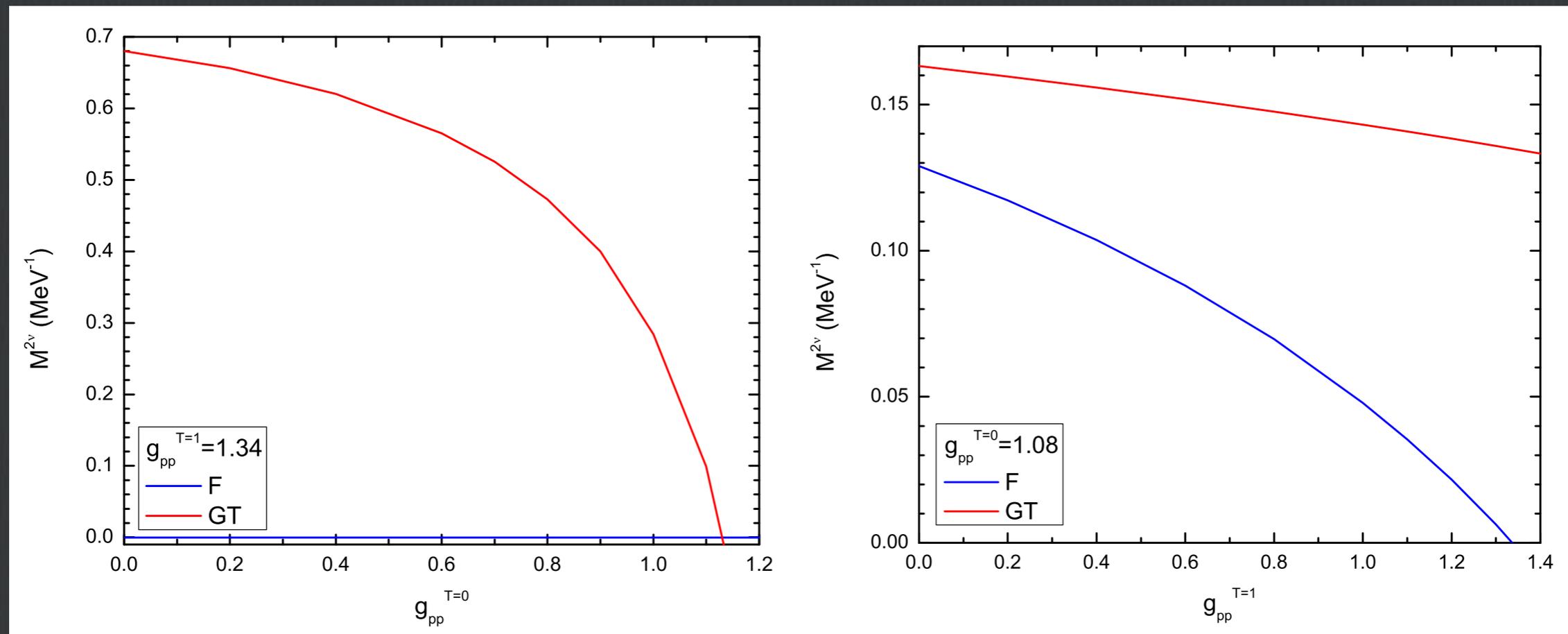
DLF et al. PRC83,034320(2011)



□ Comparison of results from different wave functions

# Results

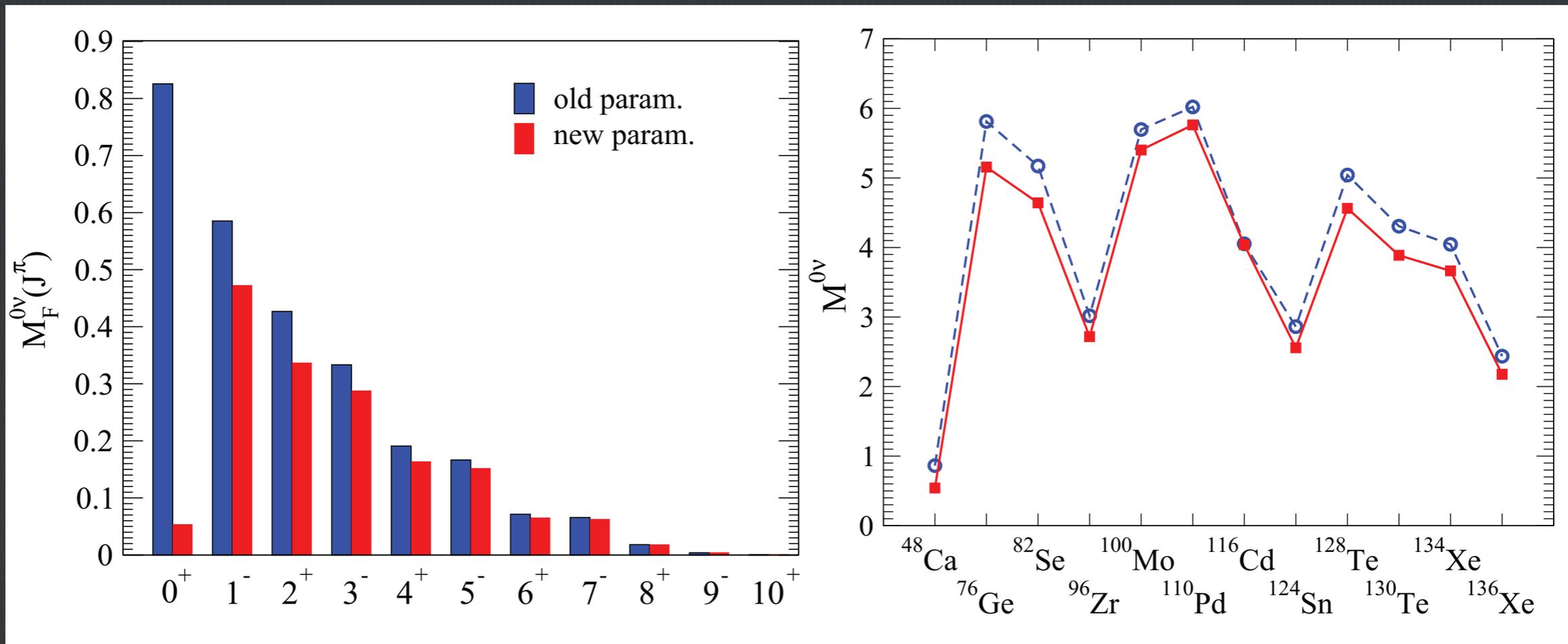
V. Rodin and A. Faessler PRC84,014322(2011)



□ Restoration of isospin symmetry  $M_F^{2v}=0$

# Results

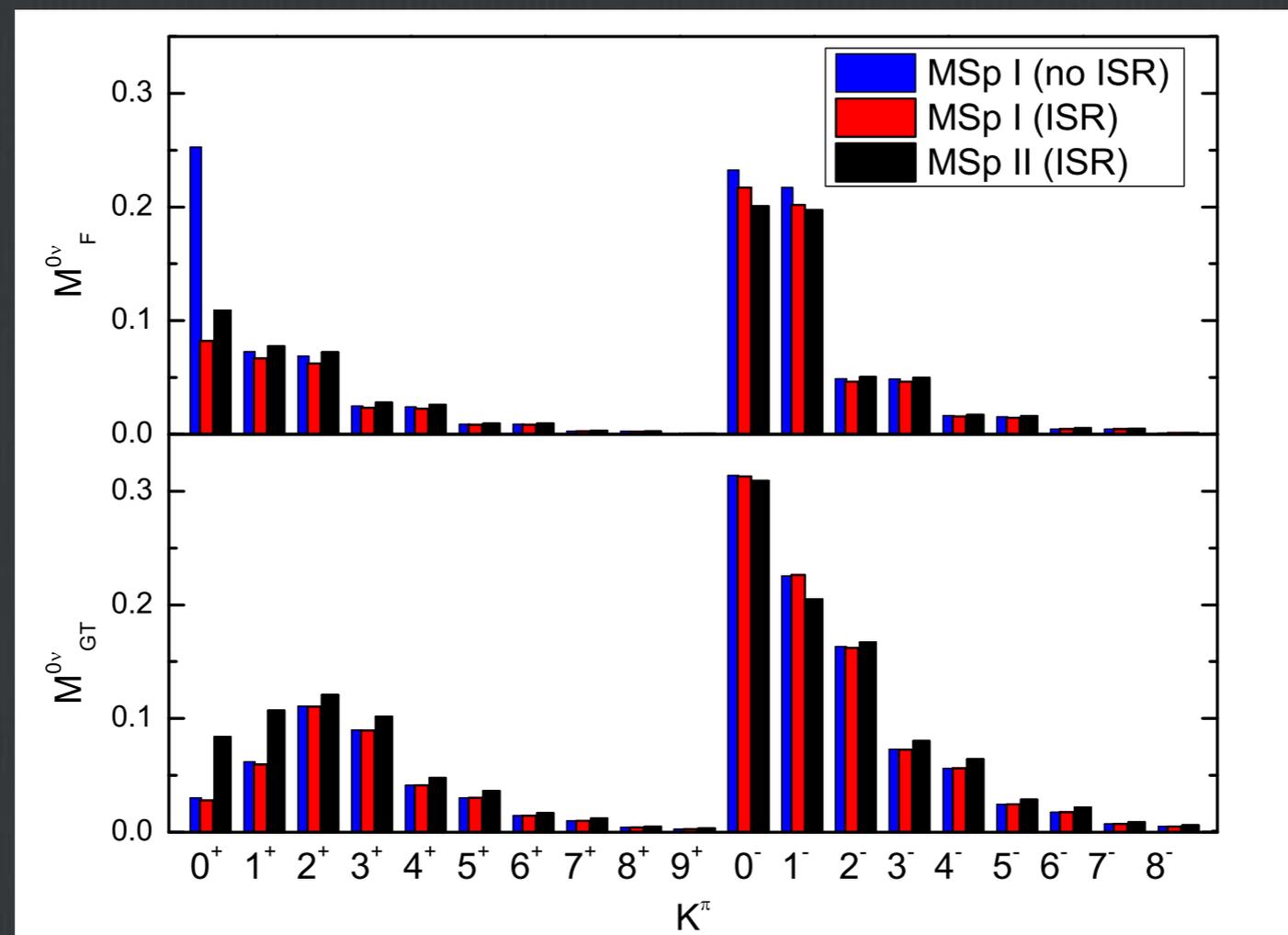
F. Simkovic et al. PRC87,045501(2013)



