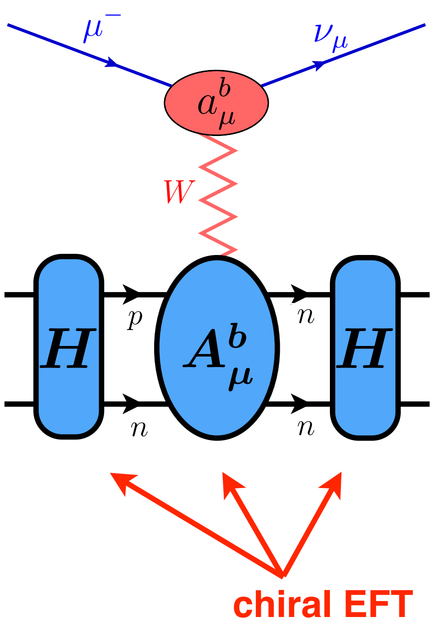
Evgeny Epelbaum, RUB

Progress and Challenges in Neutrinoless Double Beta Decay
ECT*, Trento, Italy, April 23-27, 2018

Electroweak currents in chiral EFT



- **Review of nuclear currents for 1 EW probe**
  — progress, problems, challenges
- **Applications: EM FFs of ^2H, ^3H β-decay**
- **Some thoughts on 0νββ currents**
- **Summary and conclusions**

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Review of chiral EFT (finite-regulator approach)

- The framework (especially conceptual issues)
- NN sector: a brief overview
- Electroweak currents
- Summary

Framework in a nutshell

- Write down the **effective chiral Lagrangian** for π, N (if needed, + Δ and external fields…)

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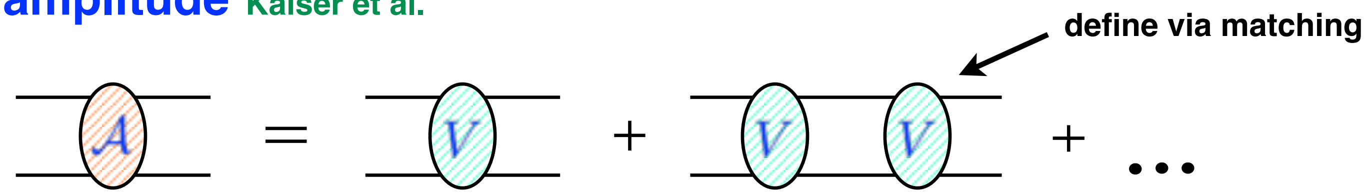
Matching to the amplitude Kaiser et al.

Feynman graphs →



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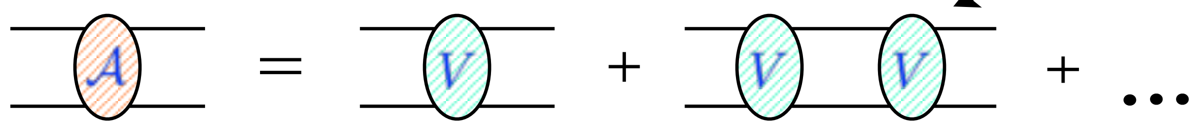

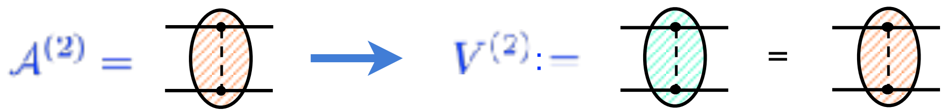

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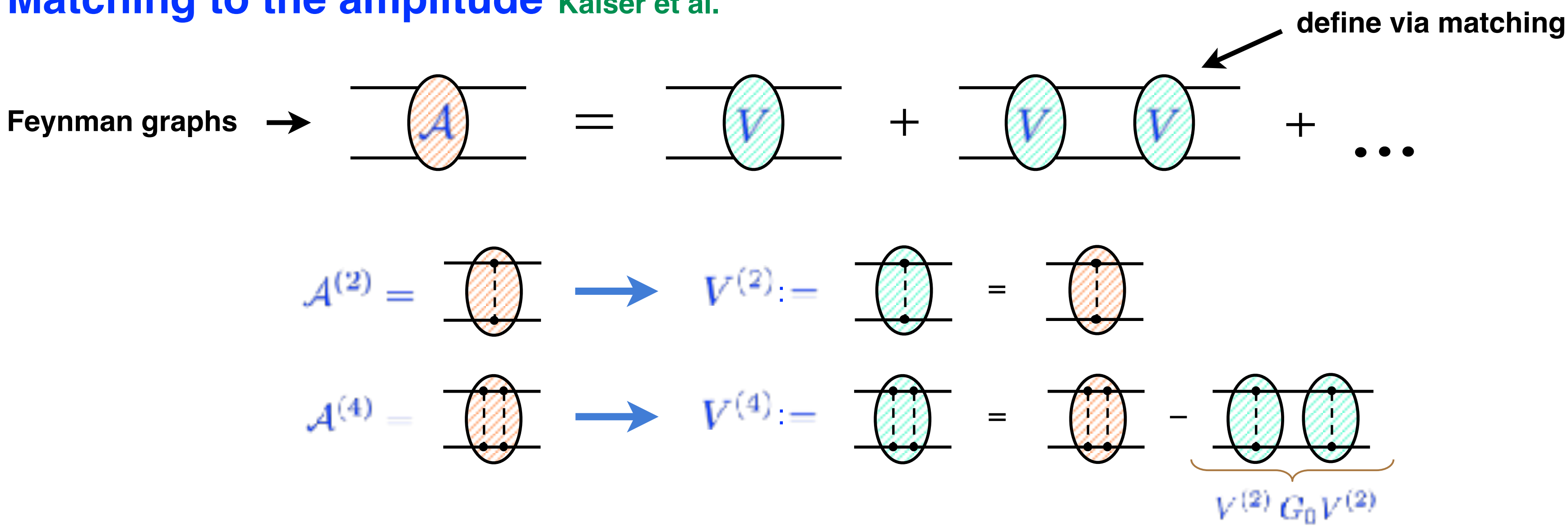

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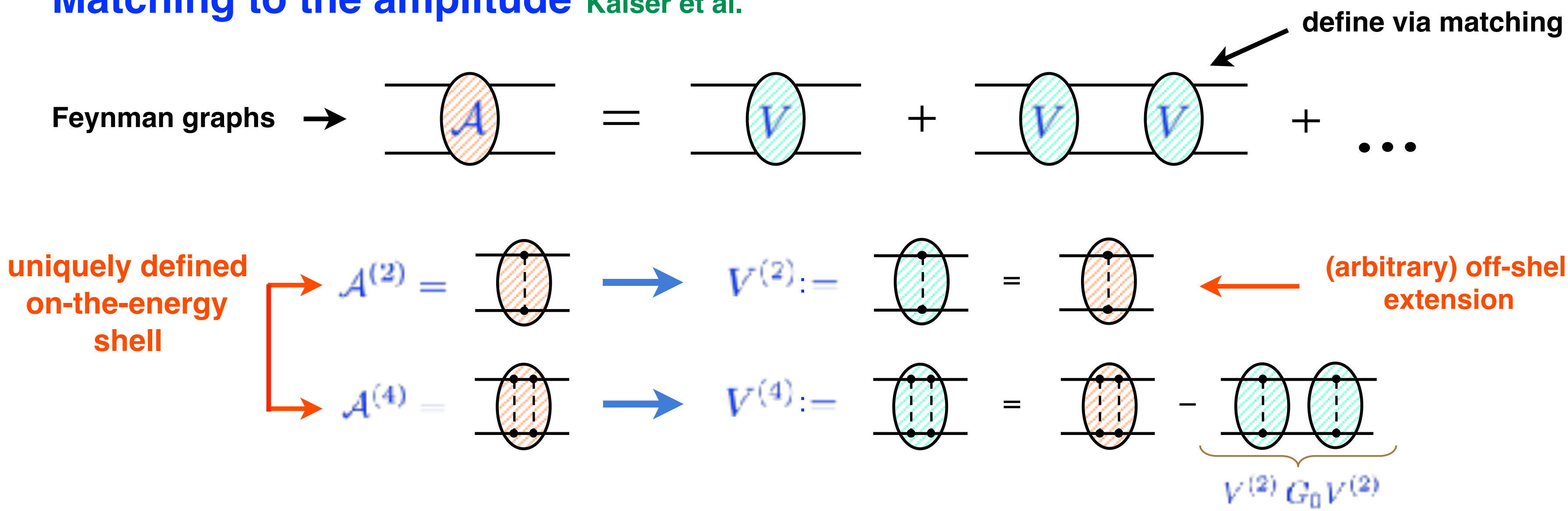

Higher-order terms in the Hamiltonian „know“ about the choice made for the off-shell extension (consistency...)

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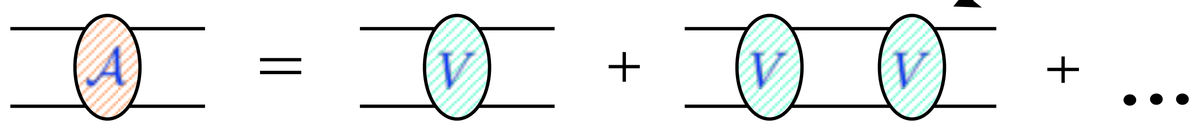
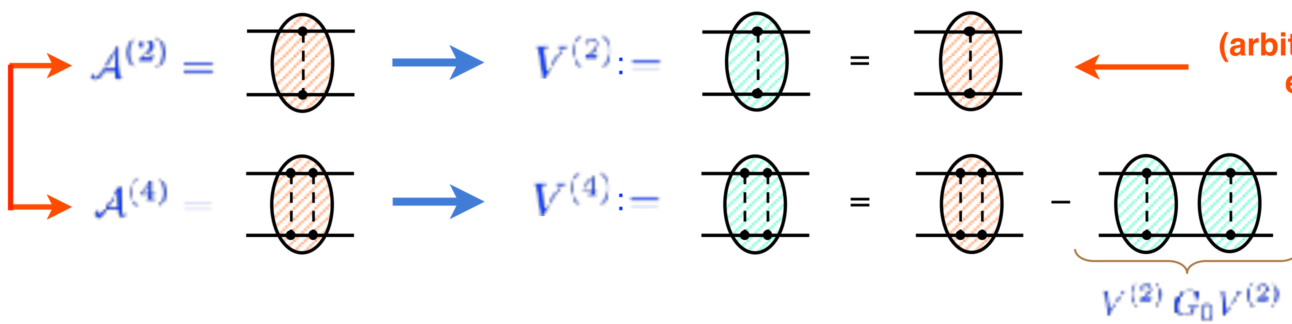


uniquely defined on-the-energy shell

(arbitrary) off-shell extension



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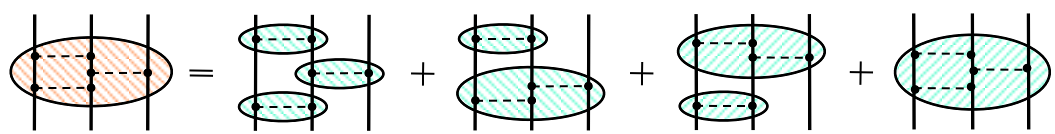
Are nuclear potentials well-defined (i.e. finite)?

not necessarily UV finite

UV finite →



So far, it was always possible to renormalize nuclear forces by systematically exploiting their unitary ambiguity…

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still consistent beyond the NN system?

