Double beta matrix elements in light nuclei

Xiaobao Wang

ECT*, 2019

Collaborate with J.A. Carlson, A.C. Hayes, E. Mereghetti,
S. Pastore, R.B. Wiringa, G.X. Dong

Outline

Background

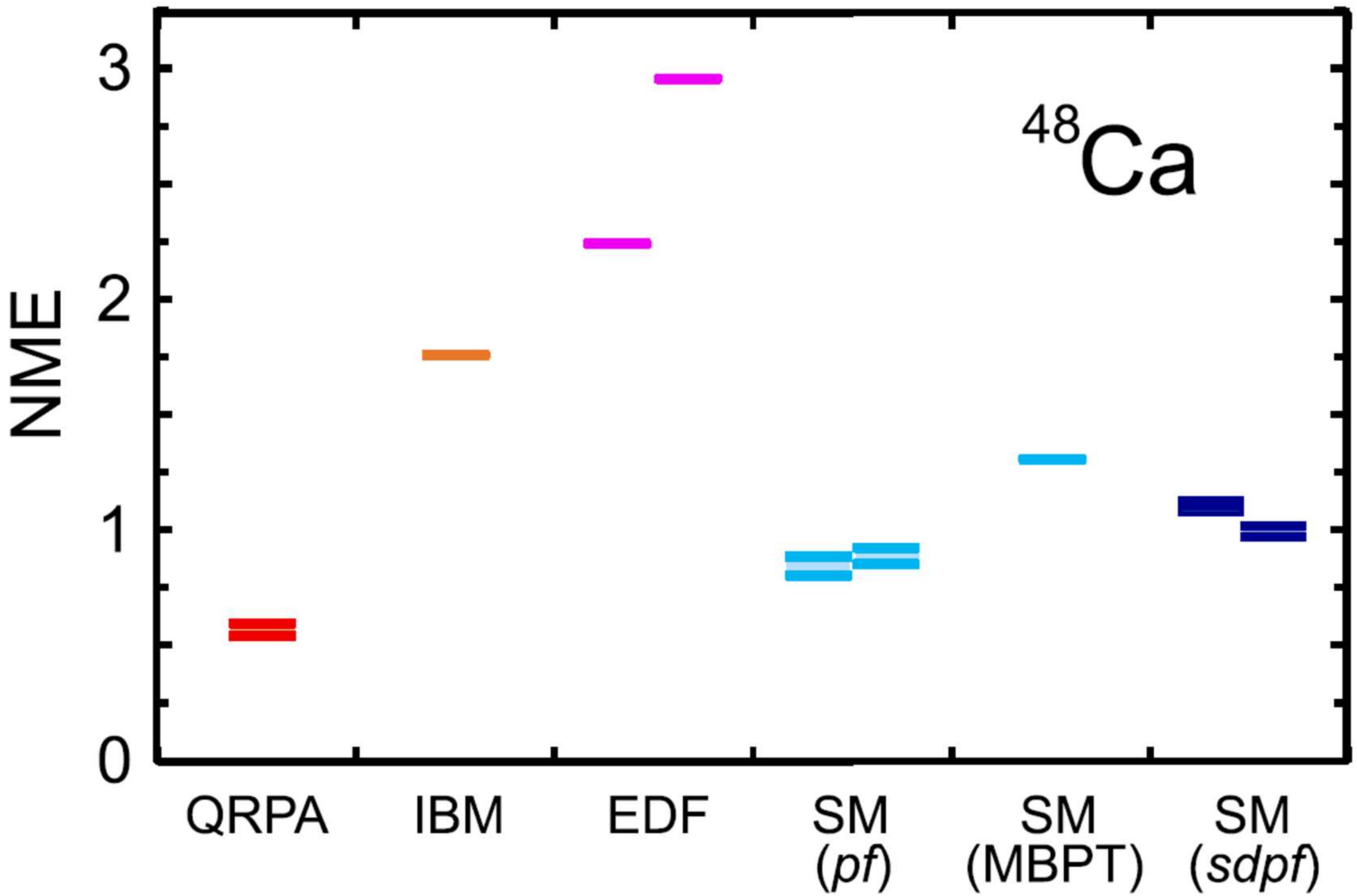
Normalizations: model space & radial wavefunctions

Short-range correlations

Results

Conclusion

Background



Large-Scale Shell-Model Analysis of the Neutrinoless ββ Decay of 48Ca
Y. Iwata, N. Shimizu, T. Otsuka, Y. Utsuno, J. Menéndez, M. Honma, and T. Abe,PRL 116, 112502 (2016).

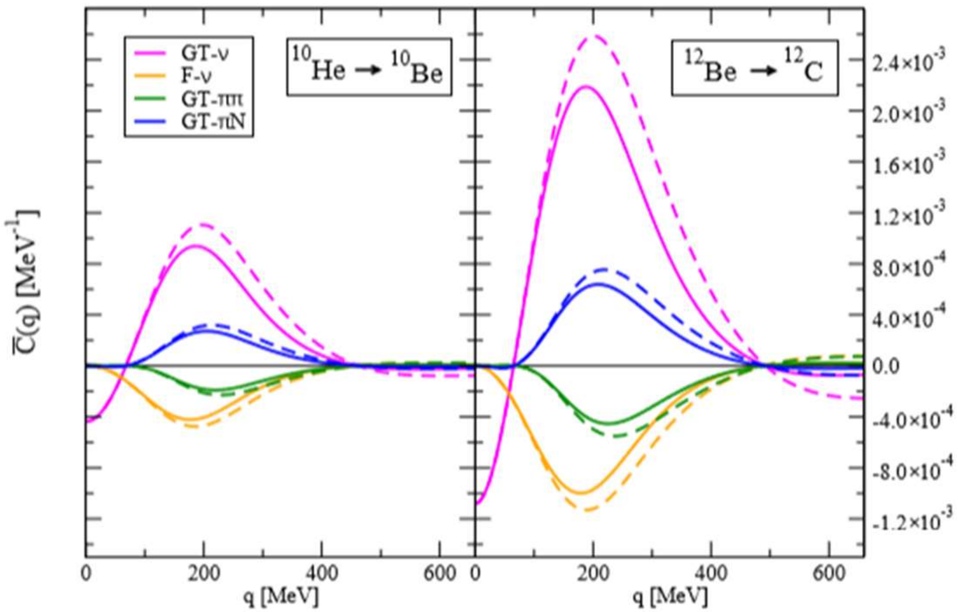
Background

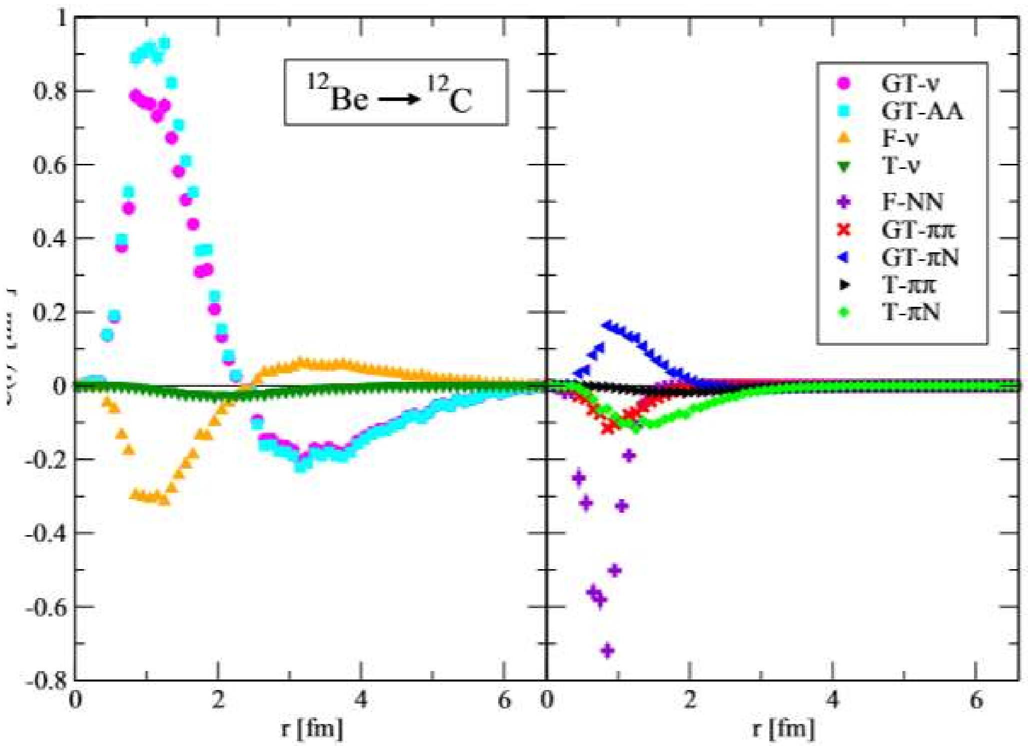
PHYSICAL REVIEW C **97**, 014606 (2018)

Neutrinoless double-β decay matrix elements in light nuclei

S. Pastore,[1] J. Carlson,[1] V. Cirigliano,[1] W. Dekens,[1,2] E. Mereghetti,[1] and R. B. Wiringa[3]
[1] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
[2] *New Mexico Consortium, Los Alamos Research Park, Los Alamos, New Mexico 87544, USA*
[3] *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

Provide benchmarks, to check the prediction power for different models..





VMC method

PHYSICAL REVIEW C **97**, 014606 (2018)

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assume a functional form for the variational wavefunction (*trial wavefunction*) that depends on a set of parameters $\{p\}$

variational theorem: the expectation value of the Hamiltonian computed on a trial wavefunction is always an upper bound to the true ground state energy of the system

$$\frac{\langle\psi_T(\alpha,\{p\})\,|H|\psi_T(\alpha,\{p\})\rangle}{\langle\psi_T(\alpha,\{p\})\,|\psi_T(\alpha,\{p\})\rangle} = E_T(\{p\}) \geq E_0$$

→ stochastic integration, M(RT)^2 algorithm

VMC implies a minimization of $E_T(\{p\})$ with respect to the parameters $\{p\}$ in order to find the optimal trial wavefunction that better approximates the ground state wavefunction

VMC method

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$$E_V = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \geq E_0$$

$$H = -\frac{\hbar^2}{2m}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}.$$

