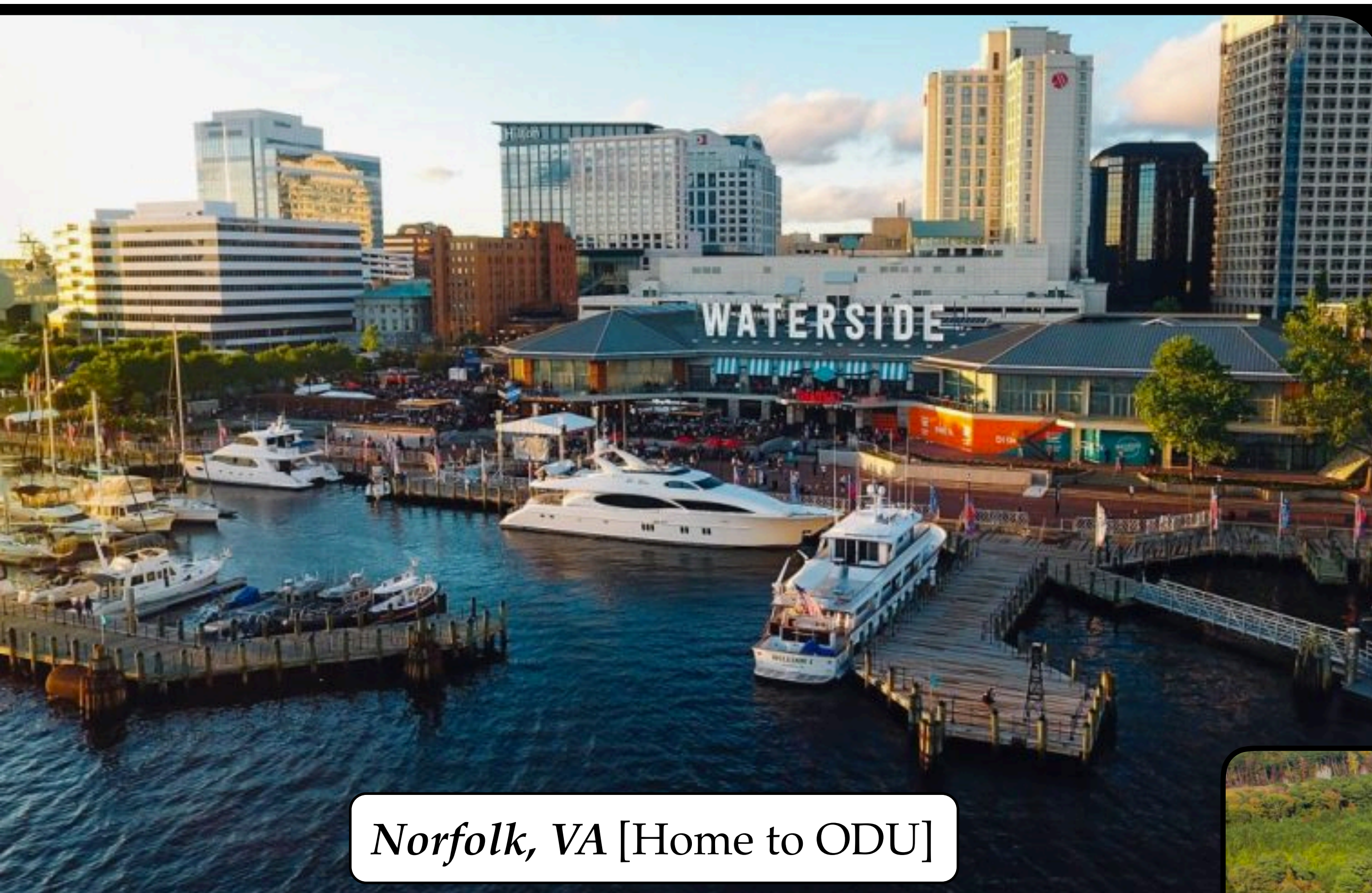
Two-body matrix elements in LQCD

Raúl Briceño - http://bit.ly/rbricenoPhD



Norfolk, VA [Home to ODU]

OLD DOMINION UNIVERSITY

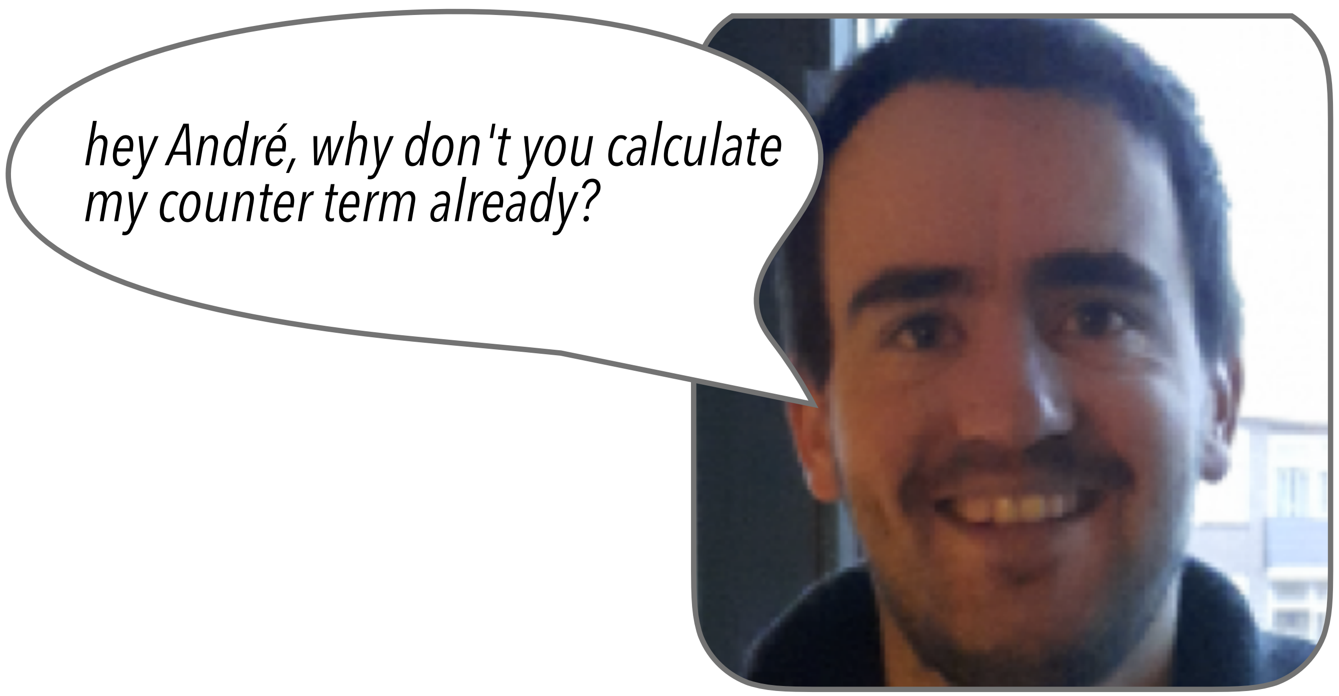
Jefferson Lab

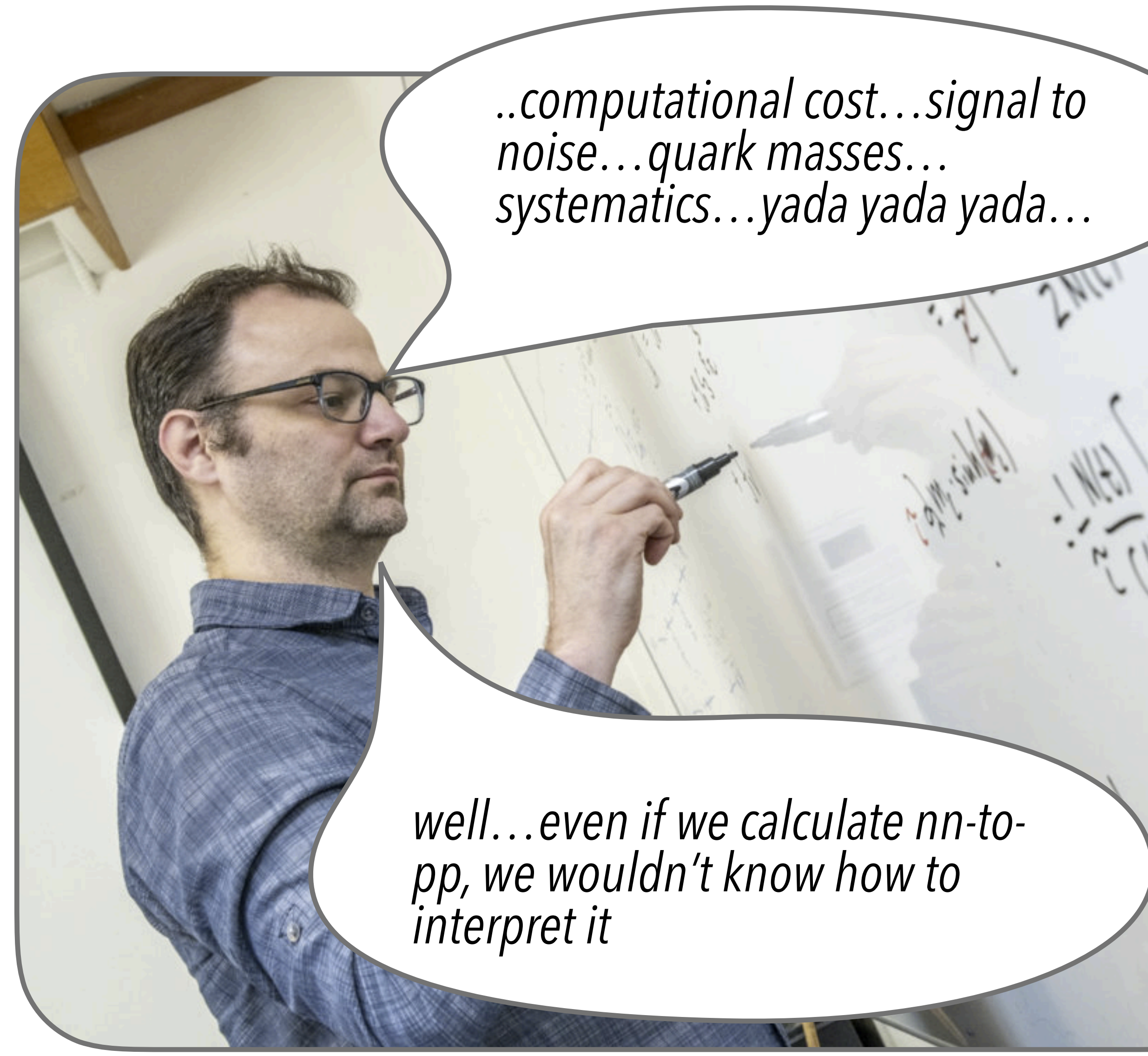
hadspec

ECT, 2019*

JLab, VA

$0\nu\beta\beta$ from lattice QCD





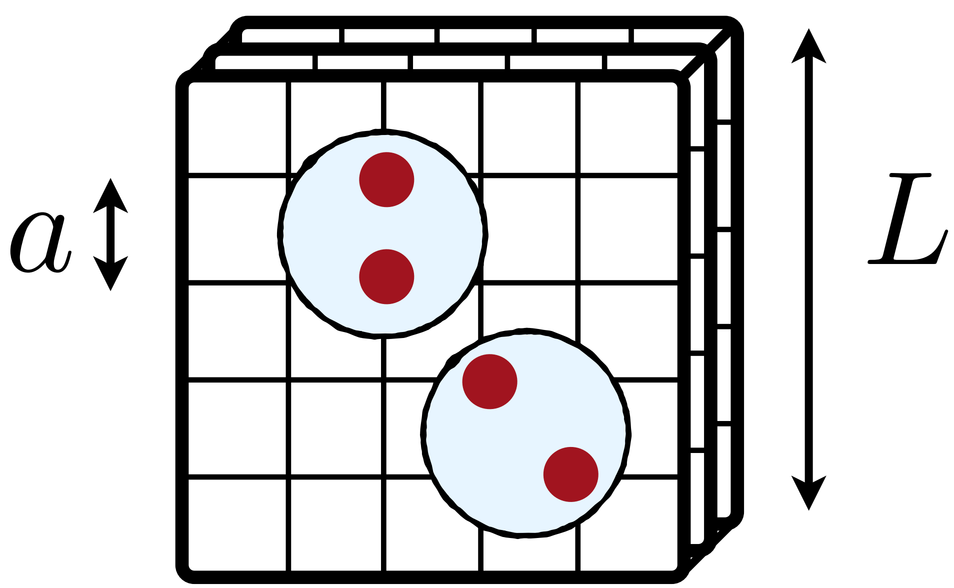
$0\nu\beta\beta$ from lattice QCD

Largely correlated challenges ahead

- contraction cost
- Euclidean spacetime: $t_M \rightarrow -it_E$
- finite volume
- quark masses: $m_q \rightarrow m_q^{\text{phys.}}$

focus of this and the next talk

and no, no Quantum computing needed 😏



$0\nu\beta\beta$ from lattice QCD

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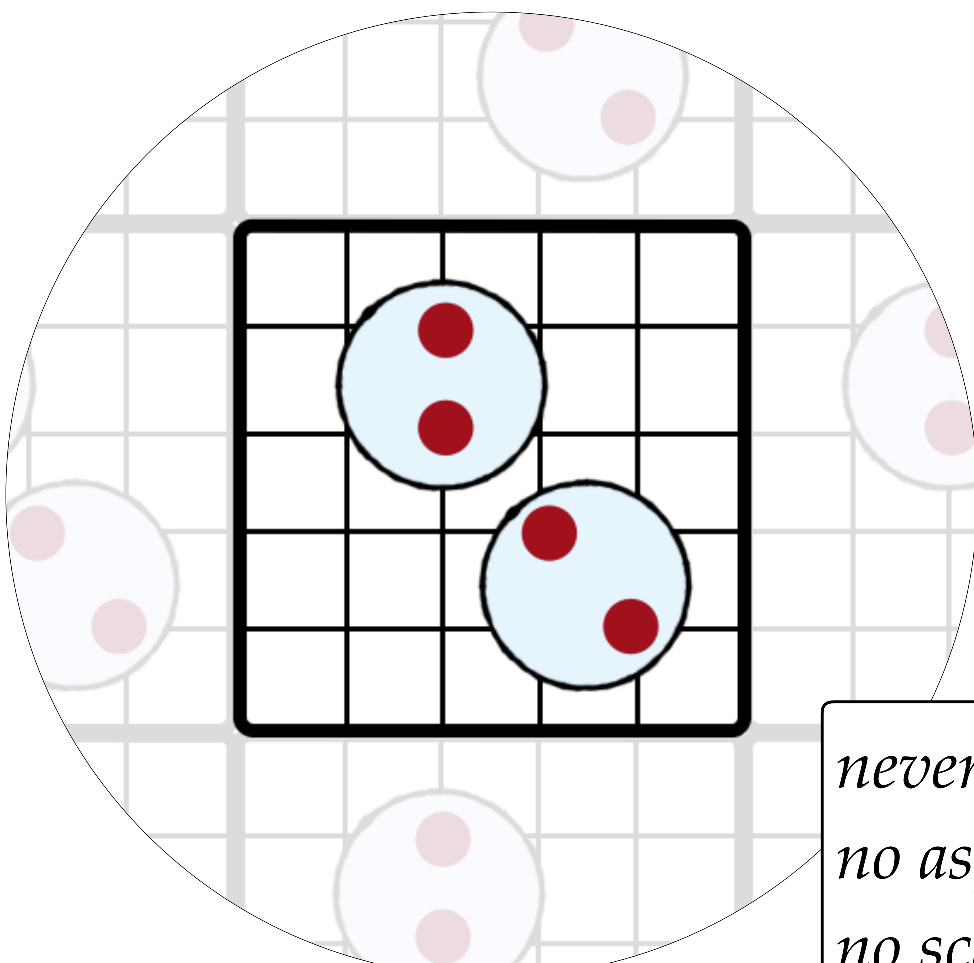
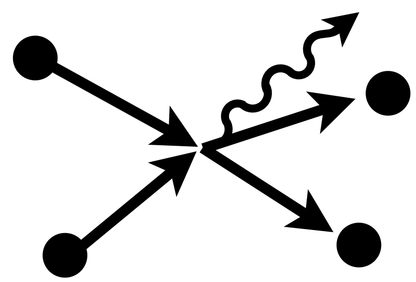


never free
no asymptotic staes
no scattering

two-nucleon electroweak process

Consider a process 2-nucleons coupling via a *local current* to a final 2-nucleon state

electroweak amplitudes



Matthias will discuss non-local currents

Schindler

two-nucleon electroweak process

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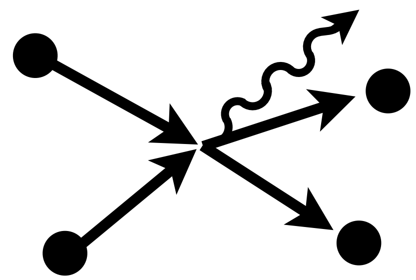
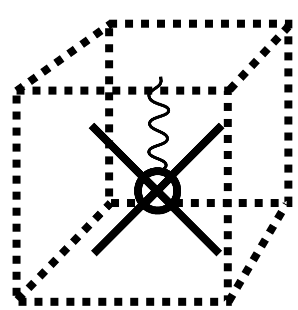


We would naturally expect that these may be accessed via appropriately designed three-point correlators

finite-volume matrix elements



$$C_{3\text{pts.}} = \langle \mathcal{O}_f(t_f)\, \mathcal{J}(t_c)\, \mathcal{O}_i^\dagger(0) \rangle$$
$$= \sum_{n,m} f_{n,m}(t_f, t_c; L)\, \langle n; L | \mathcal{J}(0) | n; L \rangle$$

$$f_{n,m}(t_f, t_c; L) \equiv \langle 0 | \mathcal{O}_f(0) | n; L \rangle \langle m; L | \mathcal{O}_i^\dagger(0) | 0 \rangle \; e^{-t_f E_n - t_c(E_m - E_n)}$$

two-nucleon electroweak process

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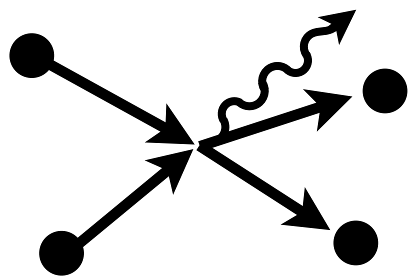

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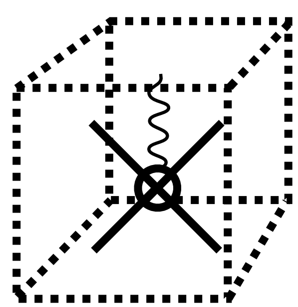
finite-volume matrix elements



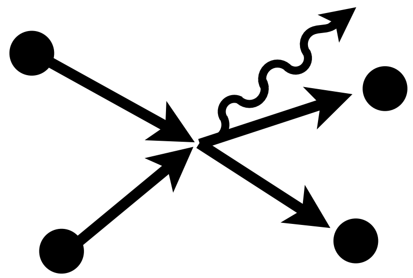
$$
\begin{aligned}
C_{\text{3pts.}} &= \langle \mathcal{O}_f(t_f)\, \mathcal{J}(t_c)\, \mathcal{O}_i^\dagger(0) \rangle \\
&= \sum_{n,m} f_{n,m}(t_f, t_c; L)\, \langle n; L | \mathcal{J}(0) | n; L \rangle \\
&\approx \sum_{n,m} f_{B,B}(t_f, t_c; \infty)\, \langle B; \infty | \mathcal{J}(0) | B; \infty \rangle + \cdots
\end{aligned}
$$

all matrix nuclear elements calculations have been of deeply bound states (see Zohreh's talk)

two-nucleon electroweak process

Consider a process 2-nucleons coupling via a *local current* to a final 2-nucleon state

electroweak amplitudes



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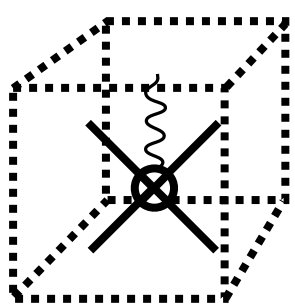
finite-volume matrix elements



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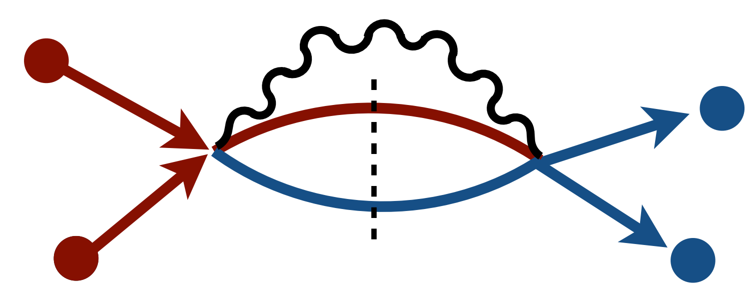
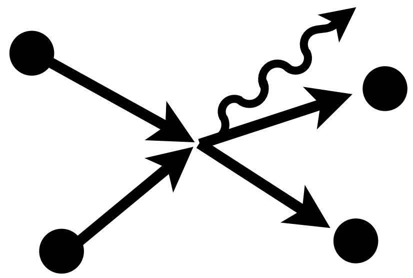
\begin{aligned}
C_{3\text{pts.}} &= \langle \mathcal{O}_f(t_f)\, \mathcal{J}(t_c)\, \mathcal{O}_i^\dagger(0)\rangle \\
&= \sum_{n,m} f_{n,m}(t_f, t_c; L)\, \langle n; L|\mathcal{J}(0)|n; L\rangle
\end{aligned}
$$

$0\nu\beta\beta$ requires understanding this for unbound states



two-nucleon electroweak process

electroweak amplitudes

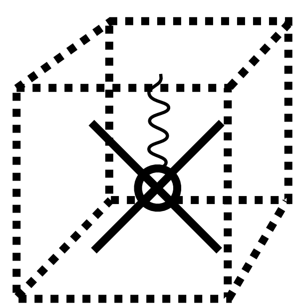


seemingly reasonable

EFTs

$$\mathcal{L}_{\chi PT} = \mathcal{L}_{\pi} + \mathcal{L}_{N\pi} + \mathcal{L}_{NN} + \cdots$$

finite-volume matrix elements



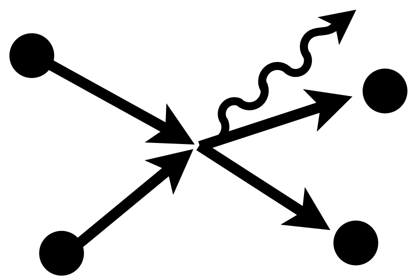
not good enough:

- requires well defined EFT
  - convergence?
  - heavy quark masses?
- perturbative
- kinematic restrictions
- strange sector of QCD

i.e. how much would your final result depend on pure QCD and how much depends on your choice and order of the EFT?

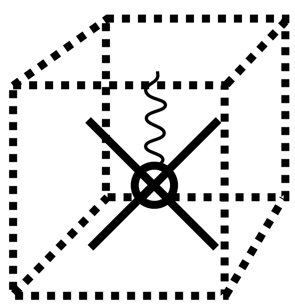
two-nucleon electroweak process

electroweak amplitudes



all-orders mapping

finite-volume matrix elements



Baroni

Hansen

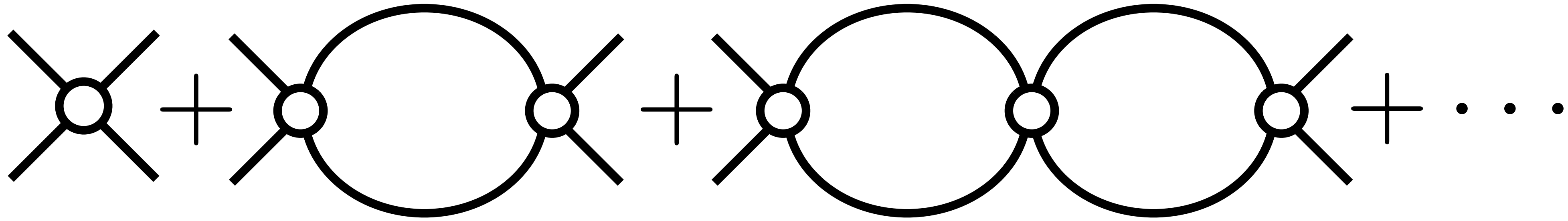
Jackura

Ortega

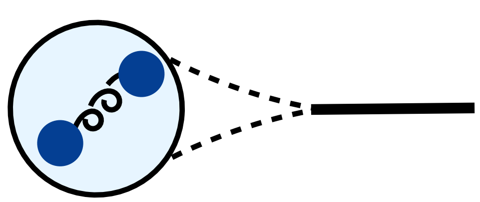
- RB & Hansen (2015)
- Baroni, RB, Hansen, Ortega (2018)

Two-body scattering

Unitarity using all orders perturbation theory:

$i\mathcal{M} =$ 

IR limit of QCD, only interested in hadronic d.o.f.