□ Impact of Isospin restoration on  $0\nu\beta\beta$

# Results

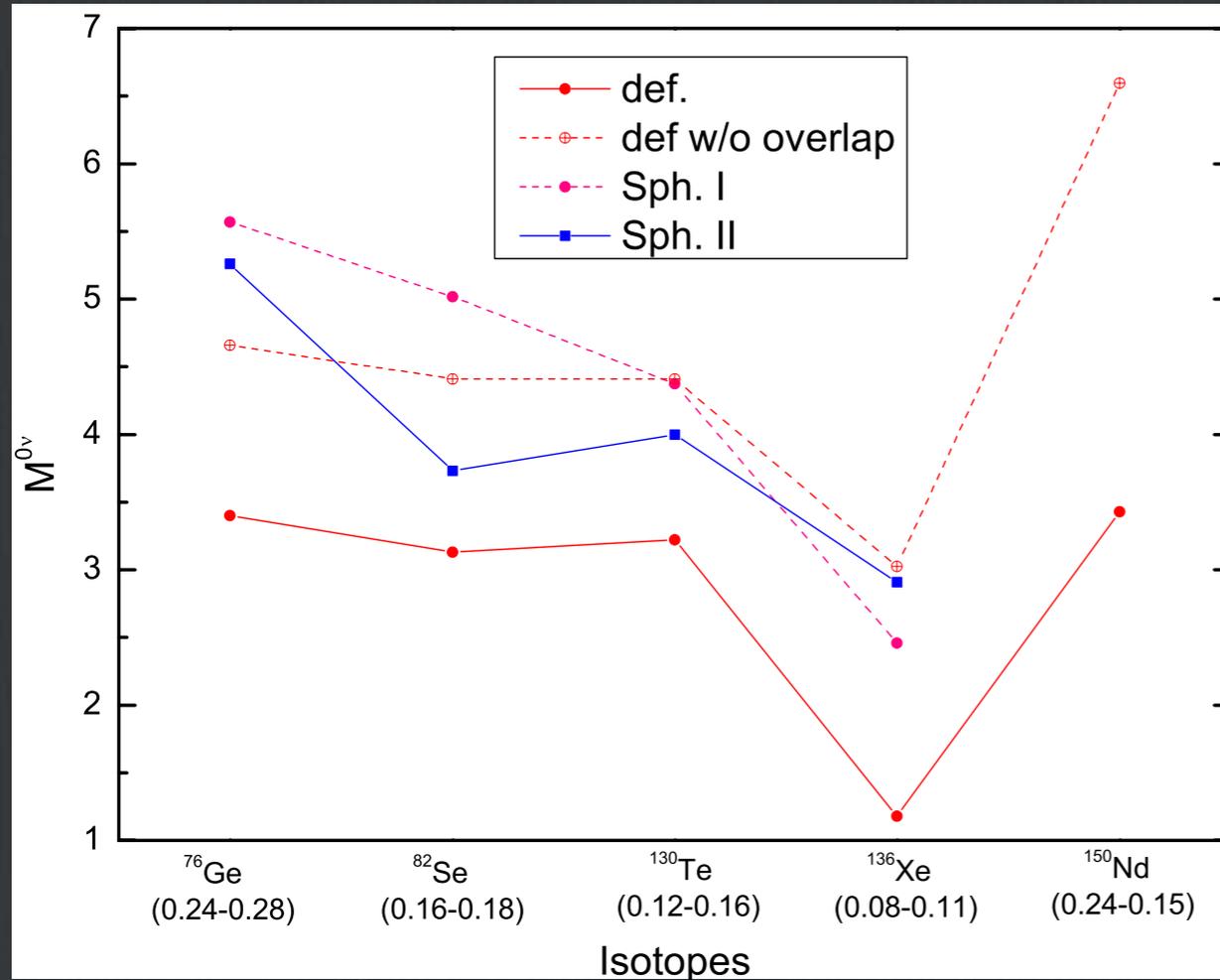
DLF et al. PRC92,044301 (2015)



□  $0\nu\beta\beta$  matrix elements with isospin symmetry restoration

# Results

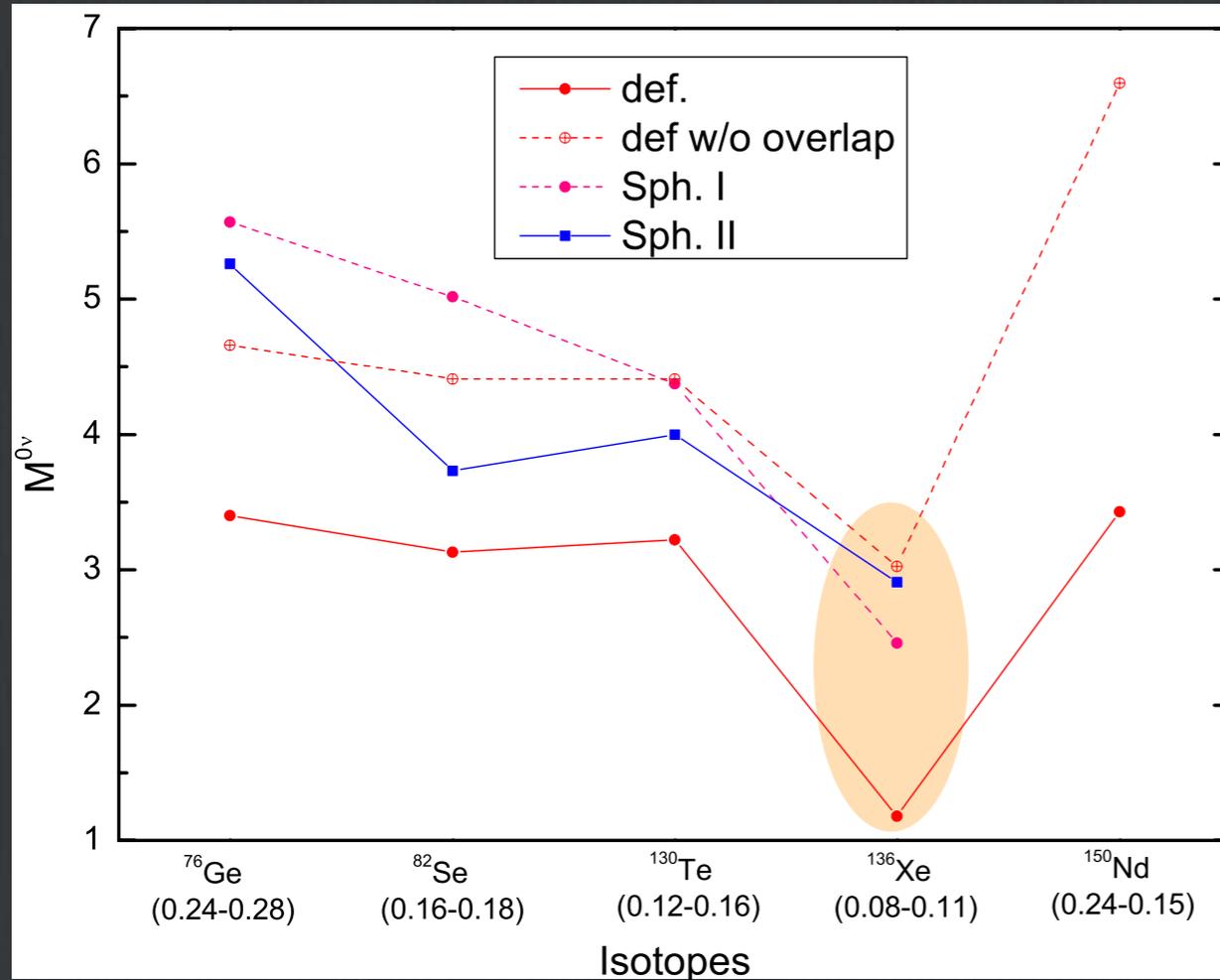
DLF et al. PRC97,045503(2018)



□ NME of double beta decay and role of deformation and overlap factors (is  $^{136}\text{Xe}$  reasonable?)

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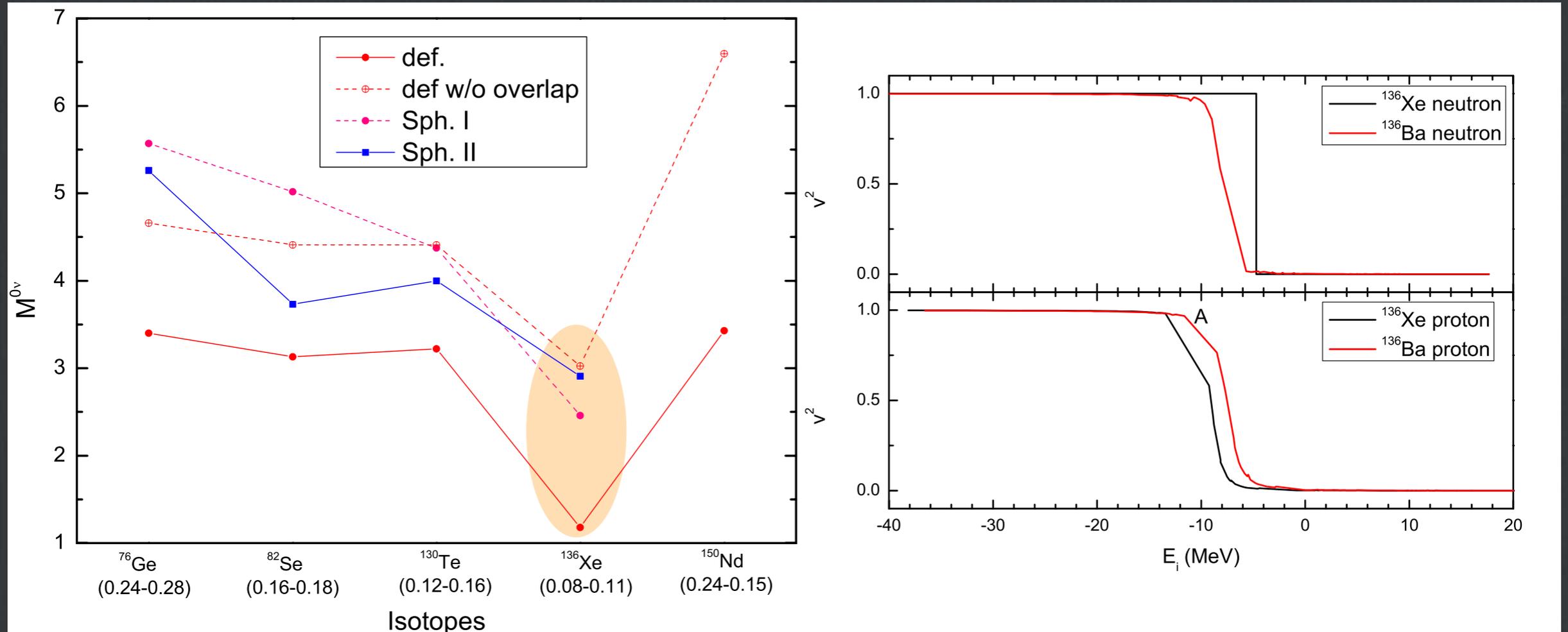
DLF et al. PRC97,045503(2018)



