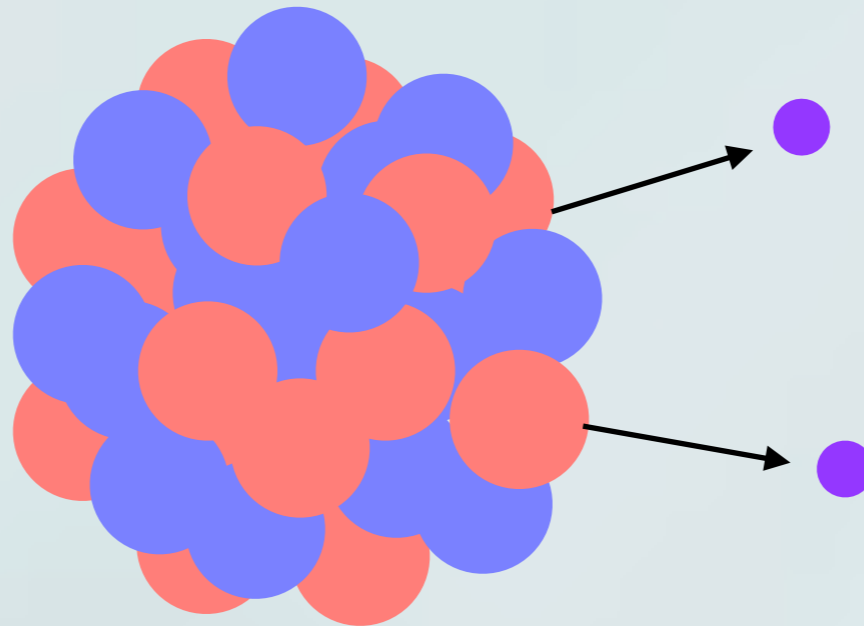


# LATTICE QCD FOR NEUTRINOLESS DOUBLE-BETA DECAY

**ZOHREH DAVOUDI**

UNIVERSITY OF MARYLAND AND RIKEN FELLOW



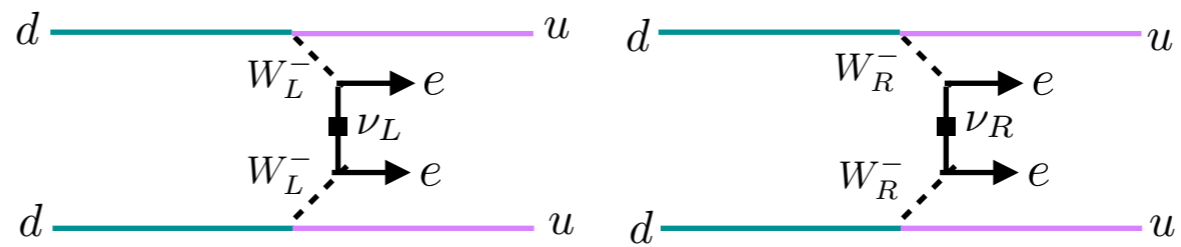
PROGRESS AND CHALLENGES IN NEUTRINOLESS DOUBLE BETA DECAY

ECT\*, Trento, Italy, July 15-19, 2019

# THE ROADMAP FOR $0\nu BB$ DECAY

$$\Lambda > \text{TeV}$$

START WITH A GIVEN HIGH-SCALE MODEL, e.g., LEFT-RIGHT SYMMETRIC MODEL:

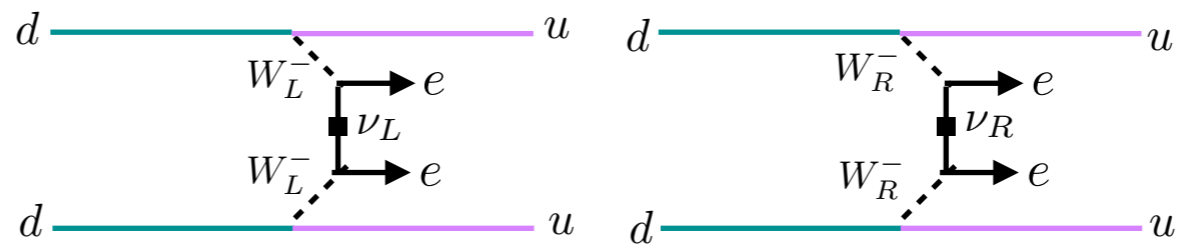


RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:

# THE ROADMAP FOR $0\nu BB$ DECAY

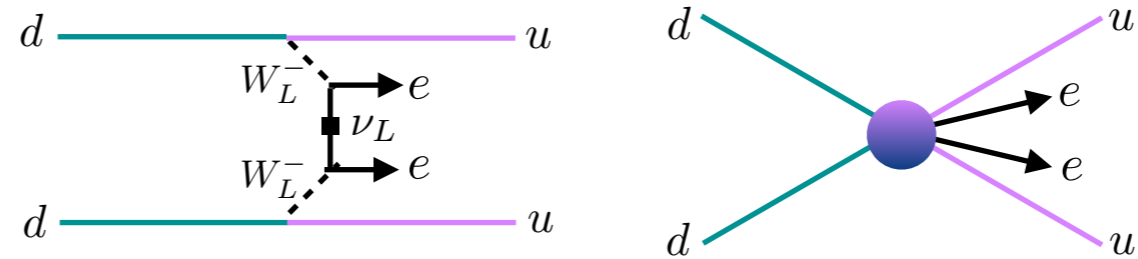
$$\Lambda > \text{TeV}$$

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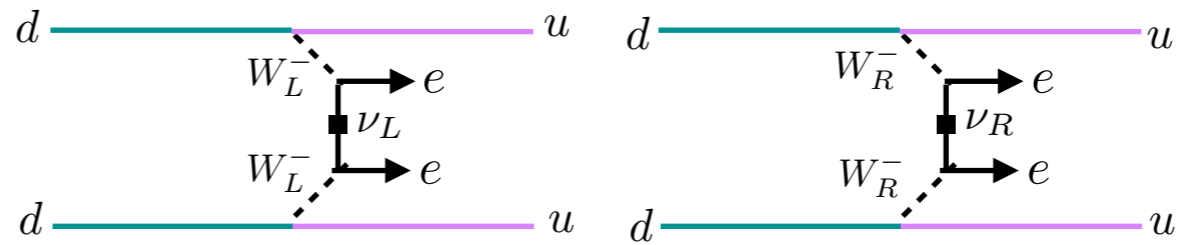
$$\Lambda \sim 10^2 \text{ GeV}$$



# THE ROADMAP FOR $0\nu BB$ DECAY

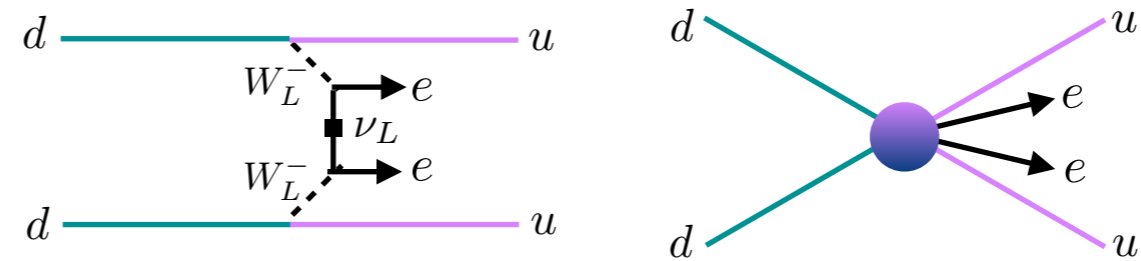
$$\Lambda > \text{TeV}$$

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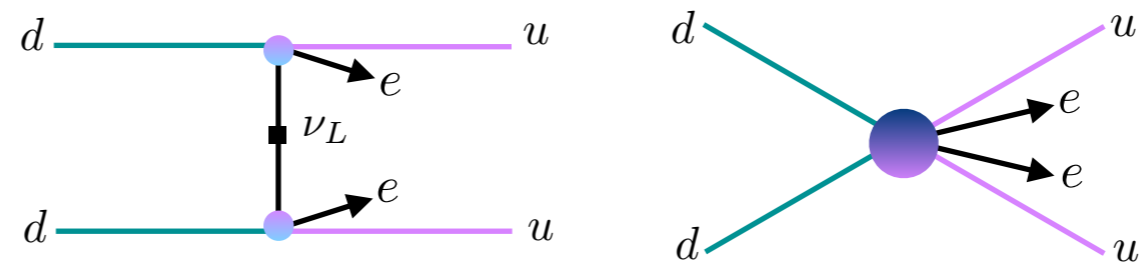
$$\Lambda \sim 10^2 \text{ GeV}$$

RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:



$$\Lambda \sim 2 \text{ GeV}$$

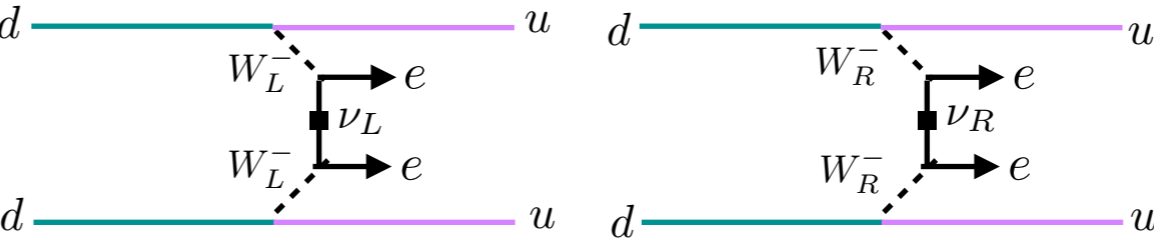
RUN IT DOWN TO THE SCALE WHERE QCD IS STILL PERTURBATIVE:



THE ROADMAP FOR  $0\nu BB$  DECAY

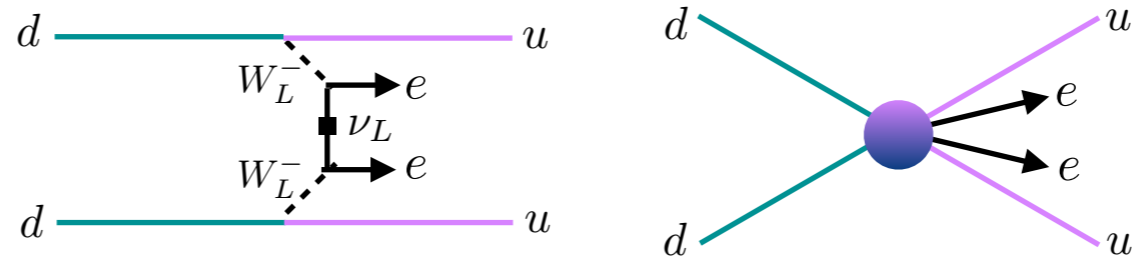
$\Lambda > \text{TeV}$

START WITH A GIVEN HIGH-SCALE MODEL, e.g., LEFT-RIGHT SYMMETRIC MODEL:



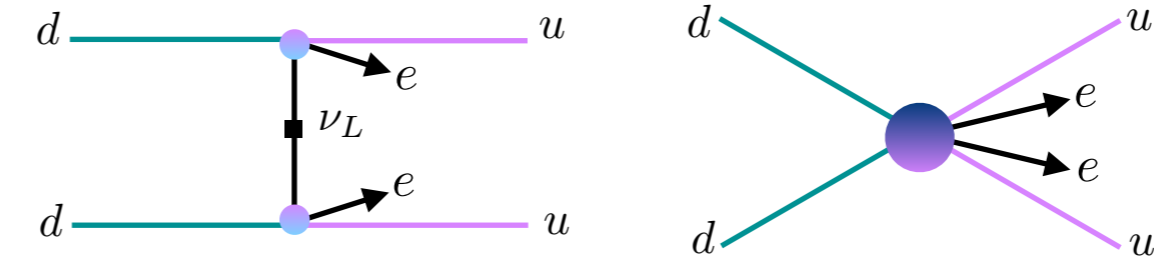
$\Lambda \sim 10^2 \text{ GeV}$

RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:



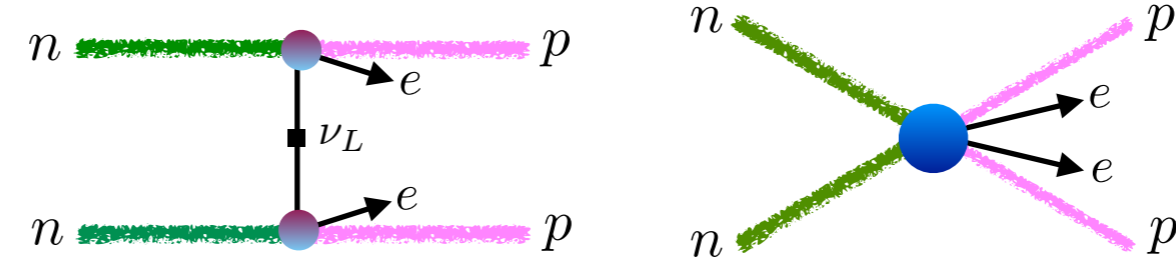
$\Lambda \sim 2 \text{ GeV}$

RUN IT DOWN TO THE SCALE WHERE QCD IS STILL PERTURBATIVE:



$\Lambda < \text{GeV}$

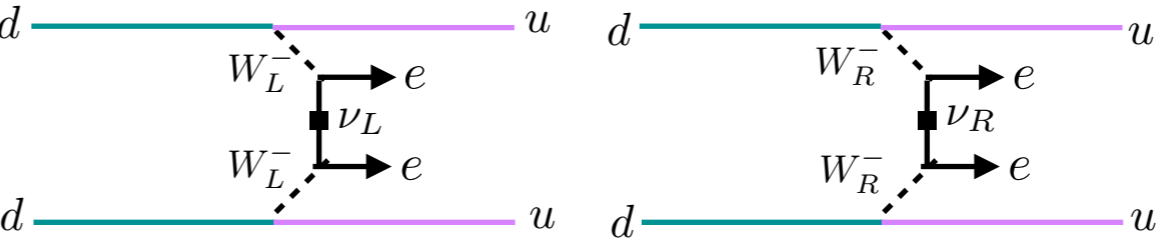
RUN IT DOWN TO HADRONIC SCALE:



THE ROADMAP FOR  $0\nu BB$  DECAY

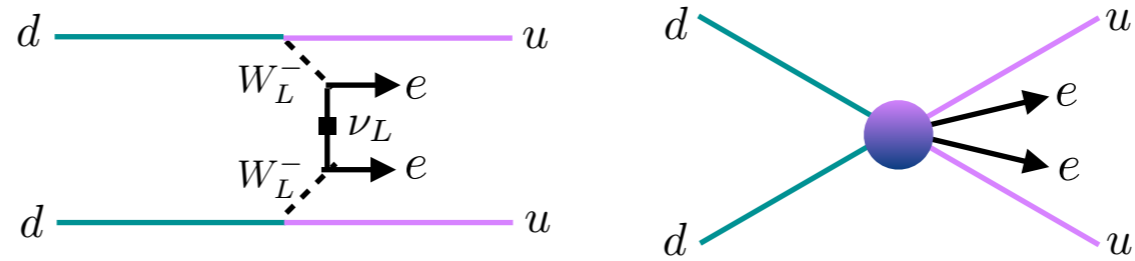
$\Lambda > \text{TeV}$

START WITH A GIVEN HIGH-SCALE MODEL, e.g., LEFT-RIGHT SYMMETRIC MODEL:



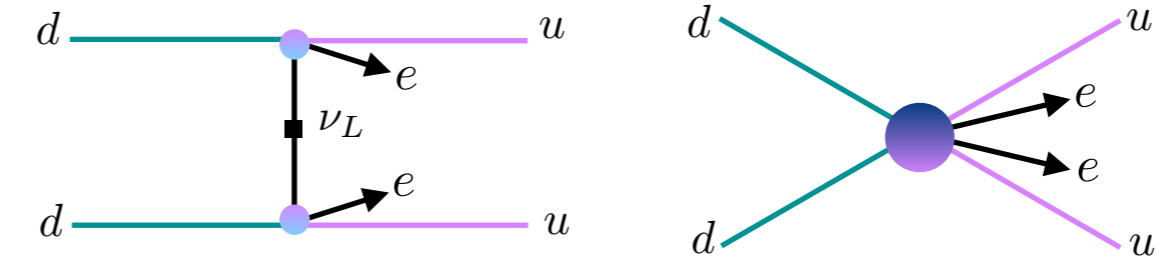
$\Lambda \sim 10^2 \text{ GeV}$

RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:



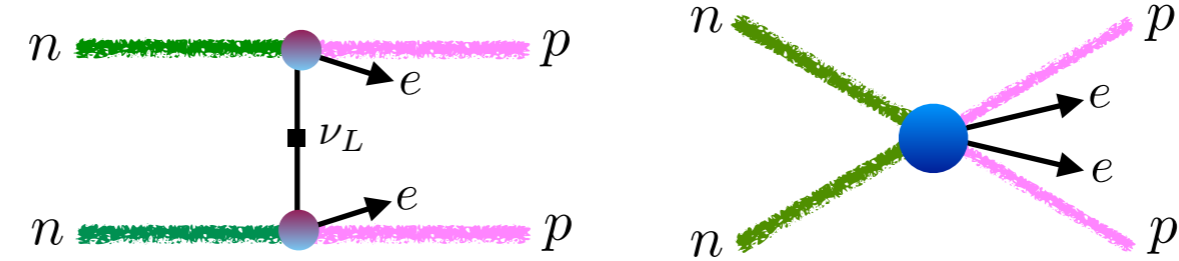
$\Lambda \sim 2 \text{ GeV}$

RUN IT DOWN TO THE SCALE WHERE QCD IS STILL PERTURBATIVE:



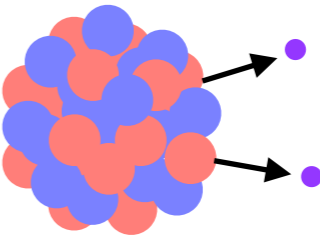
$\Lambda < \text{GeV}$

RUN IT DOWN TO HADRONIC SCALE:



$\Lambda < \text{MeV}$

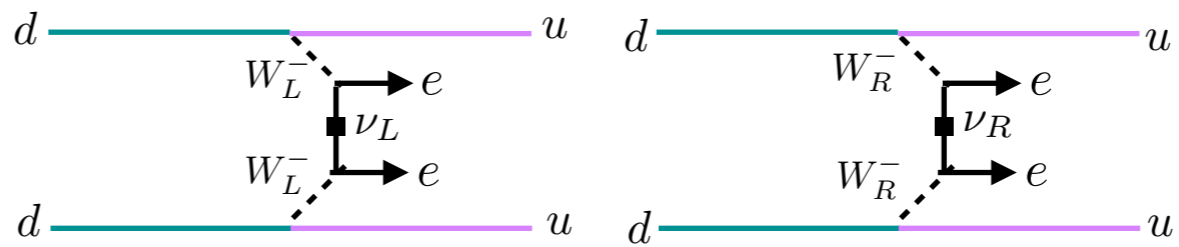
USE AB INITIO NUCLEAR MANY-BODY CALCULATIONS TO MATCH TO NUCLEAR MATRIX ELEMENTS:



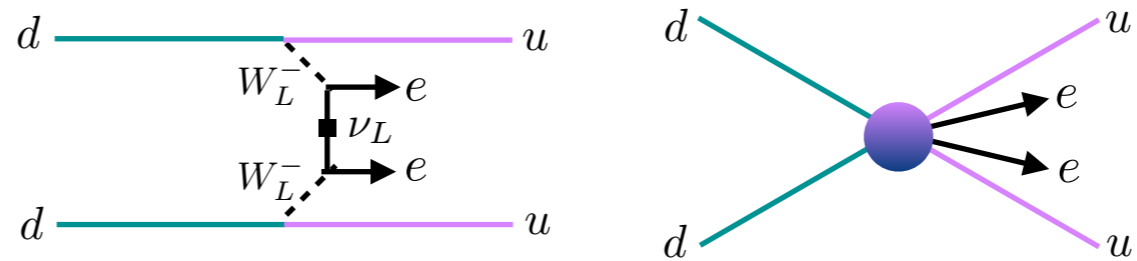
# THE ROADMAP FOR $0\nu BB$ DECAY

$$\Lambda > \text{TeV}$$

START WITH A GIVEN HIGH-SCALE MODEL, e.g., LEFT-RIGHT SYMMETRIC MODEL:



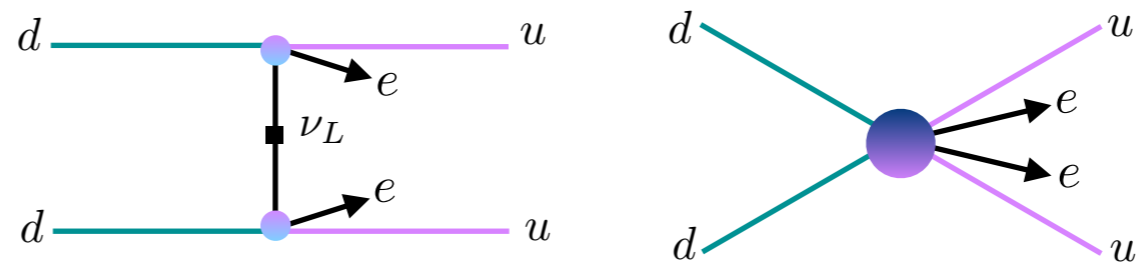
RUN IT DOWN TO THE SCALE WHERE THE HIGH-SCALE PHYSICS CAN BE INTEGRATED OUT:



(1)

$$\Lambda \sim 10^2 \text{ GeV}$$

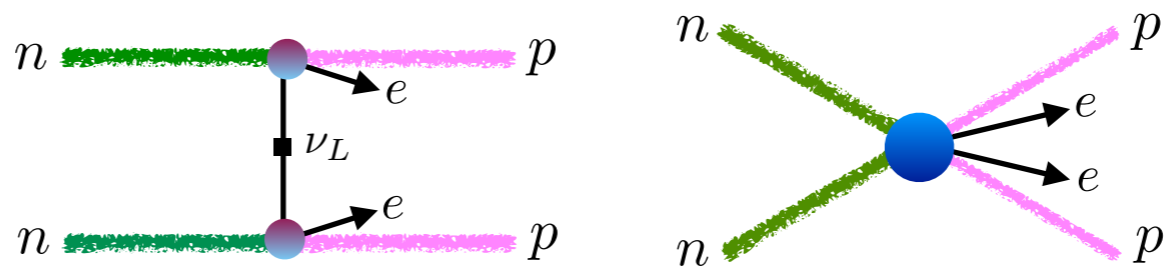
RUN IT DOWN TO THE SCALE WHERE QCD IS STILL PERTURBATIVE:



(2)

$$\Lambda \sim 2 \text{ GeV}$$

RUN IT DOWN TO HADRONIC SCALE:

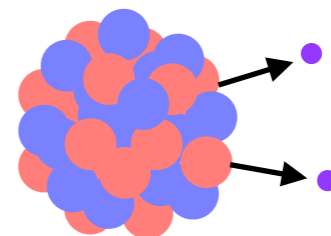


(3)

$$\Lambda < \text{GeV}$$

USE AB INITIO NUCLEAR MANY-BODY CALCULATIONS TO MATCH TO NUCLEAR MATRIX ELEMENTS:

$$\Lambda < \text{MeV}$$



(4)

