Short Range Operator Contributions to $0\nu\beta\beta$ decay from LQCD

Henry Monge-Camacho

University of North Carolina at Chapel Hill

Progress and Challenges in Neutrinoless Double Beta Decay
Trento, Italy
July 19, 2019

LBNL: Chia Cheng Chang, André Walker-Loud,

W&M and LLNL: David Brantley

BNL: Enrico Rinaldi

FZJ: Evan Berkowitz

JLab: Bálint Jóo

Liverpool Univ.: Nicolas Garron

LLNL: Pavlos Vranas,Arjun Gambhir

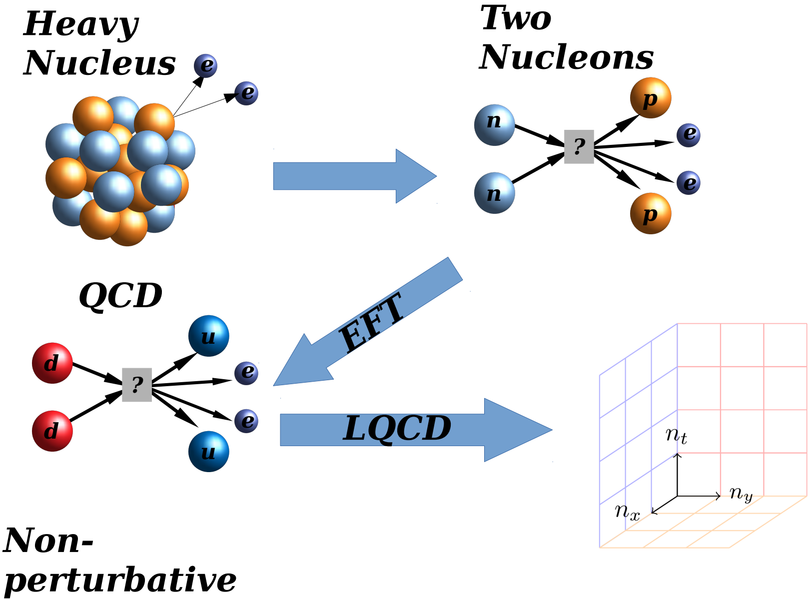
NERSC: Thorsten Kurth

UNC: Amy Nicholson

nVidia: Kate Clark

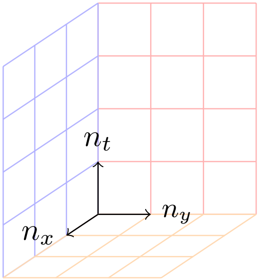
Funded by: Nuclear Theory for Double-Beta Decay and Fundamental Symmetries (DBD Collaboration, DOE)

LQCD for $0\nu\beta\beta$



LQCD Basics

Lattice QCD main idea: continous 4D space → hypercubical lattice.



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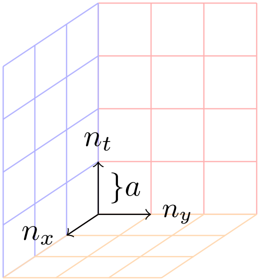

Lattice spacing acts as a UV regulator

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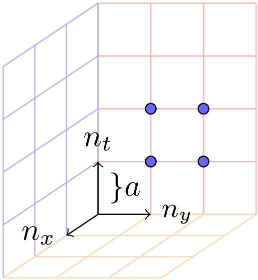

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Quark Fields defined at Lattice points

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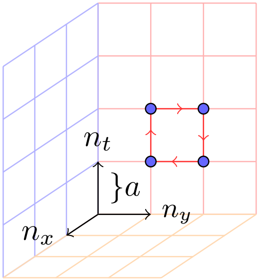

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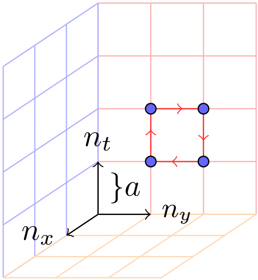

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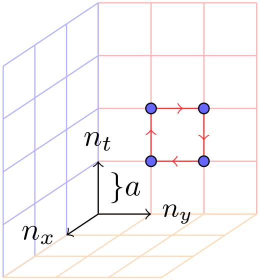

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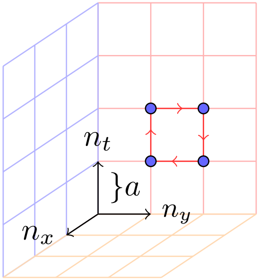
Volume is finite

$$S = \overbrace{\int d^4x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} \left(\gamma_\mu \partial^\mu + m \right) \psi \right)}^{\text{Discretize version for the lattice}}$$

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S}$$

Lattice QCD main idea: continous 4D space → hypercubical lattice.



