

Two-body matrix elements in LQCD

Raúl Briceño - <http://bit.ly/rbricenoPhD>



Norfolk, VA [Home to ODU]

had spec

ECT*, 2019



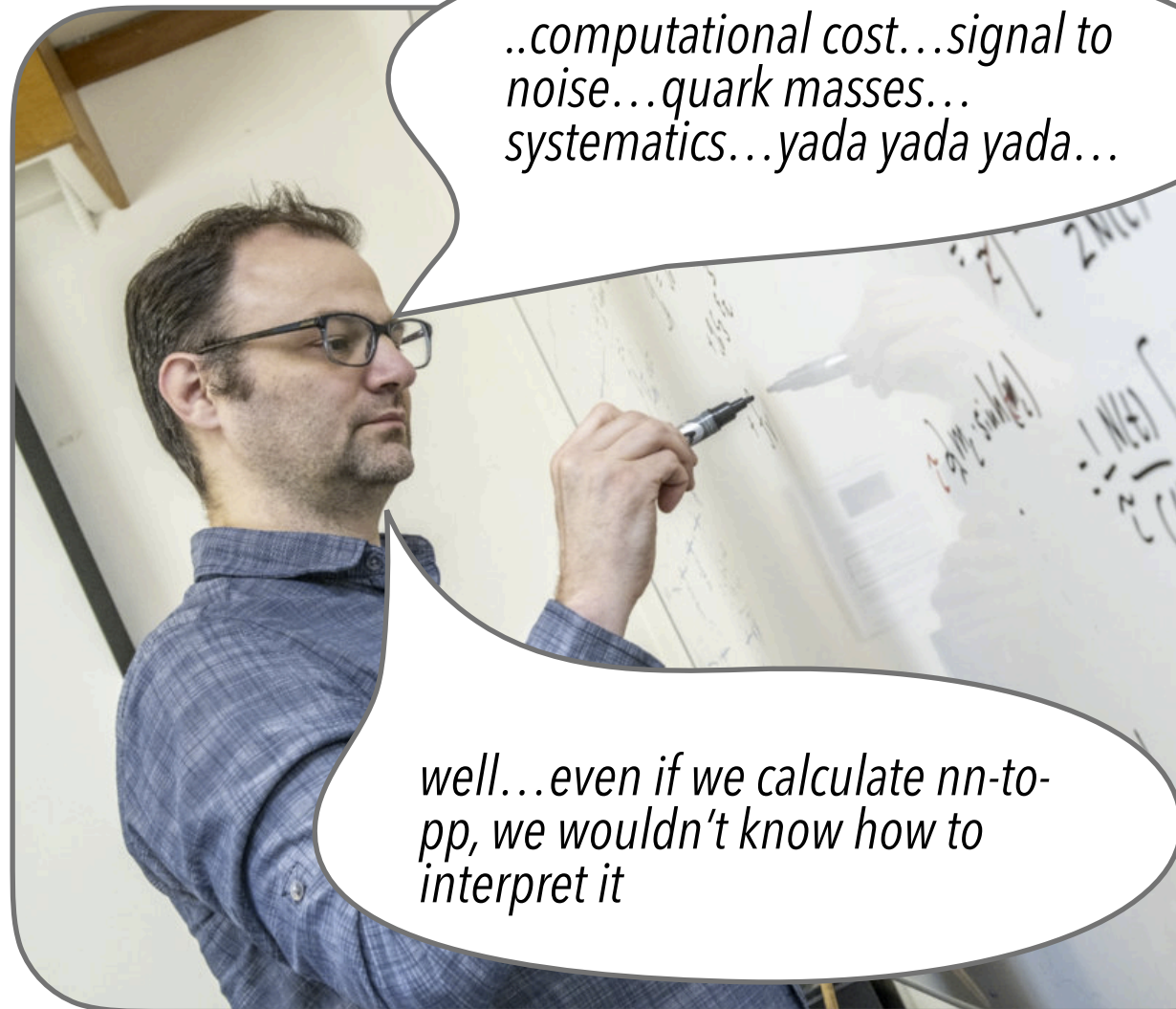
JLab, VA

$0\nu\beta\beta$ from lattice QCD

hey André, why don't you calculate my counter term already?



..computational cost...signal to noise...quark masses...systematics...yada yada yada...



well...even if we calculate nn-to-pp, we wouldn't know how to interpret it

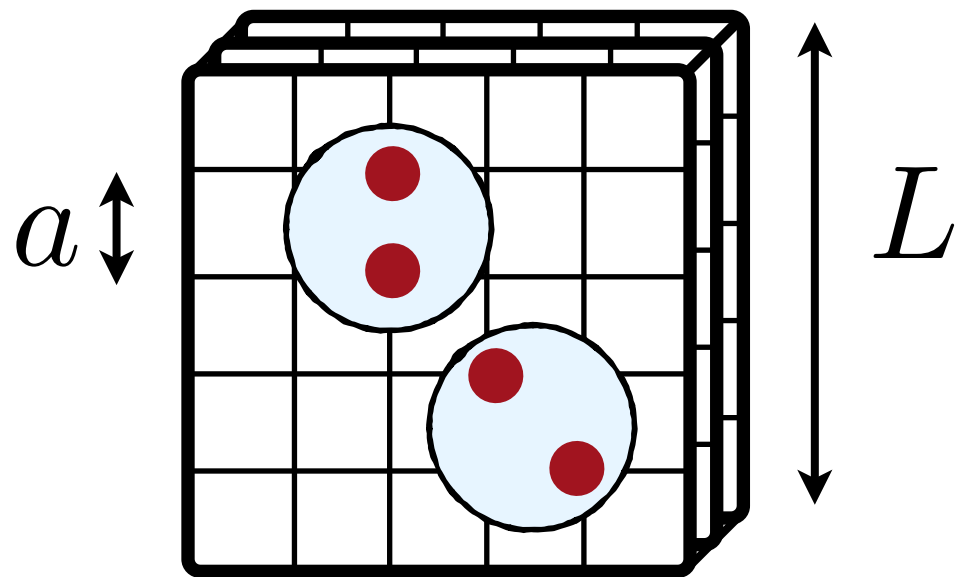
$0\nu\beta\beta$ from lattice QCD

Largely correlated challenges ahead

- 📌 contraction cost
- [
 - 📌 Euclidean spacetime: $t_M \rightarrow -it_E$
 - 📌 finite volume]
- 📌 quark masses: $m_q \rightarrow m_q^{\text{phys.}}$

focus of this and
the next talk

and no, no Quantum
computing needed 😊



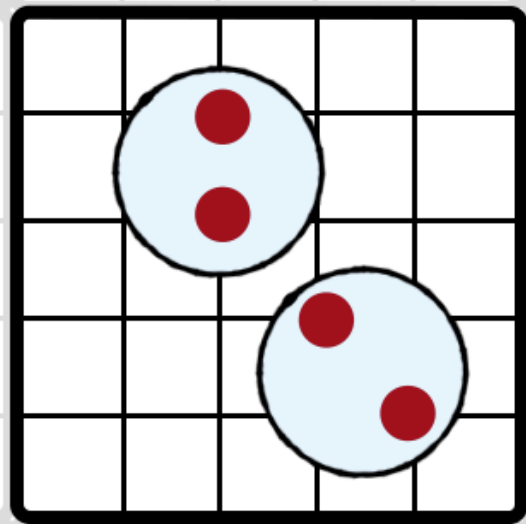
$0\nu\beta\beta$ from lattice QCD

Largely correlated challenges ahead

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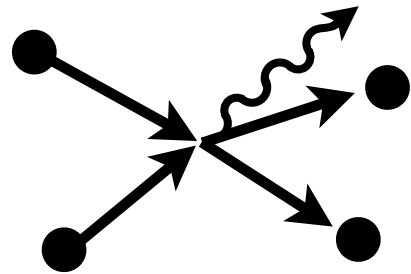


never free
no asymptotic states
no scattering

two-nucleon electroweak process

Consider a process 2-nucleons coupling via a local current to a final 2-nucleon state

electroweak
amplitudes



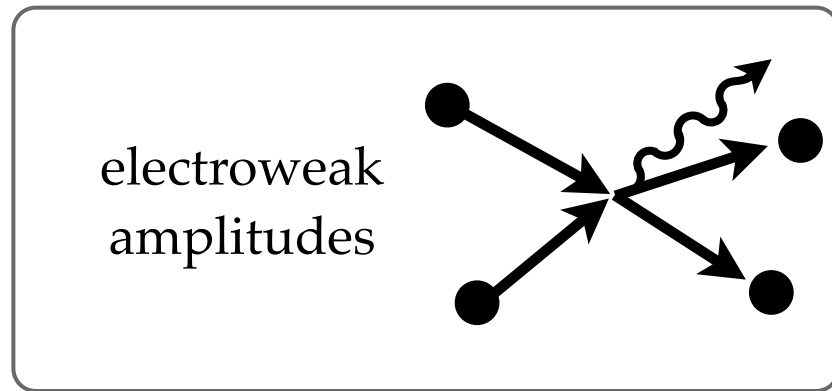
Matthias will discuss
non-local currents



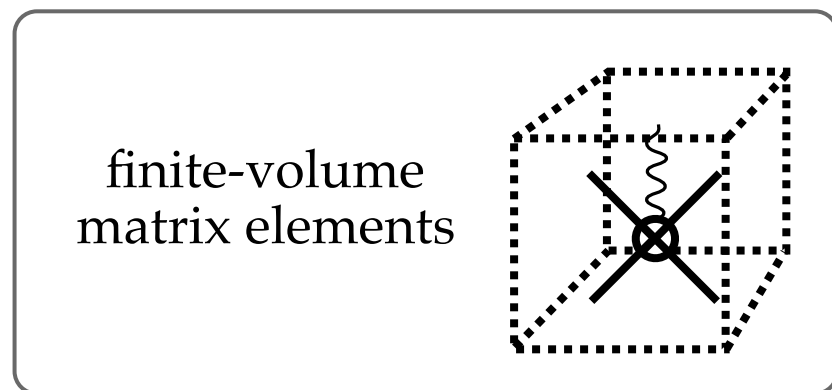
Schindler

two-nucleon electroweak process

Consider a process 2-nucleons coupling via a *local current* to a final 2-nucleon state



We would naturally expect that these may be accessed via appropriately designed three-point correlators

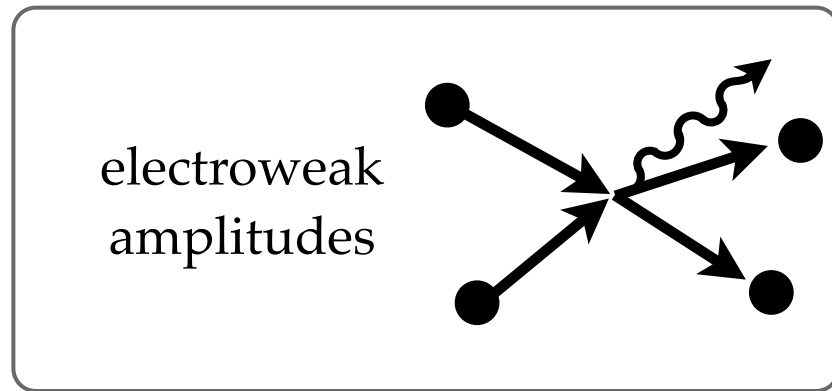


$$\begin{aligned} C_{3\text{pts.}} &= \langle \mathcal{O}_f(t_f) \mathcal{J}(t_c) \mathcal{O}_i^\dagger(0) \rangle \\ &= \sum_{n,m} f_{n,m}(t_f, t_c; L) \langle n; L | \mathcal{J}(0) | n; L \rangle \end{aligned}$$

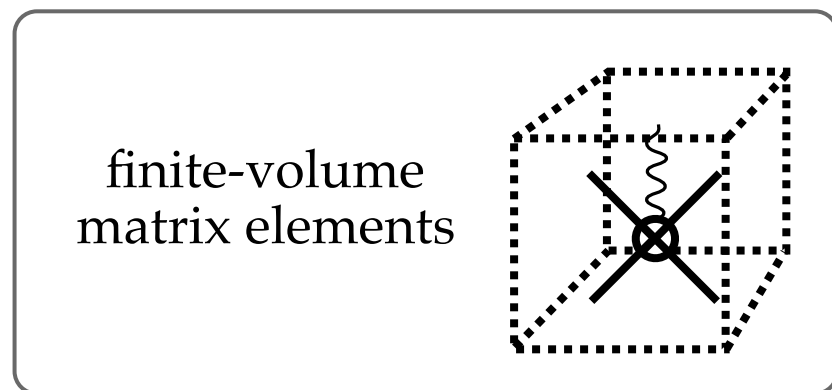
$$f_{n,m}(t_f, t_c; L) \equiv \langle 0 | \mathcal{O}_f(0) | n; L \rangle \langle m; L | \mathcal{O}_i^\dagger(0) | 0 \rangle e^{-t_f E_n - t_c (E_m - E_n)}$$

two-nucleon electroweak process

Consider a process 2-nucleons coupling via a *local current* to a final 2-nucleon state



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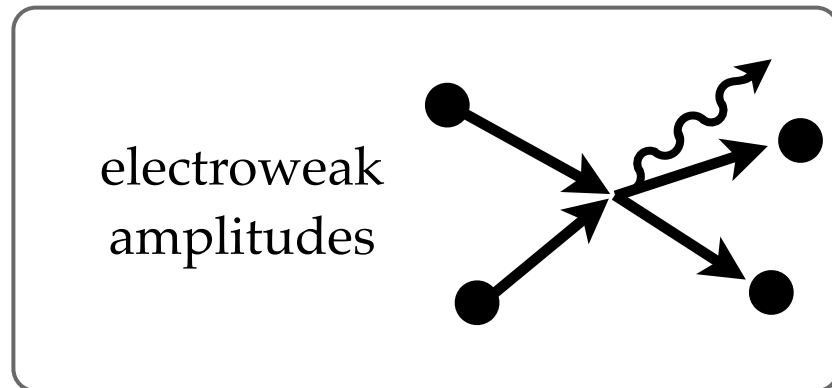
$$\begin{aligned} C_{3\text{pts.}} &= \langle \mathcal{O}_f(t_f) \mathcal{J}(t_c) \mathcal{O}_i^\dagger(0) \rangle \\ &= \sum_{n,m} f_{n,m}(t_f, t_c; L) \langle n; L | \mathcal{J}(0) | n; L \rangle \\ &\approx \sum_{n,m} f_{B,B}(t_f, t_c; \infty) \langle B; \infty | \mathcal{J}(0) | B; \infty \rangle + \dots \end{aligned}$$



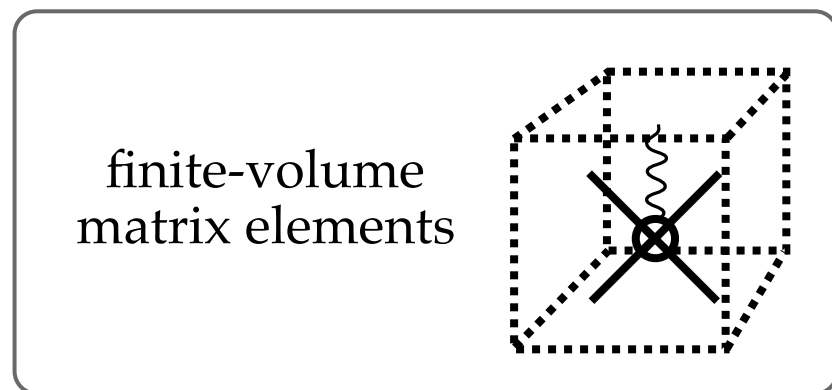
all matrix nuclear elements calculations
have been of deeply bound states
(see Zohreh's talk)

two-nucleon electroweak process

Consider a process 2-nucleons coupling via a *local current* to a final 2-nucleon state

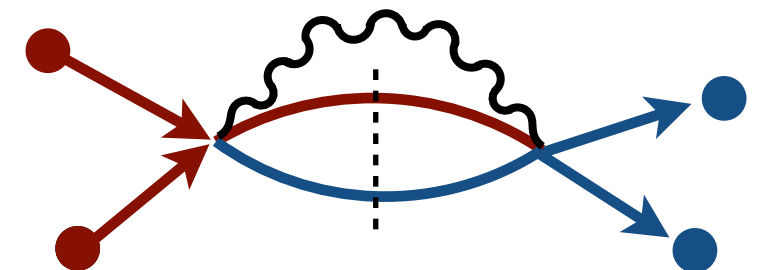


We would naturally expect that these may be accessed via appropriately designed three-point correlators



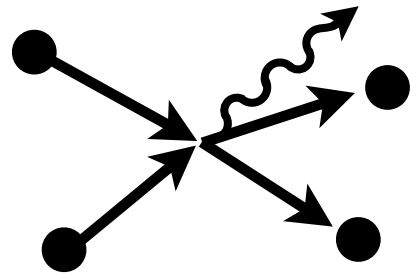
$$\begin{aligned} C_{3\text{pts.}} &= \langle \mathcal{O}_f(t_f) \mathcal{J}(t_c) \mathcal{O}_i^\dagger(0) \rangle \\ &= \sum_{n,m} f_{n,m}(t_f, t_c; L) \langle n; L | \mathcal{J}(0) | n; L \rangle \end{aligned}$$

$0\nu\beta\beta$ requires understanding
this for unbound states



two-nucleon electroweak process

electroweak
amplitudes

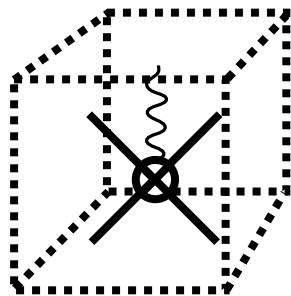


seemingly reasonable

EFTs

$$\mathcal{L}_{\chi PT} = \mathcal{L}_{\pi} + \mathcal{L}_{N\pi} + \mathcal{L}_{NN} + \dots$$

finite-volume
matrix elements

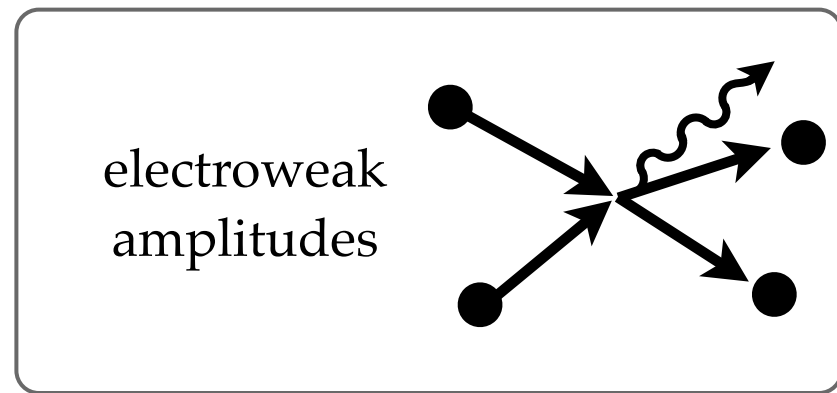


not good enough:

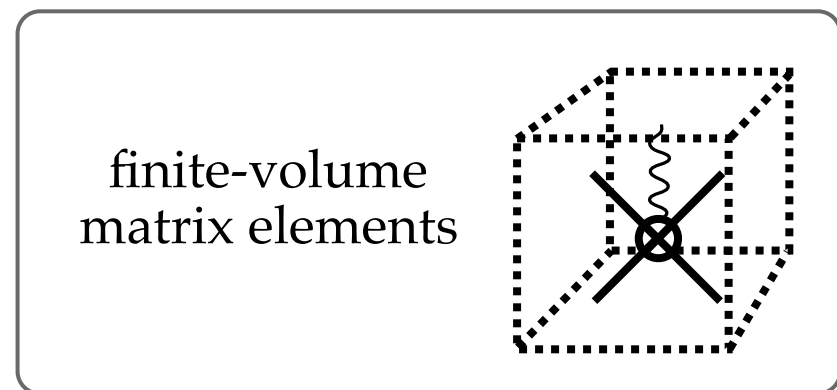
- requires well defined EFT
- convergence?
- heavy quark masses?
- perturbative
- kinematic restrictions
- strange sector of QCD

*i.e. how much would your final result depend on pure QCD
and how much depends on your choice and order of the EFT?*

two-nucleon electroweak process



all-orders mapping



Baroni

Hansen

Jackura

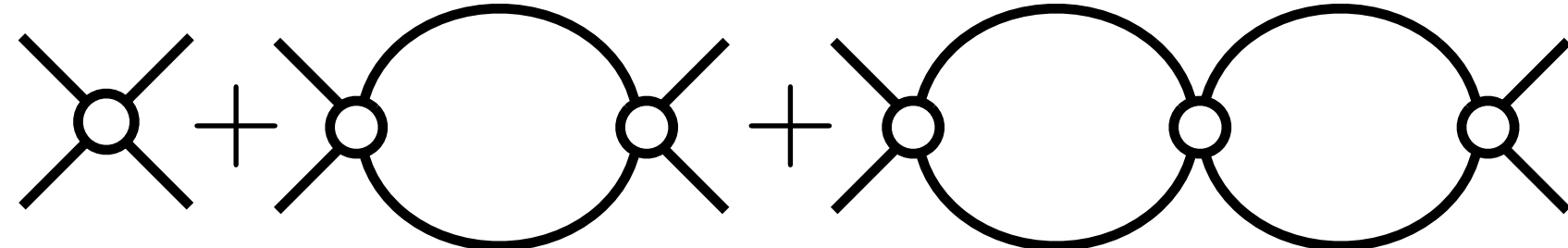
Ortega



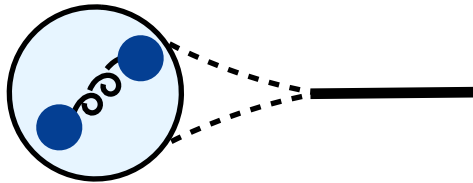
- 📌 RB & Hansen (2015)
- 📌 Baroni, RB, Hansen, Ortega (2018)

Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$
The equation shows the perturbative expansion of the scattering amplitude $i\mathcal{M}$. The first term is a tree-level diagram with four external lines meeting at a central point. The second term is a one-loop diagram consisting of a tree-level vertex connected to a loop, which then connects to another tree-level vertex. The third term is a two-loop diagram with two loops connected in series. The series continues with an ellipsis.

*IR limit of QCD, only interested in
hadronic d.o.f.*



Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \underbrace{\text{X} + \text{X} \circ \text{X} + \text{X} \circ \text{X} \circ \text{X} + \dots}_{\left\{ \text{X} + \text{---} + \text{X} + \text{X}' + \dots + \text{X} \circ \text{X} + \dots \right\}}$$

*non-perturbative kernel including
all diagrams not shown...*

“yep, the left hand cut is there”

Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

$$\begin{aligned} \text{one-loop} &= \int \frac{d^4k}{(2\pi)^4} [iB(k, P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P - k)^2 - m^2 + i\epsilon} \\ &= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k, P)]^2}{(2\omega_k)^2} \pi \delta(E - 2\omega_k) + \text{"PV integral"} \\ &= [iB_{on}] \rho [iB_{on}] + \text{"PV integral"} \\ &= \text{cut diagram} + \text{PV diagram} \end{aligned}$$

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-body scattering

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

$$= \text{diagram 4} + \dots$$

$$\left\{ \text{diagram 1} + \text{diagram 2} \text{ PV } \text{diagram 3} + \dots \right\}$$

K-matrix

Two-body scattering

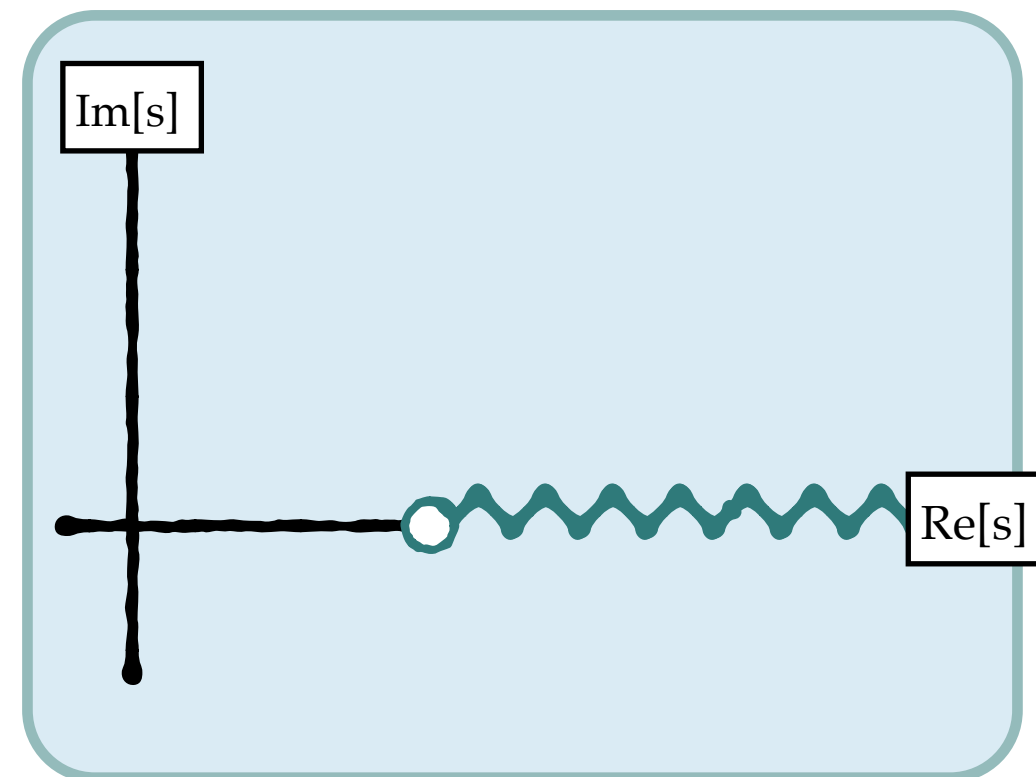
Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$
$$= \text{tree} + \text{one-loop with cut} + \dots$$

The diagrams represent the perturbative expansion of the scattering amplitude $i\mathcal{M}$. The first row shows the expansion in terms of loops: a tree-level vertex, followed by one-loop, two-loop, and higher-order terms. The second row shows the same expansion with the one-loop diagram featuring a vertical dashed line representing a branch cut, with an infinity symbol ∞ at the bottom indicating the singularity.

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

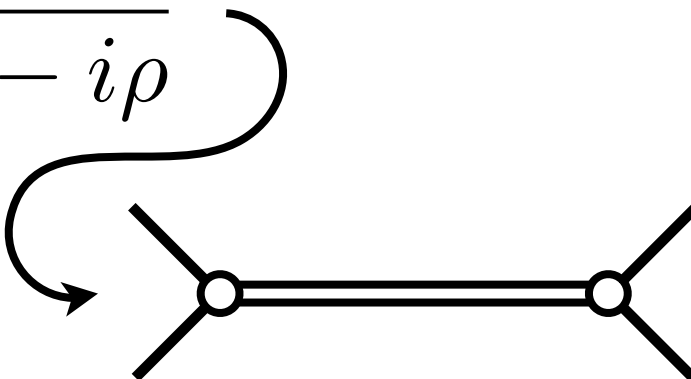


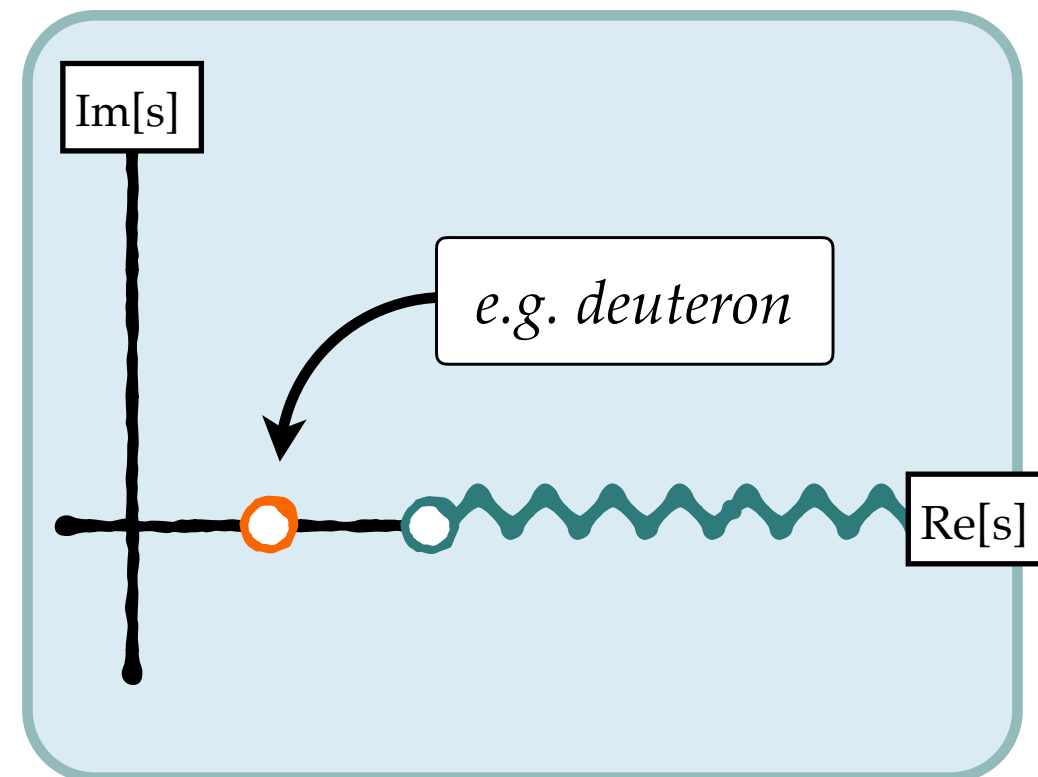
Two-body scattering

Unitarity using all orders perturbation theory:

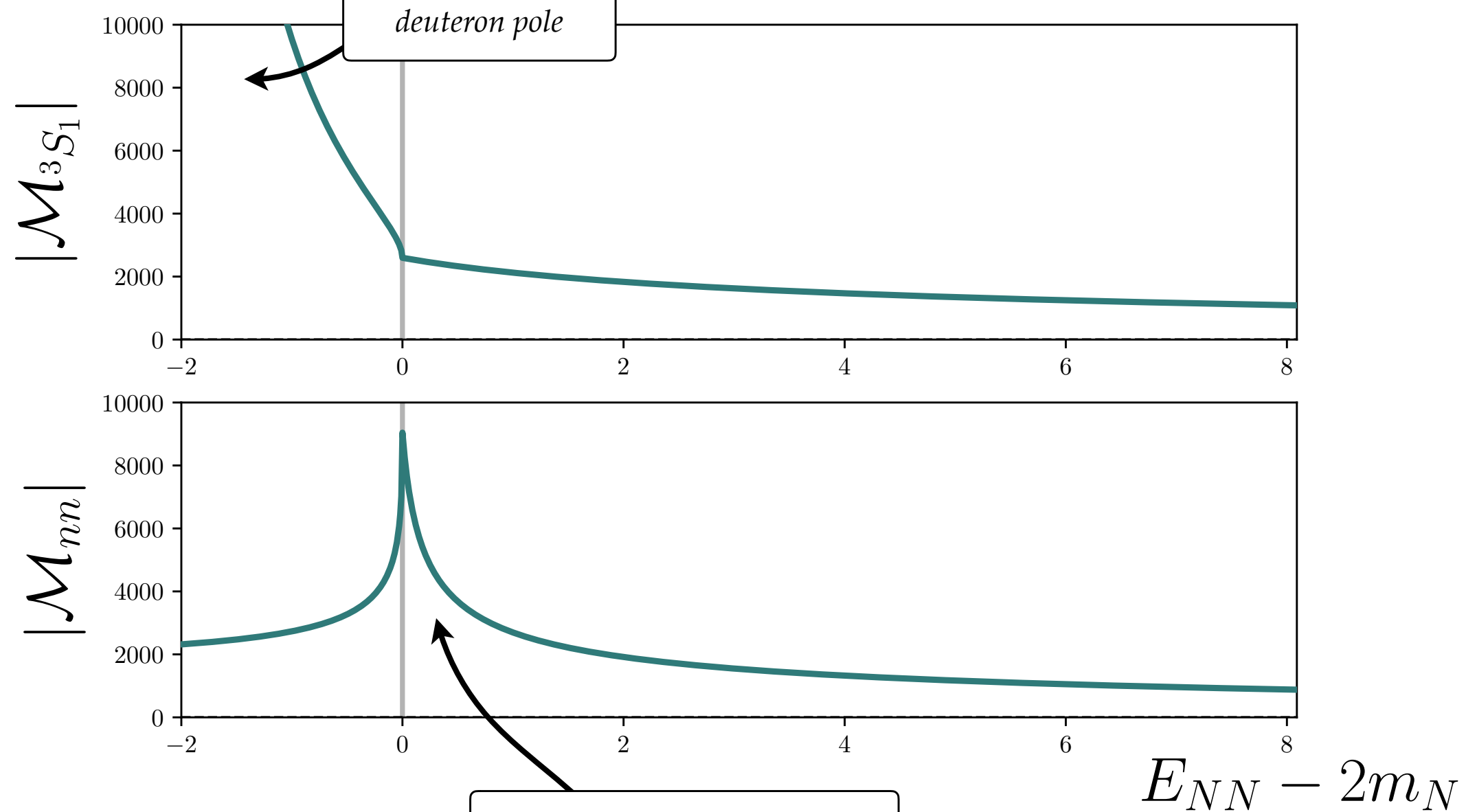
$$i\mathcal{M} = \text{tree} + \text{one-loop} + \text{two-loop} + \dots$$

$$= \text{unitarity cut} + \text{unitarity cut} + \text{unitarity cut} + \dots$$

$$= \frac{i}{\mathcal{K}^{-1} - i\rho}$$


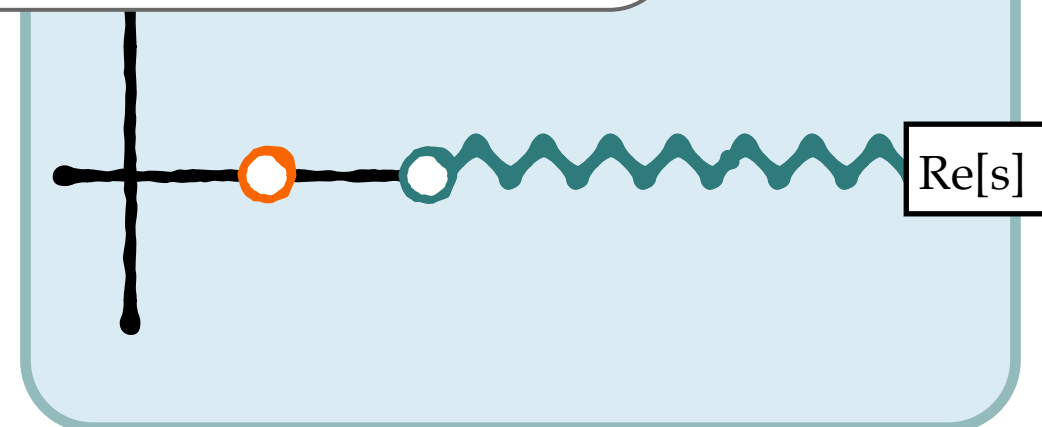


Two-body scattering



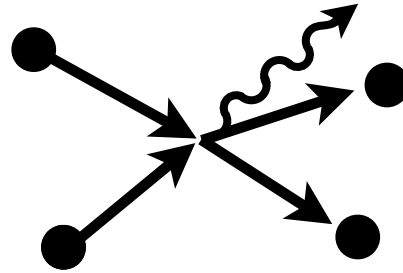
*threshold cusp dynamically
enhanced by the di-neutron pole*

*virtual bound state: second sheet
real-valued pole*



Two-to-two scattering **with current** - (full amp.)

Let's isolate all possible singularities of...

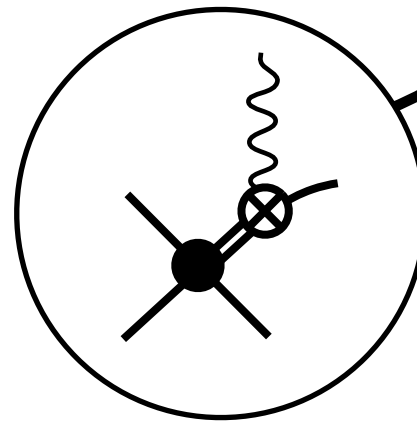
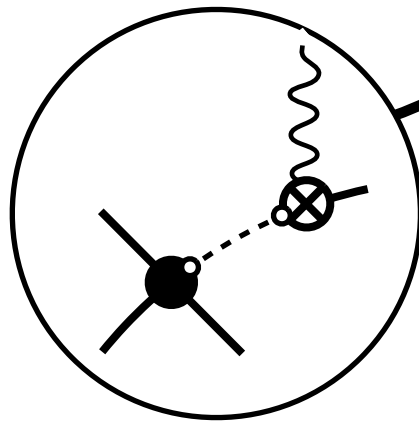


Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

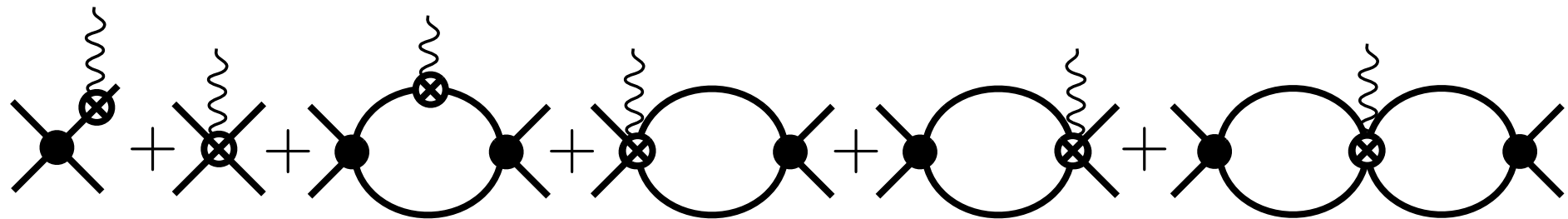
$$i\mathcal{W}_{\mu_1 \cdots \mu_n} = \text{diagram} + \cdots$$

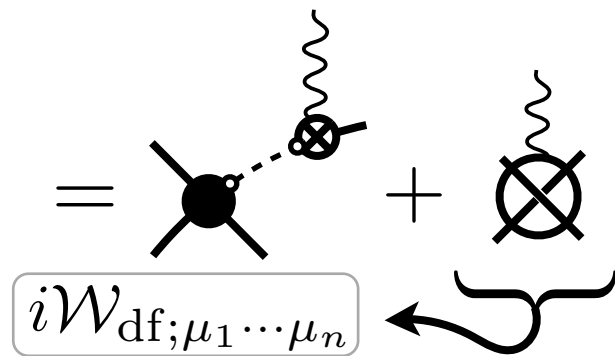
$$\text{diagram} = i\mathcal{M} \frac{i}{k^2 - m^2 + i\epsilon} i\omega_{\mu_1 \cdots \mu_n} + \text{“smooth”}$$



Two-to-two scattering **with current** - (full amp.)