## **NON-LOCAL MATRIX ELEMENTS OF TWO DIMENSION-6 FOUR-FERMION SM WEAK CURRENTS**

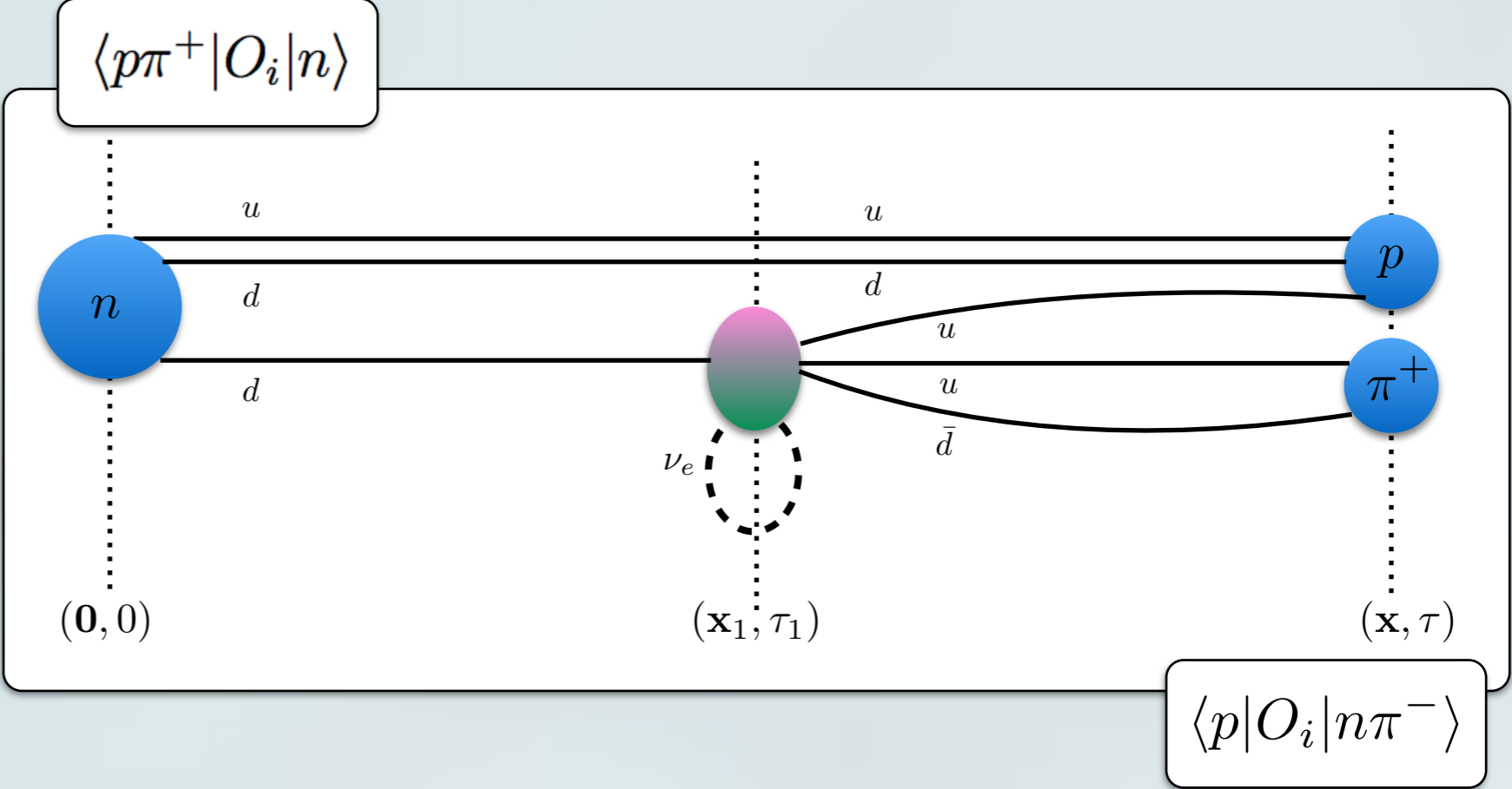
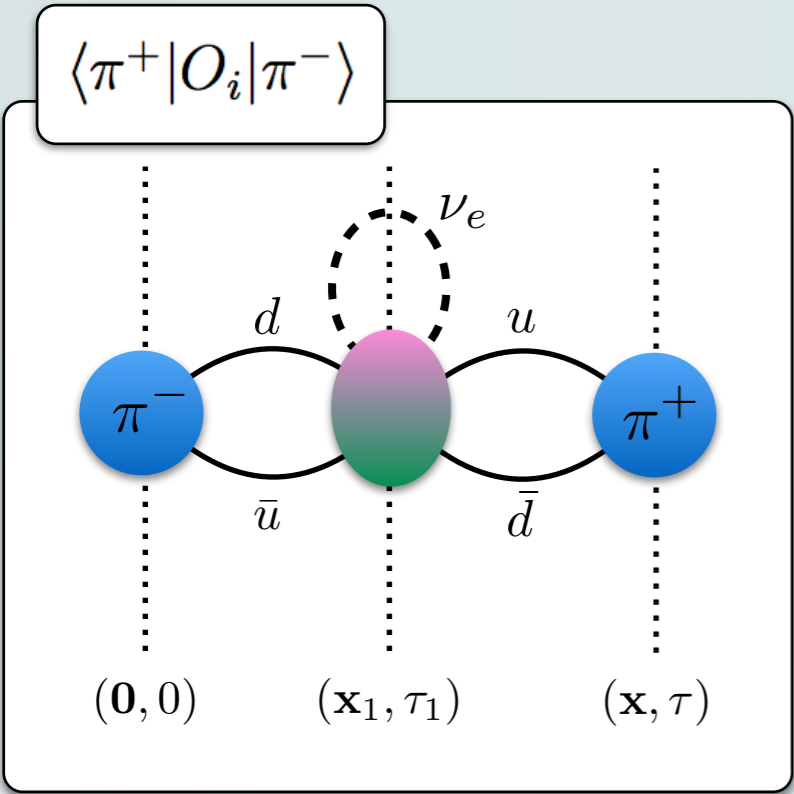
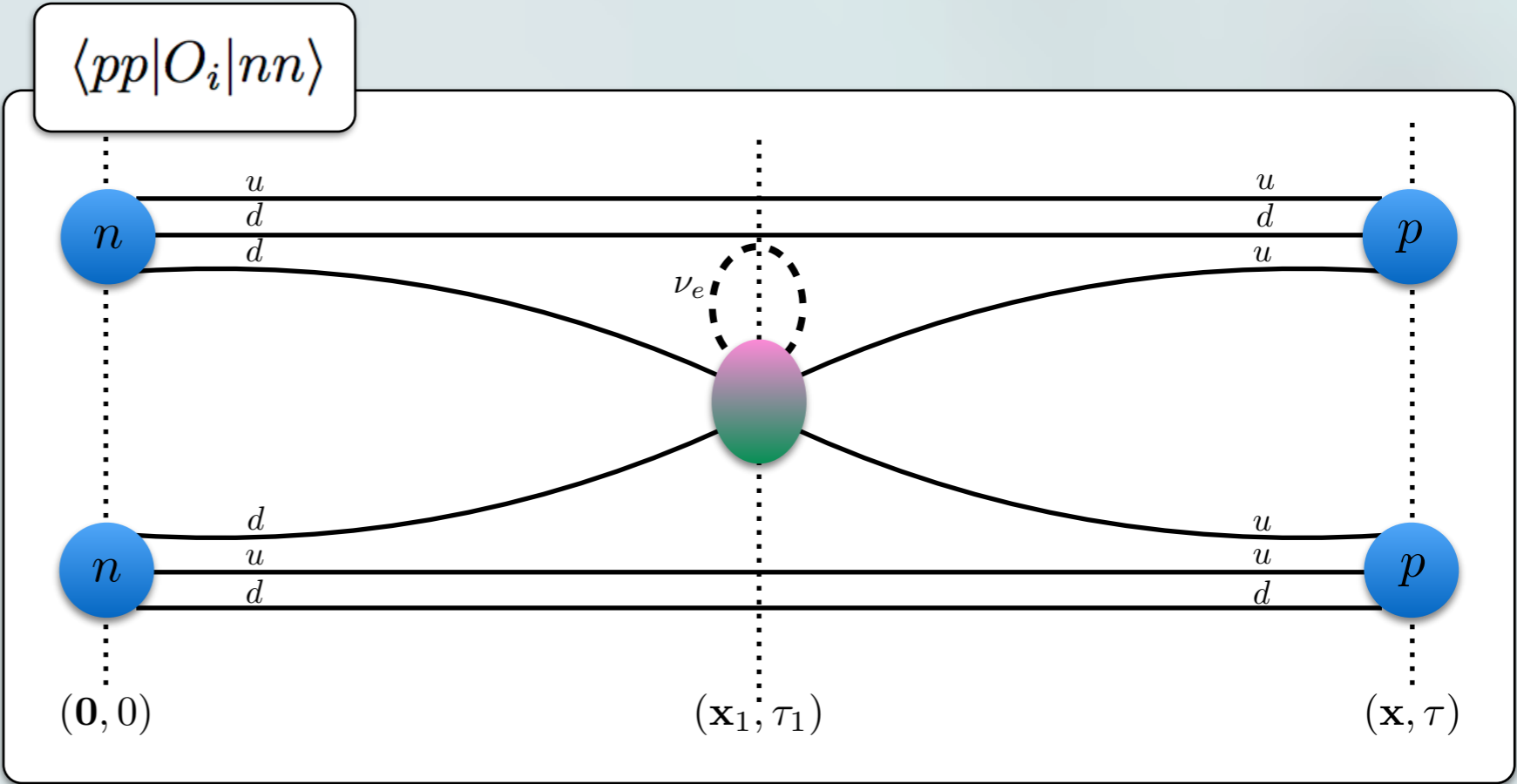
**Constrains more reliably the limits on  
the effective Majorana neutrino mass  
(a combination of masses and mixing  
angles) in the minimal extension of SM.**

## **LOCAL MATRIX ELEMENTS OF DIMENSION-9 SIX-FERMION OPERATORS**

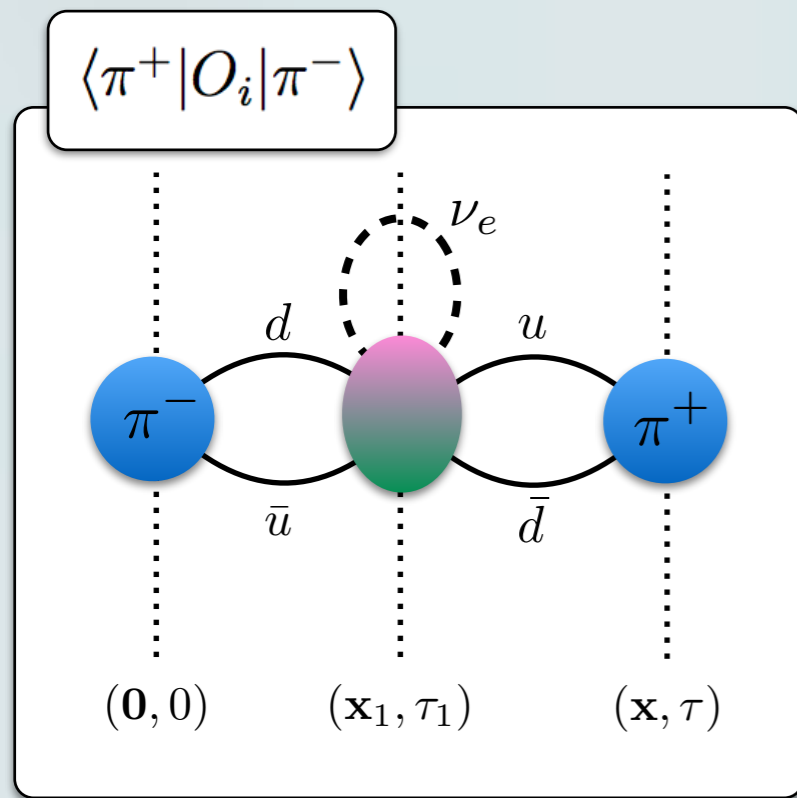
**Helps to find out if predictions of high-scale  
models are within the reach of current and  
future experimental limits. Will eventually  
constrain such models more reliably.**

**LOCAL MATRIX ELEMENTS OF  
DIMENSION-9 SIX-FERMION  
OPERATORS**

MATRIX ELEMENTS OF **LOCAL**  
FOUR-QUARK OPERATORS

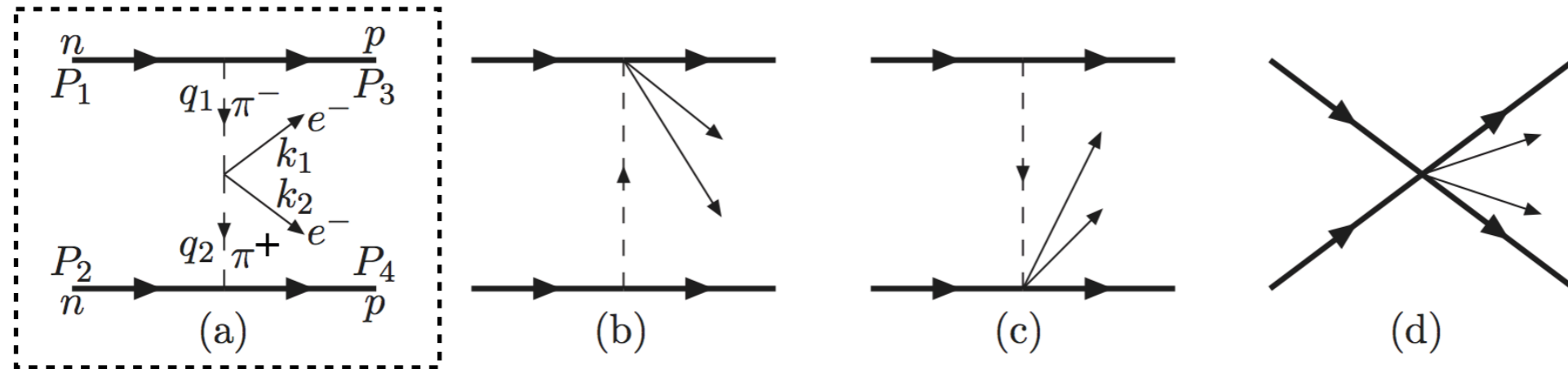


# MATRIX ELEMENTS OF **LOCAL** FOUR-QUARK OPERATORS

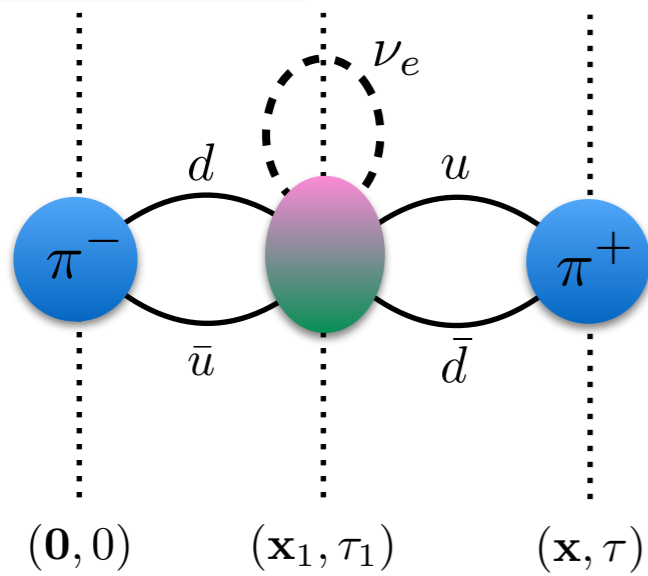


# MATRIX ELEMENTS OF LOCAL FOUR-QUARK OPERATORS

Prezeau, Ramsey-Musolf, Vogel Phys.Rev. D68 03401 (2003).

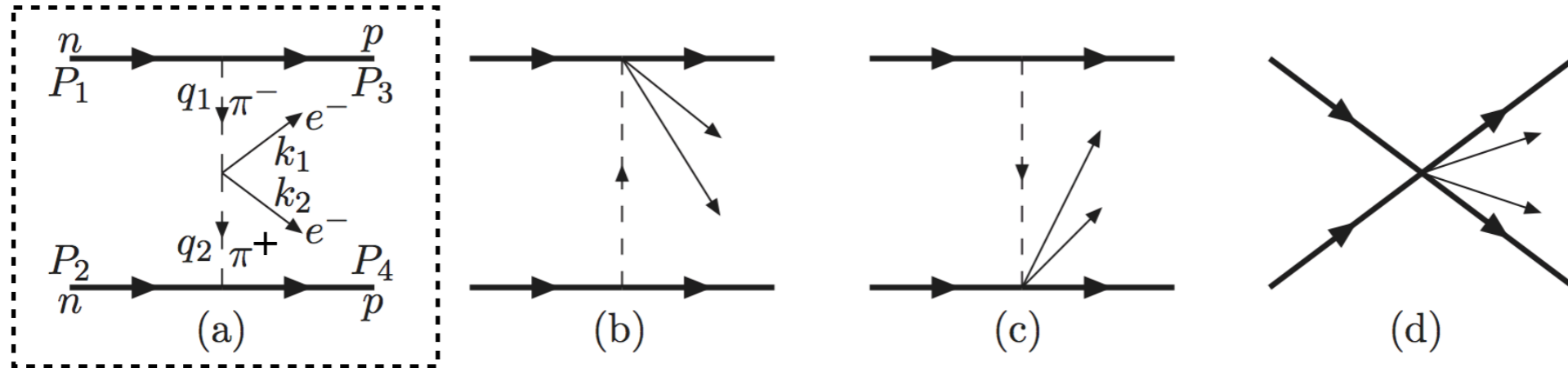


$$\langle \pi^+ | O_i | \pi^- \rangle$$

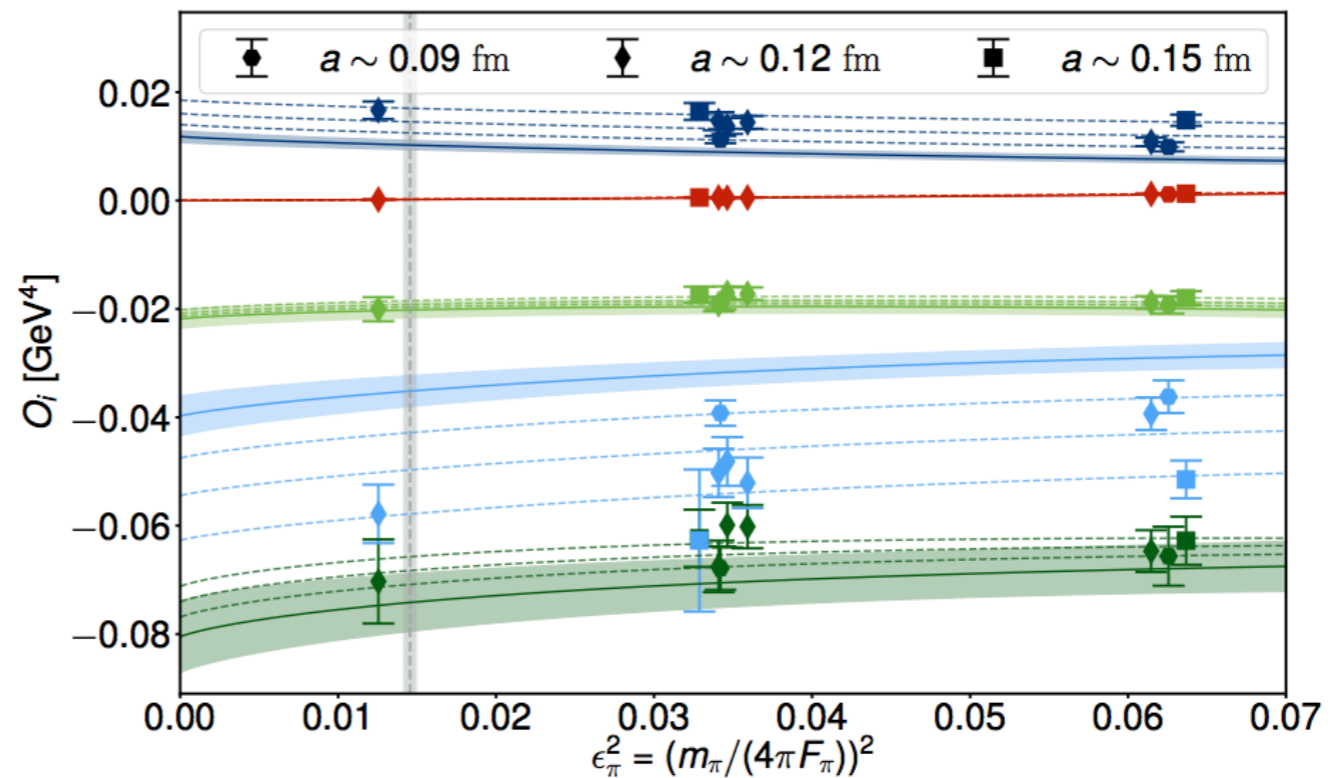
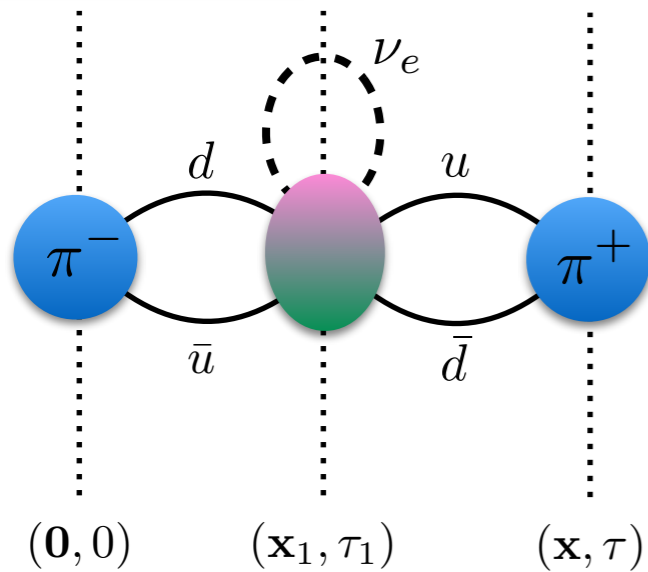


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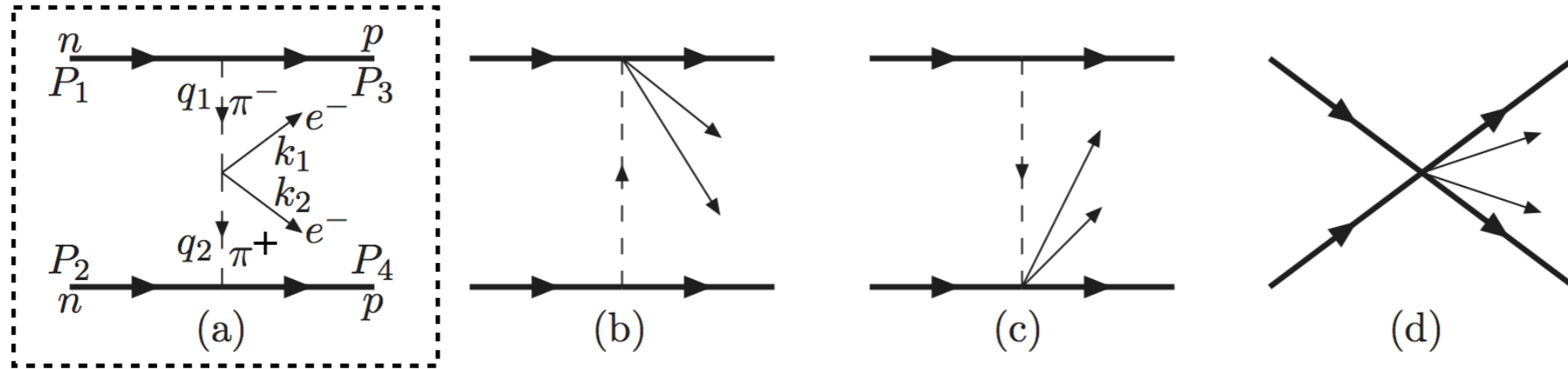


Nicholson, Berkowitz, Monge-Camacho, Brantley, Garron, Chang, Rinaldi, Clark, Joo, Kurth, Tiburzi, Vranas and Walker-Loud (CALLATT collaboration), Phys. Rev. Lett. 121, 172501 (2018), arXiv:1805.02634 [nucl-th].

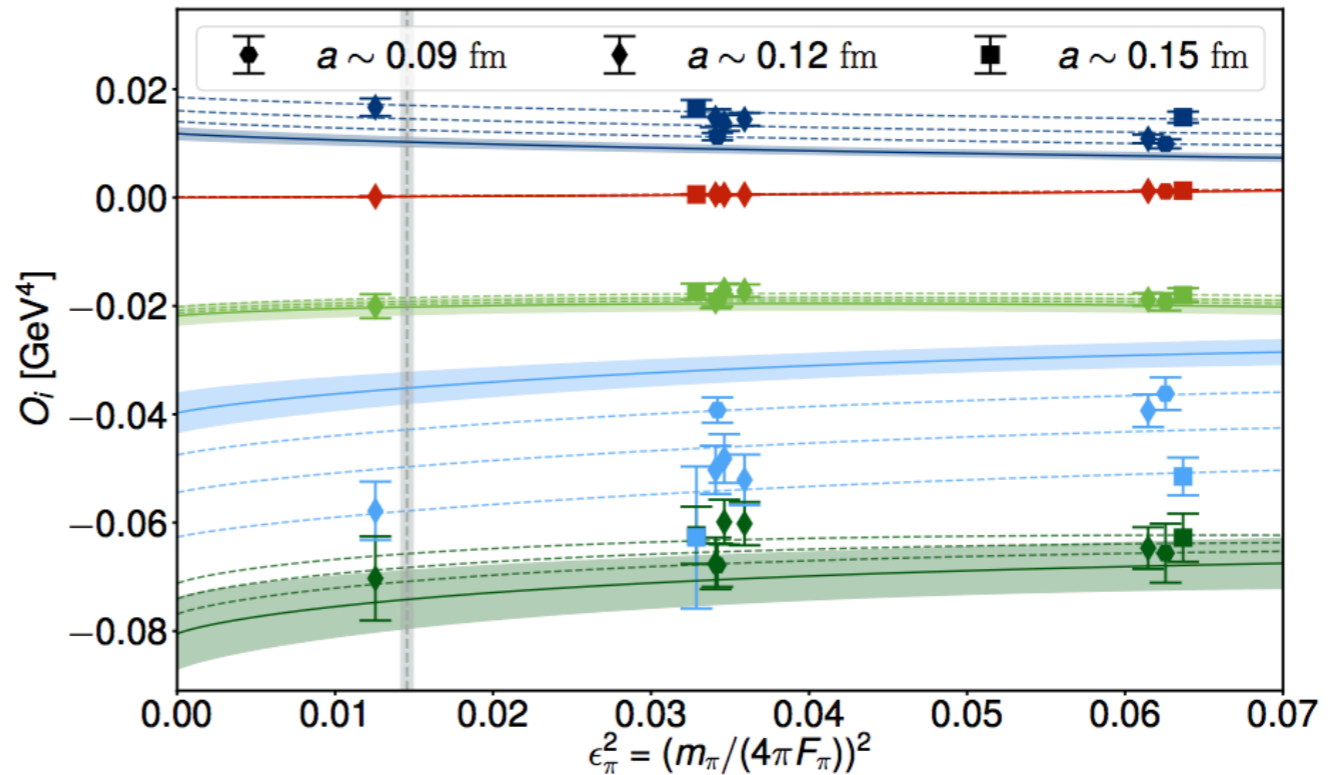
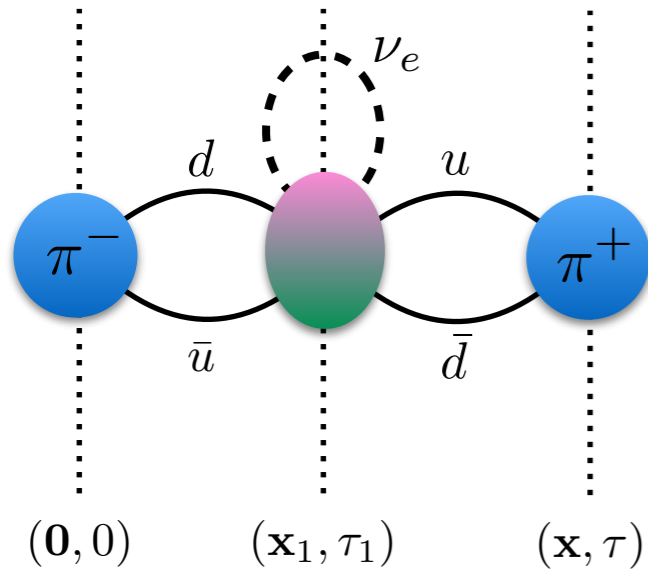
# MATRIX ELEMENTS OF LOCAL FOUR-QUARK OPERATORS

Savage, Phys. Rev. C59, 2293 (1999), arXiv:nucl-th/9811087 [nucl-th].  
Cirigliano, Dekens, Graesser and Mereghetti, PLB Volume 769, 2017,  
Pages 460-464, arXiv:1701.01443 [hep-ph].

Prezeau, Ramsey-Musolf, Vogel Phys.Rev. D68 03401 (2003).