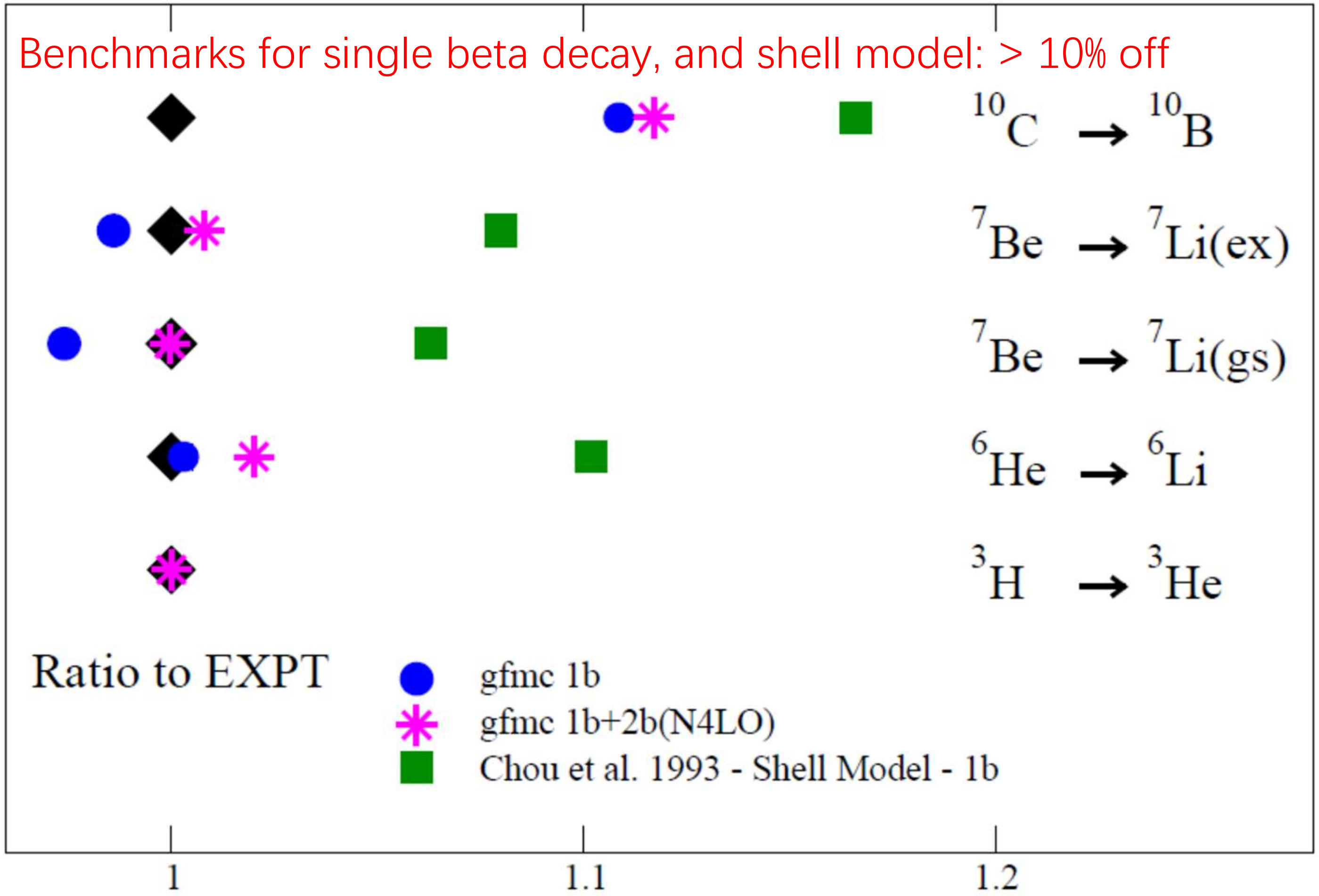
$$\langle RS|\Psi\rangle = \langle RS|\prod_{i<j} f_{ij}^1 \prod_{i<j<k} f_{ijk}^{3c} \times \left[\mathbb{1} + \sum_{i<j}\sum_{p=2}^{6} f_{ij}^p \mathcal{O}_{ij}^p f_{ij}^{3p} + \sum_{i<j<k} U_{ijk}\right]|\Phi\rangle_{J^\pi,T},$$

Background

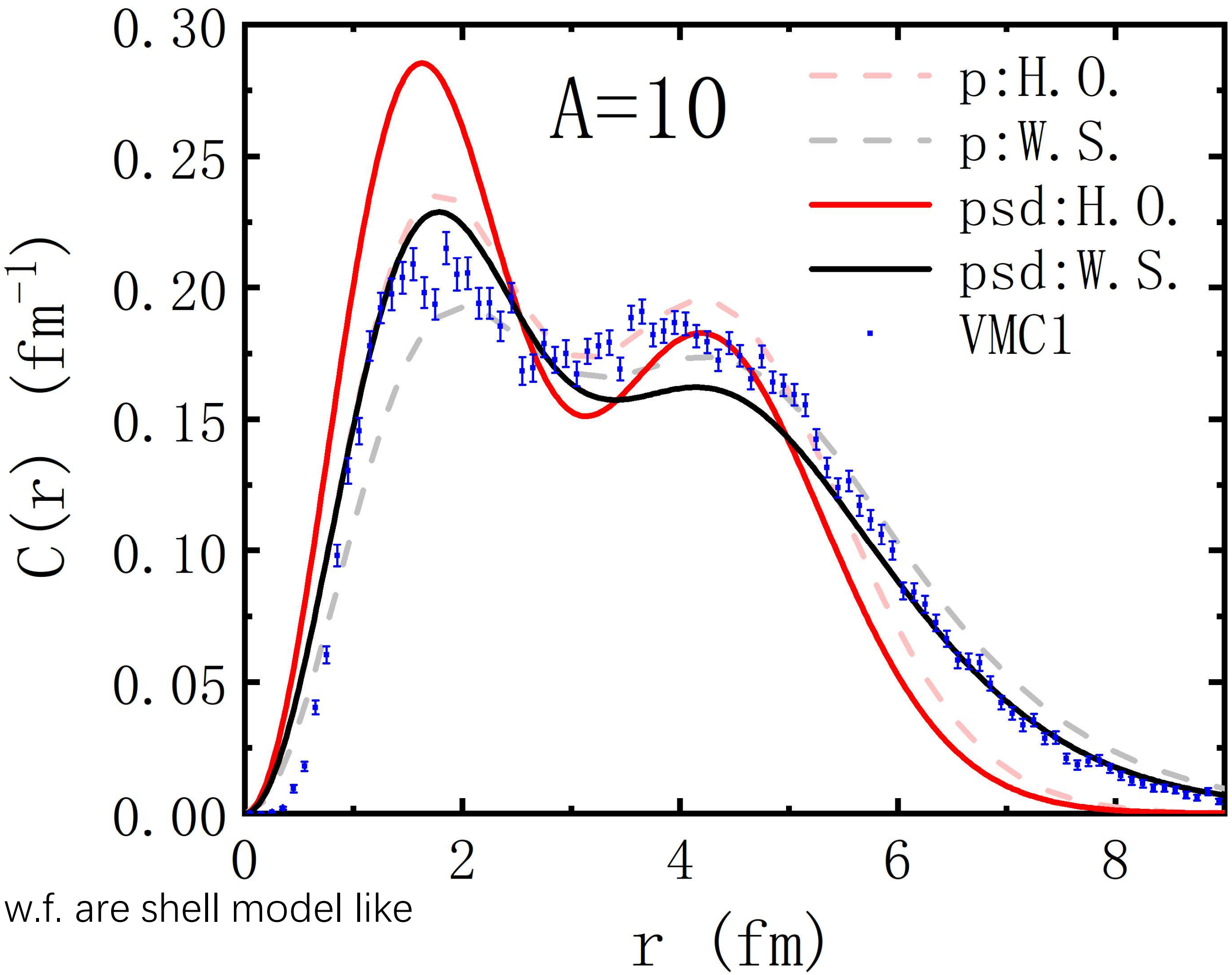
Quantum Monte Carlo calculations of weak transitions in $A=6$–10 nuclei

S. Pastore[a], A. Baroni[b], J. Carlson[a], S. Gandolfi[a], Steven C. Pieper[d], R. Schiavilla[b,c], and R.B. Wiringa[d]

Phys. Rev. C97, 022501(R) (2018)



Norm Matrix Element < i | **1** tau+tau+ | f>



VMC1: variational w.f. are shell model like

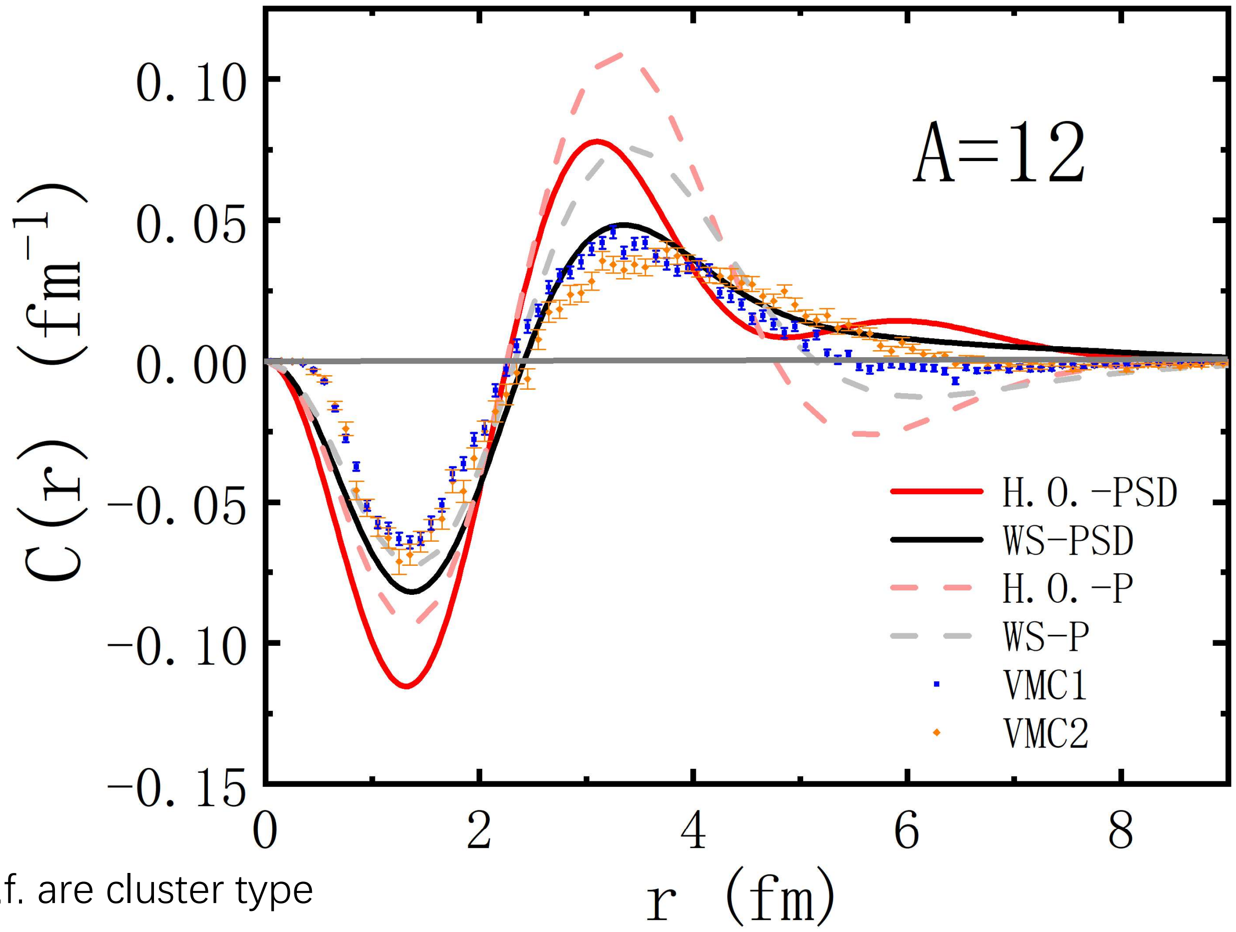
Shell model, VMC, and other models, have to give **the same value**.
But you can see the discrepancies, explicitly..

