Nuclear Matrix Elements for Neutrinoless Double Beta Decay from Lattice QCD

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July 17, 2019

ECT* Workshop on Progress and Challenges in the Theory of Neutrinoless Double Beta Decay

Background

$2\nu\beta\beta$: $nn \to ppee\overline{\nu}\overline{\nu}$ from Lattice QCD [Tiburzi et al., PRD 96 (2017) 054505]

$0\nu\beta\beta$: $\pi \to \pi ee$ from Lattice QCD

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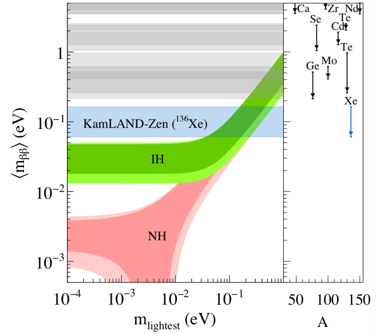
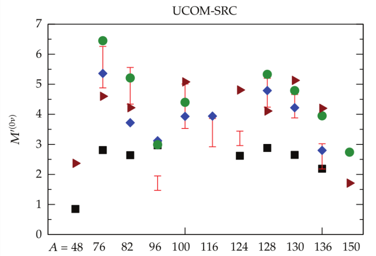
Constraints on $m_{\beta\beta}$ from $T_{1/2}^{0\nu}$ Limits

$$\left(T_{1/2}^{0\nu}\right)^{-1} \propto \left|m_{\beta\beta}\right|^2 G^{0\nu} \left|M^{0\nu}\right|^2$$



(a) KamLAND-Zen [arXiv:1605.02889]



(b) Giuliani et al. [Adv. High Energy Phys. 857016 (2012)]

- $\sim 2-3\times$ spread in different model calculations of same NME
- Realistically, lattice QCD could:
  1. Compute inputs to EFT ($\pi\pi \to ee$, $n\pi \to pee$, $nn \to ppee$ vertices)
  2. Directly test nuclear models for small nuclei

Lattice QCD in One Slide

Basic idea: compute discretized (Euclidean) PI using a computer

$$\langle \mathscr{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathscr{D}U \mathscr{D}\psi \mathscr{D}\overline{\psi} \mathscr{O}(U, \overline{\psi}, \psi) e^{-S_G[U] - \overline{\psi} \not{D}(U) \psi}$$

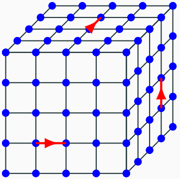
A typical lattice calculation proceeds in two steps:

1. Sample gauge field $\{U_i\}_{i=1}^N$ according to $P(U) \propto e^{-S(U)}$
2. Compute $\langle \mathscr{O} \rangle = \frac{1}{N} \sum_{i=1}^N \mathscr{O}(U_i) + \mathcal{O}(1/\sqrt{N})$

Use Wick's theorem to relate correlation functions to lattice propagators

$$\left\langle \overline{d}\gamma_5 u(y) \overline{u}\gamma_5 d(x) \right\rangle = \mathrm{Tr}\left[\gamma_5 S_u\left(x \to y\right) \gamma_5 S_d\left(y \to x\right)\right],$$

which are obtained by numerically solving the Dirac equation $\not{D}\psi = \phi$



ψ_x on sites, $U_{x,\mu}$ on links

- Solving Dirac equation is expensive (large, sparse matrix inversion)
- Many-body systems are hard
  1. Factorial growth of Wick contractions
  2. Signal-to-noise problem

Background

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Second Order Weak Processes on the Lattice

- Standard lattice QCD formalism [Bai et al., PRL 113 (2014) 112003]:
  1. Compute $\mathcal{M} = \int d^4x\, d^4y \, \langle f|\, T\{j_\mu(x) j_\mu(y)\} \,|i\rangle$ non-perturbatively
  2. Extract $M = \sum_n \frac{\langle f|j_\mu|n\rangle\langle n|j_\mu|i\rangle}{E_n - (E_i+E_f)/2}$ by fitting to lattice data
- ME can be found from slope of linear fit at large T:

$$C_{i\to f}(T) = \sum_{t_1=0}^{T}\sum_{t_2=0}^{T} \left\langle \mathscr{O}_f(t) T\left\{j_\mu(t_2) j_\mu(t_1)\right\} \mathscr{O}_i^\dagger(0)\right\rangle$$