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**NON-LOCAL MATRIX ELEMENTS OF  
TWO DIMENSION-6 FOUR-FERMION  
SM WEAK CURRENTS**

MATRIX ELEMENTS OF  
**NON-LOCAL** TWO-QUARK  
SM WEAK OPERATORS

$$S_{NL} = \int dx dy S_0(x-y) T \left( J_\alpha^+(x) J_\beta^+(y) \right) g^{\alpha\beta}$$

$$J_\alpha^+ = \bar{u} \gamma_\alpha (1 - \gamma_5) d$$

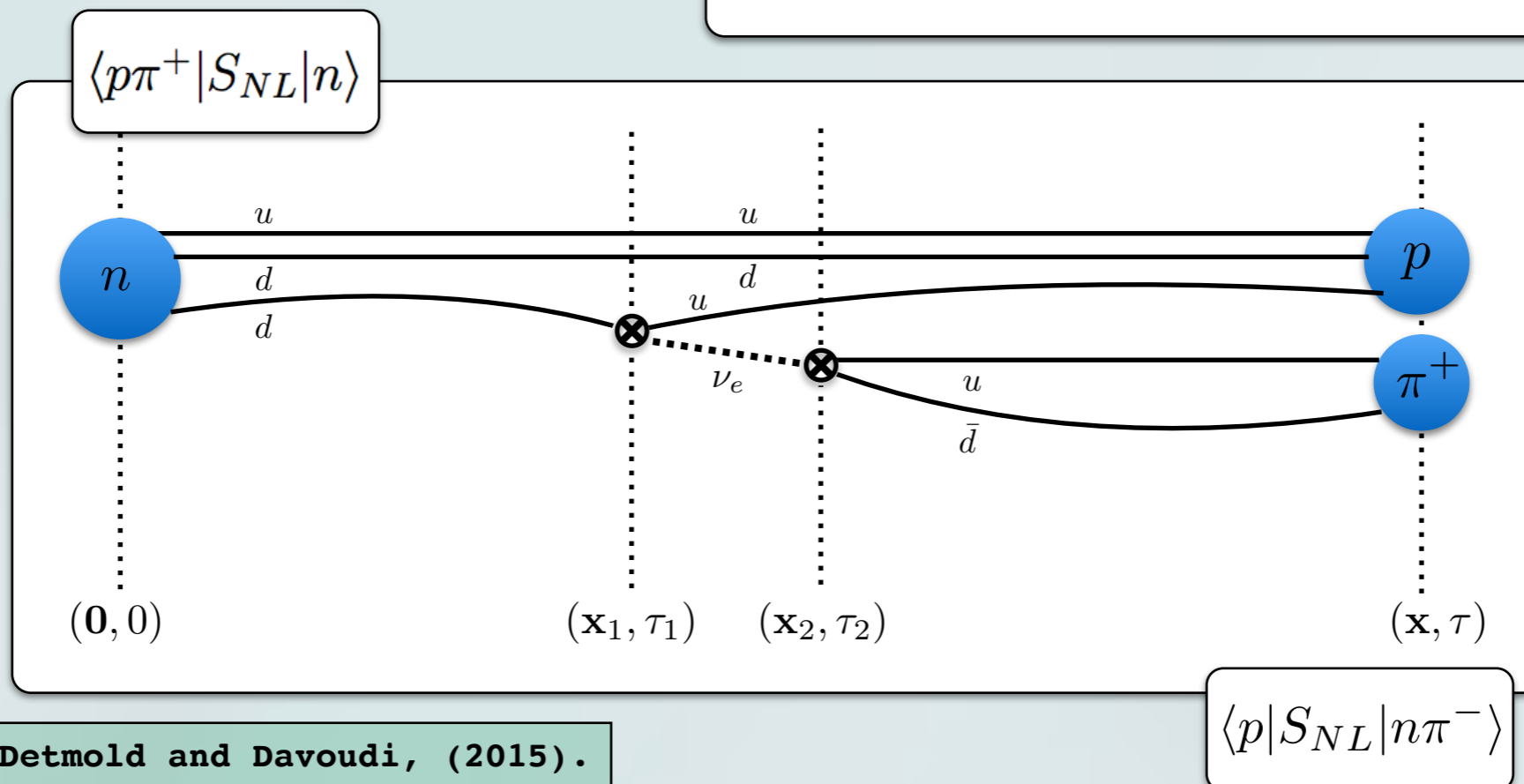
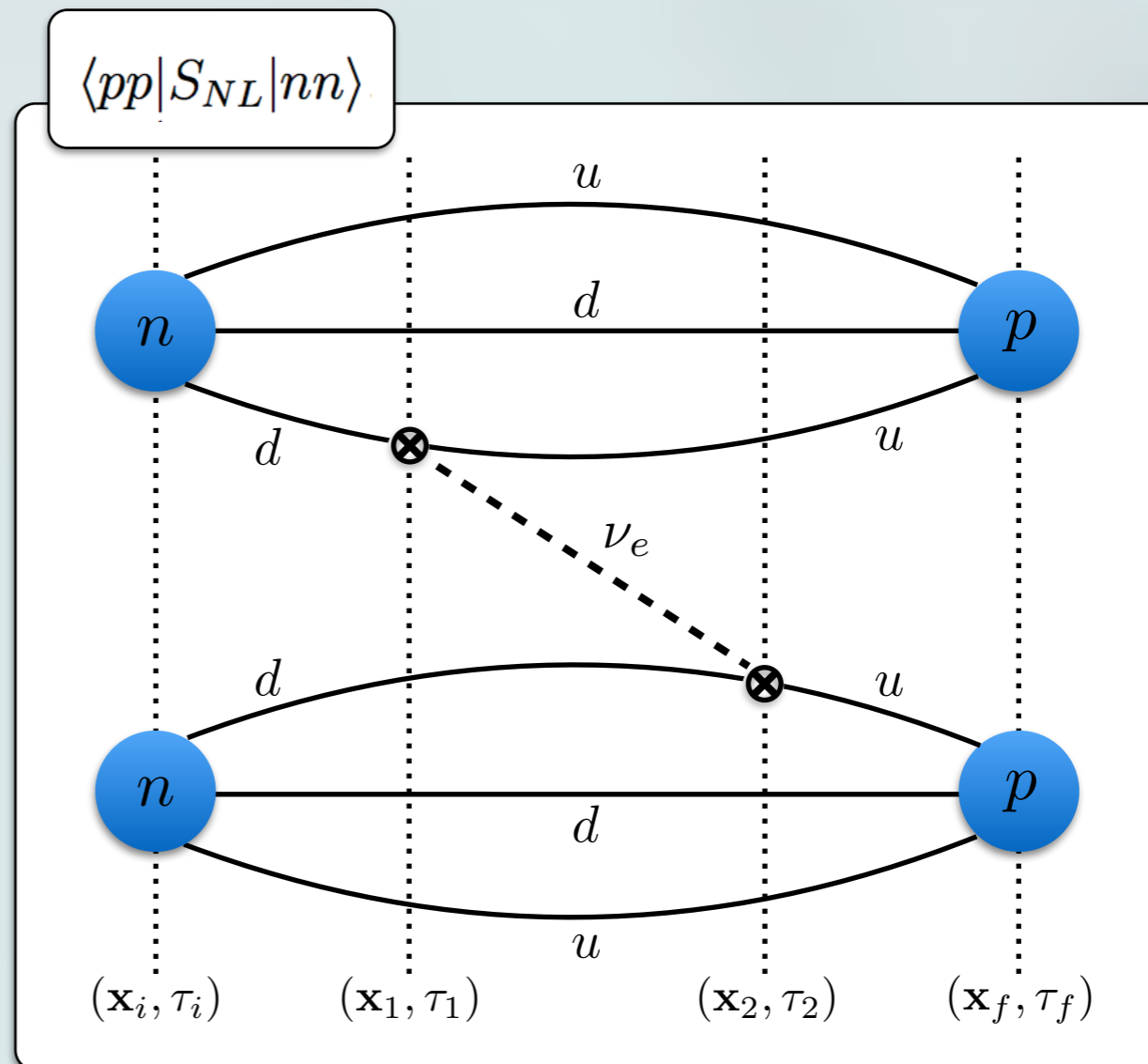
# MATRIX ELEMENTS OF NON-LOCAL TWO-QUARK SM WEAK OPERATORS

SEE JORDY'S TALKS.

$$S_{NL} = \int dx dy S_0(x - y) T (J_\alpha^+(x) J_\beta^+(y)) g^{\alpha\beta}$$

$$J_\alpha^+ = \bar{u} \gamma_\alpha (1 - \gamma_5) d$$

EFT approach makes the case for LQCD even stronger, see e.g., Cirigliano, Dekens, De Vries, Graesser, Mereghetti, Pastore, and Van Kolck, Phys. Rev. Lett. 120, 202001 (2018), arXiv:1802.10097 [hep-ph].



Detmold and Davoudi, (2015).

MATRIX ELEMENTS OF  
NON-LOCAL TWO-QUARK  
SM WEAK OPERATORS

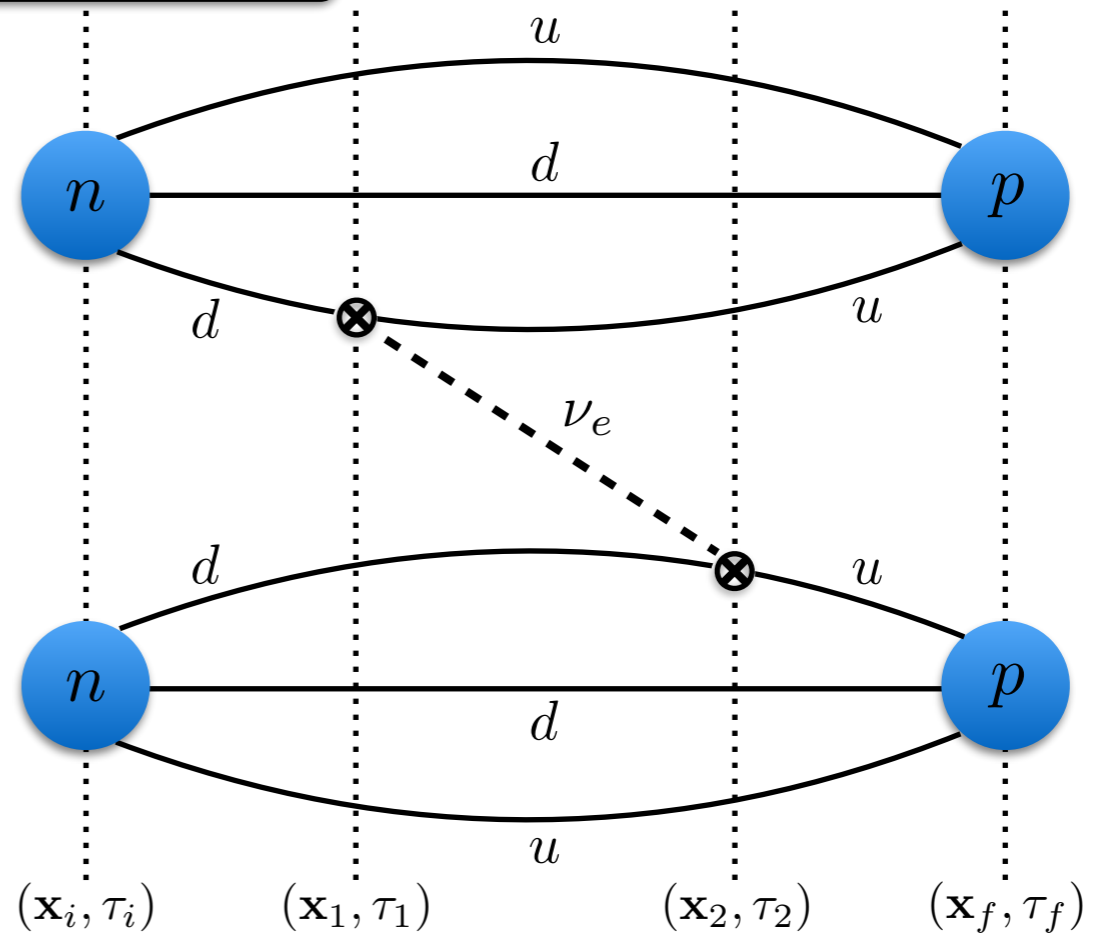
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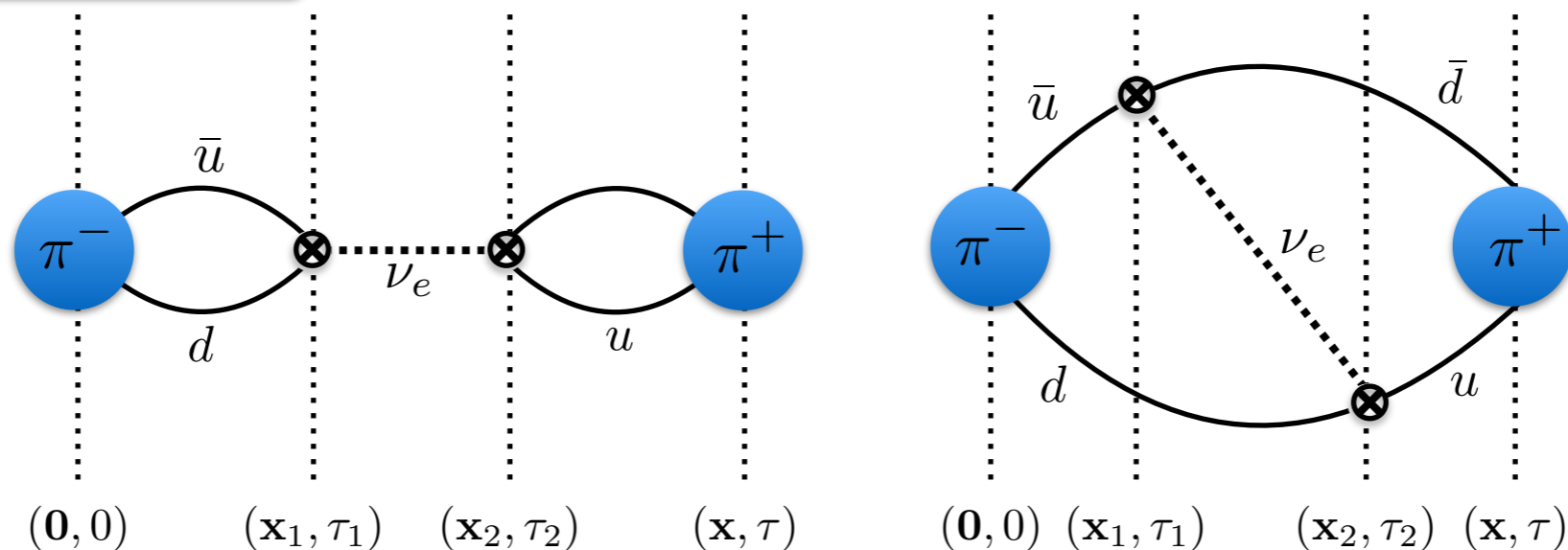
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$$\langle pp | S_{NL} | nn \rangle$$

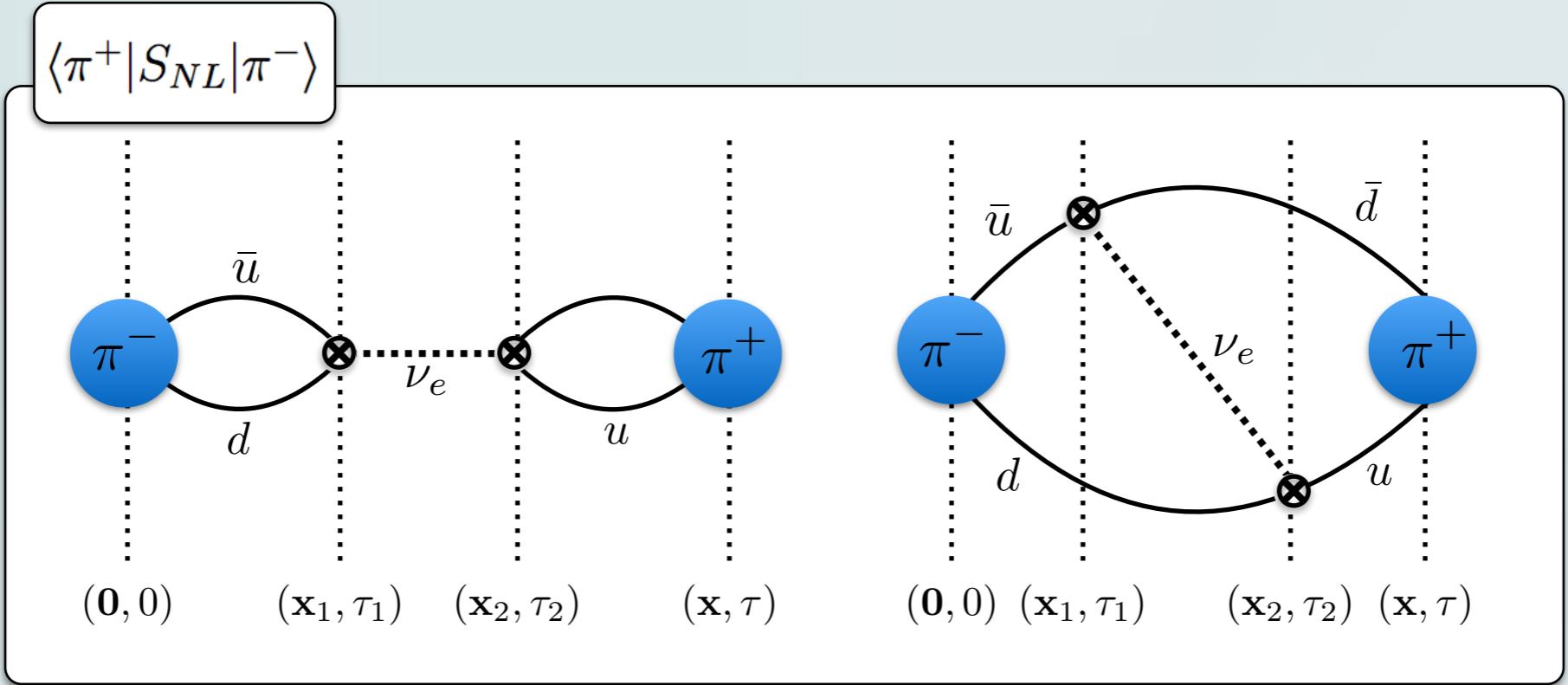


$$\langle \pi^+ | S_{NL} | \pi^- \rangle$$



Detmold and Davoudi, (2015).

MATRIX ELEMENTS OF  
NON-LOCAL TWO-QUARK  
SM WEAK OPERATORS

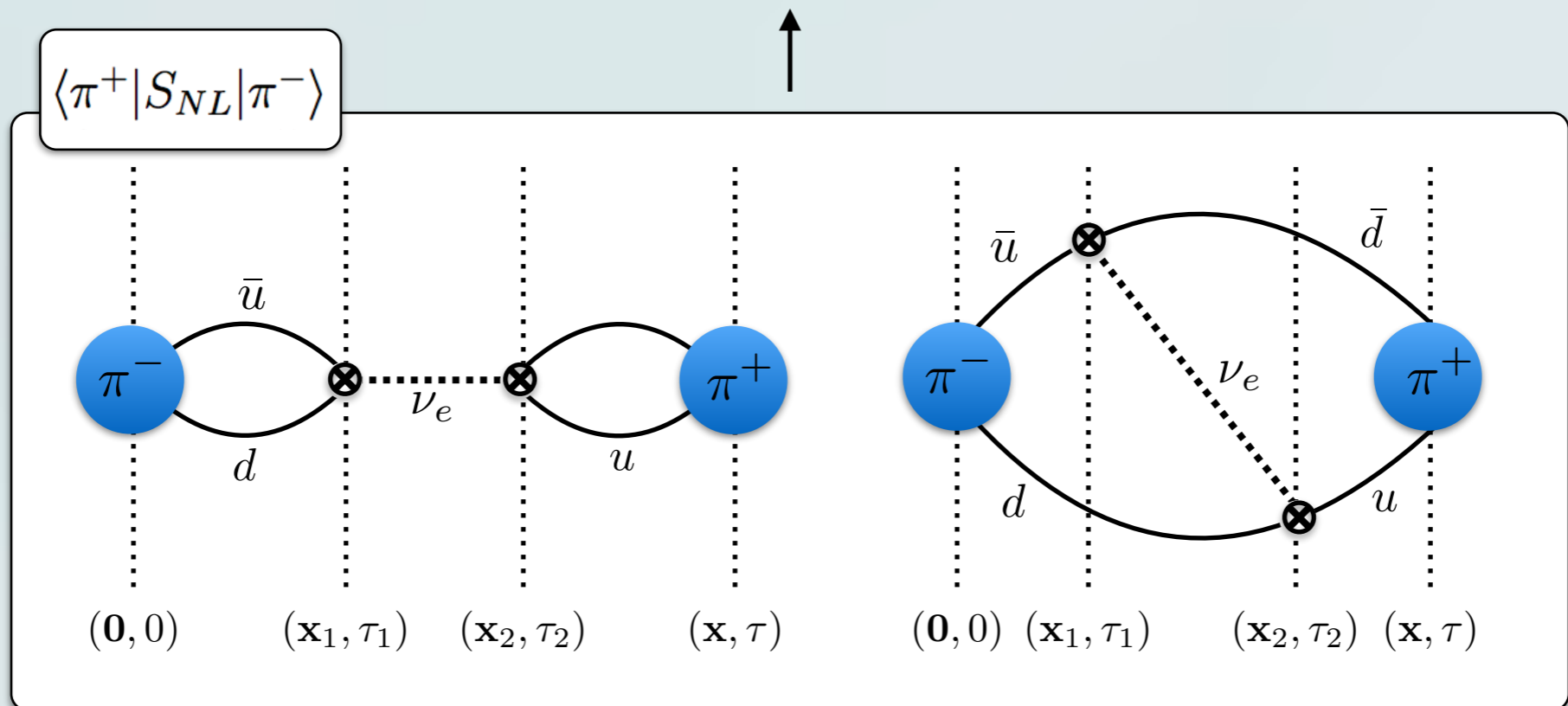
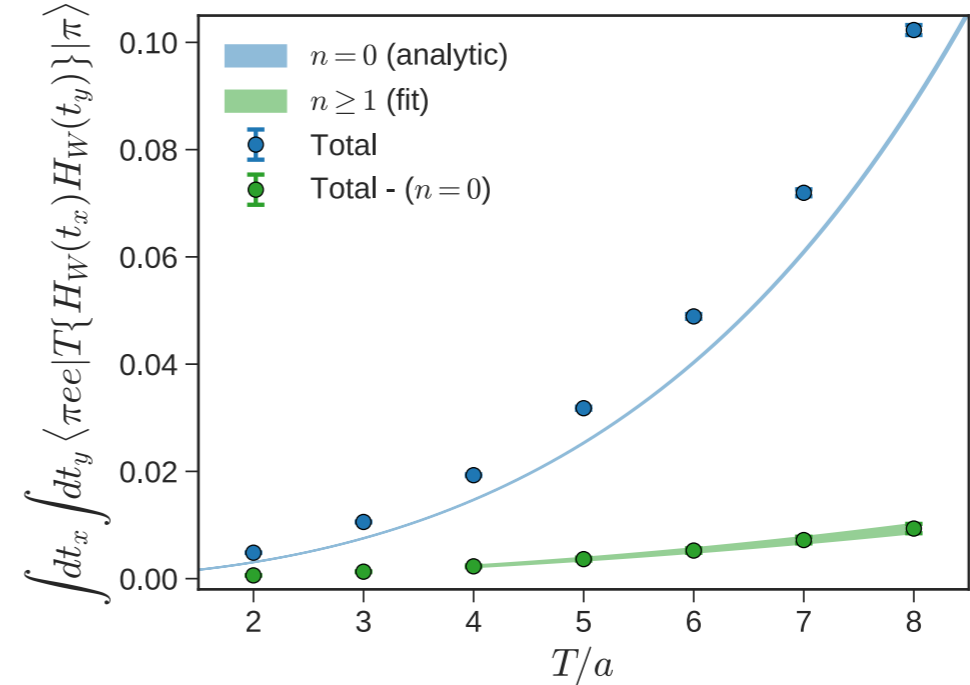
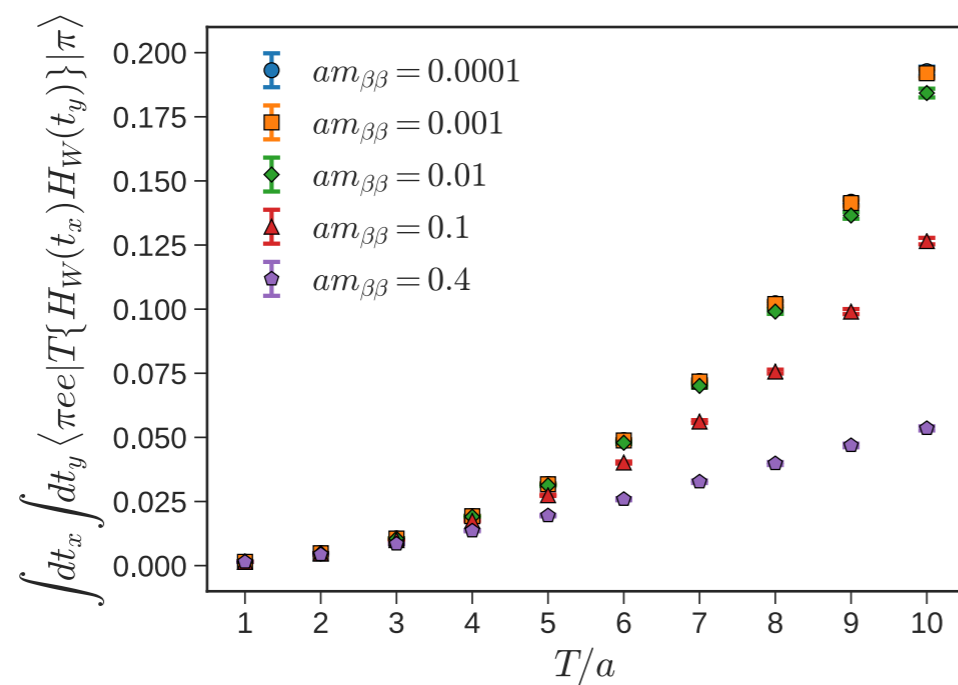


Detmold and Davoudi, (2015).

# MATRIX ELEMENTS OF NON-LOCAL TWO-QUARK SM WEAK OPERATORS

Detmold and Murphy, (2018) arXiv:1811.05554 [hep-lat].

SEE DAVID'S TALK.

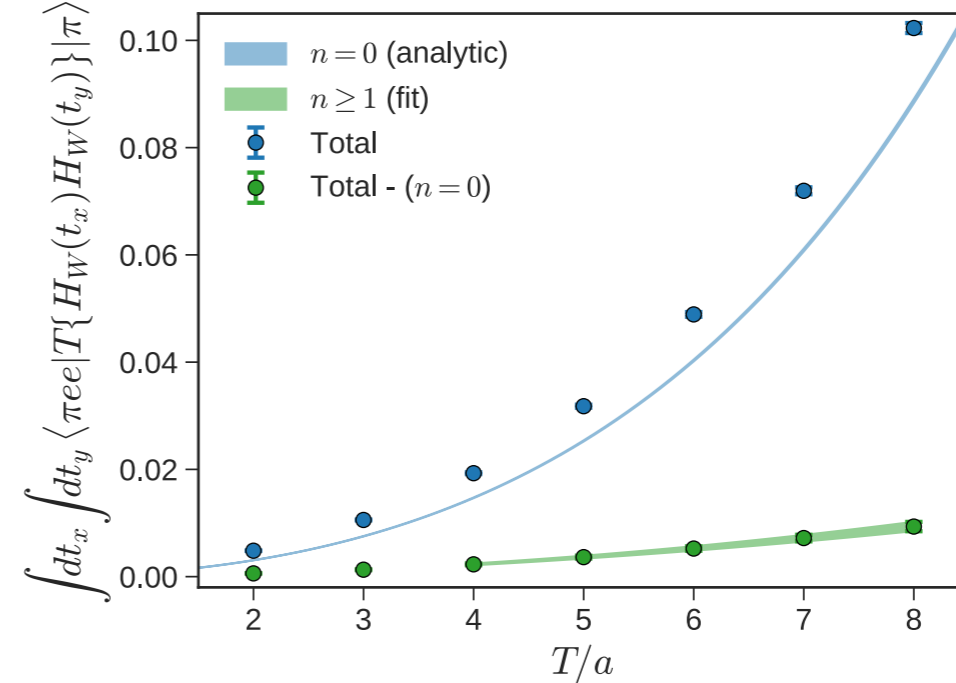
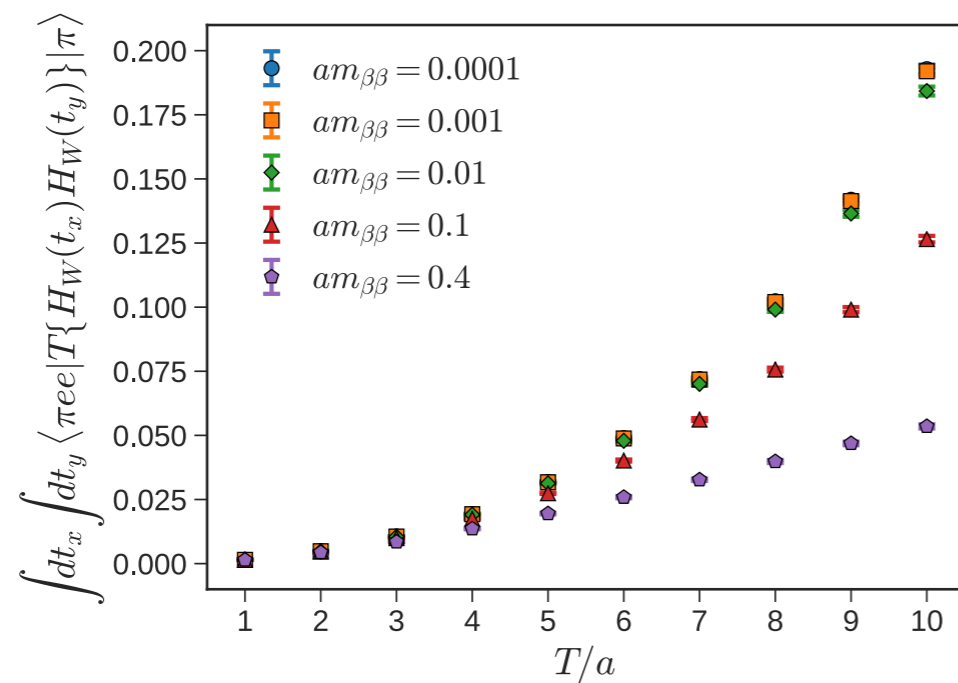


Detmold and Davoudi, (2015).