Here, there are different "choices" made for shell model:

1) **model space**;
2) **radial wavefunctions**.

A=10 : < Be10_g.s., T=1 | 1 |C10_g.s., T=1>= 1.000

Norm Matrix Element < i | **1** tau+tau+ | f>



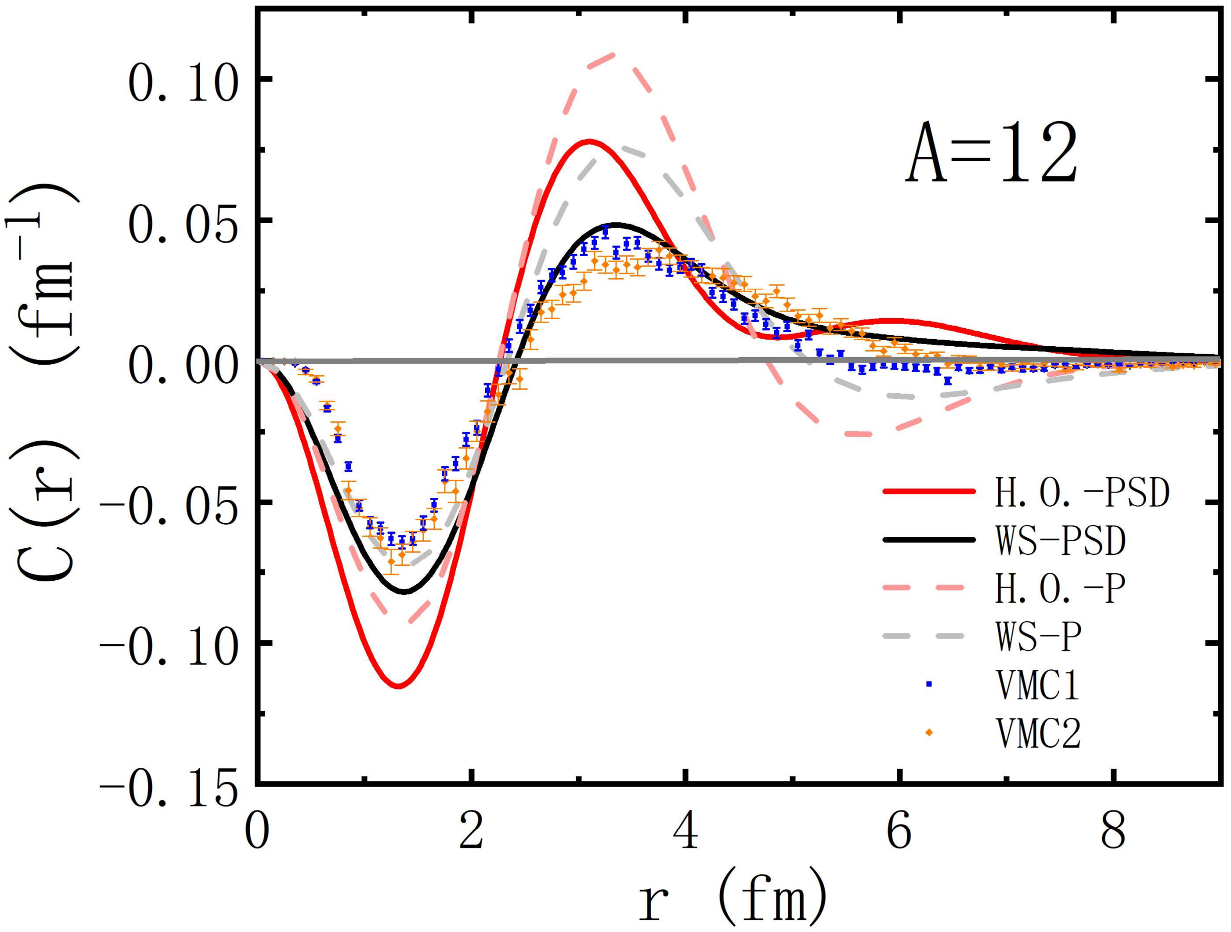
Shell model, VMC, and other models, have to give **the same value**.
But you can see the discrepancies, explicitly..

Here, there are different "choices" made for shell model:
1) **model space**;
2) **radial wavefunctions**.

VMC2: variational w.f. are cluster type

A=12: < Be12_g.s., T=2 | 1 |C12_g.s., T=0>= **0.000**

Choice of model space



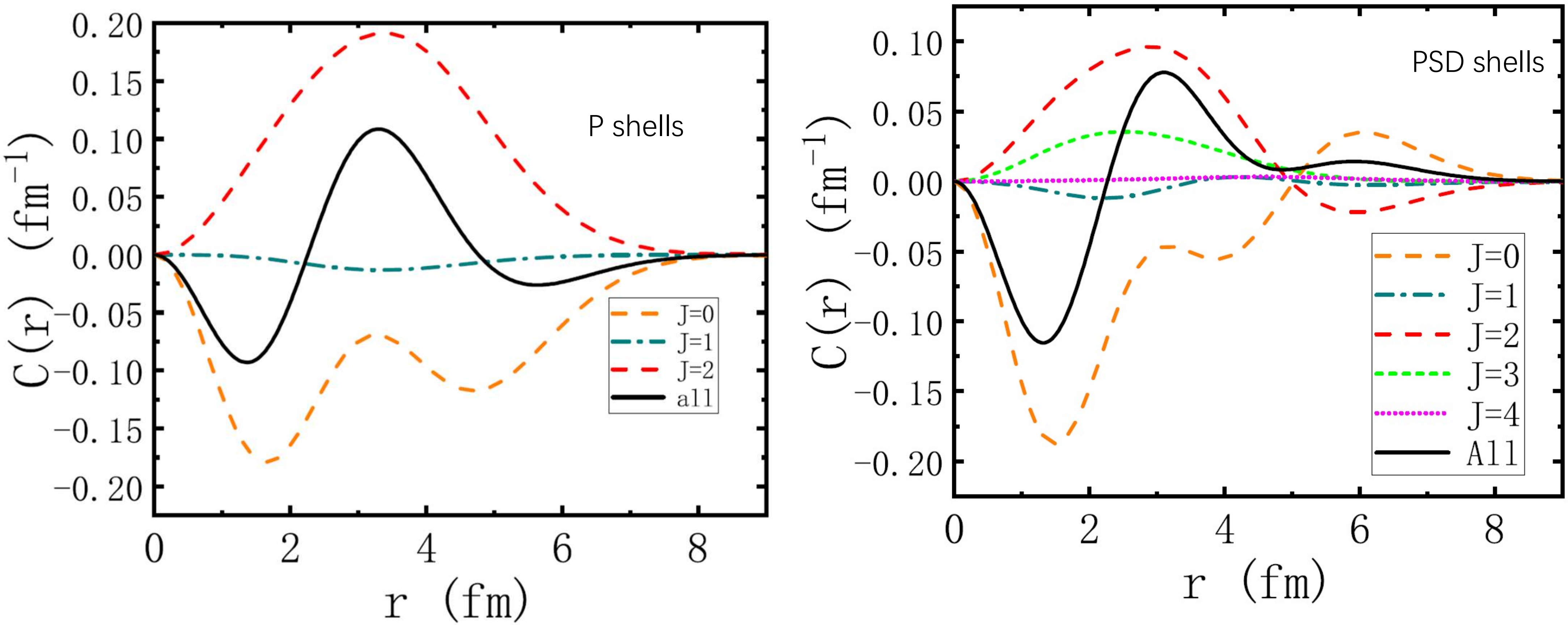
Different model space, has different nodes.

Of course, extended model space used for shell model gives better agreements with VMC method.

A=12: < Be12_g.s., T=2 | 1 |C12_g.s., T=0>= **0.000**

Choice of model space

Different J pairs, A=12, H.O.



A=12: < Be12_g.s., T=2 | 1 |C12_g.s., T=0>= **0.000**

Choice of model space

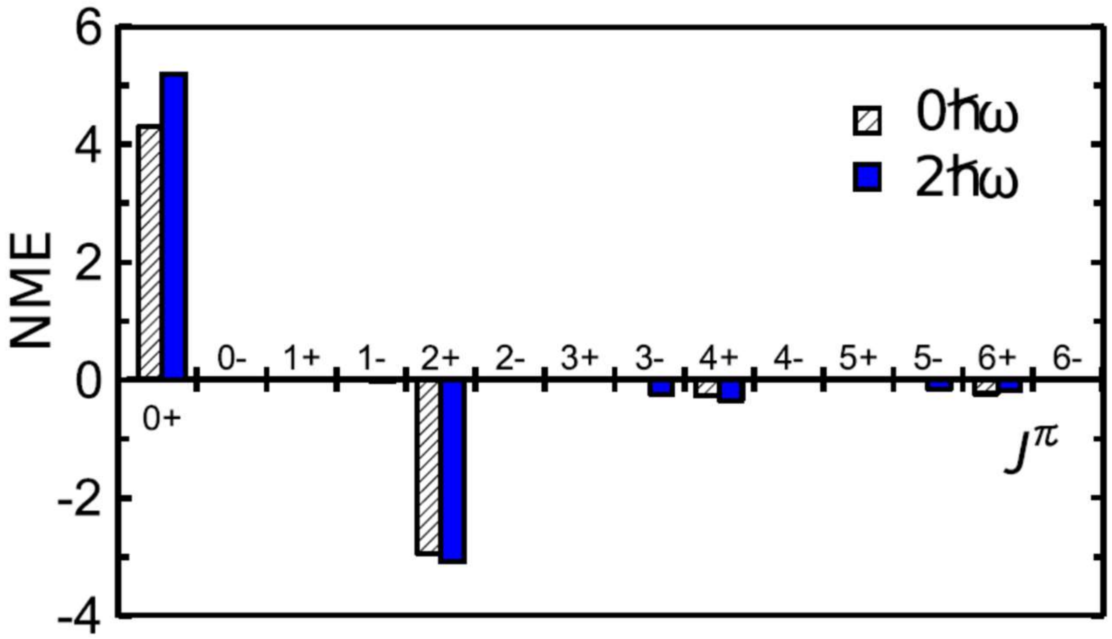
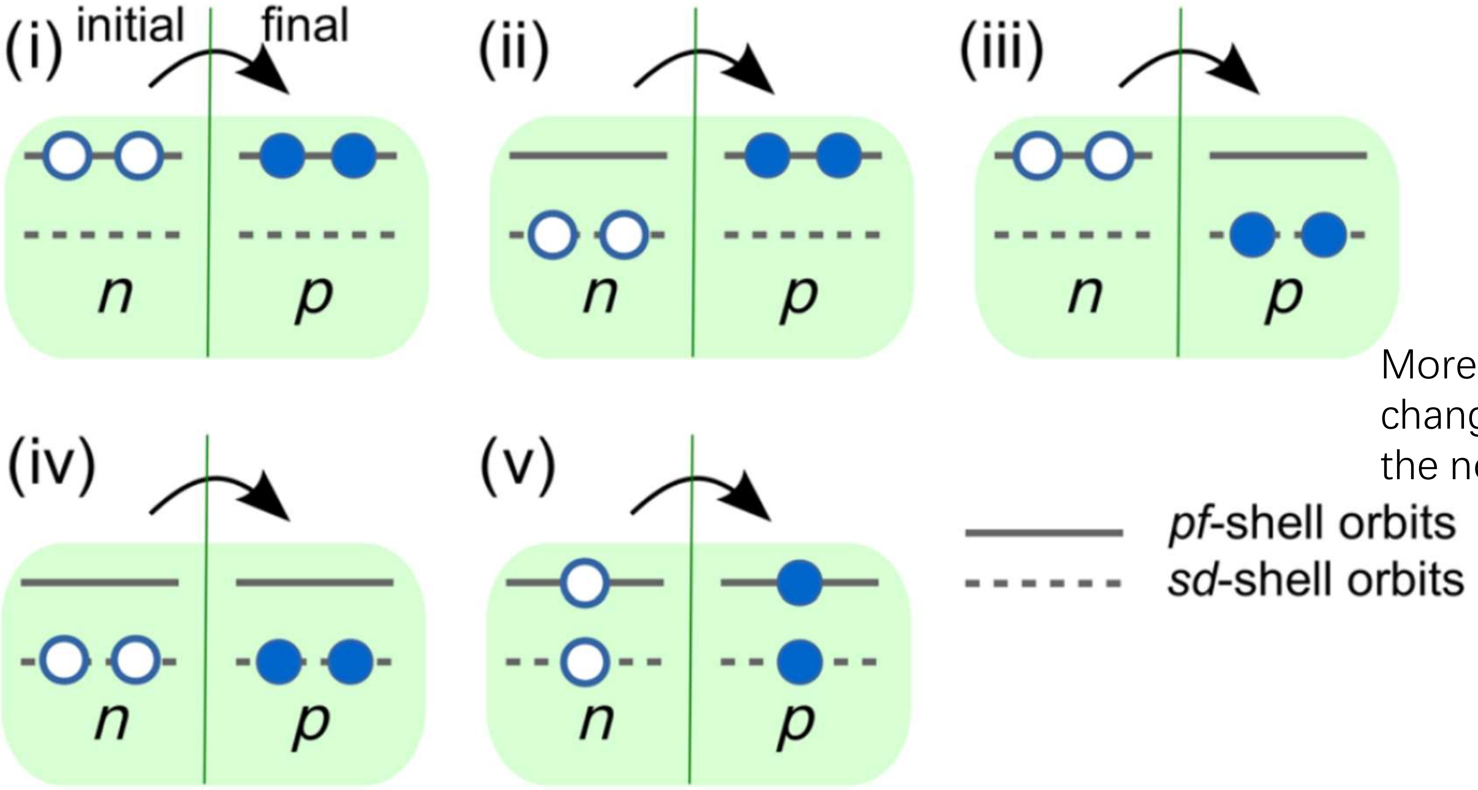