$$= \sum_{t_1=0}^{T}\sum_{t_2=0}^{T}\sum_{l,m,n} \frac{\left\langle 0\middle|\mathscr{O}_f\middle|l\right\rangle\left\langle l\middle|j_\mu\middle|m\right\rangle\left\langle m\middle|j_\mu\middle|n\right\rangle\left\langle n\middle|\mathscr{O}_i^\dagger\middle|0\right\rangle}{2E_l \cdot 2E_m \cdot 2E_n} e^{-E_l t} e^{(E_m - E_n)t_1} e^{(E_l - E_m)t_2}$$

$$\boxed{\simeq \frac{Z_i^\dagger Z_f}{4E_i^2} e^{-E_i t} \sum_{m=0}^{\infty} \frac{1}{2E_m} \frac{\langle f|\, j_\mu \,|m\rangle \langle m|\, j_\mu \,|i\rangle}{E_m - E_i} \left(T + \frac{e^{-(E_m - E_i)T} - 1}{E_m - E_i} \right)}$$

- States with $E_m \leq E_i$ require special care:
  1. $E_m < E_i$: $e^{-(E_m - E_i)T} \to \infty$ as $T \to \infty$
  2. $E_m \approx E_i$: $C(T) \propto T^2$ as $T \to \infty$
- Can remove these effects by computing relevant MEs $\langle f|j_\mu|m\rangle\langle m|j_\mu|i\rangle$

Setup

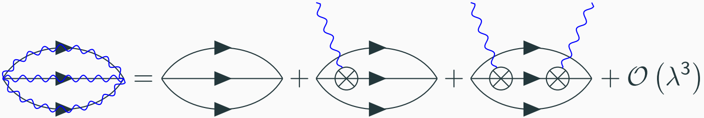
- Lattice calculation:
  - $nn \to pp$ matrix element on $32^3 \times 48$ Wilson-clover ensemble
  - $m_u = m_d = m_s^{\text{phys}}$, giving $m_\pi \approx 800$ MeV, $m_N \approx 1600$ MeV
- $2\nu\beta\beta$ decay mechanism is well-understood: $(T_{1/2}^{2\nu})^{-1} = G^{2\nu} \left|M_{GT}^{2\nu}\right|^2$, with

$$\frac{1}{6} M_{GT}^{2\nu} = \sum_{n=0}^{\infty} \frac{\langle f| A_3 |n\rangle \langle n| A_3 |i\rangle}{E_n - (E_i + E_f)/2} = \beta_A^{(2)} - \frac{|\langle pp| A_3 |d\rangle|^2}{E_{nn} - E_d},$$

- $\beta_A^{(2)}$: isotensor axial polarizability
- Trick: compute *compound propagators* in background axial field $\propto \lambda$

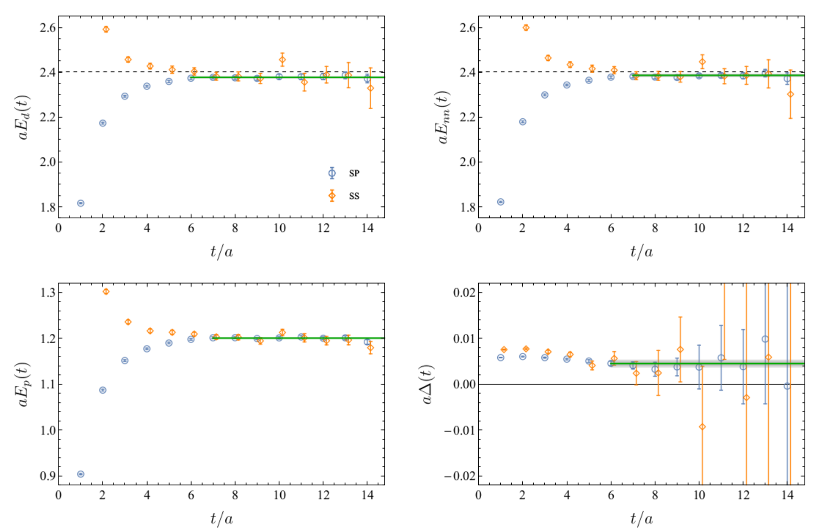
$$S_\lambda(x,y) = S(x,y) + \lambda \int d^4z\, S(x,z) A_3(z) S(z,y) + \mathcal{O}(\lambda^2)$$

- $\mathcal{O}(\lambda^n)$ compound two-point function has n axial current insertions:

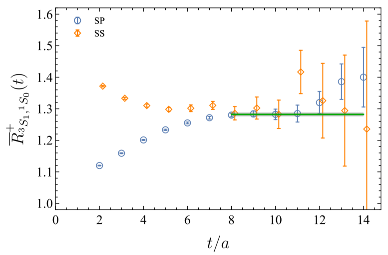


Deuteron and Dinucleon Masses at $m_\pi \approx 800$ MeV

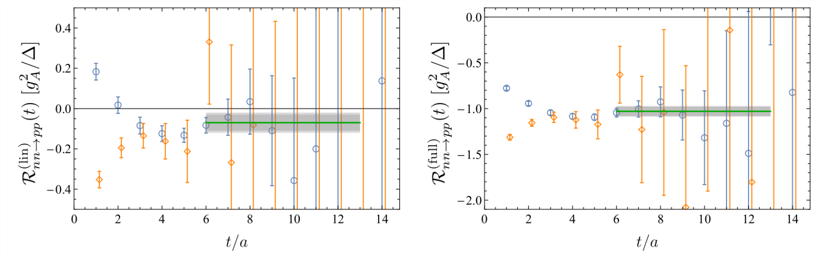
$|pp\rangle$, $|d\rangle = |pn\rangle$, and $|nn\rangle$ are bound states $\Rightarrow$ FV corrections $\sim e^{-ML}$



$\langle pp|A_3|d\rangle$, $\beta_A^{(2)}$, and $M_{GT}^{2\nu}$

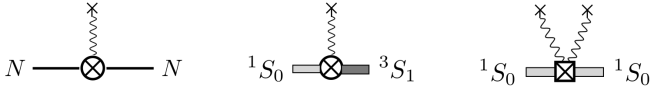


- Top: $|\langle pp|A_3|d\rangle|^2$
- Bottom left: $\beta_A^{(2)}$
  ($^1S_0$ isotensor axial polarizability)
- Bottom right: $M_{GT}^{2\nu}$

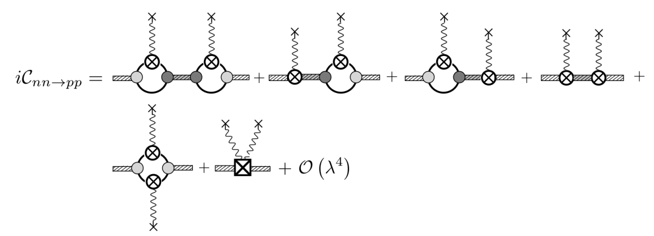


Matching to Pionless EFT

- Match to pionless EFT in background axial field
- Basic vertices (left: g_A, middle: $\mathbb{L}_{1,A}$, right: $\mathbb{H}_{2,S}$):



- $C_{nn\to pp}$ at $\mathcal{O}(\lambda^2)$:



- Result: $\mathbb{H}_{2,S} = 4.7(1.3)(1.8)$ fm
- Future calculations in EFT framework could relate to ME of larger nuclei

Summary of Results

- Successful first calculation with unphysical quark masses...
- Key results ($\Delta \equiv E_{nn} - E_d$):

$$\frac{\Delta}{g_A^2}\frac{M_{GT}^{2\nu}}{6} = -1.04(4)(4)$$
$$\frac{\Delta}{g_A^2}\frac{\left|\langle pp|A_3|d\rangle\right|^2}{\Delta} = 1.00(3)(1)$$
$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8)\ \mathrm{fm}$$

- Moving toward physical point introduces new challenges:
  - $|nn\rangle$ and $|pp\rangle$ unbound $\Rightarrow$ power law finite volume effects
  - Other intermediate states may become important
  - At some point light pions must be included in EFT
- ...however, compound propagators do not easily generalize to $0\nu\beta\beta$
  - Current insertions are connected by internal Majorana neutrino propagator
  - **New methods needed!**

Background

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$0\nu\beta\beta$: $\pi \to \pi ee$ from Lattice QCD

Setup

- Production calculations of:
  1. Leading order short-distance $\pi^- \to \pi^+ e^- e^-$ matrix elements
  2. Long-distance light Majorana exchange amplitude for $\pi^- \to \pi^+ e^- e^-$
- Series of $N_f = 2 + 1$ domain wall fermion ensembles
- Coulomb gauge-fixed wall source propagators computed on all time slices

Ensemble	a (fm)	L	T	am_s	am_{ud}	m_π (MeV)
24I	0.11	24	64	0.04	0.005	339.6
					0.01	432.2
32I	0.08	32	64	0.03	0.004	302.0
					0.006	359.7
					0.008	410.8

Table 1: Ensembles used for $0\nu\beta\beta$ calculations

SM Long-Distance, Light Majorana Exchange Mechanism

- For typical lattice scales decay is mediated by effective EW Hamiltonian

$$H_W = 2\sqrt{2} G_F V_{ud} \left(\overline{u}_L \gamma^{\mu} d_L \right) \left(\overline{e}_L \gamma_{\mu} \nu_{eL} \right)$$