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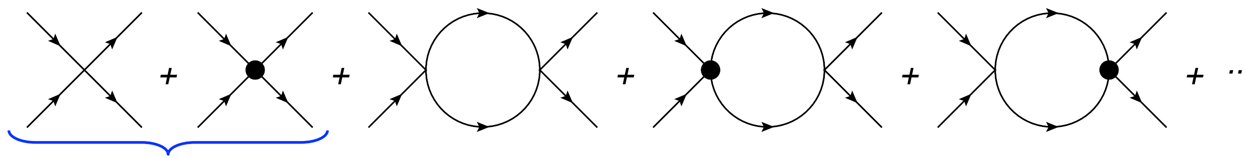

$c + c_2\left(p^2 + p'^2\right)$

taken from: EE, Gasparyan, Gegelia, Meißner, EPJA 54 (2018) 186

$$T_{\rm NLO}(q) = \frac{c_2\left[\hbar\, c_2\left(I_3 q^2 - I_5\right) - 2q^2\right] - c}{\hbar\, I\left(q^2\right)\left[c_2\left(\hbar\, c_2\left(I_5 - I_3 q^2\right) + 2q^2\right) + c\right] - \left(\hbar I_3 c_2 - 1\right)^2}$$

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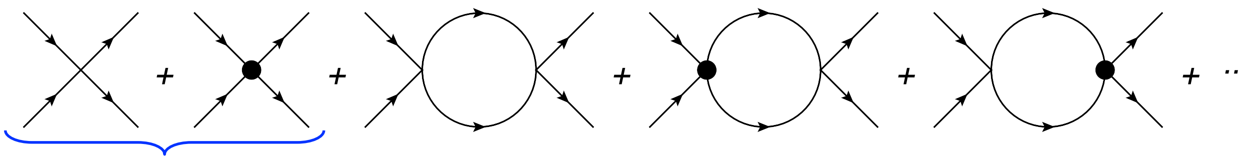

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- Subtractive renormalization: $I_n \to 0, \quad I(p^2) \to I(p^2) - I(-\mu^2), \quad c \to c_R(\mu), \quad c_2 \to c_{2R}(\mu)$

$$T^R_{\rm NLO}(q) = \frac{c_R + 2q^2 c_{2R}}{1 - \hbar\left[I(q^2) - I(-\mu^2)\right](c_R + 2q^2 c_{2R})}$$

— independent of regularization (but μ-dependent)
— involves contributions of ∞ number of c.t.; bare $V_{\rm NLO}$ available in a closed form
— exactly reproduces the infinite set of diagrams upon expansion in $\hbar$

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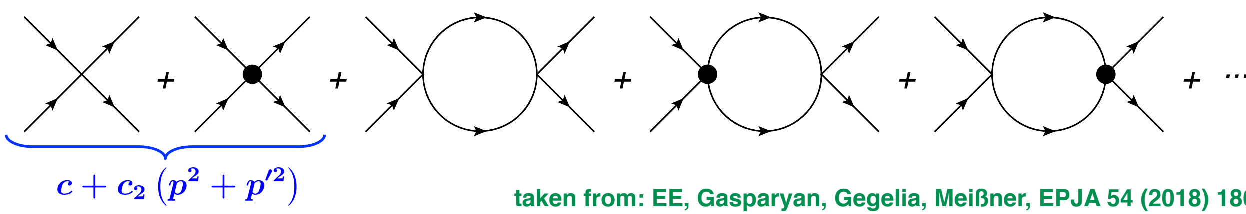

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- Implicit renormalization: express bare c(Λ), c_2(Λ) in terms of observables a, r:

$$T(q) = \frac{-4i\pi a\,[4a\hbar\Lambda + \pi\,(aq^2\, r_e + 2)]}{m\,[\pi\,(a^2 q^3\, r_e + 2aq - 2i) + 2a\hbar\Lambda(aq(2 + iq\, r_e) - 2i)]} = \frac{-4\pi/m}{-\frac{1}{a} + \frac{q^2 r_e}{2} - iq} + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

It is tempting to take the limit Λ → ∞ [Beane, Cohen, Phillips '98]. However, loop expansion yields:

$$T(q) = \frac{2\pi a\,(aq^2 r_e + 2)}{m} + \hbar\left[\frac{2a^4\Lambda q^4 r_e^2}{m} - \frac{i\pi a^2 q\,(aq^2 r_e + 2)^2}{m}\right] + \ldots$$

Not completely renormalized!
Λ-dependence OK if Λ~Λ_b~M_π; Wigner bound…

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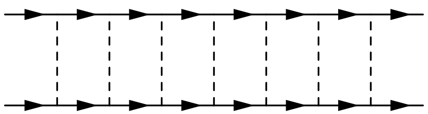
For example:  $\propto \frac{1}{d-4} \vec{p}^{\,6} m_N^6$ (spin-triplet) Savage, nucl-th/9804034

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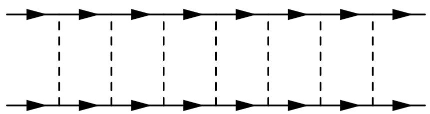
For example:  $\propto \frac{1}{d-4}\,\vec{p}^{\,6}\,m_N^6$ (spin-triplet) Savage, nucl-th/9804034

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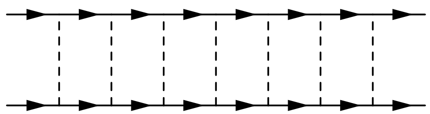
For example:  $\propto \dfrac{1}{d-4}\,\vec{p}^{\,6}\, m_N^6$ (spin-triplet) Savage, nucl-th/9804034

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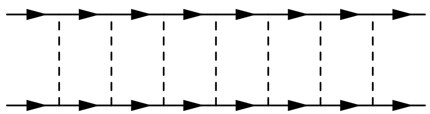
For example:  $\propto \dfrac{1}{d-4}\,\vec{p}^{\,6}\, m_N^6$ (spin-triplet) Savage, nucl-th/9804034

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What options are then left?

— relativized approach similar to EOMS-BChPT [EE, Gegelia'12]: hard calcs., convergence?

— keep Λ finite of the order of $\Lambda \sim \Lambda_b$ [Lepage '97]

Nonperturbative renormalization of singular potentials

What is the breakdown scale Λ_b?

Error plots à la Lepage suggest $\Lambda_b \sim 600$ MeV [EE, Krebs, Meißner, EPJA 51 (2015) 53]. This (or a bit larger) value is found to be statistically consistent by the Bayesian analysis for not too soft cutoffs [BUQEYE Collaboration, Furnstahl et al., PRC 92 (2015) 024005; Melendez et al., PRC 96 (2017) 024003].

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How do we organize the expansion?

We organize terms in the potential according to NDA (minimal choice for contacts). Alternatives have been proposed and can be tested by looking at the convergence pattern (requires high orders + error analysis). So far, no signs of departure found (2π-exchange, naturalness of LECs, magnitude of V_{3N}, the c_D term in the 3NF…)

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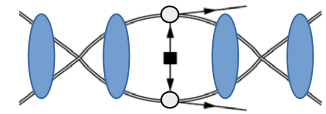
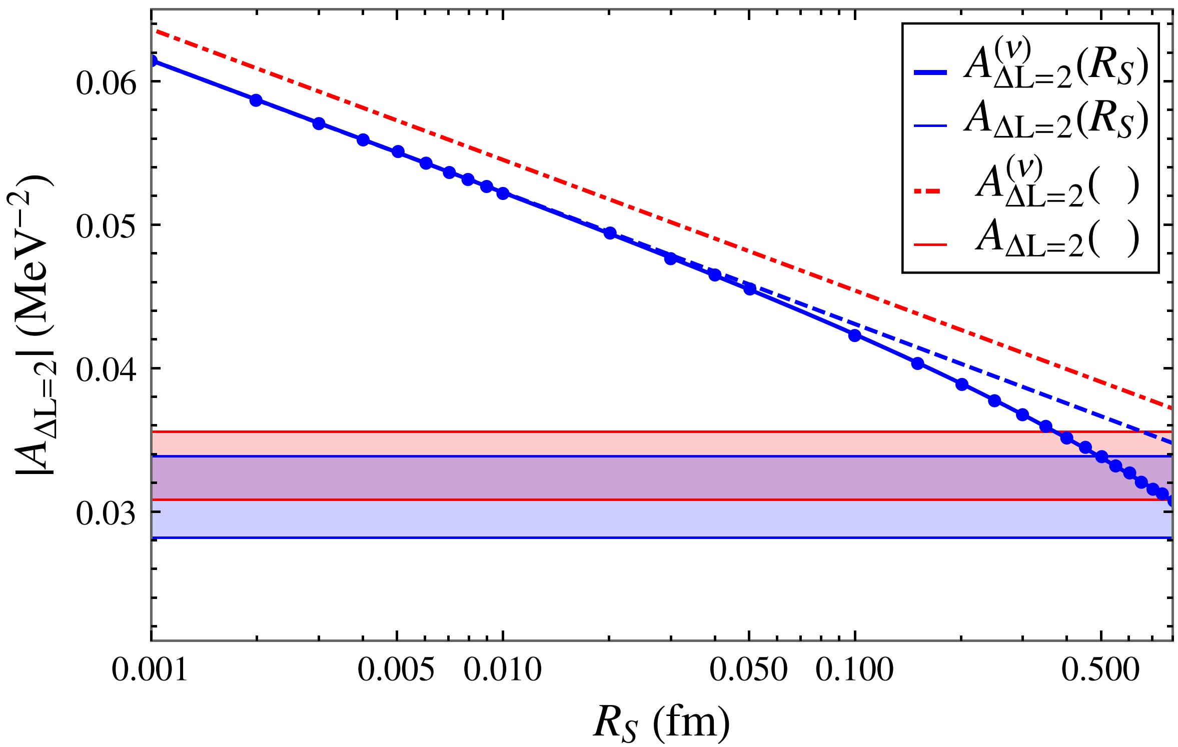
What about power counting?