FIG. 3. NME decomposition in terms of the angular momentum and parity J^π of the pair of decaying neutrons, Eq. (3). $0\hbar\omega$ (GXPF1B) and $2\hbar\omega$ (SDPFMU-DB) results are compared, without short-range correlations.

48Ca: Y. Iwata, N. Shimizu, T. Otsuka, Y. Utsuno, J. Menéndez, M. Honma, and T. Abe,PRL 116, 112502 (2016).

Choice of model space



More channels are open, which change the pairing structure, and the node structure..

48Ca: Y. Iwata, N. Shimizu, T. Otsuka, Y. Utsuno, J. Menéndez, M. Honma, and T. Abe,PRL 116, 112502 (2016).

Choice of radial wave functions

The distributions of TBME for J_0 =0 (A=12)

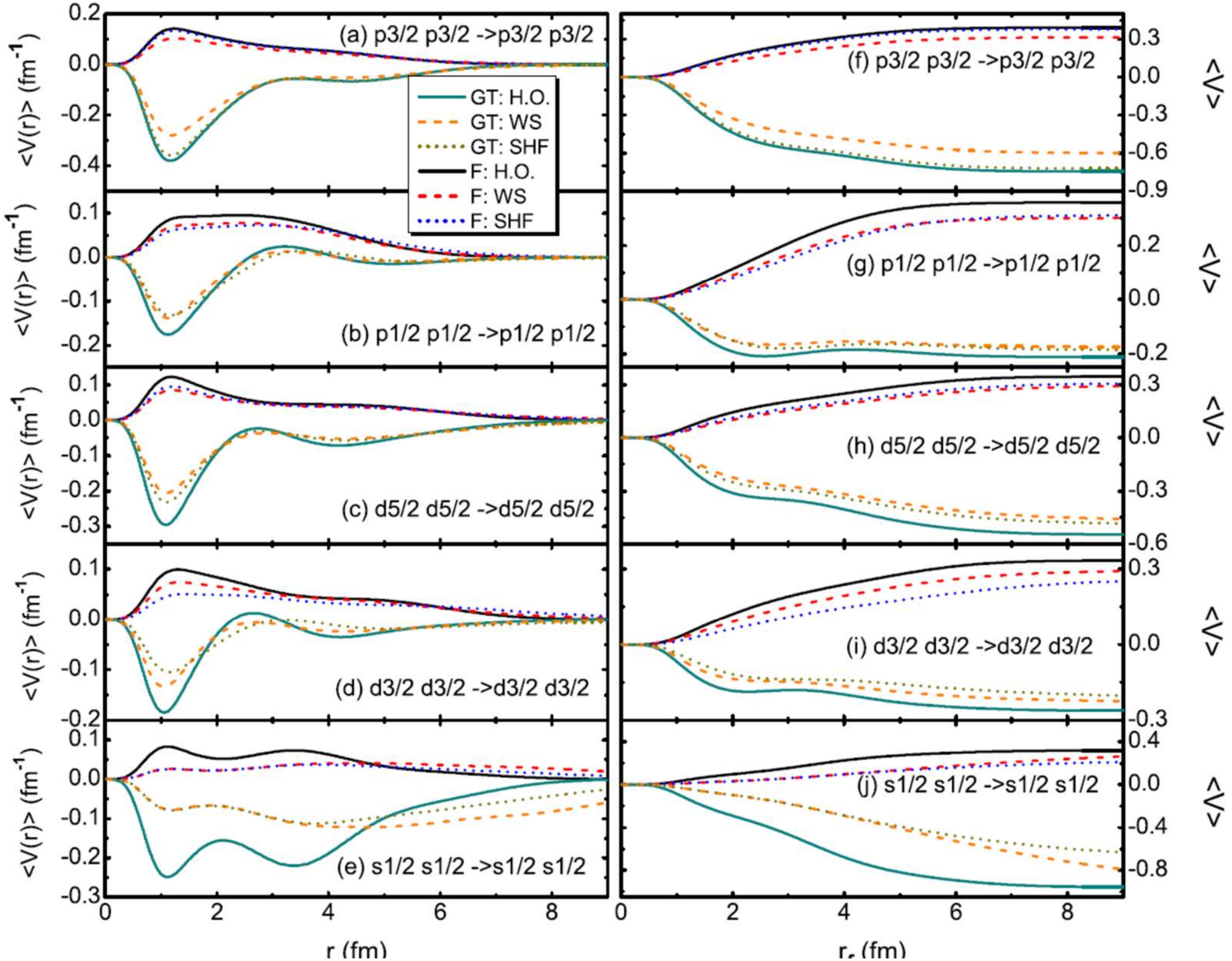
integrated matrix element, $\int_0^{r_f} C(r)dr$

as a function of the upper limit of the radial integral

H.O.: chosen most frequently, easy to use.

SHF: from Skyrme-HF

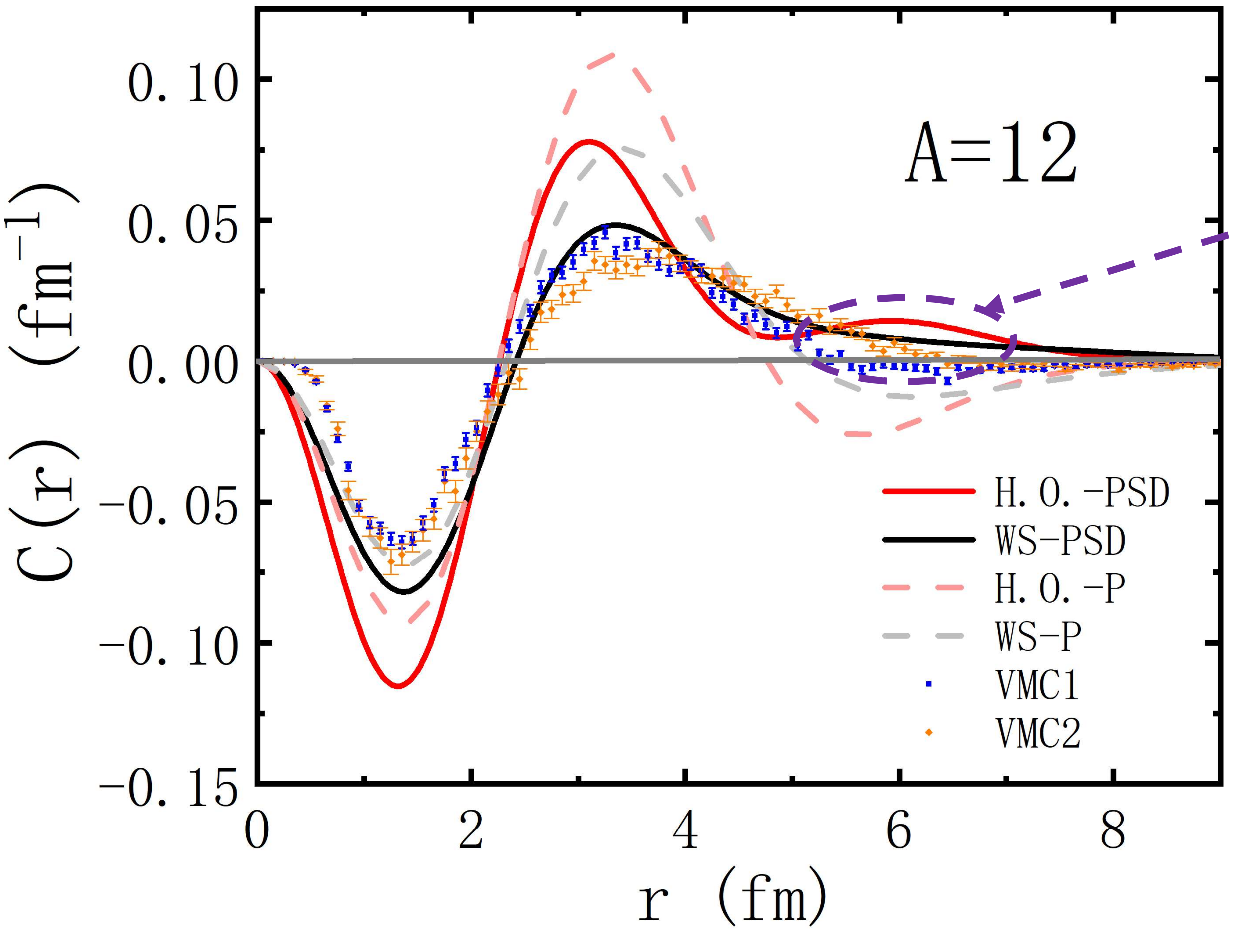
WS: from Woods-Saxon potential



H.O.: more concentrated, Larger overlap, decay faster against *r*.