- Matrix element decomposes into **leptonic** and **hadronic** pieces

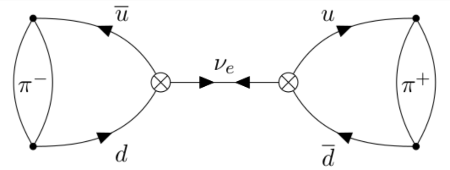
$$\int d^4x\, d^4y \left\langle fee \middle| T\{H_W(x) H_W(y)\} \middle| i \right\rangle = 4 m_{\beta\beta} G_F^2 V_{ud}^2 \int d^4x\, d^4y\, \boldsymbol{L_{\alpha\beta}(x,y) H^{\alpha\beta}(x,y)}$$

$$\boldsymbol{L_{\alpha\beta}(x,y) \equiv \overline{e}_L(p_1) \gamma_\alpha \gamma_\beta e_L^C(p_2) S_\nu(x,y) e^{-ip_1 \cdot x} e^{-ip_2 \cdot y}}$$

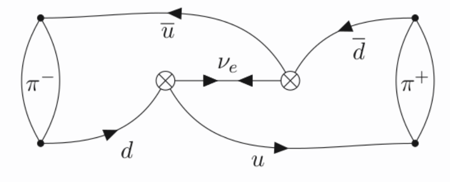
$$\boldsymbol{H_{\alpha\beta}(x,y) \equiv \langle f | T \left\{ \overline{u}_L(x) \gamma_\alpha d_L(x) \overline{u}_L(y) \gamma_\beta d_L(y) \right\} | i \rangle}$$

- Develop lattice methods by first computing $\pi^- \to \pi^+ e^- e^-$ amplitude
  - Simple Wick contractions
  - No signal-to-noise problem
- Related LD $\pi^- \pi^- \to e^- e^-$ calc. by Feng et al. [PRL 122 (2019) 022001]

Wick Contractions for $\pi^- \to \pi^+ e^- e^-$ Transition



(a) Type ①



(b) Type ②

$$① = \mathrm{Tr}\left[S_u^\dagger(t_- \to x)\gamma_\alpha (1-\gamma_5) S_d(t_- \to x)\right] \cdot \mathrm{Tr}\left[S_u^\dagger(t_+ \to y)\gamma_\beta (1-\gamma_5) S_d(t_+ \to y)\right]$$

$$② = \mathrm{Tr}\left[S_u^\dagger(t_+ \to x)\gamma_\alpha (1-\gamma_5) S_d(t_- \to x) S_u^\dagger(t_- \to y)\gamma_\beta (1-\gamma_5) S_d(t_+ \to y)\right]$$

Lattice Technology I: Second Order Weak Formalism for $0\nu\beta\beta$

- Repeat derivation including (massless) continuum neutrino propagator:

$$C_{i\to f}(T) = \sum_{t_1=0}^{T}\sum_{t_2=0}^{T}\sum_{\vec{x},\vec{y}}\int \frac{d^4q}{(2\pi)^4}\frac{e^{iq\cdot(x-y)}}{q^2}\Gamma_{\alpha\beta}\langle \mathcal{O}_f(t)T\{j_\alpha(x)j_\beta(y)\}\mathcal{O}_i^\dagger(0)\rangle$$

$$= \sum_{t_1=0}^{T}\sum_{t_2=0}^{T}\sum_{\vec{x},\vec{y}}\int \frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot(\vec{x}-\vec{y})}}{2|\vec{q}|}e^{-|\vec{q}|(t_x-t_y)}\Gamma_{\alpha\beta}\langle \mathcal{O}_f(t)T\{j_\alpha(x)j_\beta(y)\}\mathcal{O}_i^\dagger(0)\rangle$$

$$\simeq \frac{Z_i^\dagger Z_f}{4E_i^2}e^{-E_i t}\int\frac{d^3q}{(2\pi)^3}\sum_{\vec{x},\vec{y},m}\frac{\Gamma_{\alpha\beta}\langle f|\,j_\alpha(\vec{x})\,|m\rangle\langle m|\,j_\beta(\vec{y})\,|i\rangle\, e^{i\vec{q}\cdot(\vec{x}-\vec{y})}}{2E_m|\vec{q}|(E_m+|\vec{q}|+m_e-E_i)}\left(T+\frac{e^{-(E_m+|\vec{q}|+m_e-E_i)T}-1}{E_m+|\vec{q}|+m_e-E_i}\right)$$