Power counting = counting powers of soft scales (p, M_π) in diagrams after renormalization. Power counting depends on the choice of renormalization conditions. Since we work with bare LECs and perform renormalization implicitly, we cannot specify renorm. conditions…

New LO contribution to 0νββ decay

✅ The LO long-range potential ($1/q^2$) leads to a logarithmic divergence [Cirigliano et al., PRL 120 (2018) 2002001]. At LO in chiral EFT, this conclusion is not affected by the controversial renormalization issues.





❓ Cirigliano et al. then conclude that the corresponding c.t. must be promoted at LO. How does the regulator dependence for $\Lambda \sim \Lambda_b$ compare with the LO truncation error?

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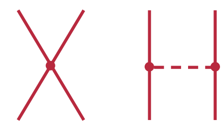
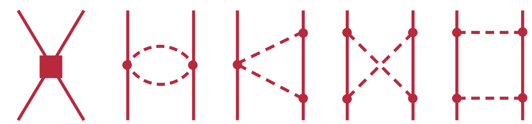
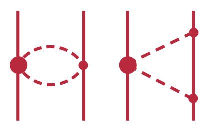
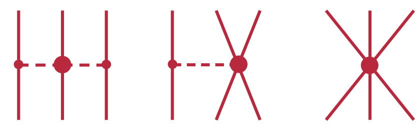
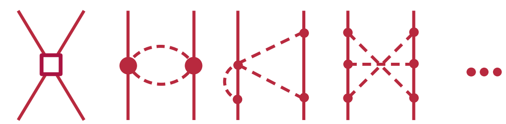
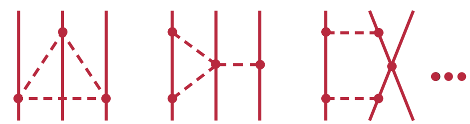
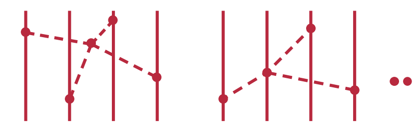
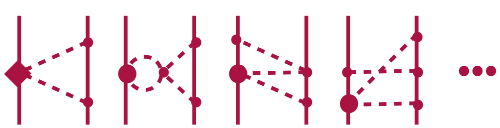
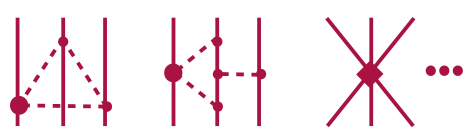
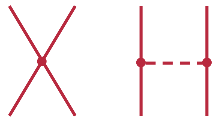
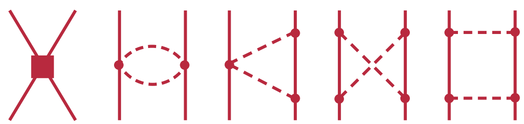
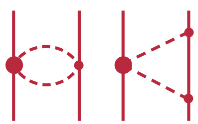
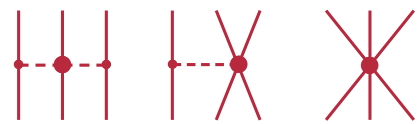
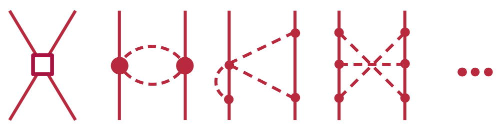
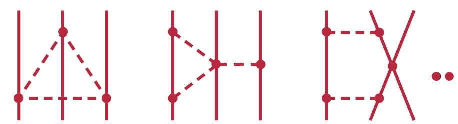
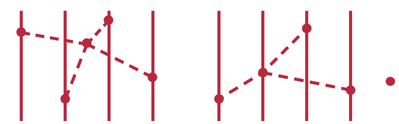
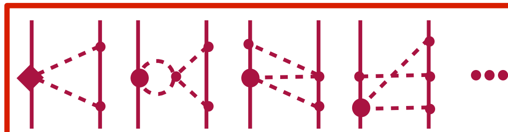
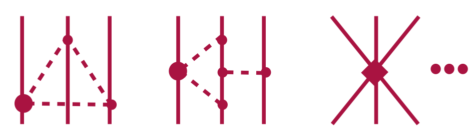
- **Error analysis and consistency checks** (naturalness, Lepage plots, …). Any observable $X^{(n)}$ calculated at order Q^n should be approximately Λ-independent: $dX^{(n)}/d\Lambda\,|_{\Lambda\sim\Lambda_b} \overset{n\to\infty}{\longrightarrow} 0$

Nuclear forces

Nuclear Hamiltonian: State-of-the-art [W-counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)	 Ordonez, van Kolck '92	—	—
N²LO (Q^3)	 Ordonez, van Kolck '92	 van Kolck '94; EE et al. '02	—
N³LO (Q^4)	 Kaiser '00 - '02	 Bernard, EE, Krebs, Meißner,'08, '11	 EE '06
N⁴LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	 Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12,'13 (short-range loop contrib. still missing)	— (preliminary)

Nuclear Hamiltonian: State-of-the-art [W-counting]

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)	 Ordonez, van Kolck '92	—	—
N^2LO (Q^3)	 **parameter-free (!): πN LECs from Roy-Steiner** Ordonez, van Kolck '92	 van Kolck '94; EE et al. '02	—
N^3LO (Q^4)	 Kaiser '00 - '02	 Bernard, EE, Krebs, Meißner,'08, '11	 EE '06
N^4LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	 Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12,'13 (short-range loop contrib. still missing)	— (preliminary)

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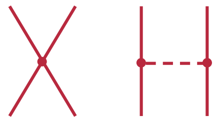
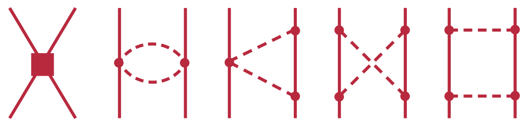
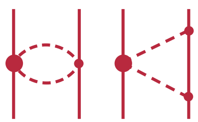
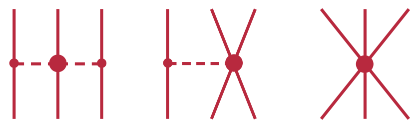
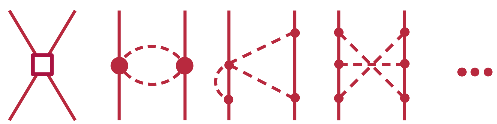
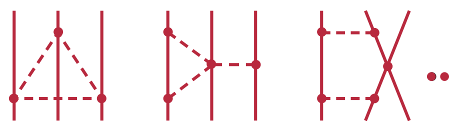
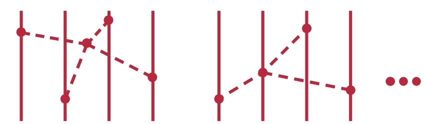
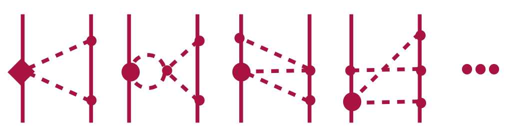
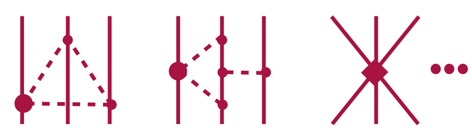
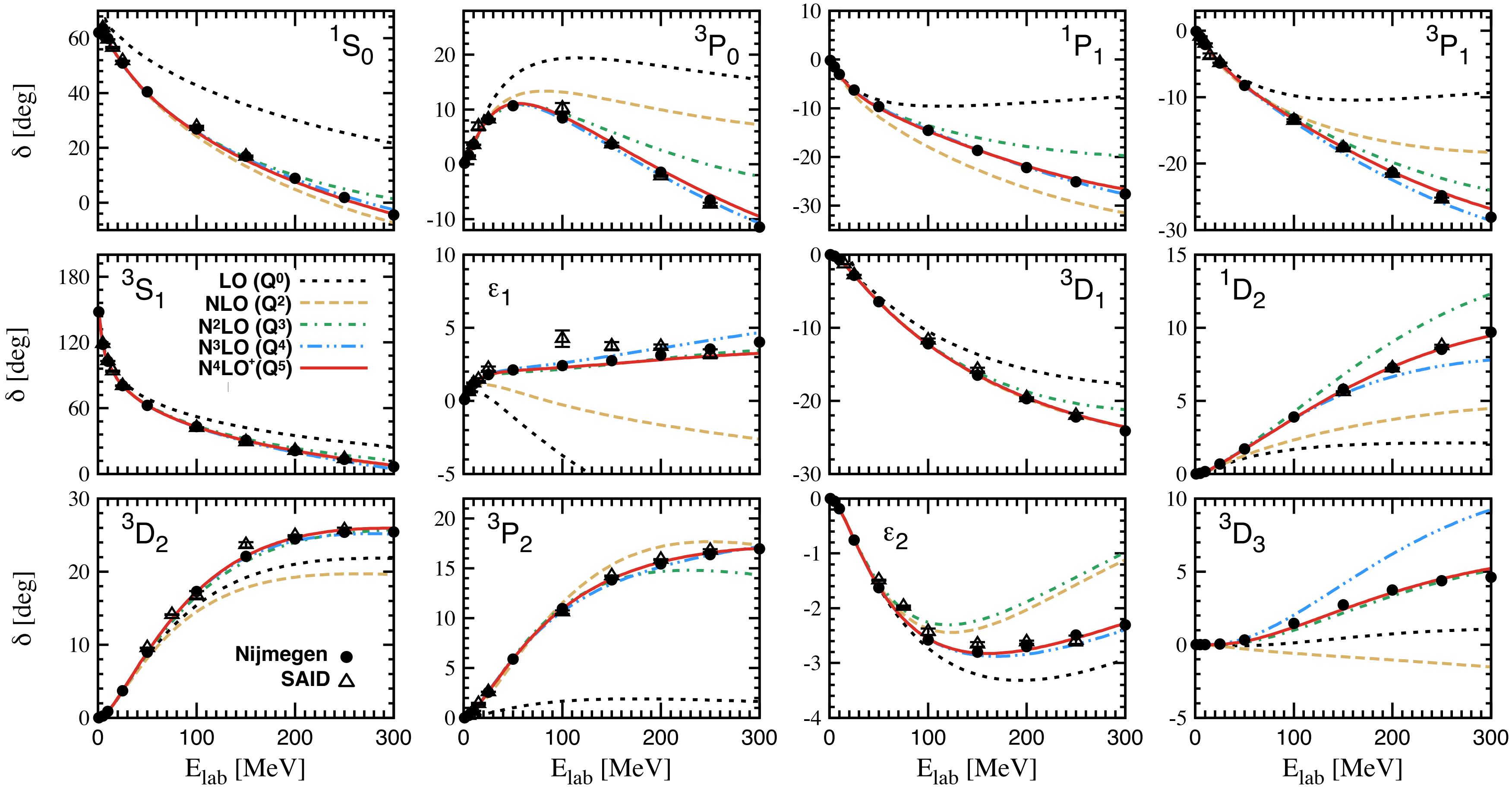
LO (Q^0)	 Weinberg '90	—	—
NLO (Q^2)	 Ordonez, van Kolck '92	—	—
N^2LO (Q^3)	 Ordonez, van Kolck '92	 van Kolck '94; EE et al. '02	—
N^3LO (Q^4)	 Kaiser '00 - '02	 Bernard, EE, Krebs, Meißner,'08, '11	 EE '06
N^4LO (Q^5)	 Entem, Kaiser, Machleidt, Nosyk '15 EE, Krebs, Meißner '15	 Girlanda, Kievsky, Viviani '11 Krebs, Gasparyan, EE '12,'13 (short-range loop contrib. still missing)	— (preliminary)

Scheme dependence (unitary ambiguity) starts showing up at N^3LO: **2 phases** ($\bar{\beta}_8$, $\bar{\beta}_9$) in the long-range relativistic corrections **+ 3 off-shell short-range terms** in the $^1S_0$, $^3S_1$ and ε_1 channels.