Kinematic divergences

$$i\mathcal{W}_{\mu_1 \cdots \mu_n} =$$


$$=$$


$i\mathcal{W}_{\text{df};\mu_1 \cdots \mu_n}$

Two-to-two scattering **with current** - (df amp.)

Divergence-free amplitude

$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5}$$


New class of diagrams:

$$\begin{aligned} & \text{diagram 2} \\ &= \left\{ 1 + \text{diagram 6} + \text{diagram 7} + \dots \right\} \text{diagram 8} \left\{ 1 + \text{diagram 9} + \text{diagram 10} + \dots \right\} \\ &= \left[\frac{1}{1 - i\mathcal{K}\rho} \right] \text{diagram 8} \left[\frac{1}{1 - \rho i\mathcal{K}} \right] \end{aligned}$$

same square-root (and possibly pole) singularities as two-body amplitudes

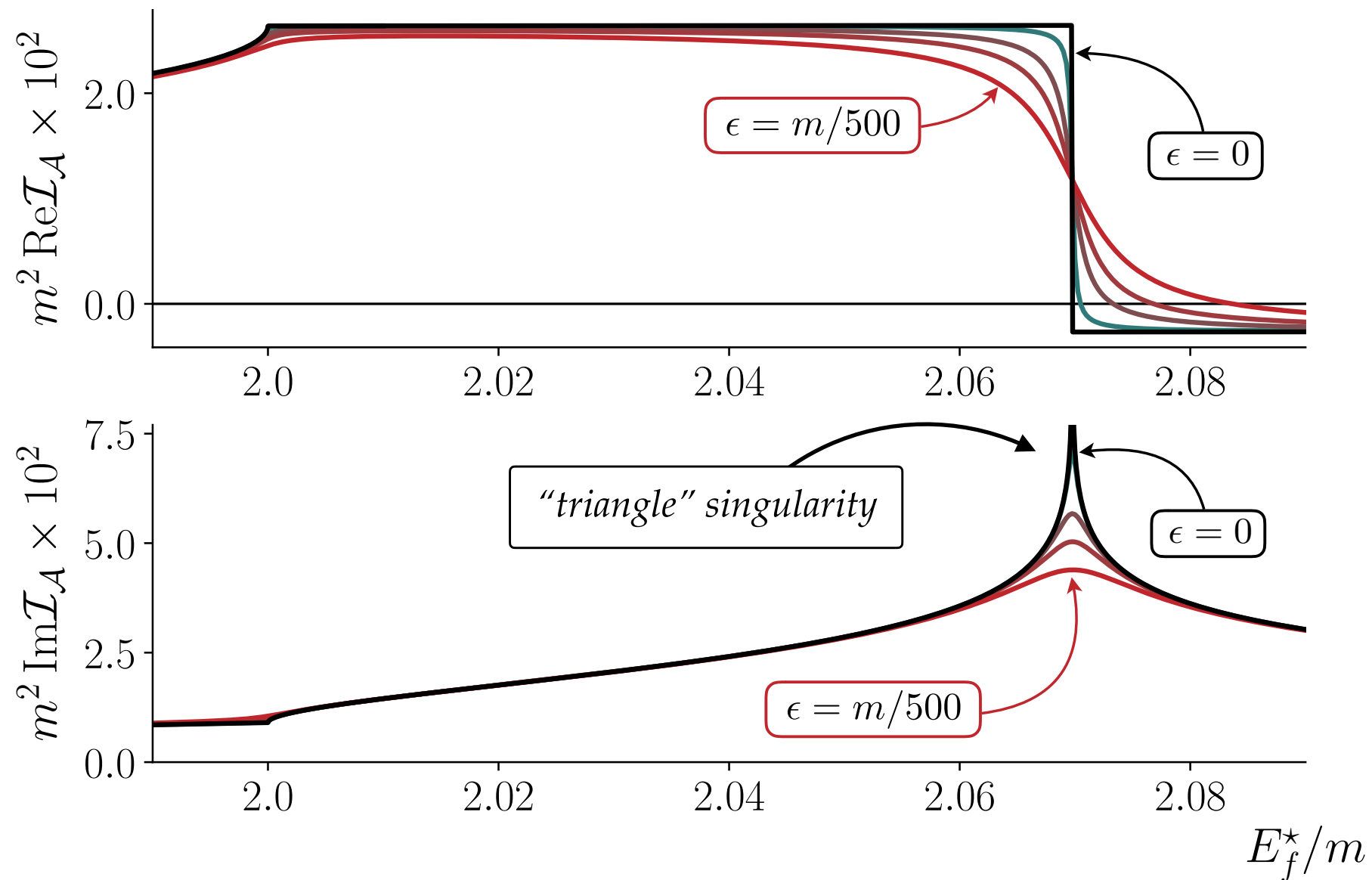
Two-to-two scattering **with current** - (df amp.)

Divergence-free amplitude

$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]}$$


Landau (1959)

New class of diagrams:



Two-to-two scattering **with current**

Divergence-free amplitude

$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5}$$

$$= i\mathcal{M} \left(\text{diagram 6} + \text{diagram 7} \right) i\mathcal{M}$$

Complex function...depending
on the one-body form factors

Naive Watson's theorem
does not apply!

finite-volume quantities **must** be
able to recover this singularity

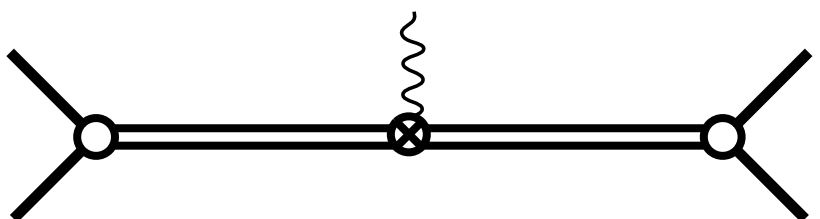
Two-to-two scattering **with current**

Divergence-free amplitude

$$i\mathcal{W}_{\text{df};\mu_1\cdots\mu_n} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]}$$

$$= i\mathcal{M} \left(\text{[diagram 6]} + \text{[diagram 7]} \right) i\mathcal{M}$$

The diagrams represent various Feynman diagrams for two-to-two scattering with a current. The first row shows five diagrams: a crossed square, a circle with a wavy line at the top, a circle with a wavy line at the left, a circle with a wavy line at the right, and a circle with a wavy line at the bottom. The second row shows the sum of two diagrams: a crossed square and a circle with two external lines.

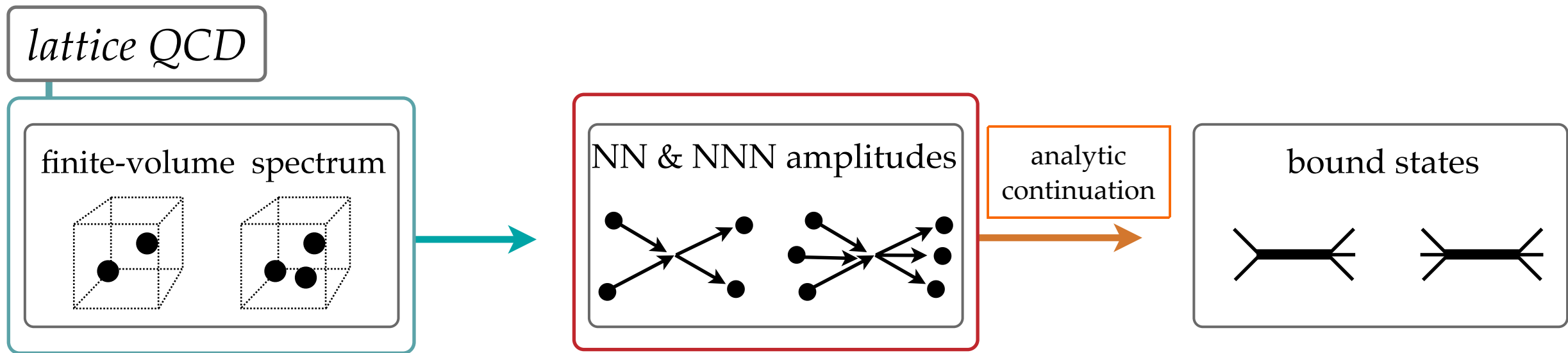
$$\xrightarrow{\lim s_i, s_f = s_B}$$


The diagram shows a horizontal line with two vertices, each having two external lines. A wavy line is attached to the middle of the horizontal line.

$$\sim \frac{g}{s_i - s_B} K_{\mu_1 \dots \mu_n} \boxed{F_R(Q^2)} \frac{g}{s_f - s_B}$$

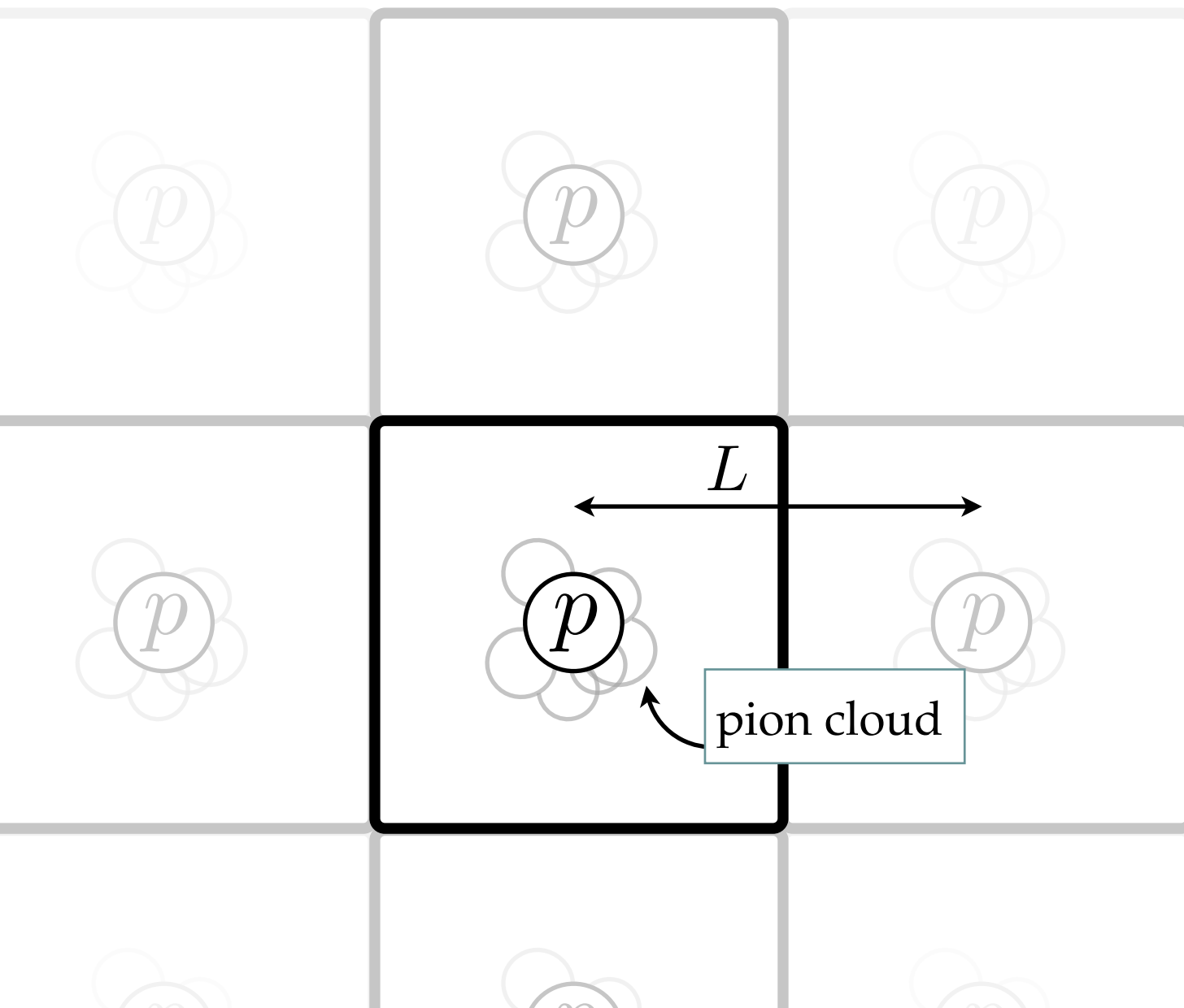
bound state form factors

few-nucleons systems in LQCD



Putting particles in a box

- Finite-volume arise from the interactions with mirror images
- Assuming $L \gg$ size of the hadrons $\sim 1/m_\pi$
 - This is a purely infrared artifact
 - We can determine these artifact using hadrons are the degrees of freedom
- Note $m_\pi L$ is a natural parameter

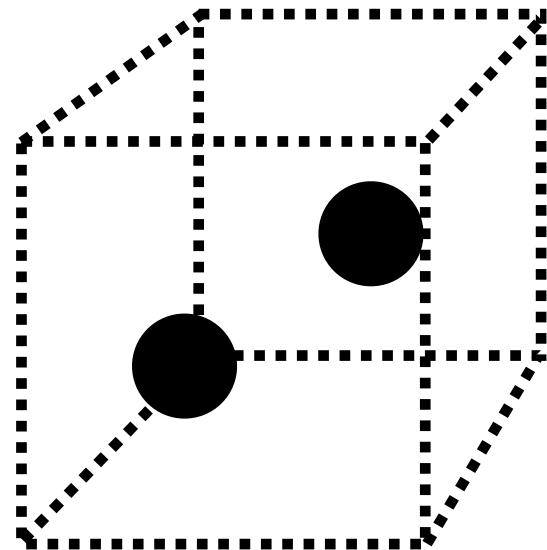


$$m_h(L) = m_h(\infty) + \mathcal{O}(e^{-m_\pi L})$$

Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) = \text{[diagram of a circle with two external lines and a vertex } V \text{]} + \text{[diagram of two circles with four external lines and two vertices } V \text{]} + \dots$$



Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) = \text{[diagram of a circle with two external lines and a vertex labeled } V \text{]} + \text{[diagram of two circles sharing a vertex, each with an external line and a vertex labeled } V \text{]} + \dots$$
$$= C_\infty(P) + \dots$$

Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) = \text{[diagram: circle with V and two external lines]} + \text{[diagram: two circles with V and four external lines]} + \dots$$

$$= C_\infty(P) + \text{[diagram: dashed circle with V - \infty and two external lines]} + \dots$$

$$\text{[diagram: dashed circle with V - \infty and two external lines]} = (iB) \left(\left[\frac{1}{L^3} \sum_{\mathbf{k}} - \int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} \right) (iB)$$

$$\equiv [iB] iF [iB]$$

F replaces ρ

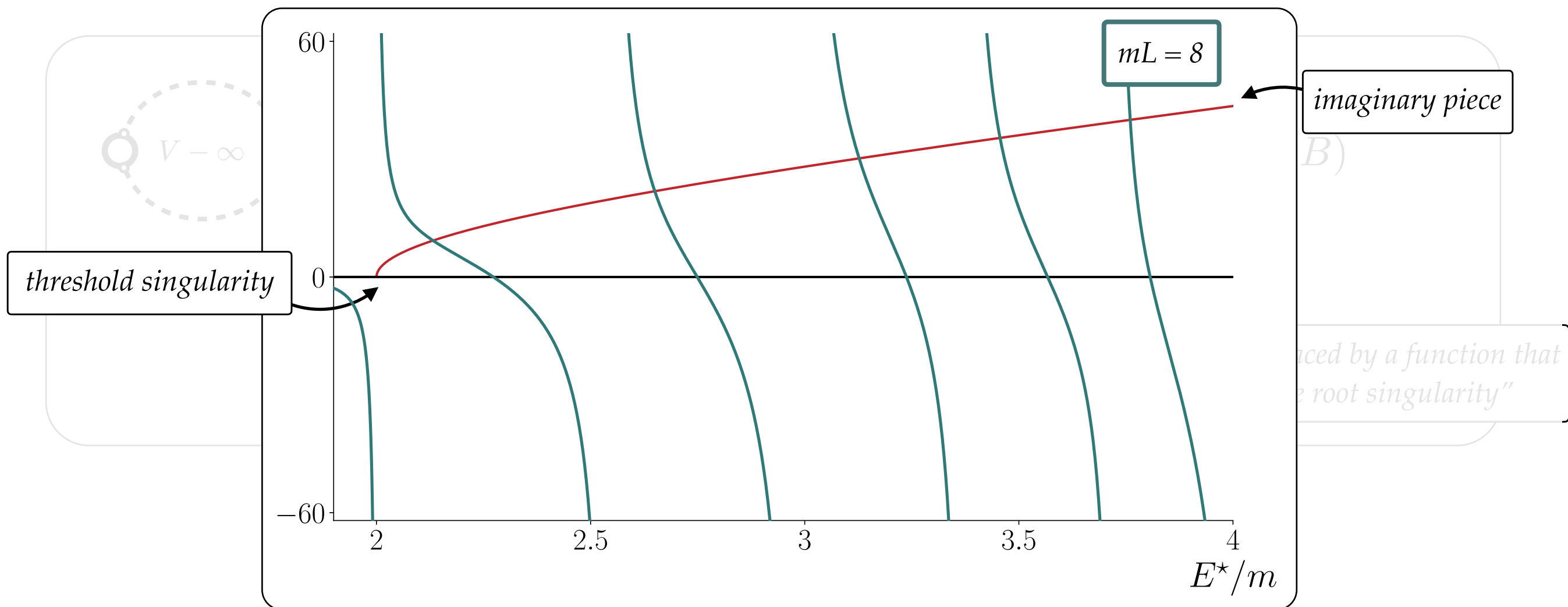
a simple square root singularity is replaced by a function that has both simple poles and the square root singularity

Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$C_L^{2pt.}(P) = \text{[diagram: circle with V]} + \text{[diagram: two circles with V]} + \dots$$

$$= C_\infty(P) + \text{[diagram: dashed circle with V - \infty]} + \dots$$



Two-particle in finite volume

Consider the finite-volume two-particle correlator ($E \sim 2m$):

$$\begin{aligned}
 C_L^{2pt.}(P) &= \text{[diagram: a circle with two external lines and label } V] + \text{[diagram: two circles connected by a line, each with two external lines and label } V] + \dots \\
 &= C_\infty(P) + \text{[diagram: a dashed circle with two external lines and label } V - \infty] + \text{[diagram: two dashed circles connected by a line, each with two external lines and label } V - \infty] + \dots \\
 &= \text{“smooth”} + A \frac{i}{F^{-1} + \mathcal{M}} B^\dagger \sim \sum_n A_n \frac{i R_n}{E - E_n} B_n^\dagger
 \end{aligned}$$

poles satisfy: $\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$

• Lüscher (1986, 1991)

• Rummukainen & Gottlieb (1995)

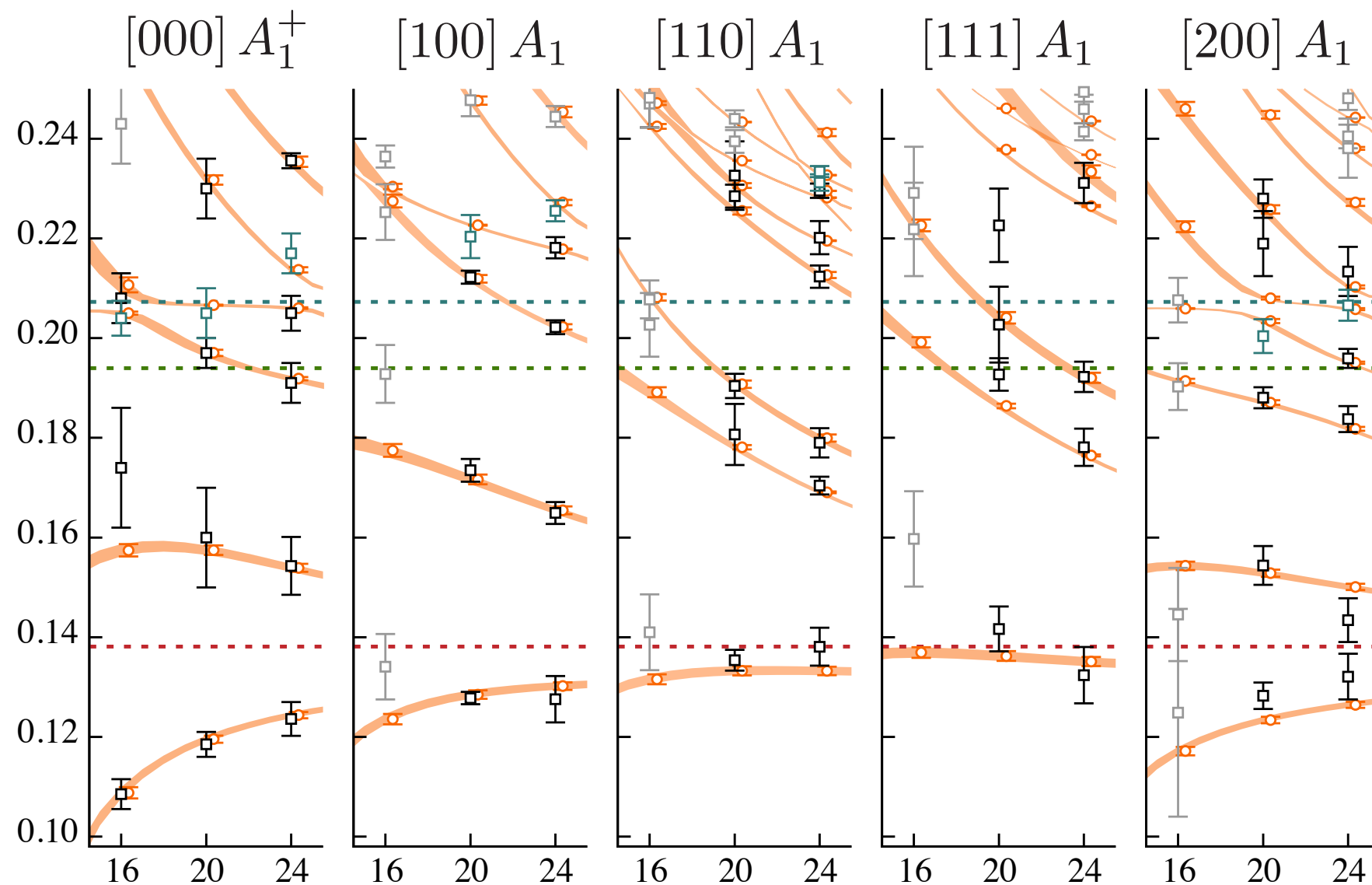
• Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005)

• Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)

• RB (2014)

These ideas in practice

Arguably the most advanced implementations of this are currently in the meson sector, where it is increasingly common to extract 30-100 energy levels

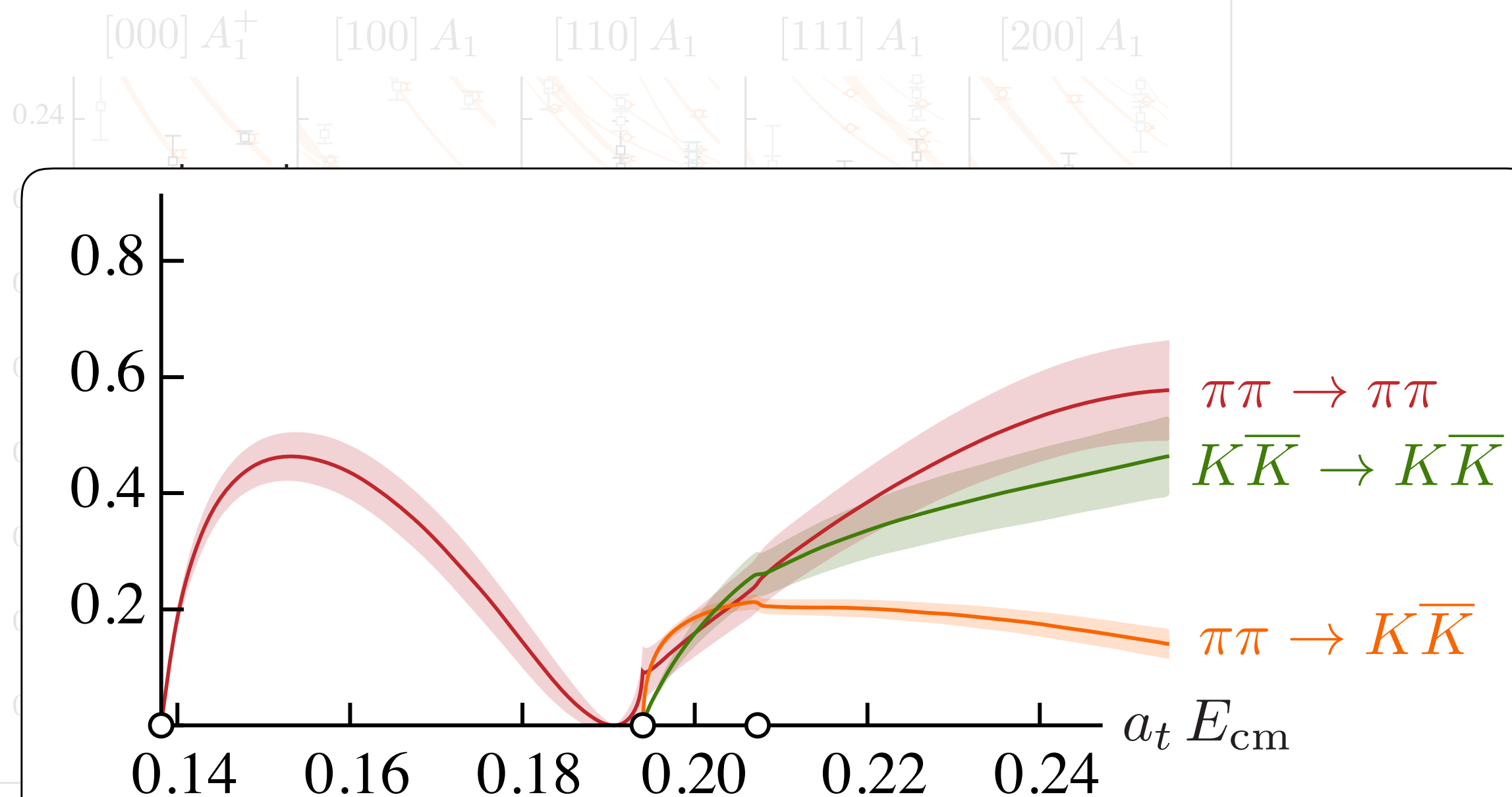


$$\chi^2/N_{\text{dof}} = \frac{44.0}{57 - 8} = 0.90$$

57 energy levels

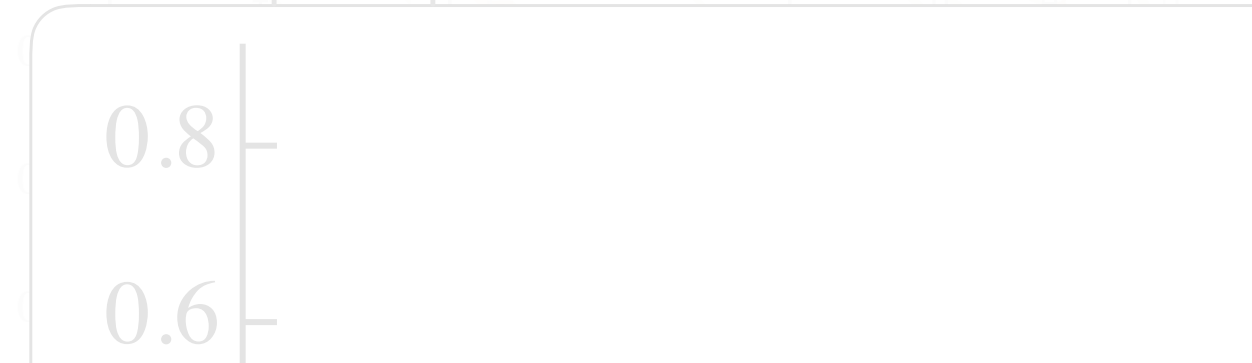
These ideas in practice

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These ideas in practice

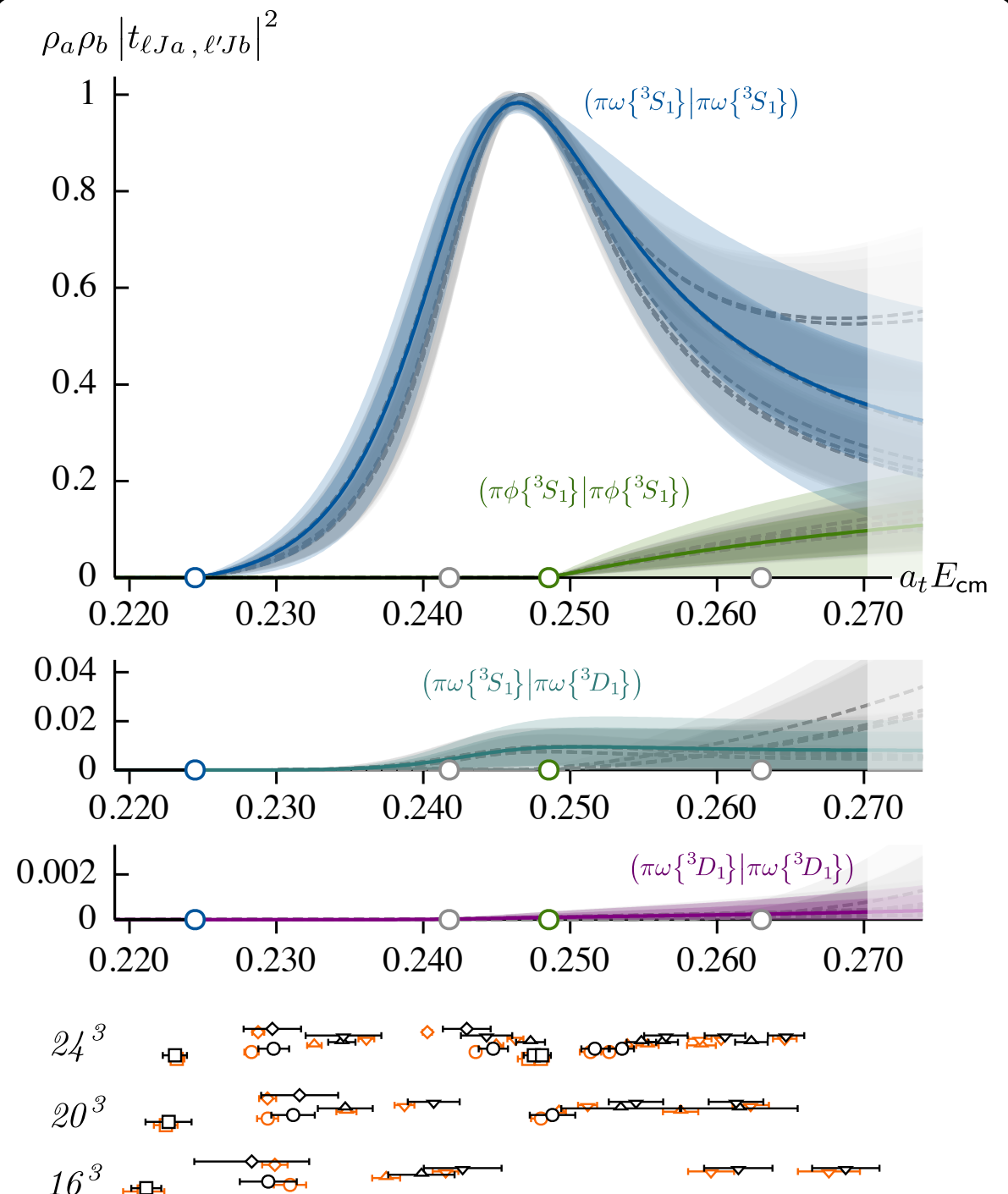
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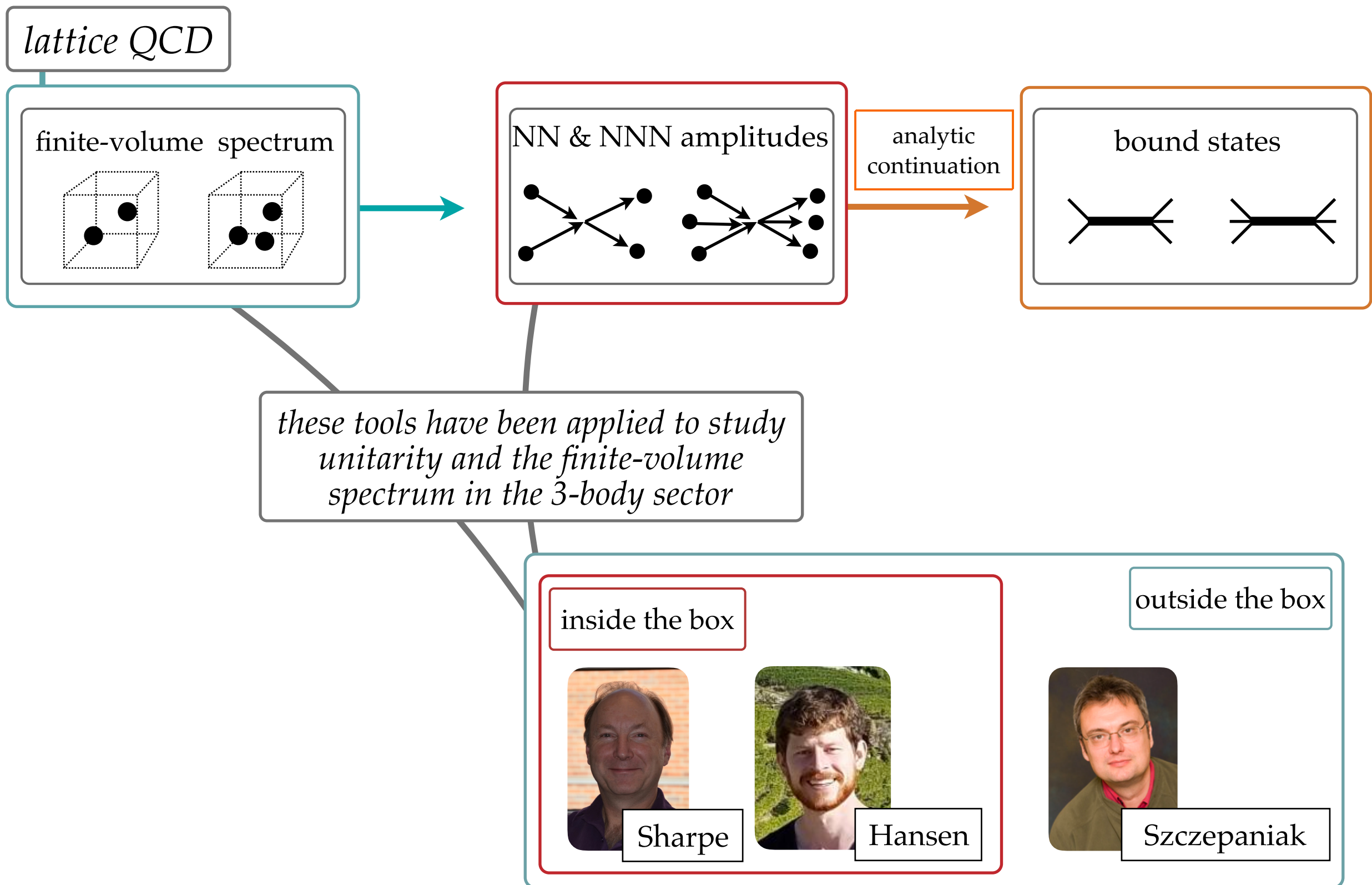
strongly coupled, multiple dynamical poles, with non-zero spin and angular momentum...

computational cost is the only barrier preventing us from applying this to NN-systems

Woss, Thomas, Dudek, Edwards & Wilson (2019)