- Can still extract $0\nu\beta\beta$ ME from asymptotic linear behavior at large T
- $\int d^3q$ and $\sum_{\vec{x},\vec{y}}$ are coupled: more difficult to isolate contributions from individual long-distance intermediate states
  - In principle need full $\vec{p}$ dependence of $\Gamma_{\alpha\beta}\langle f|j_\alpha|m(\vec{p})\rangle\langle m(\vec{p})|j_\beta|i\rangle$
  - $E_m < E_i$: still expect divergence as $T\to\infty$
  - No longer clear that $C(T)\propto T^2$ as $T\to\infty$ from states with $E_m\approx E_i$
- Long-distance intermediate states which may require special consideration in this calculation are $\langle 0|H_W|\pi\rangle$ and $\langle\pi|H_W|\pi\rangle$

Lattice Technology II: Continuum Neutrino Propagator

- Discrete neutrino propagator introduces power-law finite volume effects
- We mitigate this by directly using regulated continuum propagator

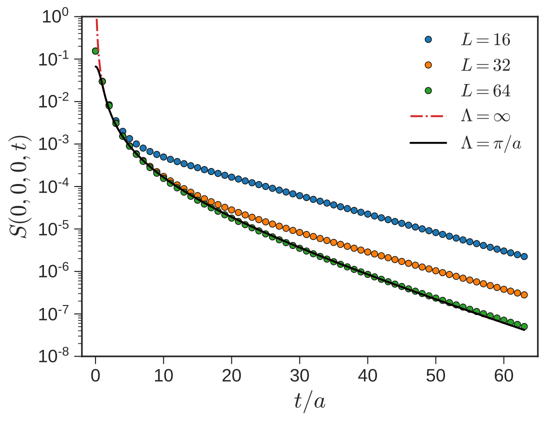


Figure 1: Lattice scalar propagator ($T=\infty$), continuum scalar propagator, and UV-regulated continuum propagator (Gaussian cutoff), with $am_\nu = 0.1$.

Lattice Technology III: Exact Double Current Integration

- Fully utilizing gauge configurations important for suppressing noise
- Build Wick contractions using 1D FFTs [Microw. Opt. Tech. Lett. 31, 28 (2001)]:
  1. Construct partially contracted "neutrino blocks" in $\mathcal{O}(V \log V)$:

$$B_\mu(x; t_1, t_2) = \int d^3y \, L_{\mu\nu}(x-y) \left[S_u^\dagger(t_1 \to y) \gamma_\nu (1-\gamma_5) S_d(t_2 \to y) \right]$$
$$= \mathscr{F}^{-1} \left[\mathscr{F}(L_{\mu\nu}) \cdot \mathscr{F}\left(S_u^\dagger \gamma_\nu (1-\gamma_5) S_d \right) \right]$$

  2. Contract remaining indices in $\mathcal{O}(V)$

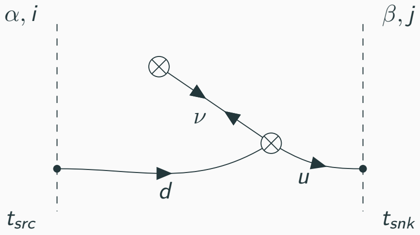


Figure 2: Neutrino block

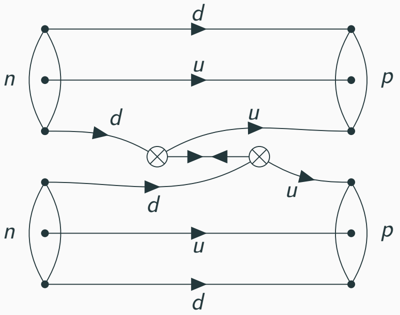


Figure 3: $nn \to ppee$

Unintegrated LD Four-Point Function

$$C_{n=0}\left(|t_x - t_y|\right) = \frac{m_\pi^2 f_\pi^2}{4 Z_A^2} \sum_{\vec{x}} \sum_{\vec{y}} \frac{1}{4\pi^2 |x-y|^2} \left(1 - e^{-\frac{\pi^2 |x-y|^2}{4}}\right) e^{(m_\pi - m_e)|t_x - t_y|}$$