Partial wave analysis of NN data

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

Convergence of the chiral expansion for np phase shifts [Λ = 450 MeV]



— Clear evidence of the parameter-free chiral 2π exchange (Roy-Steiner LECs)!

— Good convergence of the chiral expansion.

— Currently the most precise NN interaction available

Uncertainty quantification

In most cases, the uncertainty is dominated by truncation errors. Consider an observable X(p):

$$\boldsymbol{X(p) = c_0 + c_2 Q^2 + c_3 Q^3 + \ldots + c_i Q^i + c_{i+1} Q^{i+1} + \ldots} \quad \text{where} \quad \boldsymbol{Q = \max\left(\frac{\mathrm{p}}{\Lambda_\mathrm{b}}, \frac{\mathrm{M}_\pi^\mathrm{eff}}{\Lambda_\mathrm{b}}\right)}$$

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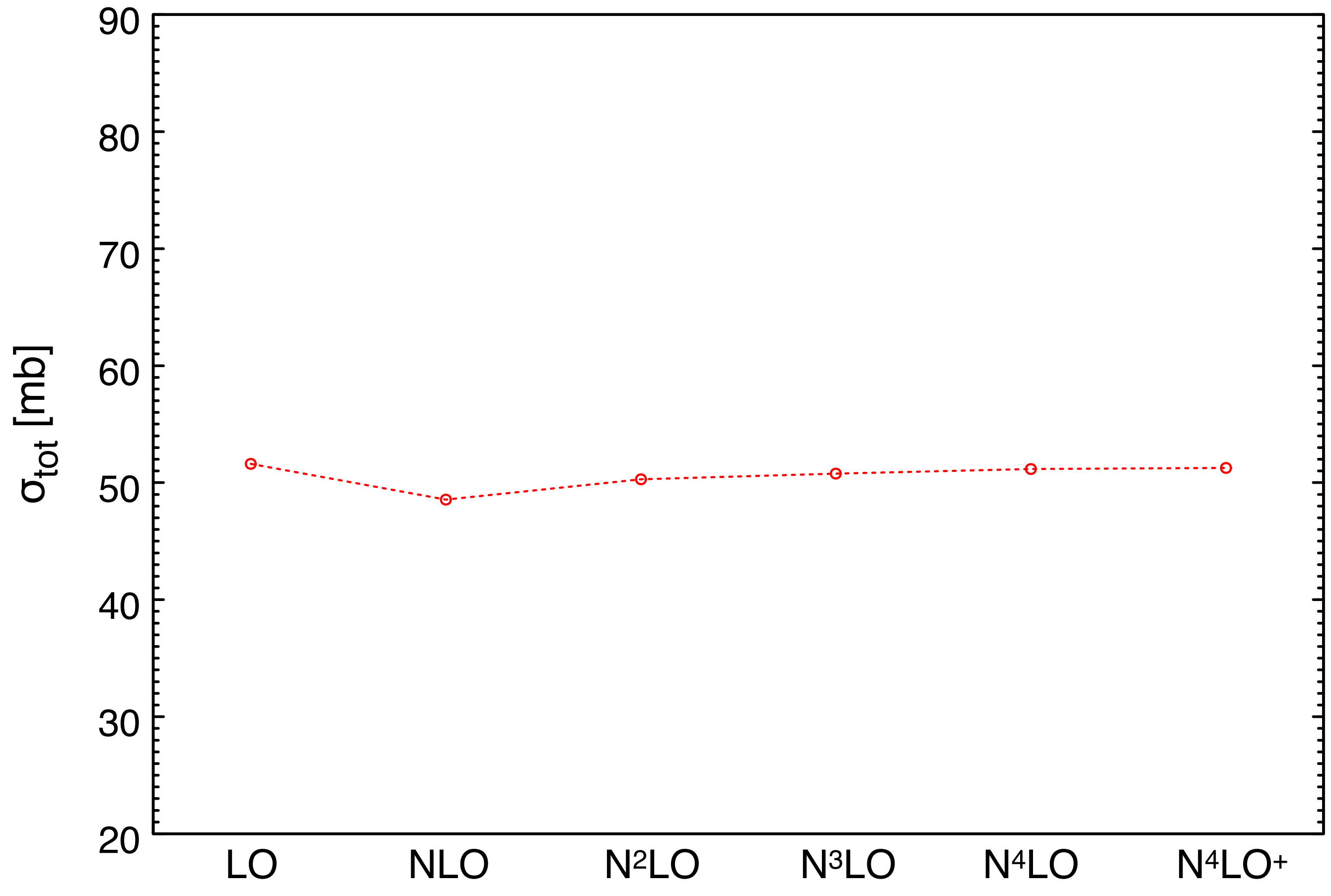
Neutron-proton total cross section at 150 MeV [Λ = 450 MeV]

$$\sigma_{\text{tot}} = 51.4_{\text{LO}} - 3.0_{\text{NLO}} + 1.7_{\text{N}^2\text{LO}} + 0.5_{\text{N}^3\text{LO}} + 0.4_{\text{N}^4\text{LO}} + 0.1_{\text{N}^4\text{LO+}}$$

$= 51.10(12)(12)(19)(6)$ mb to be compared with $\sigma_{\text{tot}}^{\text{exp.}} = 51.02 \pm 0.30$ mb

Lisowski et al. '82

Uncertainty quantification



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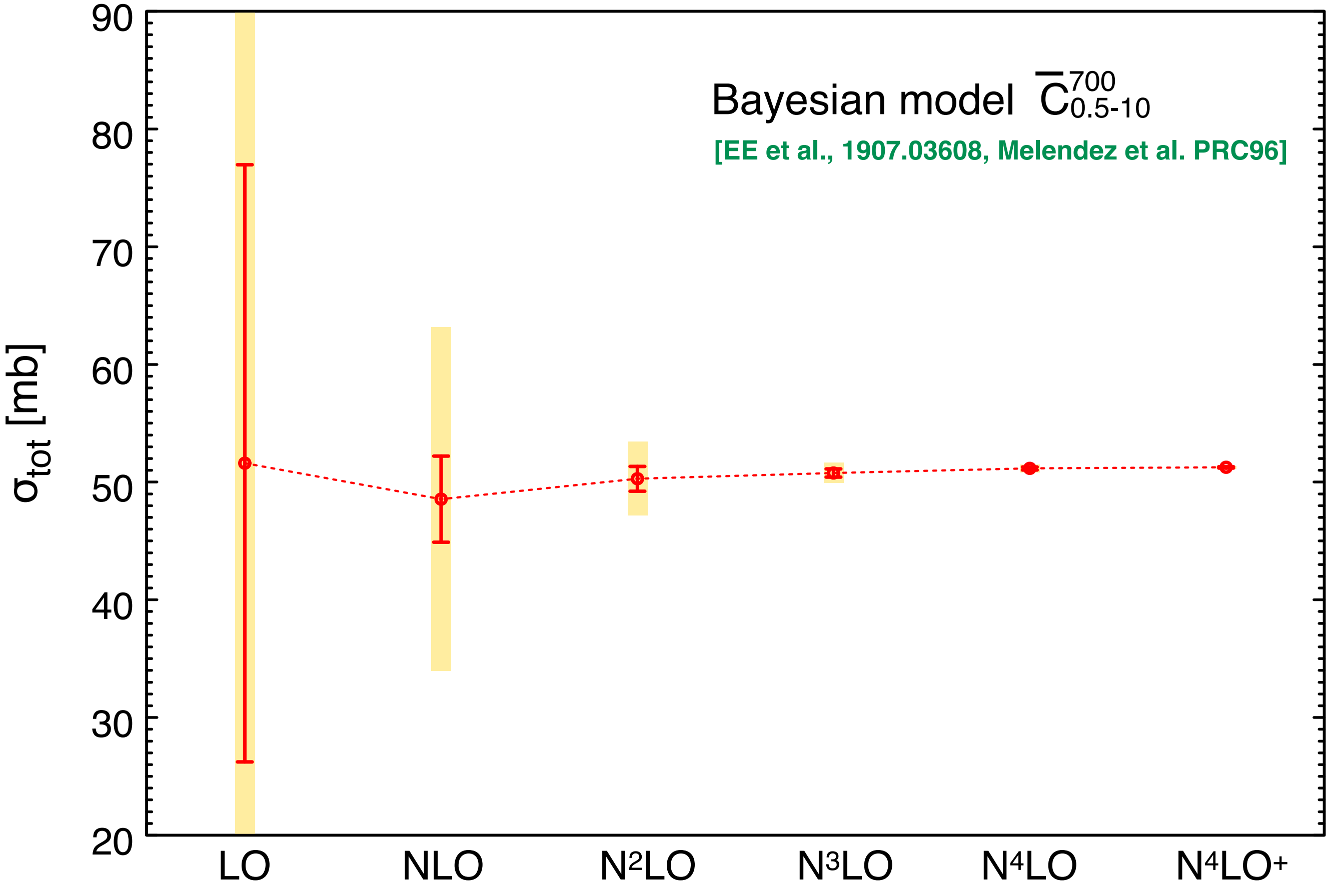