SHF/WS: smaller overlaps, Correct the asymptotic behavior of H.O.

Choice of radial wave functions



Long-range behavior become similar to VMC

A=12: < Be12_g.s., T=2 | 1 |C12_g.s., T=0>= **0.000**

Operators for 0vDB NMEs:

$$V_\nu = m_\pi \tau_a^+ \tau_b^+ \Big(\mathbf{1} \times \mathbf{1}\, V_F^\nu(z) - g_A^2\, \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b\, V_{GT}^\nu(z) \; - \; g_A^2\, S_{ab}\, V_T^\nu(z) \Big)$$

$$\begin{aligned} M_{GT}^\beta &= (4\pi R_A)\sigma_1 \cdot \sigma_2 V_{GT}^\beta(r_{12})\tau_1^+\tau_2^+ \ , \\ M_F^\beta &= (4\pi R_A) V_F^\beta(r_{12}) \ , \\ M_T^\beta &= (4\pi R_A)\left[3\,(\sigma_1 \cdot \hat{r})\,(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2\right] V_T^\beta(r_{12}) \end{aligned}$$

$R_A = 1.2A^{1/3}$ fm is the nuclear radius

Leading terms:

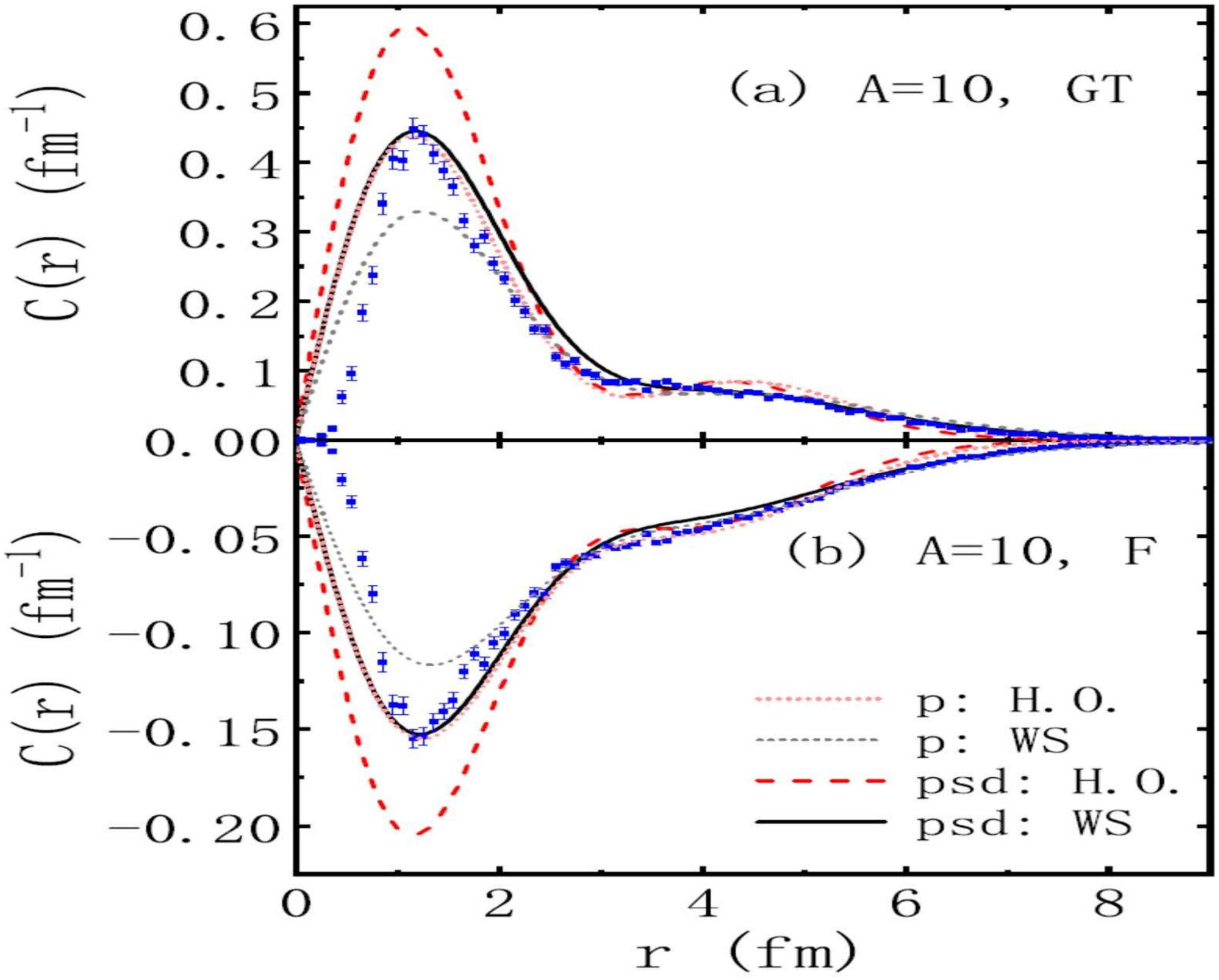
$$V_{F,\,\nu}(z) = \frac{1}{4\pi z}$$

$$V_{GT,AA}(z) = \frac{1}{4\pi z}$$

$$\begin{aligned} V_{GT,\,\nu}(z) &= V_{GT,AA}(z) + V_{GT,AP}(z) \\ &+ V_{GT,PP}(z) + V_{GT,MM}(z) \end{aligned}$$

Operator 1/r

A=10, Delta T =0



	^{10}Be(0_1^+)→^{10}C(0_1^+) F	GT
VMC-1	-1.001(40)	2.273(91)
VMC-2	—	—
$SM_{H.O.}$(w/o SRC, *p*)	-1.127	2.616
SM_{WS}(w/o SRC, *p*)	-0.980	2.269
$SM_{H.O.}$(w/o SRC, *psd*)	-1.274	3.228
SM_{WS}(w/o SRC, *psd*)	-1.100	2.783

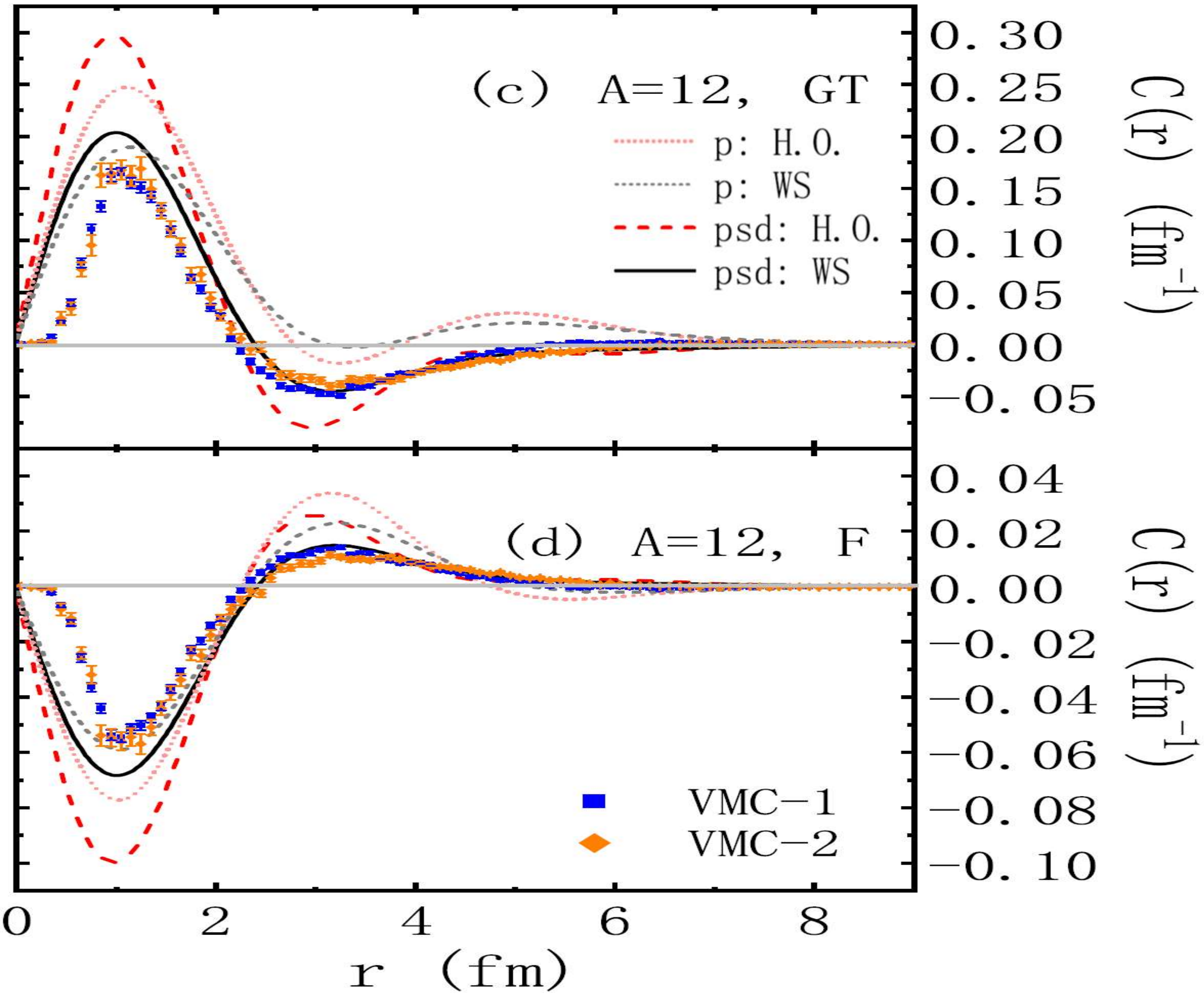
p* -> *psd

Larger model space→ more correlations→ matrix elements become larger

H.O. -> WS

W.S. r.w.f is less concentrated than H.O. ones → reduced matrix elements

Operator 1/r



A=12, Delta T =2

	^{12}Be(0_1^+)→^{12}C(0_1^+)	
	F	GT
VMC-1	-0.100(4)	0.257(10)
VMC-2	-0.113(5)	0.274(11)
$SM_{H.O.}$(w/o SRC, *p*)	-0.183	1.228
SM_{WS}(w/o SRC, *p*)	-0.147	1.023
$SM_{H.O.}$(w/o SRC, *psd*)	-0.271	0.431
SM_{WS}(w/o SRC, *psd*)	-0.198	0.570

p* -> *psd
Larger model space→ more correlations→ matrix elements can be reduced (remind: canceling effect for the normalizations)

H.O. -> WS
W.S. r.w.f is less concentrated than H.O. ones → reduced matrix elements

Test by VMC, GT, delta T=2, A=10
More correlations, reduced matrix element

PHYSICAL REVIEW C **97**, 014606 (2018)