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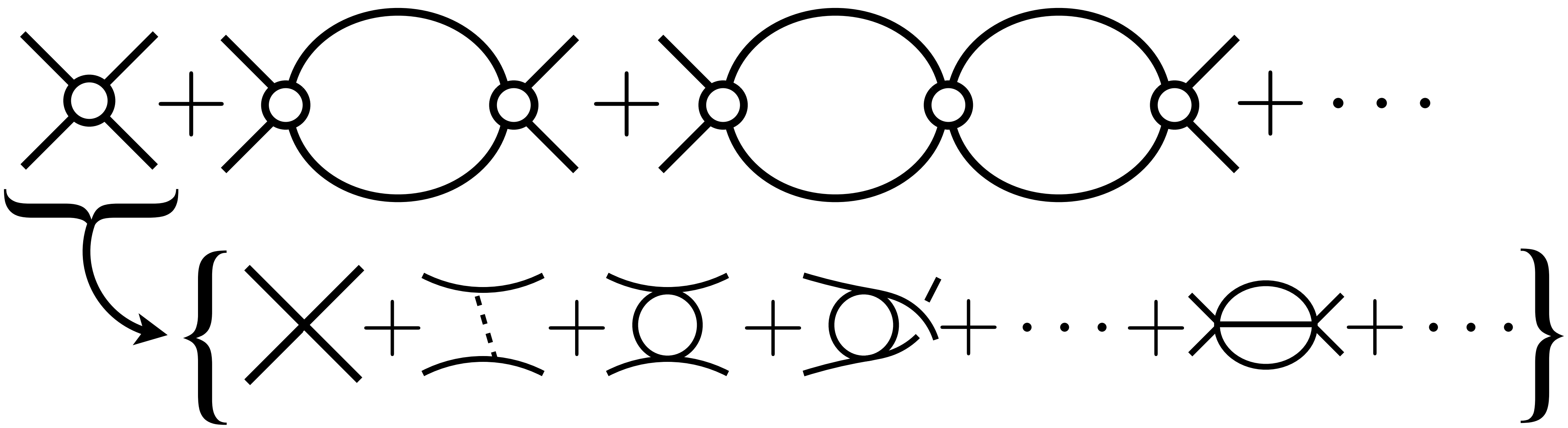

non-perturbative kernel including all diagrams not shown…

“yep, the left hand cut is there”

Two-body scattering

Unitarity using all orders perturbation theory:

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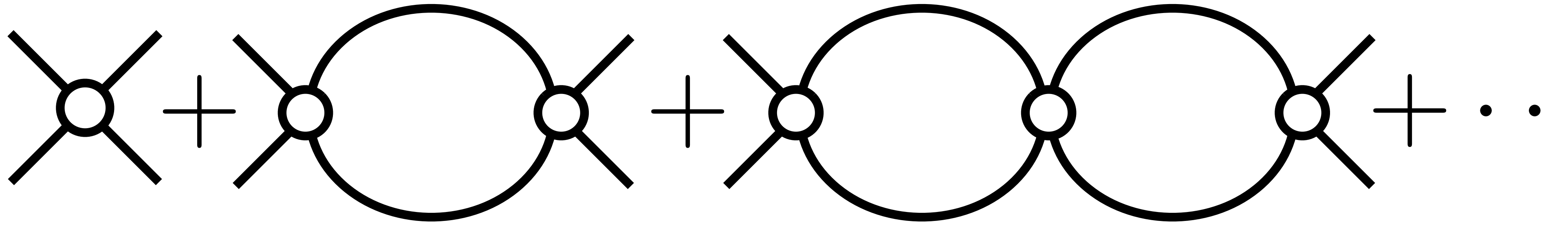
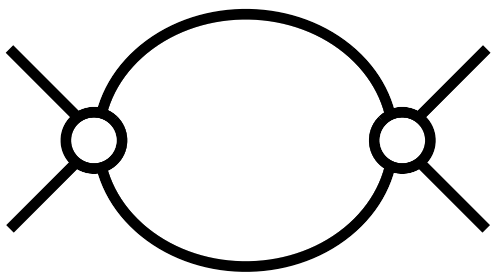


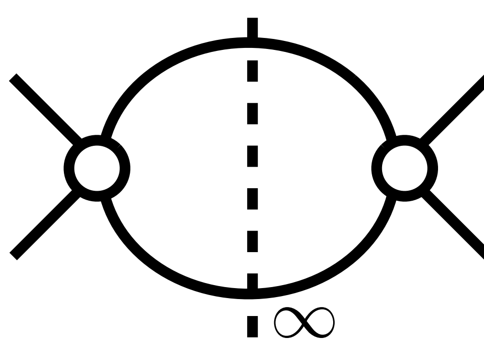


$$= \int \frac{d^4k}{(2\pi)^4} [iB(k,P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P-k)^2 - m^2 + i\epsilon}$$

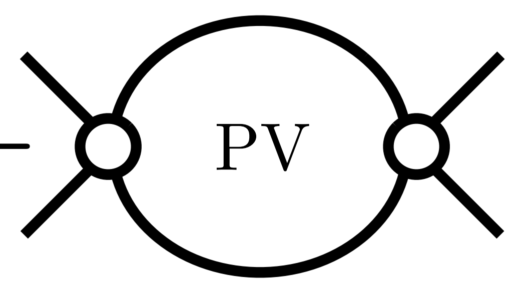
$$= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k,P)]^2}{(2\omega_k)^2} \pi\delta(E - 2\omega_k) + \text{“PV integral”}$$

$$= [iB_{on}]\rho\,[iB_{on}] + \text{“PV integral”}$$

$=$



$+$

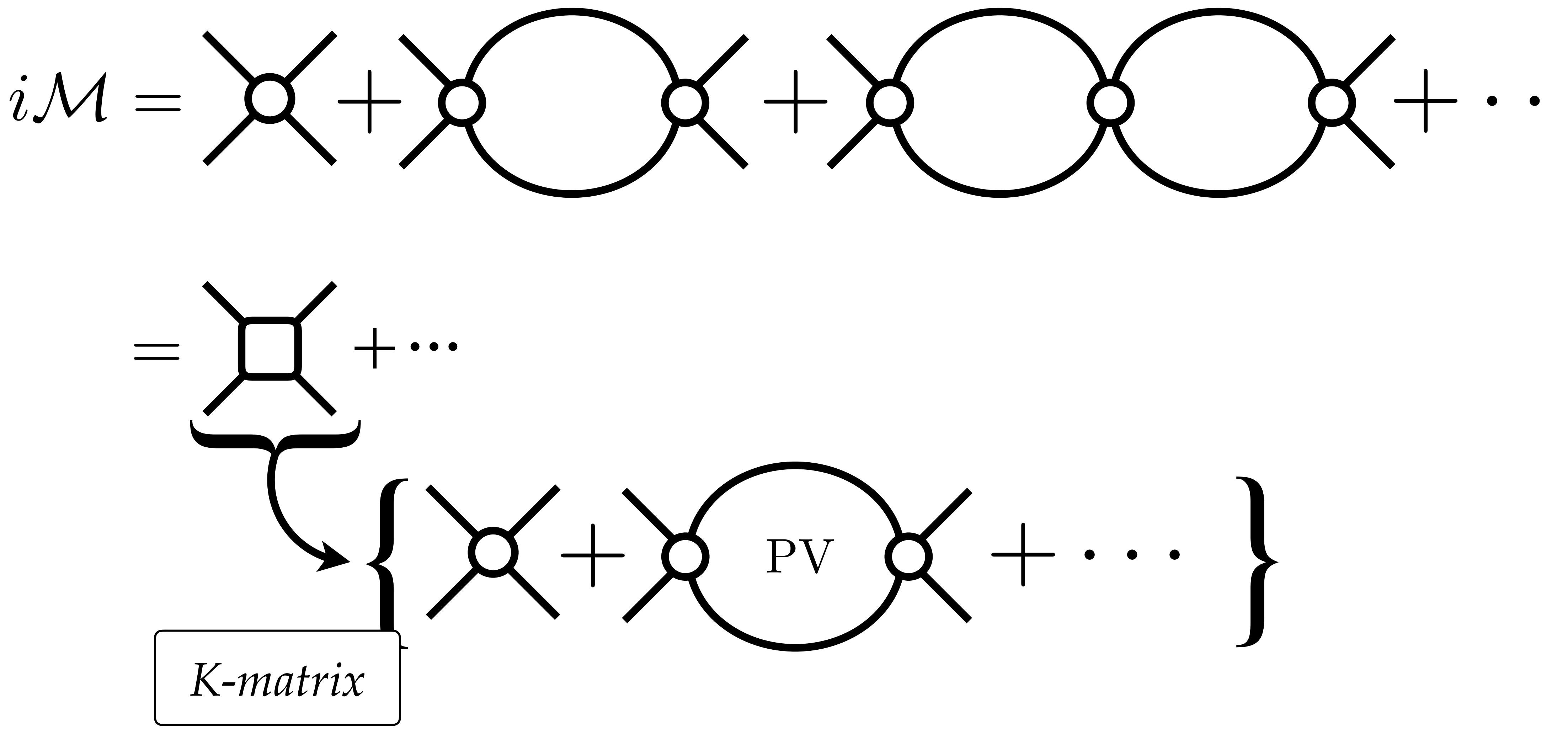


$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-body scattering

Unitarity using all orders perturbation theory:



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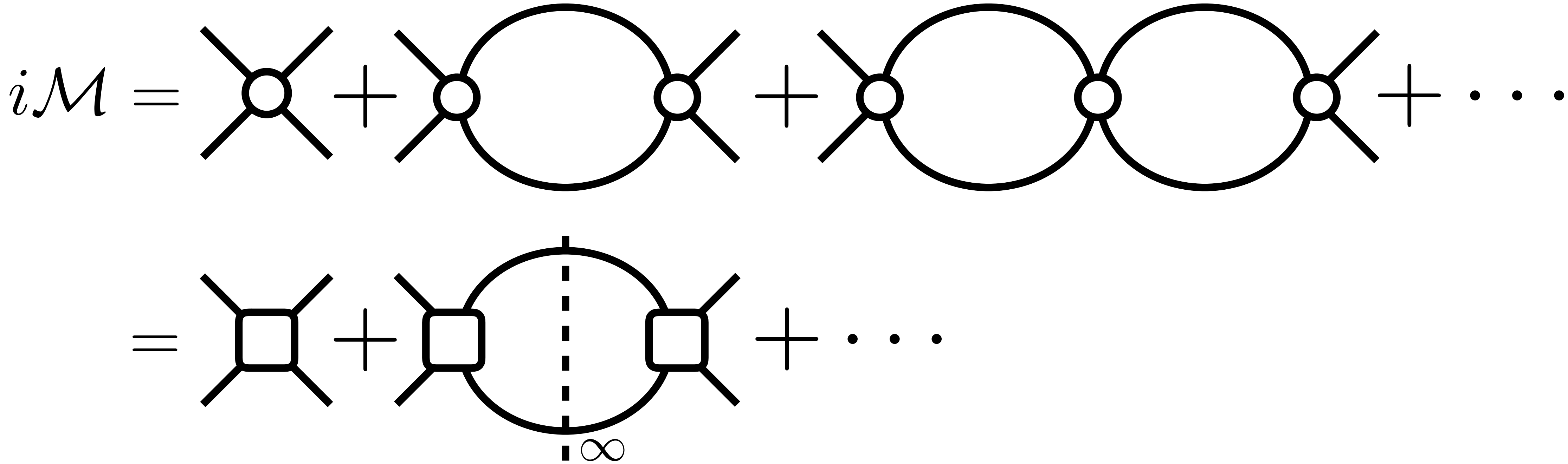

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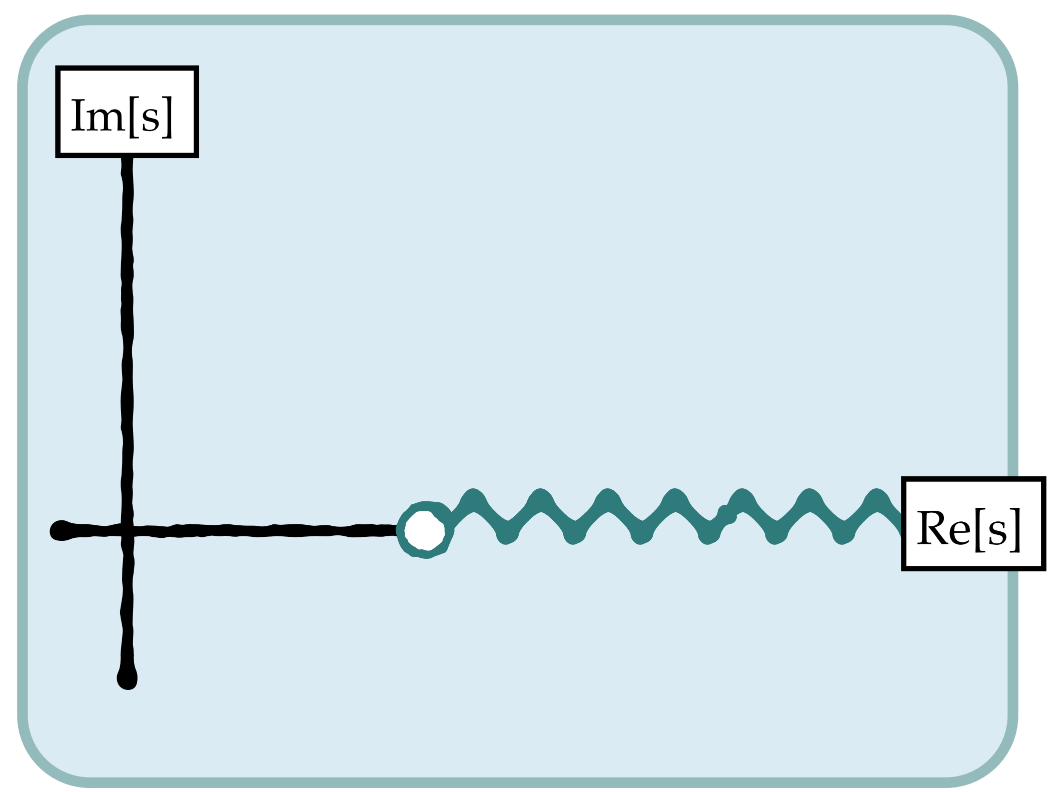
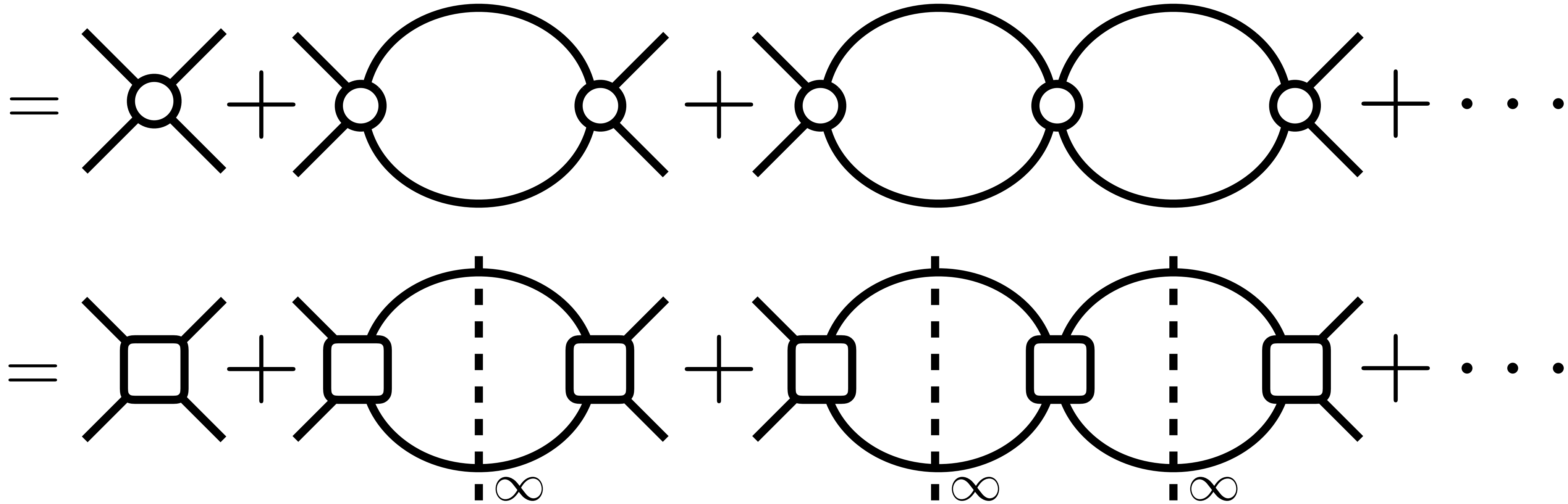


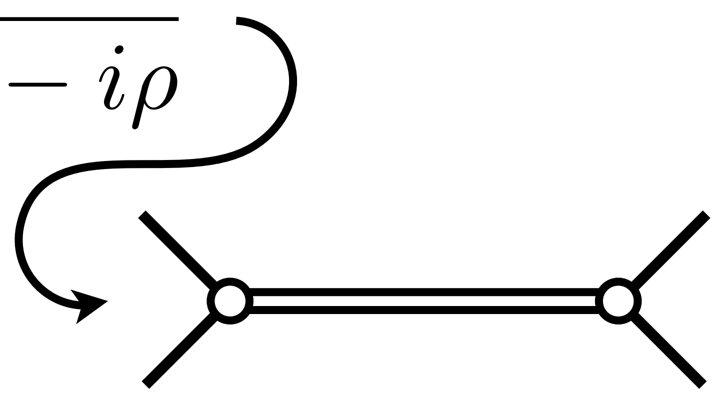
Two-body scattering

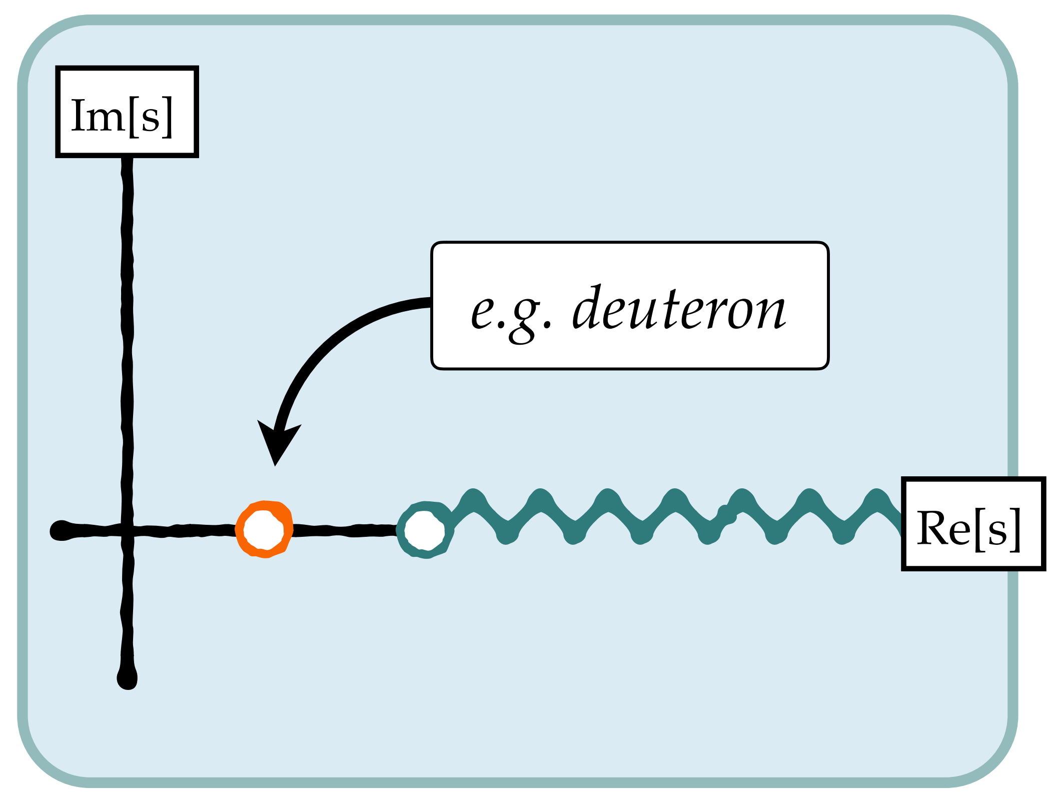
Unitarity using all orders perturbation theory:

$$i\mathcal{M} =$$

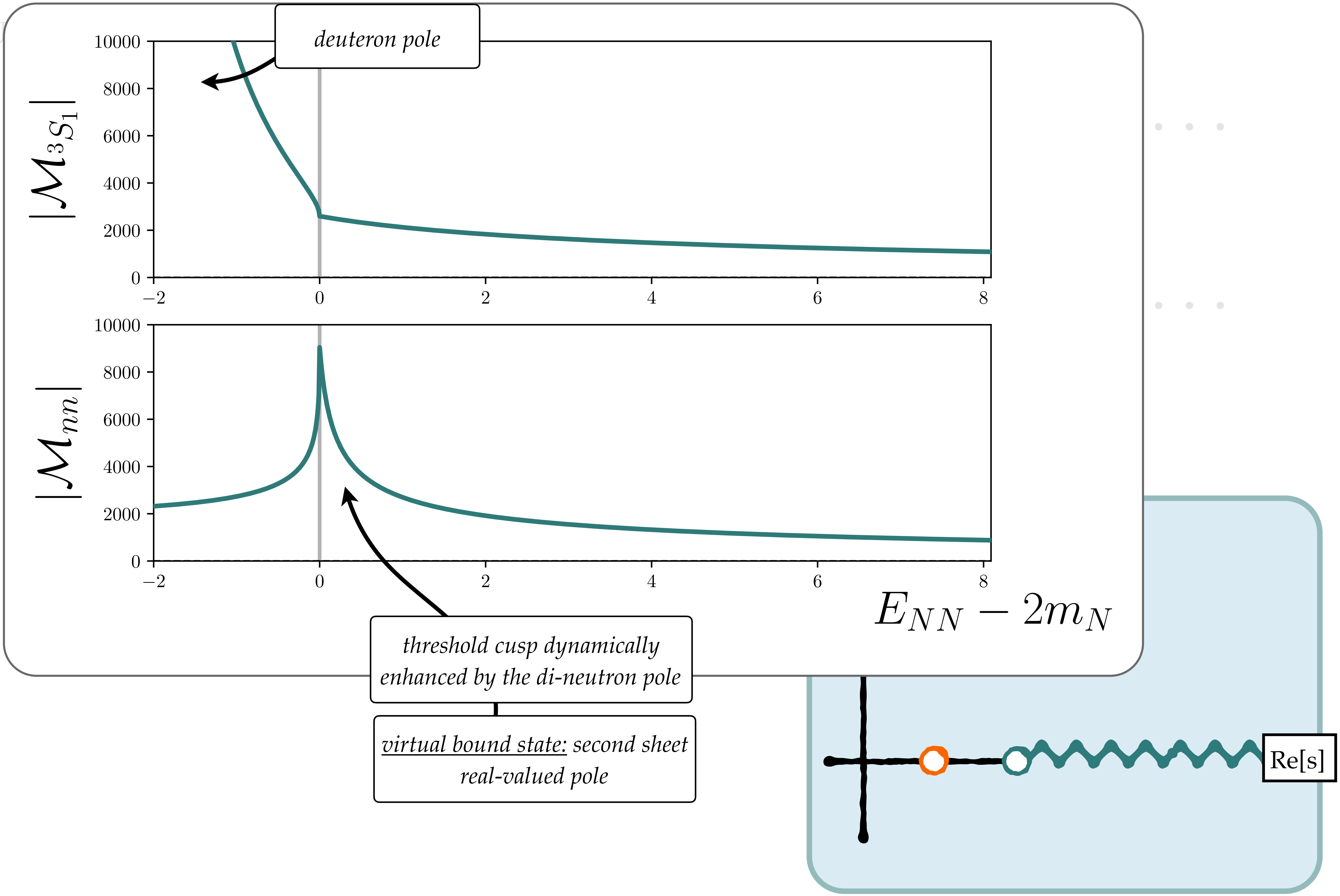


$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$

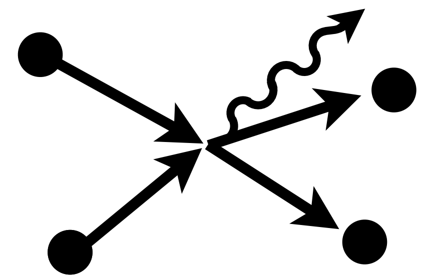




Two-body scattering

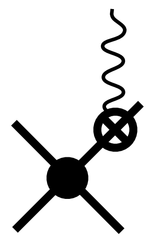


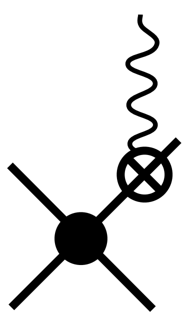
Two-to-two scattering **with current** - (full amp.)