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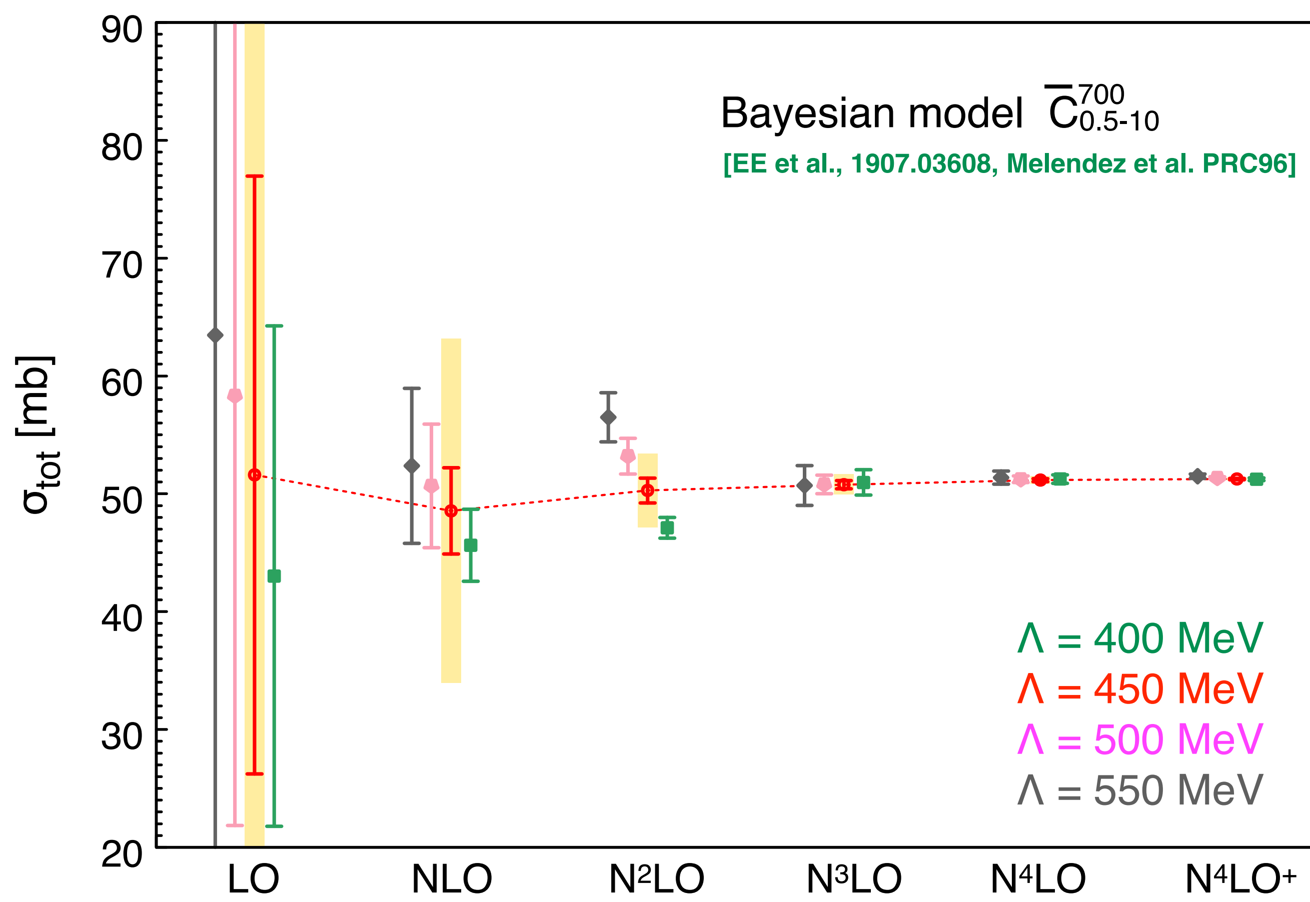

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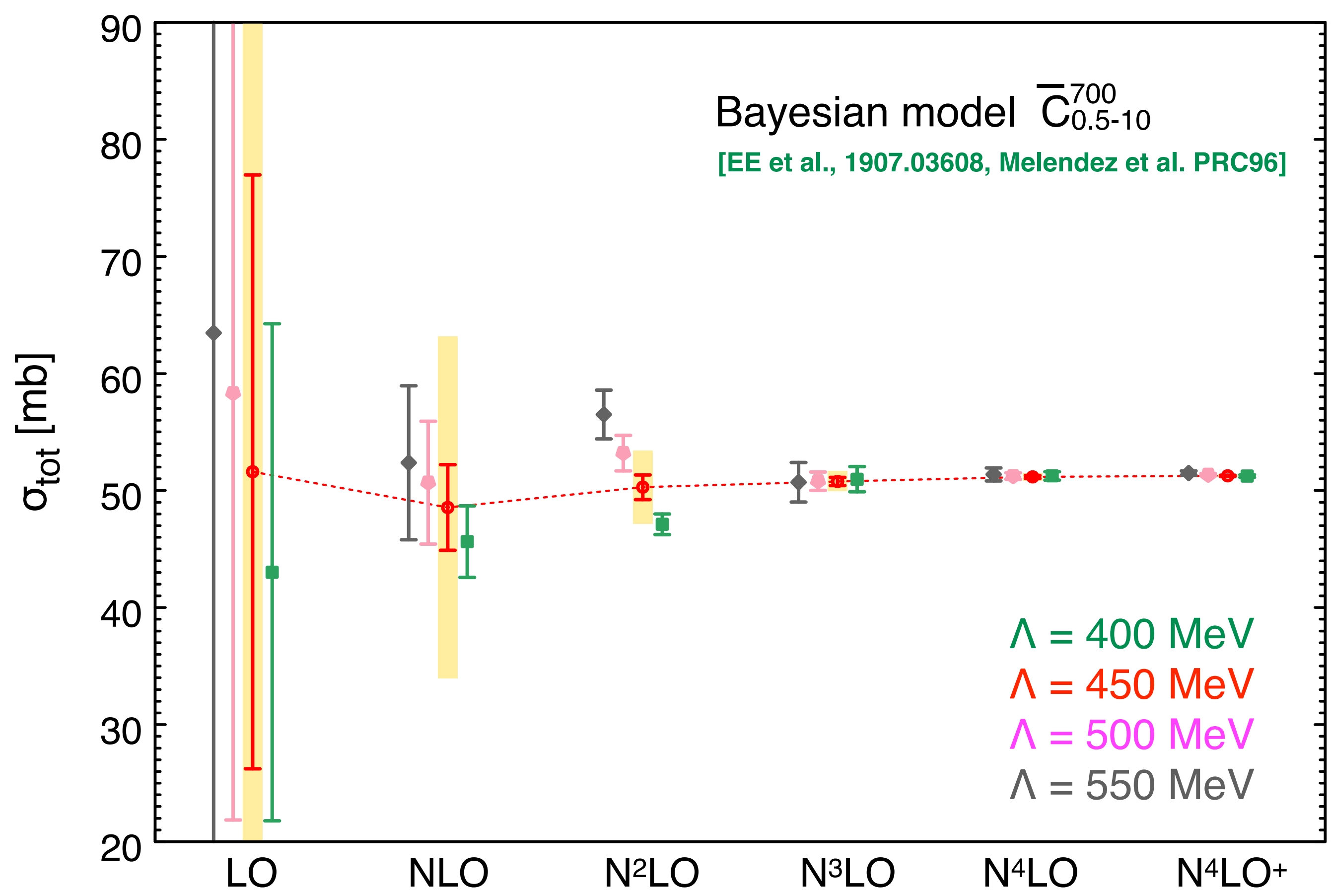
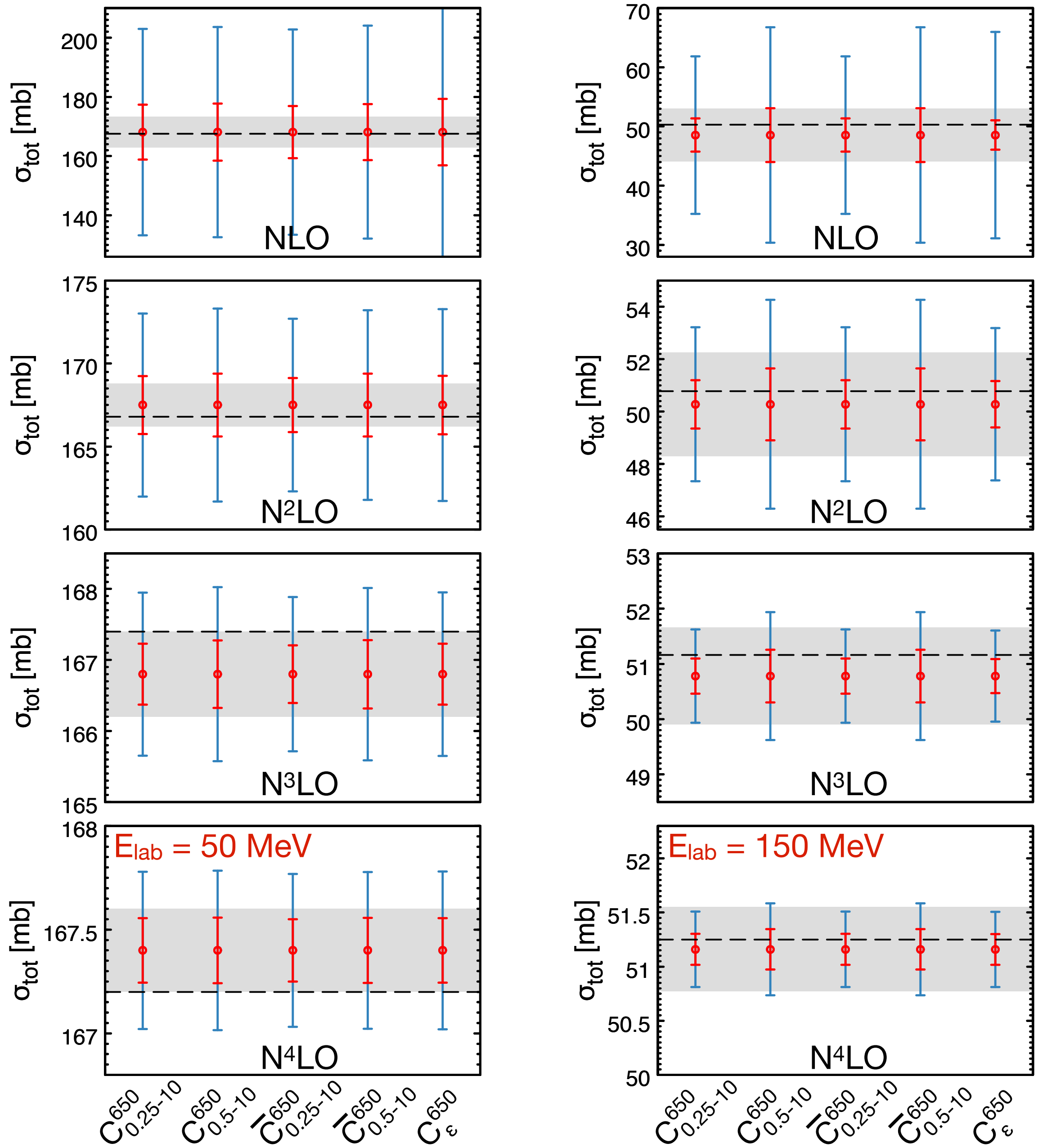


Lesson from the Bayesian analysis: can not trust LO perturbation theory.