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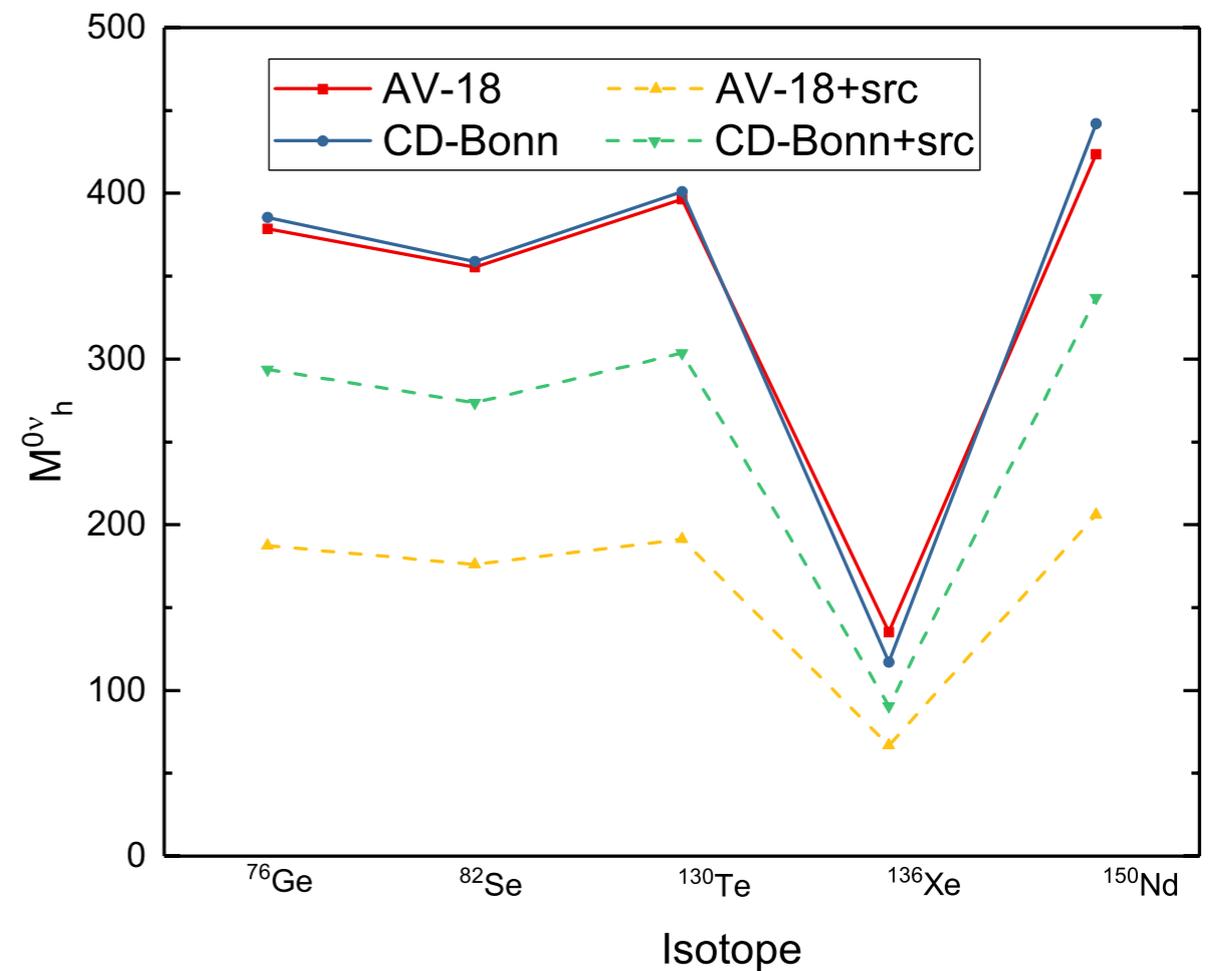
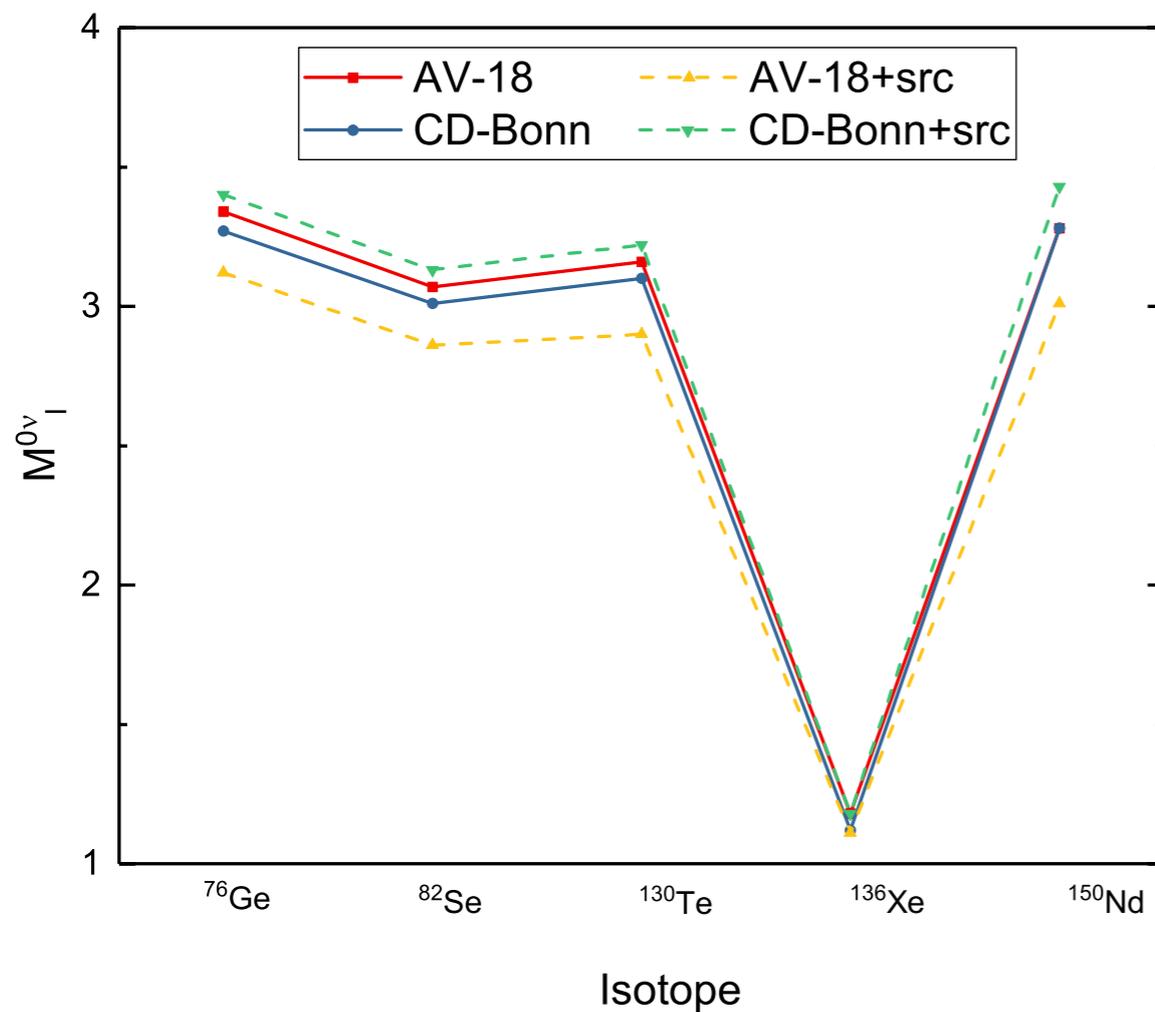
DLF et al. PRC97,045503(2018)



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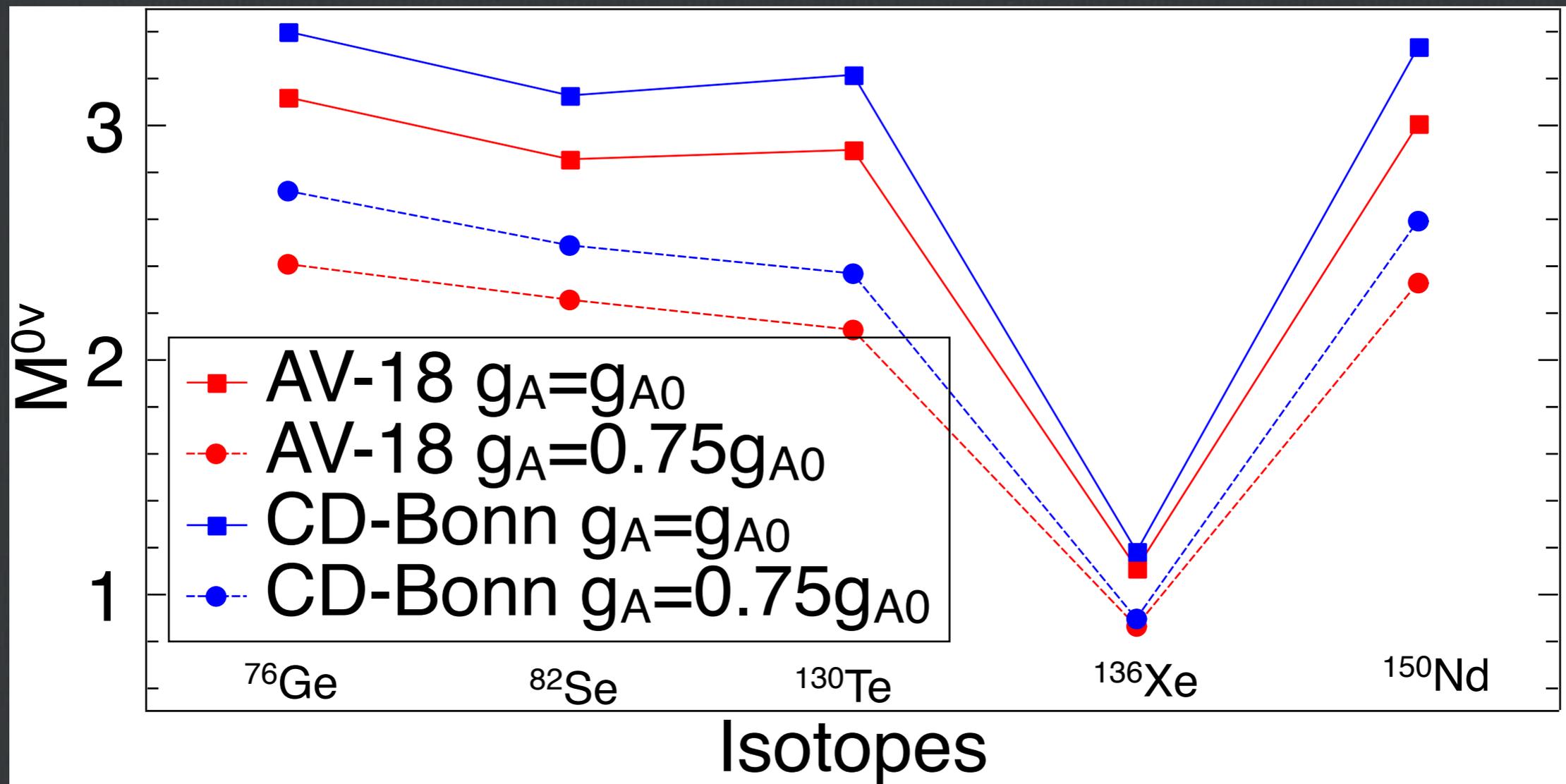
DLF et al. PRC97,045503(2018)



□ Impact from Short-Range Correlation

# Results

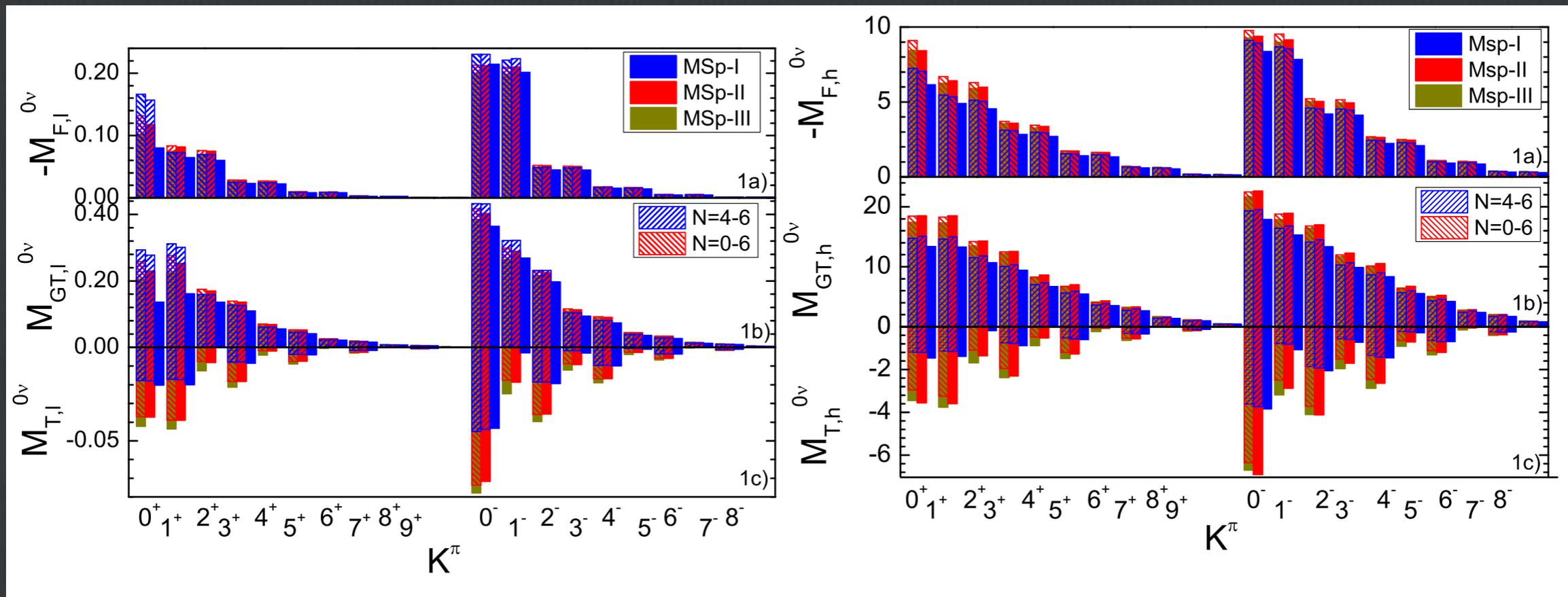
DLF et al. PRC97,045503(2018)