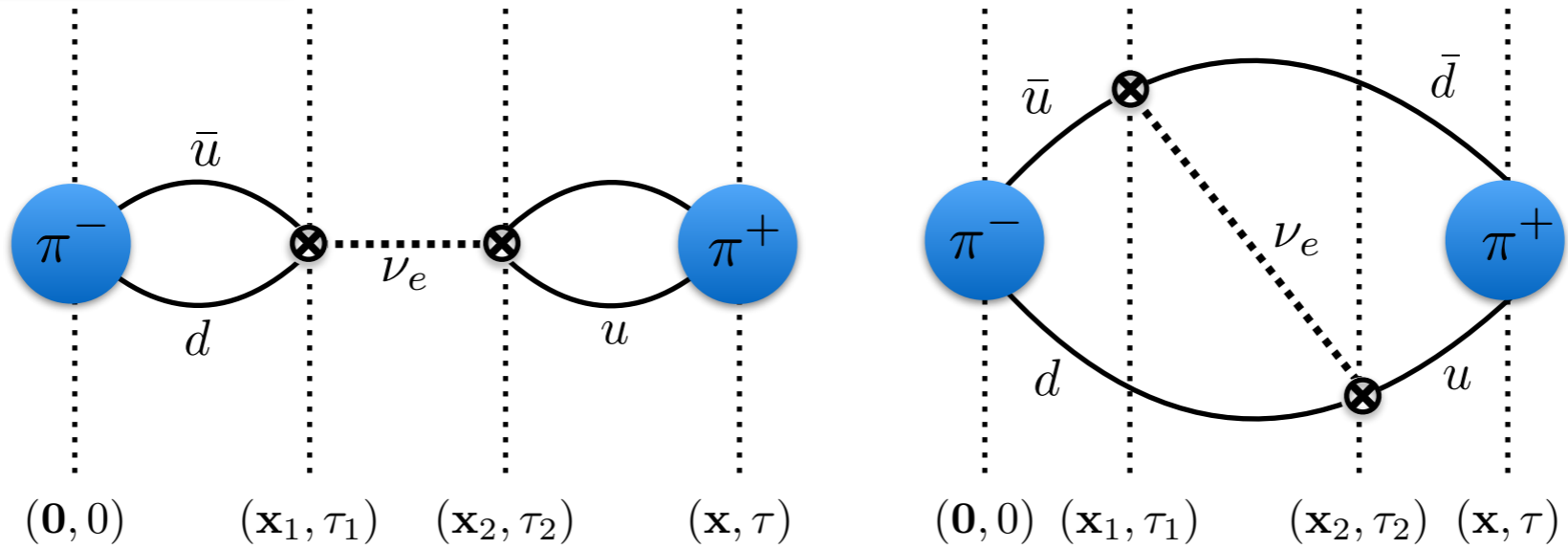
# MATRIX ELEMENTS OF NON-LOCAL TWO-QUARK SM WEAK OPERATORS

Another nice calculation on  $\pi\pi \rightarrow ee$  process: Feng, Jin, Tuo, and Xia, Phys. Rev. Lett. 122, 022001 (2019), arXiv: 1809.10511 [hep-lat].

SEE XU'S TALK.



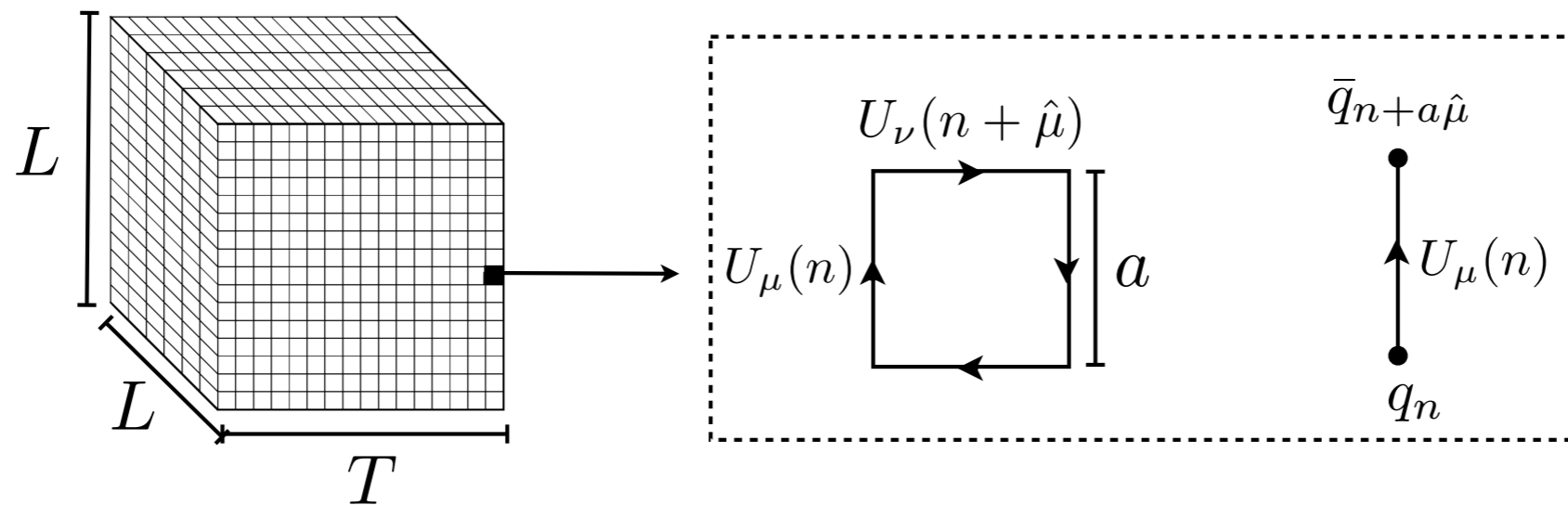
$$\langle \pi^+ | S_{NL} | \pi^- \rangle$$



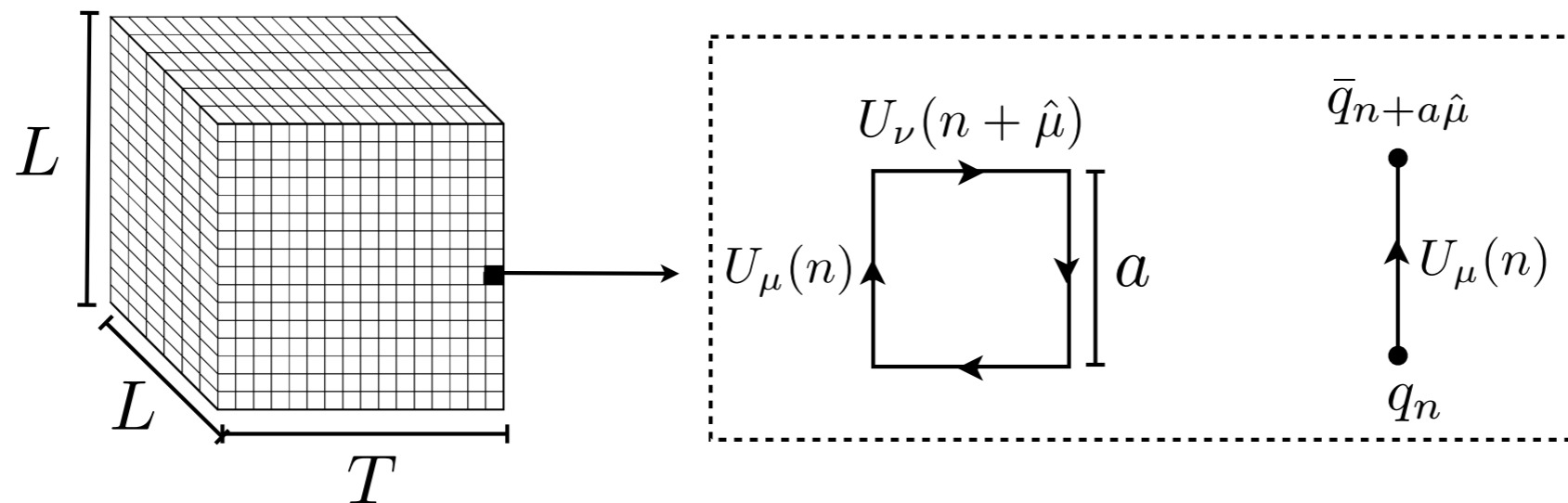
Detmold and Davoudi, (2015).

SOME BACKGROUND ON  
THE LATTICE QCD METHODOLOGY...

STEP I: DISCRETIZE THE QCD ACTION IN BOTH SPACE AND TIME. WICK ROTATE TO IMAGINARY TIMES. CONSIDER A FINITE HYPERCUBIC LATTICE.

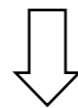


STEP I: DISCRETIZE THE QCD ACTION IN BOTH SPACE AND TIME. WICK ROTATE TO IMAGINARY TIMES. CONSIDER A FINITE HYPERCUBIC LATTICE.



STEP II: GENERATE A LARGE SAMPLE OF THERMALIZED DECORRELATED VACUUM CONFIGURATIONS.

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U_\mu \mathcal{D}q \mathcal{D}\bar{q} e^{-S_{\text{lattice}}^{(G)}[U] - S_{\text{lattice}}^{(F)}[U, q, \bar{q}]} \hat{\mathcal{O}}[U, q, \bar{q}]$$



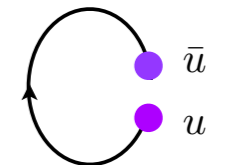
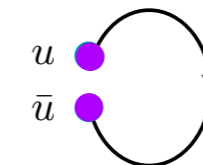
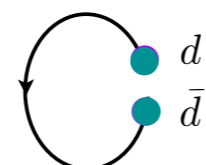
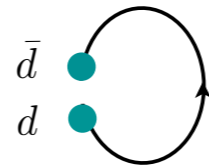
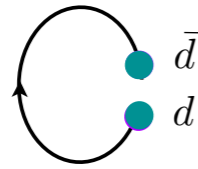
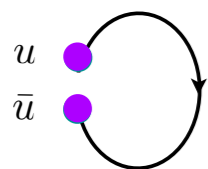
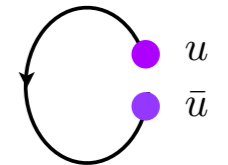
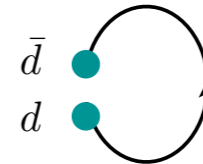
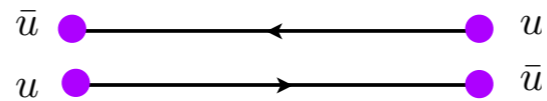
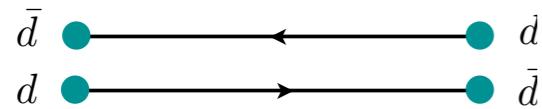
$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{N} \sum_i^N \langle \hat{\mathcal{O}} \rangle_F[U^{(i)}]$$

$U^{(i)}$  SAMPLED FROM THE DISTRIBUTION:

$$\frac{1}{\mathcal{Z}} e^{-S_{\text{lattice}}^{(G)}[U]} \prod_f \det D_f$$

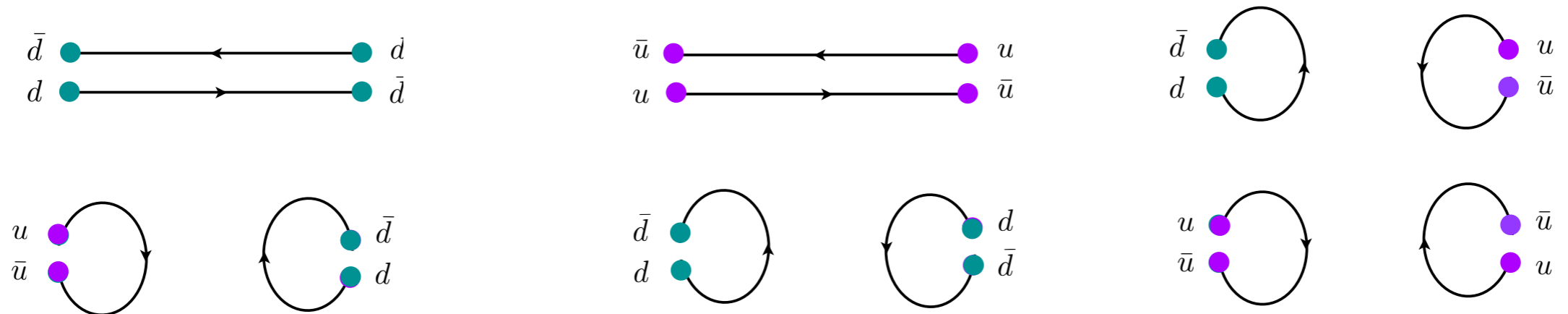
STEP III: FORM THE CORRELATION FUNCTIONS BY CONTRACTING THE QUARKS. NEED TO SPECIFY THE INTERPOLATING OPERATORS FOR THE STATE UNDER STUDY.

*e.g.*,  $\hat{O} = \frac{1}{\sqrt{2}}(\bar{u}\gamma^5 u - \bar{d}\gamma^5 d)$



STEP III: FORM THE CORRELATION FUNCTIONS BY CONTRACTING THE QUARKS. NEED TO SPECIFY THE INTERPOLATING OPERATORS FOR THE STATE UNDER STUDY.

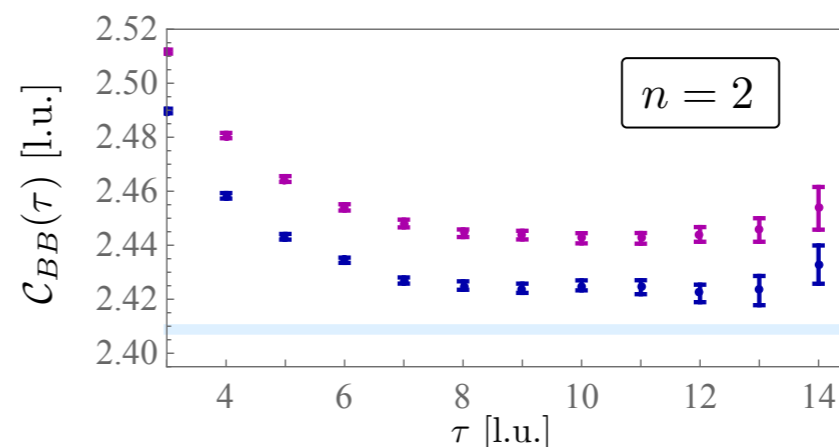
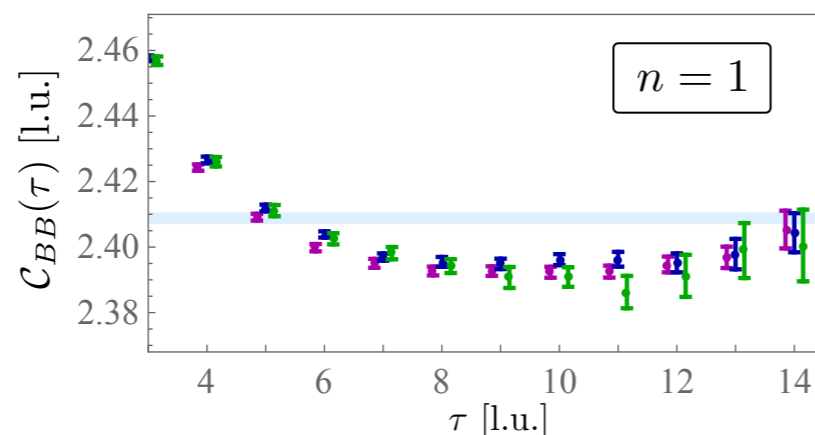
e.g.,  $\hat{O} = \frac{1}{\sqrt{2}}(\bar{u}\gamma^5 u - \bar{d}\gamma^5 d)$



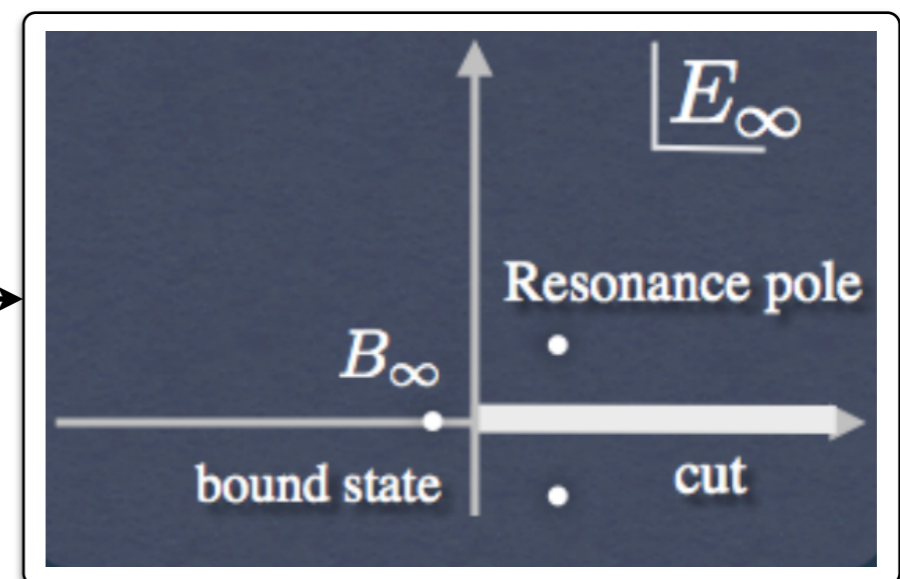
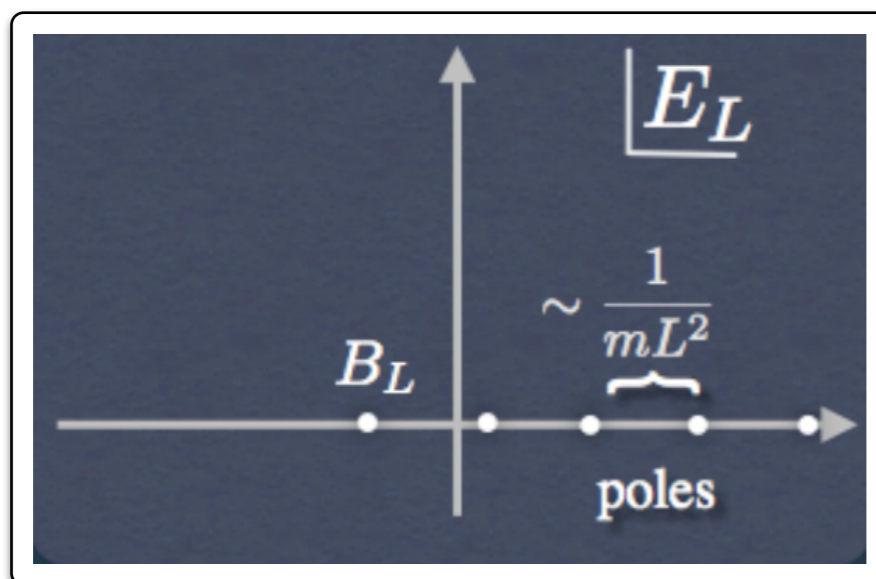
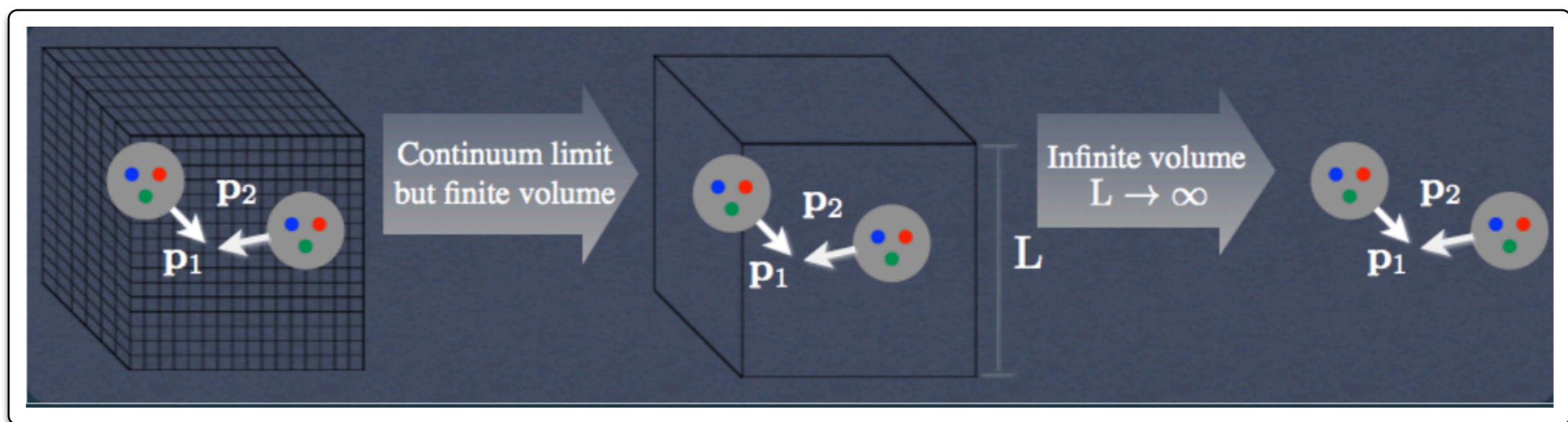
STEP IV: EXTRACT ENERGIES AND MATRIX ELEMENTS FROM CORRELATION FUNCTIONS

$$C_{\hat{O},\hat{O}'}(\tau; \mathbf{d}) = \sum_{\mathbf{x}} e^{2\pi i \mathbf{d} \cdot \mathbf{x} / L} \langle 0 | \hat{O}'(\mathbf{x}, \tau) \hat{O}^\dagger(\mathbf{0}, 0) | 0 \rangle = Z'_0 Z_0^\dagger e^{-E^{(0)}\tau} + Z'_1 Z_1^\dagger e^{-E^{(1)}\tau} + \dots$$

$NN(^1S_0)$

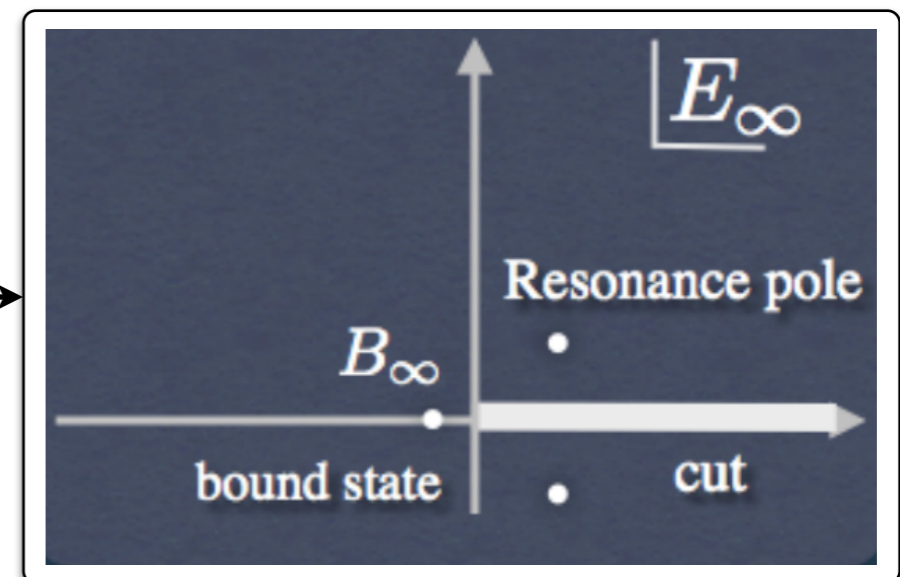
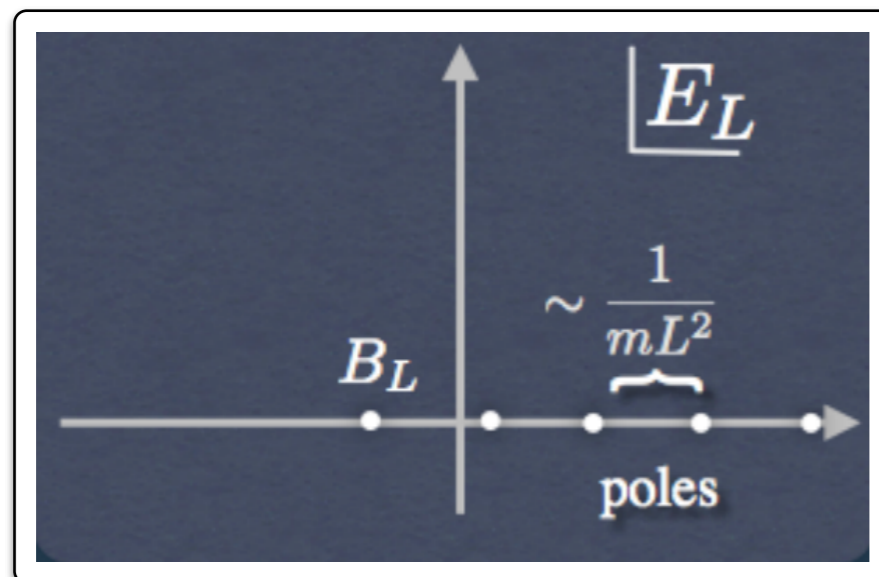
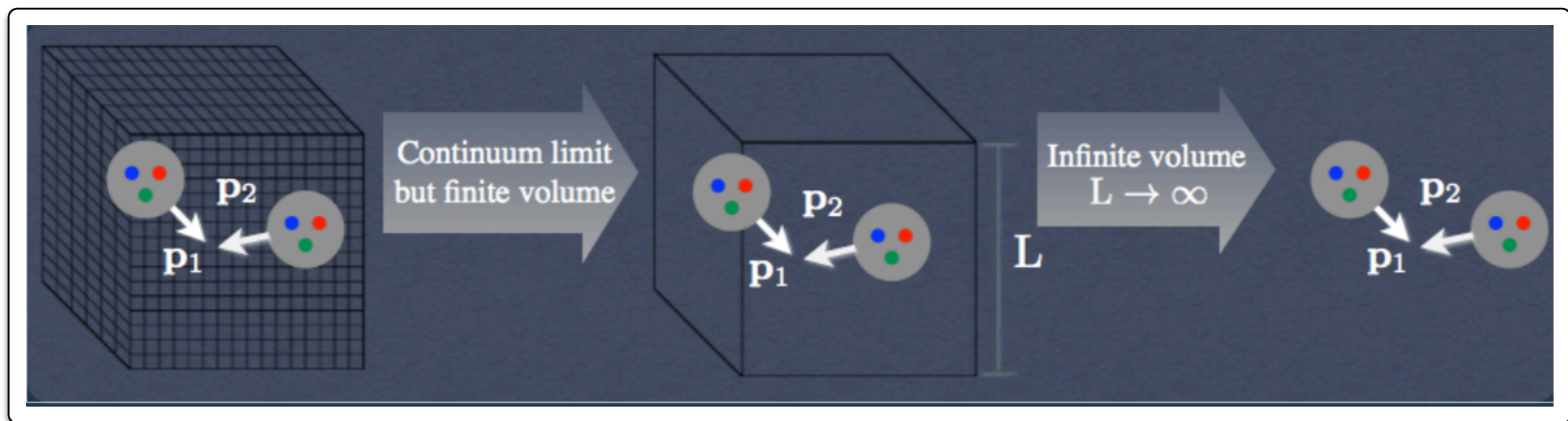


STEP V: MAKE CONNECTION TO PHYSICAL OBSERVABLES, SUCH AS SCATTERING AMPLITUDES, DECAY RATES, ETC. STILL NOT FULLY DEVELOPED AND PRESENTS CHALLENGE IN MULTI-HADRON SYSTEMS.



SEE TALKS BY RAUL AND MATTHIAS AND XU FOR TREATMENT OF THIS PROBLEM FOR MATRIX ELEMENTS RELEVANT TO  $0\nu\beta\beta$  DECAY.

STEP V: MAKE CONNECTION TO PHYSICAL OBSERVABLES, SUCH AS SCATTERING AMPLITUDES, DECAY RATES, ETC. STILL NOT FULLY DEVELOPED AND PRESENTS CHALLENGE IN MULTI-HADRON SYSTEMS.