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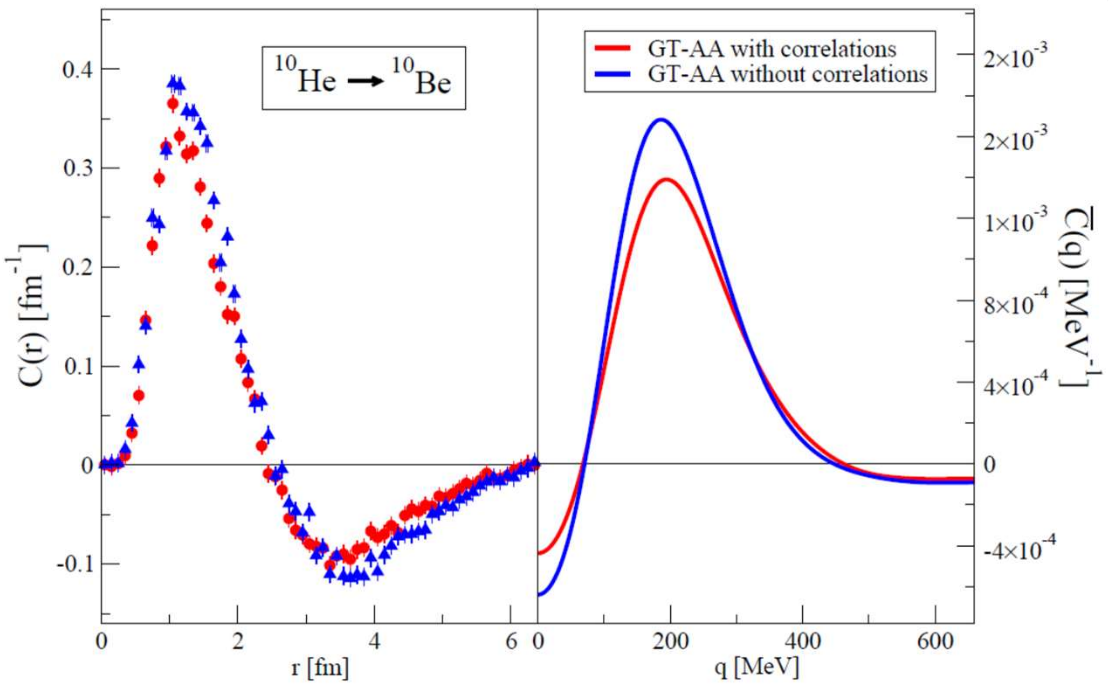
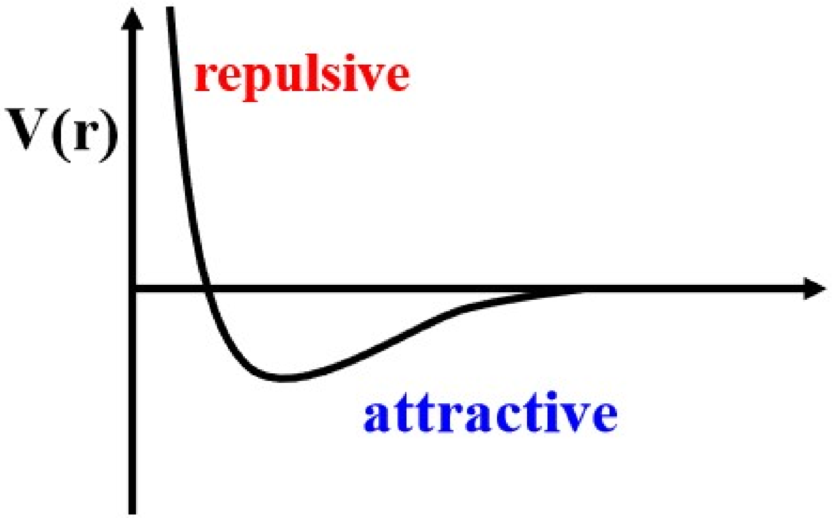


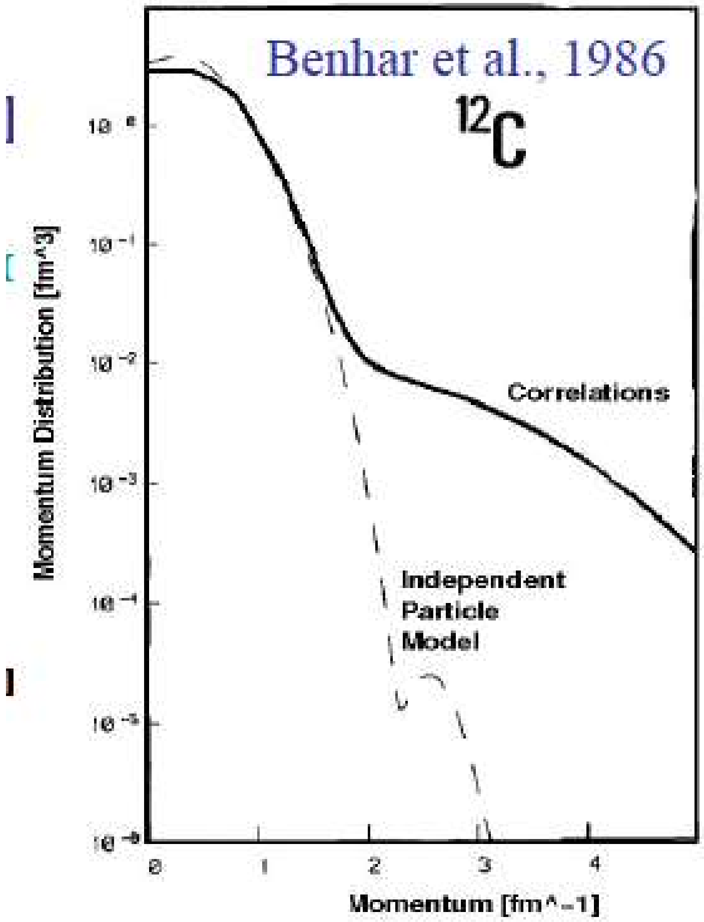
FIG. 6. The left (right) panel shows the GT-AA distribution in r-space (q-space) for the ^{10}He$\rightarrow$^{10}Be transition, with and without “one-pion-exchange-like” correlations in the nuclear wave functions. See text for explanation.

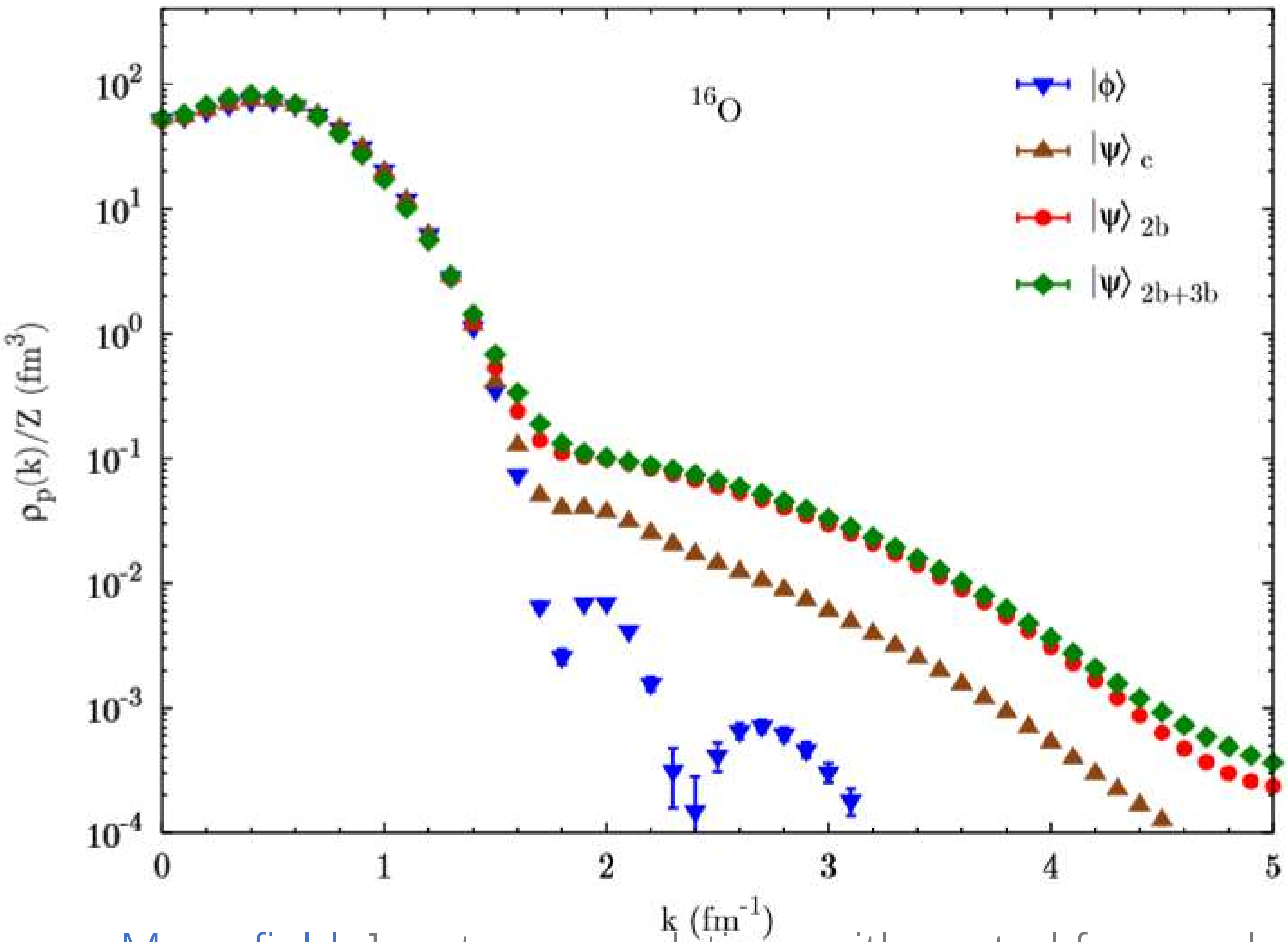
Short range correlations: *"disaster" for shell model?*

The N-N interaction is attractive at a typical distance of 2 fm, but highly repulsive at distances < 0.5 fm.