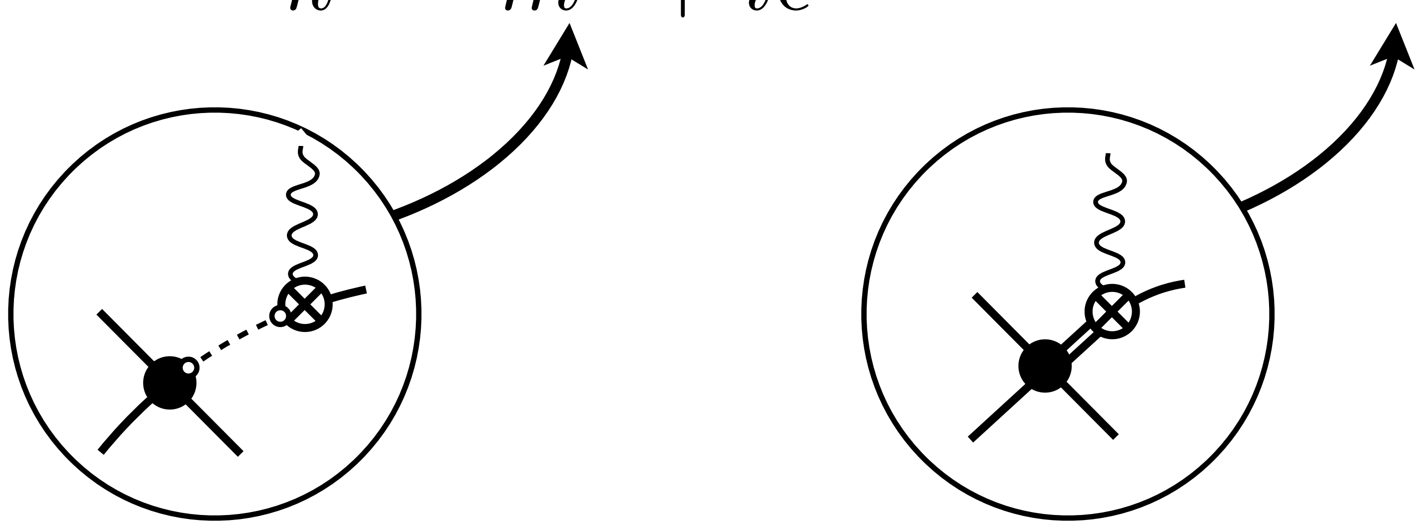
Let's isolate all possible singularities of… 

Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

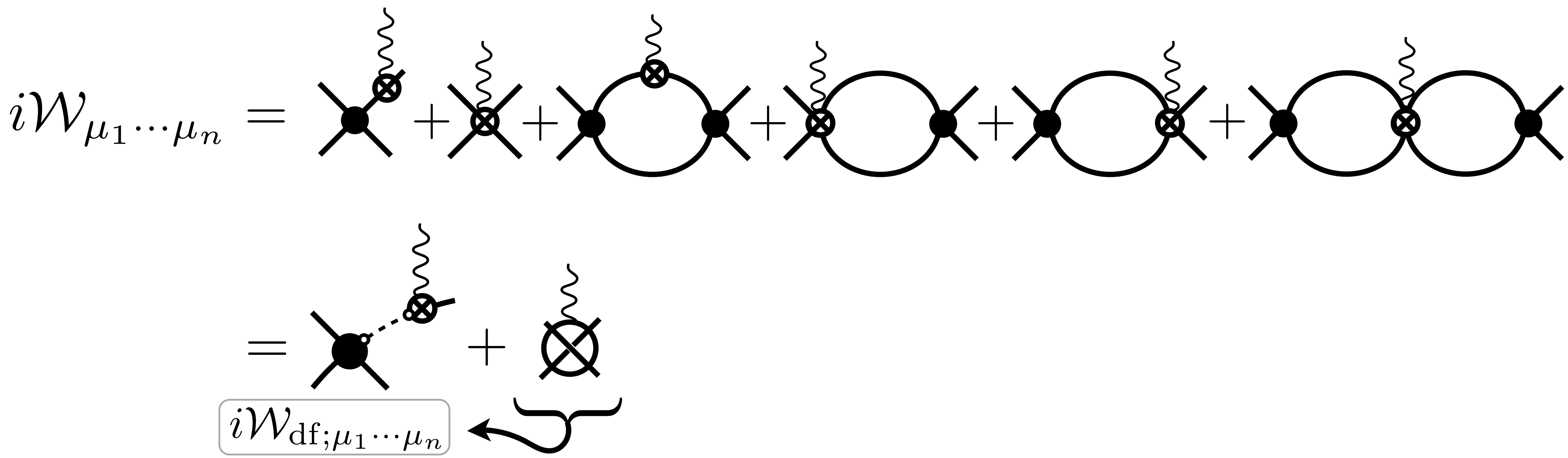
$$i\mathcal{W}_{\mu_1\cdots\mu_n} =$$  $$+\cdots$$

 $$= i\mathcal{M}\frac{i}{k^2 - m^2 + i\epsilon}\, i\omega_{\mu_1\cdots\mu_n} + \text{“smooth”}$$



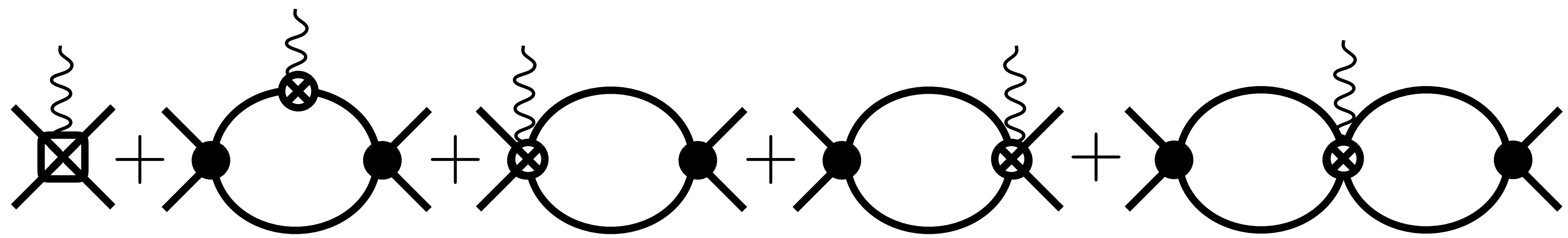
Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

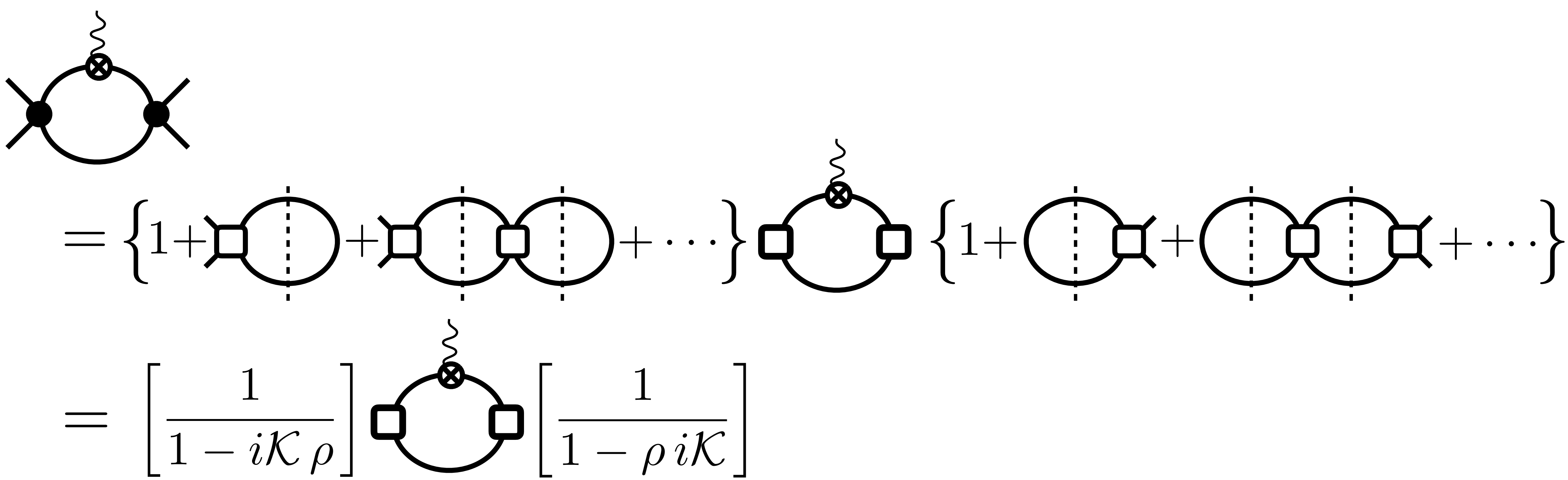


Two-to-two scattering **with current** - (df amp.)

Divergence-free amplitude

$i\mathcal{W}_{\mathrm{df};\mu_1\cdots\mu_n} =$ 

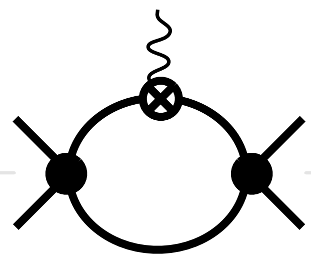
New class of diagrams:

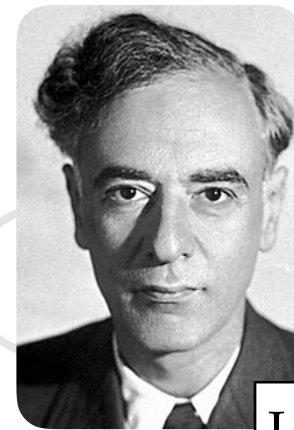


same square-root (and possibly pole) singularities as two-body amplitudes

Two-to-two scattering **with current** - (df amp.)

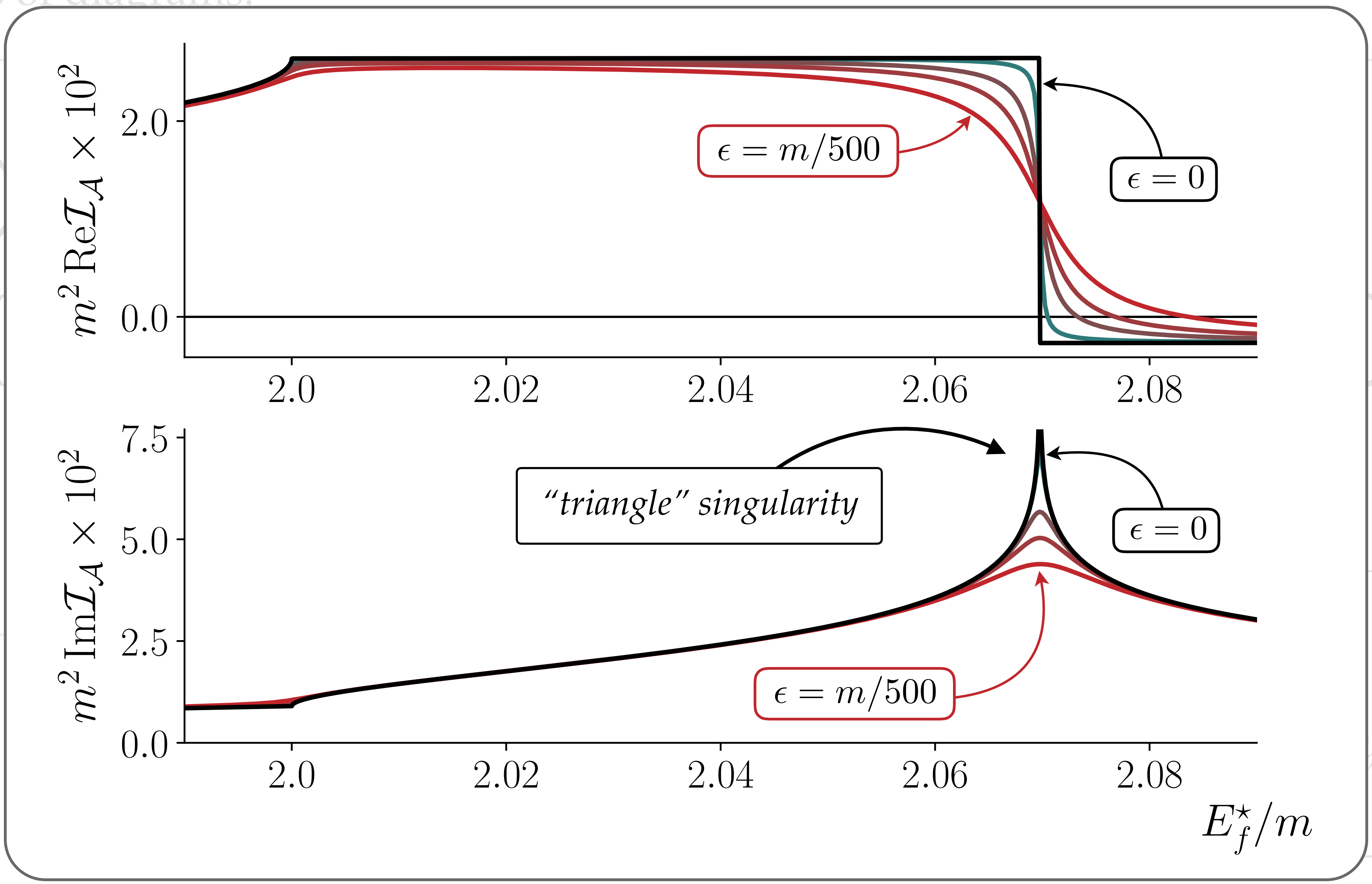
Divergence-free amplitude





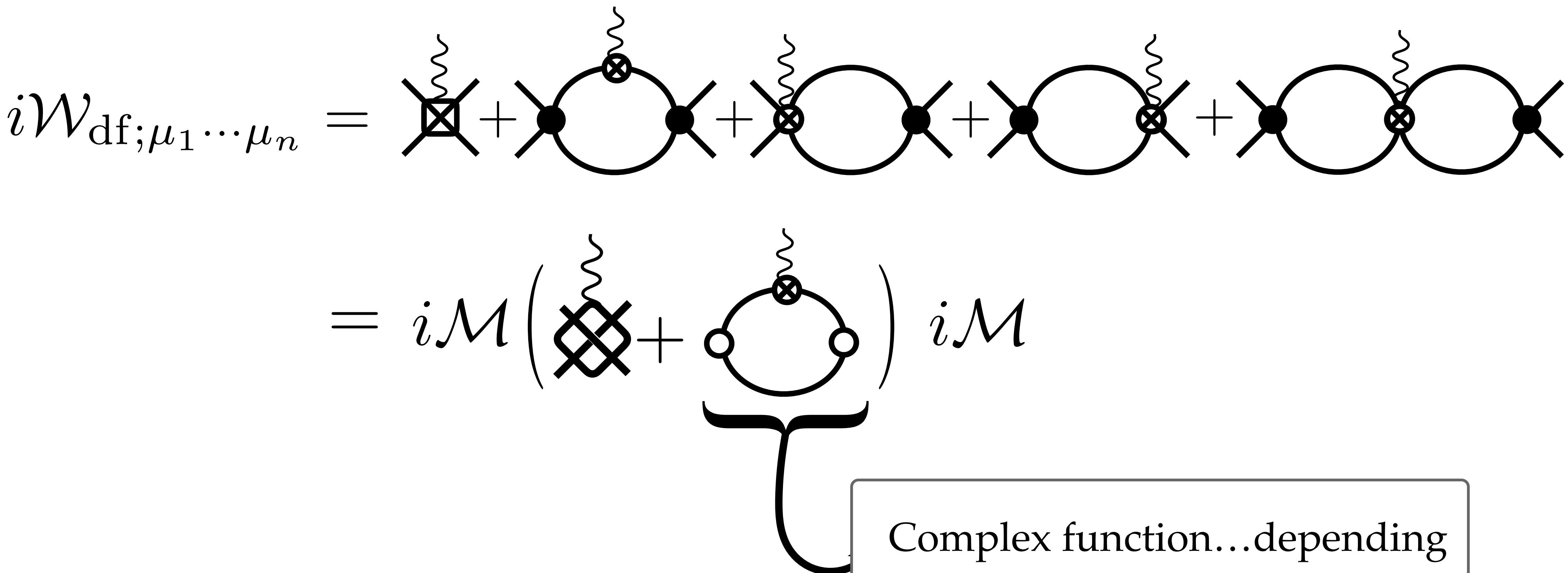
Landau (1959)

New class of diagrams:



Two-to-two scattering **with current**

Divergence-free amplitude



Complex function…depending on the one-body form factors

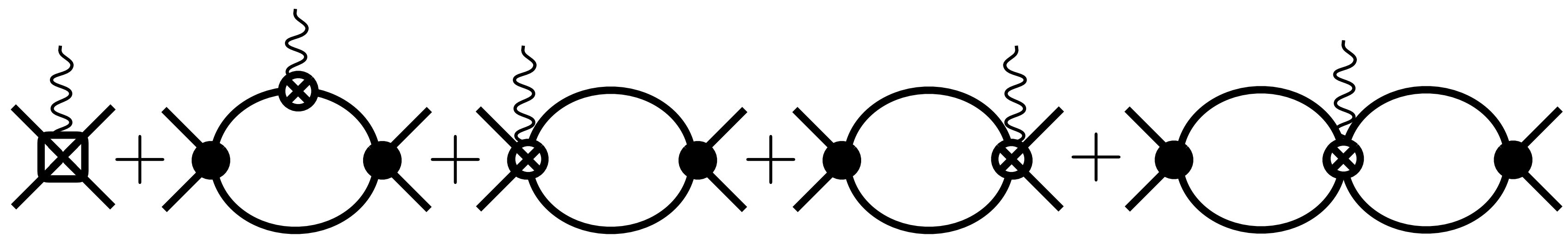
Naive Watson's theorem does not apply!

finite-volume quantities **must** be able to recover this singularity

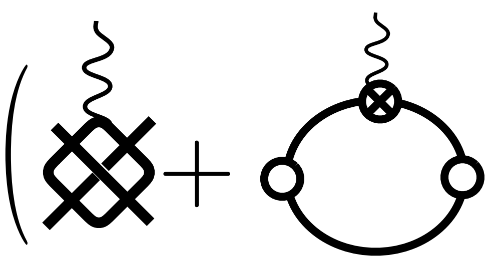
Two-to-two scattering **with current**

Divergence-free amplitude

$$i\mathcal{W}_{\mathrm{df};\mu_1\cdots\mu_n} =$$

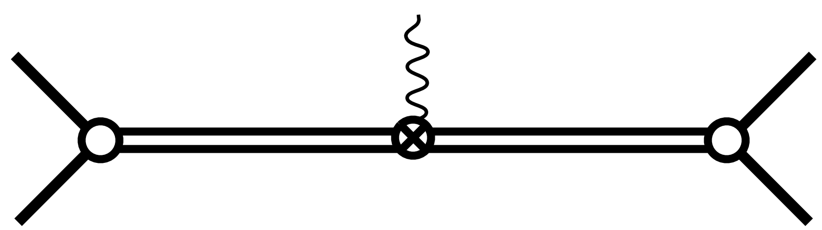


$$= i\mathcal{M}\Big($$



$$\Big)\, i\mathcal{M}$$

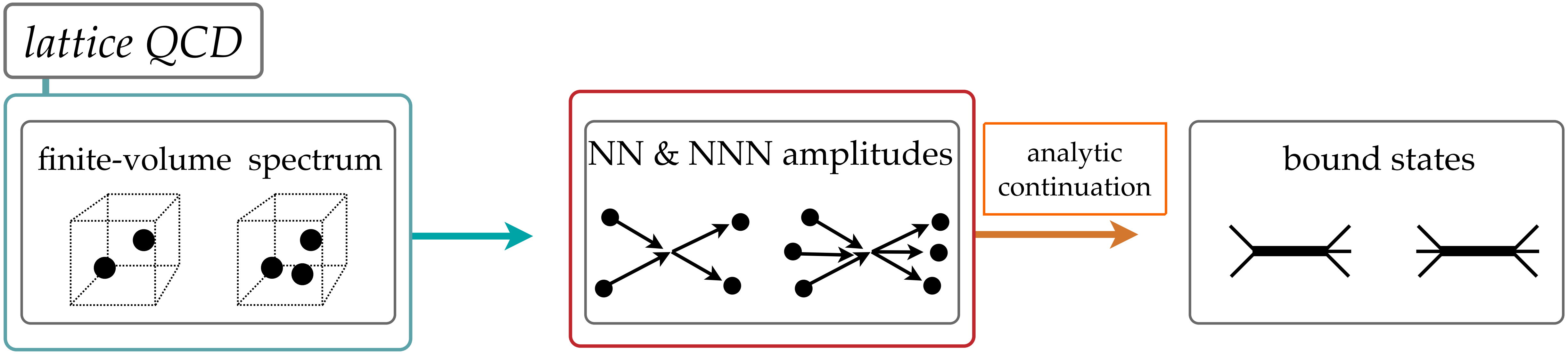
$$\xrightarrow{\lim s_i, s_f = s_B}$$



$$\sim \frac{g}{s_i - s_B} K_{\mu_1\ldots\mu_n} F_R(Q^2) \frac{g}{s_f - s_B}$$

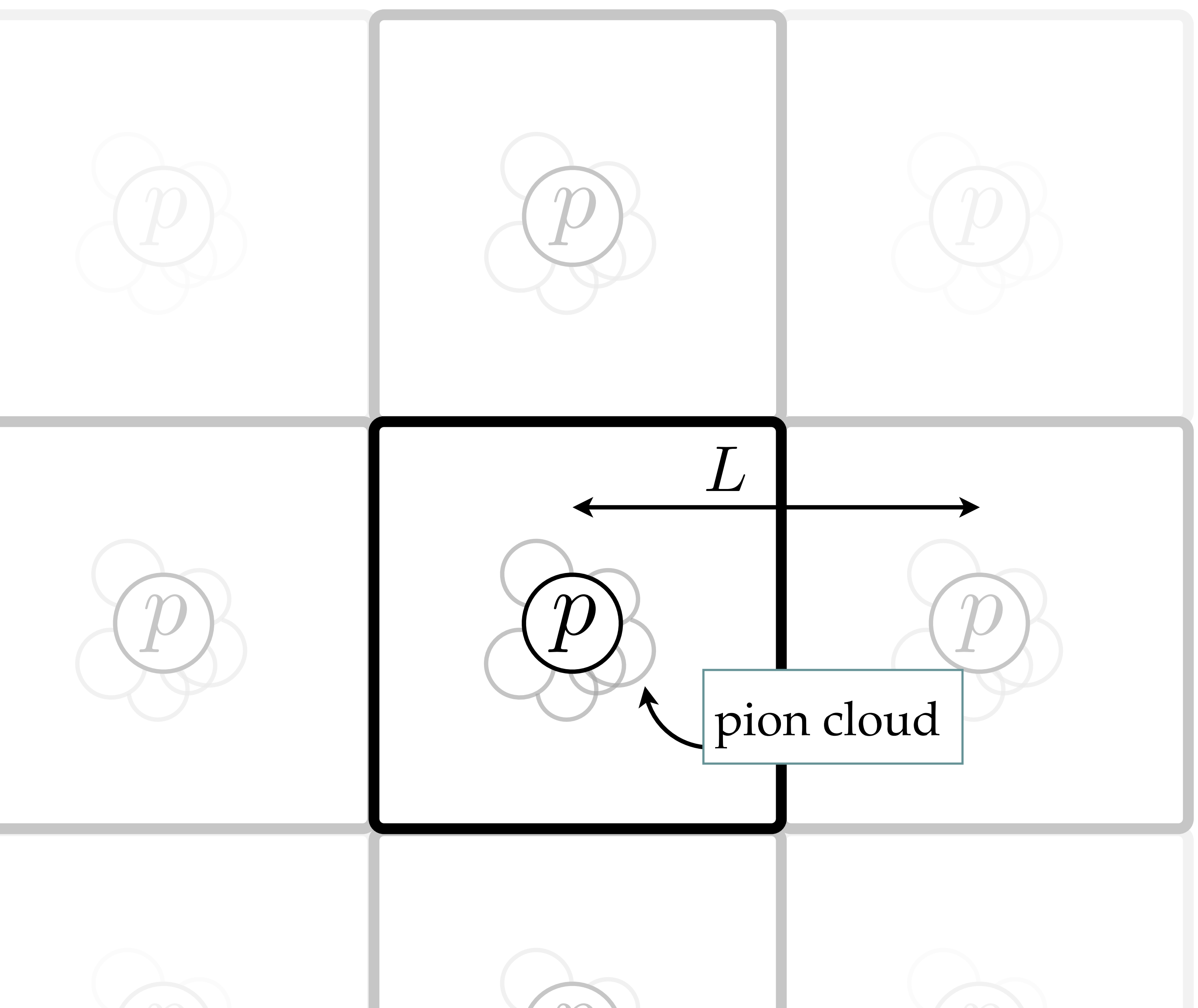
bound state form factors

few-nucleons systems in LQCD



Putting particles in a box

- Finite-volume arise from the interactions with mirror images
- Assuming $L >> $ size of the hadrons ~ $1/m_\pi$
  - This is a purely infrared artifact
  - We can determine these artifact using hadrons are the degrees of freedom
- Note $m_\pi L$ is a natural parameter