# NPLQCD APPROACH TO THE PROBLEM OF WEAK AMPLITUDES OF LIGHT NUCLEI

# ENERGIES FROM NUCLEAR CORRELATION FUNCTIONS

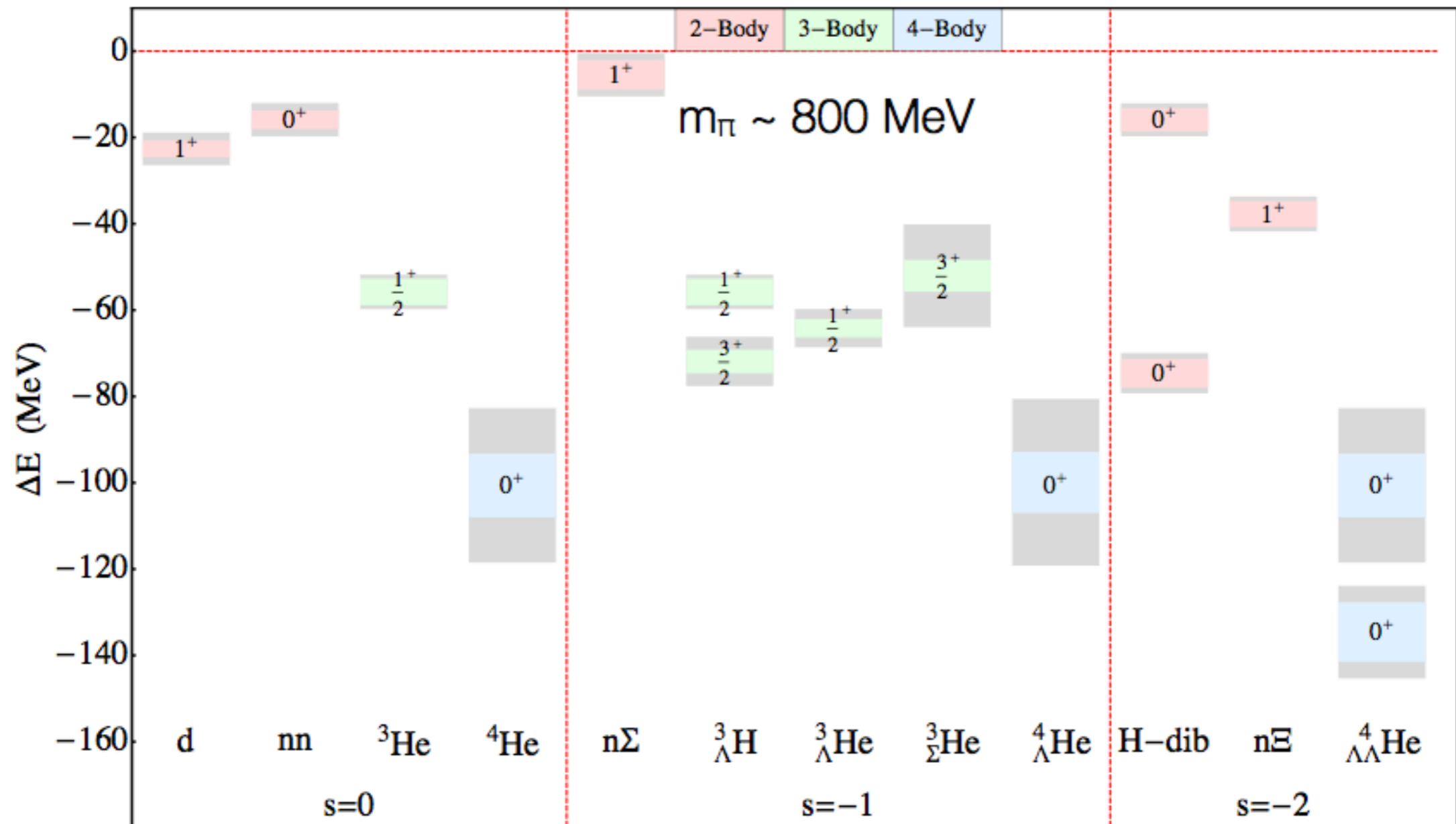
$$C(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$

The diagram illustrates the correlation function  $C(\mathbf{P}; t, t_0)$ . It shows two vertical columns of particles. The left column is labeled  $N(\mathbf{x})$  at the top and  $N(\mathbf{y})$  at the bottom, with a time label  $\tau = t$ . The right column is labeled  $N^\dagger(\mathbf{0})$  at the top and  $N^\dagger(\mathbf{0})$  at the bottom, with a time label  $\tau = 0$ . Horizontal arrows point from the left column to the right column, representing the evolution of the system. The entire diagram is enclosed in a large right-pointing arrow.

$$C_{\hat{O}, \hat{O}'}(\tau; \mathbf{d}) = \sum_{\mathbf{x}} e^{2\pi i \mathbf{d} \cdot \mathbf{x} / L} \langle 0 | \hat{O}'(\mathbf{x}, \tau) \hat{O}^\dagger(\mathbf{0}, 0) | 0 \rangle = \mathcal{Z}'_0 \mathcal{Z}_0^\dagger e^{-E^{(0)} \tau} + \mathcal{Z}'_1 \mathcal{Z}_1^\dagger e^{-E^{(1)} \tau} + \dots$$

# NUCLEI FROM QCD IN A WORLD WITH HEAVIER QUARKS

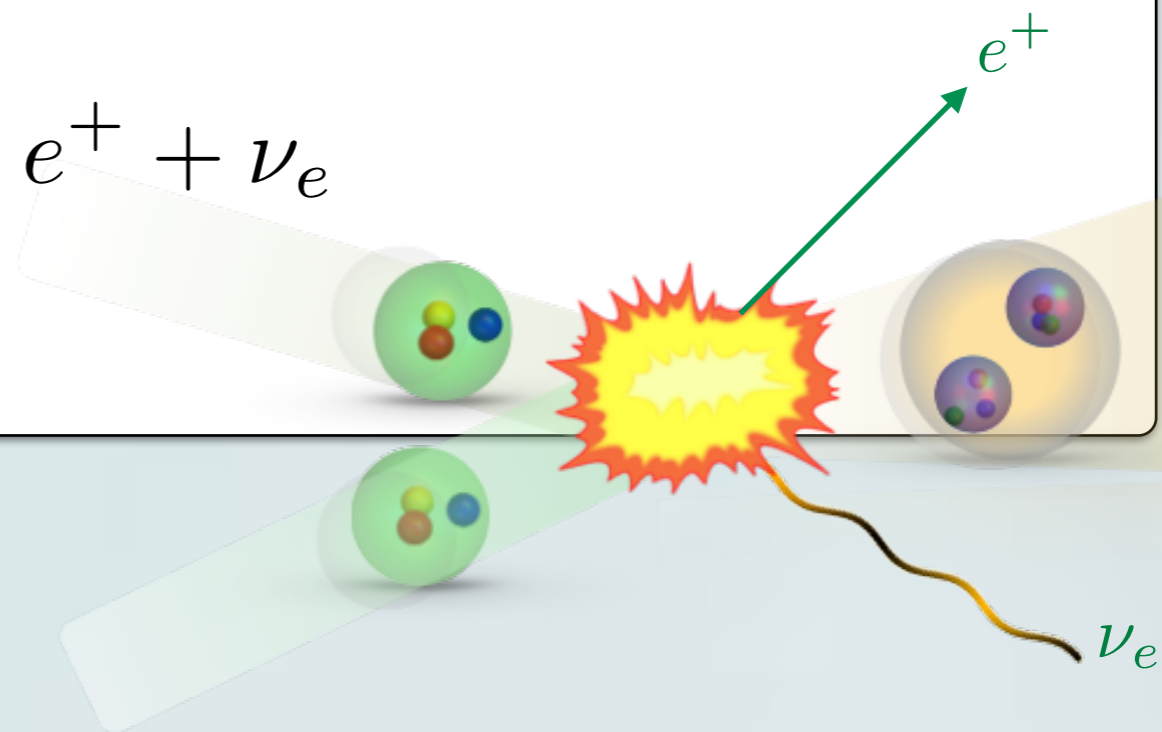
$$N_f = 3, \quad m_\pi = 0.806 \text{ GeV}, \quad a = 0.145(2) \text{ fm}$$



Beane, et al. (NPLQCD), Phys.Rev. D87 (2013), Phys.Rev. C88 (2013)

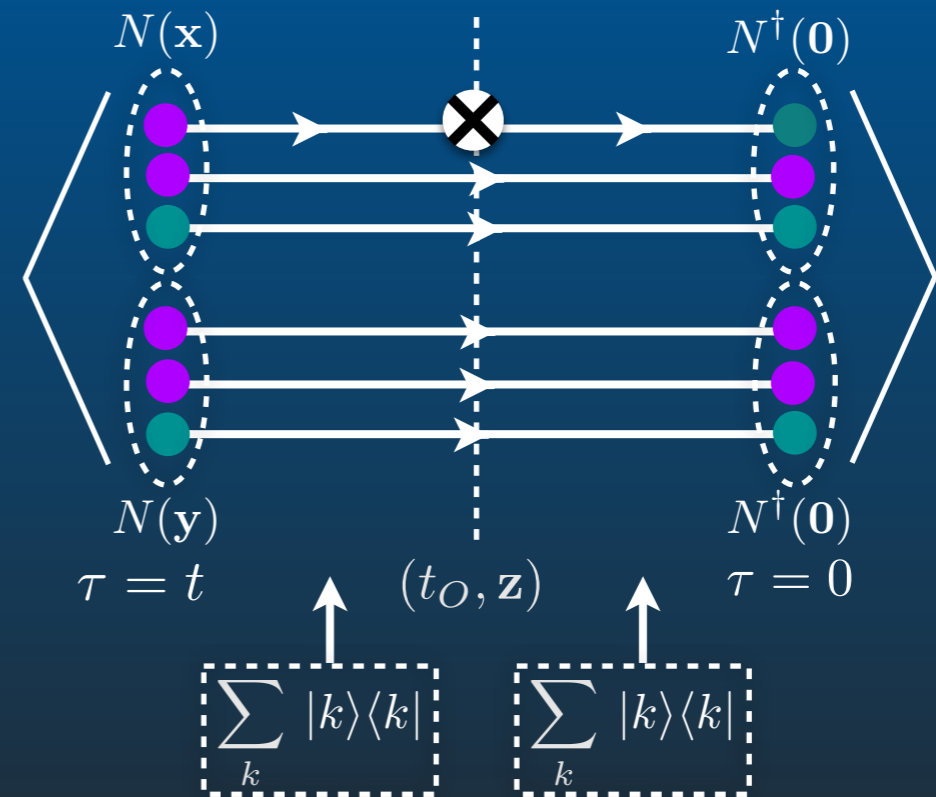
# SINGLE-WEAK PROCESSES

$$p + p \rightarrow d + e^+ + \nu_e$$



# TRADITIONAL MATRIX ELEMENT CALCULATIONS: 3-POINT FUNCTIONS

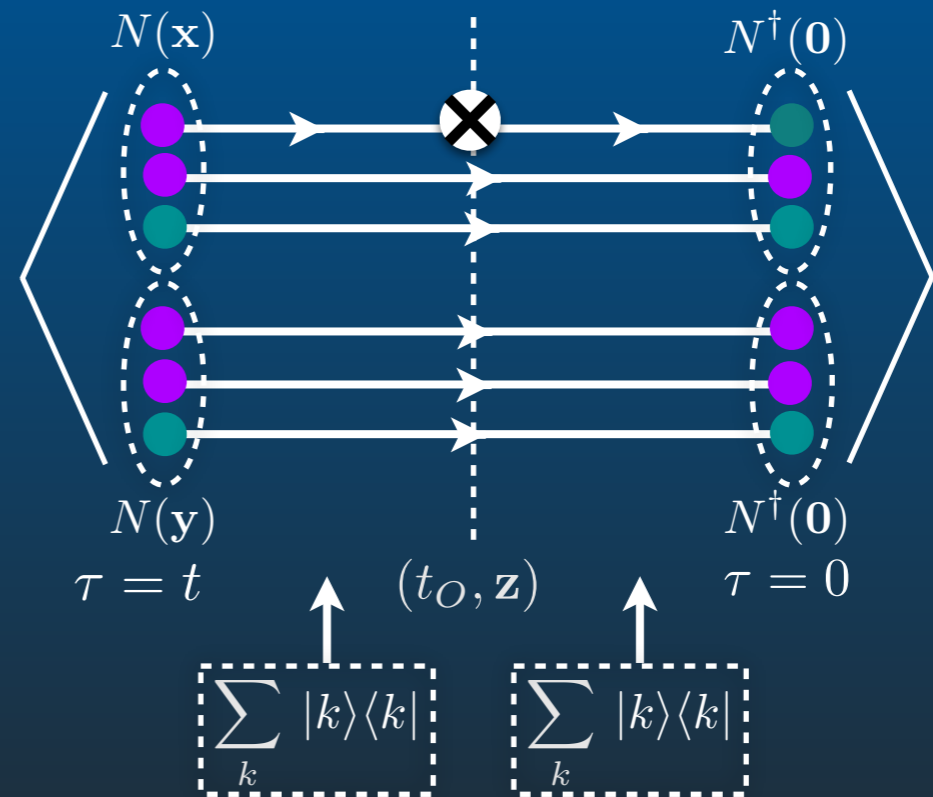
$$C(\mathbf{P}; t, t_O) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



$$= Z_{0,pp}^{\text{src}} Z_{0,d}^{\text{snk}\dagger} e^{-E_{0,pp}t_O} e^{-E_{0,d}(t-t_O)} \langle pn | A | pp \rangle_L + \dots$$

# TRADITIONAL MATRIX ELEMENT CALCULATIONS: 3-POINT FUNCTIONS

$$C(\mathbf{P}; t, t_O) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



$$= Z_{0,pp}^{\text{src}} Z_{0,d}^{\text{snk}\dagger} e^{-E_{0,pp}t_O} e^{-E_{0,d}(t-t_O)} \langle pn | A | pp \rangle_L + \dots$$

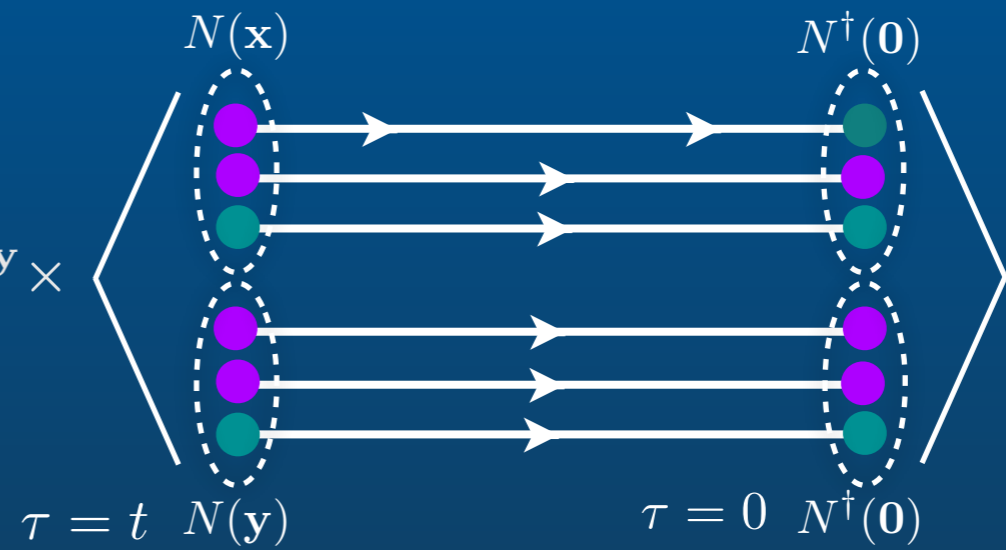
## MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD

$$S_{\lambda_q; \Gamma}^{(q)}(x, y) = S^{(q)}(x, y) + \lambda_q \int dz S^{(q)}(x, z) \Gamma S^{(q)}(z, y)$$



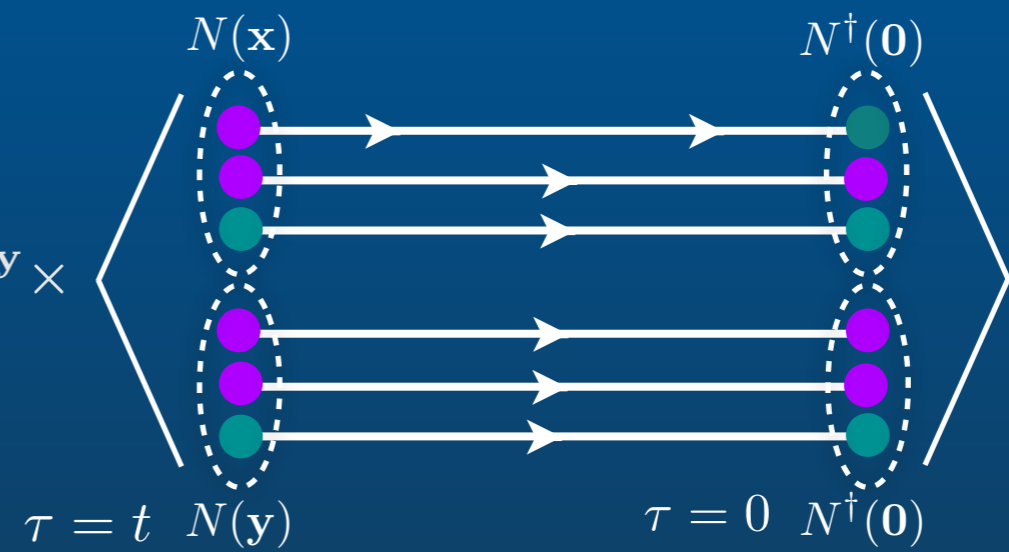
# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD

$$C_{\lambda}(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



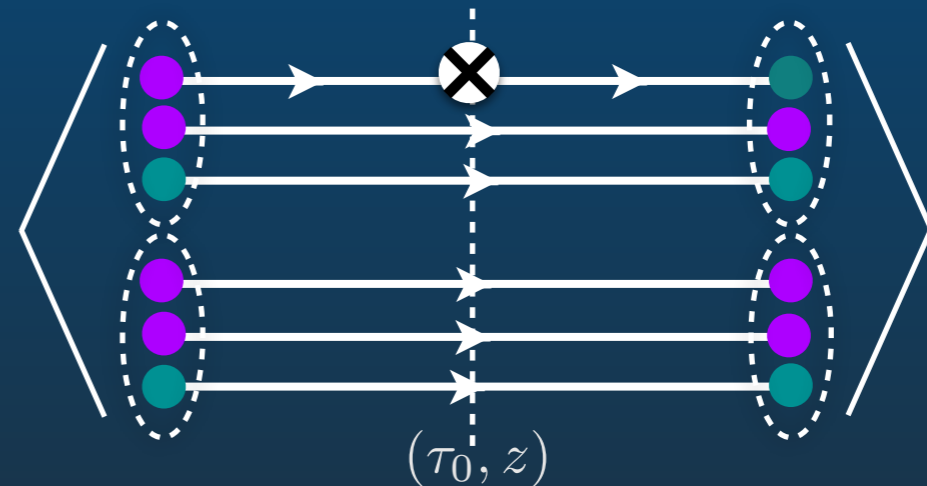
# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD

$$C_{\lambda}(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$



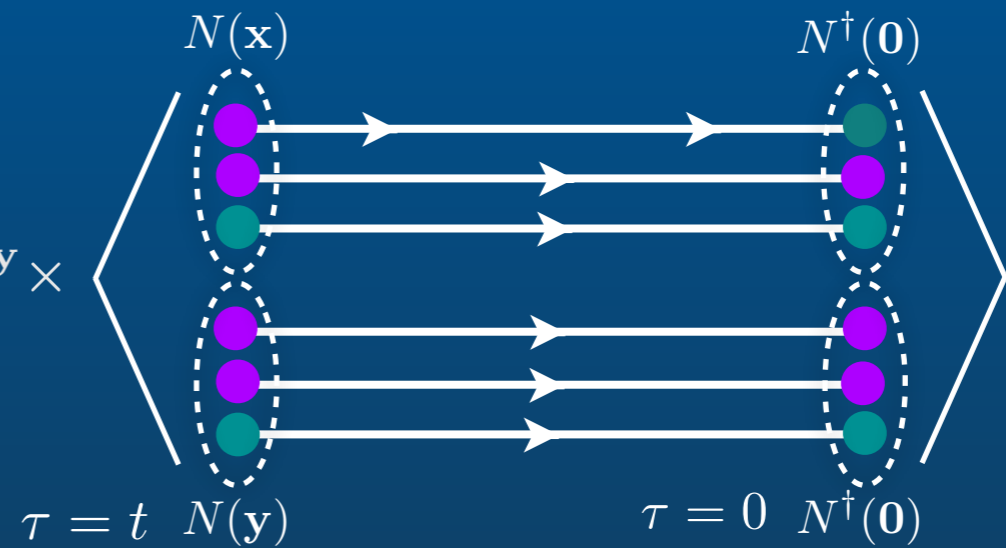
ALL  
POSSIBILITIES  $\longrightarrow$

$$+ \lambda \sum_{\tau_0=0}^T \sum_z$$

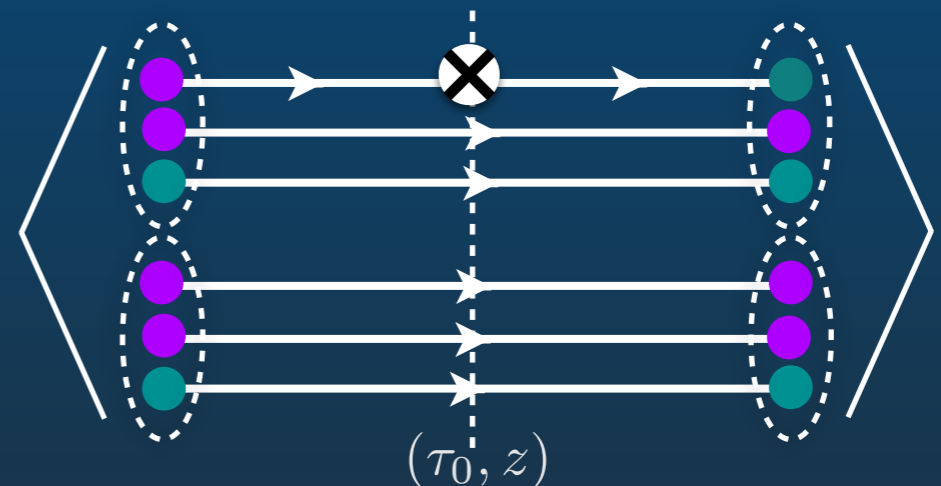


# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD

$$C_\lambda(\mathbf{P}; t, t_0) = \sum_{\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} e^{i\mathbf{p}_1 \cdot \mathbf{x} + i\mathbf{p}_2 \cdot \mathbf{y}} \times$$

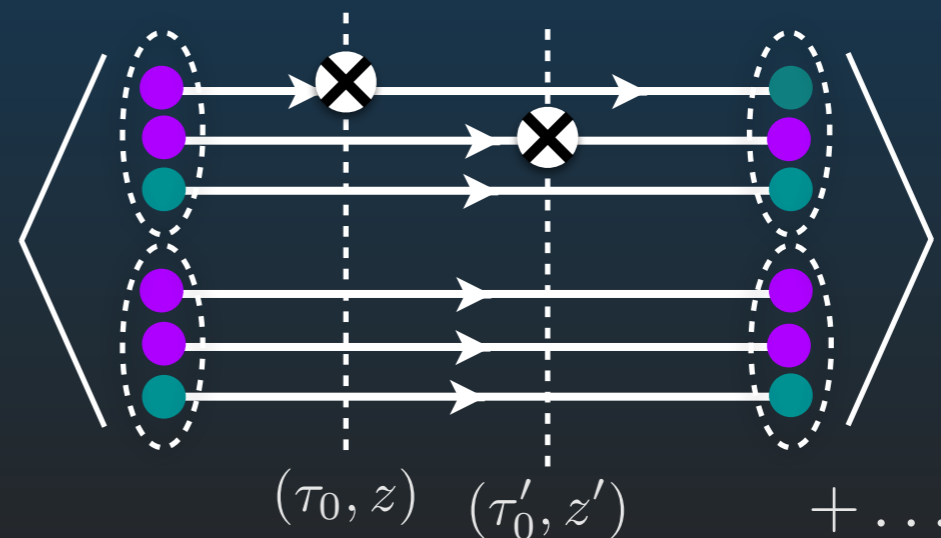


ALL  
POSSIBILITIES  $\longrightarrow$   $+ \lambda \sum_{\tau_0=0}^T \sum_z$



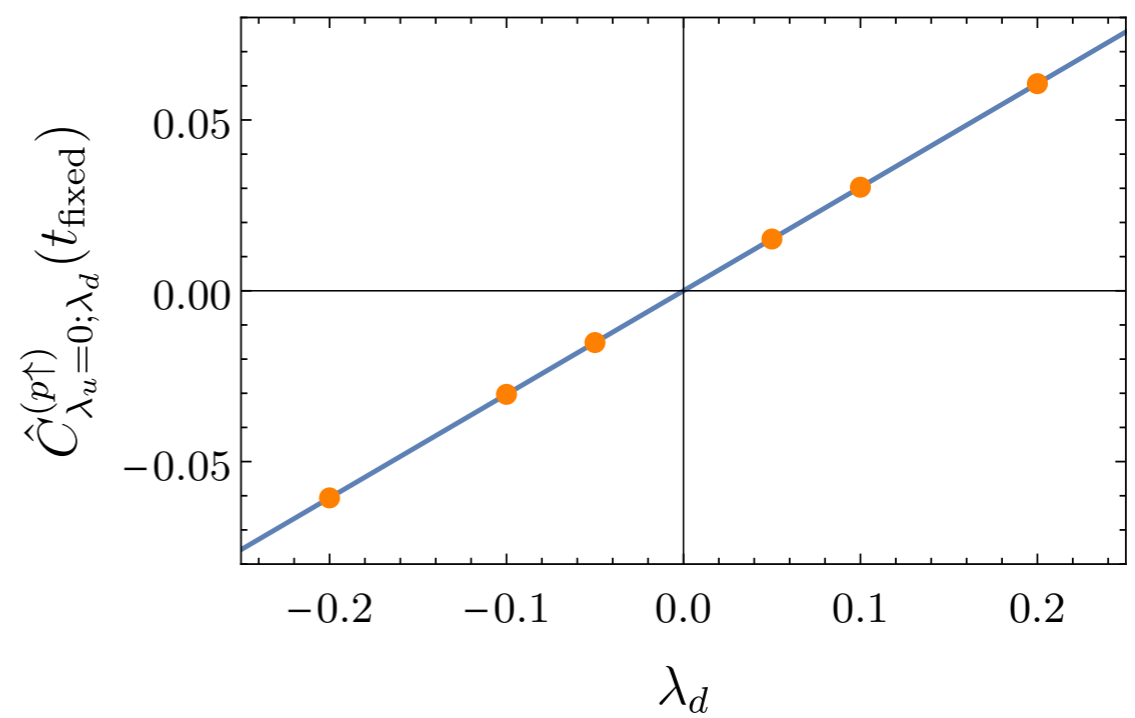
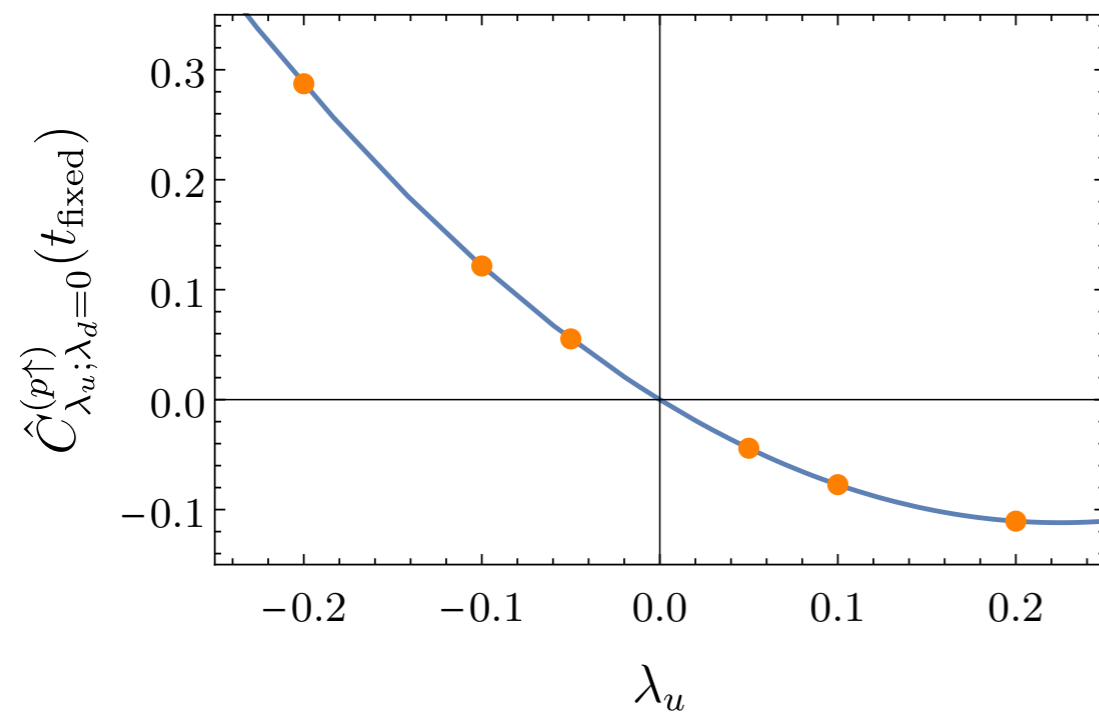
TIME-ORDERED  
PRODUCT  $\downarrow$