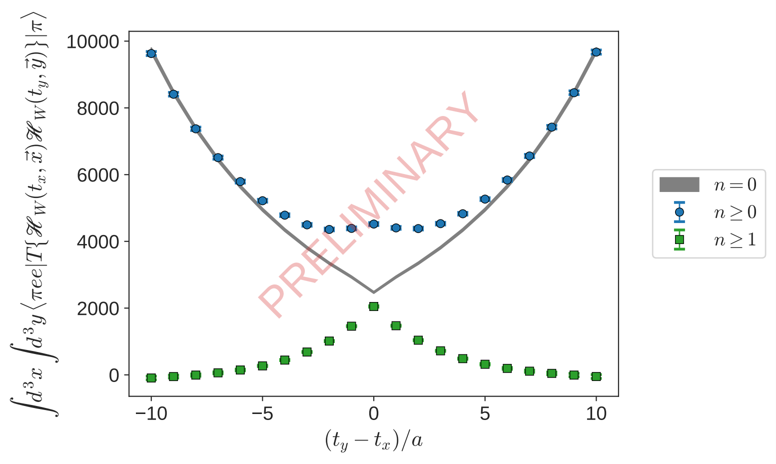


Figure 4: 24I, $m_\pi \approx 430$ MeV

Integrated LD Four-Point Function

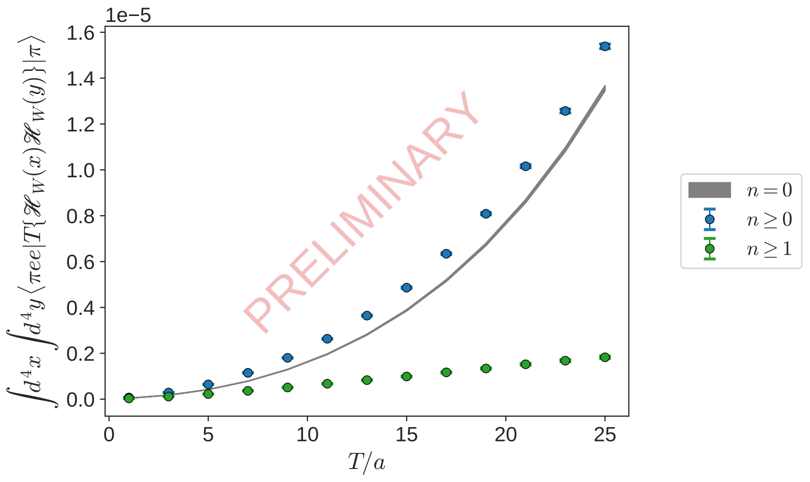
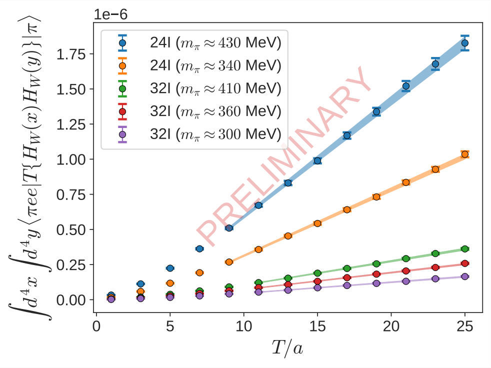


Figure 5: 24I, $m_\pi \approx 430$ MeV

- $n \geq 1$ signal appears linear at large T

Integrated LD Four-Point Function



- Next steps:
  1. Does $\langle \pi | H_W | \pi \rangle$ $(n = 1)$ contribution also need to be treated separately?
  2. Can probably be addressed using known χPT for pion vector form factor
  3. Continuum / chiral extrapolation $\to g_\nu^{\pi\pi}$ [Cirigliano et al., PRC 97 (2018) 065501]

Ensemble	m_π (MeV)	$\langle \pi | H_W | 0 \rangle \langle 0 | H_W | \pi \rangle$	$\int \frac{d^3q}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{\langle \pi | H_W | n \rangle \langle n | H_W | \pi \rangle}{2E_n |\vec{q}| (E_n + |\vec{q}| + m_e - m_\pi)}$
24I	430	$2.40(1) \times 10^{-4}$	$1.14(3) \times 10^{-3}$
24I	340	$1.29(1) \times 10^{-4}$	$6.5(1) \times 10^{-4}$
32I	410	$6.1(1) \times 10^{-5}$	$5.6(1) \times 10^{-4}$
32I	360	$4.29(3) \times 10^{-5}$	$4.1(1) \times 10^{-4}$
32I	300	$2.75(2) \times 10^{-5}$	$2.6(1) \times 10^{-4}$

Next Step: Baryonic / Nuclear Systems

- Working to implement $nn \to ppee$
  - We have developed automatic Wick contraction code
- Could in principle also consider $n\pi \to pee$
- Challenging for different reasons:
  - $n\pi \to pee$: disconnected diagrams
  - $nn \to ppee$: ~ 600 Wick contractions, expect signal-to-noise problem