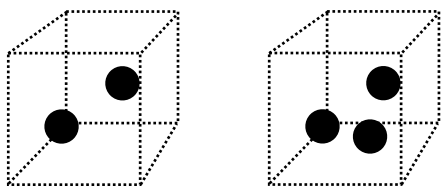
few-nucleons systems in LQCD



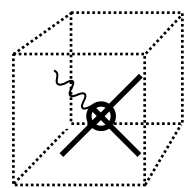
few-nucleons systems in LQCD

lattice QCD

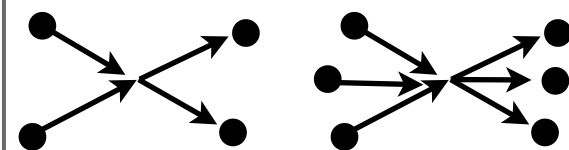
finite-volume spectrum



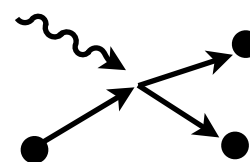
1-to-2
FV matrix
elements



NN & NNN amplitudes



electroweak
amplitudes

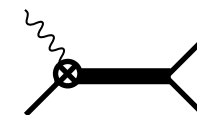


analytic
continuation

bound states

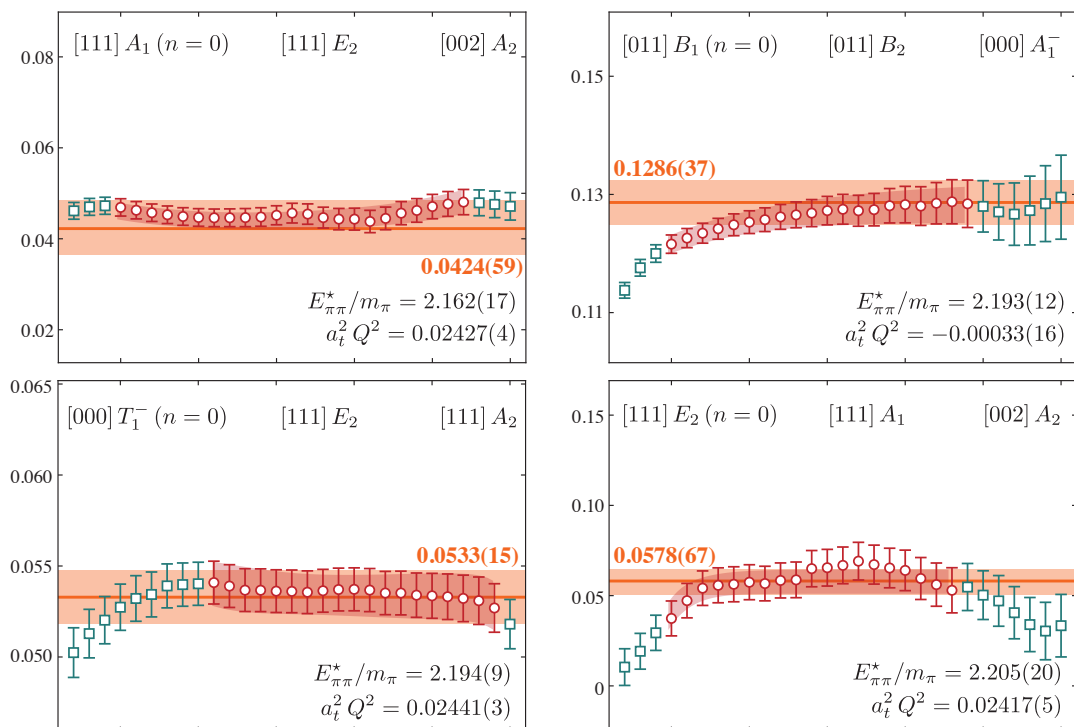


transition
form factors

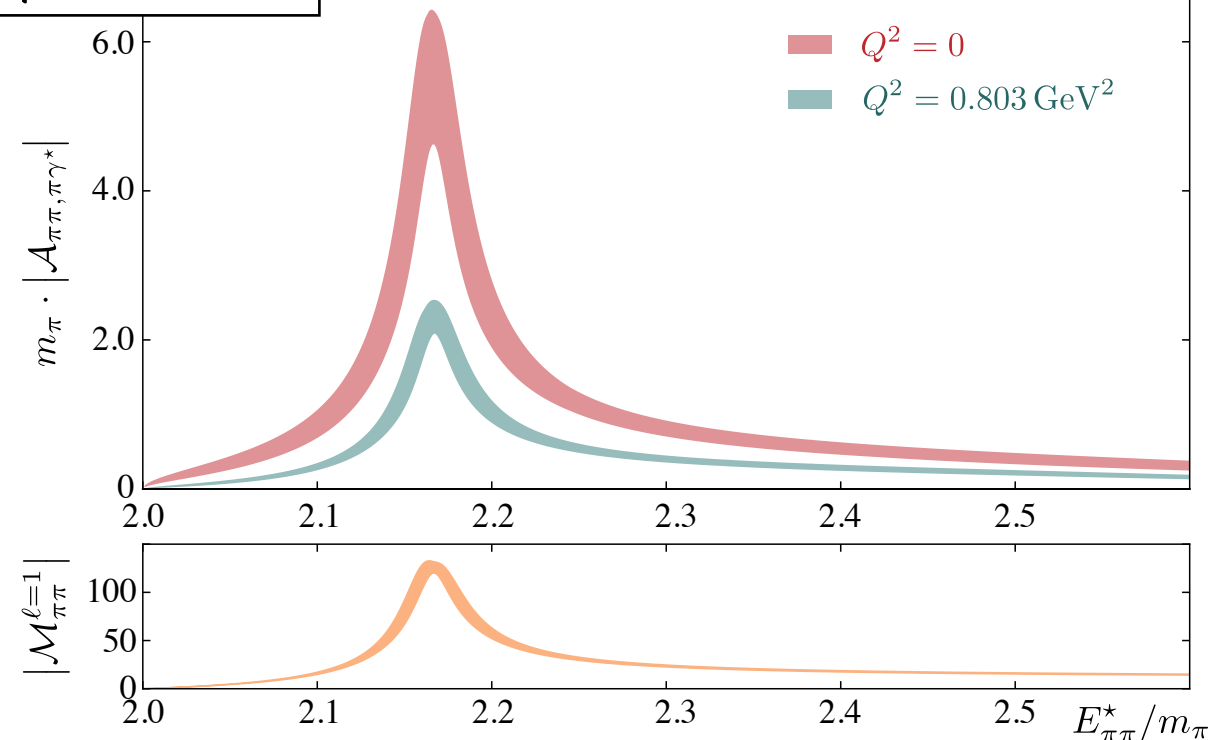


RB, Hansen & Walker-Loud (2014)

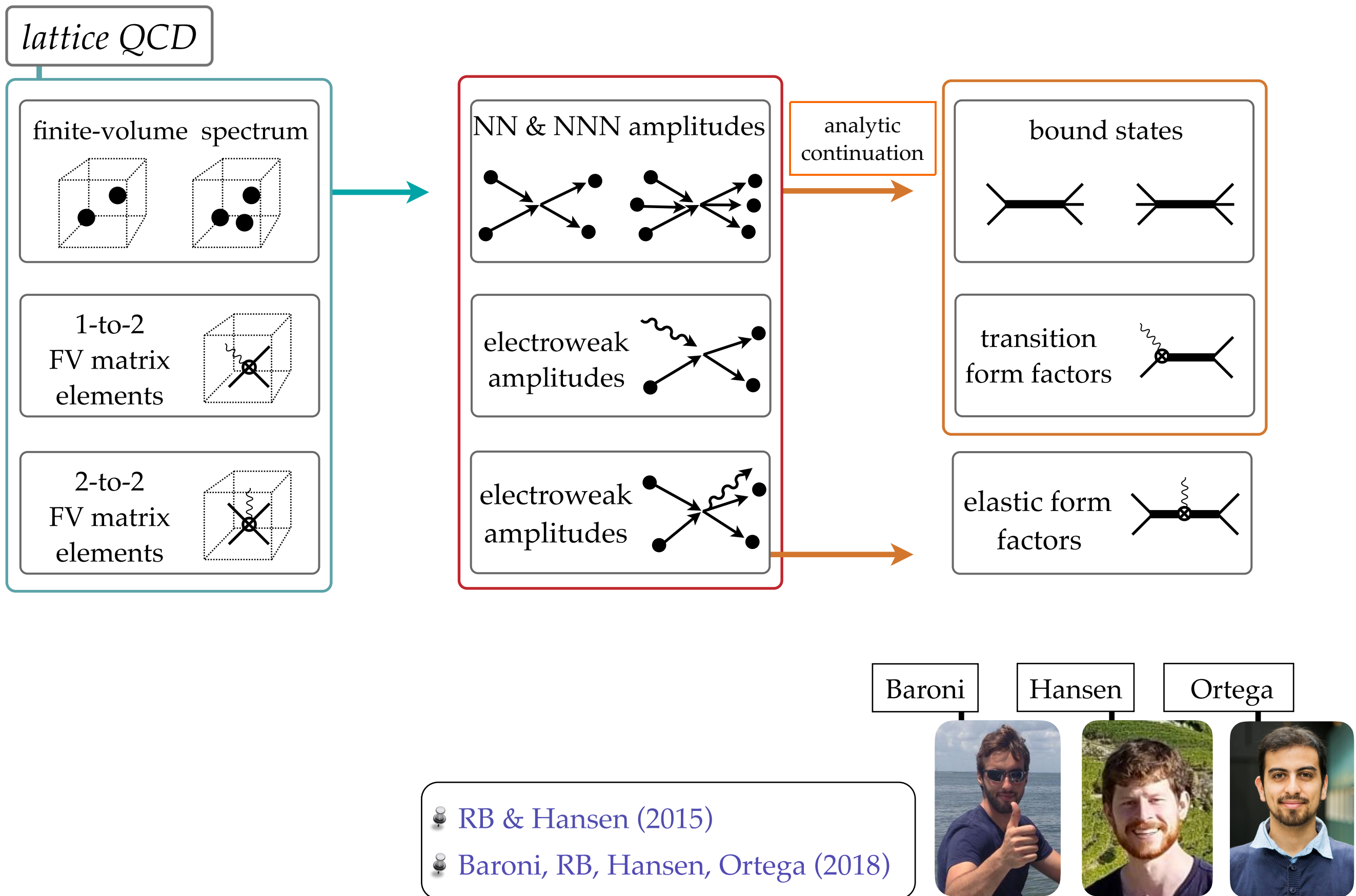
RB & Hansen (2015)



$\pi\gamma^*$ -to- $\pi\pi$



few-nucleons systems in LQCD

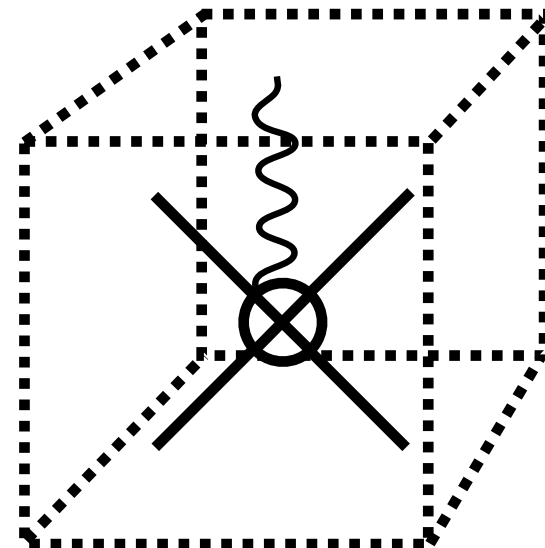


Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The equation shows a series of Feynman diagrams for a three-point correlation function $C_L^{3pt.}$ in finite volume with a current. The diagrams are separated by plus signs and followed by an ellipsis. Each diagram consists of two circles, each labeled with a V , representing a volume. The first diagram has a wavy line (representing a current) attached to the top of the first circle, with a cross inside the circle. The second diagram has a wavy line attached to the left of the first circle, with a cross inside the circle. The third diagram has a wavy line attached to the top of the first circle, with a cross inside the circle, and a second circle attached to the right of the first circle.



Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The diagrams in the equation are:

- Diagram 1: A circle with two external legs (left and right) and a wavy line (top) attached to a vertex marked with a cross. The circle is labeled V .
- Diagram 2: A circle with two external legs (left and right) and a wavy line (top) attached to a vertex marked with a cross. The circle is labeled V .
- Diagram 3: Two circles connected by a horizontal line. Each circle has a wavy line (top) attached to a vertex marked with a cross. Each circle is labeled V .

...everything is the same as before except for...

$$\text{[Diagram 1]} = \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \mathcal{O}(e^{-mL})$$

The diagrams in the equation are:

- Diagram 1: A circle with two external legs (left and right) and a wavy line (top) attached to a vertex marked with a cross. The circle is labeled $V - \infty$.
- Diagram 2: A triangle with two external legs (left and right) and a wavy line (top) attached to a vertex marked with a cross. The triangle is labeled $V - \infty$.
- Diagram 3: A triangle with two external legs (left and right) and a wavy line (top) attached to a vertex marked with a cross. The triangle is labeled $V - \infty$.
- Diagram 4: A triangle with two external legs (left and right) and a wavy line (top) attached to a vertex marked with a cross. The triangle is labeled $V - \infty$.

Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \dots$$

...everything is the same as before except for...

$$\text{[diagram 1]} = \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \mathcal{O}(e^{-mL})$$

leads to the presence of F-functions...

not too surprising that W_{df} emerges...

Two-particle in finite volume **with current**

Same as before...but with a current

$$C_L^{3pt.} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The diagrams show a series of terms for the three-point correlation function $C_L^{3pt.}$. Each term consists of a circle with two external legs (small circles) and a wavy line (current) attached to a vertex (a circle with an 'X') on the main circle. The first term has the vertex at the top. The second term has the vertex on the left. The third term has two such circles connected by a horizontal line. The series continues with an ellipsis.

...everything is the same as before except for...

$$\text{[Diagram 1]} = \text{[Diagram 4]} + \text{[Diagram 5]} + \text{[Diagram 6]} + \mathcal{O}(e^{-mL})$$

The diagram shows the first term of the series for $C_L^{3pt.}$ (a circle with two external legs and a wavy line) is equal to the sum of three diagrams. The first diagram is a triangle with dashed lines and a wavy line, with a blue box around it. The second and third diagrams are similar triangles but with solid lines. The fourth term is $\mathcal{O}(e^{-mL})$.

New finite-volume function:

$$G_{\mu_1 \dots \mu_n}(P_f, P_i, L) = \text{[Diagram 7]} - \text{[Diagram 8]}$$

The diagram shows the definition of the new finite-volume function $G_{\mu_1 \dots \mu_n}(P_f, P_i, L)$. It is equal to the difference between two diagrams. The first diagram is a dashed oval with a wavy line and a vertex, labeled V . The second diagram is a dashed oval with a wavy line and a vertex, labeled $i\epsilon$.

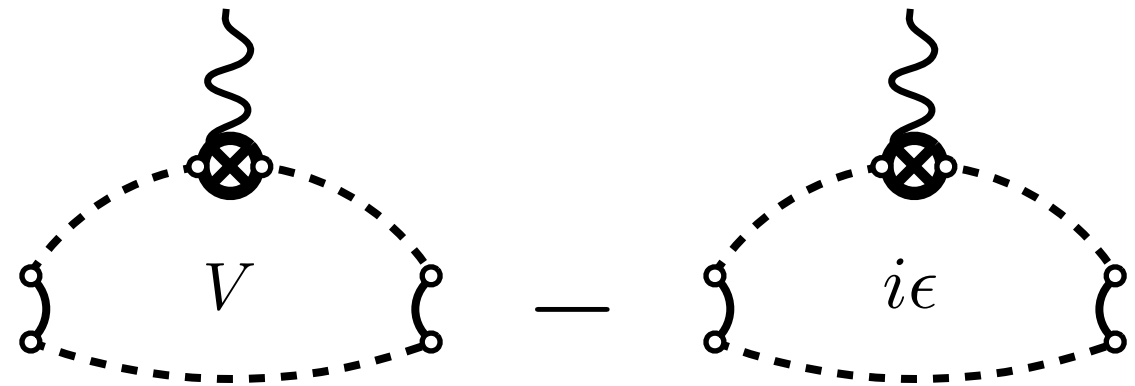
Two-particle in finite volume **with current**

Same as before...but with a current

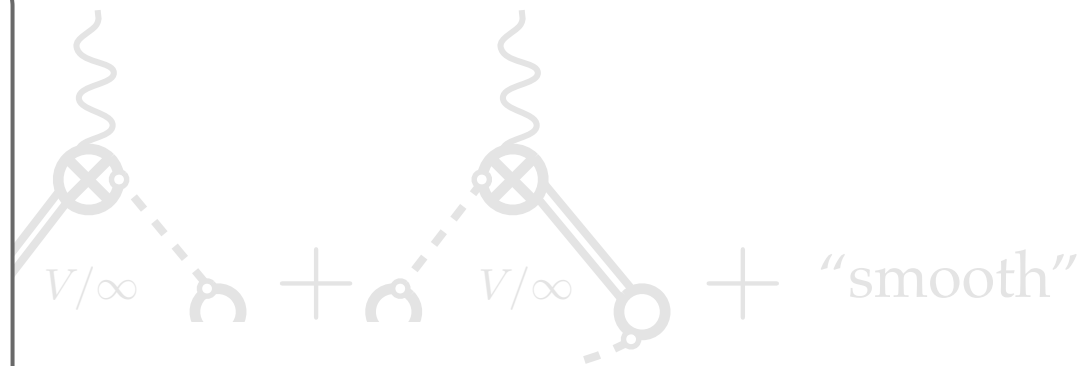
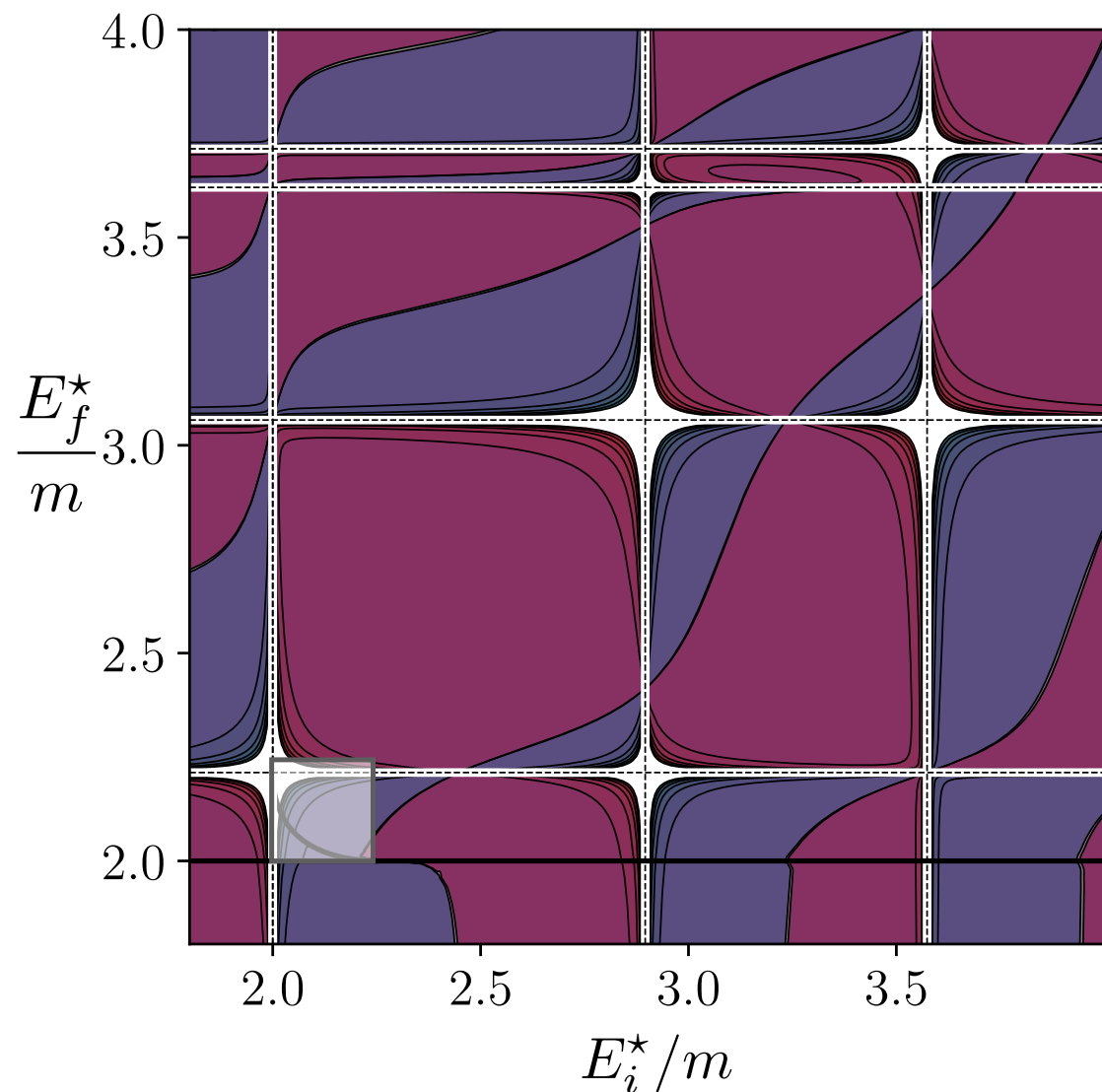
New finite-volume function:

$C_L^{3pt.}$

$$G_{\mu_1 \cdots \mu_n}(P_f, P_i, L) =$$



...everything is the same as before except for...



free-particle FV poles



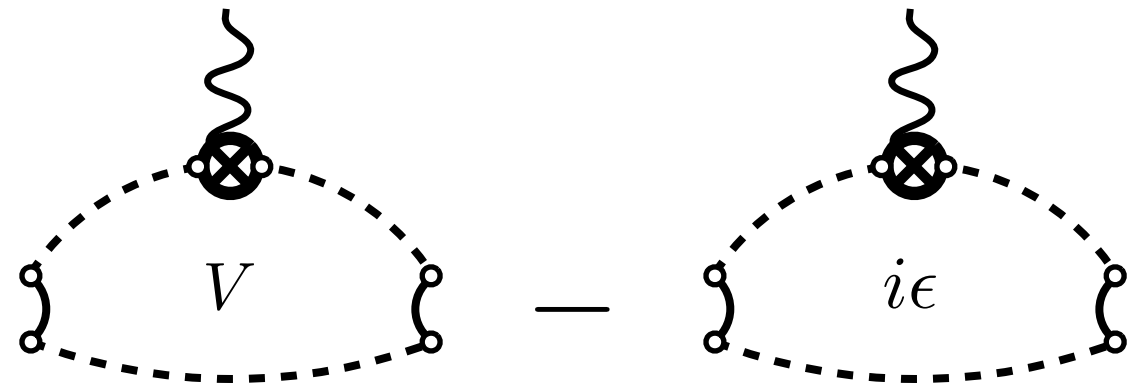
Two-particle in finite volume **with current**

Same as before...but with a current

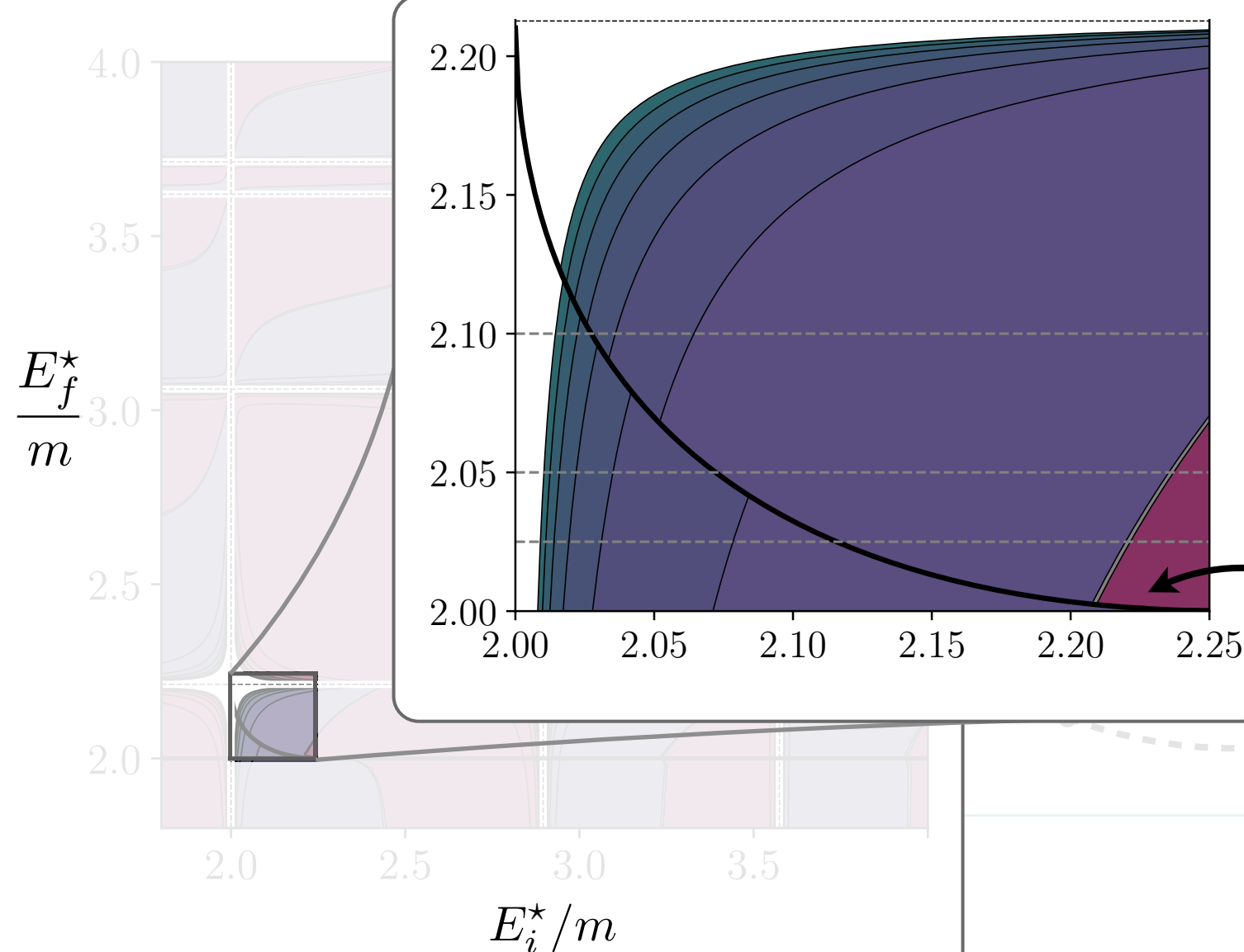
New finite-volume function:

$C_L^{3pt.}$

$$G_{\mu_1 \cdots \mu_n}(P_f, P_i, L) =$$



...everything is the same as before except for...



triangle singularity

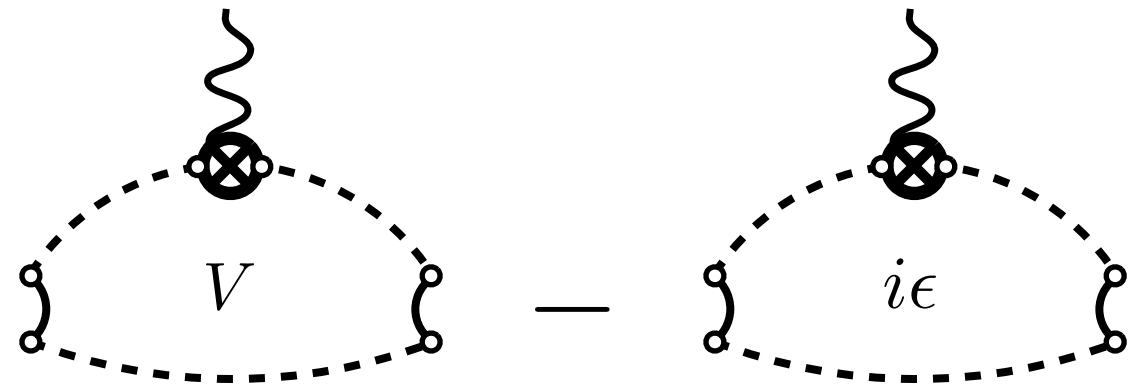
Two-particle in finite volume **with current**

Same as before...but with a current

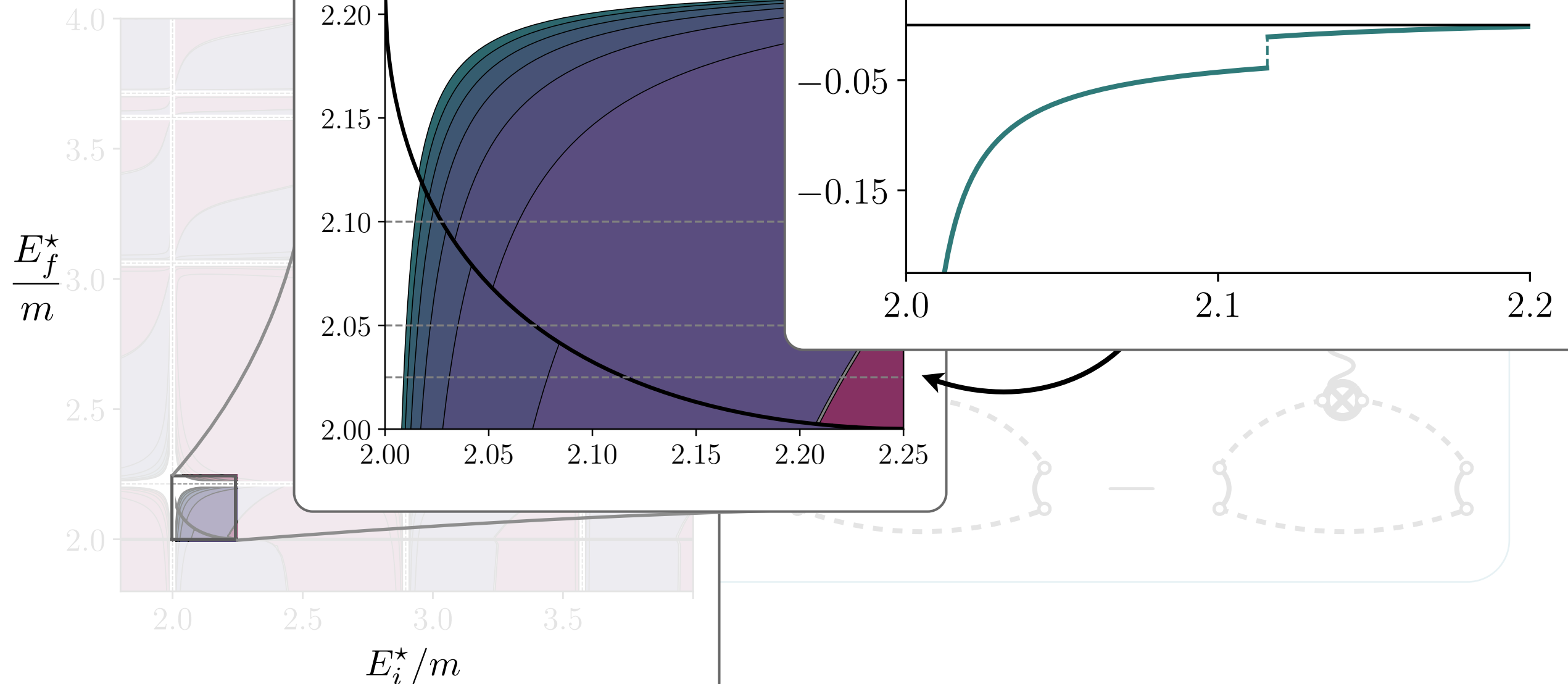
New finite-volume function:

$C_L^{3pt.}$

$$G_{\mu_1 \dots \mu_n}(P_f, P_i, L) =$$



...everything is the same as before except for...



Two-particle in finite volume **with current**

After lots of massaging...

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R} \mathcal{W}_{L,\text{df}}$$

Building block #1) Lellouch-Lüscher matrices:

$$\mathcal{R}(E_n, \mathbf{P}) \equiv \lim_{E \rightarrow E_n} \left[\frac{(E - E_n)}{F^{-1}(P, L) + \mathcal{M}(P)} \right]$$

*derivatives of amplitudes and F-function
at the finite-volume spectra*

Two-particle in finite volume **with current**

After lots of massaging...

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R} \mathcal{W}_{L,\text{df}}$$

Building block #2) stable particle form factor

$$\mathcal{W}_{L,\text{df}} - \mathcal{W}_{\text{df}}^{\mu_1 \cdots \mu_n} \sim \sum_{n'}^n \mathcal{M} [G^{(j)} \underbrace{f^{(j)}(-q^2)}] \mathcal{M}$$

form factors of single-particle states

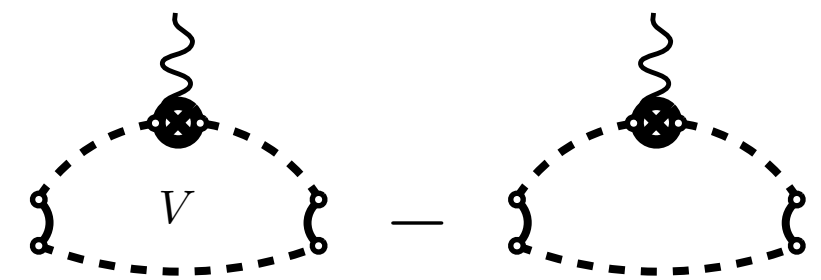
Two-particle in finite volume **with current**

After lots of massaging...

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R} \mathcal{W}_{L,\text{df}}$$

Building block #3) G-function

$$\mathcal{W}_{L,\text{df}} - \mathcal{W}_{\text{df}}^{\mu_1 \cdots \mu_n} \sim \sum_{n'}^n \mathcal{M} \left[\underbrace{G^{(j)} f^{(j)}(-q^2)} \right] \mathcal{M}$$

$$G_{\mu_1 \cdots \mu_n}(P_f, P_i, L) = \text{diagram 1} - \text{diagram 2}$$


known geometric function, which “adds missing singularities by hand”

Checks: Ward-Takahashi Identity

Ward-Takahashi identity implies (for specific kinematics), the five-point function with $\mathcal{J}_{\text{QED}}^0$ is determined from the four-point function and the QED charge

$$\mathcal{W}_{\text{df}}^0 = 2P^0 Q \frac{d}{ds} \mathcal{M}$$

It is not obvious, but this implies

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R} \mathcal{W}_{L,\text{df}} = \frac{Q}{2E}$$

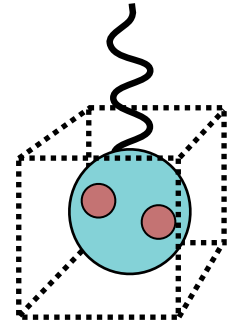
i.e., the QED charge is protected, even for unbound states



Jackura (IU)

Checks: Bound-state limit

Intuitively, in the limit that our state is bound, all these effects must vanish *exponentially fast*.

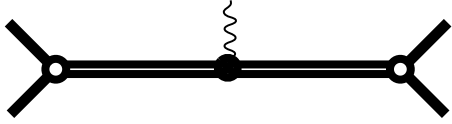


Well known for the spectrum: $E_L = E_B + \mathcal{O}(e^{-\kappa L})$

Also applies for the matrix elements:

$$L^3 \langle 2 | \mathcal{J} | 2 \rangle_L = \mathcal{R} \mathcal{W}_{L,\text{df}} = \frac{f_B(Q^2)}{2\sqrt{E_i E_f}} + \mathcal{O}(e^{-\kappa L})$$

remember, this is doubly singular

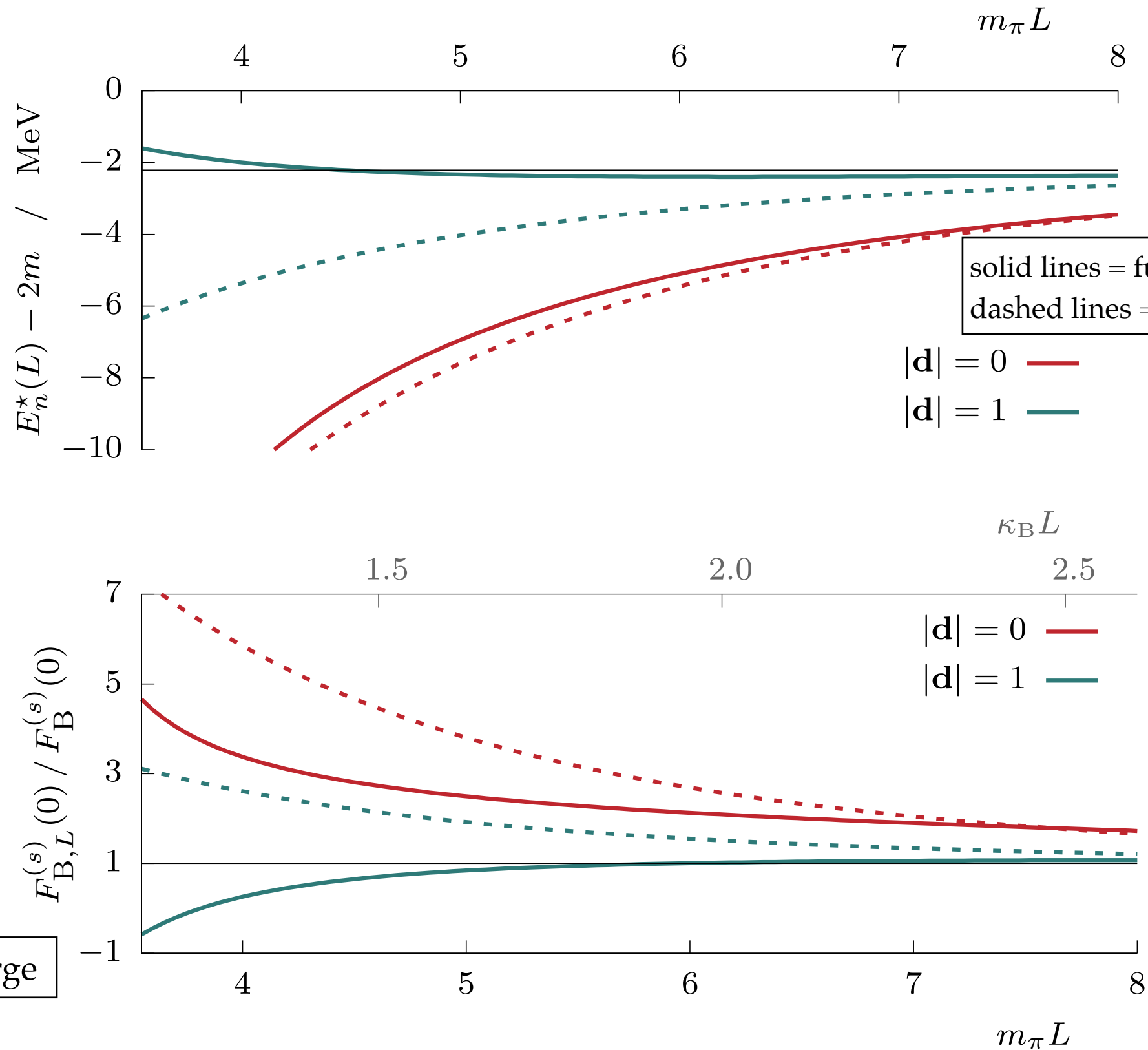
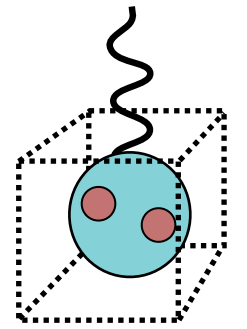

$$\sim \frac{g}{s_i - s_B} F_R(Q^2) \frac{g}{s_f - s_B}$$



Jackura (IU)