ALL  
POSSIBILITIES  $\longrightarrow$   $+ \lambda^2 \sum_{\tau_0=0}^T \sum_{\tau'_0=0}^T \sum_z \sum_{z'}$

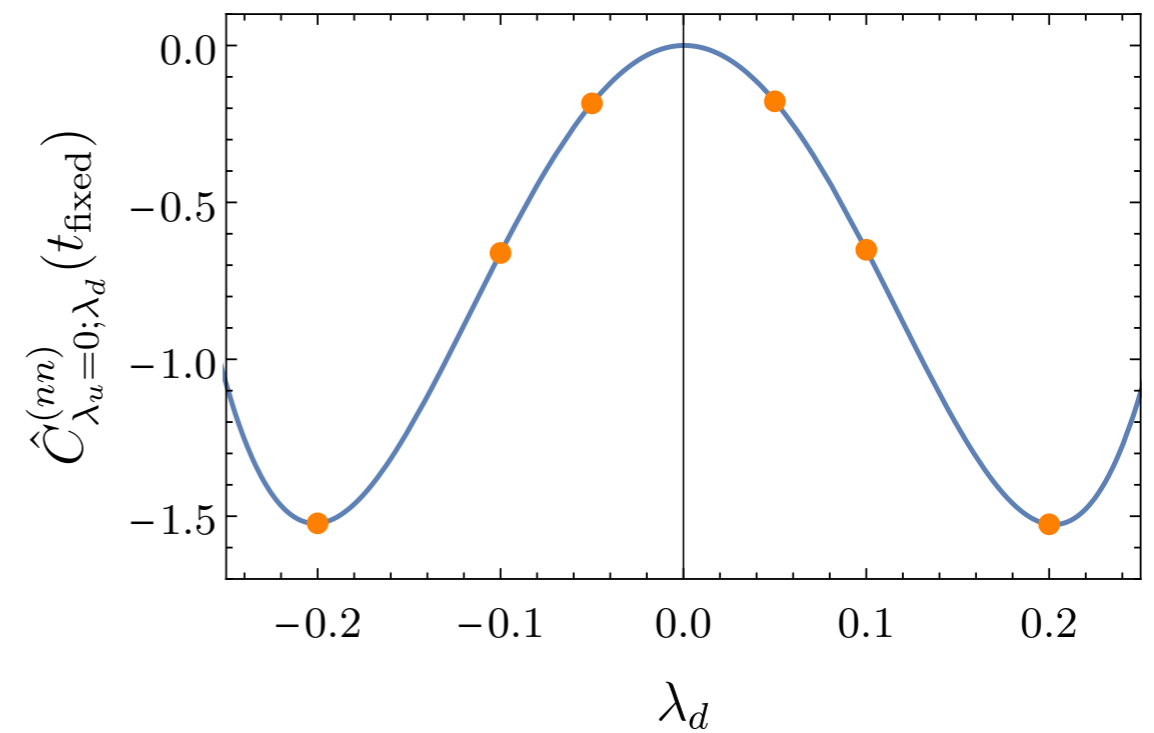
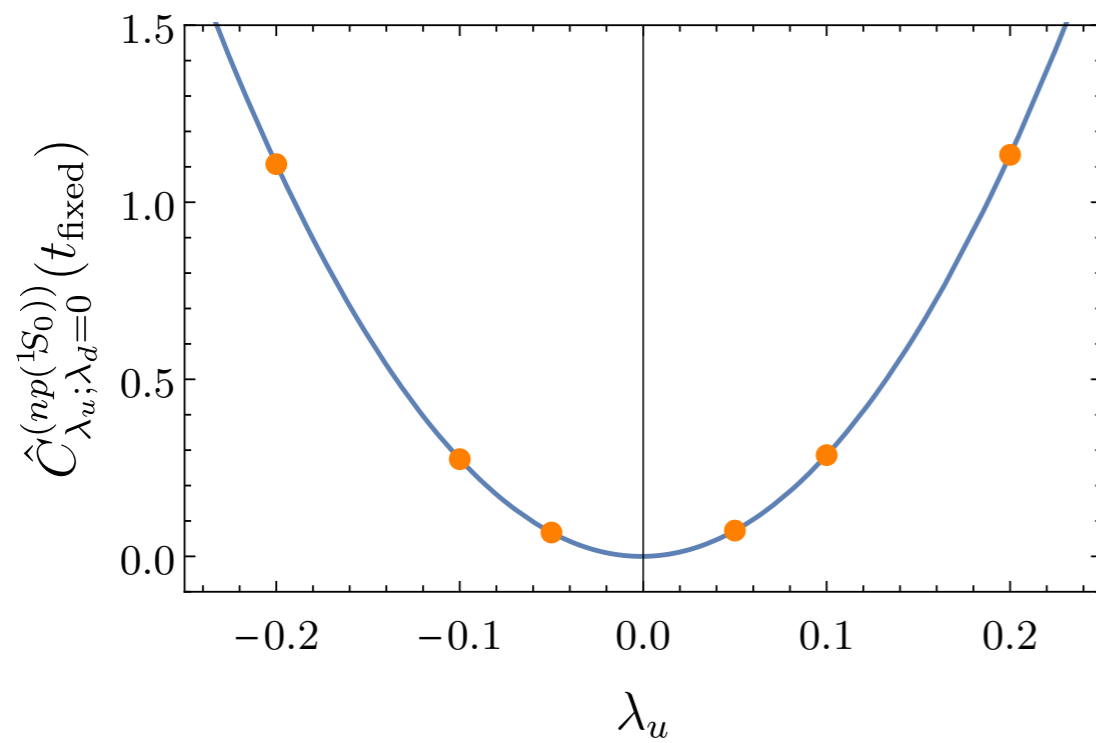
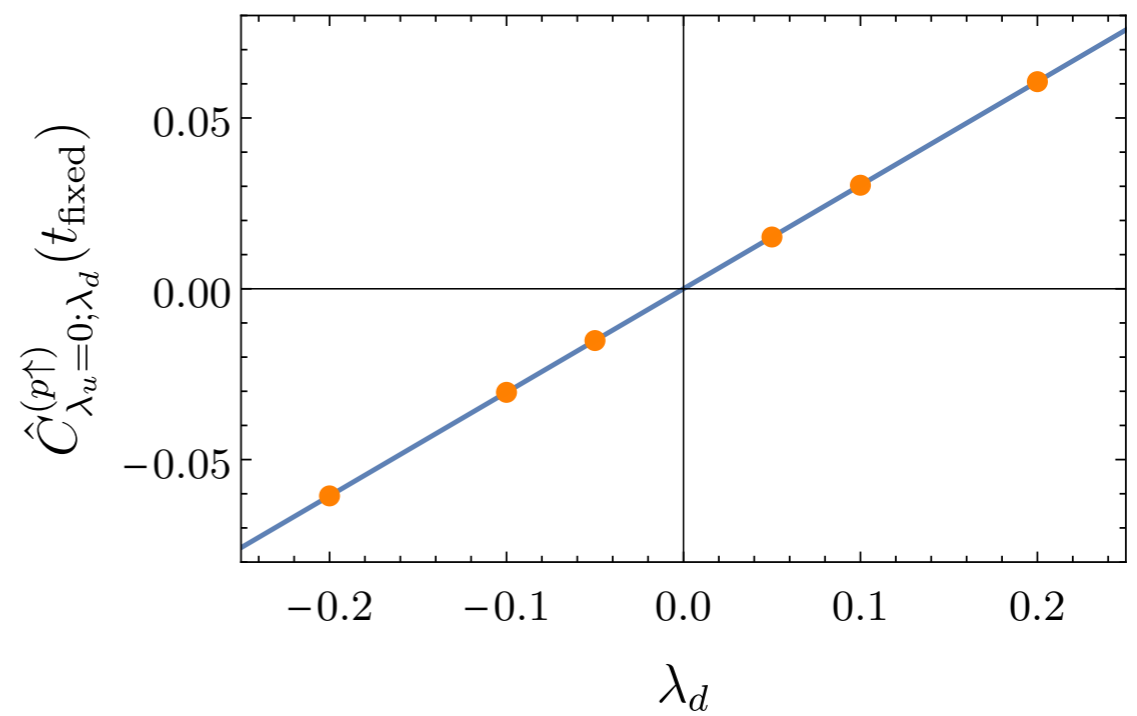
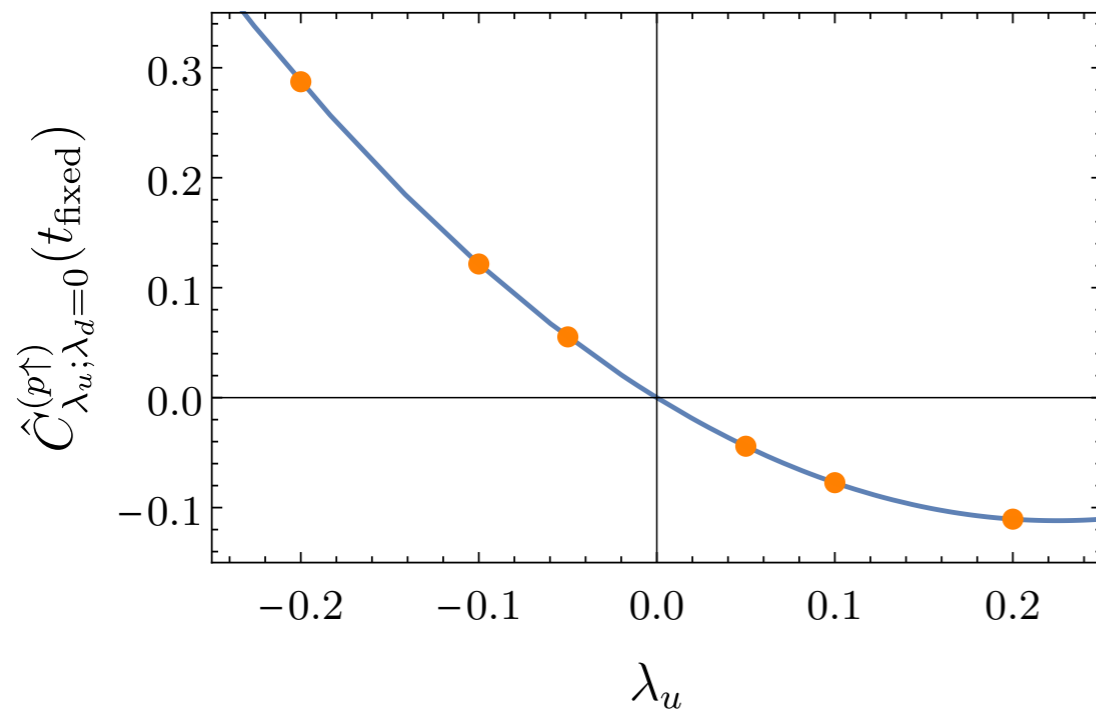


DOUBLE-CURRENT MES ARE EXACT  
FOR ISOTENSOR QUANTITIES.

# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD



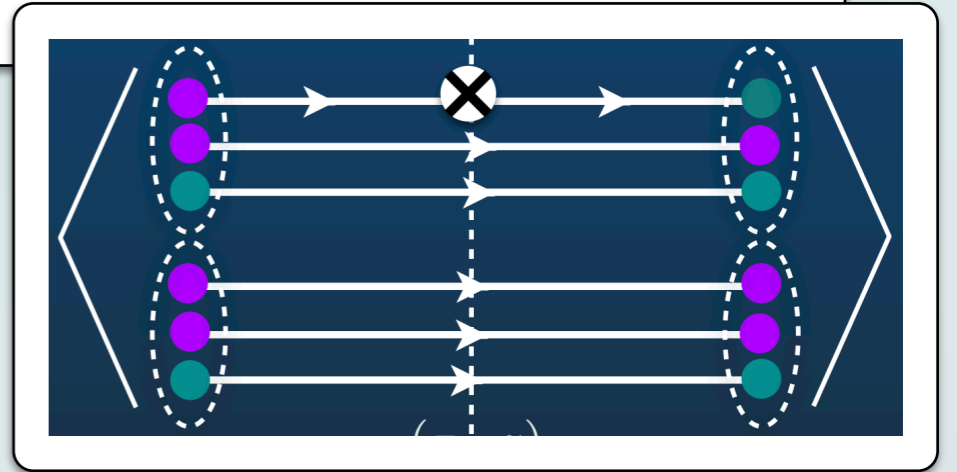
# MATRIX ELEMENTS FROM A COMPOUND PROPAGATOR/BACKGROUND FIELD



# FIRST-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD

$$C_{\lambda_u; \lambda_d=0}^{(^3S_1, ^1S_0)}(t) = \lambda_u \sum_{t_1=0}^t \sum_{\mathbf{x}, \mathbf{y}} \langle 0 | \chi_{^3S_1}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) \chi_{^1S_0}^\dagger(0) | 0 \rangle + c_2 \lambda_u^2 + c_3 \lambda_u^3$$

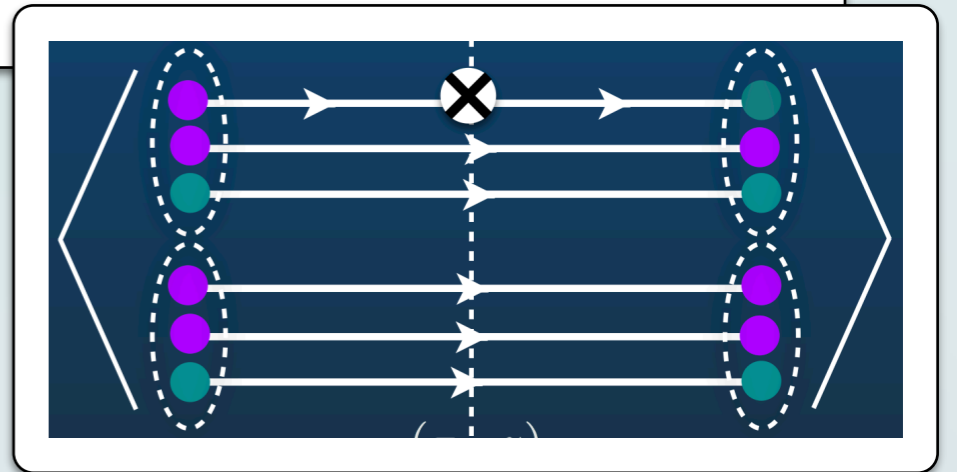
$$C_{\lambda_u=0; \lambda_d}^{(^3S_1, ^1S_0)}(t) = \lambda_d \sum_{t_1=0}^t \sum_{\mathbf{x}, \mathbf{y}} \langle 0 | \chi_{^3S_1}(\mathbf{x}, t) J_3^{(d)}(\mathbf{y}, t_1) \chi_{^1S_0}^\dagger(0) | 0 \rangle + b_2 \lambda_d^2 + b_3 \lambda_d^3,$$



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$$C_{\lambda_u=0; \lambda_d}^{(^3S_1, ^1S_0)}(t) = \lambda_d \sum_{t_1=0}^t \sum_{\mathbf{x}, \mathbf{y}} \langle 0 | \chi_{^3S_1}(\mathbf{x}, t) J_3^{(d)}(\mathbf{y}, t_1) \chi_{^1S_0}^\dagger(0) | 0 \rangle + b_2 \lambda_d^2 + b_3 \lambda_d^3,$$



$$C_{\lambda_u; \lambda_d=0}^{(^3S_1, ^1S_0)}(t) \Big|_{\mathcal{O}(\lambda_u)} = \underbrace{Z_d Z_{np(^1S_0)}^\dagger}_{\text{OVERLAP FACTORS}} e^{-\bar{E}t} \left[ \sinh\left(\frac{\Delta t}{2}\right) \left\{ \frac{\langle d | \tilde{J}_3^{(u)} | np(^1S_0) \rangle}{a\Delta/2} + \underbrace{c_-}_{\text{DEPEND ON EXCITED-STATES}} \right\} + \cosh\left(\frac{\Delta t}{2}\right) \underbrace{c_+}_{\text{DEPEND ON EXCITED-STATES}} + \mathcal{O}(e^{-\tilde{\delta}t}) \right]$$

DINEUTRON-DEUTERON MASS DIFFERENCE      GROUND-STATE MATRIX ELEMENT

SINCE  $a\Delta < 0.01$ , WE ARE ABLE TO FIT THE GROUND-STATE ME UP TO A SMALL SYSTEMATICS.

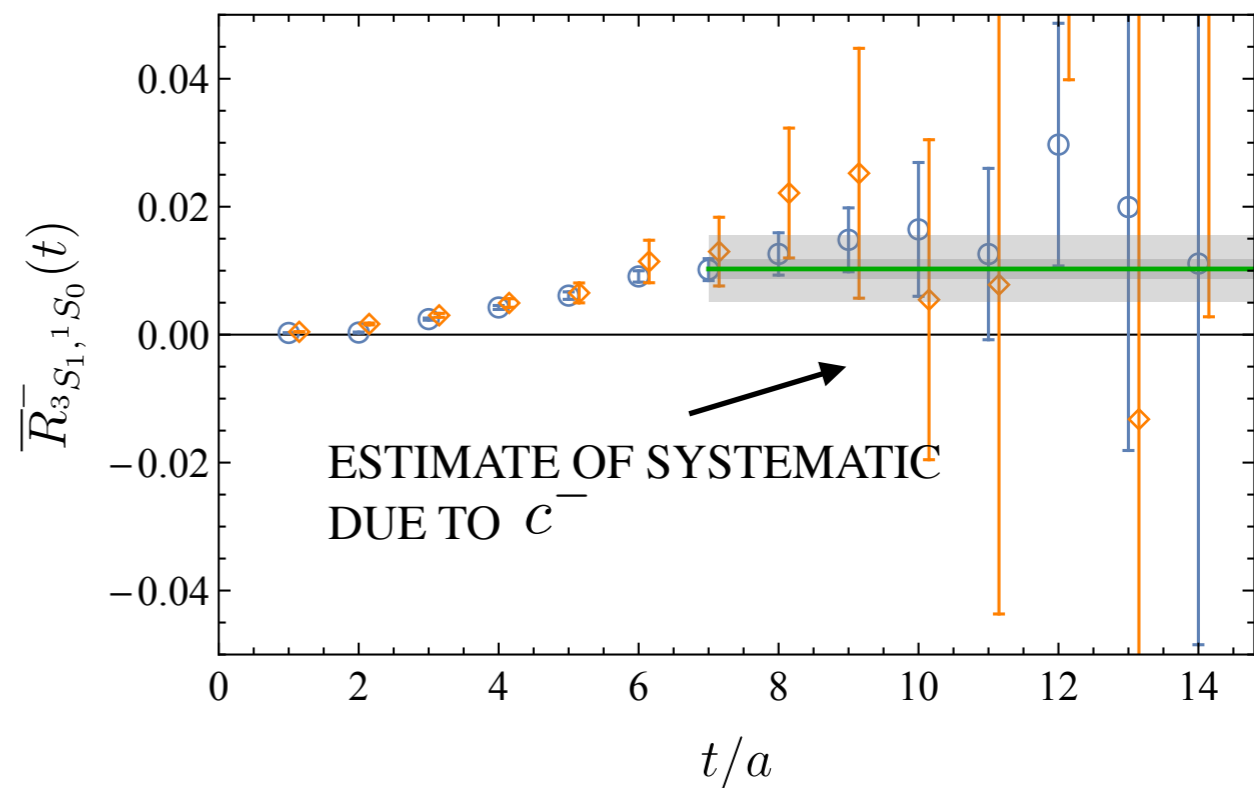
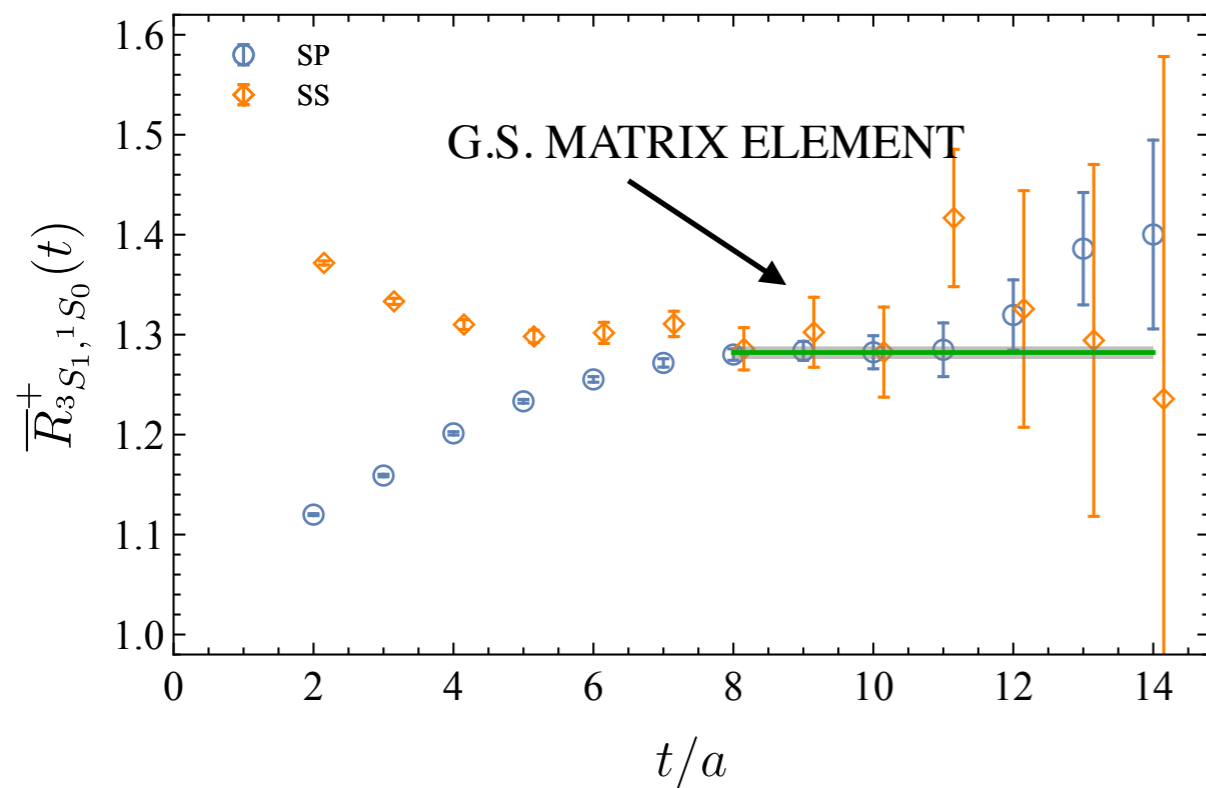
# MATRIX ELEMENT FROM QCD

AVERAGE AND DIFFERENCE OF CORRELATOR  
AND TIME-REVERSED CORREALTOR

$$R_{3S_1, 1S_0}^{\pm}(t) = \frac{1}{2} \frac{C_{\lambda_u; \lambda_d=0}^{\pm}(t) \Big|_{\mathcal{O}(\lambda_u)} - C_{\lambda_u=0; \lambda_d}^{\pm}(t) \Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C_{0;0}^{(3S_1)}(t) C_{0;0}^{(1S_0)}(t)}}$$

# MATRIX ELEMENT FROM QCD

$$N_f = 3, \quad m_\pi = 0.806 \text{ GeV}, \quad a = 0.145(2) \text{ fm}$$



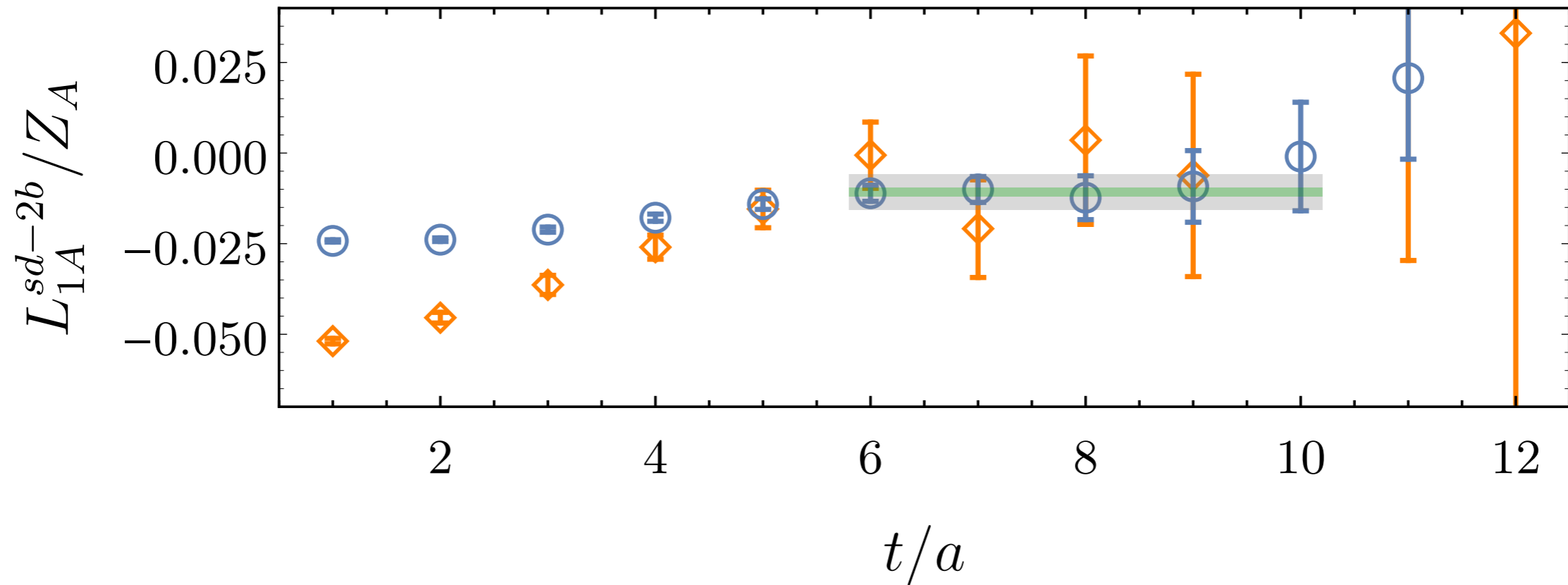
Savage et al (NPLQCD), Phys. Rev. Lett. 119, 062002 (2017).