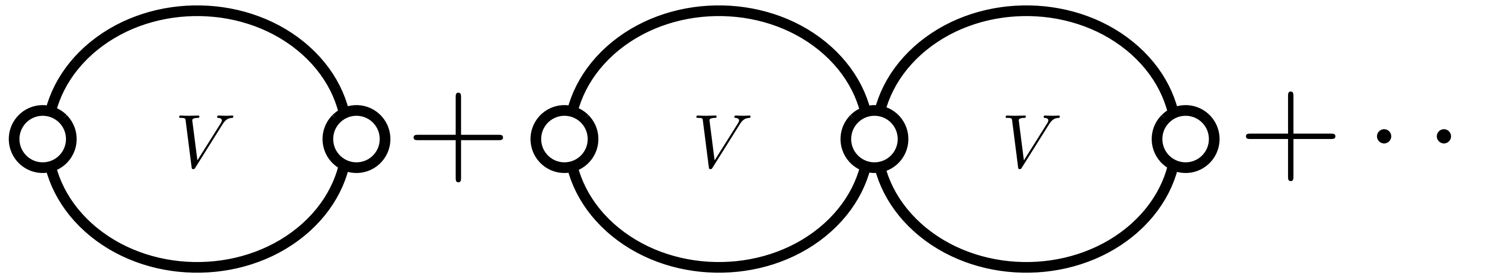


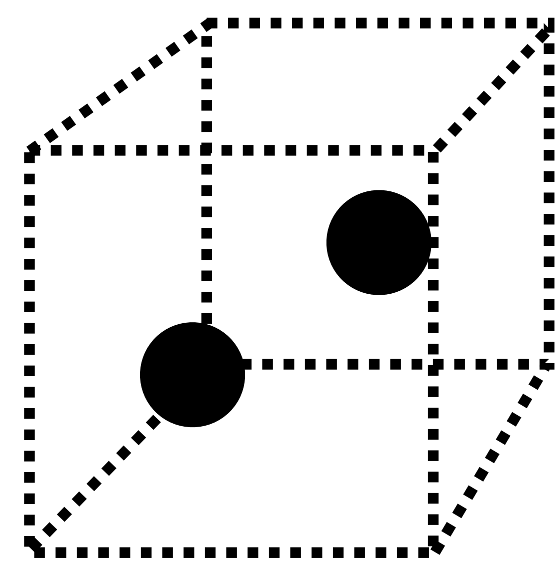
$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

$$C_L^{2pt.}(P) =$$





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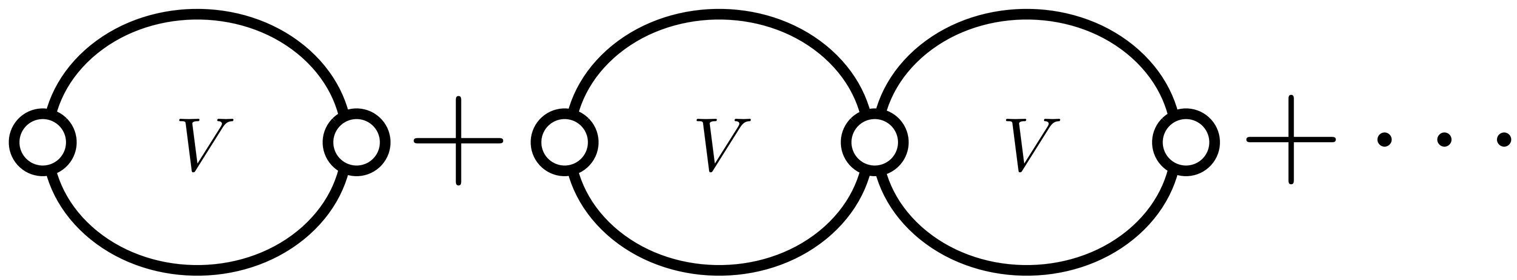

$$= C_\infty(P) + \cdots$$

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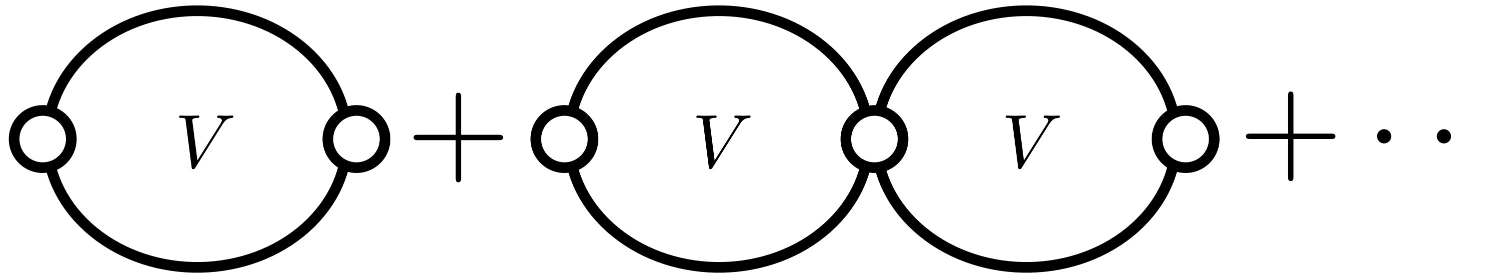
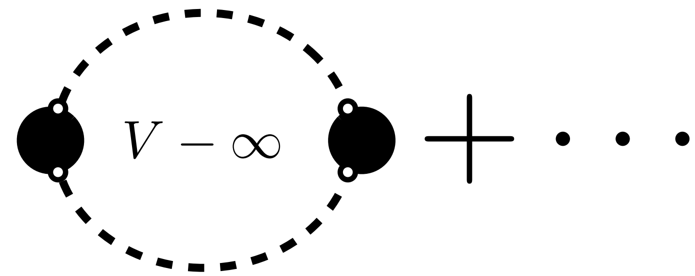
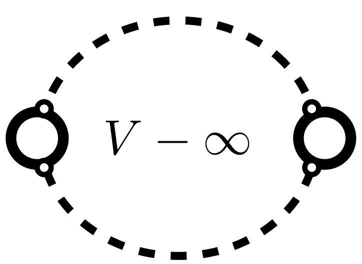


$$= C_\infty(P) +$$





$$= (iB)\left(\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3}\right]\frac{1}{(2\omega_k)^2}\frac{i}{E-2\omega_k+i\epsilon}\right)(iB)$$

$$\equiv [iB]\, iF\, [iB]$$

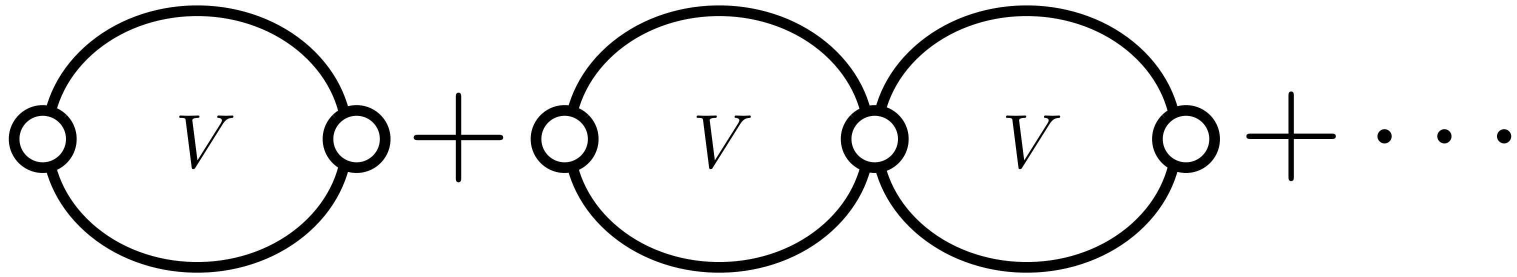
F replaces ρ

a simple square root singularity is replaced by a function that has both simples poles and the square root singularity

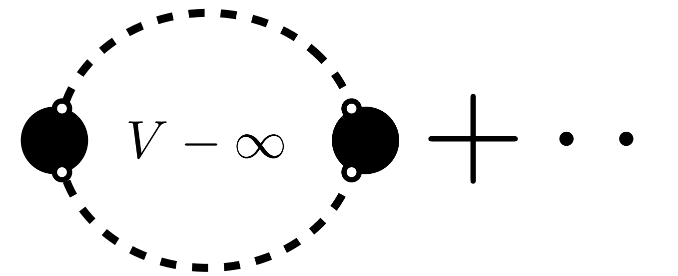
Two-particle in finite volume

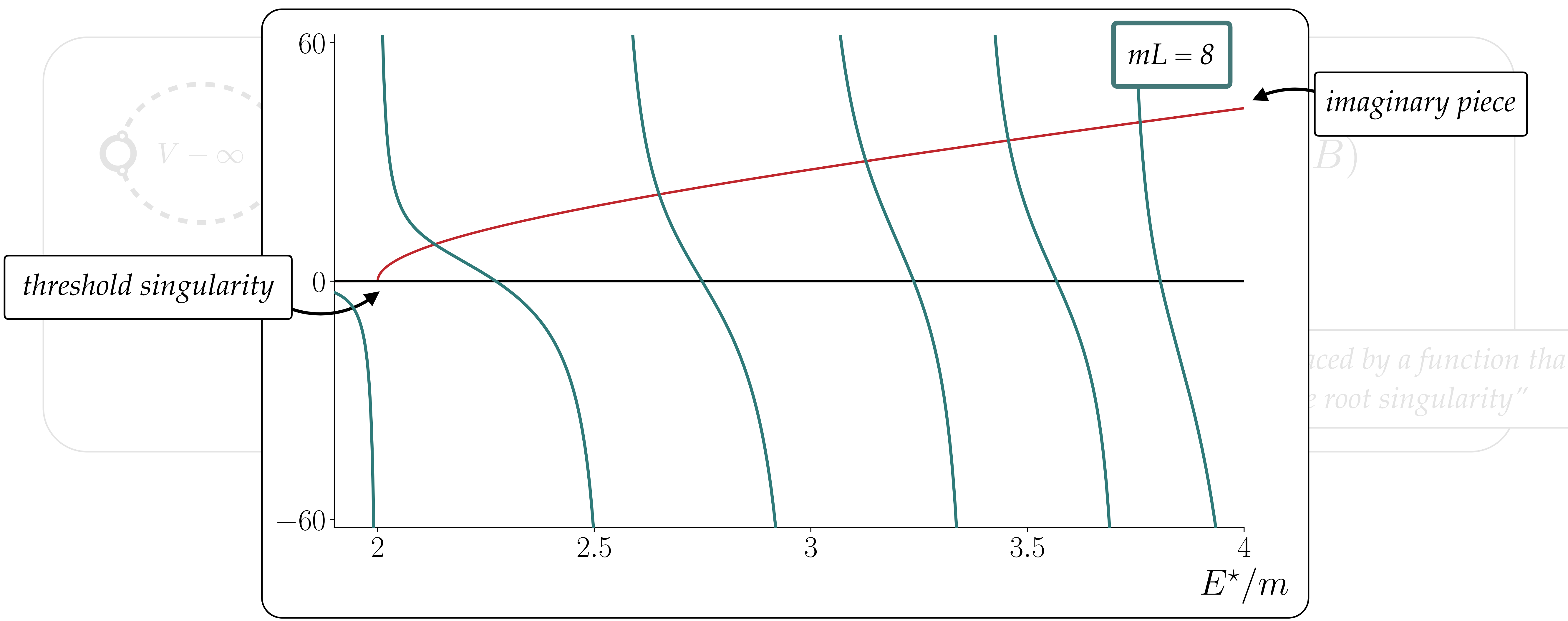
Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) =$$



$$= C_\infty(P) +$$





Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

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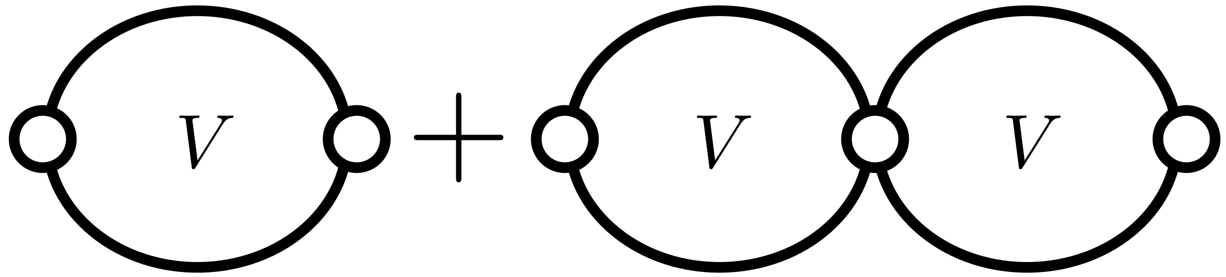
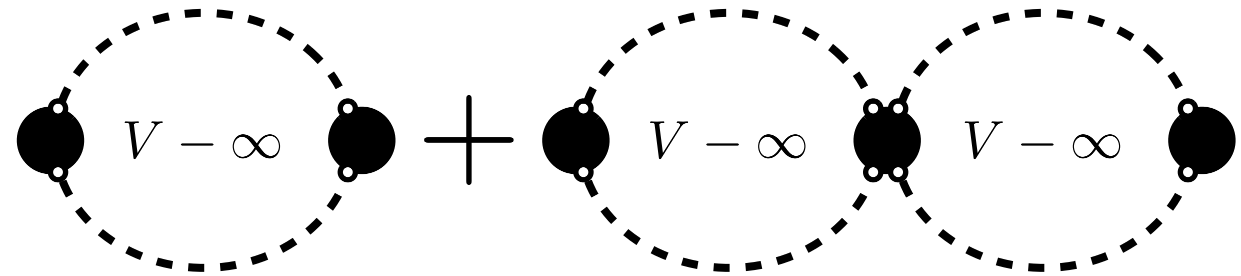


$$+ \cdots$$

$$= C_\infty(P) +$$



$$+ \cdots$$

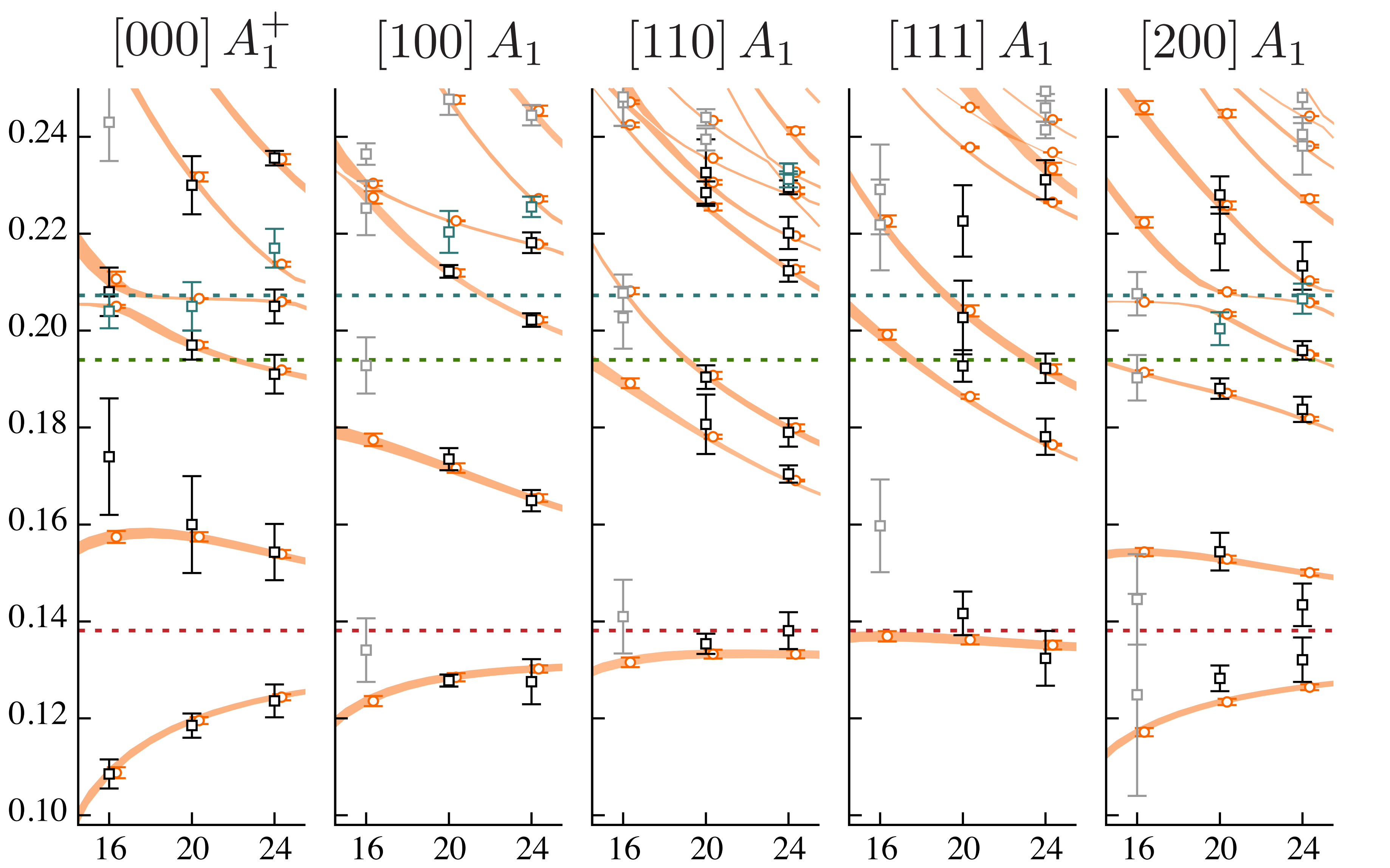
$$= \text{“smooth”} + A \frac{i}{F^{-1} + \mathcal{M}} B^\dagger \sim \sum_n A_n \frac{i R_n}{E - E_n} B_n^\dagger$$

poles satisfy: $\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

These ideas in practice

Arguably the most advanced implementations of this are currently in the meson sector, where it is increasingly common to extract 30-100 energy levels



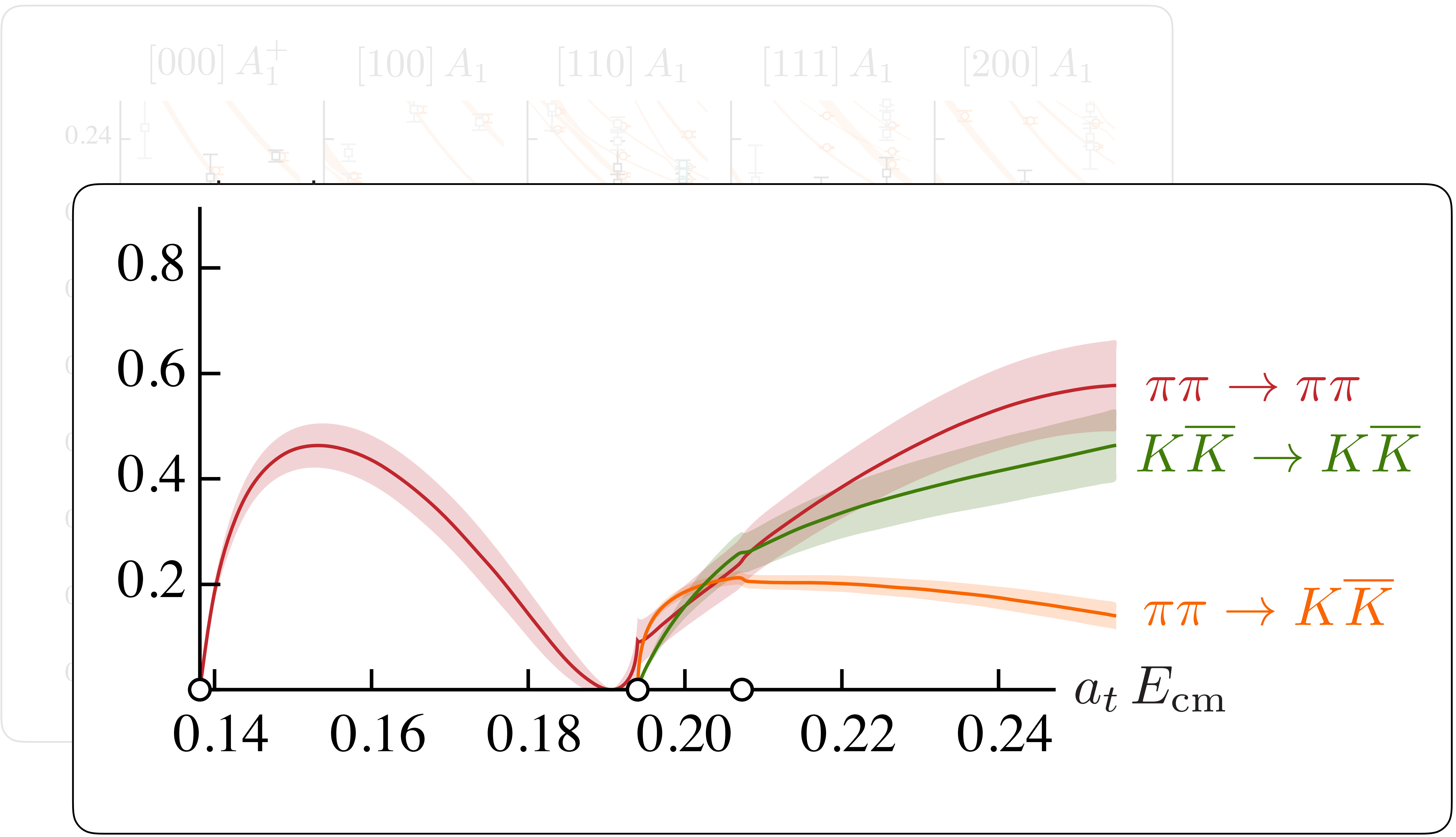
$$\chi^2/N_{\rm dof} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

RB, Dudek, Edwards & Wilson (2017)