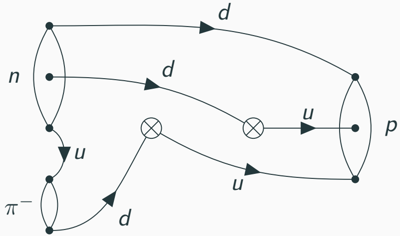


Figure 6: $n\pi \to pee$

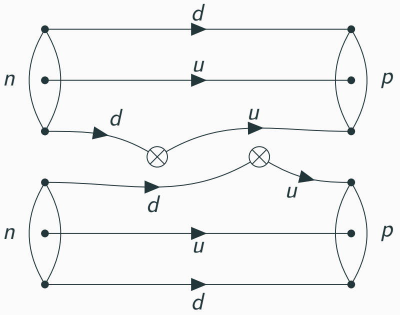


Figure 7: $nn \to ppee$

Conclusions

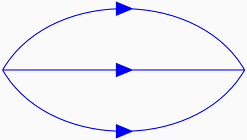
- We have previously computed the $2\nu\beta\beta$ decay amplitude for the process $nn \to ppee\overline{\nu}_e\overline{\nu}_e$
- We have also recently explored techniques for computing long-distance contributions to $0\nu\beta\beta$ decays
- In some cases, we have improved on these techniques:
    1. UV-regulated continuum neutrino propagator
    2. Explicit integration over locations of both current insertions via FFTs
- We have recently finished computing full LD (light Majorana exchange) and SD contributions to $\pi^- \to \pi^+ e^- e^-$ on a series of DWF ensembles
    - SD: renormalization in $\overline{\mathrm{MS}}$
    - LD: matching to χPT $\to$ extract $g_\nu^{\pi\pi}$
- Beginning to compute SD and LD $nn \to ppee$ amplitudes

Thank you!

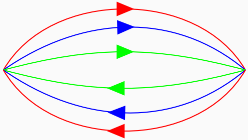
Extra slides

Challenges for Many-Body Systems

- Two major obstacles to lattice QCD calculations of many-body systems:
    1. Factorial growth of Wick contractions
    2. Signal-to-noise problem
- Cost of contractions can be improved with better algorithms
- SNR issue is harder (LePage, 1980's):



$C(t) \sim e^{-m_n t}$



$C^\dagger C(t) \sim e^{-3m_\pi t}$

$$\mathrm{SNR}(t) \sim \frac{C(t)}{\sqrt{C^\dagger C(t)}} \sim \exp\left(-A\left(m_n - \frac{3}{2}m_\pi\right)t\right)$$

- Mitigated by simulating at heavier-than-physical quark masses

Challenges for Many-Body Systems

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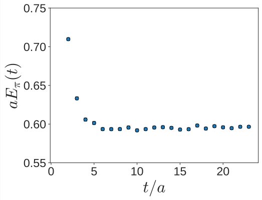
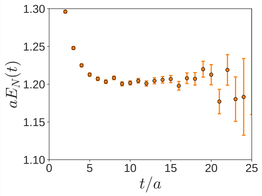
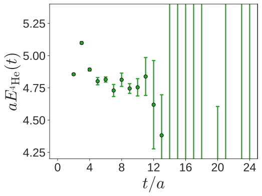


(a) Pion



(b) Nucleon



(c) $^4\mathrm{He}$

$$\mathrm{SNR}(t) \sim \frac{C(t)}{\sqrt{C^\dagger C(t)}} \sim \exp\left(-A\left(m_n - \frac{3}{2}m_\pi\right)t\right)$$

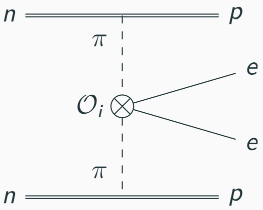
- Mitigated by simulating at heavier-than-physical quark masses

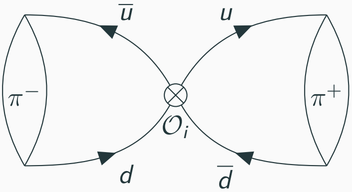
Leading Order Short-Distance $0\nu\beta\beta$