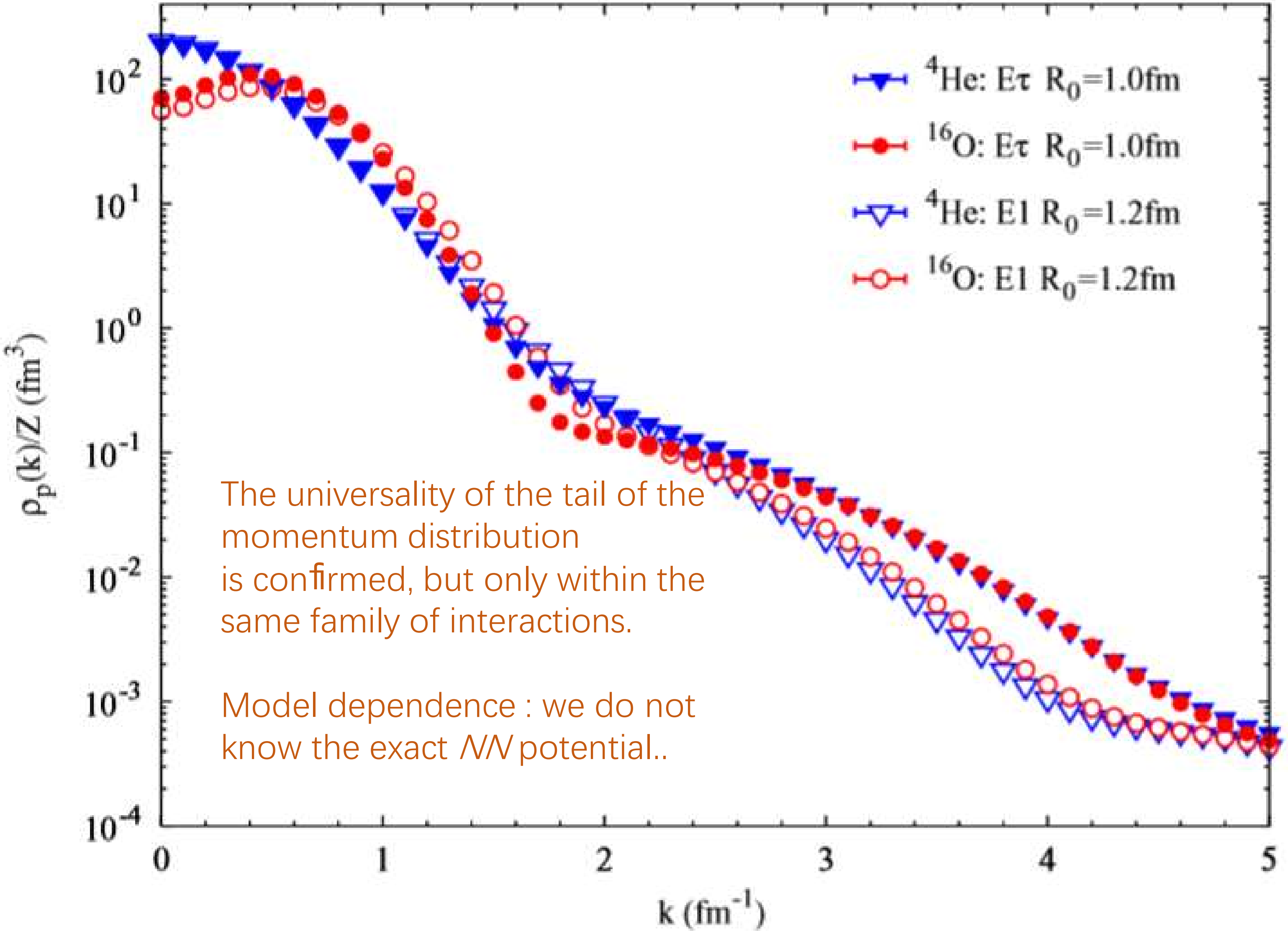
Short range repulsion and High momentum tail





Mean field; Jawstraw correlations with central force only;
Full two body; 2+3 bd force

D.Lonardoni, S. Gandolfi, X. B. Wang, and J. Carlson, PRC 98, 014322 (2018)

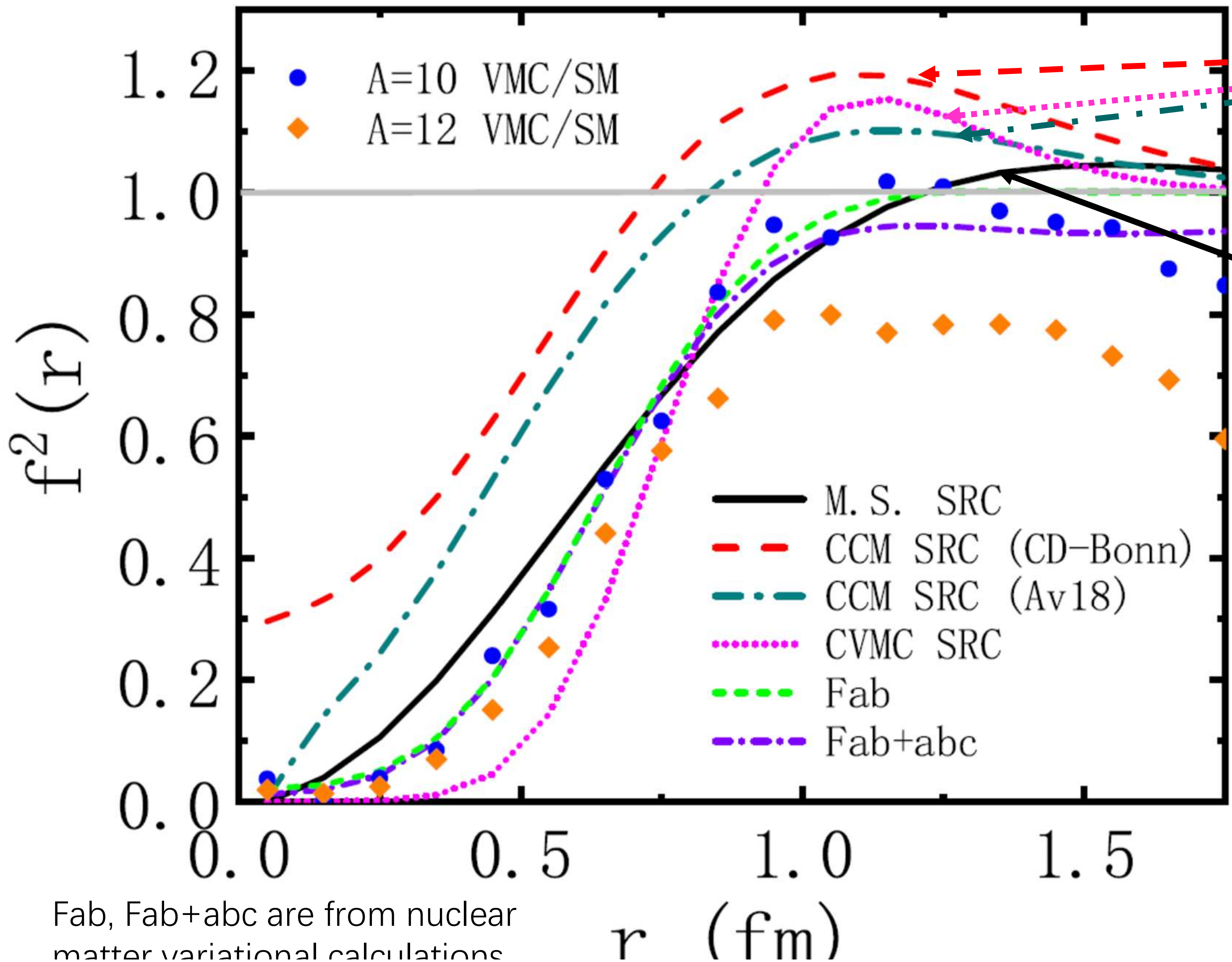


The universality of the tail of the momentum distribution
is confirmed, but only within the same family of interactions.

Model dependence : we do not know the exact *NN* potential..

Wang, and
J. Carlson, PRC 98, 014322 (2018)

A collection of short range correlations:



Fab, Fab+abc are from nuclear matter variational calculations

Overshoot "unit"

To preserve the Normalization & isospin symmetry

PhysRevC.83.034317, J. Engel, J. Carlson, and R. B. Wiringa

M.S. SRC is the traditional one (1976)

$$f(r) = 1 - ce^{-ar^2}(1 - br^2)$$

SRC parameters	A	B	C
Miller-Spencer SRC	1.10	0.68	1.00
CCM SRC (Av18)*	1.59	1.45	0.92
CCM SRC (CD-Bonn) *	1.52	1.88	0.46

CCM SRC F. Simkovic, A. Faessler, H. Muther, V. Rodin, and M. Stauf, Phys. Rev. **C79**, 055501 (2009)

CCM SRC is fitted to Correlated 2-bd wavefunction of CCM (S_2 correlation) / H.O. 2-bd wavefunction in the relative Coordinate, in the S_0 channel with node as 0 (R_{n=0,l=0}).

To get rid off the node dependence of the correlated wavefunction? SRC will change if the other choices are made.

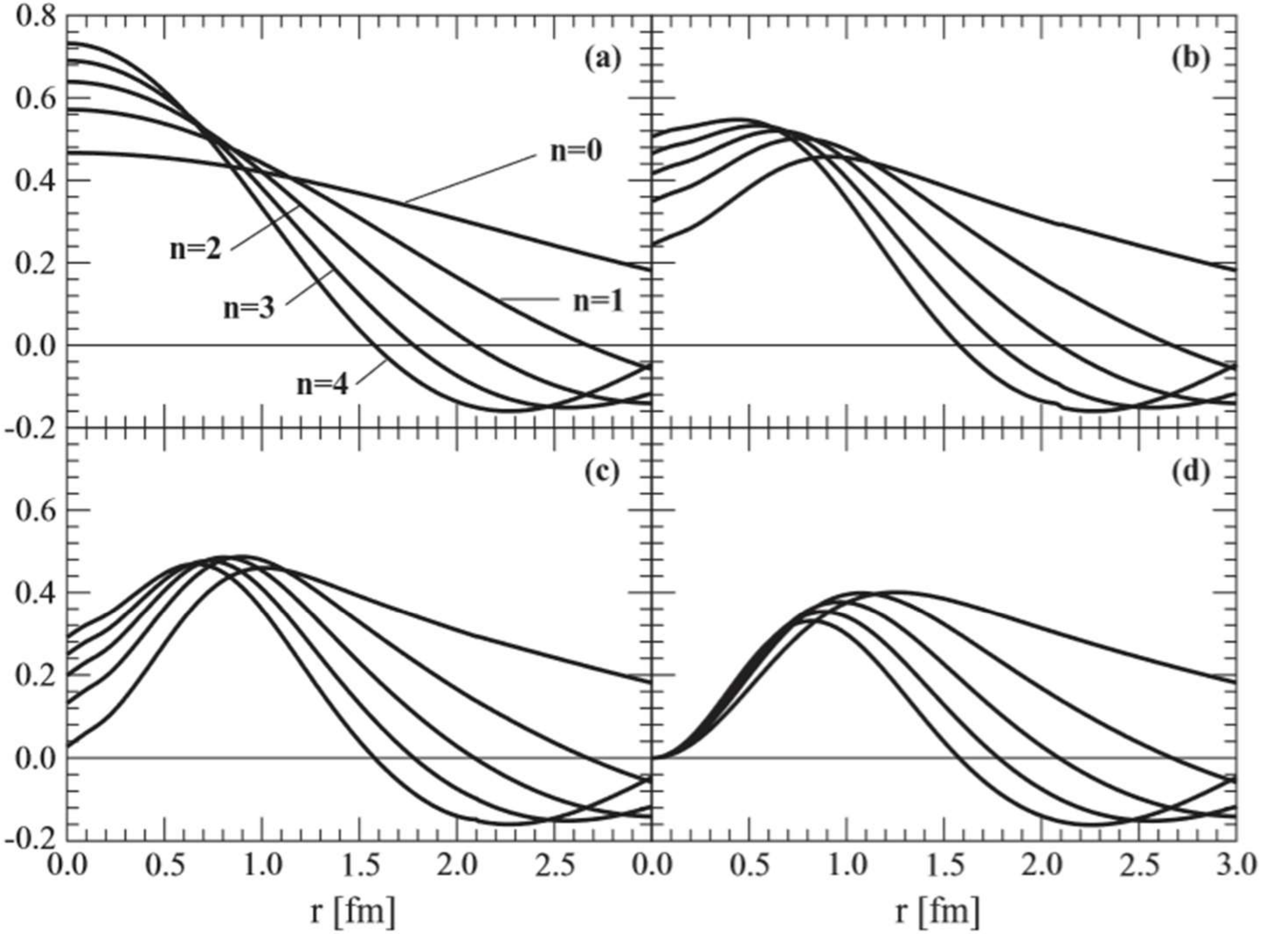


FIG. 1. Two-nucleon wave functions as a function of the relative distance for the $^1S_0$ partial wave and radial quantum numbers $n = 0, 1, 2, 3$, and 4. The results are for the (a) uncorrelated two-nucleon wave functions, (b) coupled-cluster method with CD-Bonn potential, (c) coupled-cluster method with Argonne potential, and (d) Miller-Spencer Jastrow short-range correlations. The harmonic oscillator parameter b is 2.18 fm.