These ideas in practice

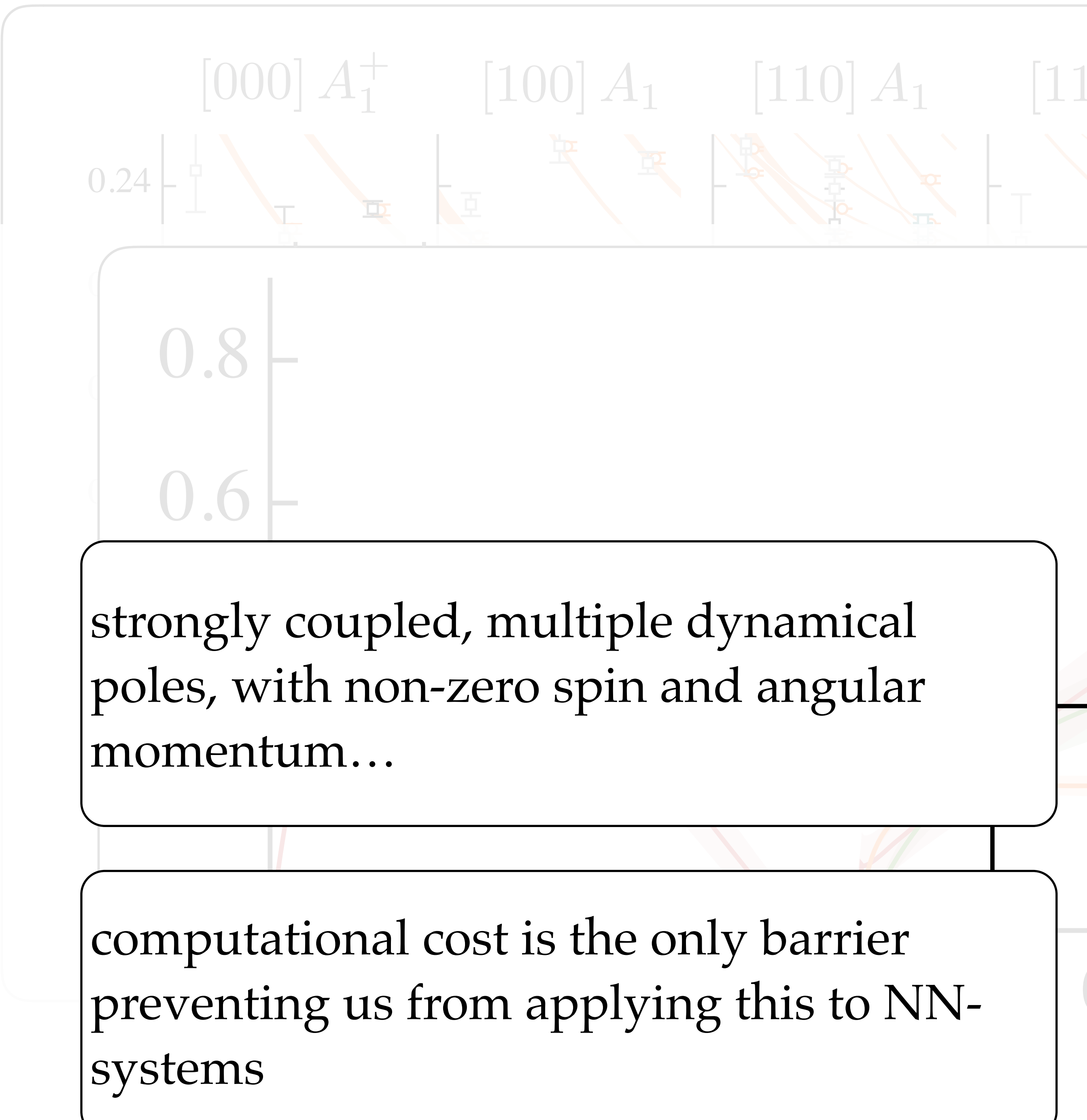
Arguably the most advanced implementations of this are currently in the meson sector, where it is increasingly common to extract 30-100 energy levels



RB, Dudek, Edwards & Wilson (2017)

These ideas in practice

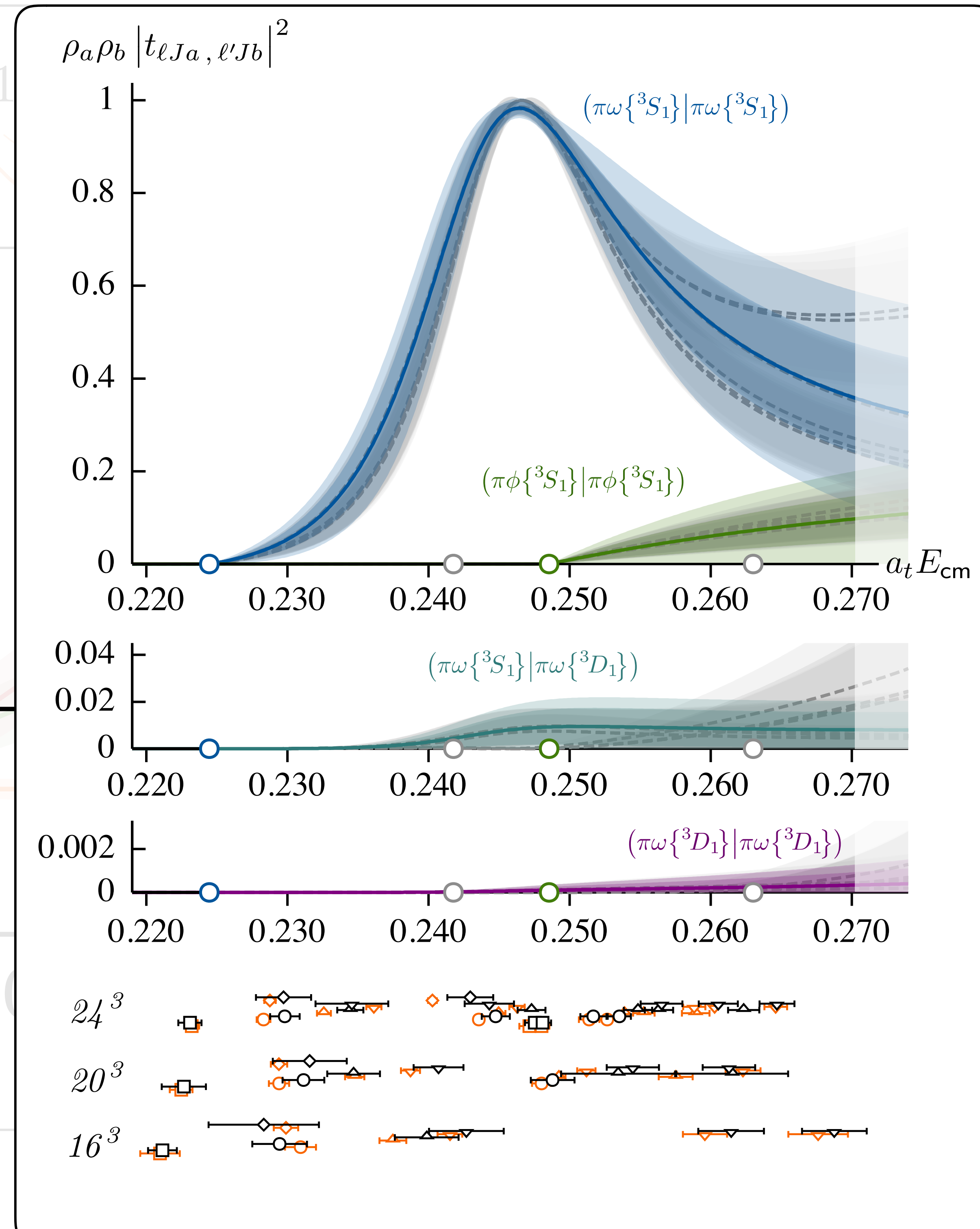
Arguably the most advanced implementations of this are currently in the meson sector, where it is increasingly common to extract 30-100 energy levels



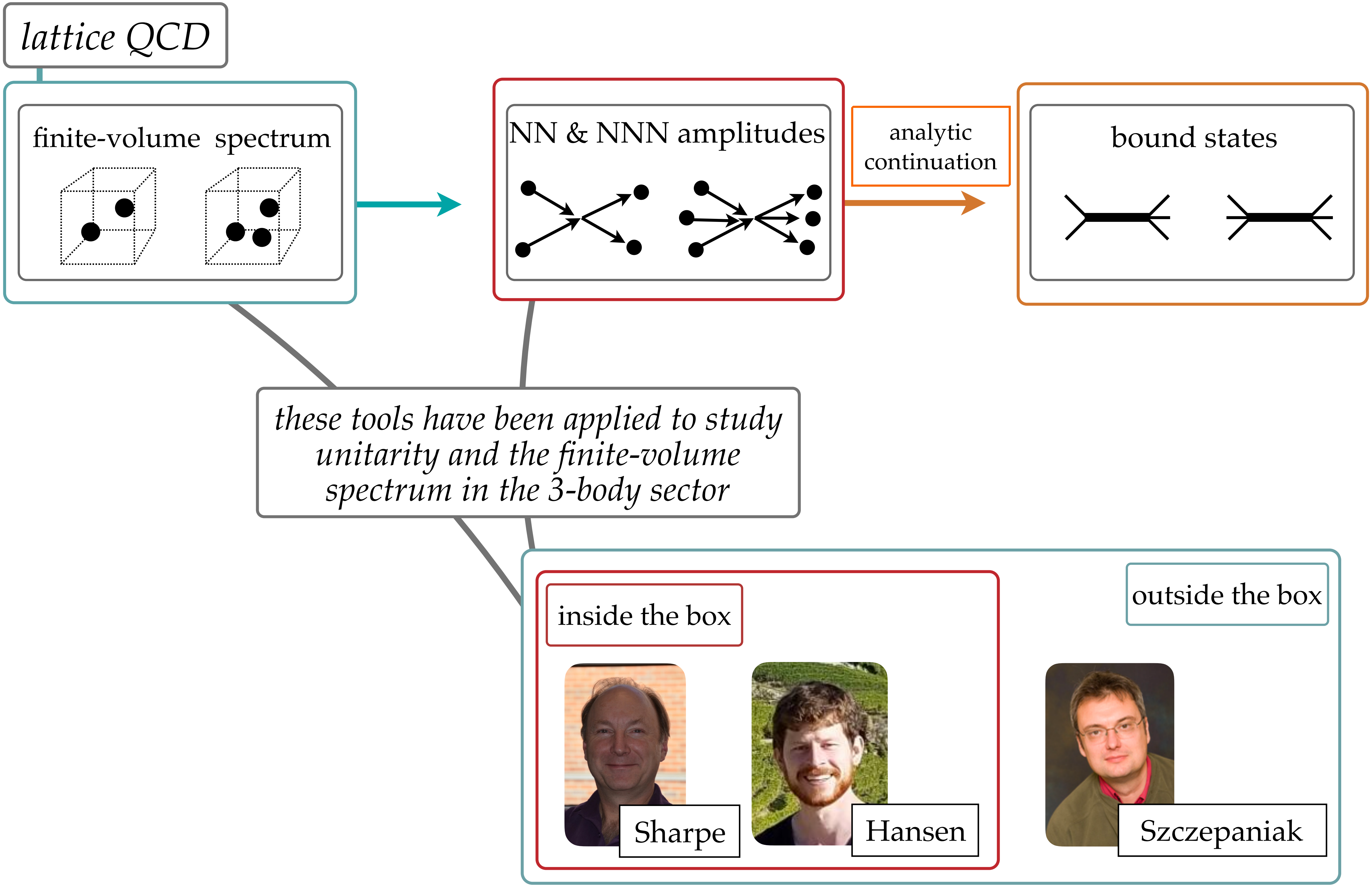
strongly coupled, multiple dynamical poles, with non-zero spin and angular momentum…

computational cost is the only barrier preventing us from applying this to NN-systems

Woss, Thomas, Dudek, Edwards & Wilson (2019)

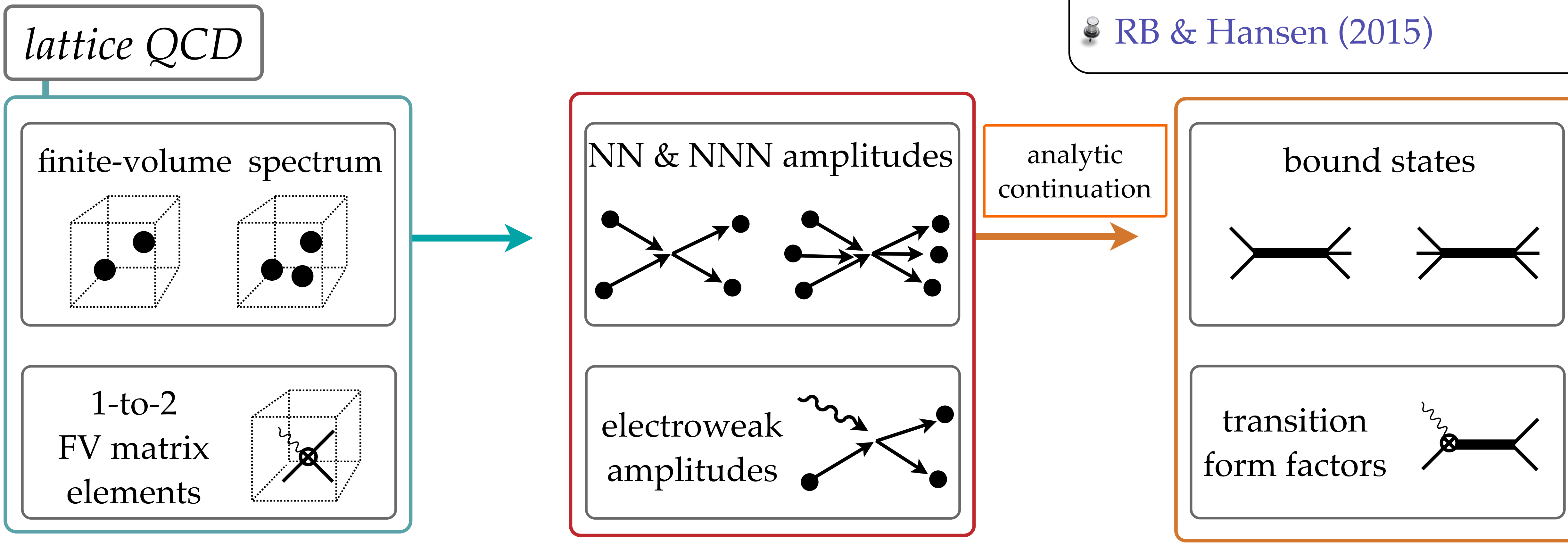


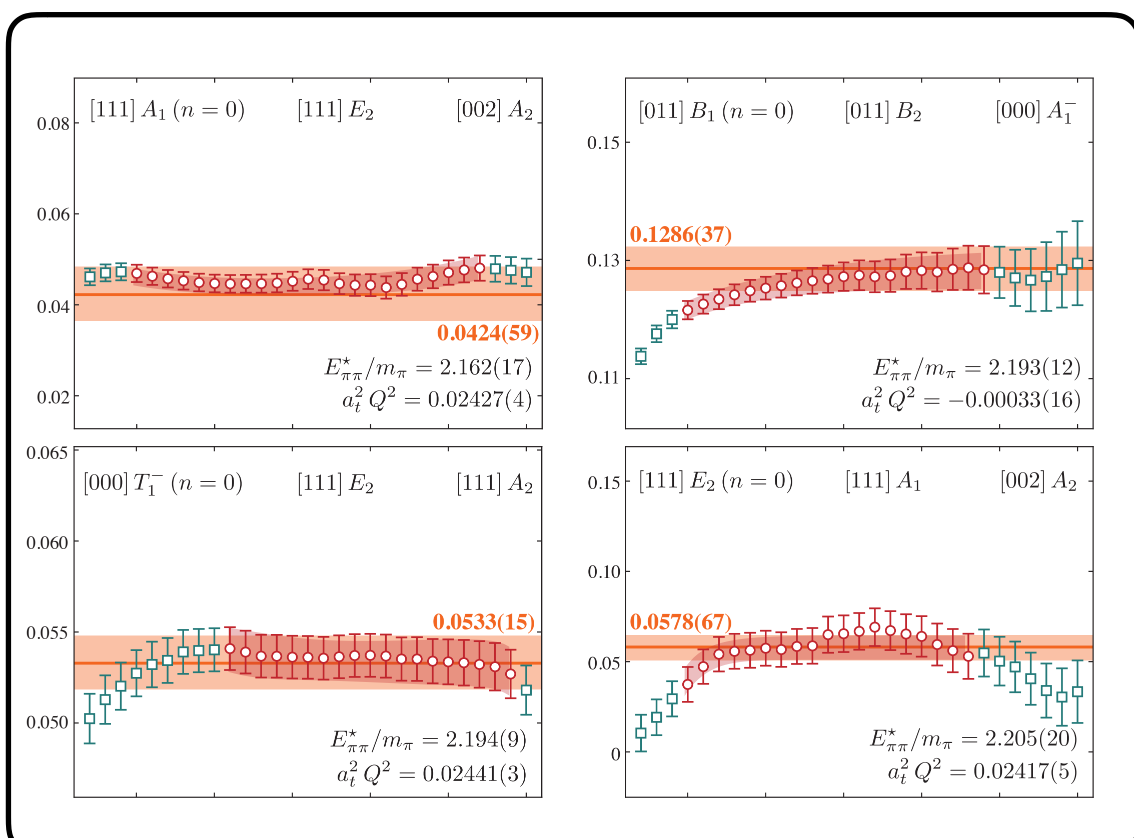
few-nucleons systems in LQCD

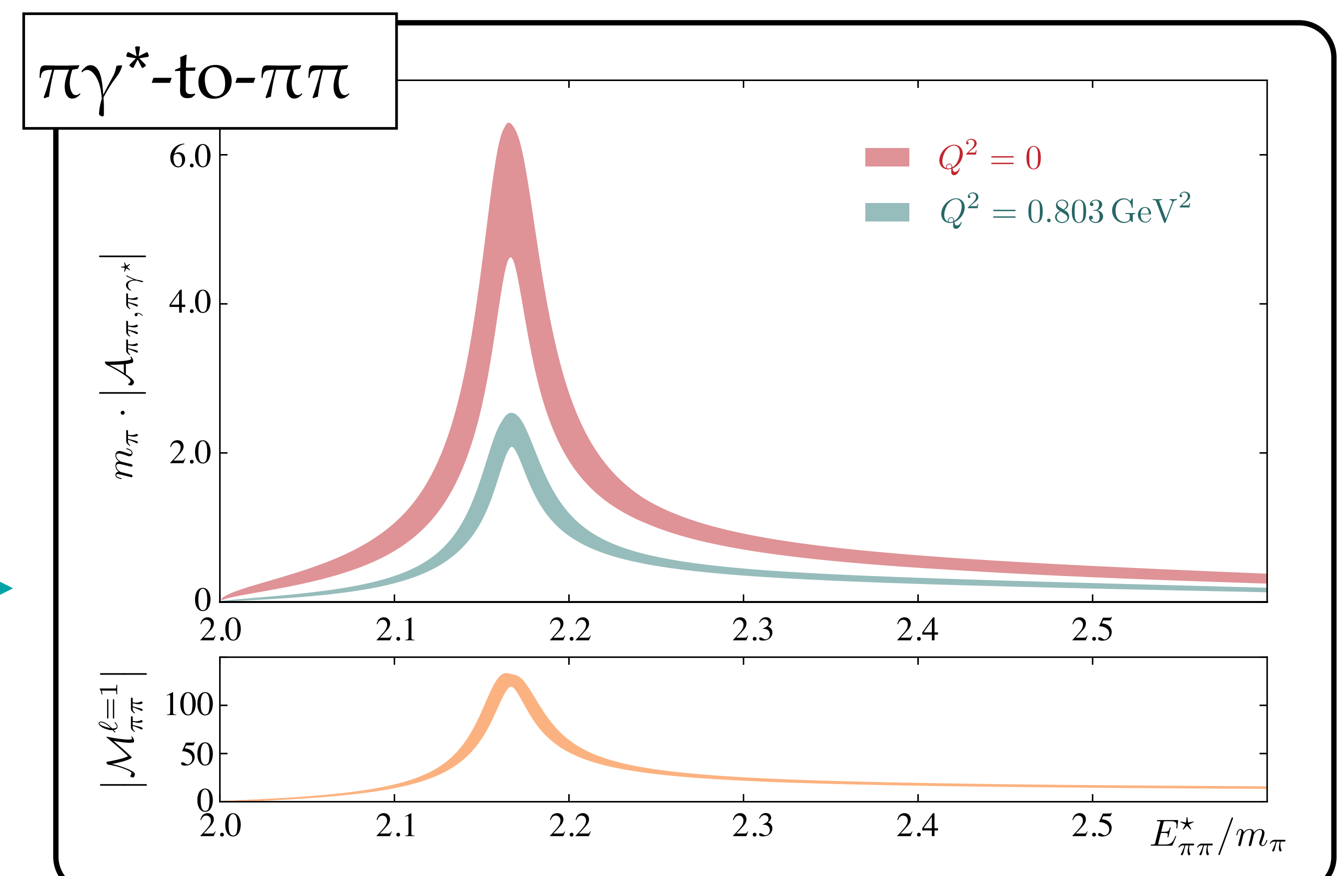


few-nucleons systems in LQCD

- RB, Hansen & Walker-Loud (2014)
- RB & Hansen (2015)







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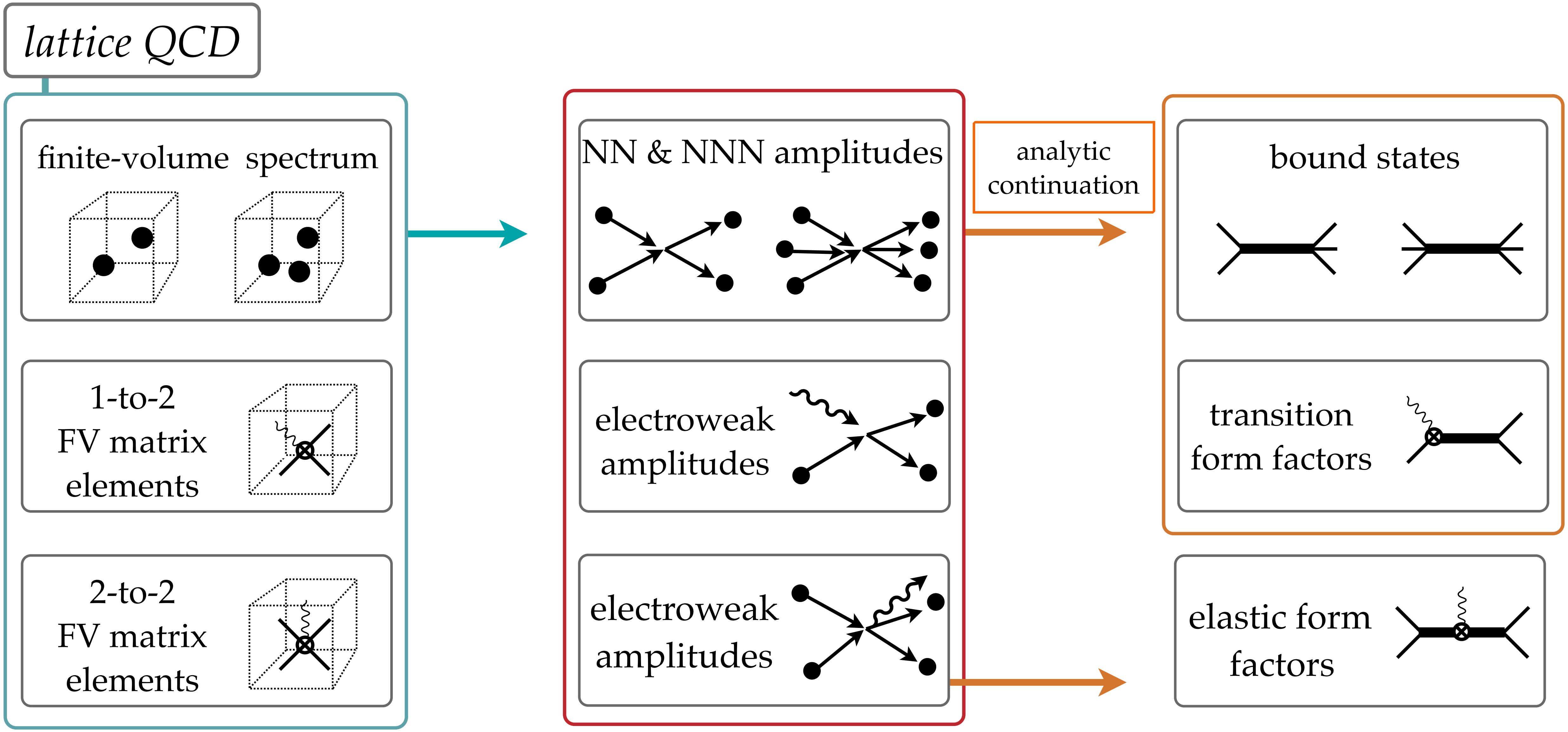
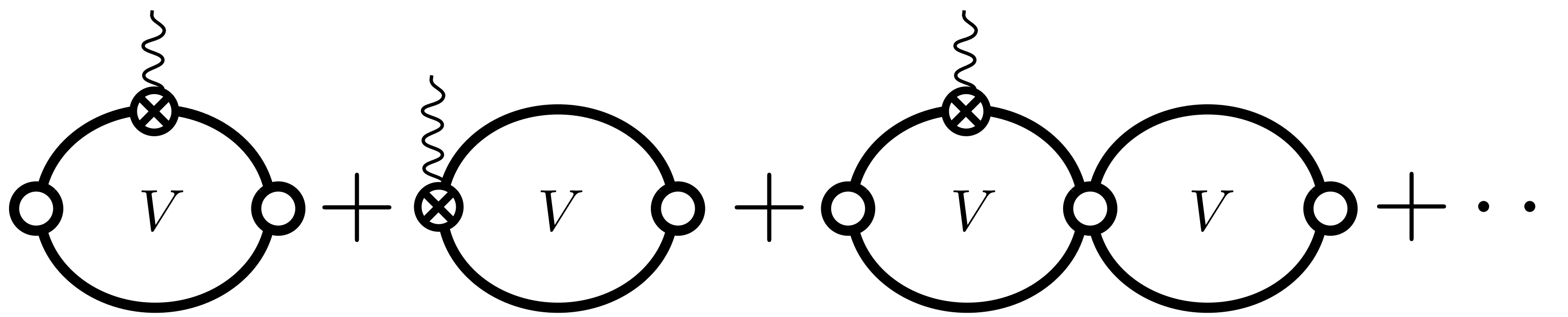
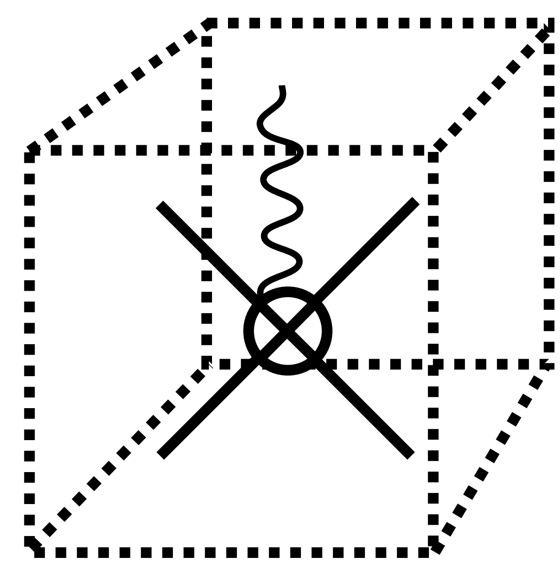