□ The quenching of  $g_A$

# Results

DLF et al. PRC97,045503(2018)

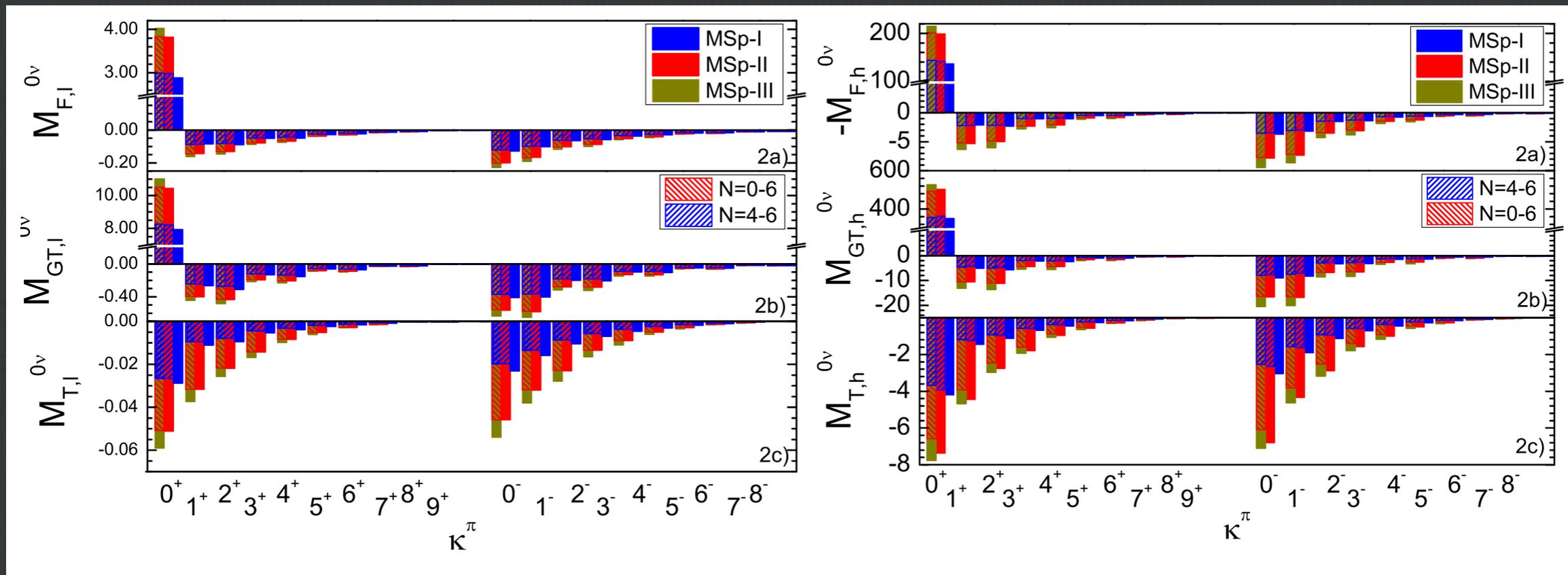


□ Contribution from different intermediate states

□ Different dependence on model space

# Results

DLF et al. PRC97,045503(2018)

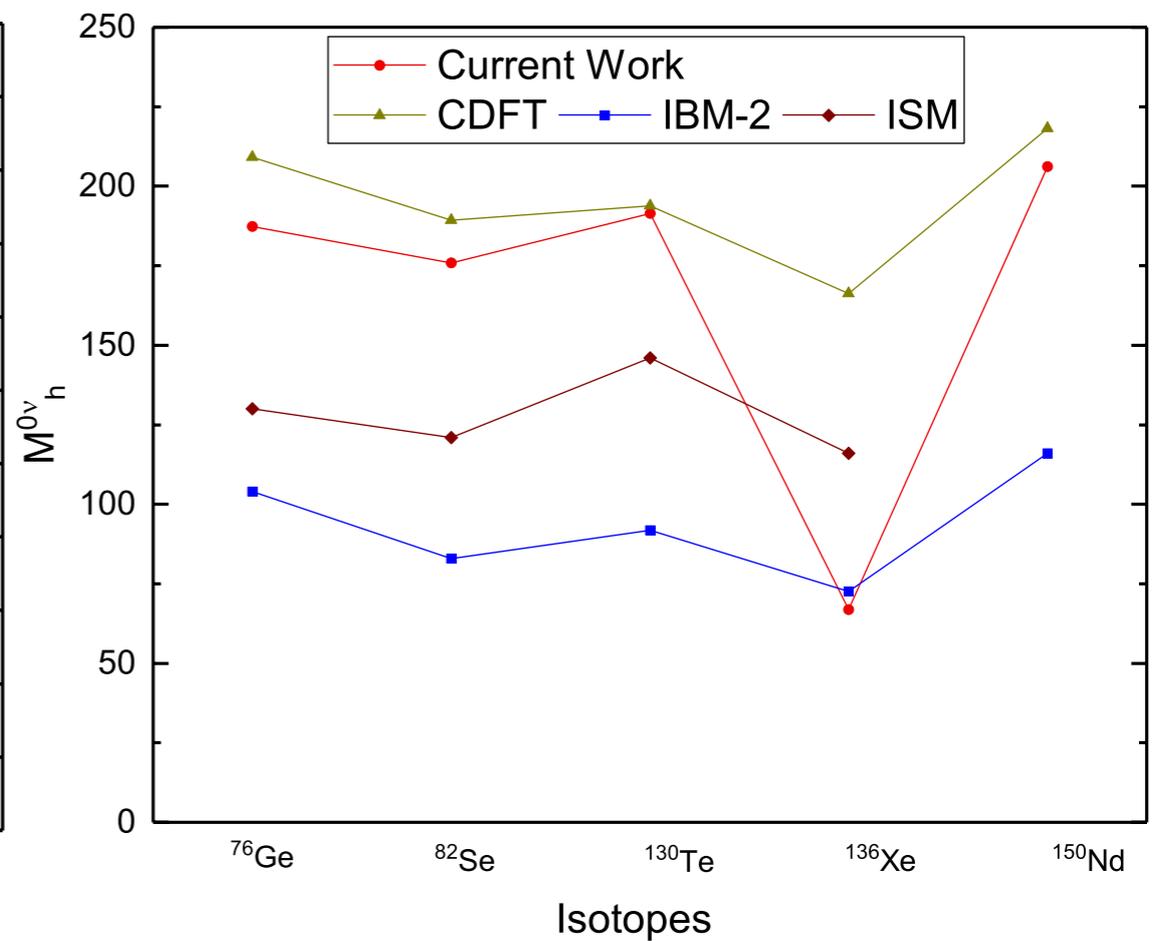
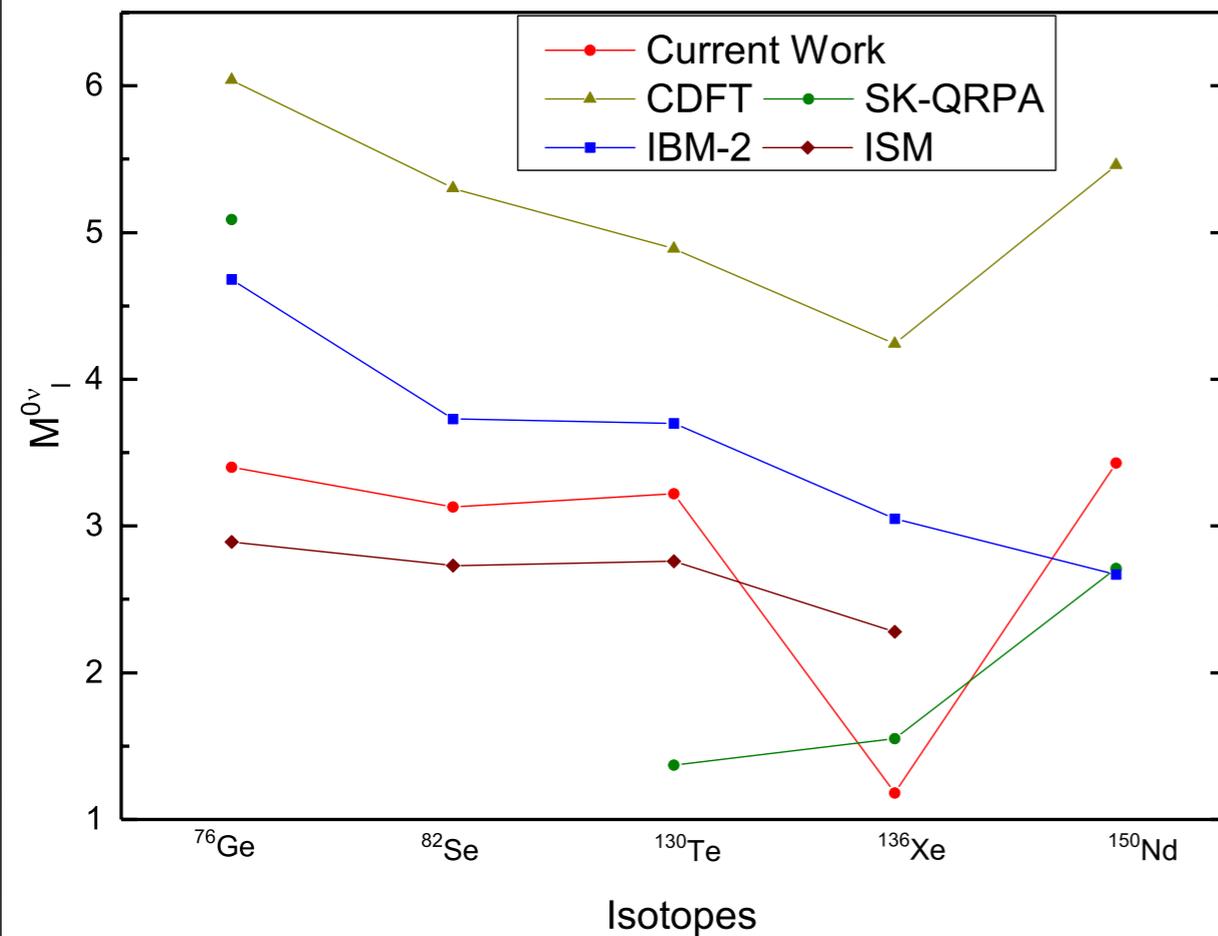


□ Contribution from different nucleon pairs

□  $0^+$  dominance

# Results

DLF et al. PRC97,045503(2018)



□ Results from different models

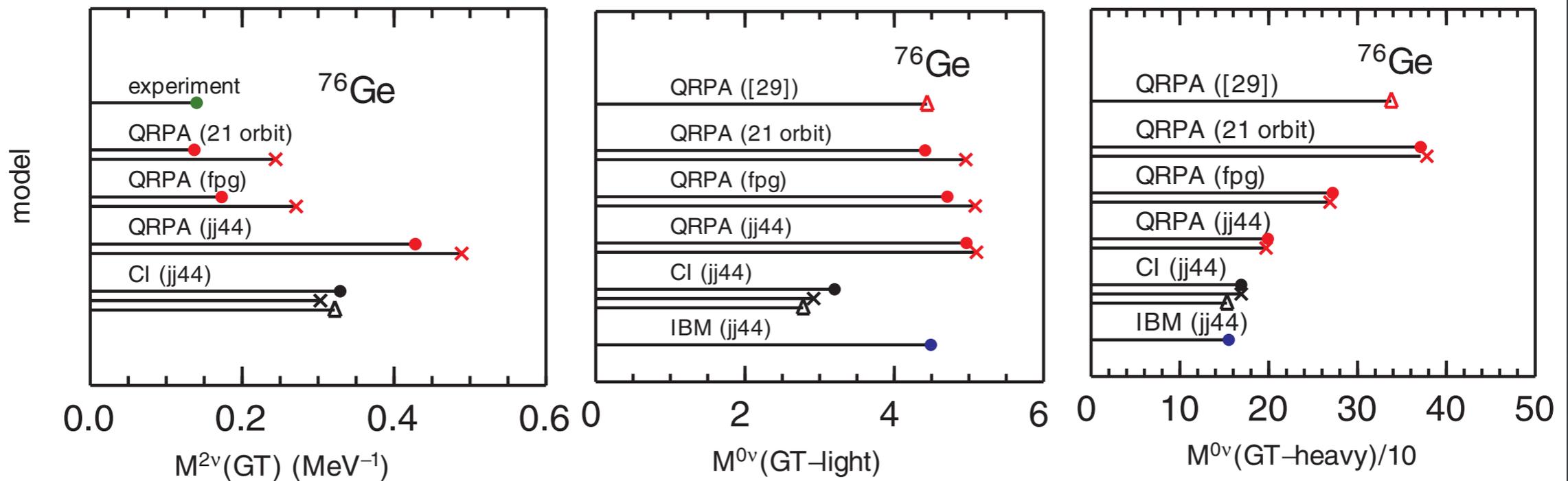
# Error Analysis

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- To compare different many-body approaches, we need some assumptions:
  - Different approaches have something in common
  - The some approximation plays similar role
    - For example: inclusion of pp force will reduce the  $2\nu\beta\beta$  NME
- Narrow the deviations brought by the method itself