- LO SMEFT operators appear at dim. 9 [M.L. Graesser, JHEP (2017) 99]
- 5 are relevant at LO in χPT [G. Prézeau et al., PRD 68 (2003) 034016]

$$\begin{aligned}
\mathcal{O}_{1+}^{++} &= \left(\overline{q}_L \tau^+ \gamma^\mu q_L\right) \left[\overline{q}_R \tau^+ \gamma_\mu q_R\right] \\
\mathcal{O}_{2+}^{++} &= \left(\overline{q}_R \tau^+ q_L\right) \left[\overline{q}_R \tau^+ q_L\right] + \left(\overline{q}_L \tau^+ q_R\right) \left[\overline{q}_L \tau^+ q_R\right] \\
\mathcal{O}_{3+}^{++} &= \left(\overline{q}_L \tau^+ \gamma^\mu q_L\right) \left[\overline{q}_L \tau^+ \gamma_\mu q_L\right] + \left(\overline{q}_R \tau^+ \gamma^\mu q_R\right) \left[\overline{q}_R \tau^+ \gamma_\mu q_R\right] \\
\mathcal{O'}_{1+}^{++} &= \left(\overline{q}_L \tau^+ \gamma^\mu q_L\right] \left[\overline{q}_R \tau^+ \gamma_\mu q_R\right) \\
\mathcal{O'}_{2+}^{++} &= \left(\overline{q}_R \tau^+ q_L\right] \left[\overline{q}_R \tau^+ q_L\right) + \left(\overline{q}_L \tau^+ q_R\right] \left[\overline{q}_L \tau^+ q_R\right)
\end{aligned}$$

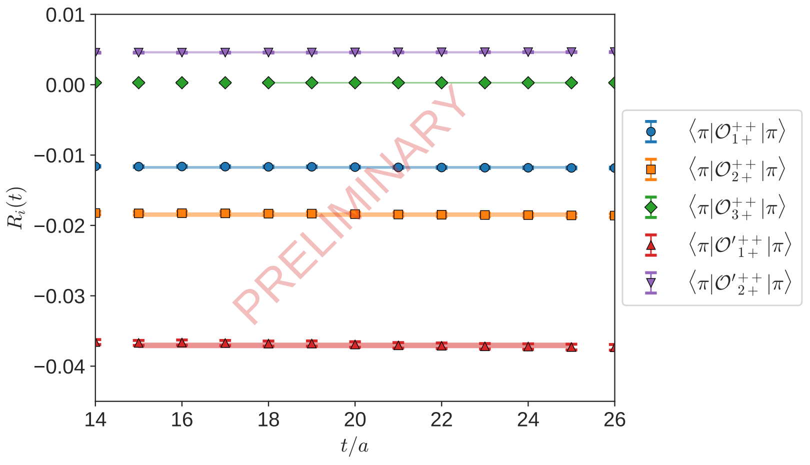
- Extract $\langle \pi | \mathcal{O}_i | \pi \rangle$ MEs from lattice 3-point functions $C_{\pi \mathcal{O}_i \pi}(t_-, t_{\mathcal{O}}, t_+)$





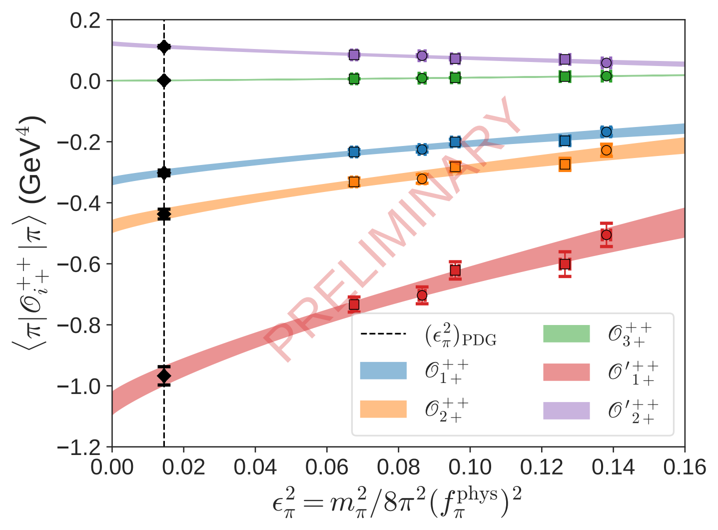
- Previously computed by A. Nicholson et al. [PRL 121 (2018) 172501]

SD Preliminary Lattice Results (32I, $m_\pi \approx 300$ MeV)



$$R_i(t) = \frac{C_{\pi \mathcal{O}_i \pi}(0, t, 2t)}{C_\pi(2t)} \overset{t \gg 1}{\simeq} \frac{\langle \pi | \mathcal{O}_i | \pi \rangle}{2 m_\pi}$$

SD Preliminary Physical Point Extrapolation



- Fit ansatz: (continuum NLO χPT) + (NLO χPTFV) + ($c_a a^2$)
- (Unrenormalized) results appear consistent with Nicholson et al.