For a related discussion see Furnstahl et al., PRC 92 (2015) 024005; Melendez et al., PRC 96 (2017) 024003

Uncertainty quantification



The dependence on the assumptions within a Bayesian model (priors) gets small at high orders

[Furnstahl et al. '15]

Figure taken from:

[EE et al., 1907.03608]

Electroweak currents

Kölling, EE, Krebs, Meißner '09,'12; Krebs, EE, Meißner '16,'19

see Hermann Krebs, Nuclear currents in chiral effective field theory, to appear in EPJA

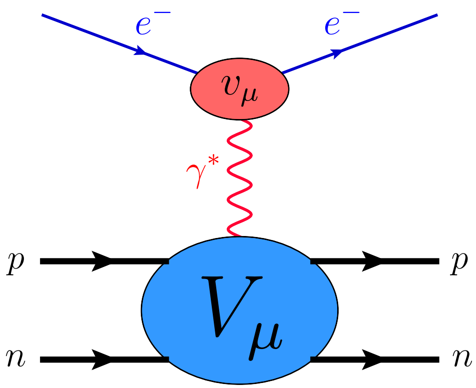
Current operators

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \;\to\; r'_\mu = R\, r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \;\to\; l'_\mu = L\, l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \;\to\; s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \;\to\; s' - i\,p' = L(s - i\,p)R^\dagger$$

- Decouple π's to get (nonlocal) nuclear $H_{\rm eff}[a, v, s, p]$ (MUT) & get currents via

$$V_\mu^a(\vec x\,) = \frac{\delta H_{\rm eff}}{\delta v_a^\mu(\vec x, t)}, \quad A_\mu^a(\vec x\,) = \frac{\delta H_{\rm eff}}{\delta a_a^\mu(\vec x, t)}$$ calculated at $a = v = p = 0,\ s = m_q$.



Park, Min, Rho '95
Pastore et al. (TOPT) '08 — '11
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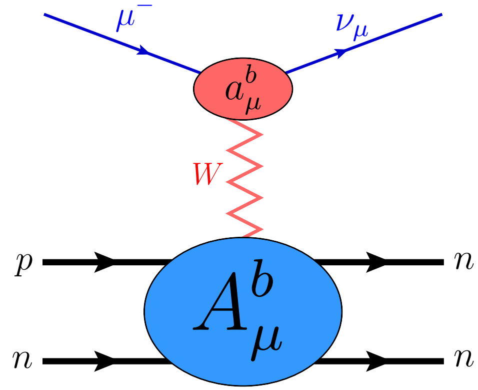

Park, Min, Rho '93
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Krebs, EE, Meißner (MUT) '17: complete (1 loop) & renormalized, also derived pseudoscalar currents

— about 250 topologies
— 2-loop/1-loop/tree for 1N/2N/3N operators

Also derived scalar currents, Krebs et al., in preparation

Current operators

$$\vec{k}\cdot\vec{A}^i(\vec{k},0) = \Big[H_{\rm str},\,A_0^i(\vec{k},0) - \frac{\partial}{\partial k_0}\Big(\vec{k}\cdot\vec{A}^i(k) + [H_{\rm str},\,A_0^i(k)] + im_q P^i(k)\Big)\Big] + im_q P^i(\vec{k},0)$$

$$\vec{k}\cdot\vec{V}^i(\vec{k},0) = \Big[H_{\rm str},\,V_0^i(\vec{k},0) - \frac{\partial}{\partial k_0}\Big(\vec{k}\cdot\vec{V}^i(k) + [H_{\rm str},\,V_0^i(k)]\Big)\Big]$$

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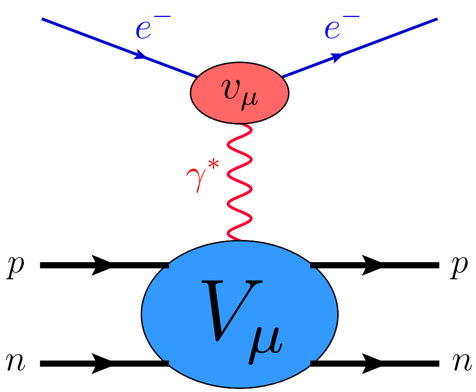

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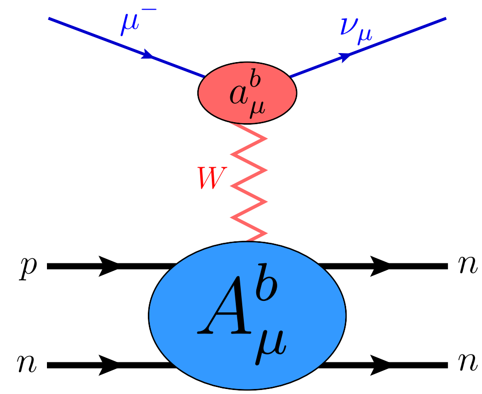

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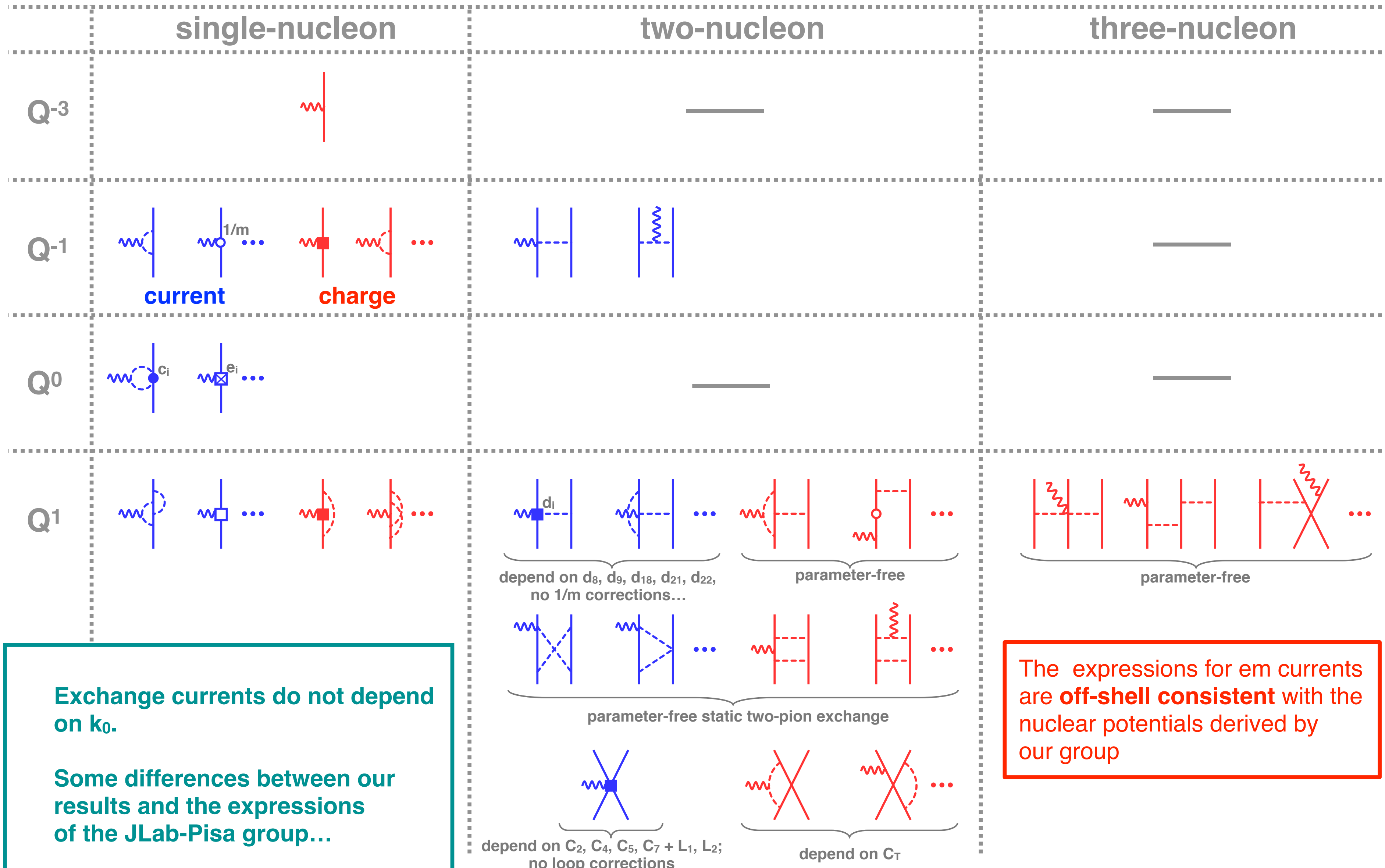
Chiral expansion of electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001; Krebs, EE, Meißner, Few Body Syst 60 (19) 31

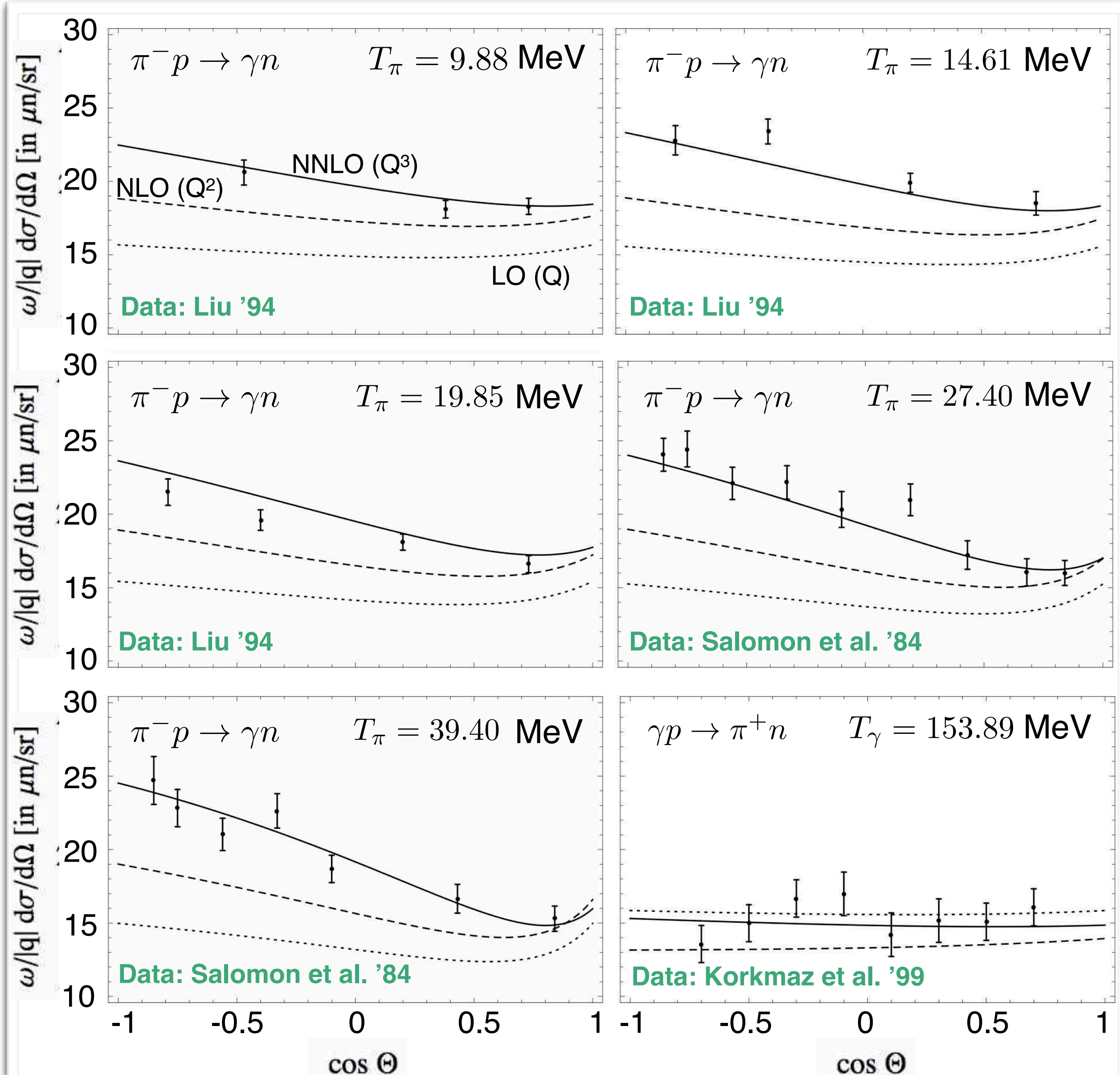


Exchange currents do not depend on k_0.