# Error Analysis

B.A. Brown et. al. PRC92,041301(2015)



$$M^{2\nu} = 0.140(5) = [0.31(3)][0.45][1][1] \quad M^{0\nu} = [3.0(3)][1.2(2)][0.97(3)][1.12(7)] = 3.9(8)$$

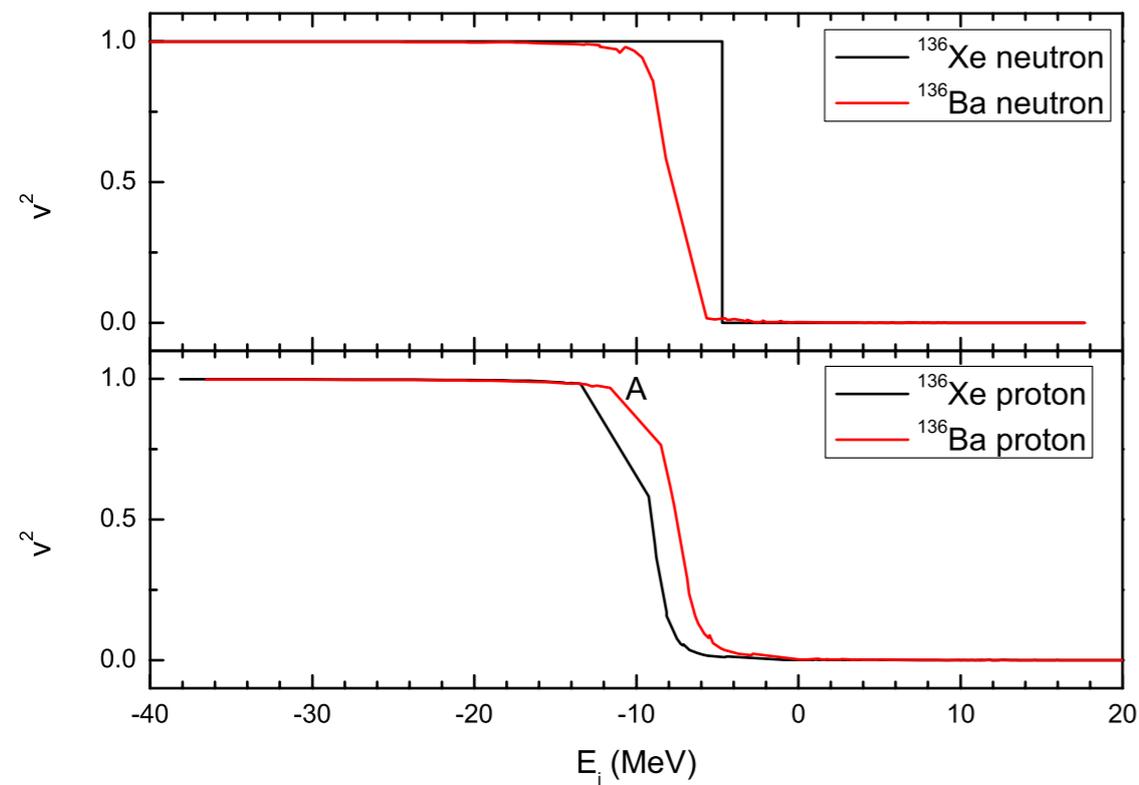
$$M = [M_{\text{GT}}(\text{CI})][R_V][R_S][R_{\text{GT}}]$$

$$M^{0\nu} = [155(10)][1.65(25)][0.80(20)][1.13(13)] = 232(80)$$

□ The roles of model space

# Error Analysis

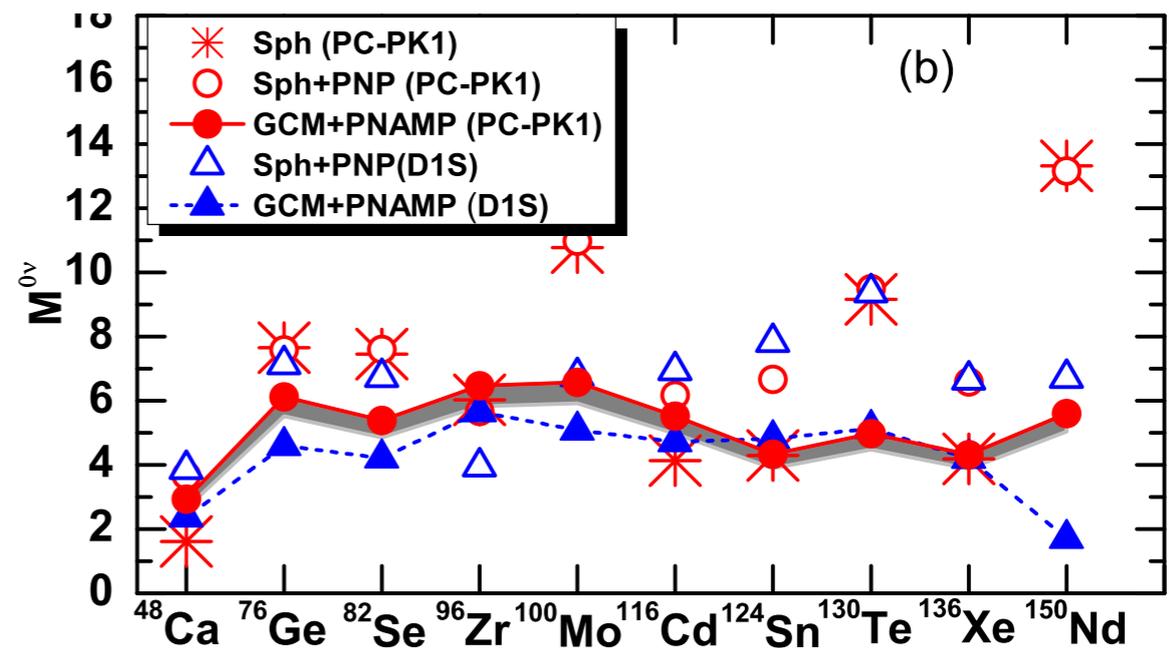
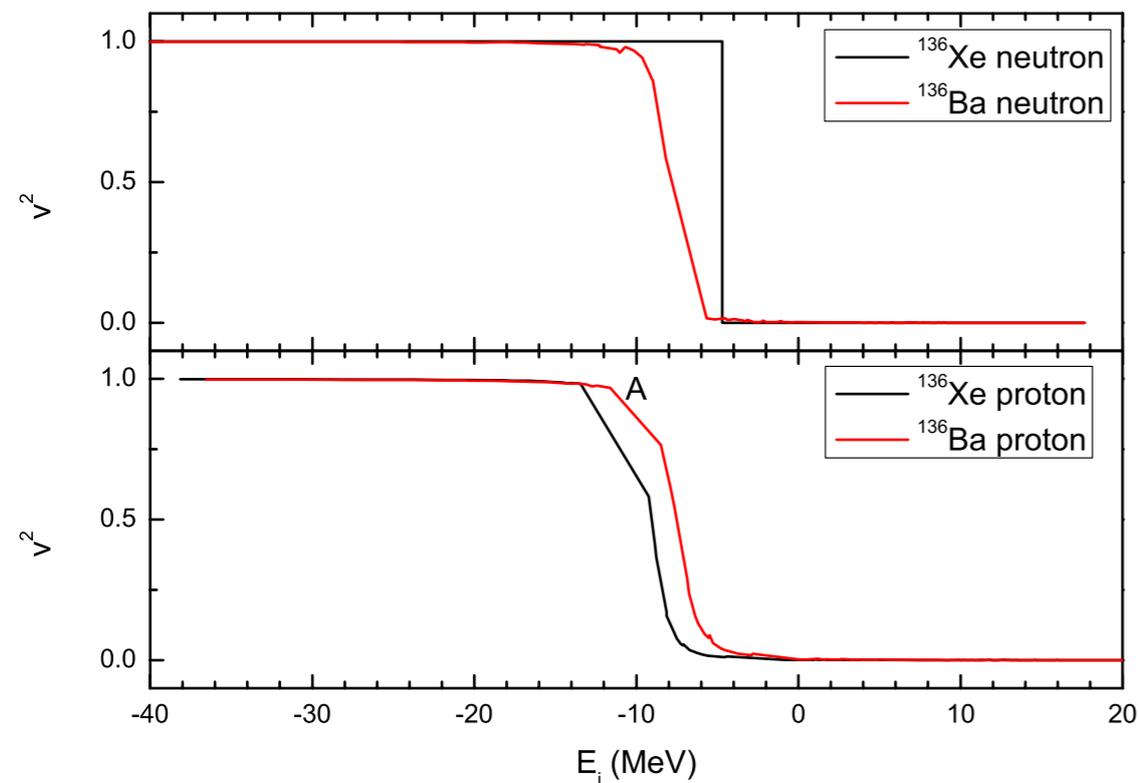
J. M. Yao et. al. PRC91,024306(2015)



Is small values of NME for  $^{136}\text{Xe}$  reasonable?

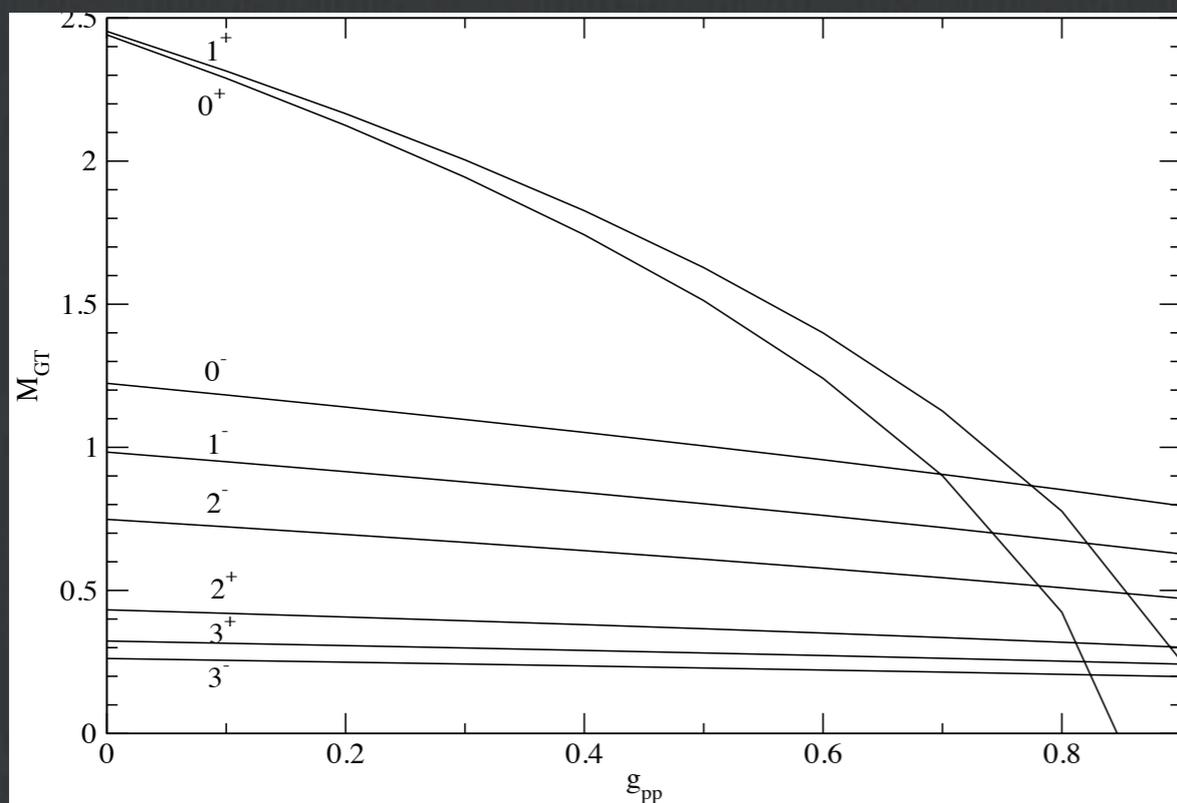
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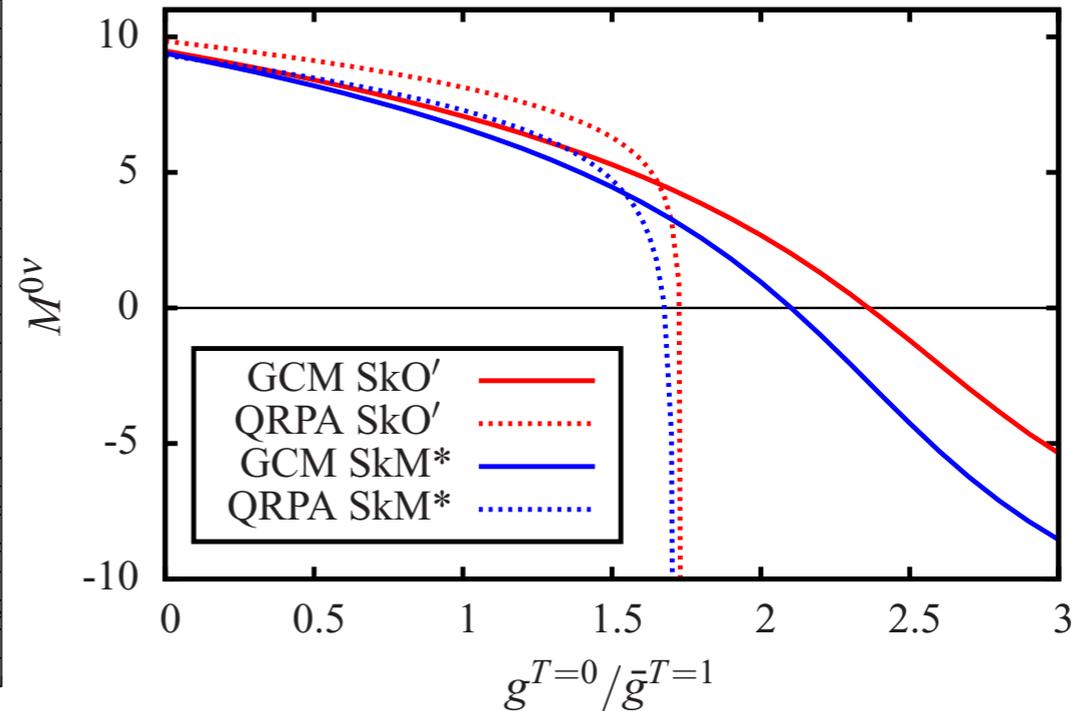
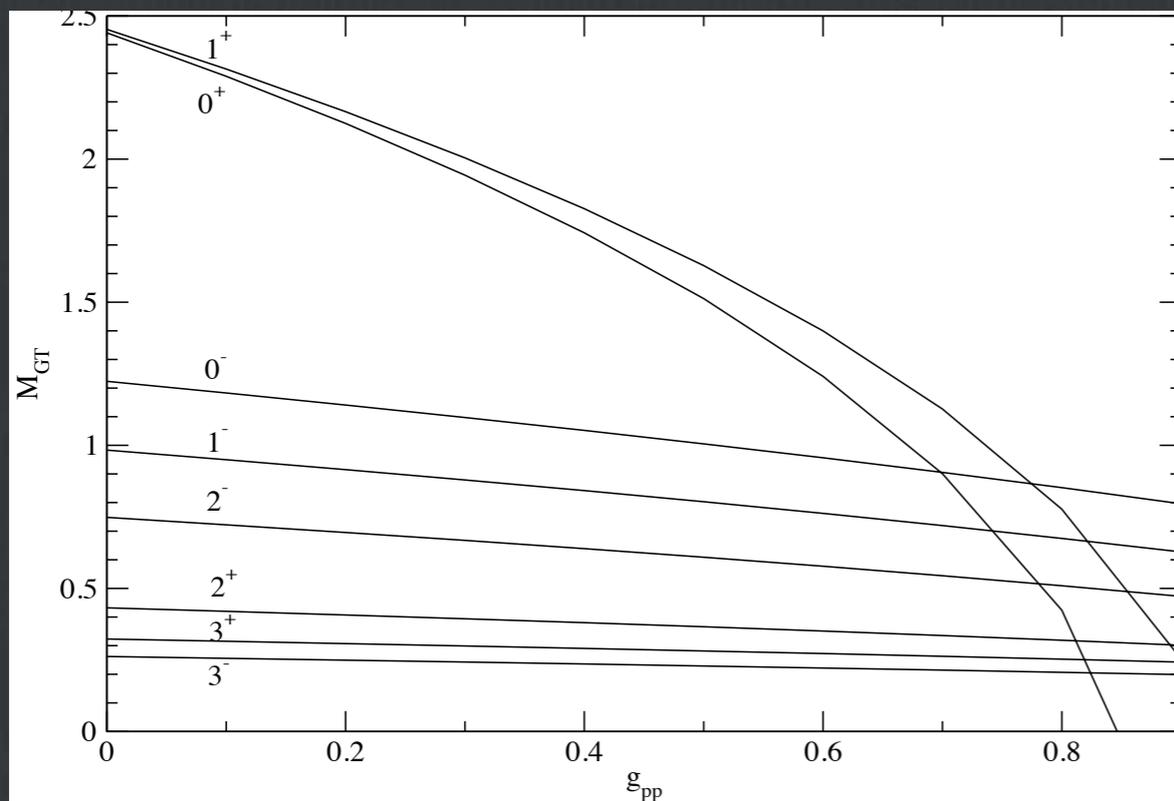
# Error Analysis



- Proton-neutron particle-particle interactions(or protons-neutron pairing) play important role
- Is this the reason for deviations?

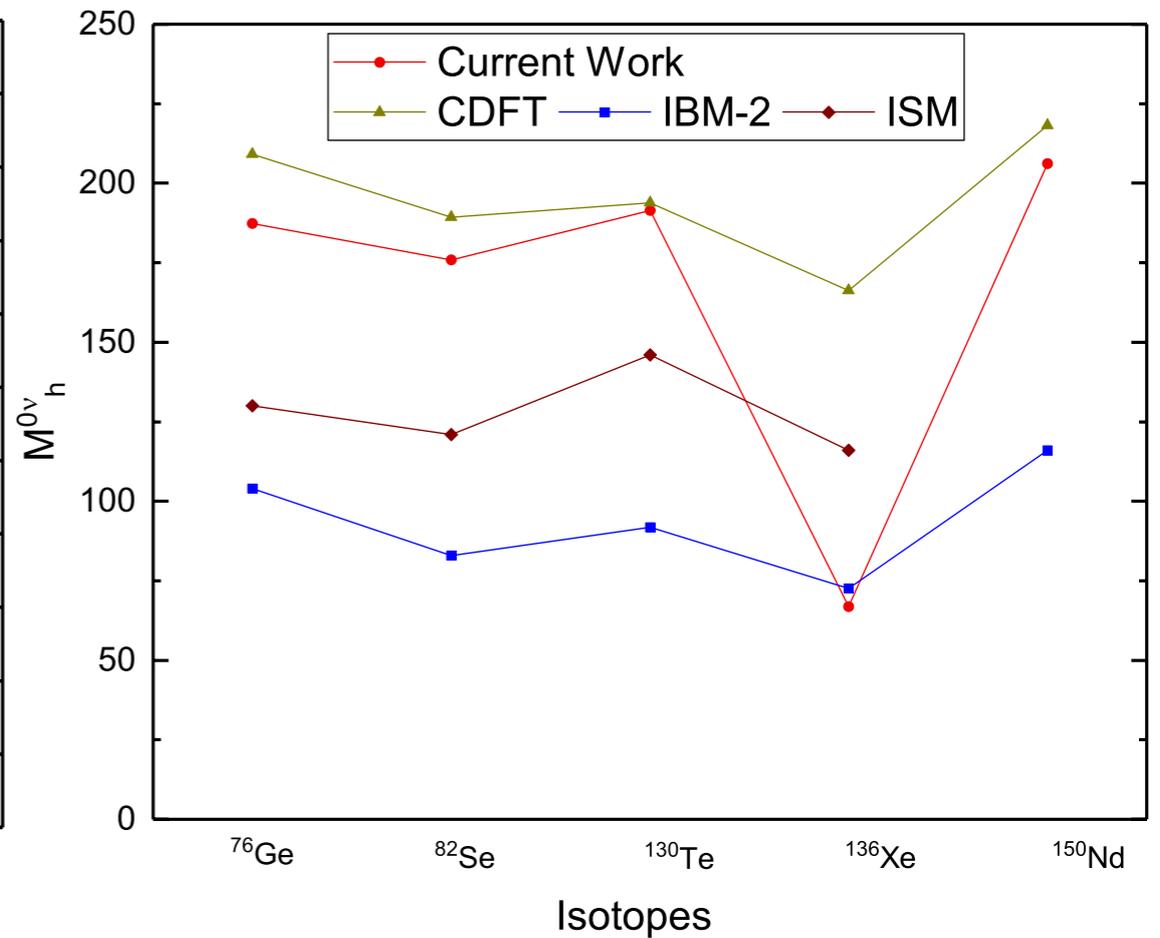
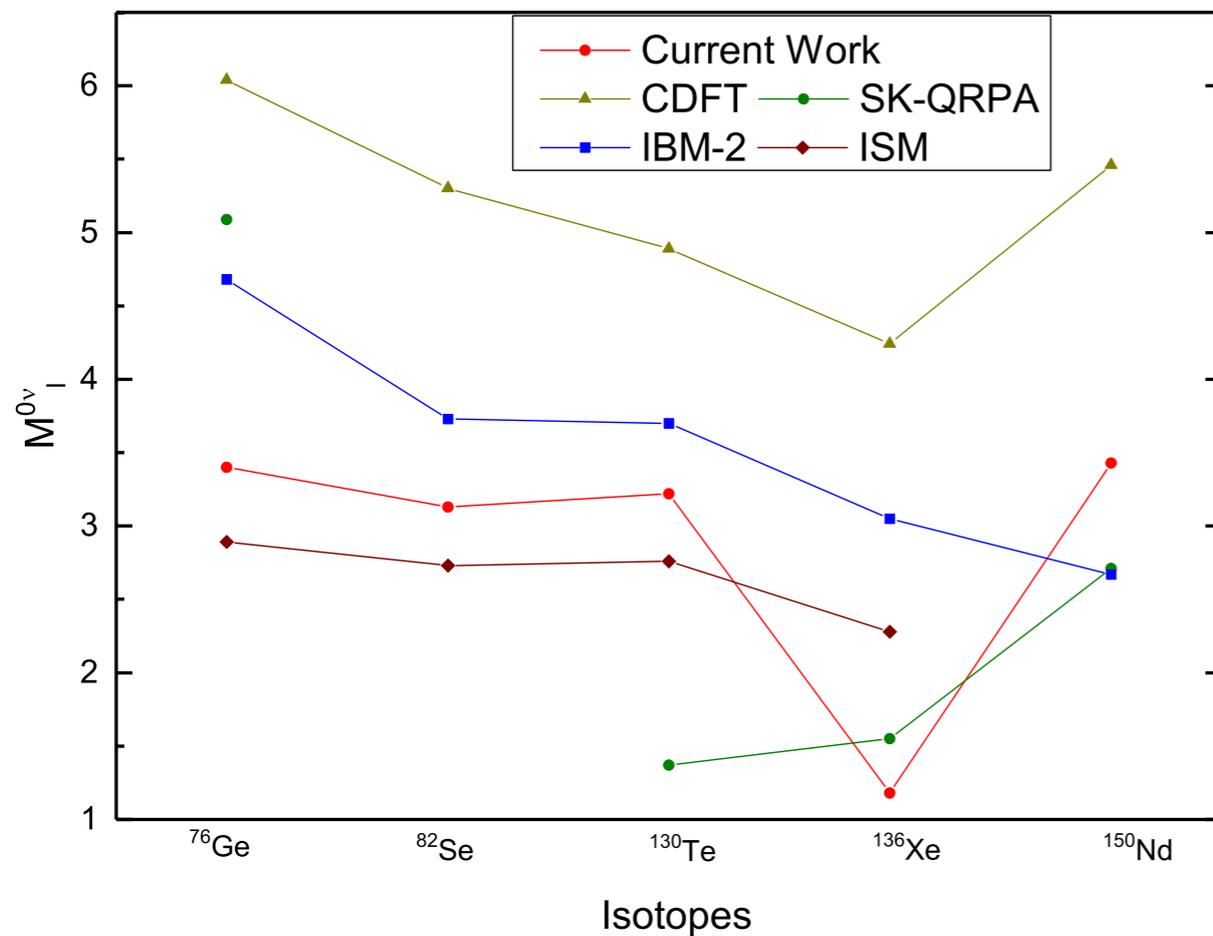
# Error Analysis

N. Hinohara et. al. PRC90,031301(2014)



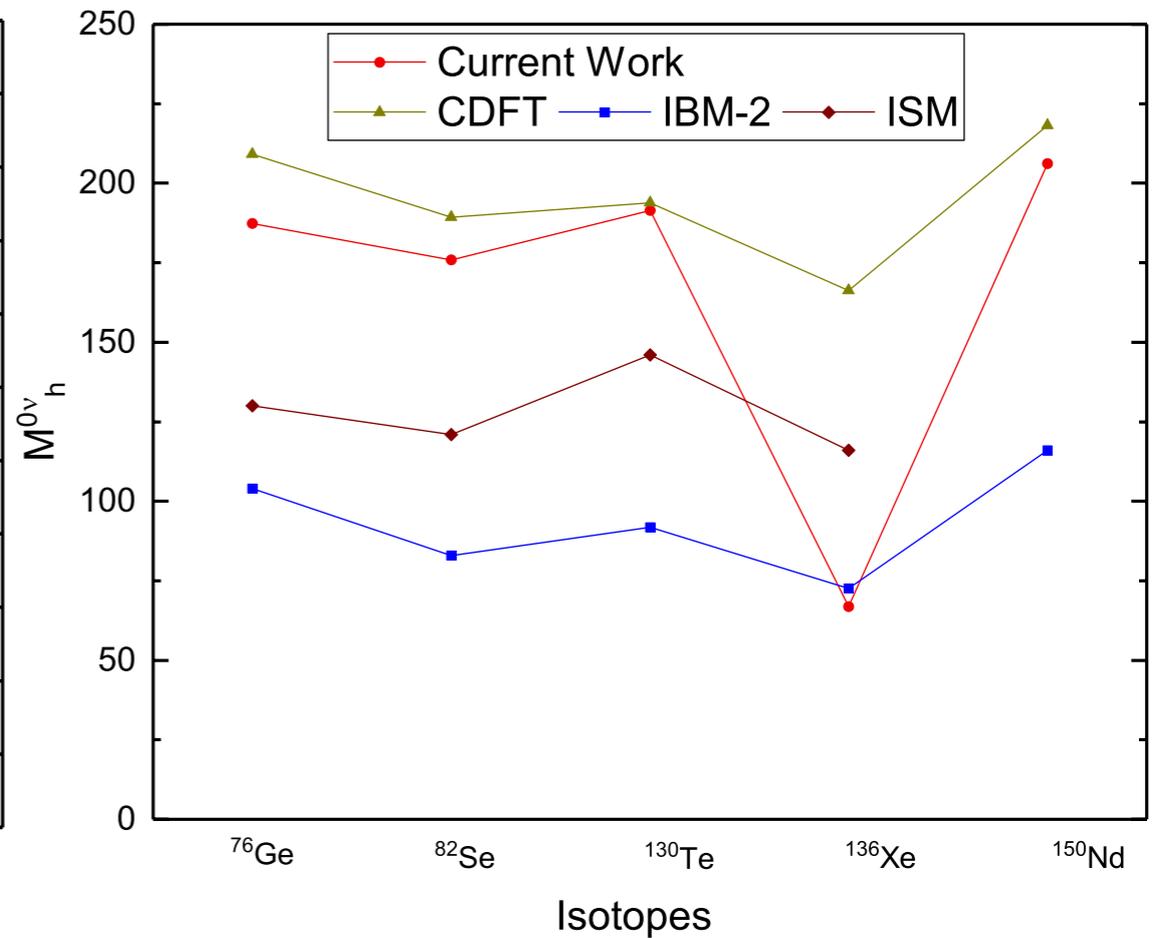
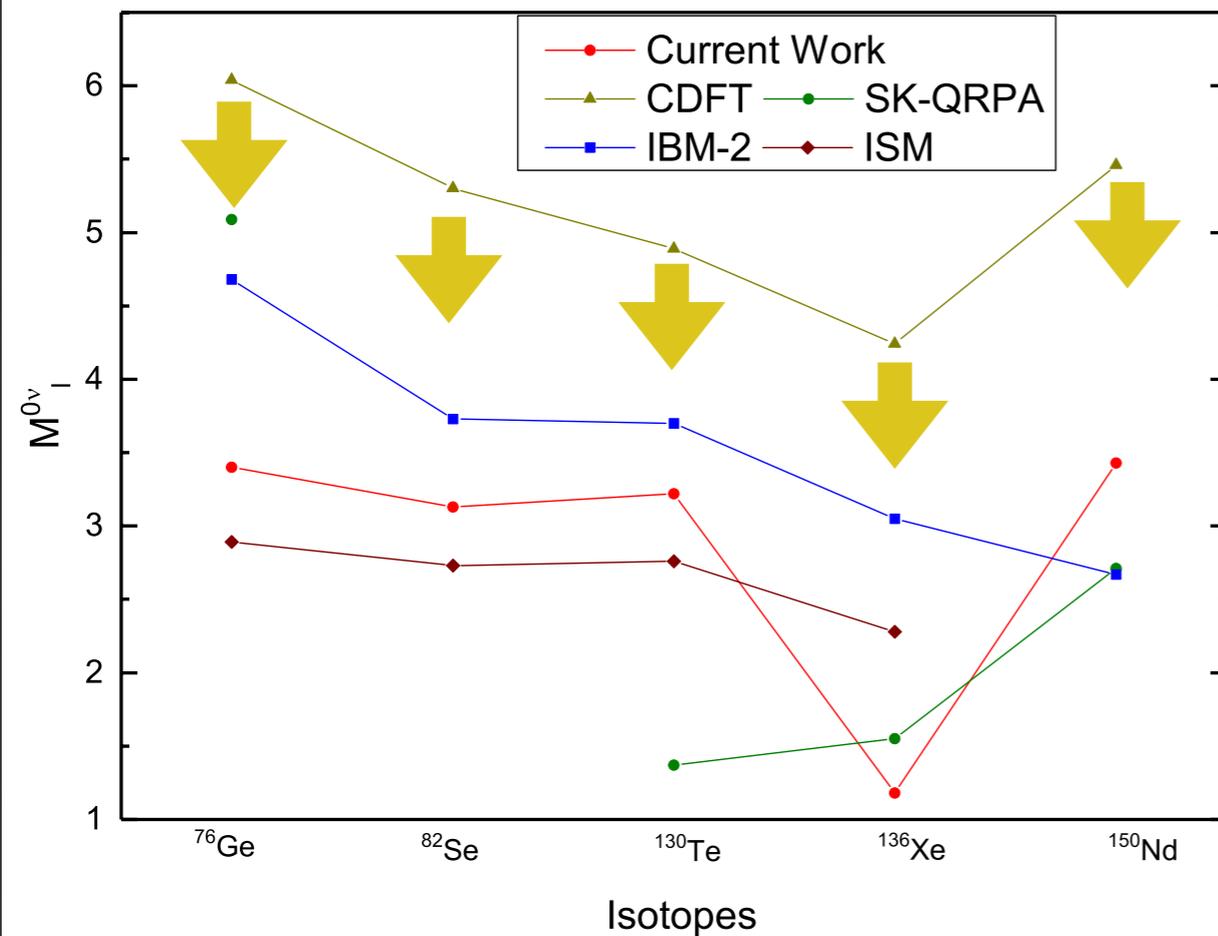
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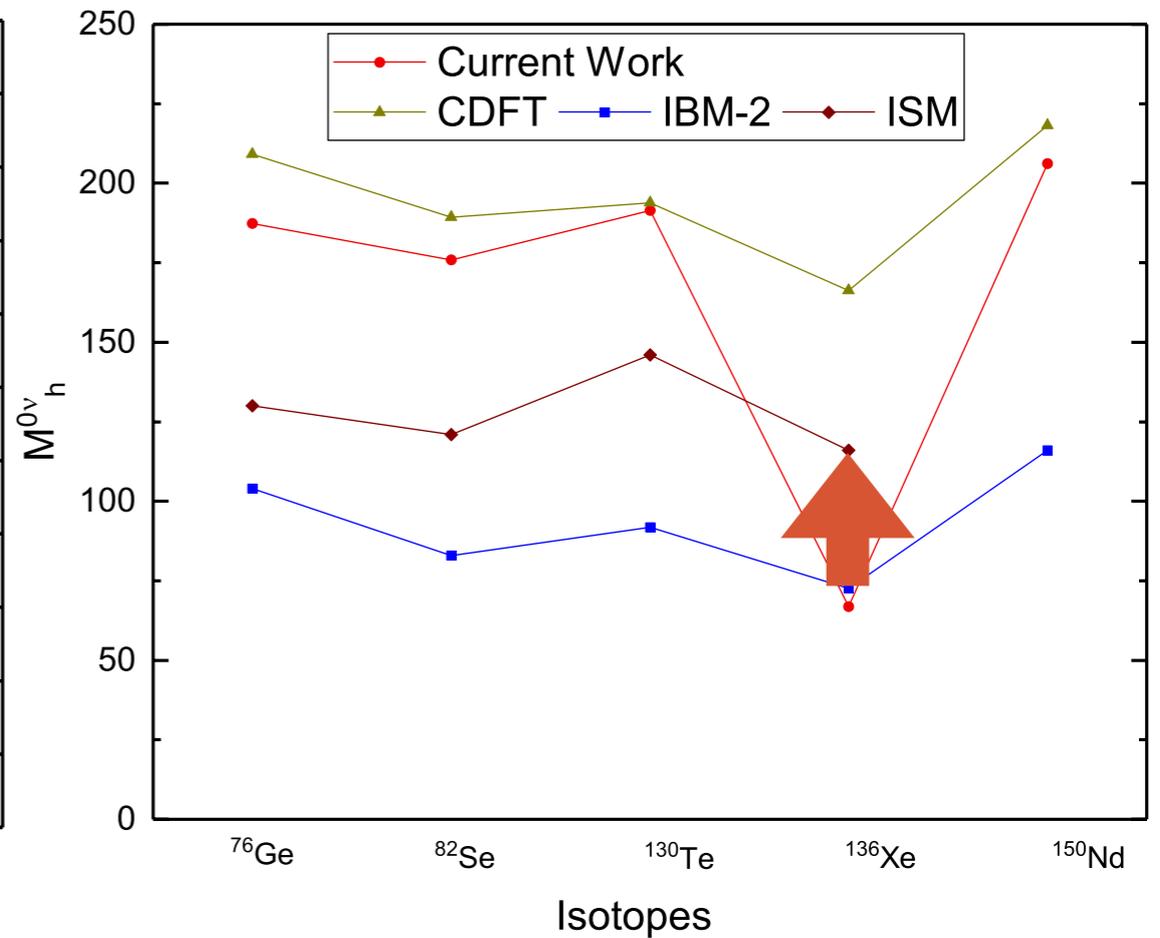
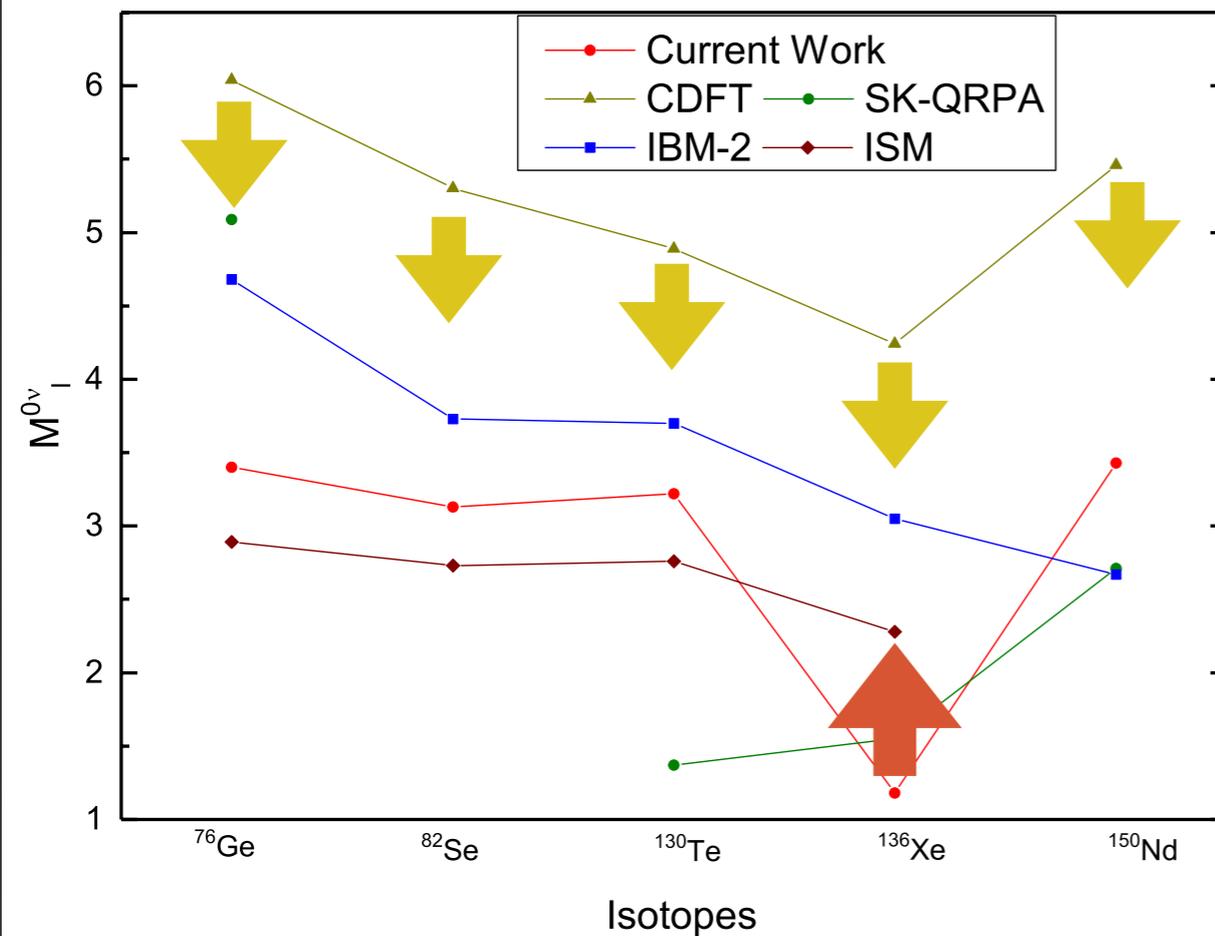
Results from different models converge?

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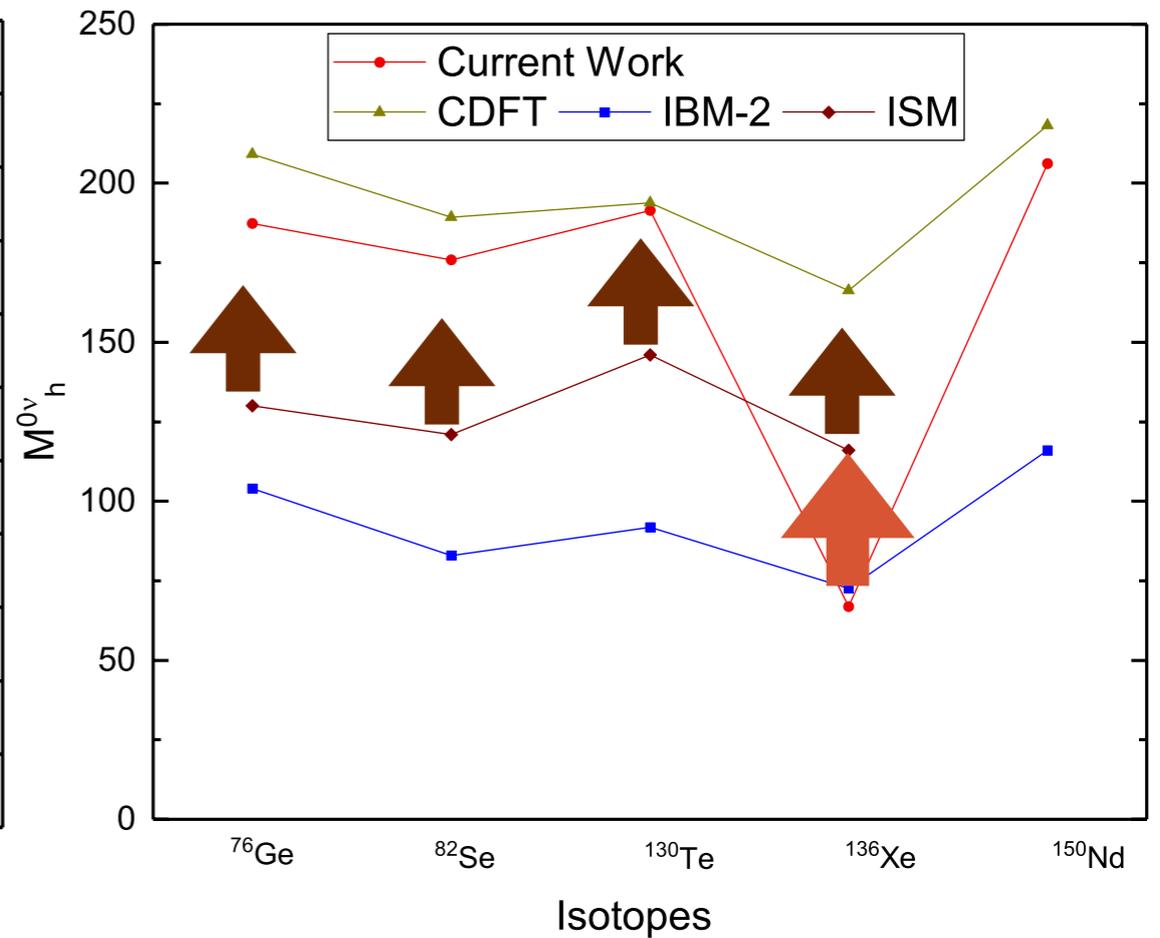
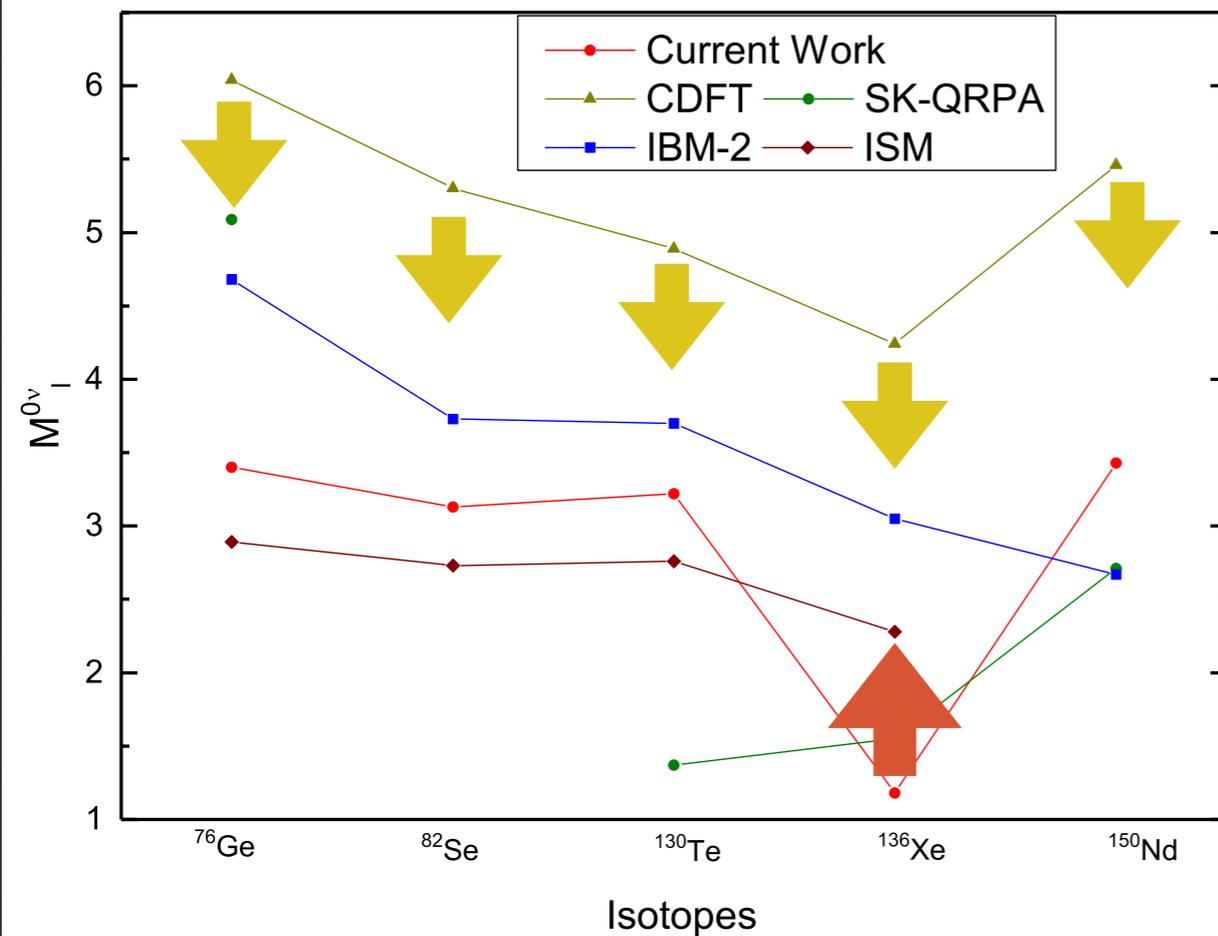
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Results from different models converge?

# Error Analysis



□ Results from different models converge?

# Open Questions

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- Inputs needed by many-body approaches:**
- Beyond “impulse approximation”—the chiral two body currents?**
- The  $g_A$  quenching problem—is it originated from the choice of hadron current**
- More degrees of freedom for hard neutrino?**

# Conclusion

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- We adopted deformed QRPA method with realistic force for the calculation of nuclear matrix elements for double beta decay
- The major effects of deformation comes from the BCS overlaps
- Possible errors are analyzed

**Thanks**