- RB & Hansen (2015)
- Baroni, RB, Hansen, Ortega (2018)

Two-particle in finite volume **with current**

Same as before…but with a current

$C_L^{3pt.} =$ 



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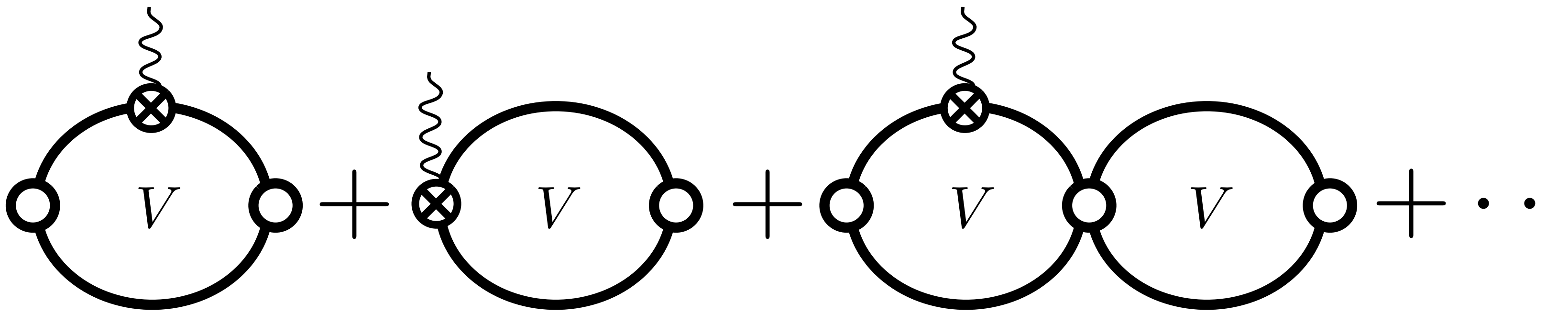
$C_L^{3pt.} =$ 

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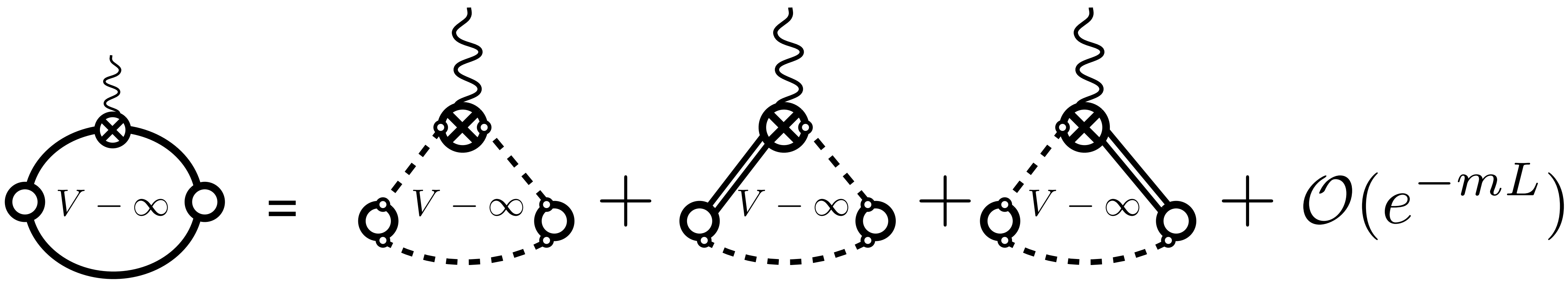

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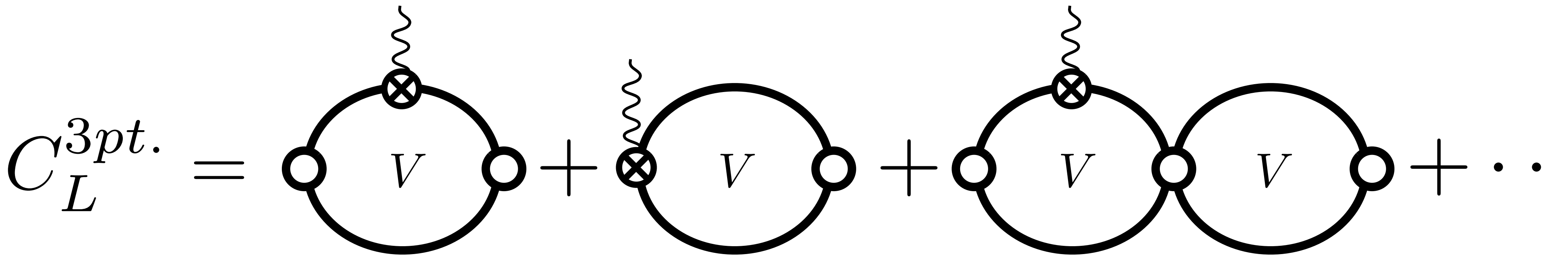

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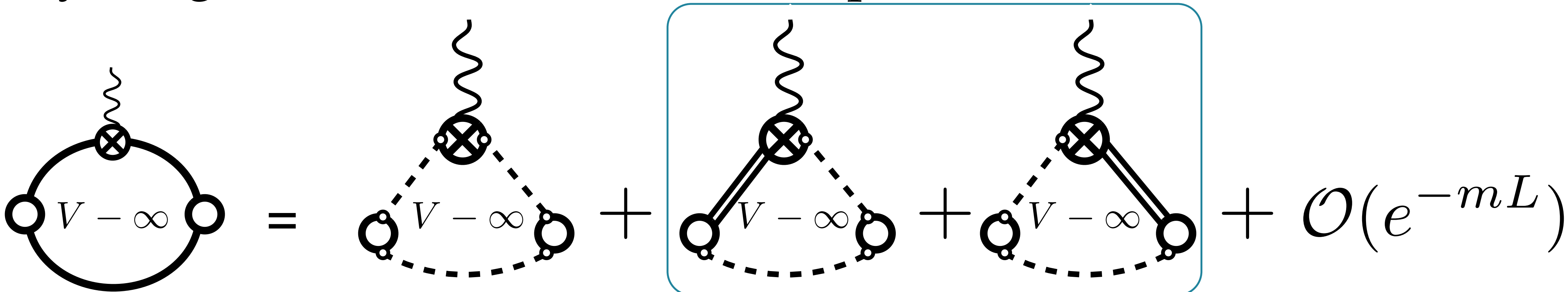

leads to the presence of F-functions…

not too surprising that W_{df} emerges…

Two-particle in finite volume **with current**

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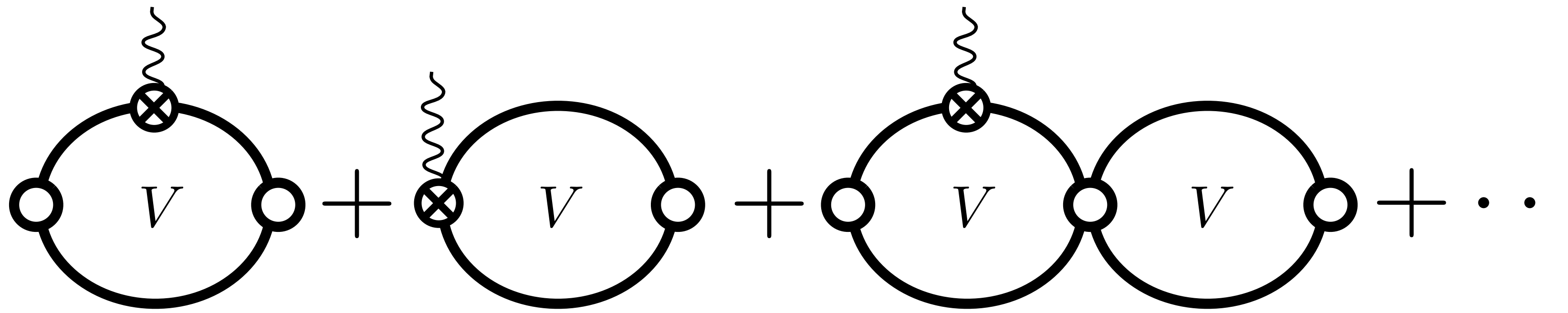

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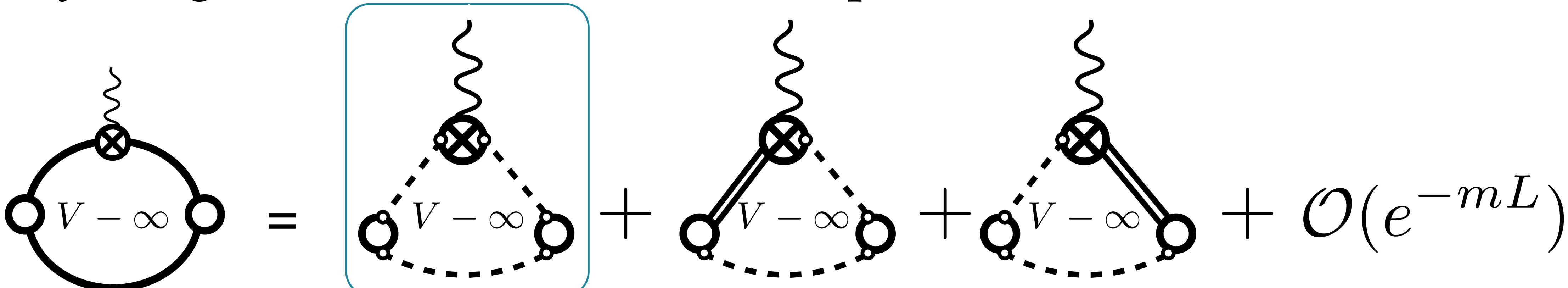
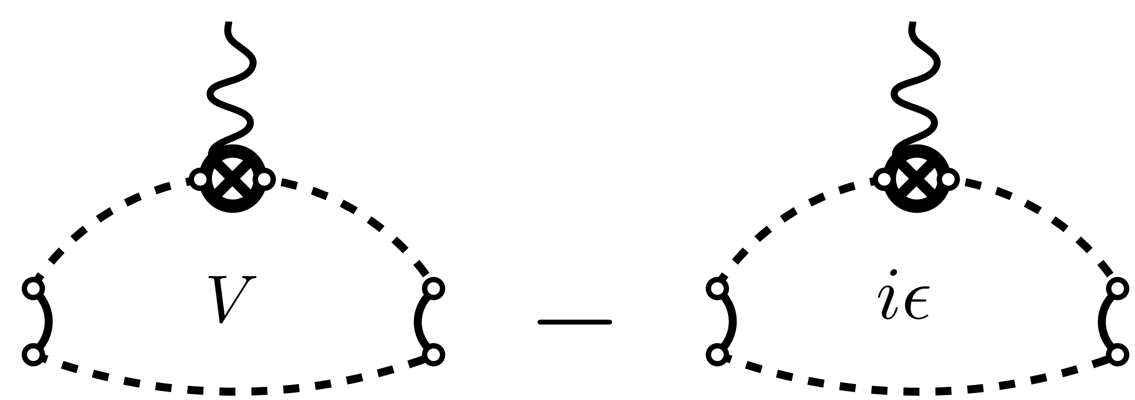


New finite-volume function:

$G_{\mu_1\cdots\mu_n}(P_f, P_i, L) =$



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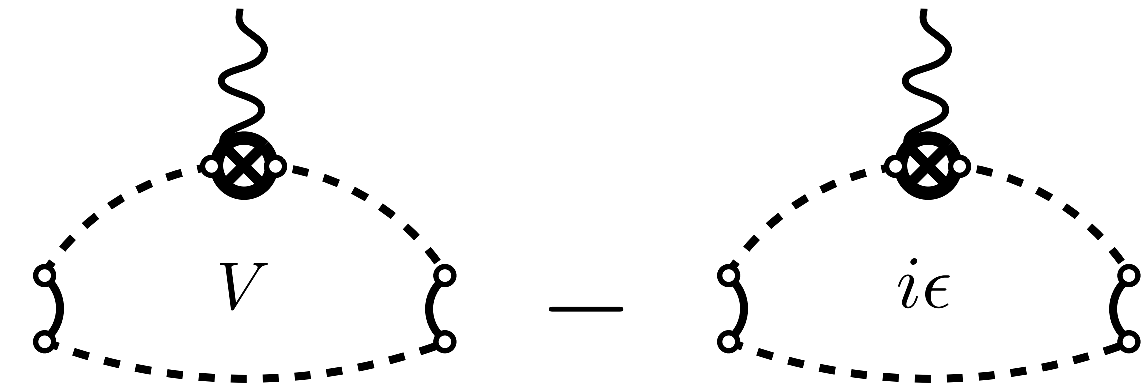

$C_L^{3pt.}$

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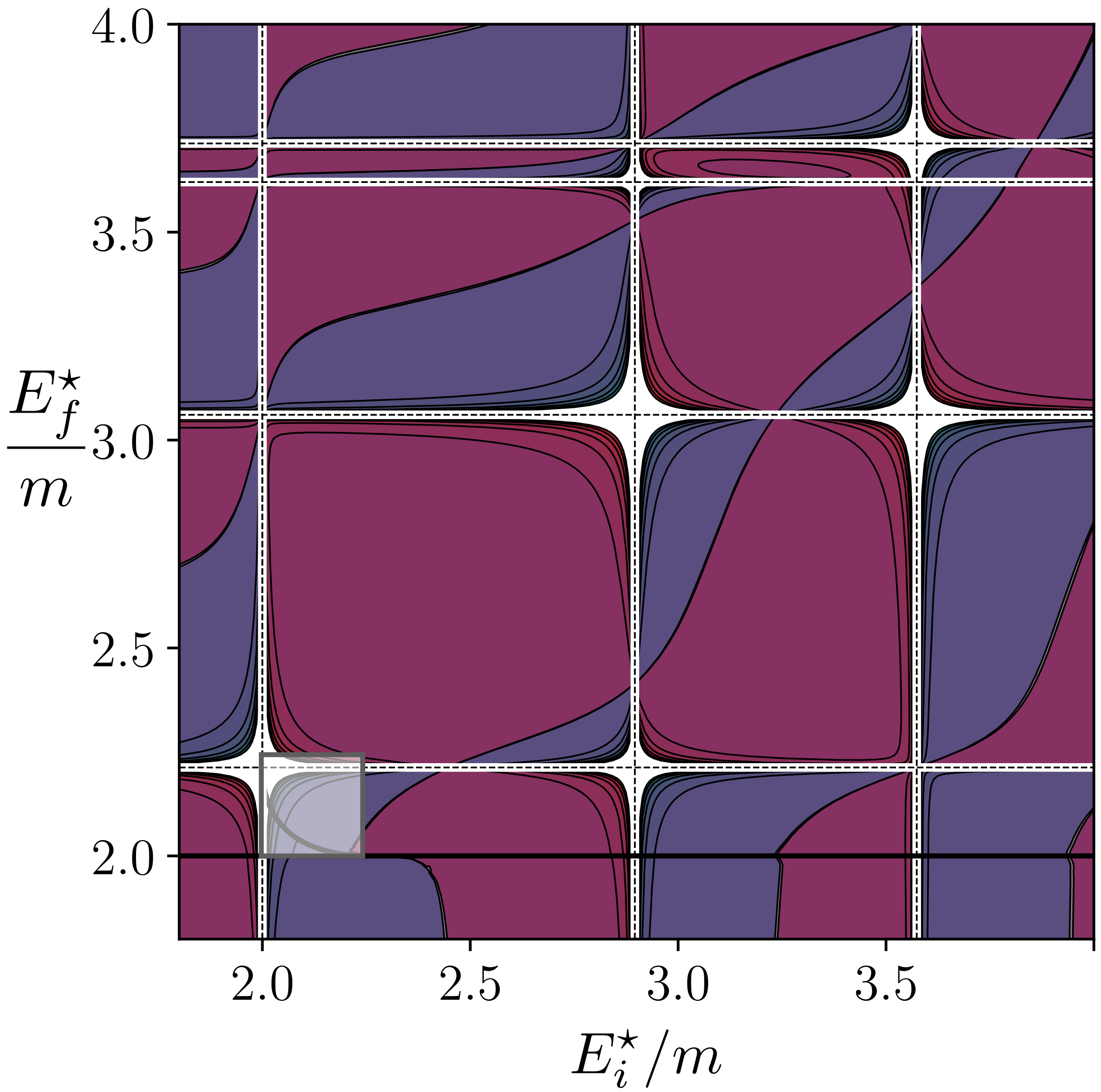


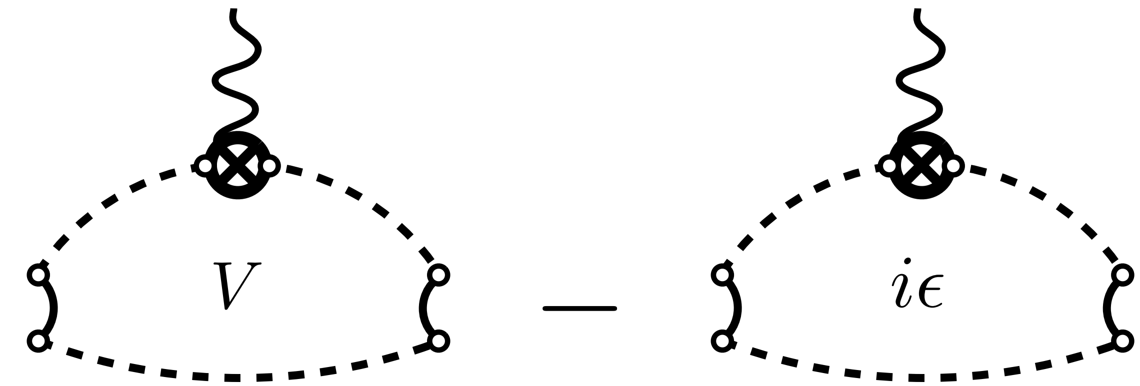
free-particle FV poles

Two-particle in finite volume **with current**

Same as before…but with a current

New finite-volume function:

$$G_{\mu_1 \cdots \mu_n}(P_f, P_i, L) =$$



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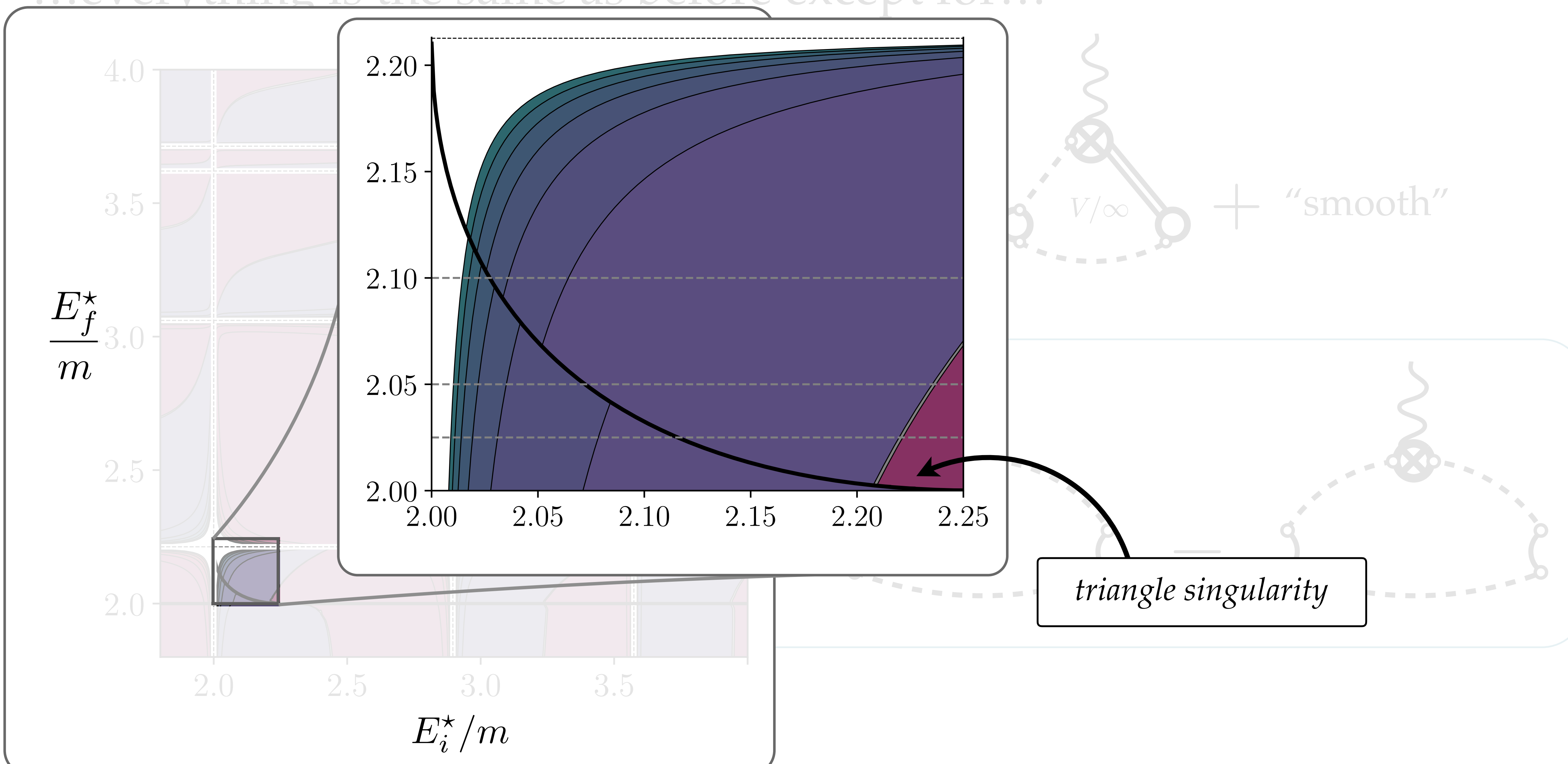