Some differences between our results and the expressions of the JLab-Pisa group…

The expressions for em currents are **off-shell consistent** with the nuclear potentials derived by our group

πN low-energy constants: HB



LECs entering the 1π current:

$$\bar{l}_6,\ \bar{d}_8,\ \bar{d}_9,\ \bar{d}_{18},\ \bar{d}_{21},\ \bar{d}_{22}$$

$\bar{l}_6$ - known from the π sector

$\bar{d}_{18}$ - known from GTD

$\bar{d}_{22}$ - from the axial radius:

$$\bar{d}_{22} = 2.2 \pm 0.2\ \mathrm{GeV}^{-2}$$

$\bar{d}_9,\ \bar{d}_{21},\ \bar{d}_{22}$ - contribute to charged pion photoproduction (radiative capture)

Fearing et al.'00
Till Wolf, master thesis, Bochum, 2013

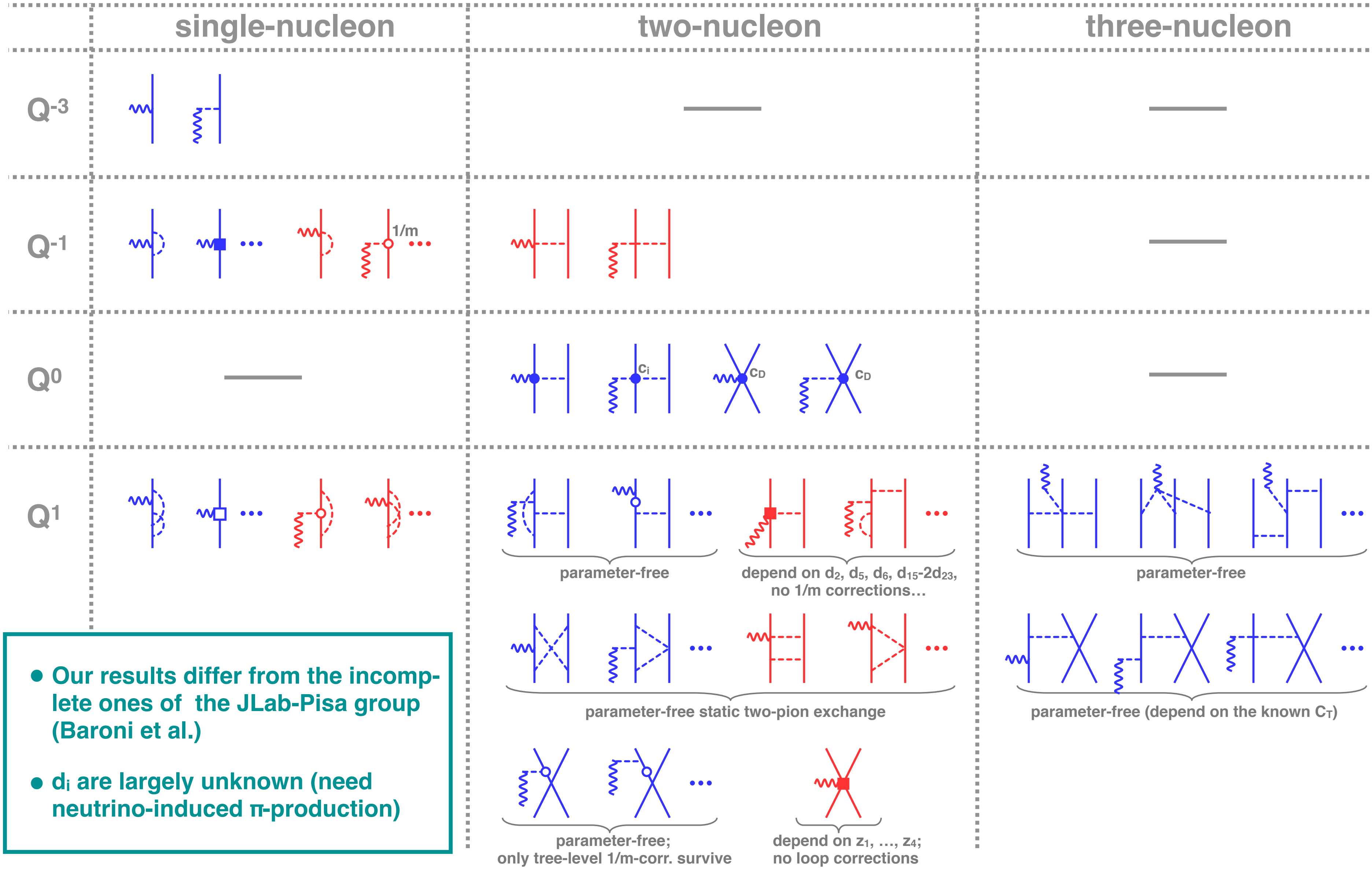
LEC [GeV⁻²]	Fearing *et al.*	Wolf
$\bar{d}_9$	2.5 ± 0.8	2.2 ± 0.9
$\bar{d}_{20}$	-1.5 ± 0.5	-3.2 ± 0.5
$2\bar{d}_{21} - \bar{d}_{22}$	5.7 ± 0.8	6.8 ± 1.0

Some d_i's have been determined by Gasparyan, Lutz '10 (ChPT + disp. relations)

Axial currents

Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators



- Our results differ from the incomplete ones of the JLab-Pisa group (Baroni et al.)
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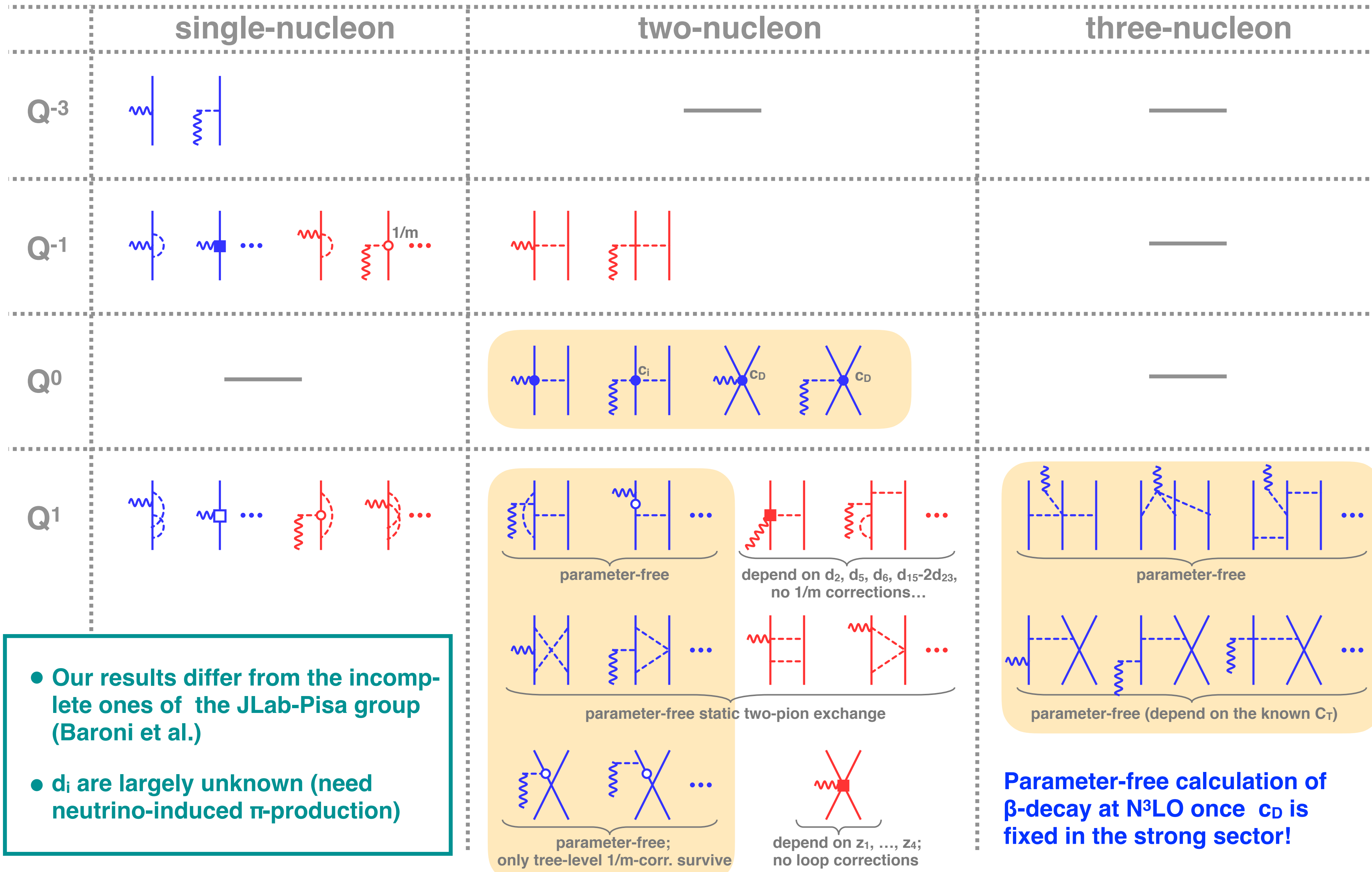

- Our results differ from the incomplete ones of the JLab-Pisa group (Baroni et al.)
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Parameter-free calculation of β-decay at N³LO once c_D is fixed in the strong sector!