Checks: Implications for the physical deuteron

The deuteron is loosely bound, so these effects will be big

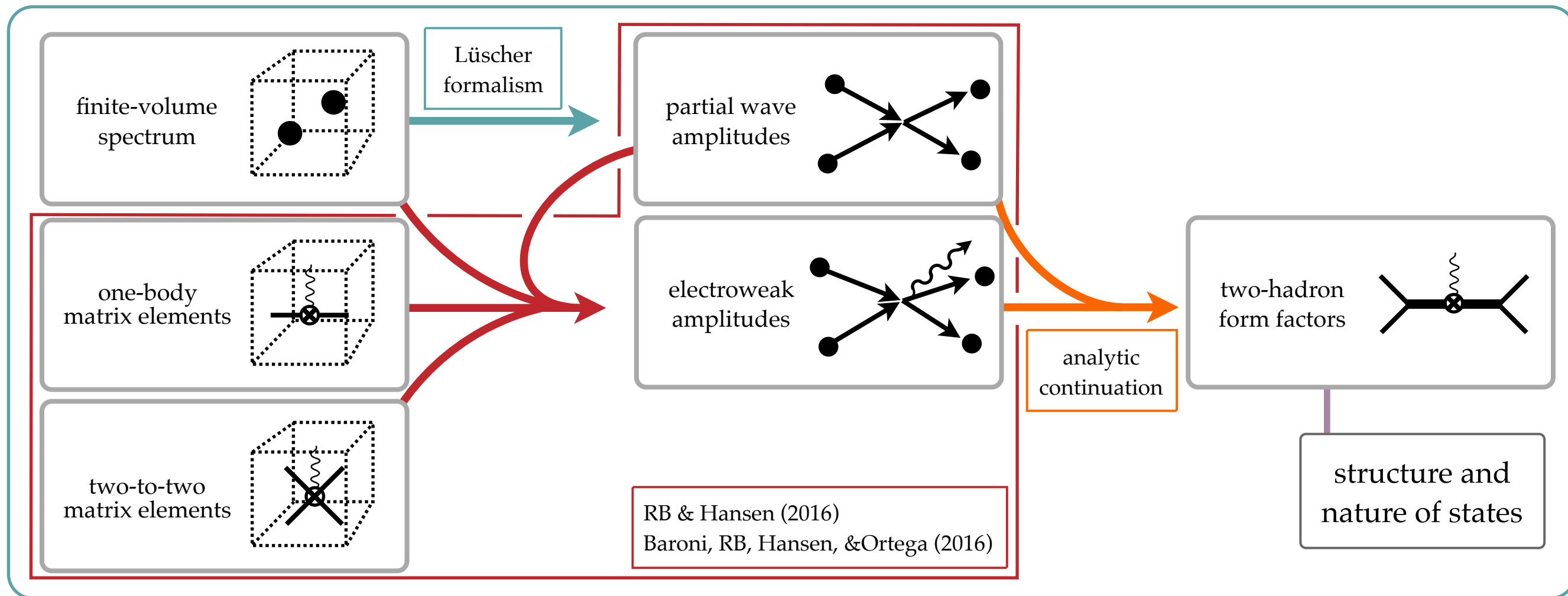


scalar charge



Jackura (IU)







Take-home message: it *can be done*



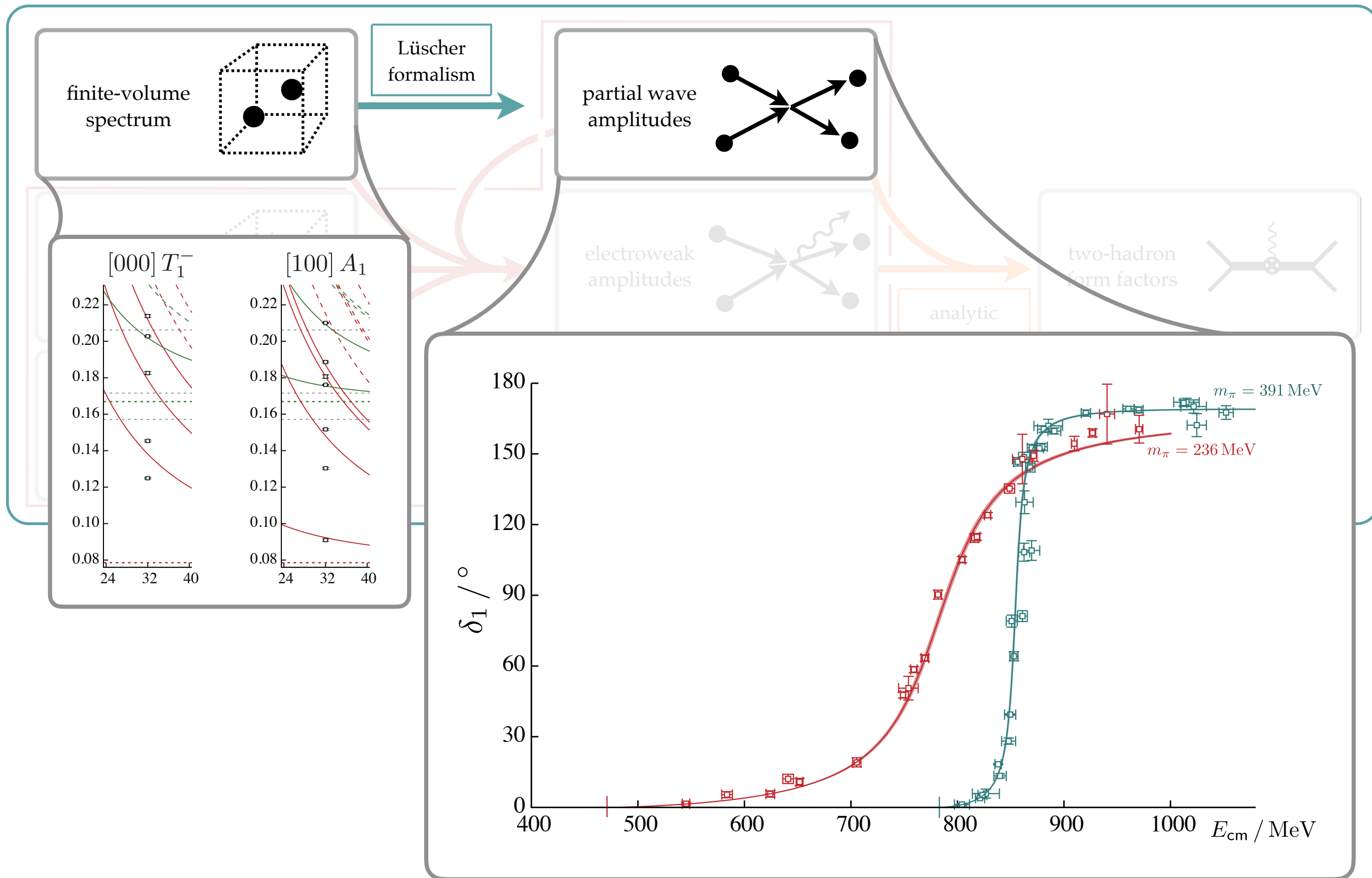
Multiple birds with one stone:

$$\langle 2 | \mathcal{J} | 2 \rangle_L \sim \mathcal{R} \cdot [\mathcal{W}_{\text{df}} + f \mathcal{M} G \mathcal{M}]$$

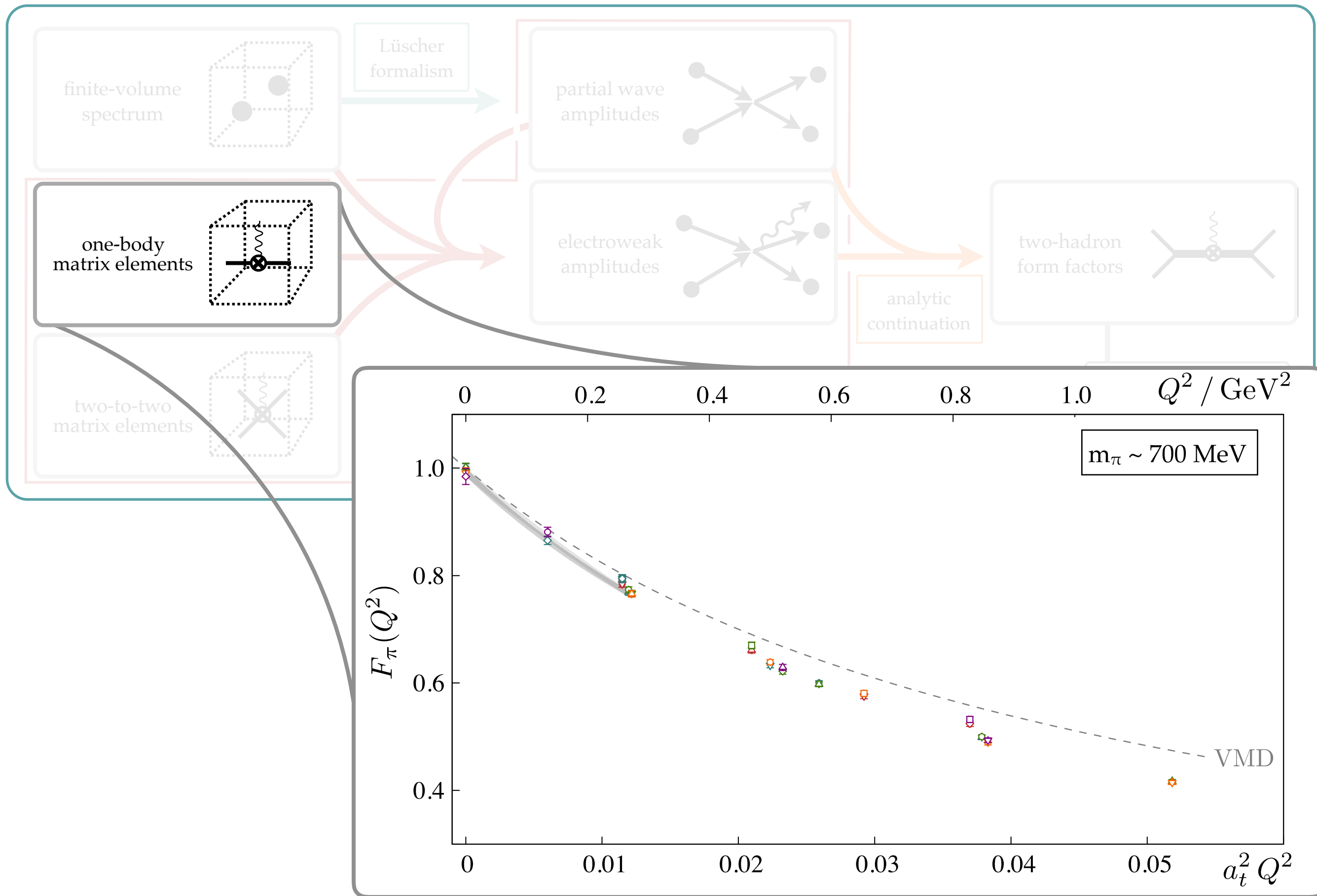
Access:

-  transition electroweak amplitudes
-  elastic electroweak amplitudes
-  structural information composite states:
 -  bound states
 -  resonance
-  remove all finite-volume systematics

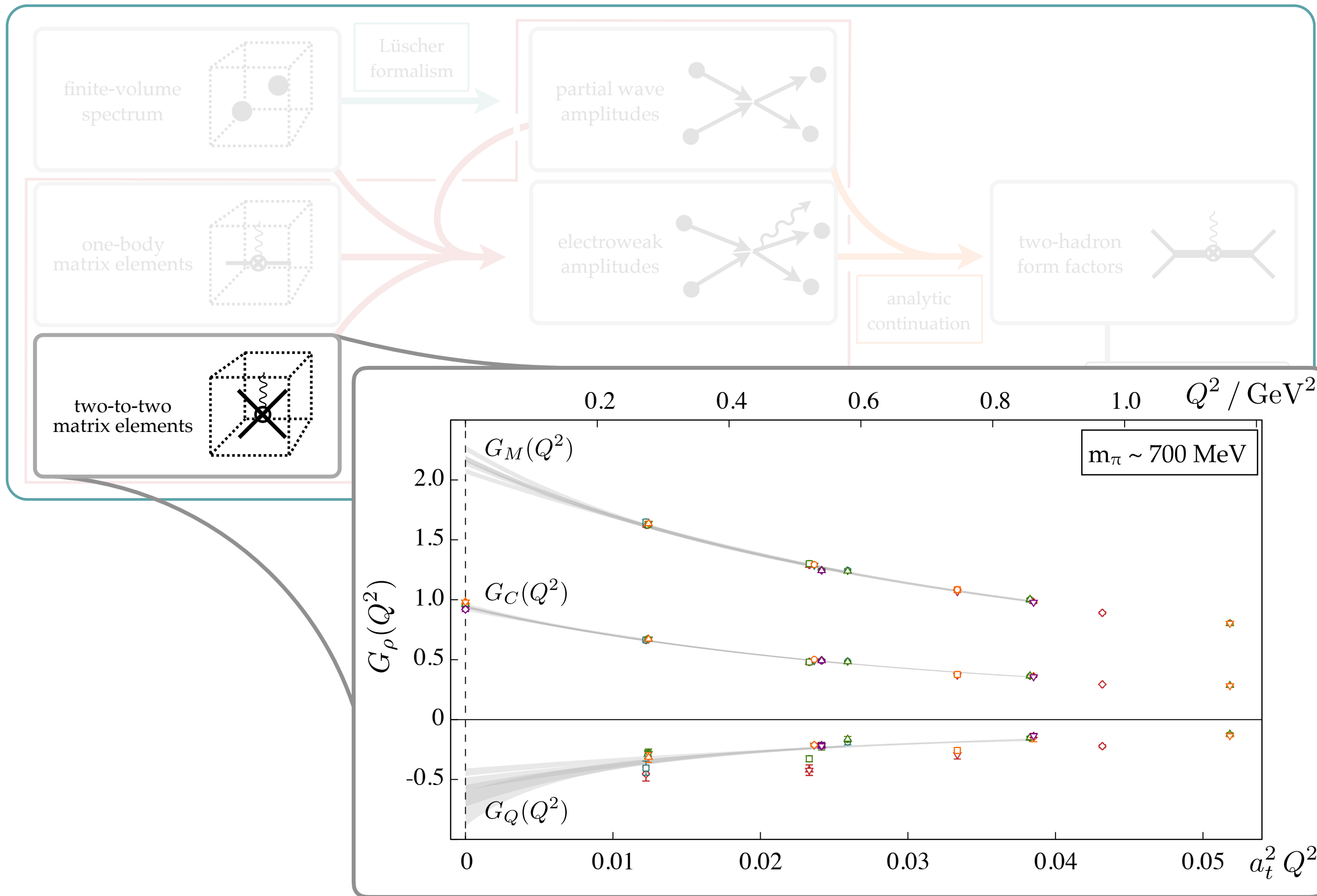
Take-home message: it *can be done*



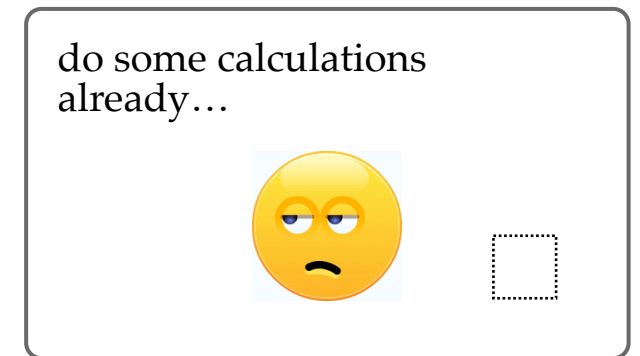
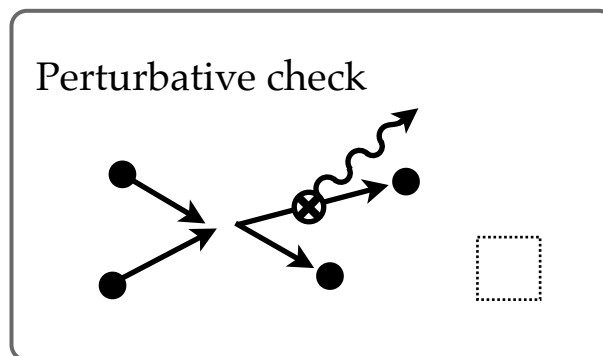
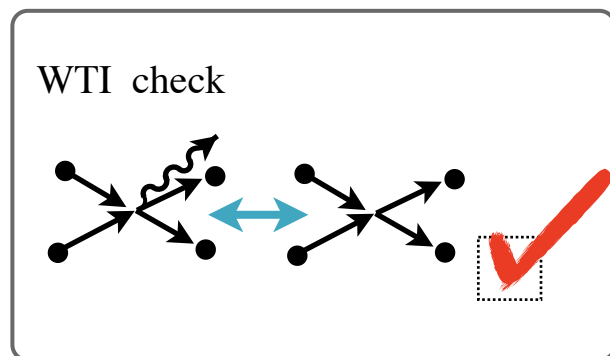
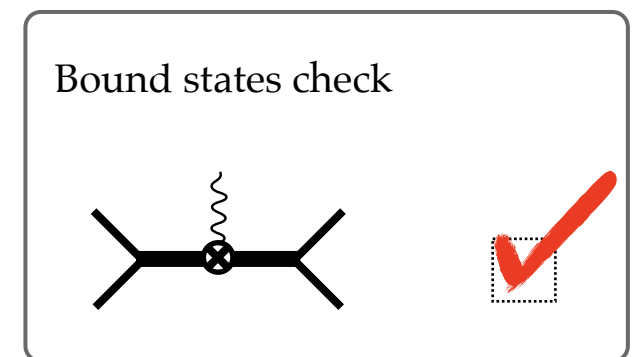
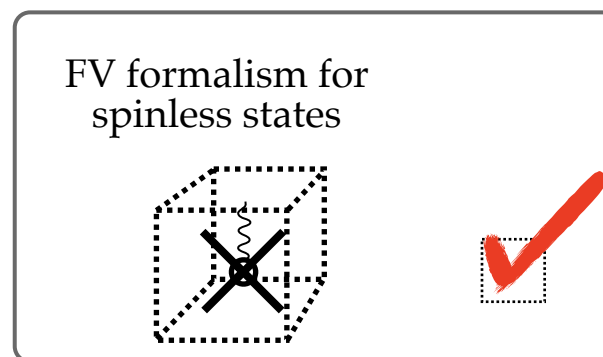
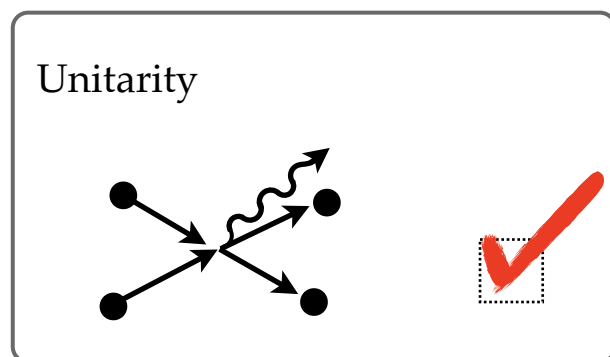
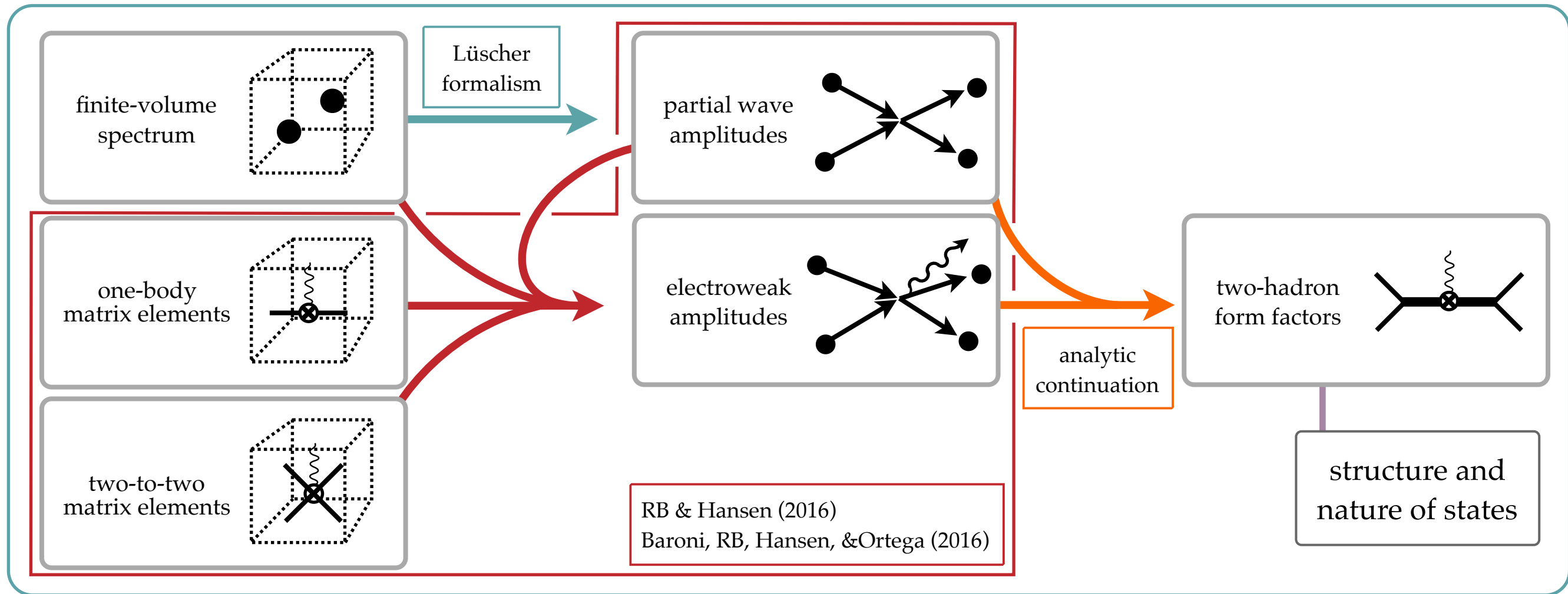
Take-home message: it *can be done*



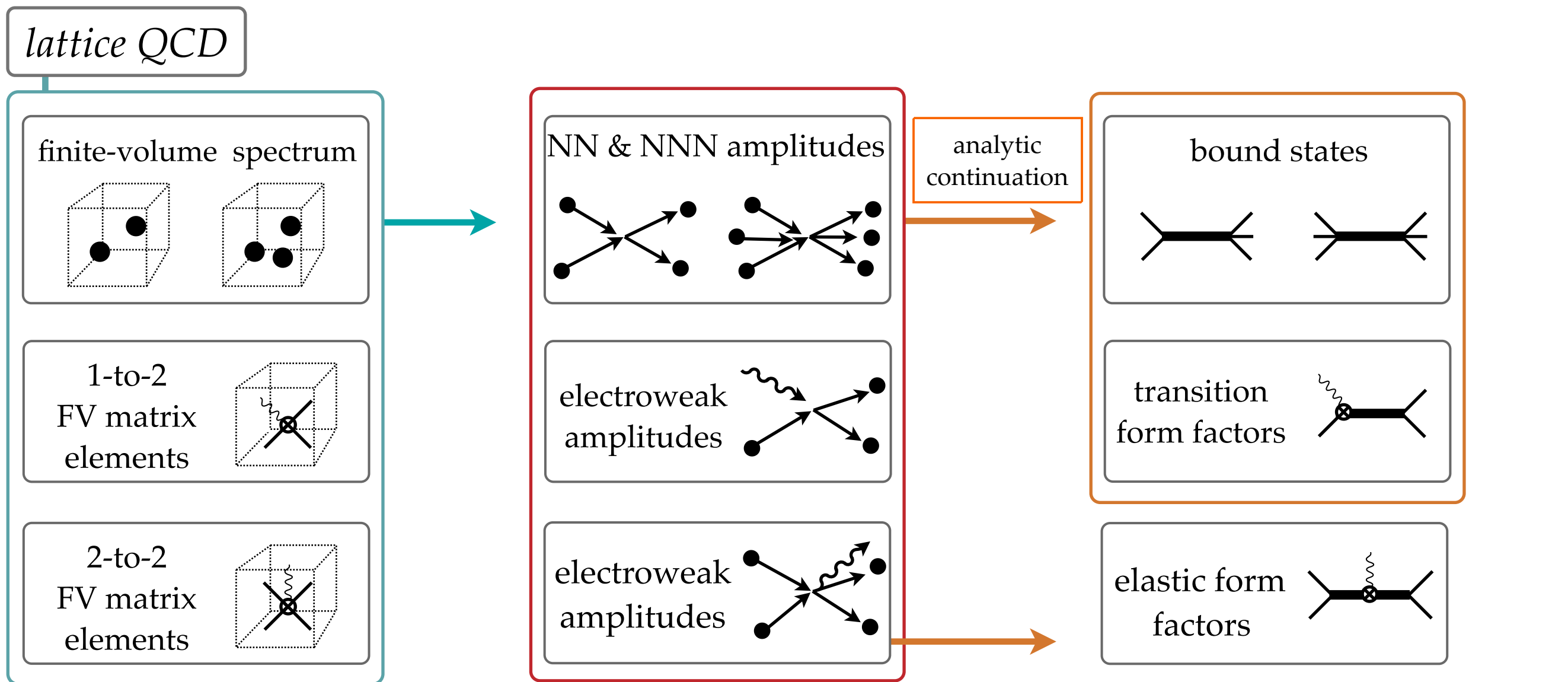
Take-home message: it *can be done*



Take-home message: it *can be done*



few-nucleons systems in LQCD



Baroni

Hansen

Jackura

Ortega

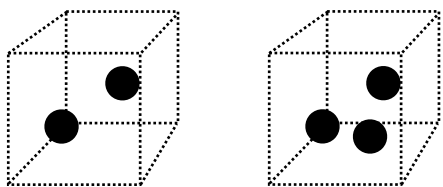


- RB & Hansen (2015)
- Baroni, RB, Hansen, Ortega (2018)

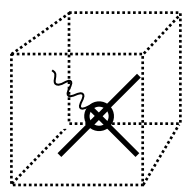
few-nucleons systems in LQCD

lattice QCD

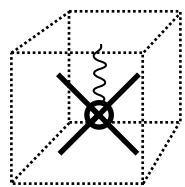
finite-volume spectrum



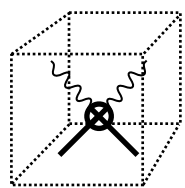
1-to-2
FV matrix
elements



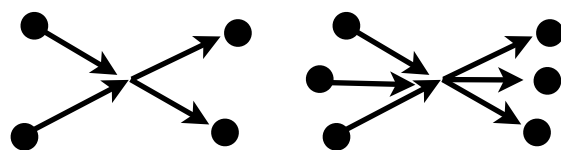
2-to-2
FV matrix
elements



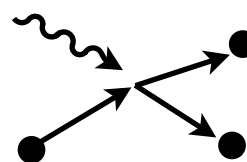
1-to-1
with two
currents



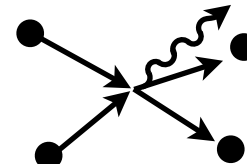
NN & NNN amplitudes



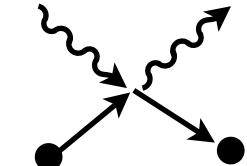
electroweak
amplitudes



electroweak
amplitudes



electroweak
amplitudes

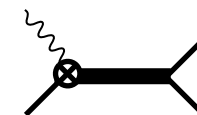


analytic
continuation

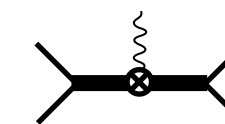
bound states



transition
form factors



elastic form
factors



Baroni

Hansen

Schindler

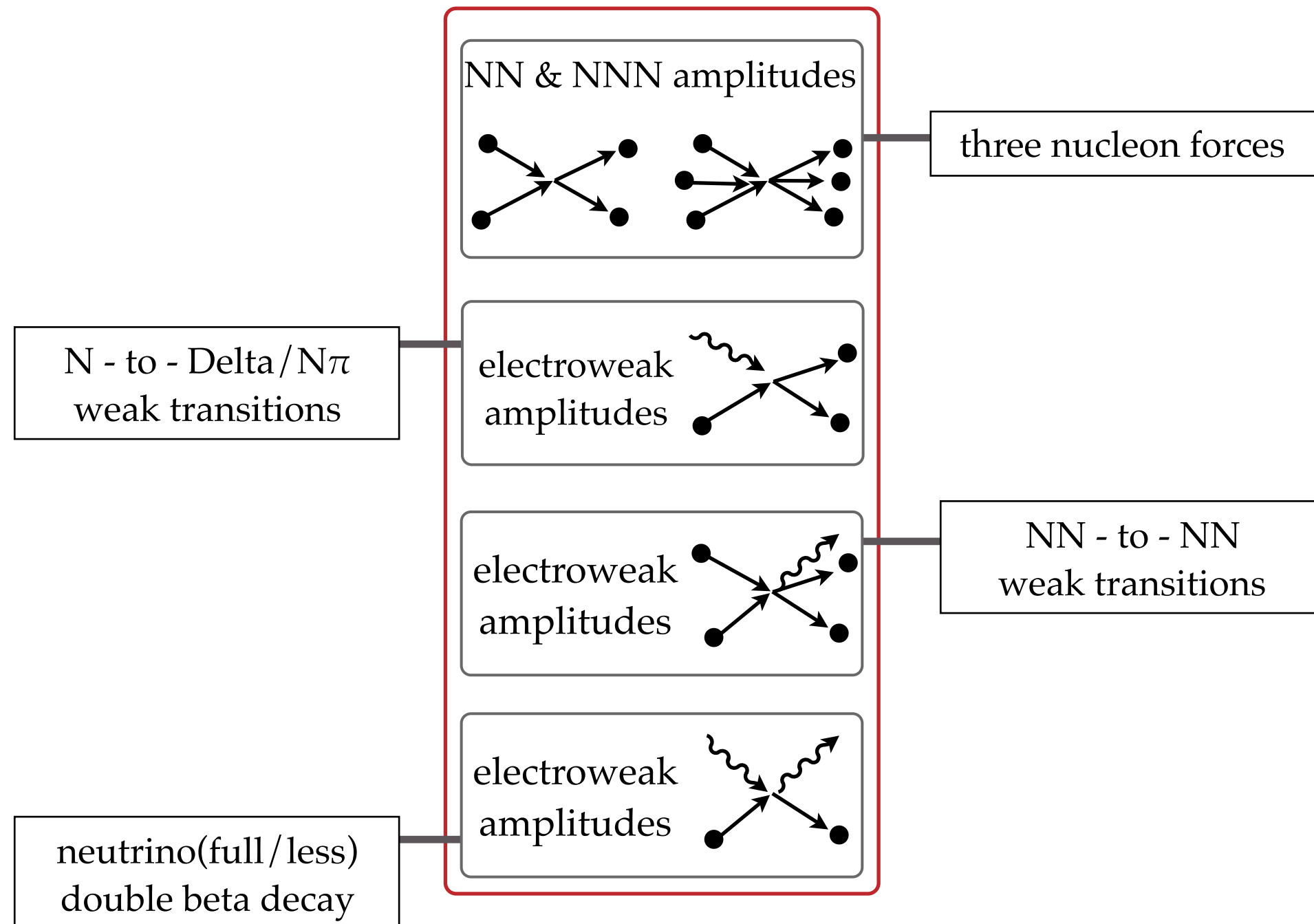


(to appear)

Davoudi

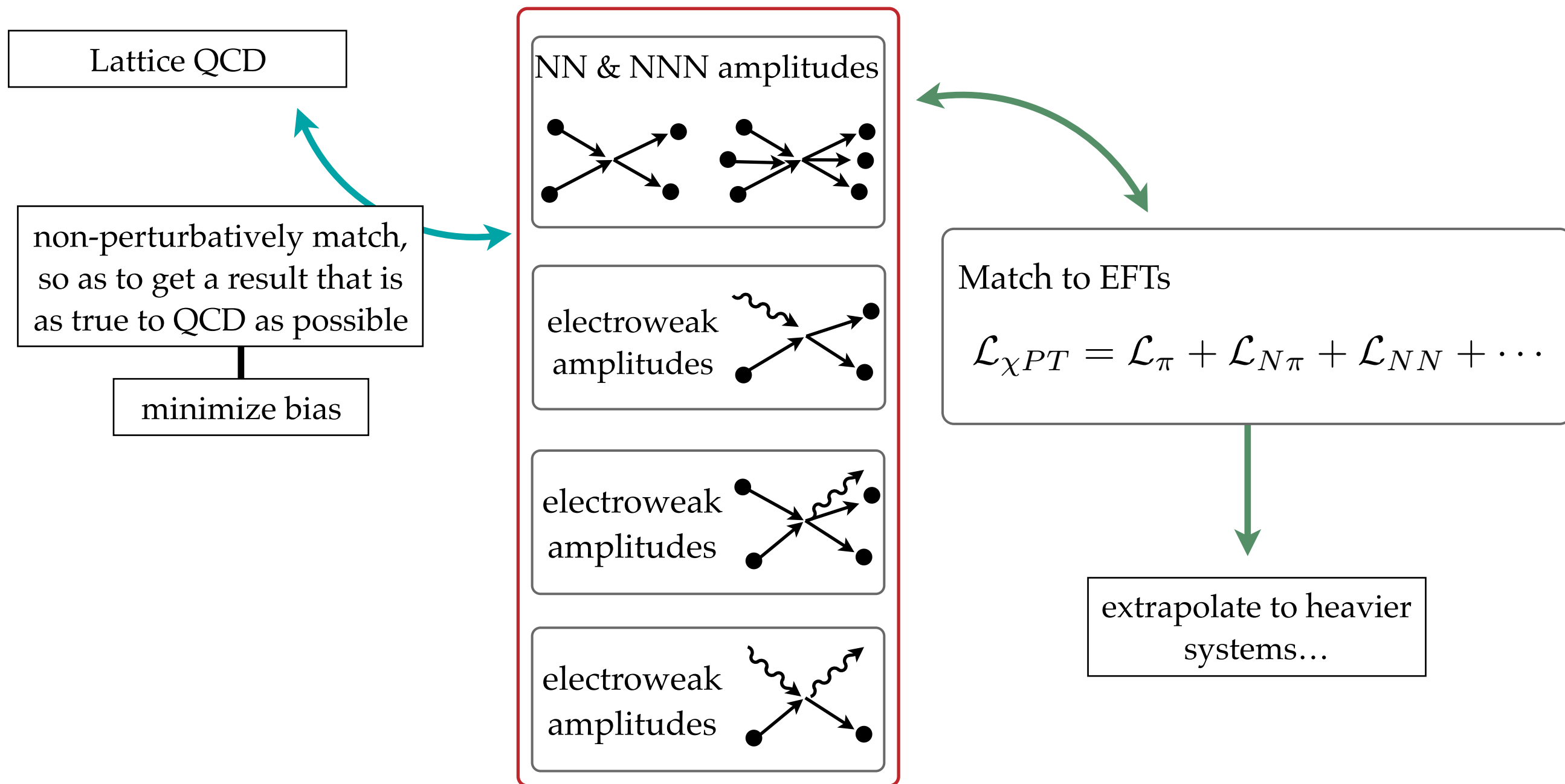
few-nucleons systems in LQCD

These techniques are being tested and implemented for $A=0$ systems first, but they are necessary and will be applied for light nuclear systems...



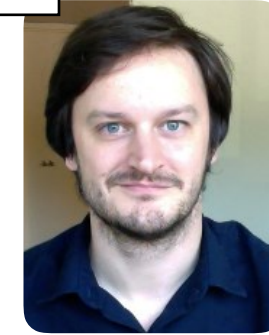
few-nucleons systems in LQCD

Extrapolating to bigger systems



Introduction to the field

Dudek



Young



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Scattering processes and resonances from lattice QCD

Raúl A. Briceño, Jozef J. Dudek, and Ross D. Young
Rev. Mod. Phys. **90**, 025001 – Published 18 April 2018

Article

References

Citing Articles (26)

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ABSTRACT

The vast majority of hadrons observed in nature are not stable under the strong interaction; rather they are resonances whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy nonperturbative region, and in addition many probes of the limits of the electroweak sector of the standard model consider processes which feature hadron