AVERAGE AND DIFFERENCE OF CORRELATOR  
AND TIME-REVERSED CORREALTOR

$$R_{3S_1, 1S_0}^\pm(t) = \frac{1}{2} \frac{C_{\lambda_u; \lambda_d=0}^\pm(t) \Big|_{\mathcal{O}(\lambda_u)} - C_{\lambda_u=0; \lambda_d}^\pm(t) \Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C_{0;0}^{(3S_1)}(t) C_{0;0}^{(1S_0)}(t)}}$$

# MATRIX ELEMENT FROM QCD

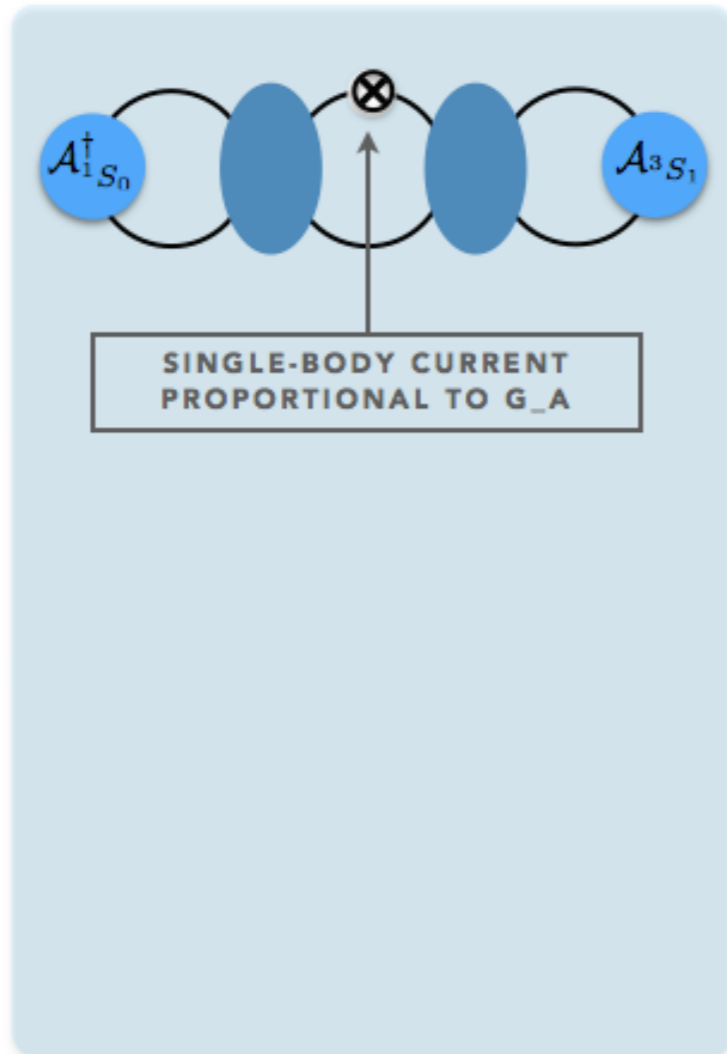
$N_f = 3, m_\pi = 0.806 \text{ GeV}, a = 0.145(2) \text{ fm}$



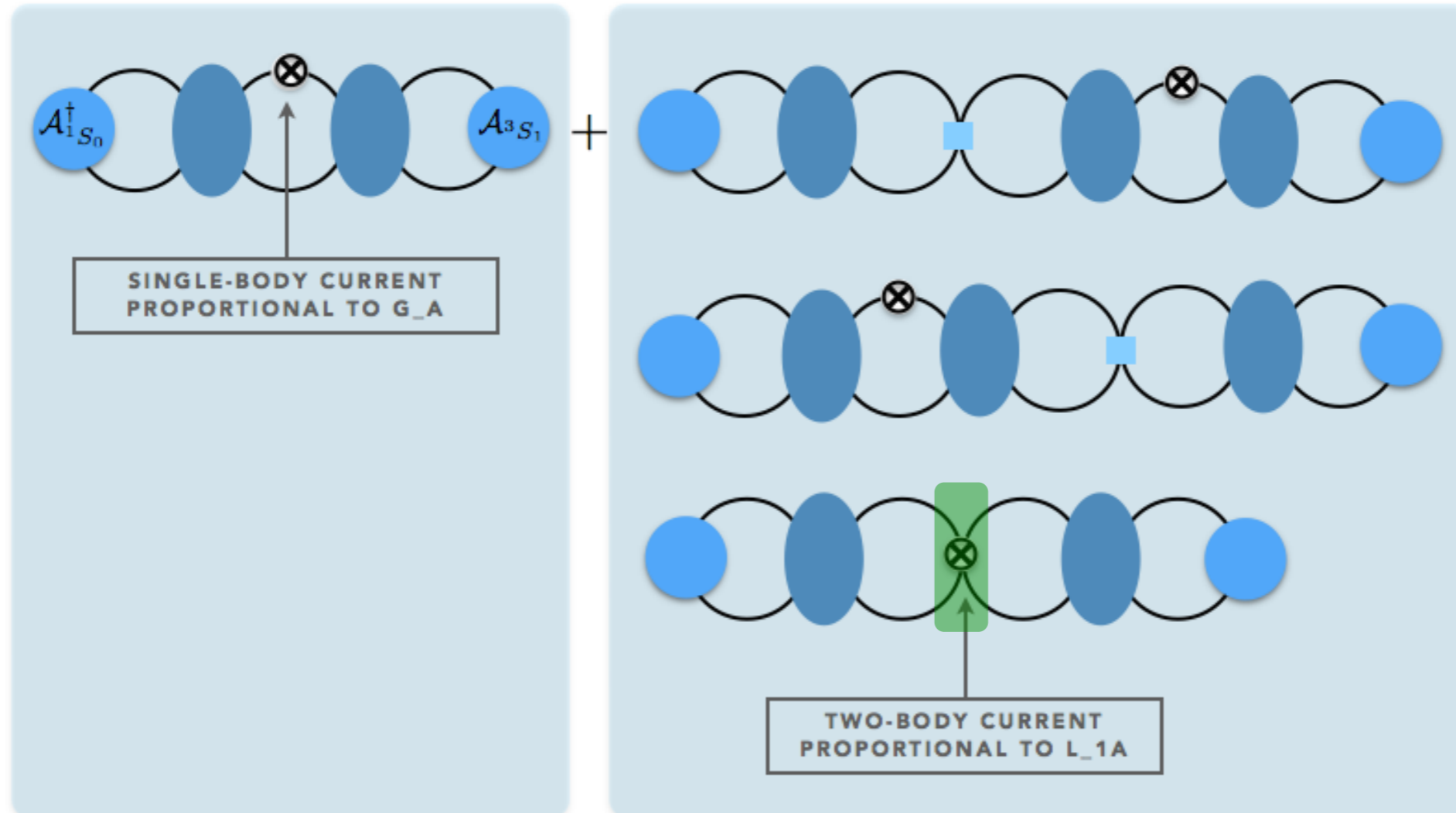
Savage et al (NPLQCD), Phys. Rev. Lett. 119, 062002 (2017).

$$L_{1,A}^{sd-2b} \equiv \frac{|\langle pp | A_3^+ | d \rangle| - g_A}{Z_A} = -0.011(01)(15)$$

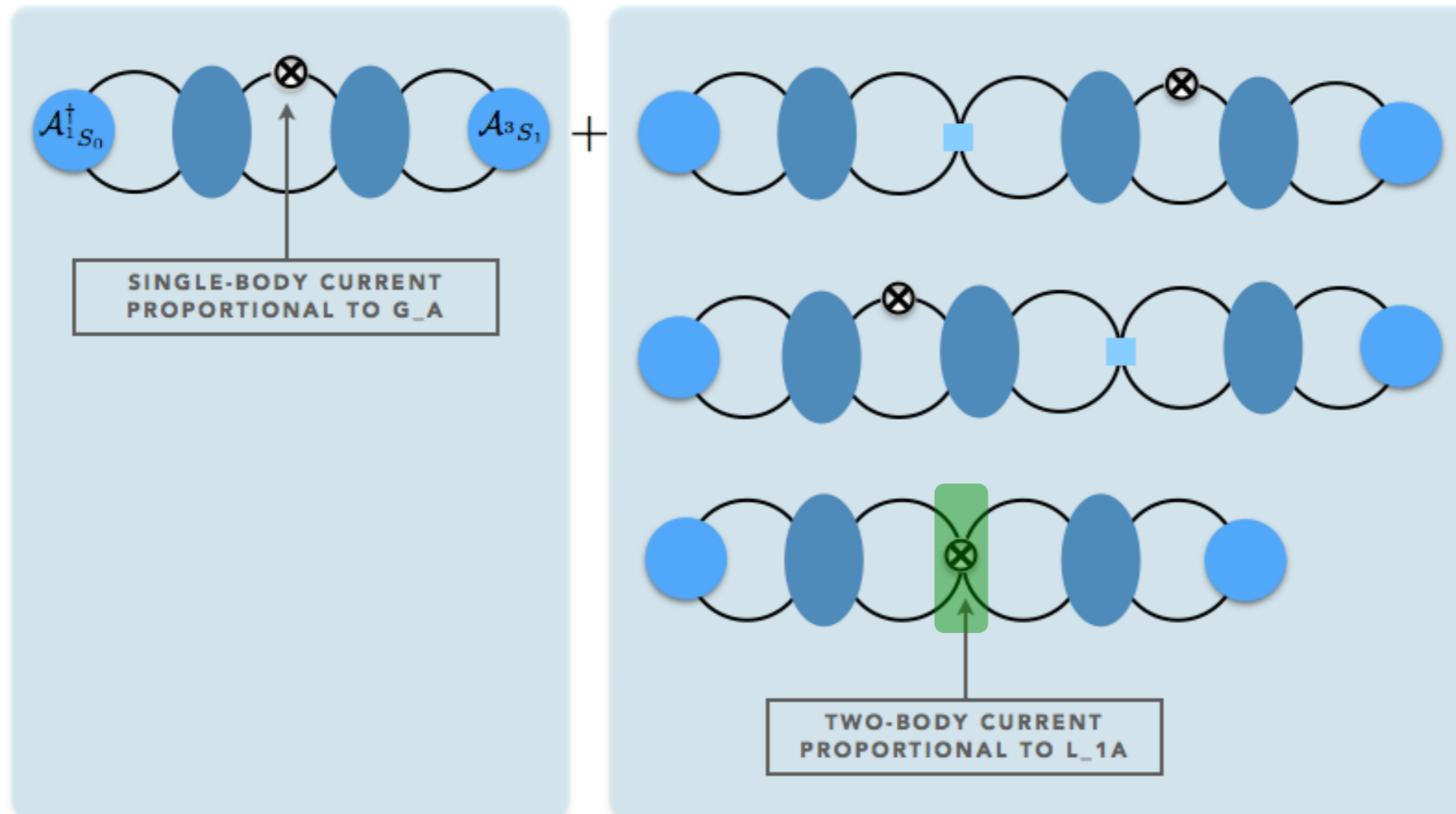
# MATRIX ELEMENT FROM EFT



# MATRIX ELEMENT FROM EFT

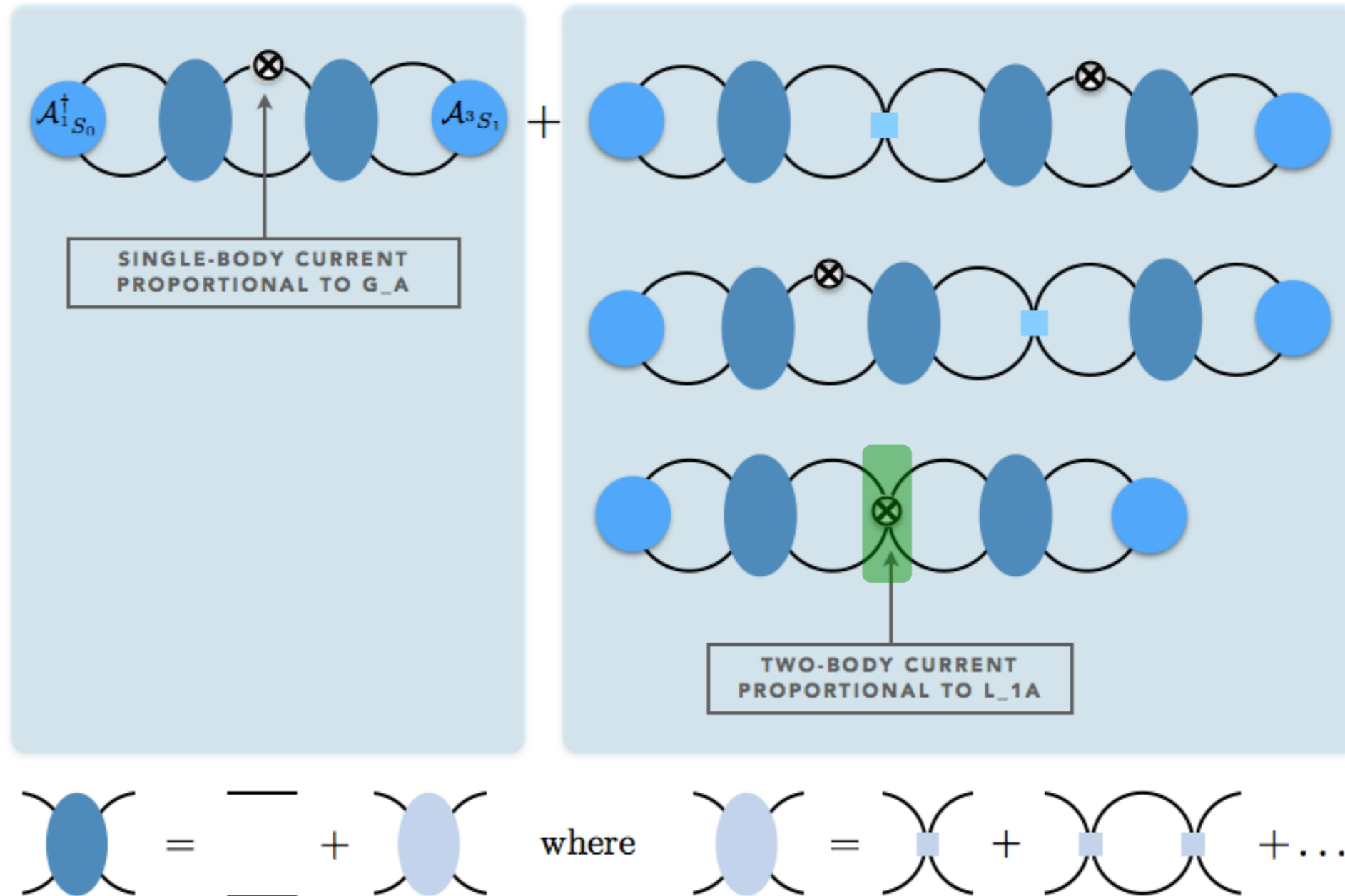


# MATRIX ELEMENT FROM EFT



$$\text{Blue Oval} = \text{Two Parallel Lines} + \text{Light Blue Oval} \quad \text{where} \quad \text{Light Blue Oval} = \text{Crossed Lines} + \text{Crossed Lines with Squares} + \dots$$

# MATRIX ELEMENT FROM EFT



Detmold and Savage, Nucl.Phys. A743 (2004) 170.  
Briceno and ZD, Phys. Rev. D 88, 094507 (2013).

Chen et al, Phys.Rev. C67 (2003) 025801.

$$|\langle d; j | A_k^- | pp \rangle| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}} \Lambda(p) \delta_{jk}$$

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}} \{e^\chi - \gamma a_{pp} [1 - \chi e^\chi \Gamma(0, \chi)] + \frac{1}{2} \gamma^2 a_{pp} \sqrt{r_1 \rho}\} - \frac{1}{2g_A} \gamma a_{pp} \sqrt{1-\gamma\rho} L_{1,A}^{sd-2b}$$

# TWO-NUCLEON SHORT-DISTANCE COUPLING

FROM TRITON LIFETIME:

$$L_{1,A} \approx 2.0(2.4) \text{ fm}^3 \quad @ \quad \mu = m_{\pi}^{\text{phys.}} = 140 \text{ MeV}$$

De-Leon, Platter and Gazit, arXiv:1611.10004 (2016).

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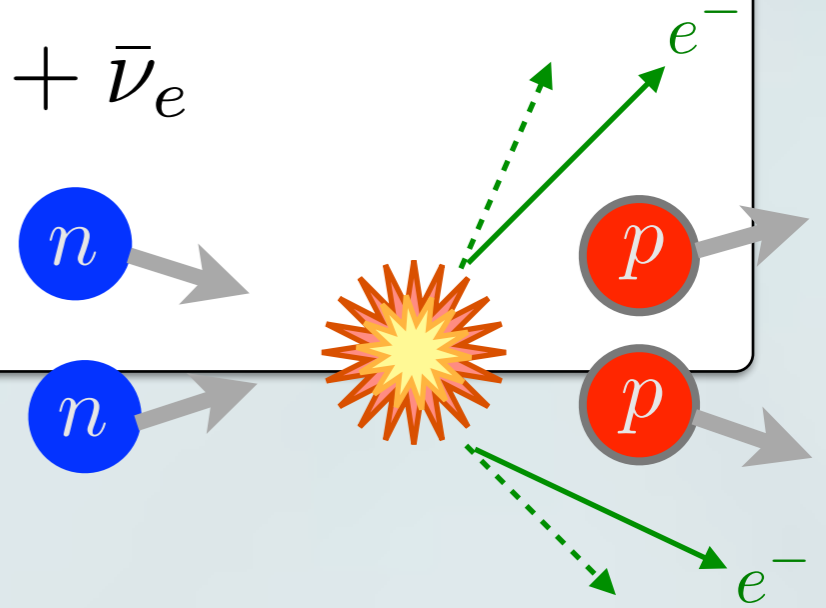
THIS WORK:

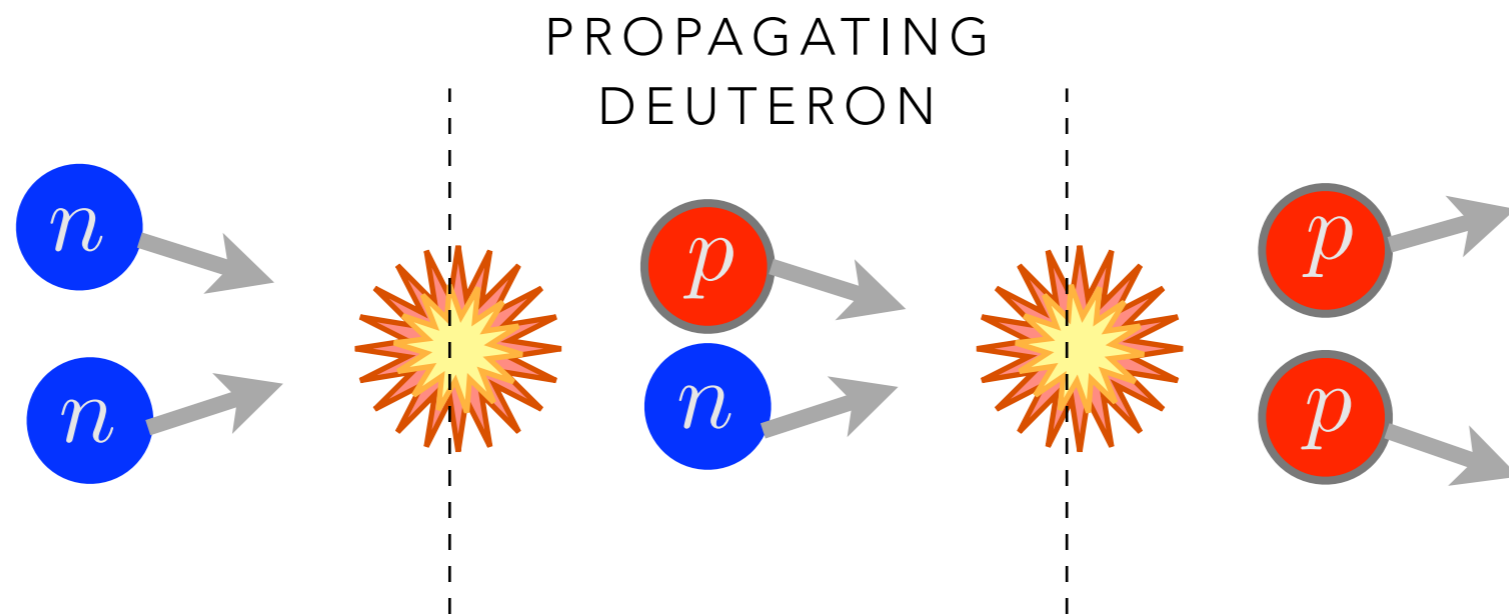
$$L_{1,A} \approx 3.9(0.2)(1.0)(0.4)(0.9) \text{ fm}^3 \text{ @ } \mu = m_{\pi}^{\text{phys.}} = 140 \text{ MeV}$$

Savage et al (NPLQCD), Phys. Rev. Lett. 119, 062002 (2017).

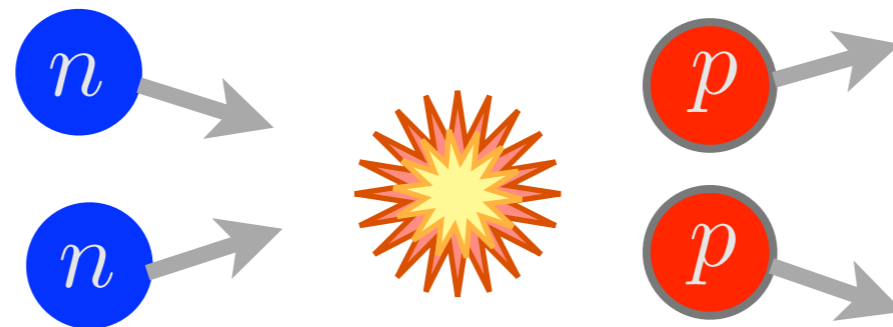
# NEUTRINOFUL DOUBLE-BETA DECAY

$$n + n \rightarrow p + p + e + e + \bar{\nu}_e + \bar{\nu}_e$$





LONG-DISTANCE PIECE



SHORT-DISTANCE PIECE

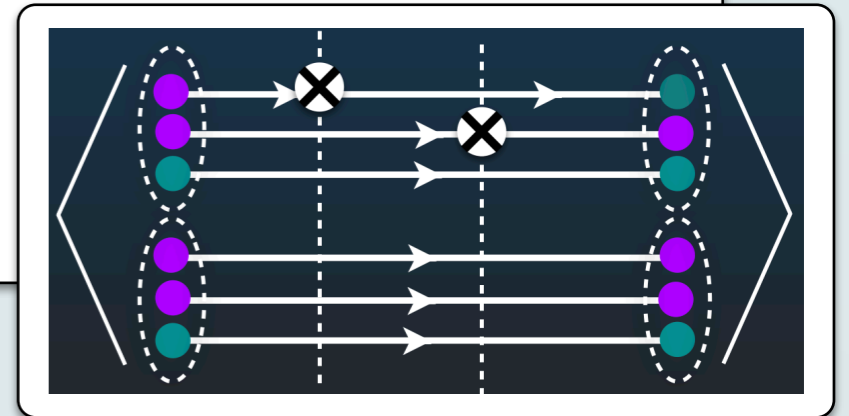
# SECOND-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD

$$C_{nn \rightarrow pp}(t) = 2 C_{\lambda_u; \lambda_d=0}^{(np(^1S_0))}(t) \Big|_{\mathcal{O}(\lambda_u^2)} - C_{\lambda_u; \lambda_d=0}^{(nn)}(t) \Big|_{\mathcal{O}(\lambda_u^2)} - C_{\lambda_u=0; \lambda_d}^{(nn)}(t) \Big|_{\mathcal{O}(\lambda_d^2)}$$



$$C_{\lambda_u; \lambda_d=0}^{(np(^1S_0))}(t) = \sum_{\mathbf{x}} \langle 0 | \chi_{np}(\mathbf{x}, t) \chi_{np}^\dagger(0) | 0 \rangle + \lambda_u \sum_{\mathbf{x}, \mathbf{y}} \sum_{t_1=0}^t \langle 0 | \chi_{np}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) \chi_{np}^\dagger(0) | 0 \rangle$$

$$+ \frac{\lambda_u^2}{2} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \sum_{t_1=0}^t \sum_{t_2=0}^t \langle 0 | \chi_{np}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) J_3^{(u)}(\mathbf{z}, t_2) \chi_{np}^\dagger(0) | 0 \rangle + g_3 \lambda_u^3,$$

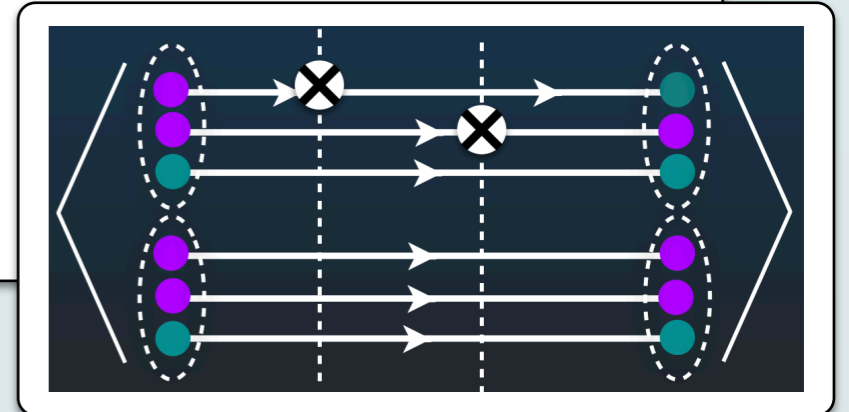


# SECOND-ORDER RESPONSE TO AN AXIAL BACKGROUND FIELD

$$C_{nn \rightarrow pp}(t) = 2 C_{\lambda_u; \lambda_d=0}^{(np(^1S_0))}(t) \Big|_{\mathcal{O}(\lambda_u^2)} - C_{\lambda_u; \lambda_d=0}^{(nn)}(t) \Big|_{\mathcal{O}(\lambda_u^2)} - C_{\lambda_u=0; \lambda_d}(t) \Big|_{\mathcal{O}(\lambda_d^2)}$$



$$C_{\lambda_u; \lambda_d=0}^{(np(^1S_0))}(t) = \sum_{\mathbf{x}} \langle 0 | \chi_{np}(\mathbf{x}, t) \chi_{np}^\dagger(0) | 0 \rangle + \lambda_u \sum_{\mathbf{x}, \mathbf{y}} \sum_{t_1=0}^t \langle 0 | \chi_{np}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) \chi_{np}^\dagger(0) | 0 \rangle \\ + \frac{\lambda_u^2}{2} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \sum_{t_1=0}^t \sum_{t_2=0}^t \langle 0 | \chi_{np}(\mathbf{x}, t) J_3^{(u)}(\mathbf{y}, t_1) J_3^{(u)}(\mathbf{z}, t_2) \chi_{np}^\dagger(0) | 0 \rangle + g_3 \lambda_u^3,$$



ALREADY CONSTRAINED FROM  
ZEROTH AND FIRST ORDERS!