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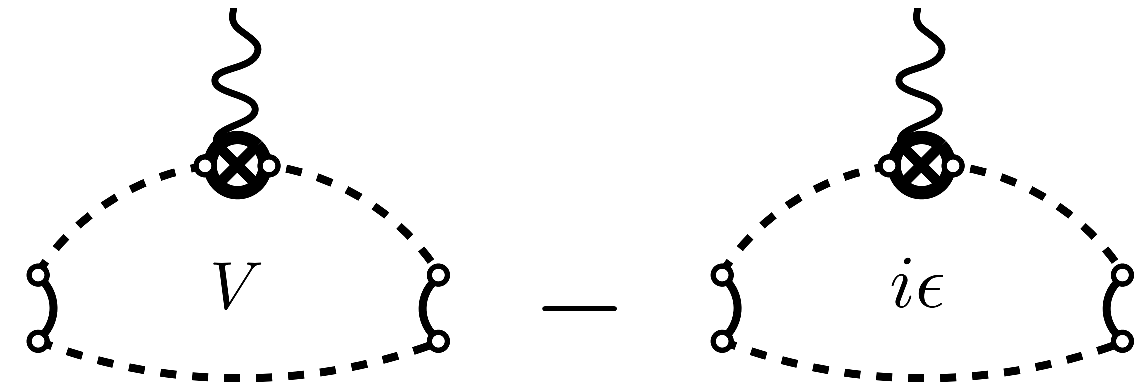
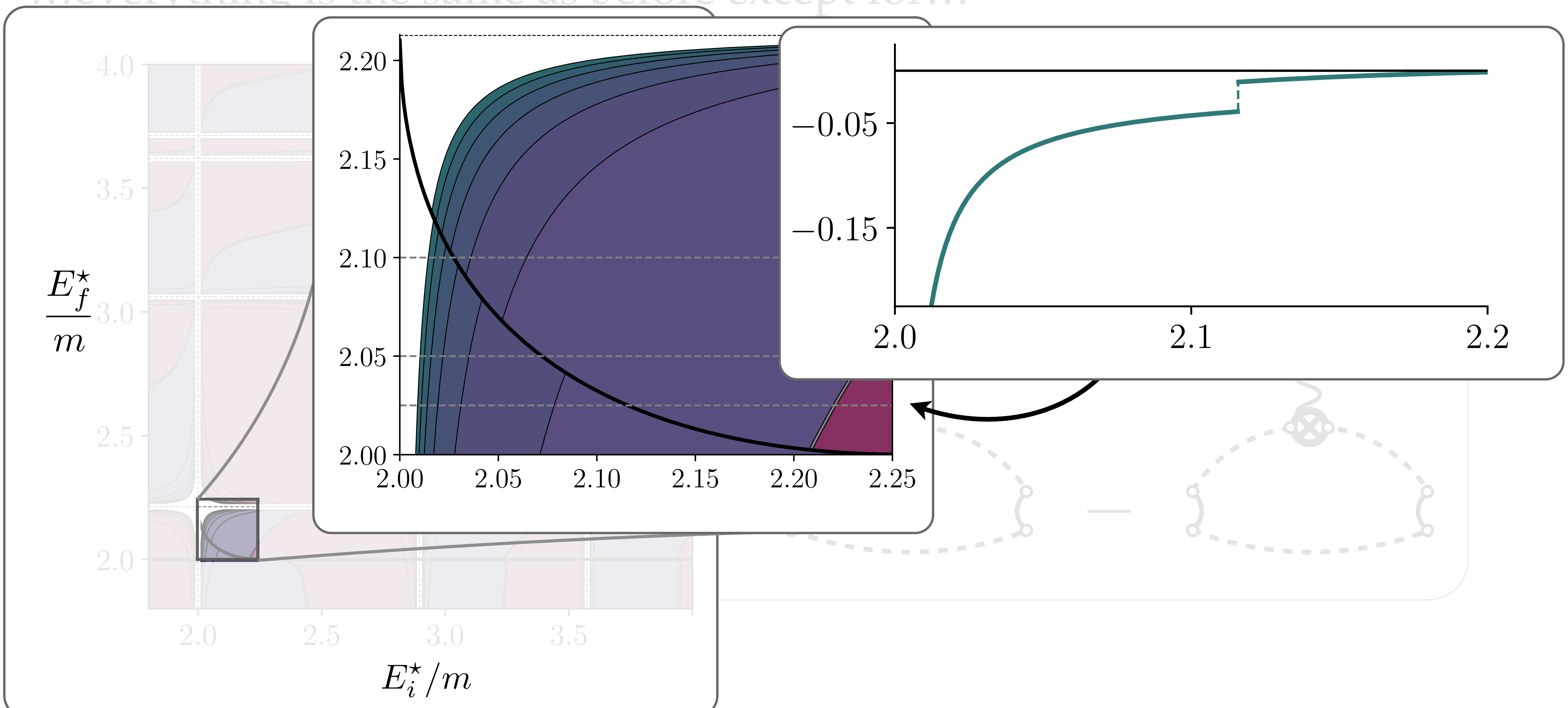




Two-particle in finite volume **with current**

After lots of massaging…

$$L^3 \langle 2|\mathcal{J}|2\rangle_L = \mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}$$

Building block #1) Lellouch-Lüscher matrices:

$$\mathcal{R}(E_n, \mathbf{P}) \equiv \lim_{E \to E_n} \left[\frac{(E - E_n)}{F^{-1}(P, L) + \mathcal{M}(P)} \right]$$

derivatives of amplitudes and F-function at the finite-volume spectra

Two-particle in finite volume **with current**

After lots of massaging…

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R}\, \mathcal{W}_{L,\mathrm{df}}$$

Building block #2) stable particle form factor

$$\mathcal{W}_{L,\mathrm{df}} - \mathcal{W}_{\mathrm{df}}^{\mu_1 \cdots \mu_n} \sim \sum_{n'}^{n} \mathcal{M}\ \left[G^{(j)}\ f^{(j)}(-q^2) \right]\ \mathcal{M}$$

form factors of single-particle states

Two-particle in finite volume **with current**

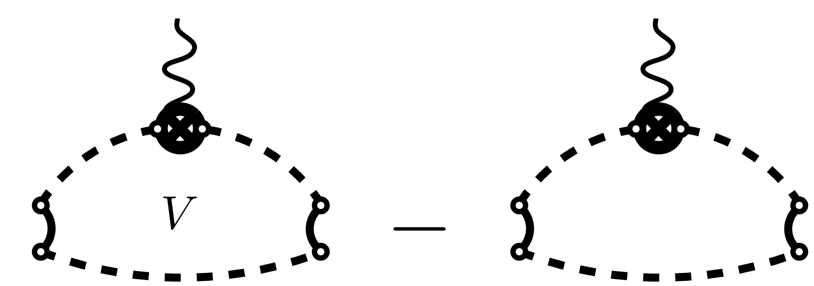
After lots of massaging…

$$L^3 \langle 2|\mathcal{J}|2\rangle_L = \mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}$$

Building block #3) G-function

$$\mathcal{W}_{L,\mathrm{df}} - \mathcal{W}_{\mathrm{df}}^{\mu_1\cdots\mu_n} \sim \sum_{n'}^{n} \mathcal{M}\,\left[G^{(j)}\, f^{(j)}(-q^2)\right]\mathcal{M}$$

$$G_{\mu_1\cdots\mu_n}(P_f, P_i, L) =$$



known geometric function, which "adds missing singularities by hand"

Checks: Ward-Takahashi Identity

Ward-Takahashi identity implies (for specific kinematics), the five-point function with $\mathcal{J}^0_{\mathrm{QED}}$ is determined from the four-point function and the QED charge

$$\mathcal{W}^0_{\mathrm{df}} = 2P^0\, Q \frac{\mathrm{d}}{\mathrm{d}s} \mathcal{M}$$

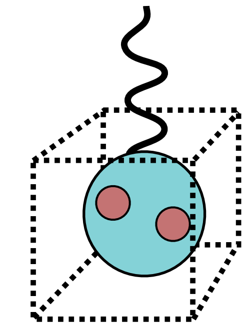
It is not obvious, but this implies

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R}\, \mathcal{W}_{L,\mathrm{df}} = \frac{Q}{2E}$$

i.e., the QED charge is protected, even for unbound states

Jackura (IU)

Checks: Bound-state limit



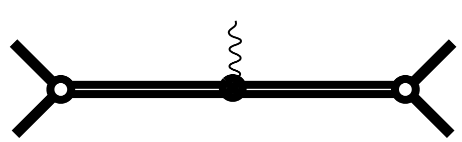
Intuitively, in the limit that our state is bound, all these effects must vanish *exponentially fast*.

Well known for the spectrum:
$$E_L = E_B + \mathcal{O}(e^{-\kappa L})$$

Also applies for the matrix elements:

$$L^3 \langle 2|\mathcal{J}|2\rangle_L = \mathcal{R}\,\mathcal{W}_{L,\mathrm{df}} = \frac{f_B(Q^2)}{2\sqrt{E_i E_f}} + \mathcal{O}(e^{-\kappa L})$$

remember, this is doubly singular

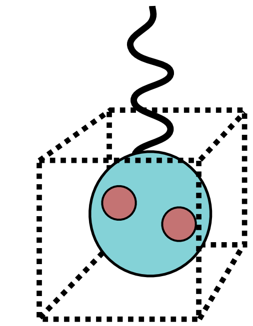


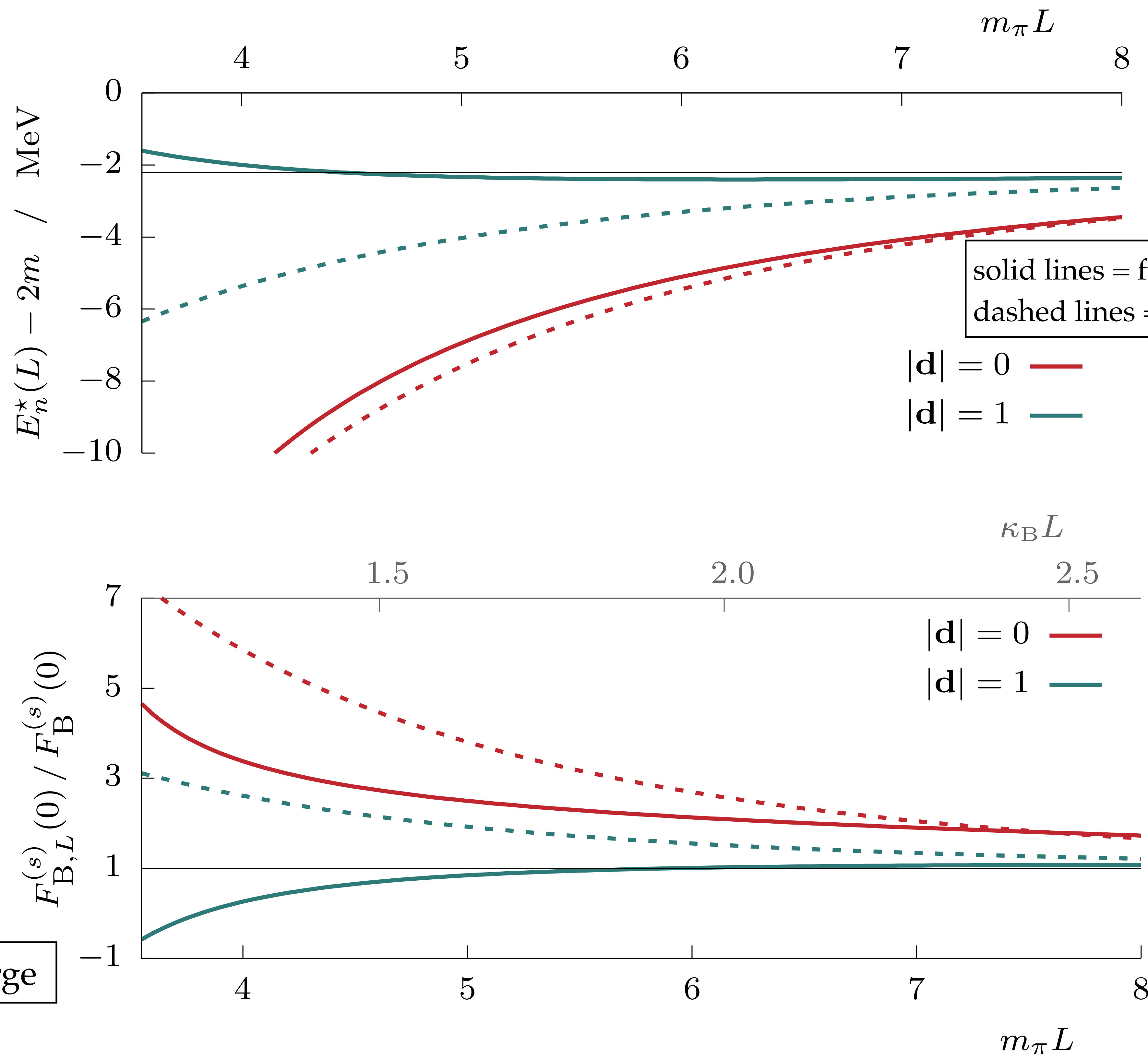
$$\sim \frac{g}{s_i - s_B} F_R(Q^2) \frac{g}{s_f - s_B}$$

Jackura (IU)

Checks: Implications for the physical deuteron

The deuteron is loosely bound, so these effects will be big



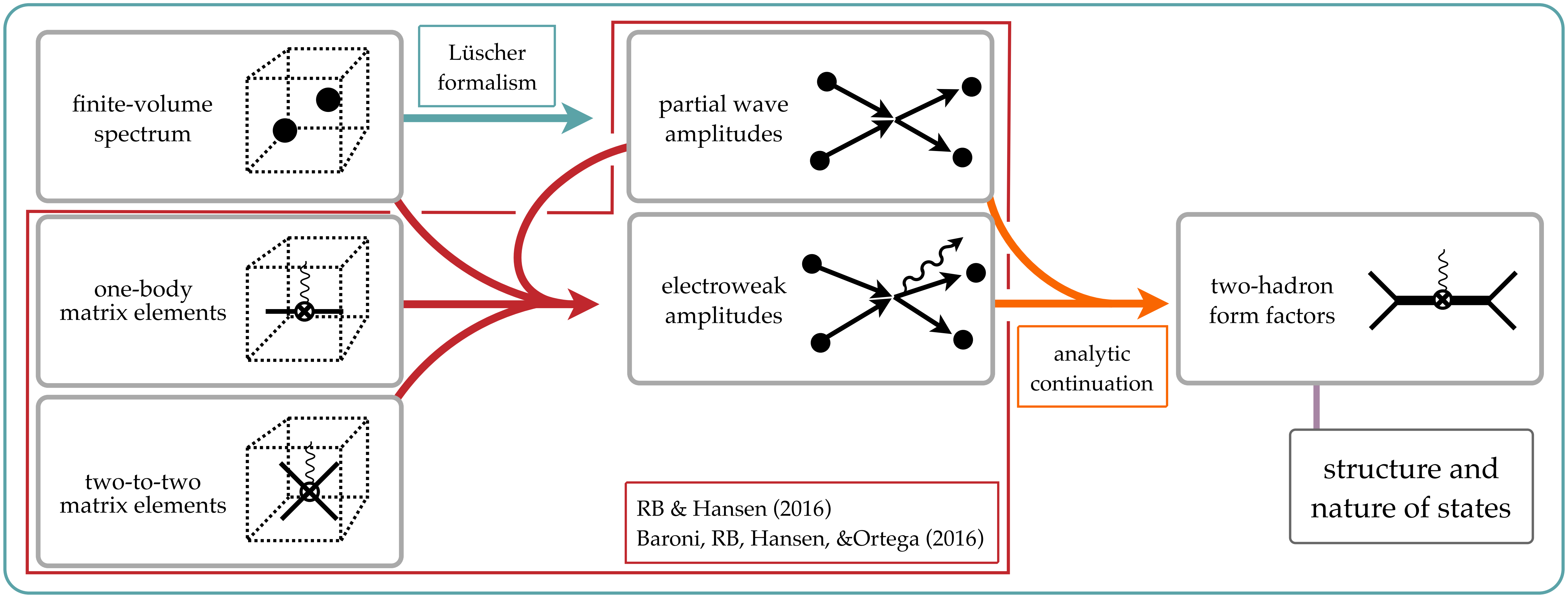


solid lines = full answer
dashed lines = keeping only $\mathcal{O}(e^{-\kappa_B L})$

scalar charge

Jackura (IU)

Take-home message: it *can be done*



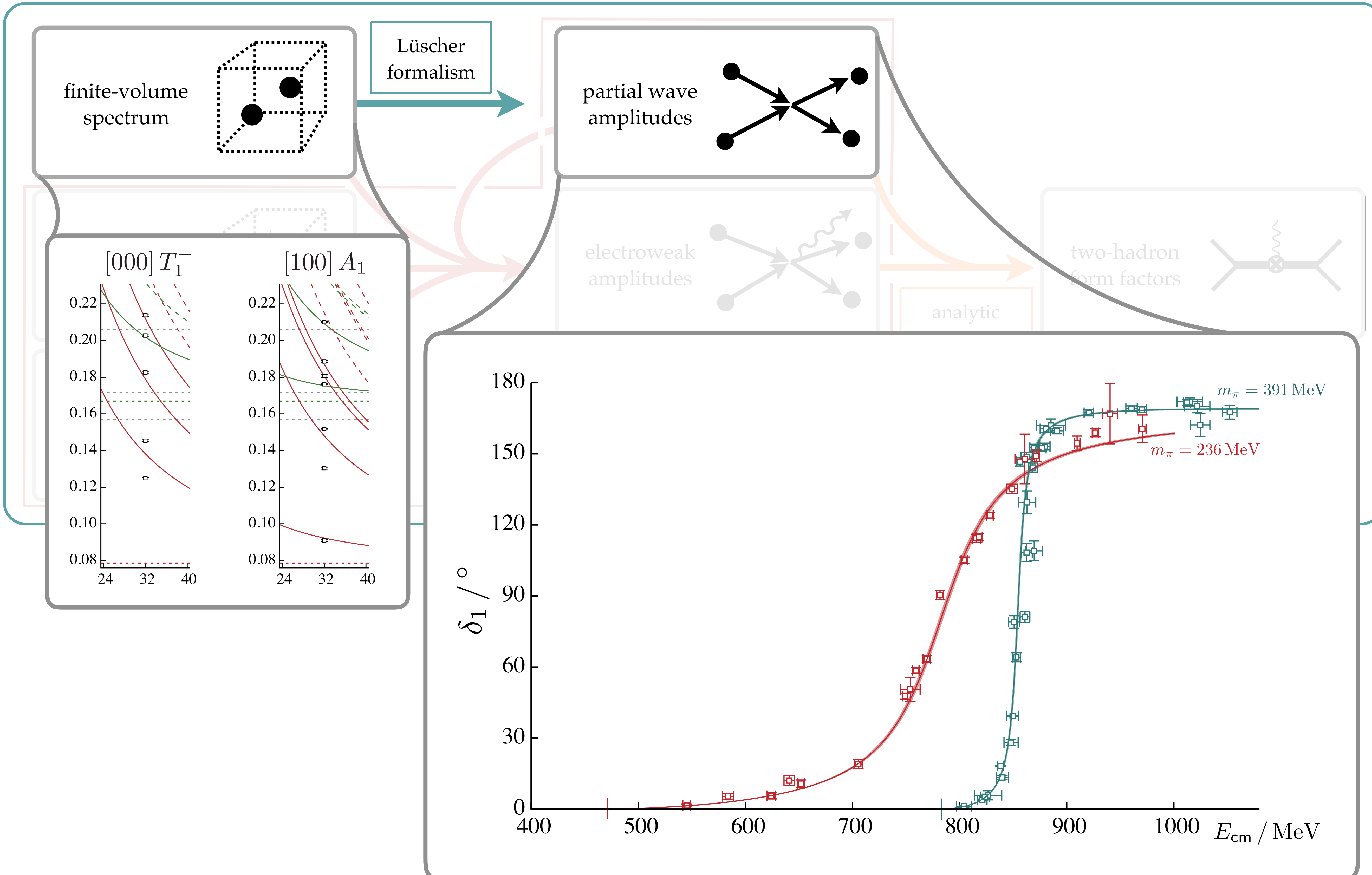
Multiple birds with one stone:

Access:

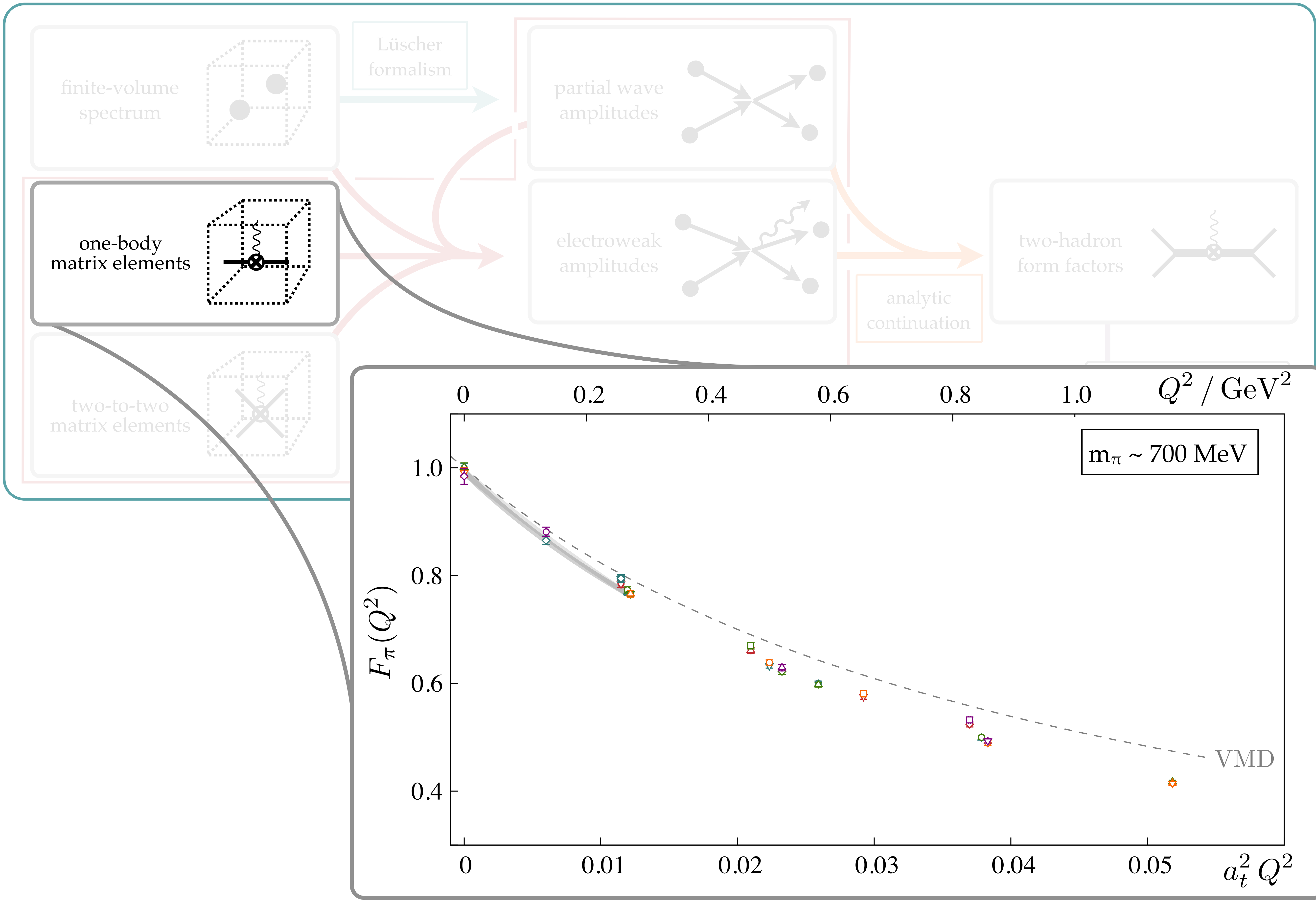
- transition electroweak amplitudes
- elastic electroweak amplitudes
- structural information composite states:
  - bound states
  - resonance
- remove all finite-volume systematics

$$\langle 2|\mathcal{J}|2\rangle_L \sim \mathcal{R}\cdot[\,\mathcal{W}_{\rm df} + f\,\mathcal{M}\,G\,\mathcal{M}]$$