Towards nuclear forces & currents beyond N²LO

still consistent beyond the NN system?

- **Introduce a cutoff Λ** to make the few-N Schrödinger equation well defined.
  - long range: $$\frac{1}{\vec{q}^2 + M_\pi^2} \longrightarrow \frac{1}{\vec{q}^2 + M_\pi^2} e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} \simeq \frac{1}{\vec{q}^2 + M_\pi^2} \left(1 + \text{short-range terms}\right)$$
  - short range: nonlocal Gaussian regulator

[Reinert, Krebs, EE, EPJA 54 (2018) 88]

Towards consistent 3NF and MECs

Hermann Krebs, EE, in preparation

Regularization of the 3NF, 4NF and MEC at N^{3}LO and beyond is nontrivial!

Standard approach: Take expressions obtained in DR and multiply with some cutoff: finite-Λ artifacts are expected to be removed by contacts terms (adjusted to data). Is it true?

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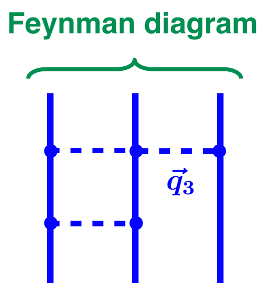

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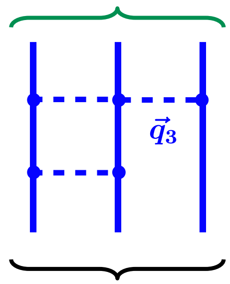
Feynman diagram



(finite in DR)

Linearly divergent: $\propto \Lambda \dfrac{q_3^a \, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$

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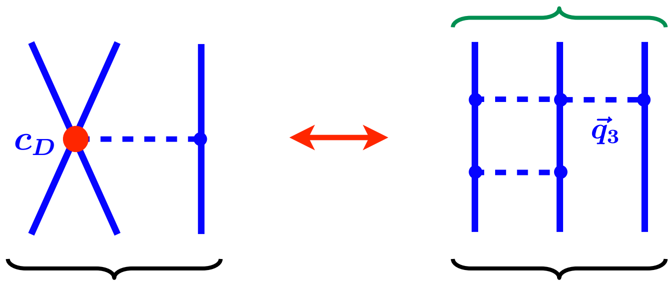

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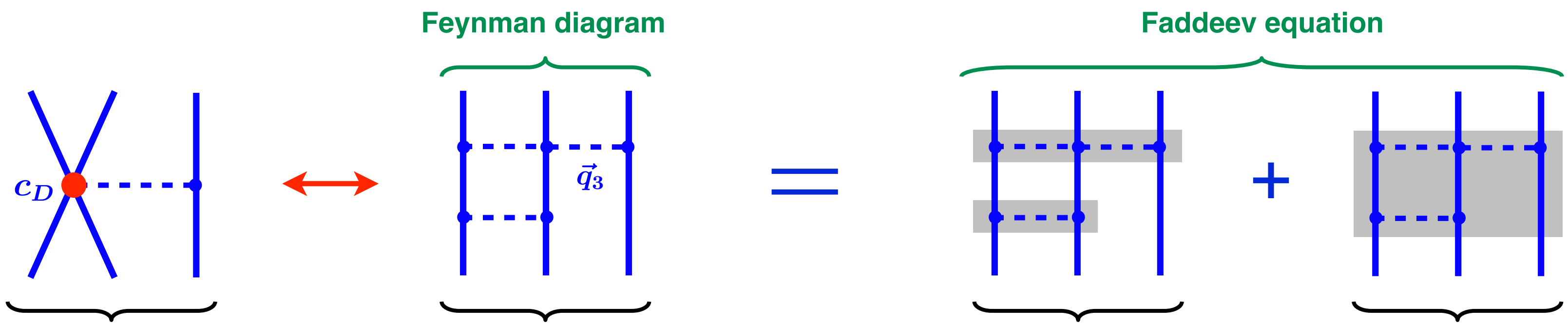

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[Bernard et al.’08]

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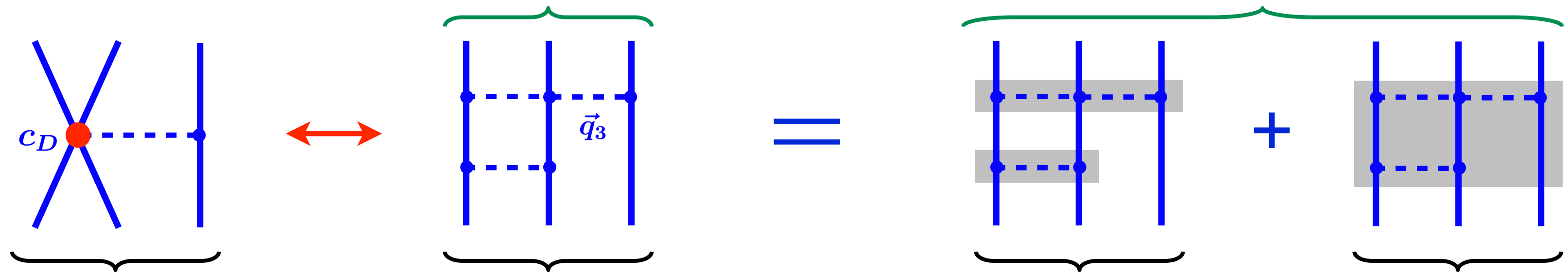

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Renormalization of the iteration requires χ-symmetry breaking counter terms!

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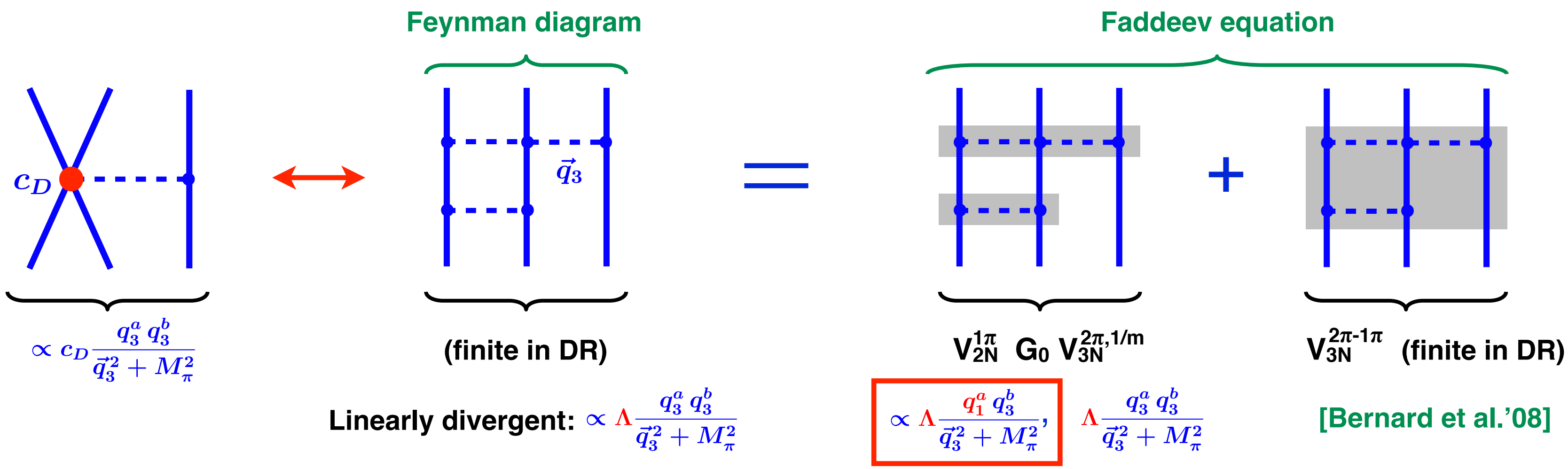

Renormalization of the iteration requires χ-symmetry breaking counter terms!

- The problematic divergence cancels out if $\mathbf{V}_{3N}^{2\pi\text{-}1\pi}$ is calculated using cutoff regularization.
- Irrelevant for $\mathbf{V}_{2N}$: momentum dependence of 2N contacts is not constrained by χ-symm.
- Regularization of $\mathbf{V}_{3N}$ must be **consistent** to maintain matching (of finite pieces).
- Can one enforce renormalizability of V_{3N} (i.e. remove problematic divergences) by systematically exploiting unitary ambiguities? This indeed seems to be possible!

Regularization and the chiral symmetry

The same problems affect loop contributions to the exchange charge/current operators.

Is it enough to recalculate all loop contributions to the 3NF/exchange currents by modifying the pion propagators via $(\vec{q}^{\,2} + M_\pi^2)^{-1} \longrightarrow \exp[-(\vec{q}^{\,2} + M_\pi^2)/\Lambda^2]\,(\vec{q}^{\,2} + M_\pi^2)^{-1}$?

Solution: higher-derivative regularization [Slavnov, Nucl. Phys. B31 (1971) 301]

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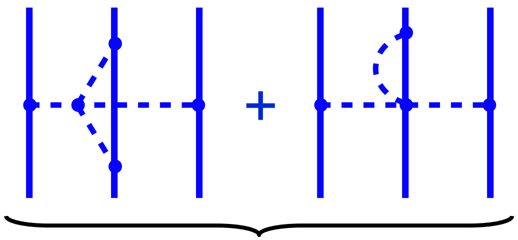
Not quite… Have to ensure that regularization maintains the chiral symmetry.

$$U(\vec{\pi}) = 1 + \frac{i}{F_\pi}\vec{\tau}\cdot\vec{\pi} - \frac{1}{2F_\pi^2}\vec{\pi}^{\,2} - \frac{i\alpha}{F_\pi^3}(\vec{\tau}\cdot\vec{\pi})^3 - \frac{8\alpha - 1}{8F_\pi^4}\vec{\pi}^{\,4} + \ldots$$



is independent on α in DR, but not of one uses (naive) cutoff regularization

All observables should be α-independent.

Solution: higher-derivative regularization [Slavnov, Nucl. Phys. B31 (1971) 301]

(designed to coincide with the employed local regularization in the NN sector)

Summary and outlook

- Precision calculations of few-N systems at $N^{3,4}LO$ will challenge chiral EFT! (especially in the 3N continuum)
- Naive regularization of 3NF and MECs, calculated using DR, should **NOT** be applied beyond N^2LO!
- Need to recalculate loop contributions to 3NF and MECs using regularization which **maintains the chiral symmetry** and **is consistent with the NN force** (in progress…)

Thanks to:

— my Bochum collaborators on these topics:
Vadim Baru, Arseniy Filin, Ashot Gasparyan, Jambul Gegelia
Hermann Krebs, Daniel Möller, Patrick Reinert

— and the whole LENPIC

LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration