Towards lattice QCD determinations of double beta decay matrix elements

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Progress and Challenges in Neutrinoless Double Beta Decay

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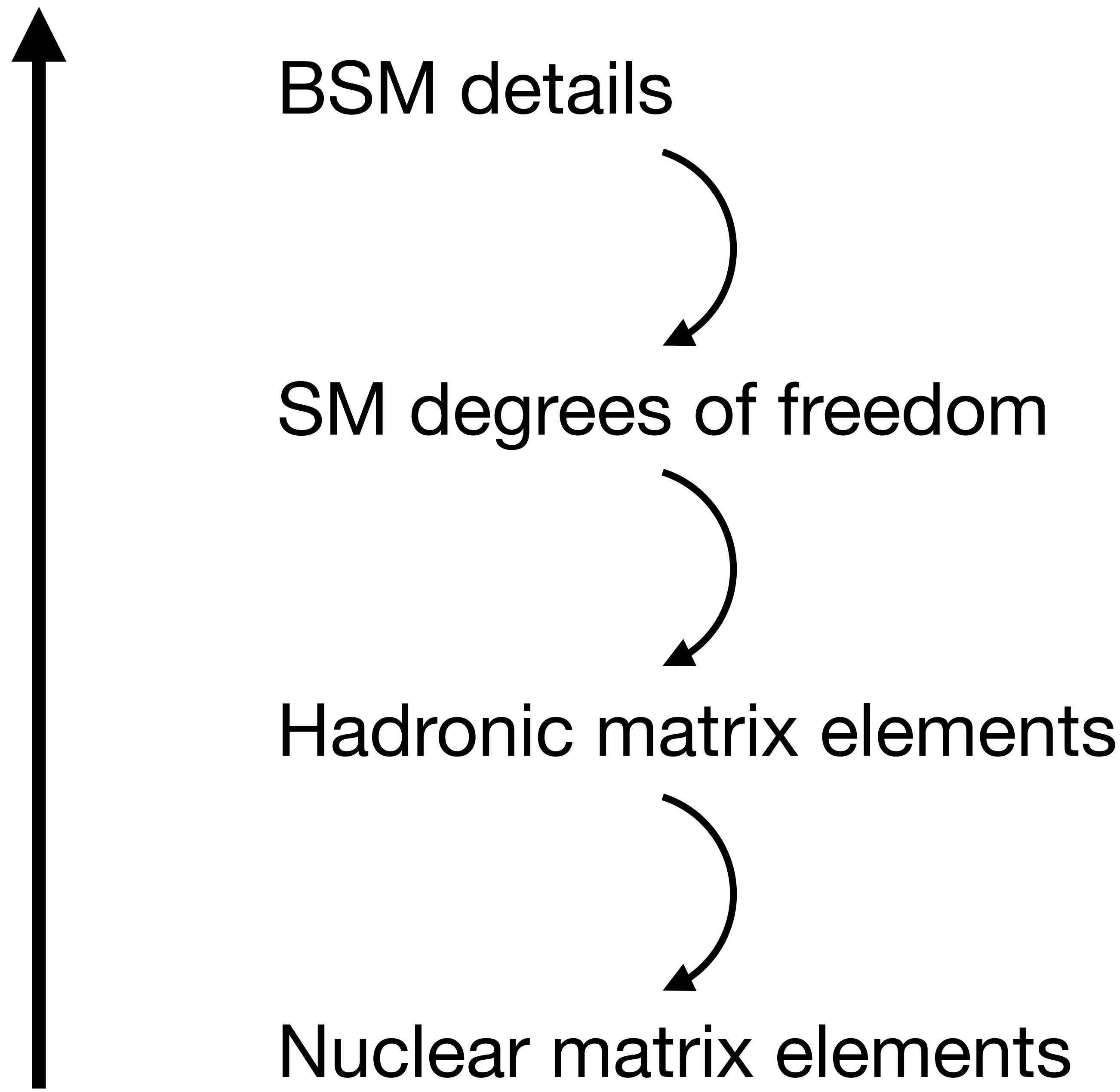
Scales and interactions



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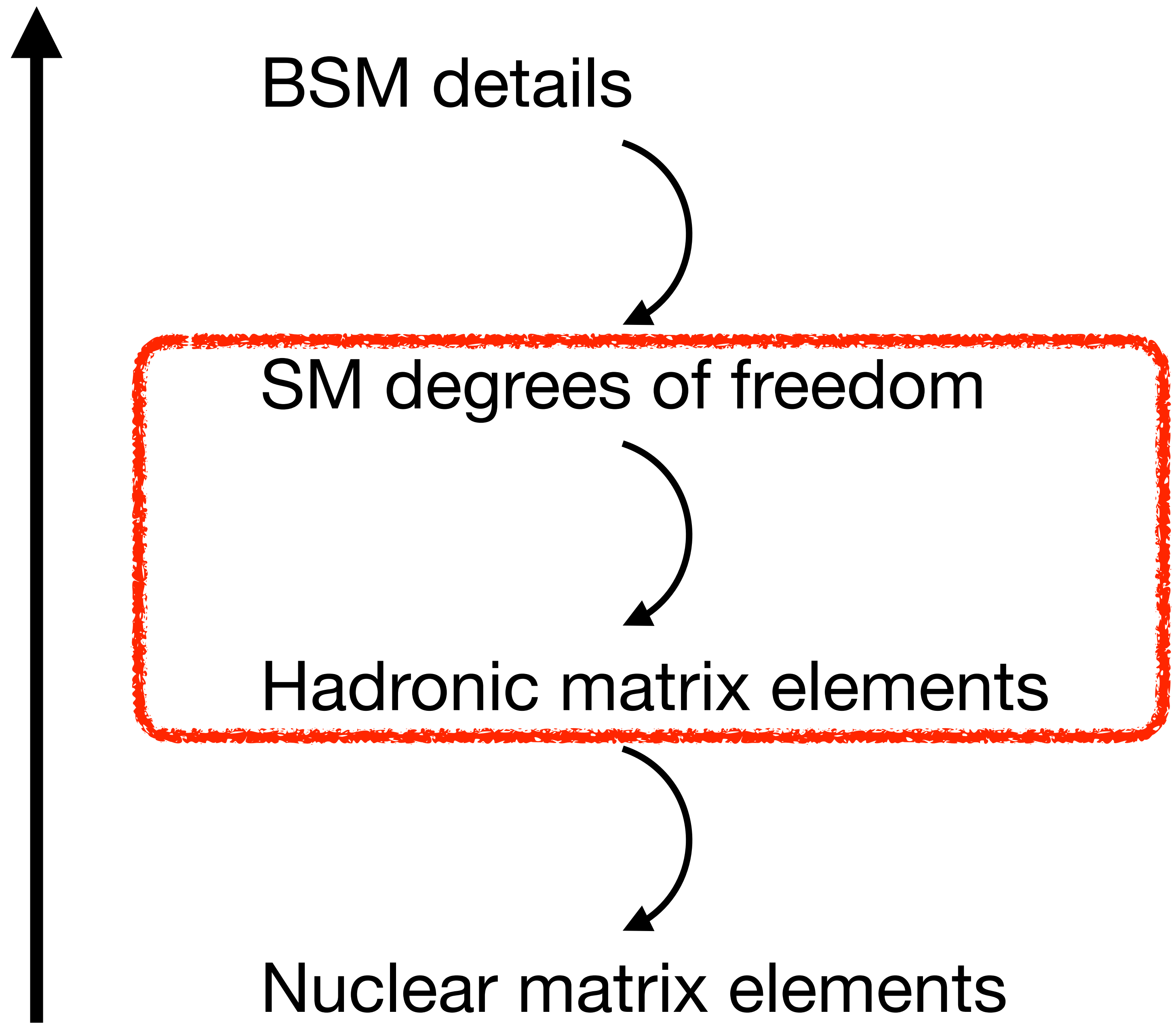
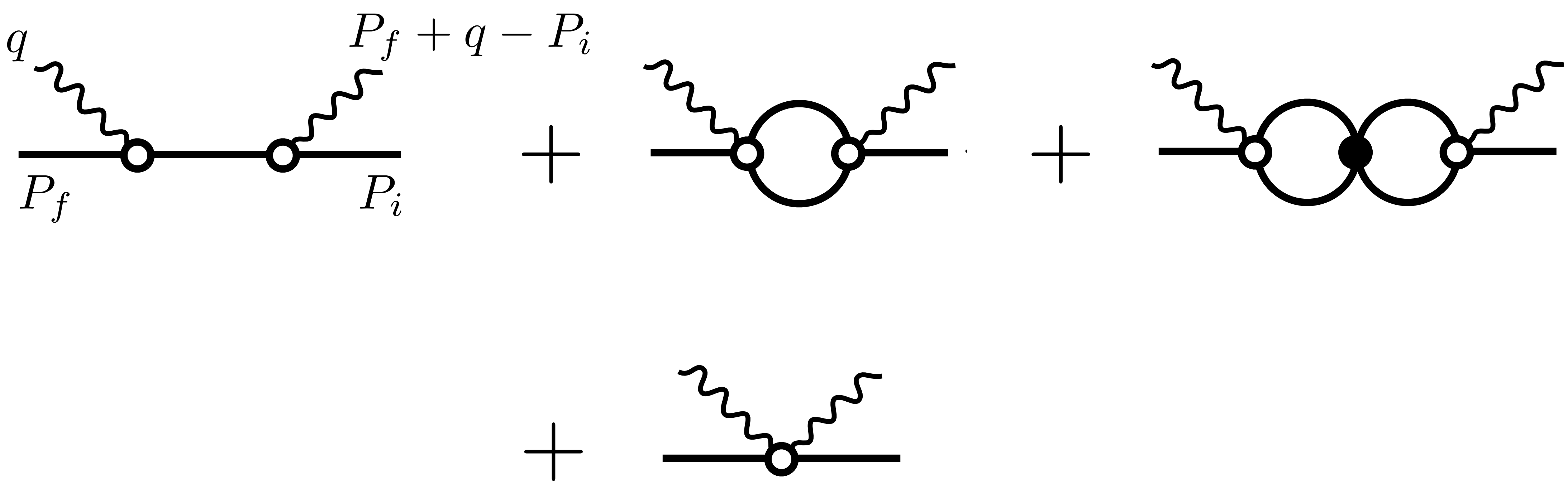


“Towards”

Things I will **not** talk about

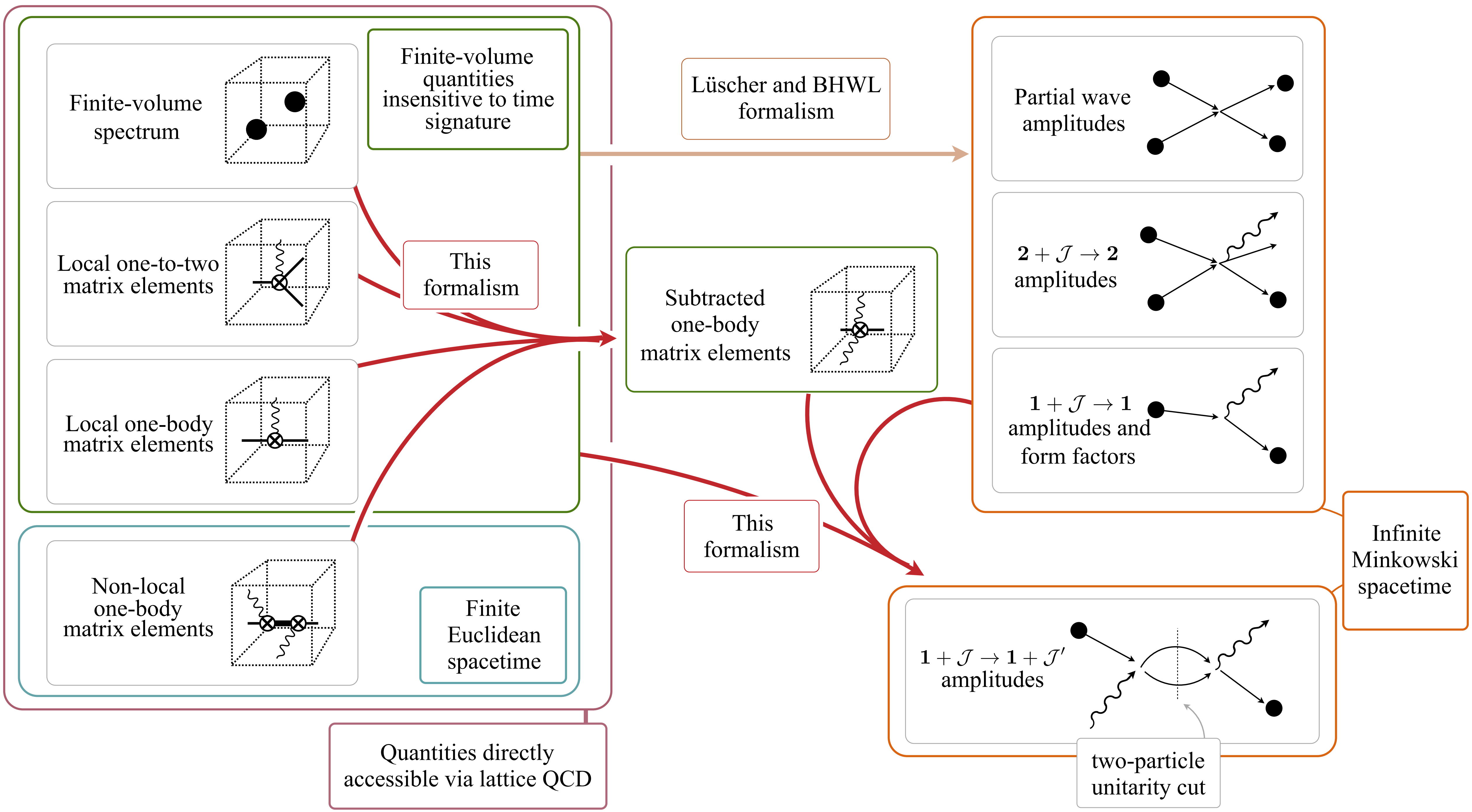
- 2-hadron initial and final states
- Particular form of interactions
- Propagating neutrinos
- BSM models

“Compton” amplitude



- Hadronic initial and final states
- Two current insertions
- Hadronic interactions
- On-shell 2-particle intermediate states $\rightarrow 0\nu\beta\beta$ decay

Overview



Finite-volume spectrum
Finite-volume quantities insensitive to time signature
Local one-to-two matrix elements
This formalism
Local one-body matrix elements
Non-local one-body matrix elements
Finite Euclidean spacetime
Quantities directly accessible via lattice QCD
Lüscher and BHWL formalism
Subtracted one-body matrix elements
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Partial wave amplitudes
$\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes
$\mathbf{1} + \mathcal{J} \to \mathbf{1}$ amplitudes and form factors
Infinite Minkowski spacetime
$\mathbf{1} + \mathcal{J} \to \mathbf{1} + \mathcal{J}'$ amplitudes
two-particle unitarity cut

What we want

Cross section related to amplitude

$$\frac{d\sigma}{d\Omega} \sim |\mathcal{M}|^2$$

Amplitude calculated in

- Minkowski space
- Infinite volume

How to get there

Nonperturbative problem → Lattice QCD

- Finite volume
- Discretized spacetime
- Euclidean time

Complications

- Energies and momenta quantized
- No asymptotic states
- Analytic structure of correlation functions very different

Goals

Address three key aspects:

- Euclidean vs Minkowski time
- Finite-volume effects
  - Exponentially suppressed → assume under control
  - Power-law corrections
- Large-t properties of correlation functions

see also Christ et al. (14,15)

Infinite-volume, Minkowski amplitude

Interested in matrix element

$$\mathcal{T}_{AB}(q, P_f, P_i) \equiv i \int d^4x \, e^{-iq\cdot x} \, \langle P_f | \, \mathrm{T}\{\mathcal{J}_A(x)\mathcal{O}_B(0)\} \, | P_i \rangle$$

where

$|P_{i/f}\rangle$: hadronic states, $\mathcal{J}_A(x), \mathcal{O}_B(0)$: currents

Finite-volume Minkowski amplitude

$$\mathcal{T}_{AB}(q, P_f, P_i, L) \equiv i \int d^4x\, e^{-iq\cdot x} \langle P_f, L|\, \mathrm{T}\{\mathcal{J}_A(x)\mathcal{O}_B(0)\}\, |P_i, L\rangle$$

$$= i \sum_n \int dt\, e^{[\omega-(E_n-E_f)+i\epsilon]t} \langle P_f, L|\, \mathcal{J}_A(0, \boldsymbol{q})|P_n, L\rangle\langle P_n, L|\mathcal{O}_B(0)\,|P_i, L\rangle + \cdots$$

$$\sim -\sum_n \frac{c_{n,AB}}{\omega - (E_n - E_f) + i\epsilon} \cdots$$

- Spectral decomposition of Minkowski amplitude
- Integral well defined

FV, Euclidean matrix element

From lattice (dropping some normalization factors)

$$G_{AB}(\tau, \boldsymbol{q}, L) \equiv \int_L d^3\boldsymbol{x}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \, \langle P_f, L| \, \mathrm{T}_E\{\mathcal{J}_A^E(\tau, \boldsymbol{x})\mathcal{O}_B^E(0)\} \, |P_i, L\rangle$$

Euclidean currents

$$\mathcal{J}_A^E(0, \boldsymbol{x}) = \mathcal{J}_A(0, \boldsymbol{x}), \; \mathcal{O}_B^E(0, \boldsymbol{x}) = \mathcal{O}_B(0, \boldsymbol{x})$$

FV, Euclidean matrix element

From lattice (dropping some normalization factors)

$$G_{AB}(\tau, \boldsymbol{q}, L) \equiv \int_L d^3\boldsymbol{x}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \langle P_f, L|\, \mathrm{T}_E\{\mathcal{J}_A^E(\tau, \boldsymbol{x})\mathcal{O}_B^E(0)\}\, |P_i, L\rangle$$

$$= \Theta(\tau) \sum_{n=0}^{\infty} c_{n,AB}\, e^{-[E_n(L,\boldsymbol{P}_f+\boldsymbol{q})-E_f]|\tau|} + \Theta(-\tau)\ldots$$

$$c_{n,AB} \equiv \int_L d^3\boldsymbol{x}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \langle P_f, L|\mathcal{J}_A^E(0, \boldsymbol{x})|n, L\rangle\langle n, L|\mathcal{O}_B^E(0)|P_i, L\rangle$$

FV, Euclidean amplitude

Integrate to get amplitude

$$
\begin{aligned}
T_{AB}(\omega, \boldsymbol{q}, L) &\equiv \int_{-\infty}^{\infty} d\tau \, e^{\omega\tau} \, G_{AB}(\tau, \boldsymbol{q}, L) \\
&= \sum_{n=0}^{\infty} \int_0^{\infty} d\tau \, c_{n,AB} \, e^{-[E_n - E_f - \omega]\tau} \\
&\quad + \cdots
\end{aligned}
$$

FV, Euclidean amplitude

Integrate to get amplitude

$$T_{AB}(\omega, \boldsymbol{q}, L) \equiv \int_{-\infty}^{\infty} d\tau \, e^{\omega\tau} \, G_{AB}(\tau, \boldsymbol{q}, L)$$

$$= \sum_{n=0}^{\infty} \int_0^{\infty} d\tau \, c_{n,AB} \, e^{-[E_n - E_f - \omega]\tau}$$

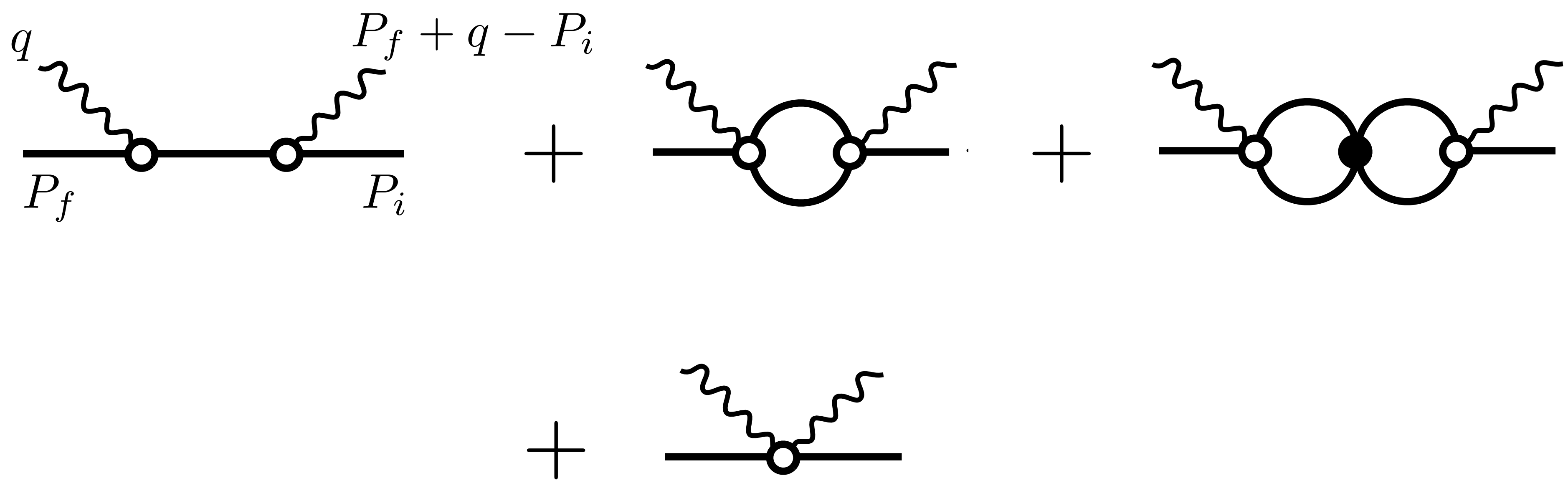
$$+ \cdots$$

Converges for ω “small enough”

$$= -\sum_{n=0}^{\infty} \frac{c_{n,AB}}{[\omega - (E_n - E_f)]} + \cdots$$

Agrees with spectral decomposition of Minkowski amplitude

ω small enough



No intermediate on-shell multi-particle states

FV Euclidean ME = IV Minkowski ME

Ji, Jung (01)

ω not small enough

- Integral no longer convergent

- On-shell intermediate states $\Rightarrow$ power-law FV corrections

ω not small enough

- Integral no longer convergent

For given kinematics consider only convergent piece:

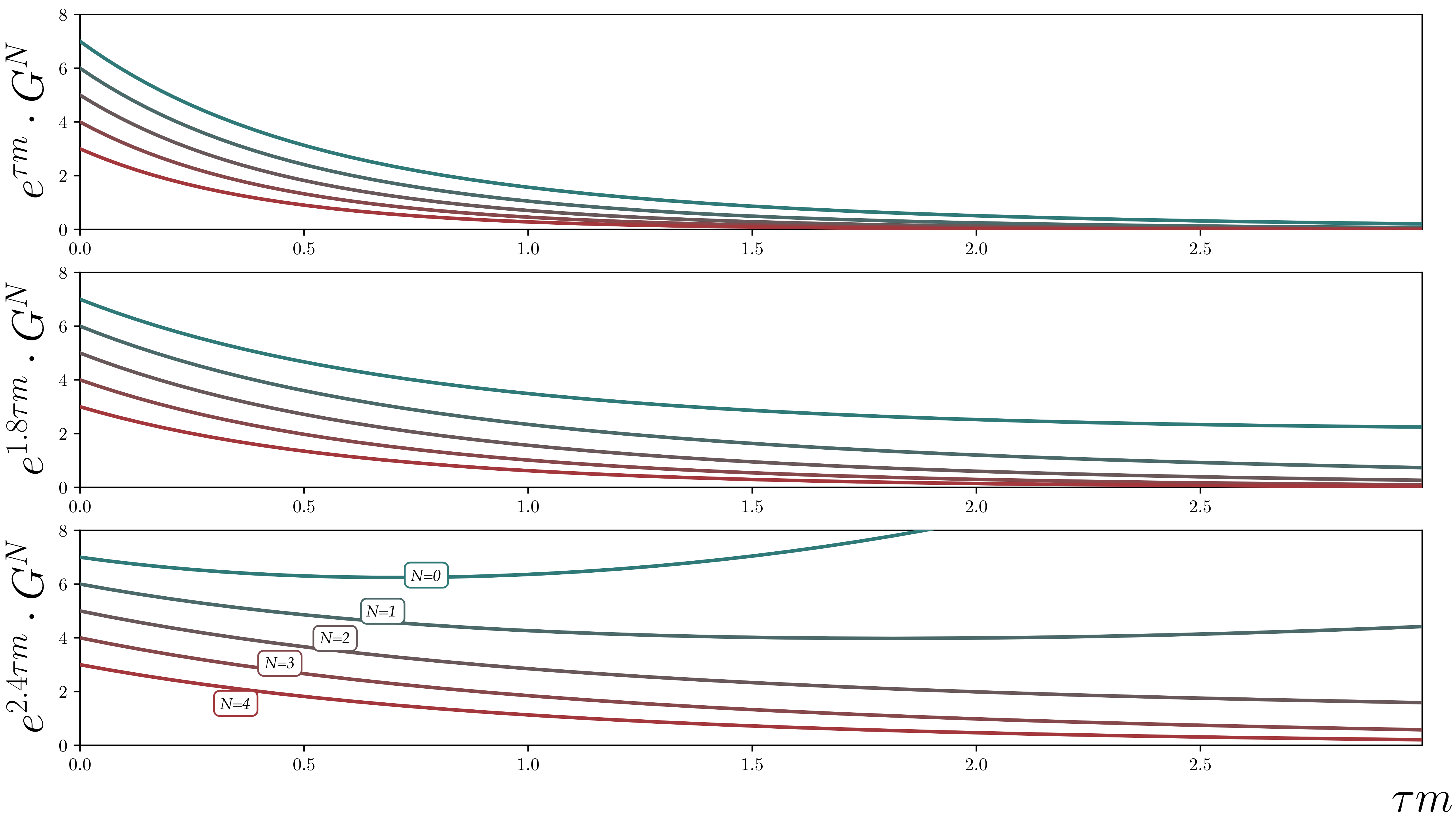
$$G_{AB}^{\geq N}(\tau, \boldsymbol{q}, L) \equiv \sum_{n=N}^{\infty} c_{n,AB}\, \Theta(\tau)\, e^{-[E_n(L, \boldsymbol{P}_f + \boldsymbol{q}) - E_f]|\tau|} + \cdots$$

$$T_{AB}^{\geq N}(\omega, \boldsymbol{q}, L) \equiv \int_{-\infty}^{\infty} d\tau\, e^{\omega\tau}\, G_{AB}^{\geq N}(\tau, \boldsymbol{q}, L)\,,$$

$$= - \sum_{n=N}^{\infty} \frac{c_{n,AB}}{\omega - [E_n - E_f]} + \cdots$$

Added benefit: convergence

Can subtract as many states as desired



Missing terms

$$T_{AB}^{<N}(\omega, \boldsymbol{q}, L) \equiv -\sum_{n=0}^{N-1} \frac{c_{n,AB}}{\omega - [E_n - E_f]} + \cdots$$

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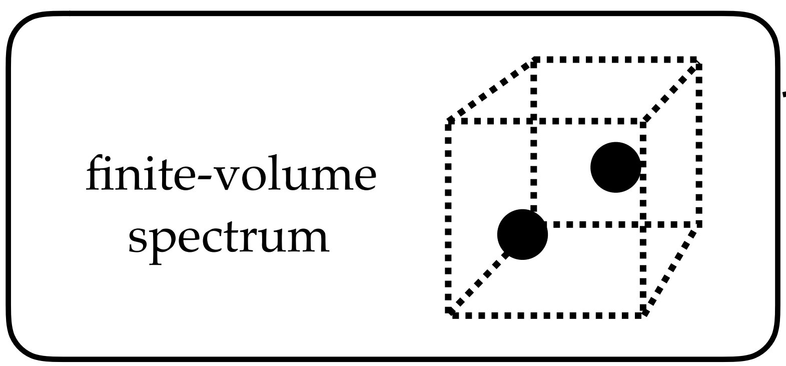
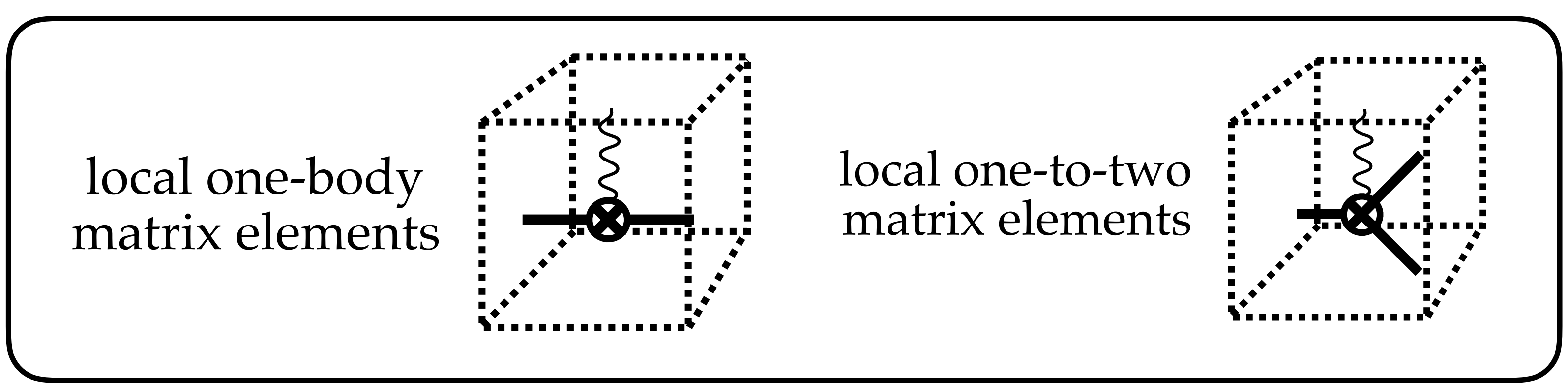
$$T_{AB}^{<N}(\omega, \boldsymbol{q}, L) \equiv -\sum_{n=0}^{N-1} \frac{c_{n,AB}}{\omega - [E_n - E_f]} + \cdots$$

Known for 1- and 2-particle states





Minkowski vs Euclidean

$$T_{AB}(\omega, \boldsymbol{q}, L) = T_{AB}^{<N}(\omega, \boldsymbol{q}, L) + T_{AB}^{\geq N}(\omega, \boldsymbol{q}, L)$$

matches spectral decomposition of Minkowski

$$\mathcal{T}_{AB}(q, P_f, P_i, L)$$

Power-law finite-volume effects

$$\mathcal{T}_{AB}(q, P_f, P_i) = T_{AB}(\omega, \boldsymbol{q}, L) + \Delta T_{AB}(\omega, \boldsymbol{q}, L)$$

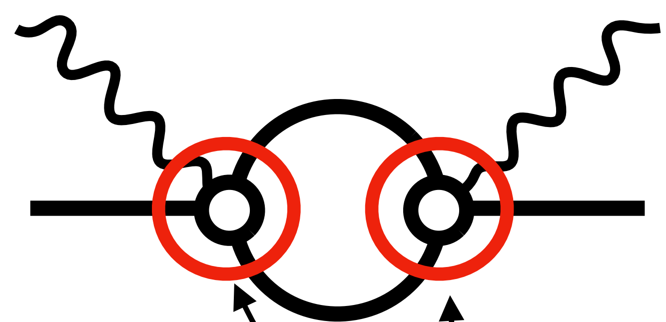
Finite volume correction $\Delta T_{AB}(\omega, \boldsymbol{q}, L)$

- Not from single-particle intermediate states
- For kinematics with on-shell 2-, but not 3-,4-particle states

$\Rightarrow$ use existing formalism

Lellouch, Lüscher (01); Lin et al. (01); Kim et al. (05); Briceño, Hansen (15);…

Power-law finite-volume effects



$$\Delta T_{AB}(\omega, \boldsymbol{q}, L) \equiv \mathcal{H}_{\text{in}}^{[\mathcal{J}]}(P_f)\,\frac{1}{F^{-1}+\mathcal{M}}\,\mathcal{H}_{\text{out}}^{[\mathcal{O}]}(P_i) + \cdots$$

$\mathcal{H}^{[\mathcal{J}]} :\ 1+\mathcal{J}\to 2$ transition amplitude

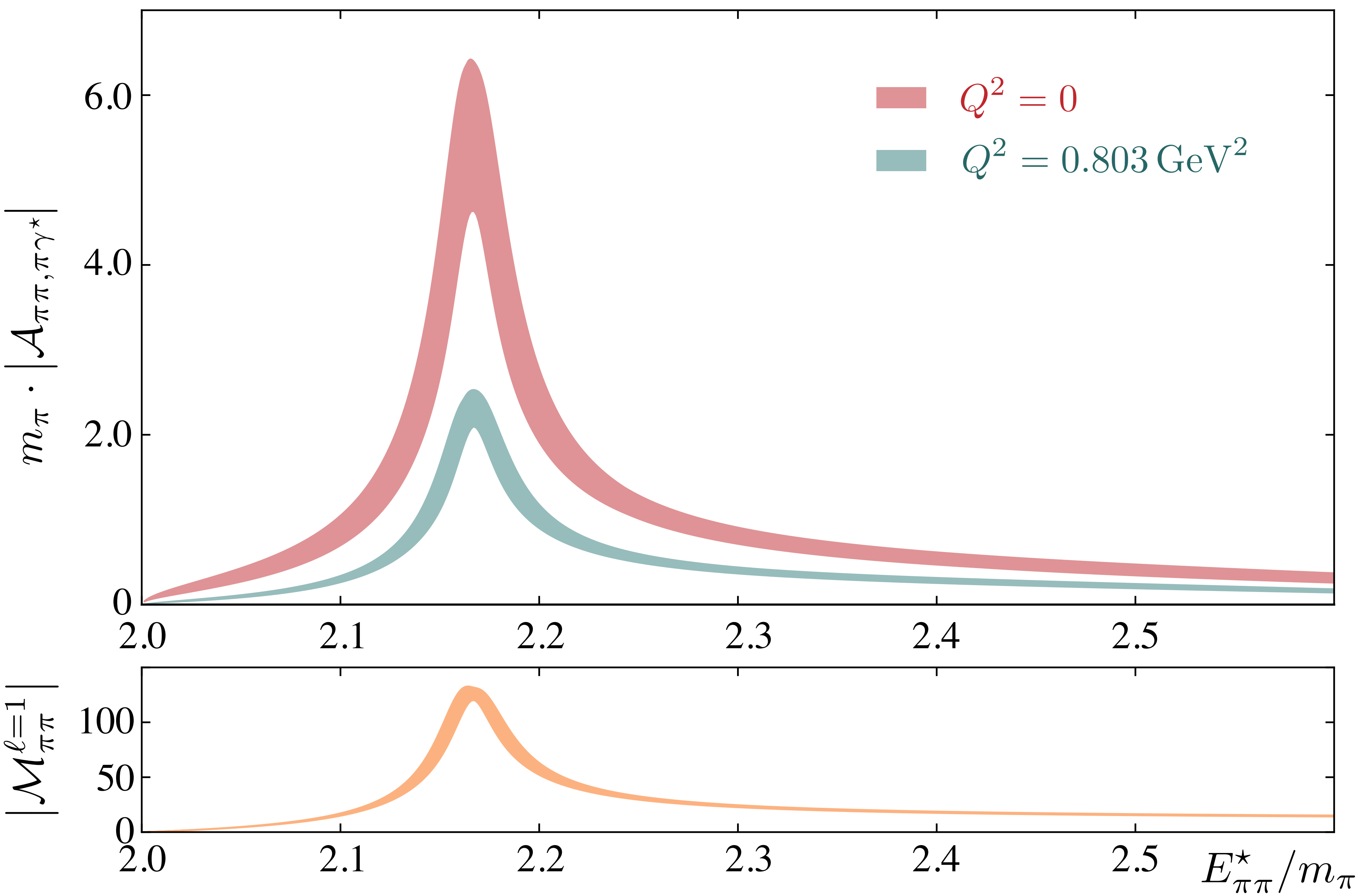
get from dedicated lattice calculation

Result

$$\mathcal{T}_{AB} = T_{AB}^{\geq N} + T_{AB}^{<N} + \mathcal{H}\,\frac{1}{F^{-1} + \mathcal{M}}\,\mathcal{H}$$

- All quantities on RHS accessible on lattice
- Several dedicated lattice QCD calculations required
  - Four-point function
  - 1/2-particle spectrum
  - 1-particle form factors
  - $1 + \mathcal{J} \to 2$ transition amplitude

Existing formalism



Briceño et al. (16)

Example: Analytic structure

2-particle scattering amplitude:

$$\mathcal{M}(P^2) = \mathcal{K}(P^2)\frac{1}{1 - i\rho\mathcal{K}(P^2)}$$

$1 + \mathcal{J} \to 2$ transition amplitude:

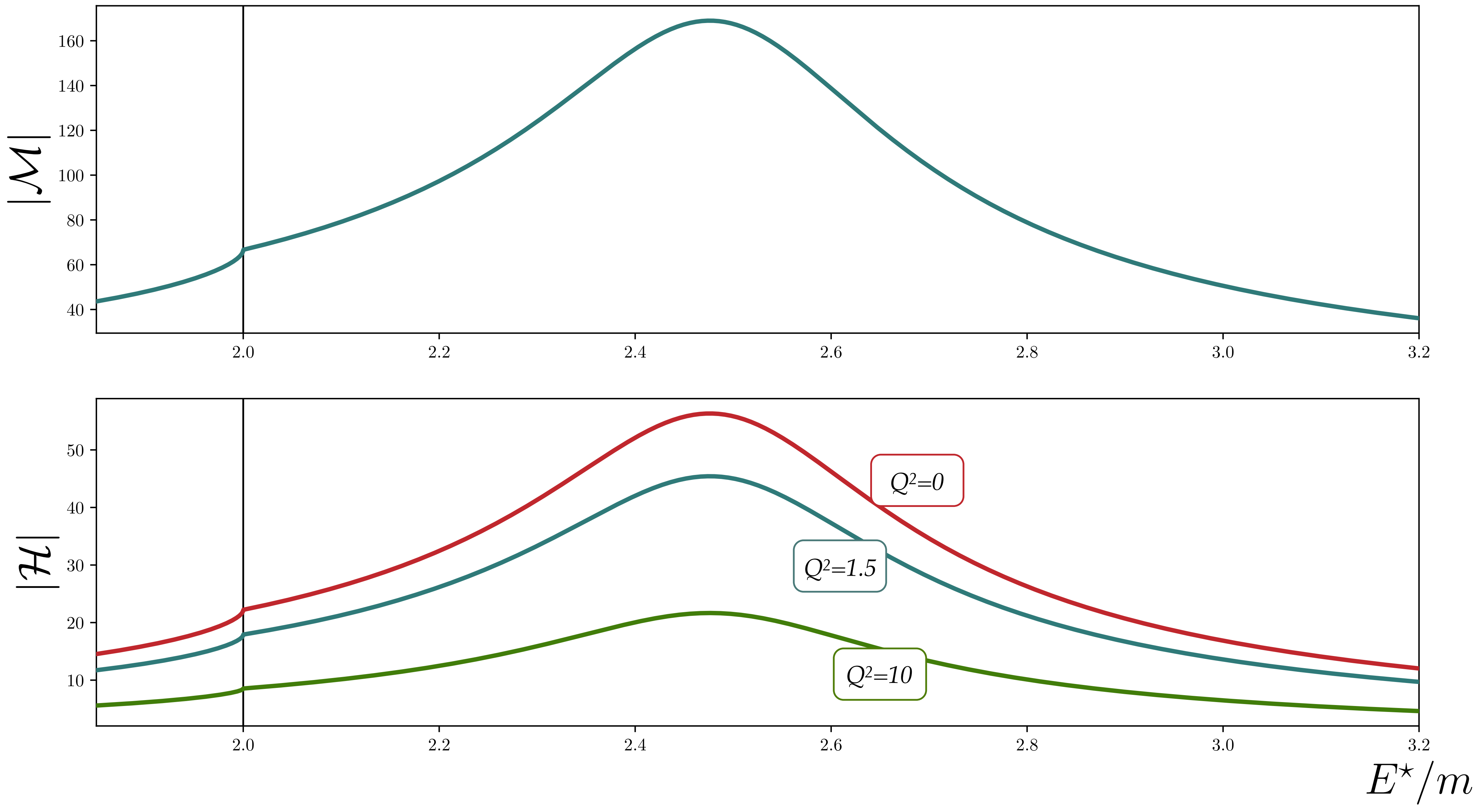
$$\mathcal{H}(Q^2, P^2) = \mathbf{F}(Q^2, P^2)\frac{\mathcal{K}(P^2)}{1 - i\rho\mathcal{K}(P^2)}$$

Compton amplitude:

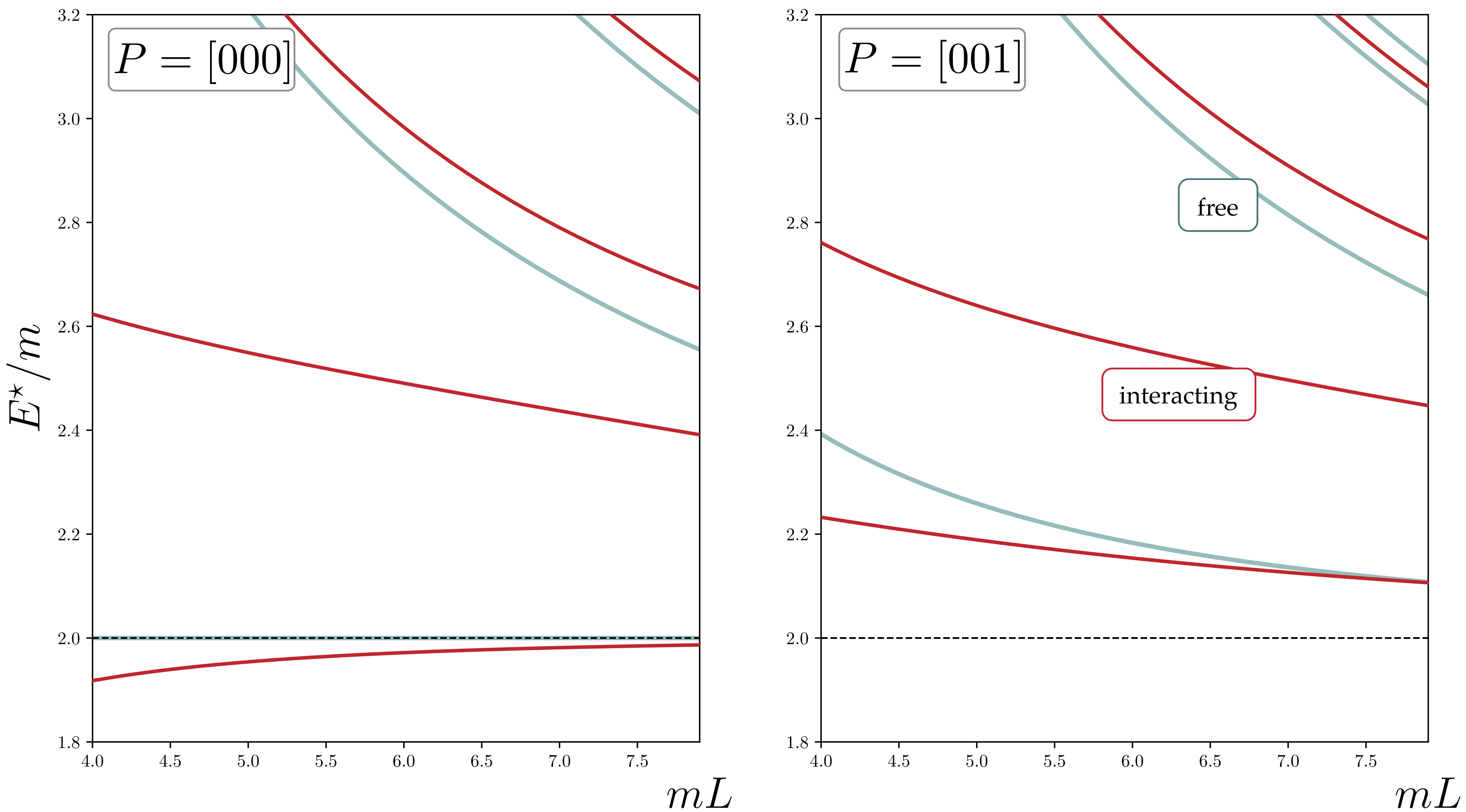
$$i\mathcal{C} = if\frac{i}{s - m_\pi^2 + i\epsilon}if + i\mathcal{H}\left[1 - i\rho\mathcal{K}\right]\rho\, i\mathcal{H} + \boxed{i\mathbf{C}} + \cdots$$

principle-value analogue

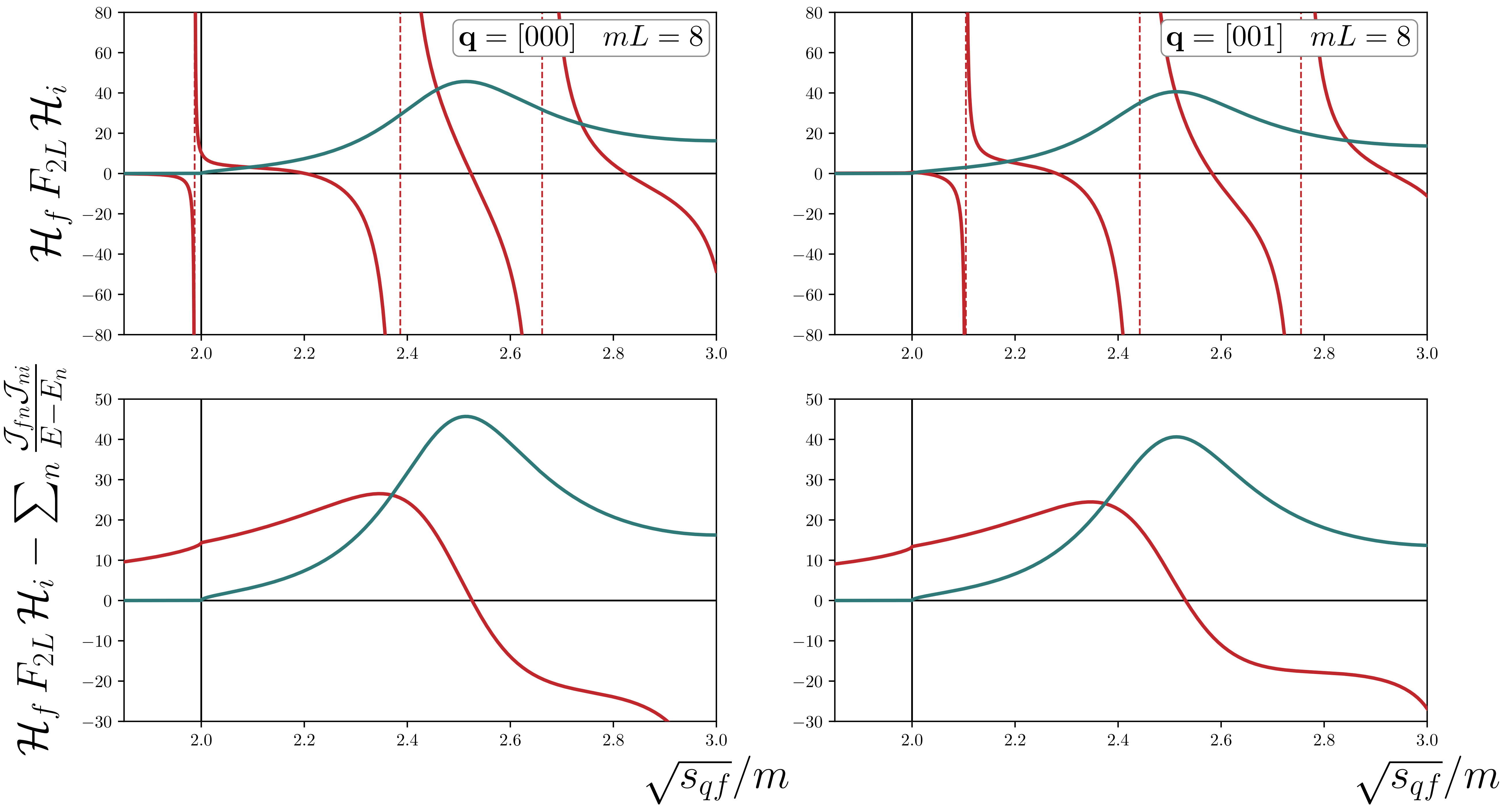
Numerical example



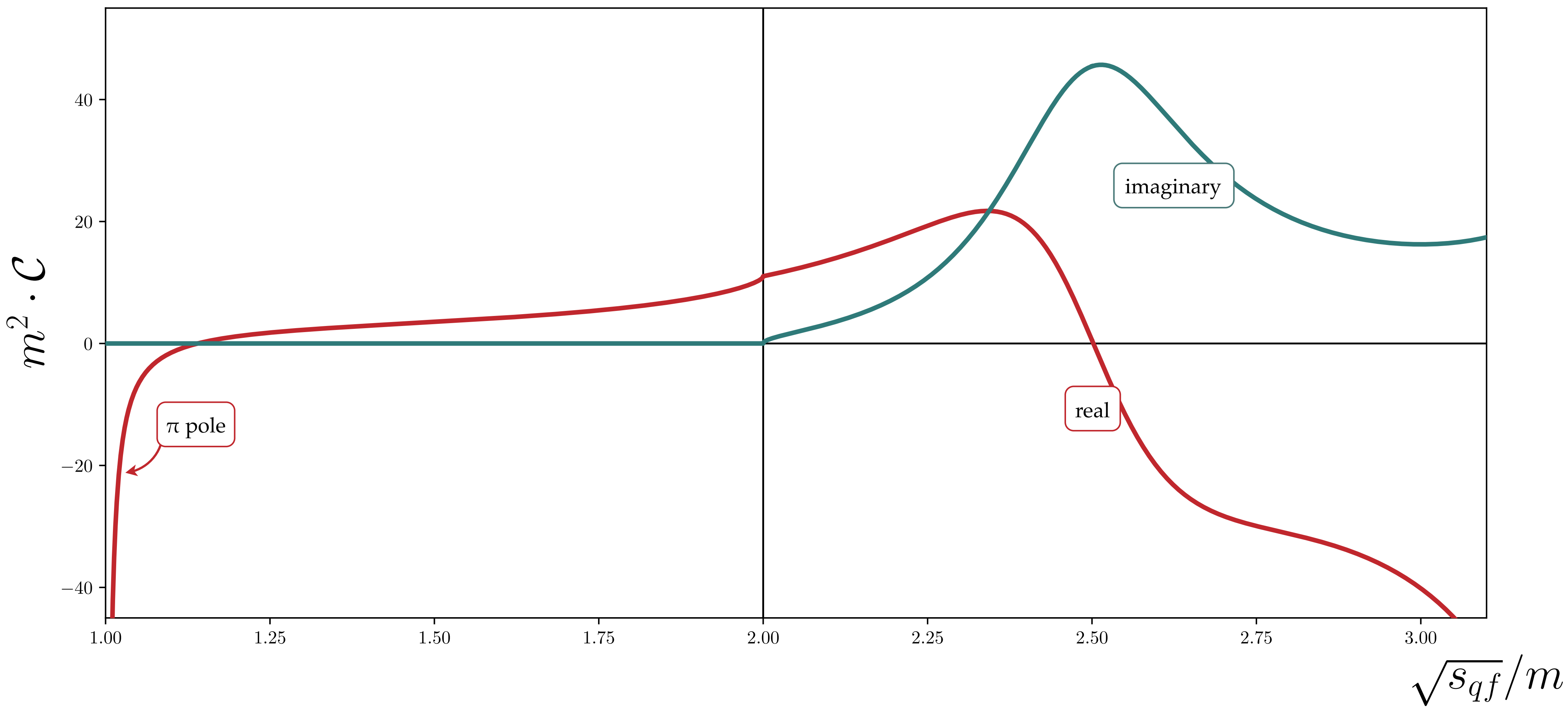
Finte volume energy levels



Subtracted correlation functions



Compton amplitude

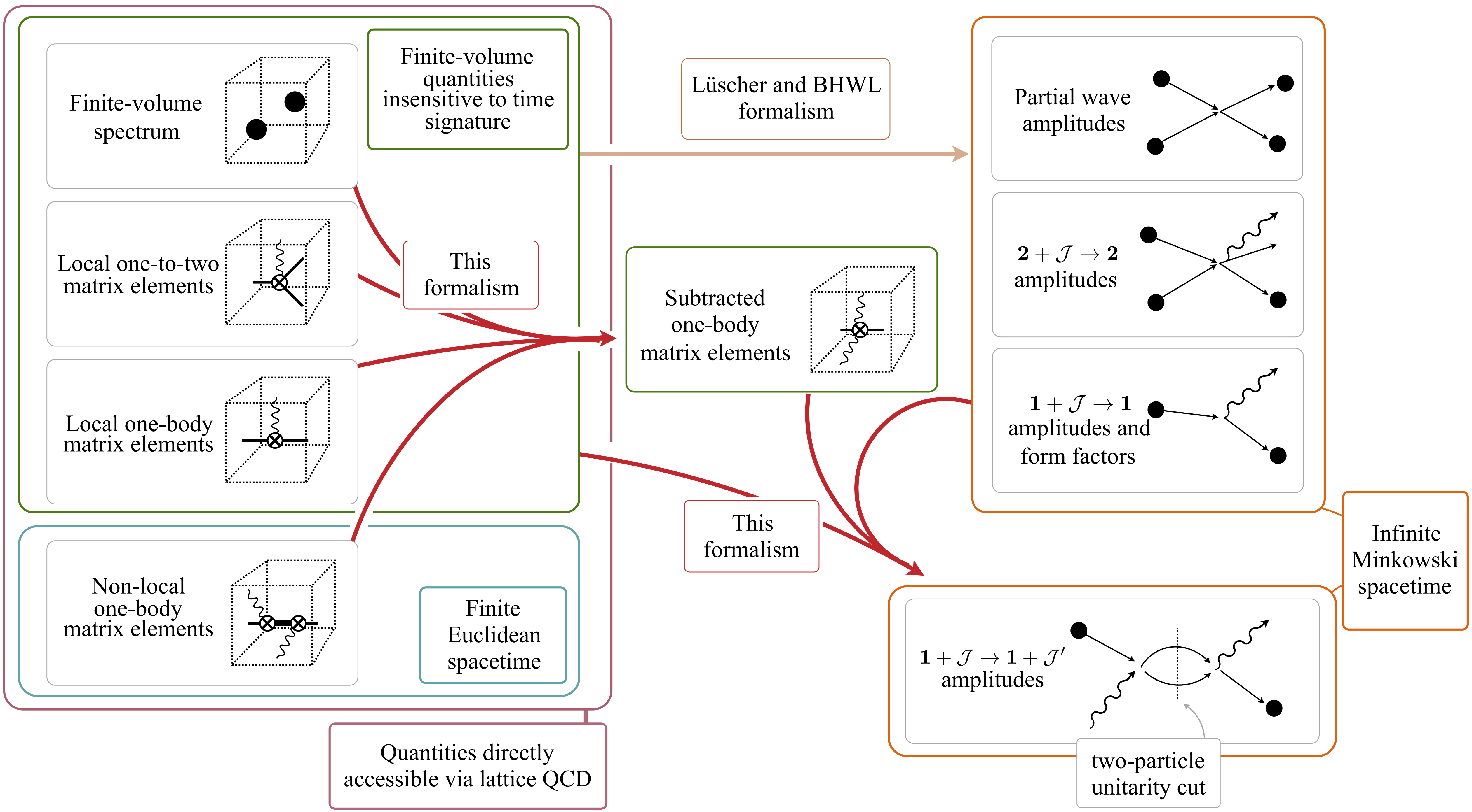


Summary

Towards 2-current matrix elements from lattice QCD

- Map Euclidean finite volume to Minkowski infinite volume
- Subtract intermediate on-shell contributions
- Determine from dedicated lattice calculations
- Subtractions improve asymptotic properties of correlators
- Applicable to particles with spin, coupled channels,…

Summary



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“Towards” $0\nu\beta\beta$

- Two-hadron initial and final states

  $\rightarrow$ finite-volume effects

- Triangle and box diagrams