Introduce a new parameter "c":

It means that at r =0, the 2-bd w.f. is not zero (not eliminated by the hard core).

CCM SRC

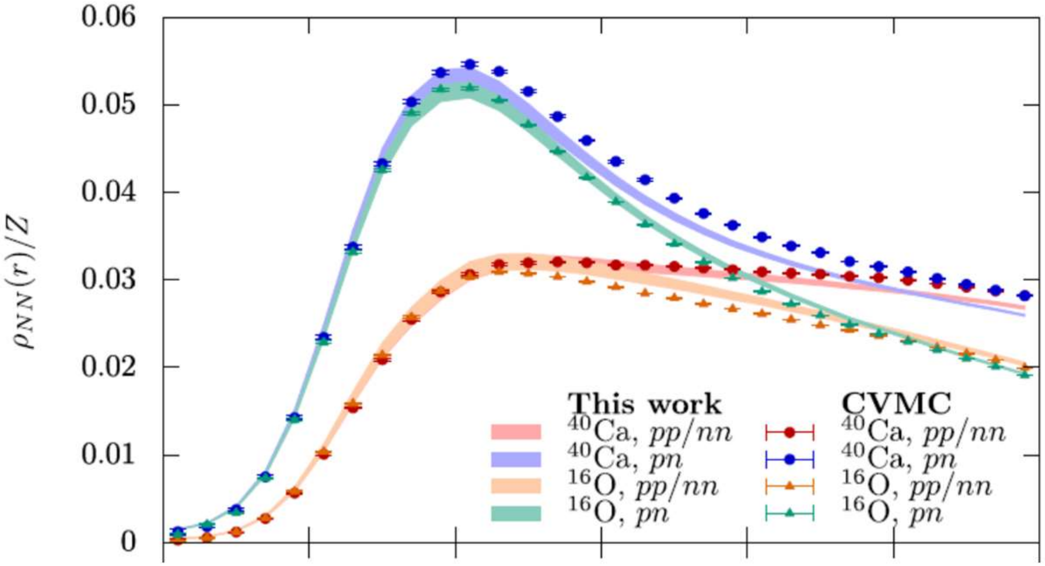
- There is systematic difference between CCM SRC and traditional SRC (MS SRC):

(1) CCM SRC's peak is at 1.0 fm, but MS SRC's peak is at 1.5 fm. So MS SRC will shift the peak of NME distribution toward 1.5 fm (NME w/o SRC peak at 1.0 fm), but CCM SRC does not shift NME distribution. So CCM SRC maintain the original peak position.

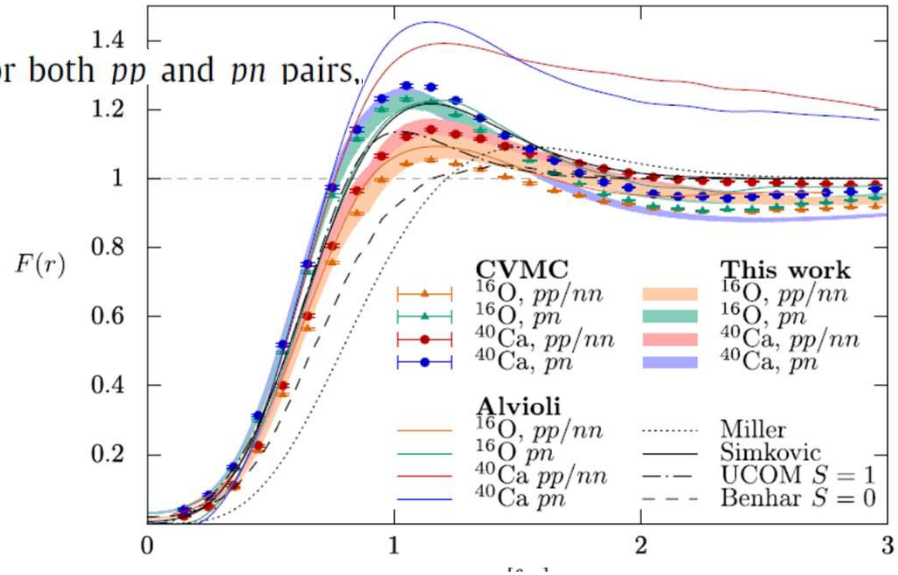
(2) MS SRC eliminate the distribution at r=0 completely (C parameter =0); CCM SRC does not.

CVMC SRC

R. Cruz-Torres, A. Schmidt, G. Miller, L. Weinstein, N. Barnea, R. Weiss, E. Piasetzky, and O. Hen, Physics Letters B 785, 304 (2018).



correlation functions for both pp and pn pairs,



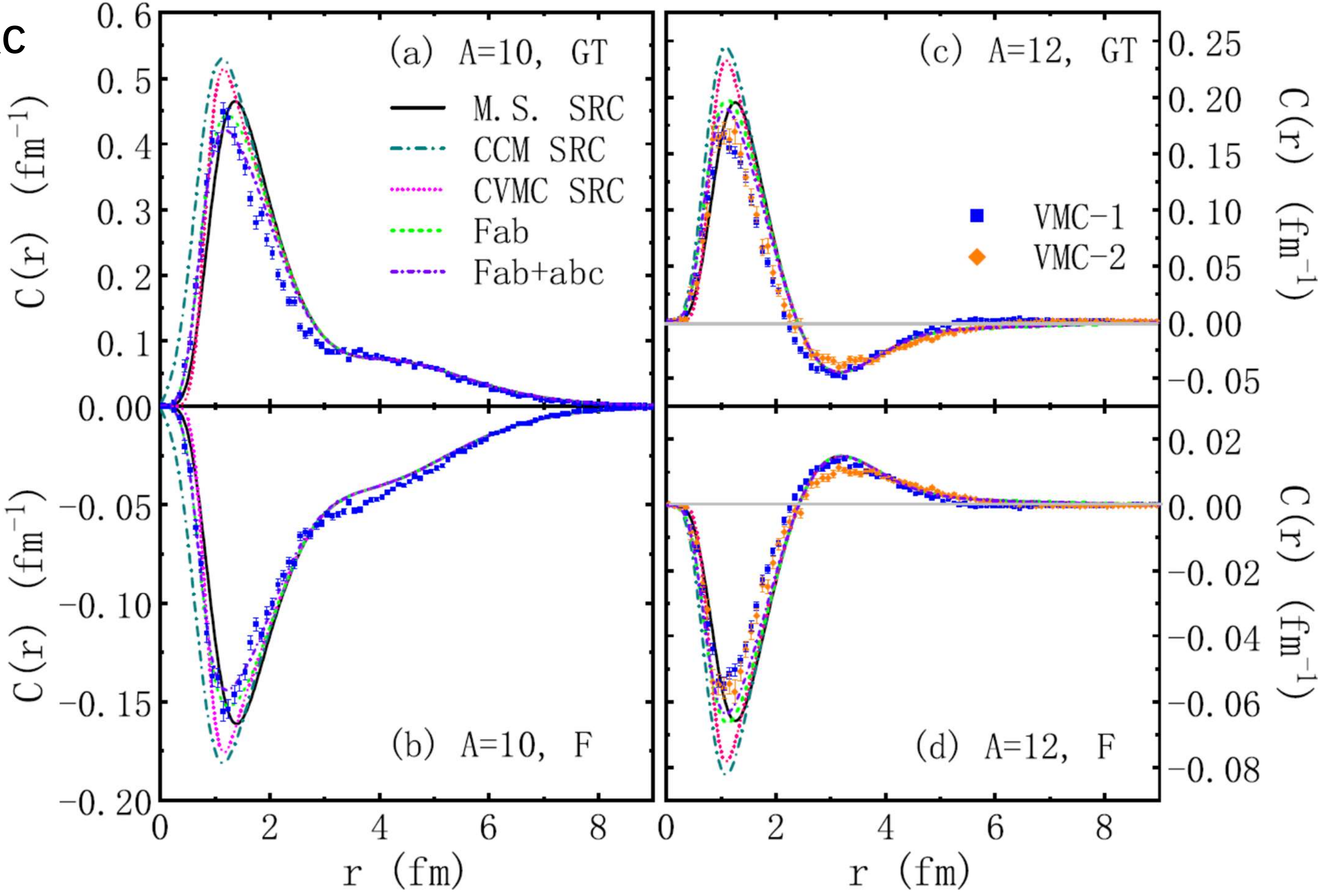
For convenience, we provide the following parameterization for the pp/nn and pn correlation functions determined from CVMC:

$$F(r) = 1 - e^{-\alpha r^2} \times \left(\gamma + r \sum_{i=1}^{3} \beta_i r^i \right) \tag{11}$$

Table 1
Parameters describing $F(r)$, using the functional form of equation (11).

Parameter	Units	Value (pp/nn)	Value (pn)
α	fm^{-2}	3.17	1.08
γ	–	0.995	0.985
β_1	fm^{-2}	1.81	−0.432
β_2	fm^{-3}	5.90	−3.30
β_3	fm^{-4}	−9.87	2.01

With SRC



(a) A=10, GT
(b) A=10, F
(c) A=12, GT
(d) A=12, F
M.S. SRC
CCM SRC
CVMC SRC
Fab
Fab+abc
VMC-1
VMC-2
C(r) (fm^{-1})
r (fm)

Results

	$^{10}Be(0_1^+)\rightarrow{}^{10}C(0_1^+)$		$^{12}Be(0_1^+)\rightarrow{}^{12}C(0_1^+)$	
	F	GT	F	GT
VMC-1	-1.001(40)	2.273(91)	-0.100(4)	0.257(10)
VMC-2	—	—	-0.113(5)	0.274(11)
SM_{WS}(M.S. SRC, *psd*)	-0.967	2.381	-0.122	0.342
SM_{WS}(CCM SRC, *psd*)	-1.069	2.683	-0.175	0.499
SM_{WS}(CVMC SRC, *psd*)	-0.992	2.457	-0.141	0.398
SM_{WS}(Fab, *psd*)	-0.988	2.449	-0.138	0.388
SM_{WS}(Fab+abc, *psd*)	-0.957	2.362	-0.128	0.361

Conclusions from the study of light nuclei

- 1. The use of H.O. radial wave functions will likely lead
- to an overestimate of matrix elements.

- 2. Limited size model space calculations could affect the magnitude of the predicted $0\nu\beta\beta$ matrix elements, particularly for calculations constrained to a single shell.

- 3. The inclusion of a SRC function is needed.
- The best choice for this function requires further study.

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THANKS!