Take-home message: it *can be done*



Take-home message: it *can be done*



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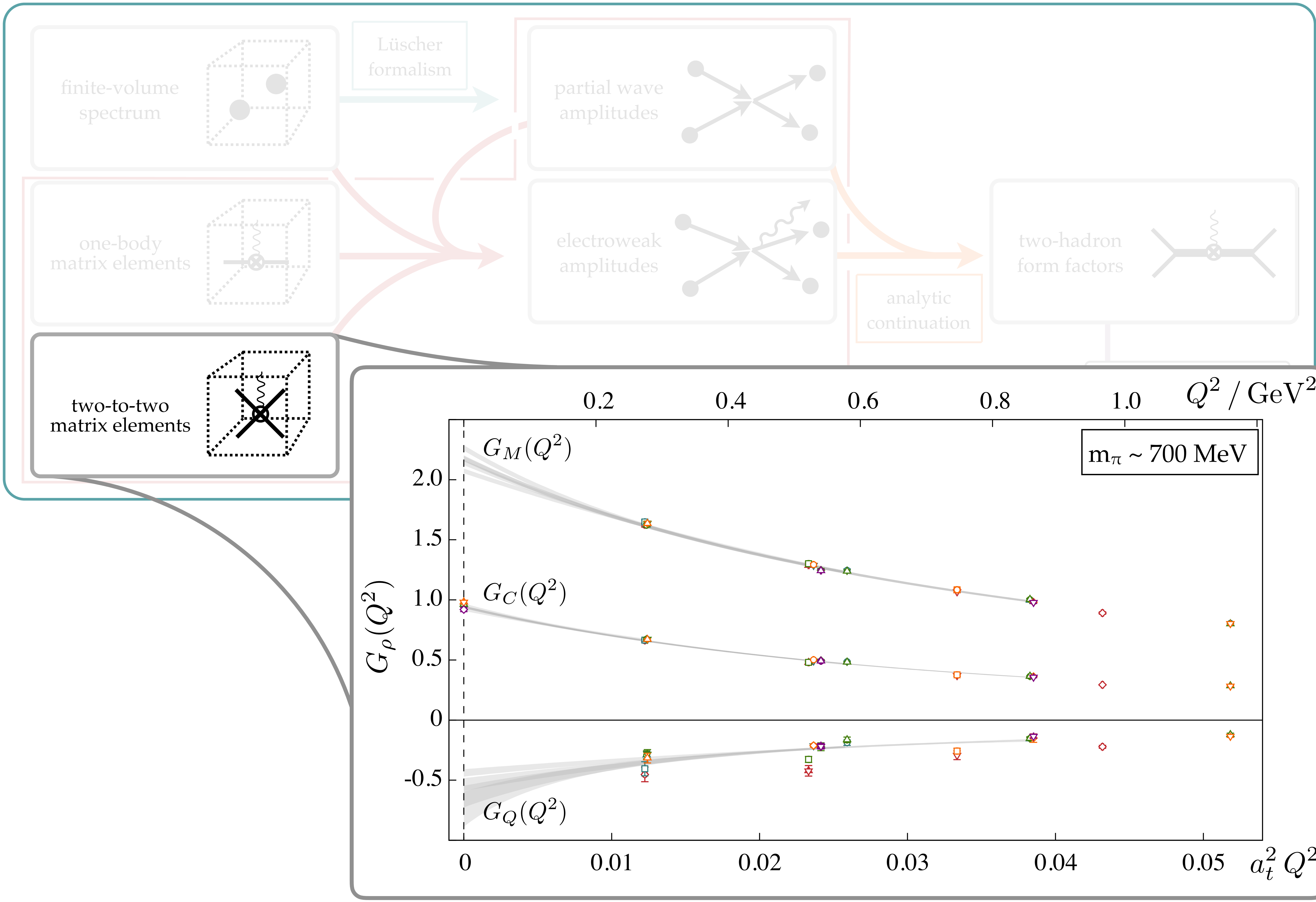
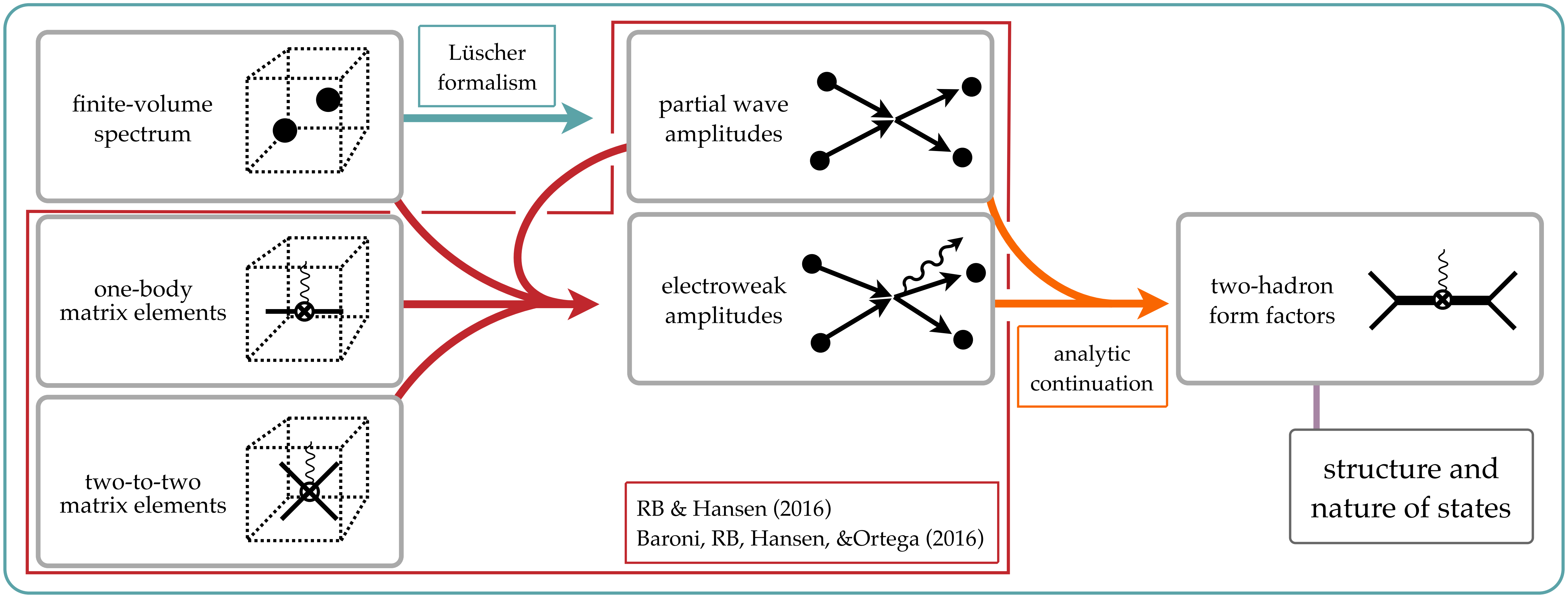
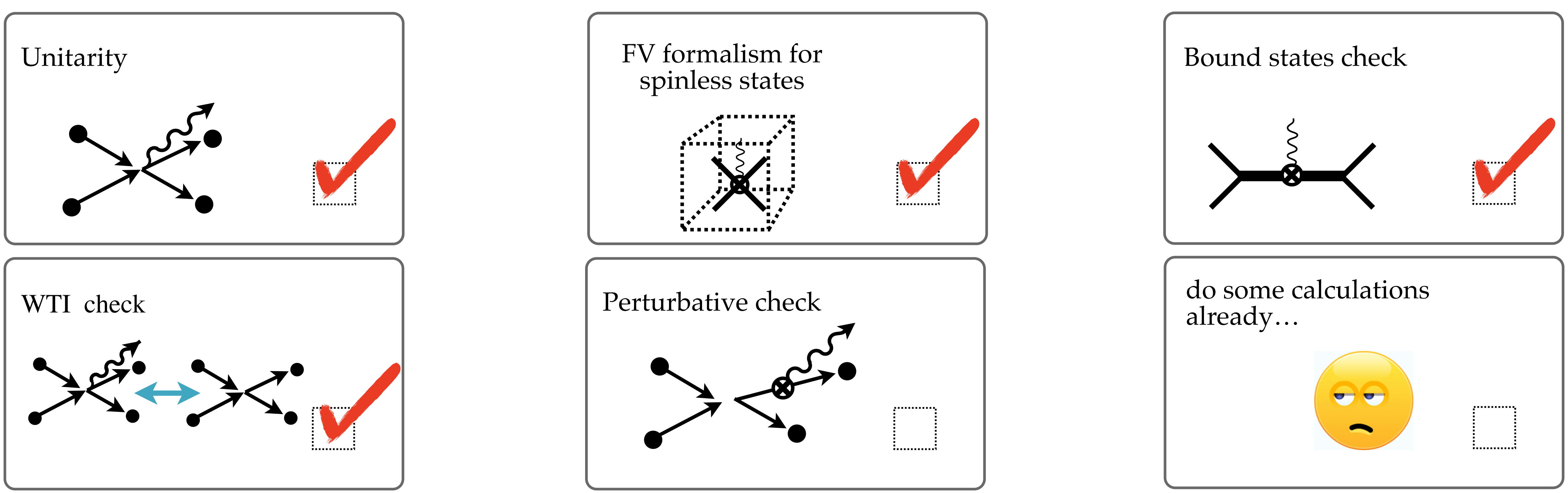
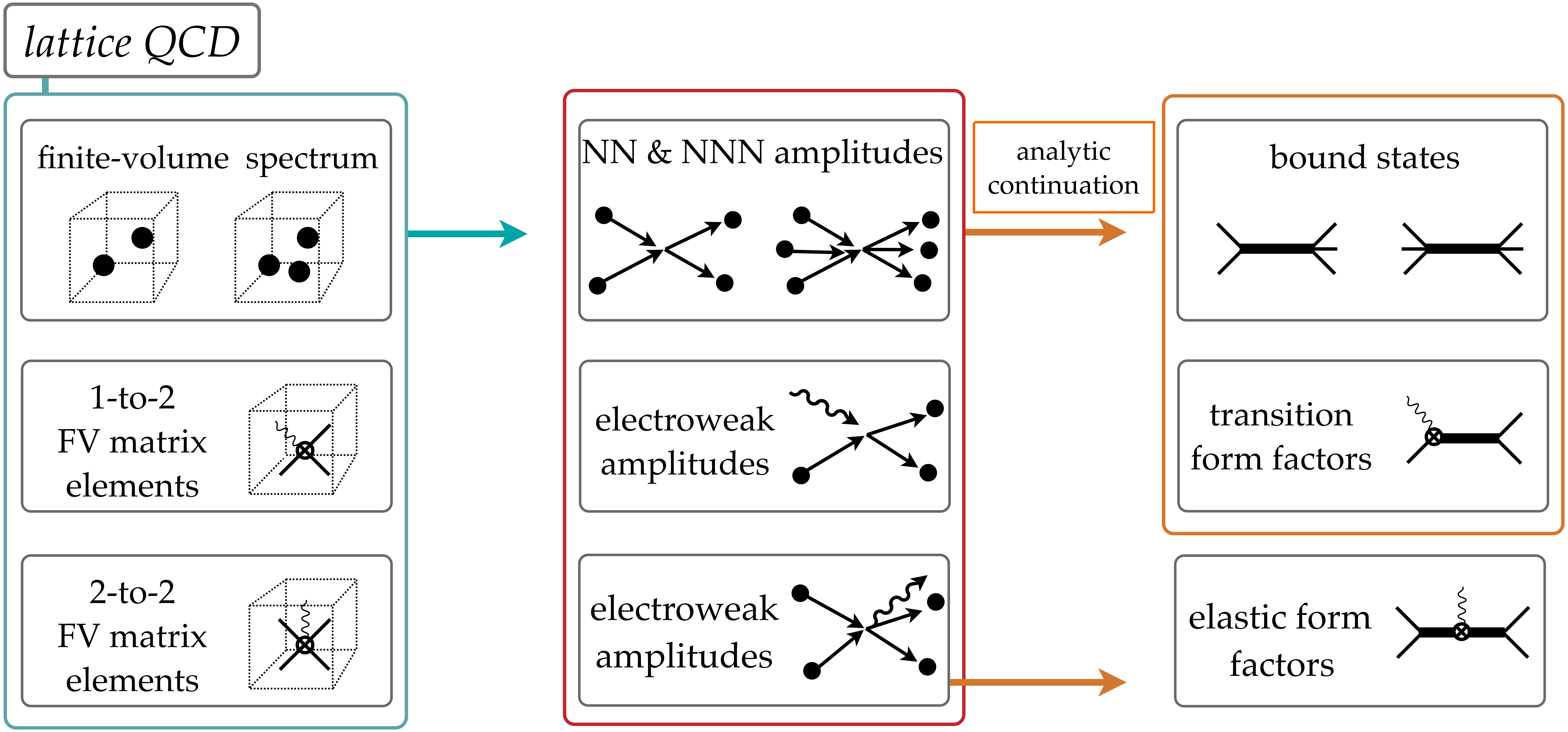


Take-home message: it *can be done*





few-nucleons systems in LQCD



- RB & Hansen (2015)
- Baroni, RB, Hansen, Ortega (2018)

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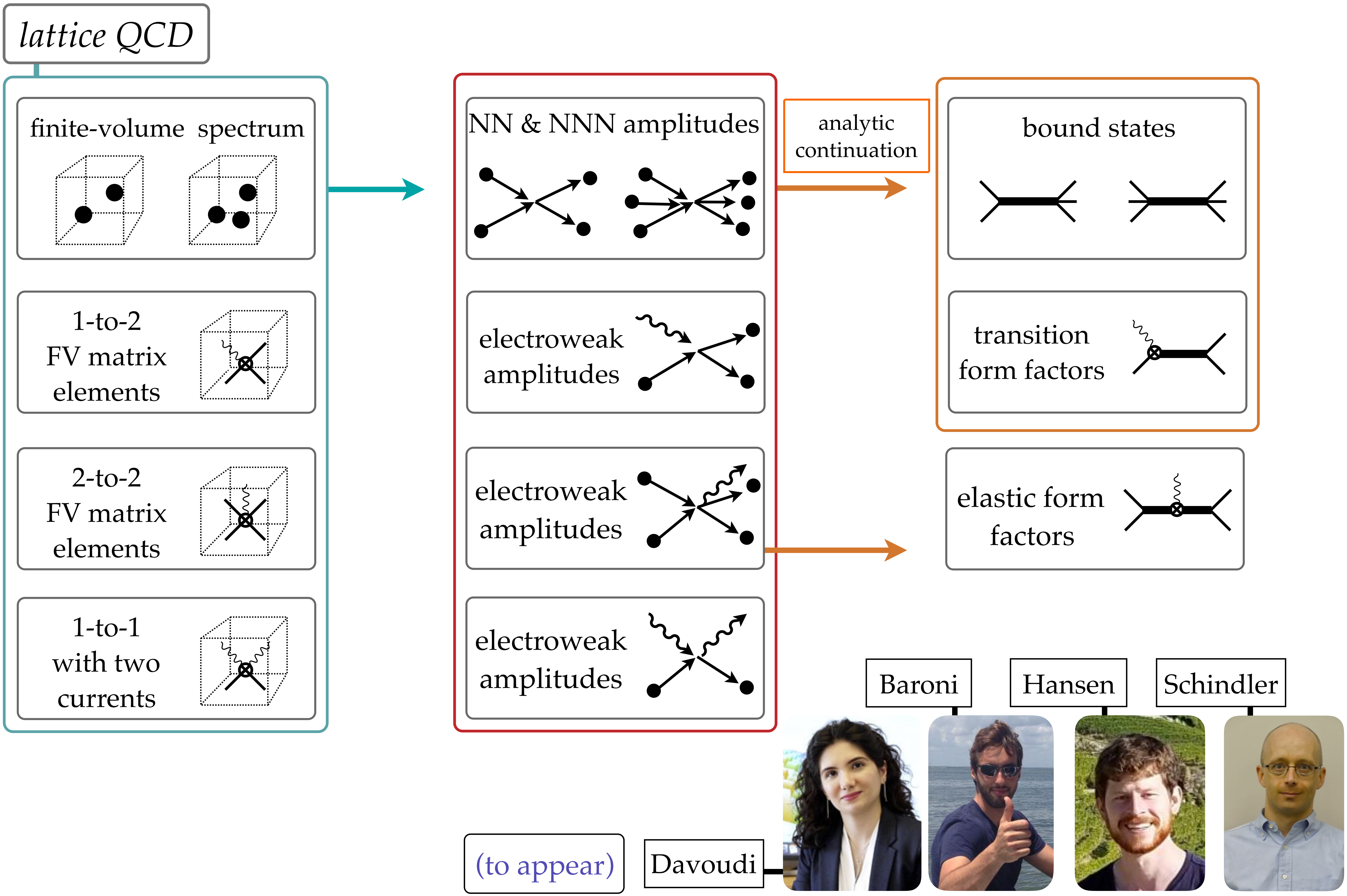
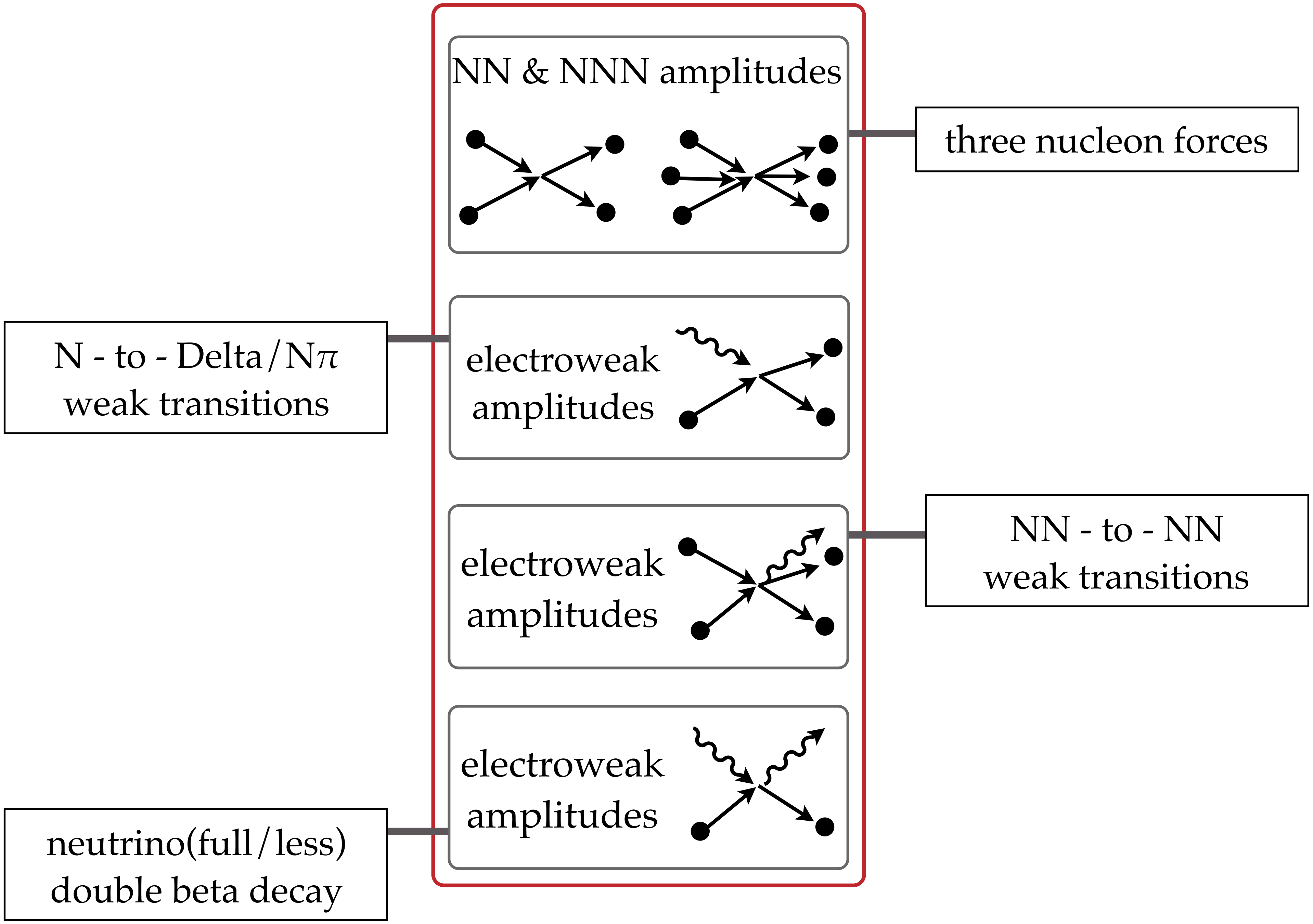


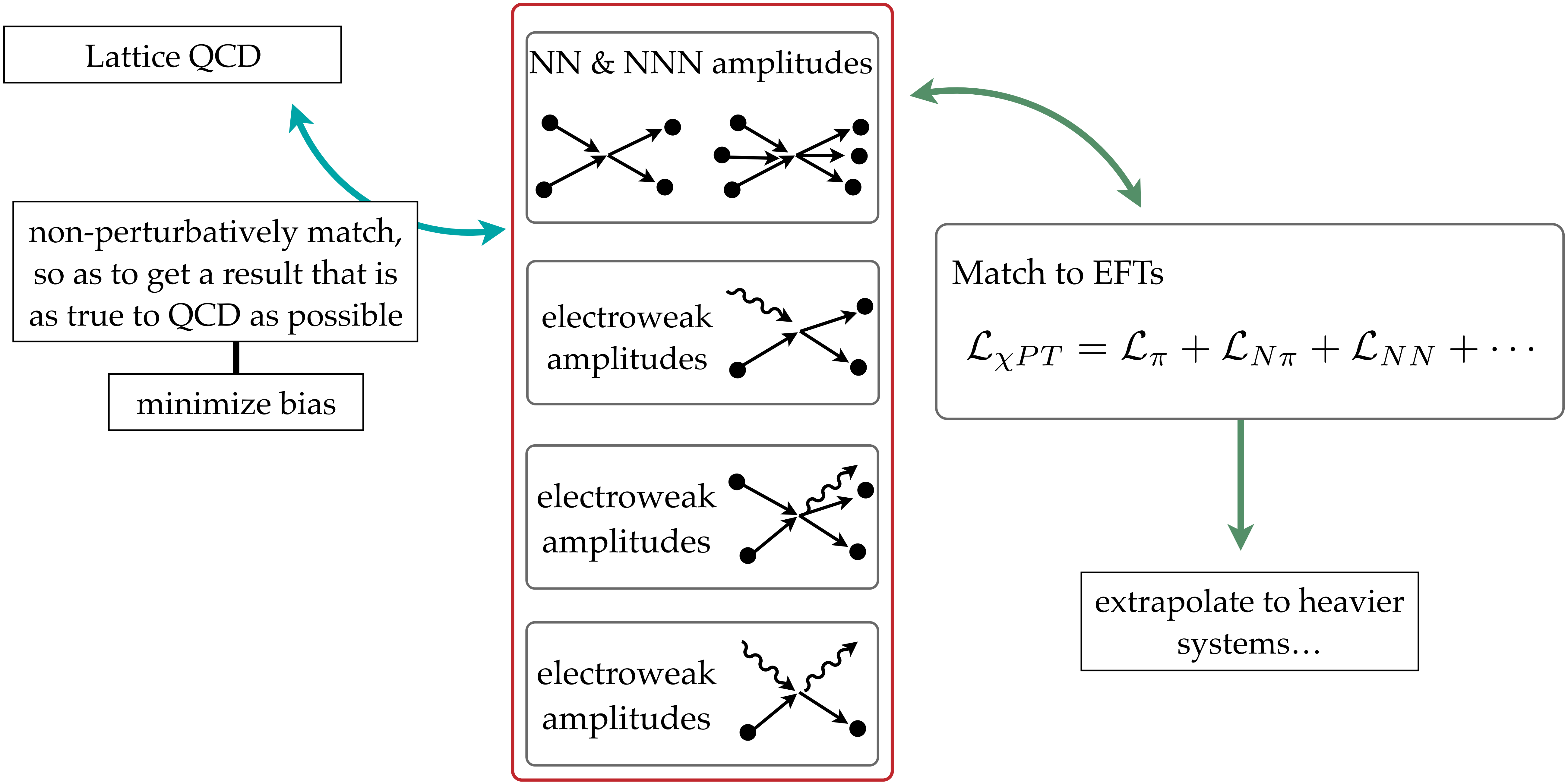
few-nucleons systems in LQCD

These techniques are being tested and implemented for *A=0* systems first, but they are necessary and will be applied for light nuclear systems…



few-nucleons systems in LQCD

Extrapolating to bigger systems



Introduction to the field

Dudek

Young

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Scattering processes and resonances from lattice QCD

Raúl A. Briceño, Jozef J. Dudek, and Ross D. Young

Rev. Mod. Phys. **90**, 025001 – Published 18 April 2018

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ABSTRACT

The vast majority of hadrons observed in nature are not stable under the strong interaction; rather they are resonances whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy nonperturbative region, and in addition many probes of the limits of the electroweak sector of the standard model consider processes which feature hadron