SHORT-DISTANCE G.S. TO G.S. ME

$$a^2 \mathcal{R}_{nn \rightarrow pp}(t) = \left[ -t + \frac{e^{\Delta t} - 1}{\Delta} \right] \frac{\langle pp | \tilde{J}_3^+ | d \rangle \langle d | \tilde{J}_3^+ | nn \rangle}{\Delta} + t \sum_{l' \neq d} \frac{\langle pp | \tilde{J}_3^+ | l' \rangle \langle l' | \tilde{J}_3^+ | nn \rangle}{\delta_{l'}} \\ + \text{STUFF THAT DEPEND ON EXCITED STATES} + c + d e^{\Delta t} + \mathcal{O}(e^{-\delta t}, e^{-\delta' t}),$$

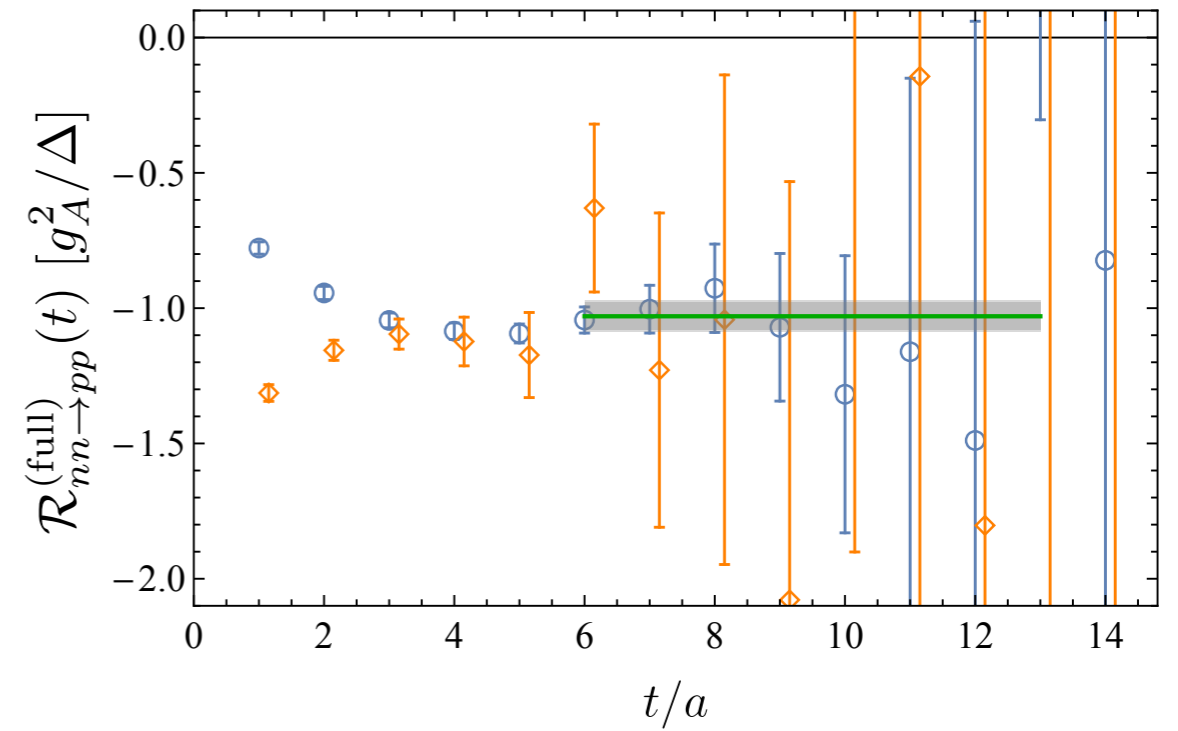
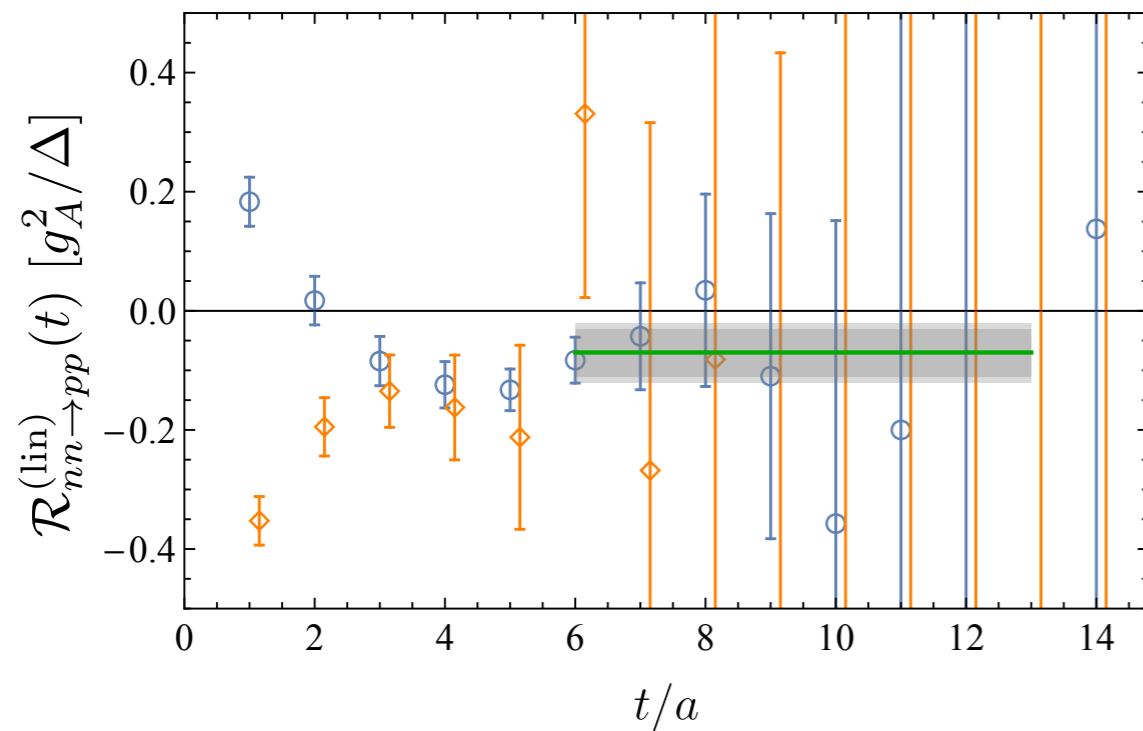
HERE THE FACT THAT  $\Delta \neq 0$  RESCUES US!

Tiburzi et al (NPLQCD), Phys. Rev.D96,054505(2017)  
Shanahan et al (NPLQCD), Phys. Rev. Lett.119,062003(2017).

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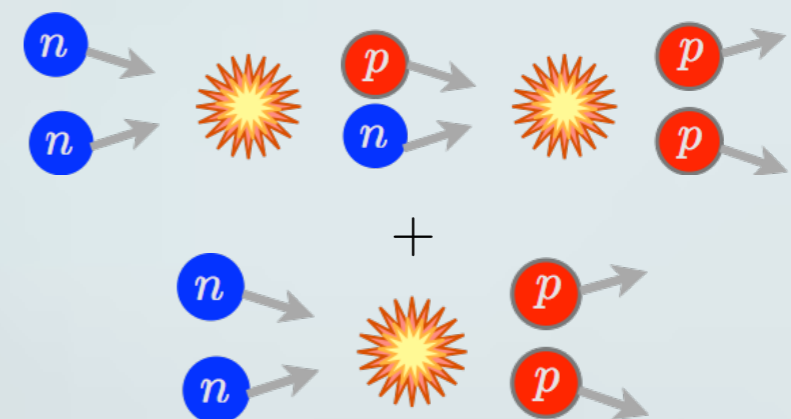
Tiburzi et al (NPLQCD), Phys. Rev.D96,054505(2017)  
 Shanahan et al (NPLQCD), Phys. Rev. Lett.119,062003(2017).



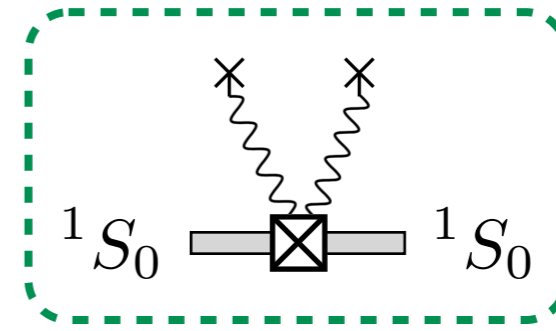
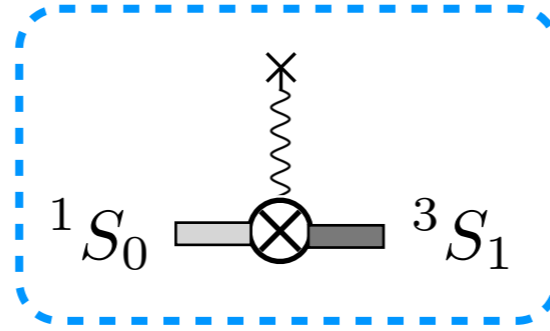
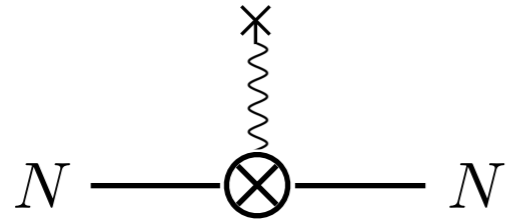
SHORT-DISTANCE CONTRIBUTION



FULL CONTRIBUTION



# EFT VERTICES AND CORRELATION FUNCTIONS USING DIBARYONS



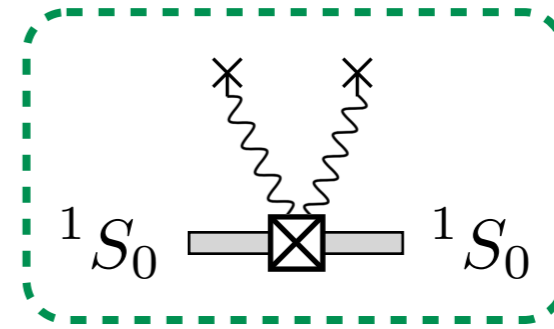
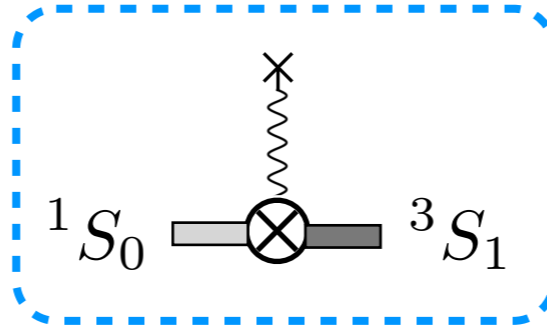
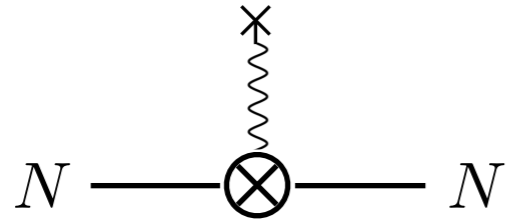
$$i\mathcal{C}_{nn \rightarrow pp} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} +$$

Diagram 1: A sequence of vertices (circles with X) connected by thick lines, labeled  $^1S_0$ ,  $^3S_1$ , and  $^1S_0$ .  
 Diagram 2: A sequence of vertices (circles with X) connected by thick lines, labeled  $^1S_0$  and  $^3S_1$ .  
 Diagram 3: A sequence of vertices (circles with X) connected by thick lines, labeled  $^1S_0$  and  $^3S_1$ .  
 Diagram 4: A sequence of vertices (circles with X) connected by thick lines, labeled  $^1S_0$  and  $^3S_1$ .

$$\text{Diagram 5} + \text{Diagram 6} + \mathcal{O}(\lambda^4)$$

Diagram 5: A vertex (circle with X) connected to four nucleons (N) via thick lines.  
 Diagram 6: A vertex (square with X) connected to four nucleons (N) via thick lines.

# EFT VERTICES AND CORRELATION FUNCTIONS USING DIBARYONS



$$i\mathcal{C}_{nn \rightarrow pp} = \text{[Four diagrams showing various nucleon-nucleon and proton-proton interactions with wavy lines]} + \dots$$

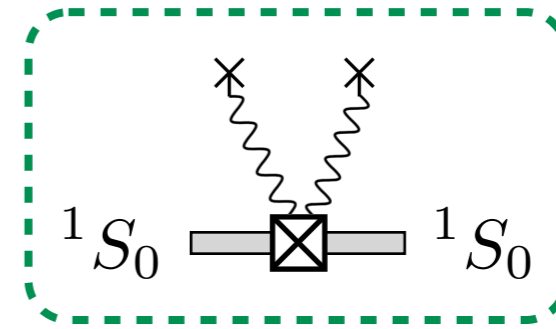
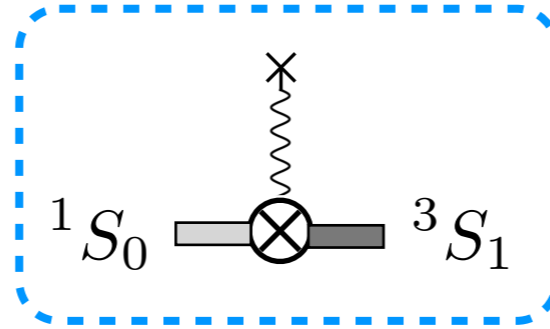
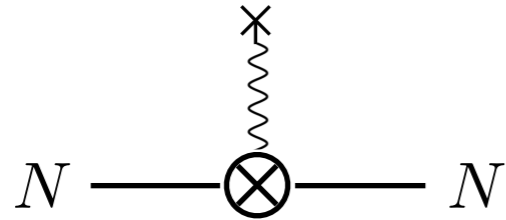
The equation shows the correlation function  $i\mathcal{C}_{nn \rightarrow pp}$  as a sum of four diagrams enclosed in a blue box. Each diagram represents a different interaction topology between two nucleons (represented by circles) and two protons (represented by circles with diagonal lines), with wavy lines representing photons. The diagrams are separated by plus signs.

Give partly the dominant long-range contribution

$$\text{[Two diagrams showing dominant long-range contributions]} + \mathcal{O}(\lambda^4)$$

The equation shows two diagrams representing dominant long-range contributions, separated by a plus sign, followed by the term  $\mathcal{O}(\lambda^4)$ . The first diagram shows a four-point interaction with two wavy lines. The second diagram shows a four-point interaction with two wavy lines and a central square vertex with an X inside.

# EFT VERTICES AND CORRELATION FUNCTIONS USING DIBARYONS



$$i\mathcal{C}_{nn \rightarrow pp} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \dots$$

Diagram 1: A sequence of vertices (circles with  $\otimes$ ) connected by shaded lines, representing a chain of interactions. The first and last vertices are labeled  $^1S_0$  and the middle one  $^3S_1$ .

Diagram 2: A sequence of vertices (circles with  $\otimes$ ) connected by shaded lines, representing a chain of interactions. The first and last vertices are labeled  $^1S_0$  and the middle one  $^3S_1$ .

Diagram 3: A sequence of vertices (circles with  $\otimes$ ) connected by shaded lines, representing a chain of interactions. The first and last vertices are labeled  $^1S_0$  and the middle one  $^3S_1$ .

Diagram 4: A sequence of vertices (circles with  $\otimes$ ) connected by shaded lines, representing a chain of interactions. The first and last vertices are labeled  $^1S_0$  and the middle one  $^3S_1$ .

Quenching of axial charge?

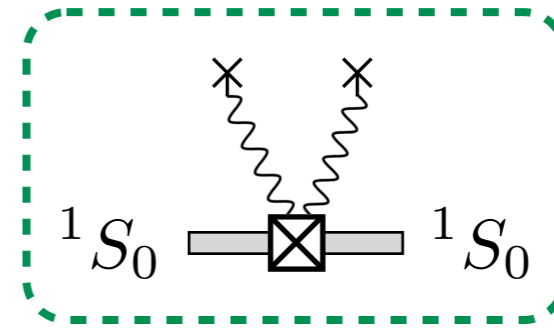
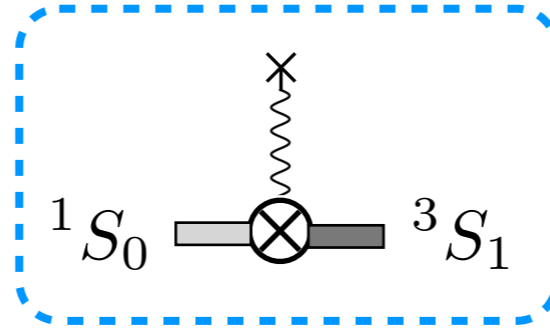
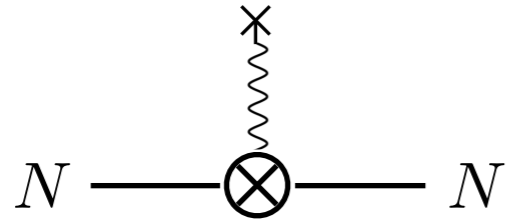
Give partly the dominant long-range contribution

$$\text{[Diagram 5]} + \text{[Diagram 6]} + \mathcal{O}(\lambda^4)$$

Diagram 5: A sequence of vertices (circles with  $\otimes$ ) connected by shaded lines, representing a chain of interactions. The first and last vertices are labeled  $^1S_0$  and the middle one  $^3S_1$ .

Diagram 6: A sequence of vertices (circles with  $\otimes$ ) connected by shaded lines, representing a chain of interactions. The first and last vertices are labeled  $^1S_0$  and the middle one  $^3S_1$ .

# EFT VERTICES AND CORRELATION FUNCTIONS USING DIBARYONS



$$i\mathcal{C}_{nn \rightarrow pp} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots$$

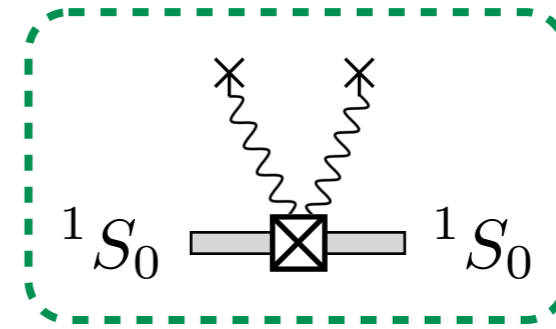
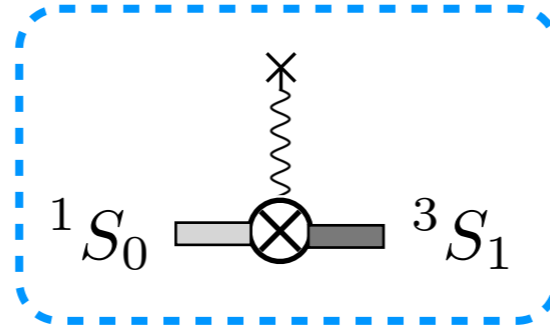
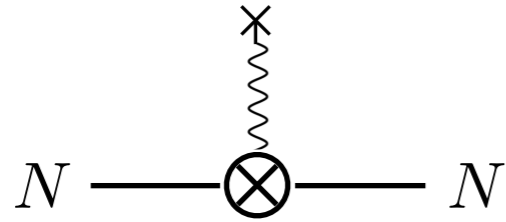
The equation shows the sum of four diagrams representing the correlation function  $i\mathcal{C}_{nn \rightarrow pp}$ . Each diagram consists of two nucleon lines (shaded) and two pion lines (wavy). The first three diagrams show different topologies of pion exchange between the nucleons, with vertices represented by circles with an  $\otimes$ . The fourth diagram shows a contact interaction between the two nucleons, represented by a square with an  $\otimes$ .

$$\text{Diagram 5} + \text{Diagram 6} + \mathcal{O}(\lambda^4)$$

The equation shows the sum of two diagrams representing higher-order contributions to the correlation function. Diagram 5 is a box diagram with two nucleon lines and two pion lines. Diagram 6 is a contact diagram with two nucleon lines and two pion lines, enclosed in a dashed green box. The term  $\mathcal{O}(\lambda^4)$  indicates higher-order corrections.

A short-range contribution  
not accounted for before

# EFT VERTICES AND CORRELATION FUNCTIONS USING DIBARYONS



$$i\mathcal{C}_{nn \rightarrow pp} = \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots$$

The equation shows the correlation function  $i\mathcal{C}_{nn \rightarrow pp}$  as a sum of four diagrams. Each diagram represents a different way two neutrons can interact via pion exchange to become two protons. The first diagram shows a sequence of  $^1S_0$ ,  $^3S_1$ , and  $^1S_0$  states. The subsequent diagrams show different topologies of pion exchange between the nucleons.

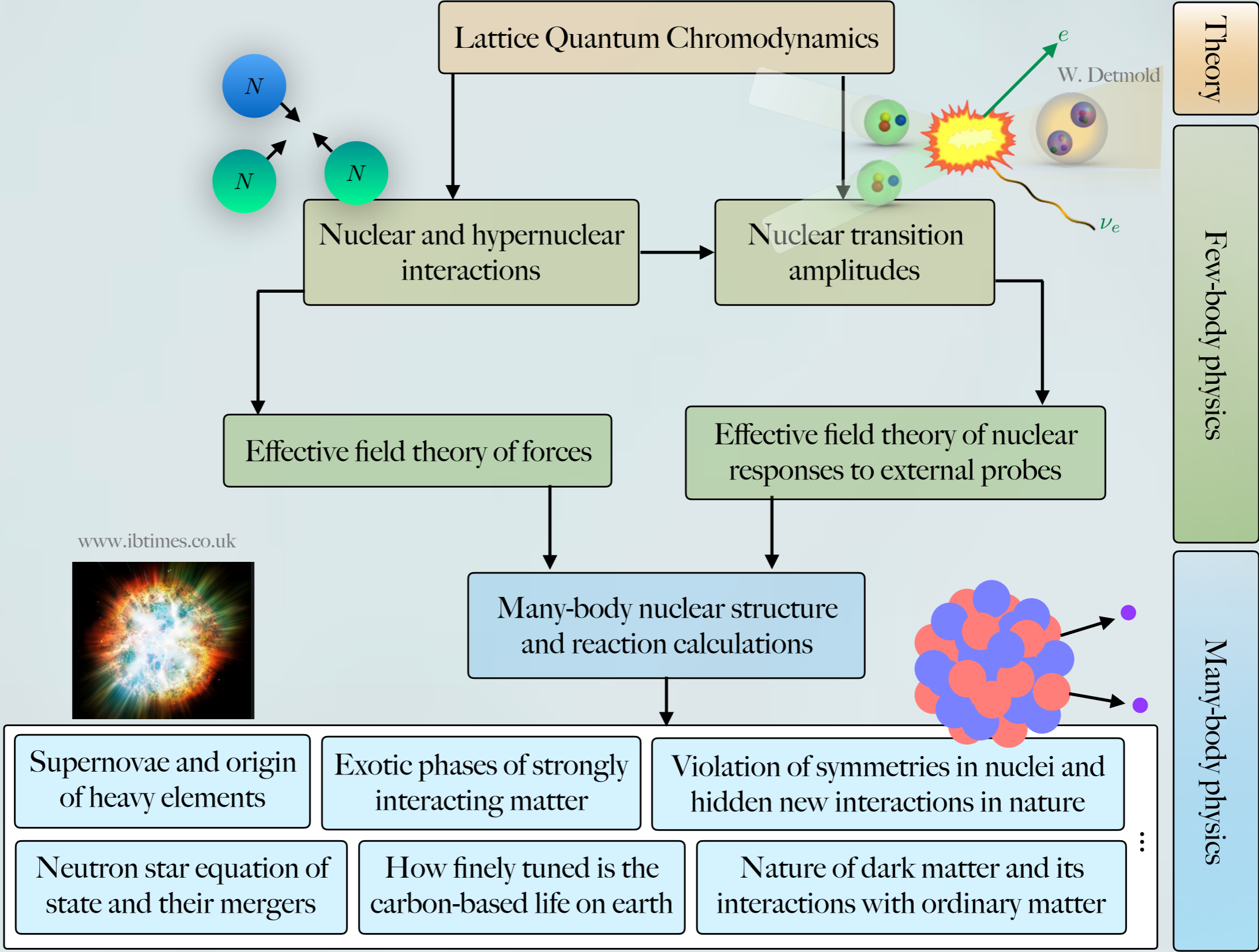
$$\text{Diagram 5} + \text{Diagram 6} + \mathcal{O}(\lambda^4)$$

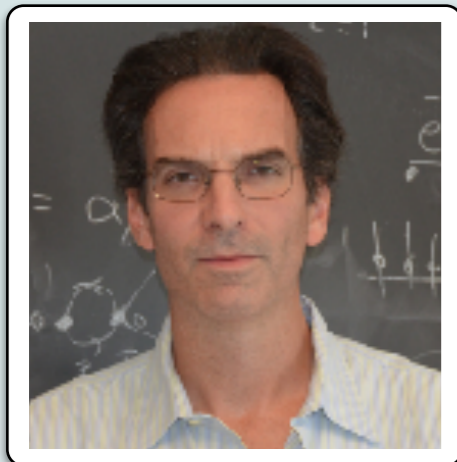
The equation shows two more diagrams. Diagram 5 is a four-pion exchange contact term. Diagram 6 is a  $^1S_0$ - $^1S_0$  dibaryon contact term, enclosed in a green dashed box. The sum is followed by  $\mathcal{O}(\lambda^4)$ .

$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8) \text{ fm}$$

$$@ m_\pi \approx 800 \text{ MeV}$$

# NUCLEAR PHYSICS FROM FIRST-PRINCIPLES ROADMAP

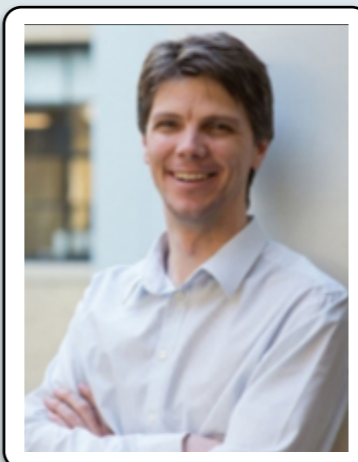




S. BEANE



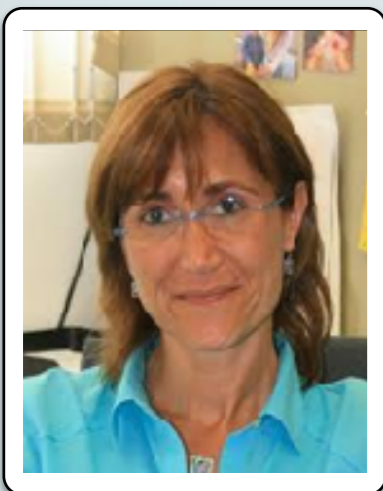
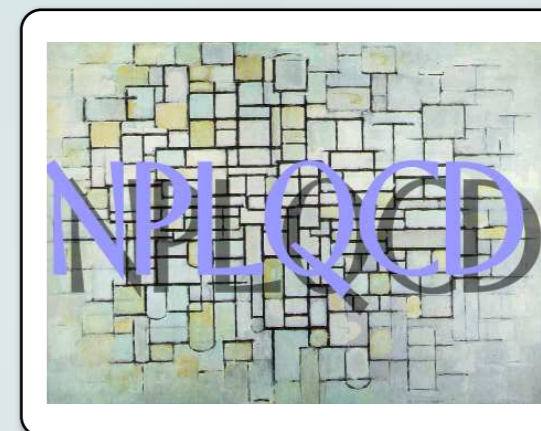
E. CHANG



W. DETMOLD



K. ORGINOS



A. PARRENO



M. SAVAGE



P. SHANAHAN



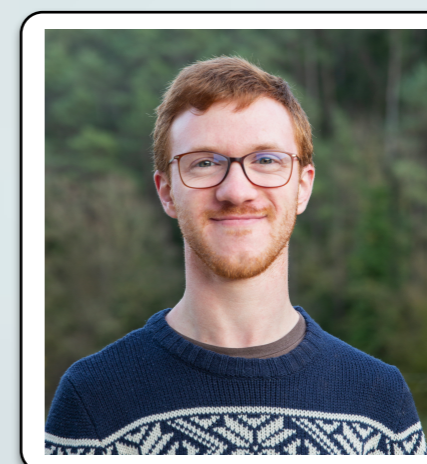
B. TIBURZI



M. WAGMAN



F. WINTER

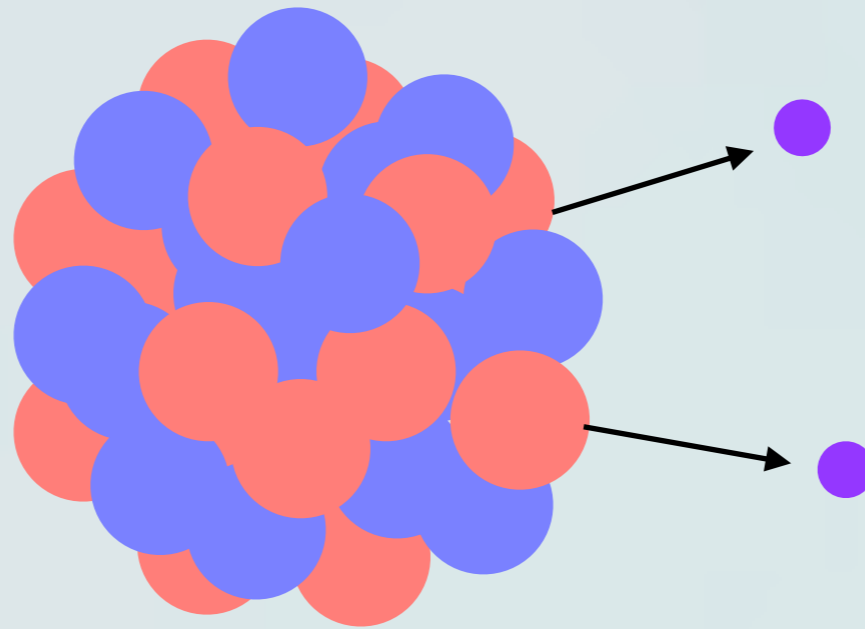


M. ILLA



D. MURPHY

NEW MEMBERS



THANK YOU