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Lattice Observables

$$S = \overbrace{\int d^4x \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} \left(\gamma_\mu \partial^\mu + m \right) \psi \right)}^{\text{Discretize version for the lattice}}$$

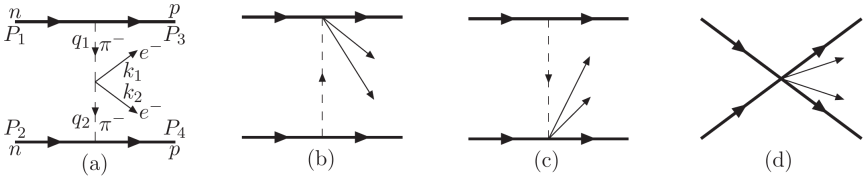
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu e^{-S} \mathcal{O}[A_\mu]$$

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S}$$

$$\langle O \rangle = \frac{1}{N_i} \sum_i \mathcal{O}[U_i]$$

Contributing Diagrams

G. Prezeau, M. Ramsey-Musolf, and P. Vogel (2003). In: *Phys. Rev.* D68, p. 034016. arXiv: hep-ph/0303205 [hep-ph] used Effective Field Theory to Integrate out heavy modes and obtain the contributing operators are found to be:



Details in W. Dekens talk

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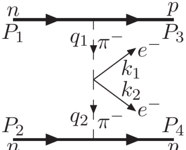
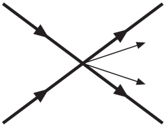


(a)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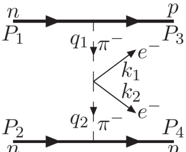

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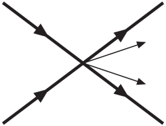
(d)

Details in W. Dekens talk

Decay Operators

$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^+ \gamma^\mu q_L)(\bar{q}_R \tau^+ \gamma^\mu q_R)$$

$$\mathcal{O}_{2\pm}^{++} = (\bar{q}_R \tau^+ q_L)\ (\bar{q}_R \tau^+ q_L) + (\bar{q}_L \tau^+ q_R)(\bar{q}_L \tau^+ q_R)$$

$$\mathcal{O}_{3\pm}^{++} = (\bar{q}_L \tau^+ q_L)(\bar{q}_L \tau^+ q_L) + (\bar{q}_R \tau^+ q_R)(\bar{q}_R \tau^+ q_R)$$

$$\mathcal{O}_{4\pm}^{++} = (\bar{q}_L \tau^+ \gamma^\mu q_L \mp \bar{q}_R \tau^+ \gamma^\mu q_R)(\bar{q}_L \tau^+ q_R - \bar{q}_R \tau^+ q_L)$$

$$\mathcal{O}_{5\pm}^{++} = (\bar{q}_L \tau^+ \gamma^\mu q_L \pm \bar{q}_R \tau^+ \gamma^\mu q_R)(\bar{q}_L \tau^+ q_R + \bar{q}_R \tau^+ q_L)$$

$$\mathcal{O}^{++}_{1+} = (\bar{q}_L\tau^+\gamma^\mu q_L)(\bar{q}_R\tau^+\gamma^\mu q_R)$$

$$\mathcal{O}^{++}_{2\pm} = (\bar{q}_R\tau^+ q_L)\,(\bar{q}_R\tau^+ q_L) + (\bar{q}_L\tau^+ q_R)(\bar{q}_L\tau^+ q_R)$$

$$\mathcal{O}^{++}_{3\pm} = (\bar{q}_L\tau^+ q_L)(\bar{q}_L\tau^+ q_L) + (\bar{q}_R\tau^+ q_R)(\bar{q}_R\tau^+ q_R)$$

Mix under renormalization

$$\mathcal{O}^{++}_{4\pm} = (\bar{q}_L\tau^+\gamma^\mu q_L \mp \bar{q}_R\tau^+\gamma^\mu q_R)(\bar{q}_L\tau^+ q_R - \bar{q}_R\tau^+ q_L)$$

$$\mathcal{O}^{++}_{5\pm} = (\bar{q}_L\tau^+\gamma^\mu q_L \pm \bar{q}_R\tau^+\gamma^\mu q_R)(\bar{q}_L\tau^+ q_R + \bar{q}_R\tau^+ q_L)$$

Suppresed by m_e

Operator Basis

$$\mathcal{O}^{++}_{1+} = (\bar{q}^a_L\tau^+\gamma^\mu q^a_L)(\bar{q}^b_R\tau^+\gamma^\mu q^b_R)$$

$$\mathcal{O}^{++}_{2+} = (\bar{q}^a_R\tau^+ q^a_L)\,(\bar{q}^b_R\tau^+ q^b_L) + (\bar{q}^a_L\tau^+ q^a_R)(\bar{q}^b_L\tau^+ q^b_R)$$

$$\mathcal{O}^{++}_{3+} = (\bar{q}^a_L\tau^+ q^a_L)(\bar{q}^a_L\tau^+ q^a_L) + (\bar{q}^a_R\tau^+ q^a_R)(\bar{q}^a_R\tau^+ q^a_R)$$

$$\mathcal{O}'^{++}_{1+} = (\bar{q}^a_L\tau^+\gamma^\mu q^b_L)(\bar{q}^b_R\tau^+\gamma^\mu q^a_R)$$

$$\mathcal{O}'^{++}_{2+} = (\bar{q}^a_R\tau^+ q^b_L)\,(\bar{q}^b_R\tau^+ q^a_L) + (\bar{q}^a_L\tau^+ q^b_R)(\bar{q}^b_L\tau^+ q^a_R)$$

V. Cirigliano, W. Dekens, J. De Vries, M. L. Graesser, E. Mereghetti, S. Pastore, and U. Van Kolck (2018). In: *Phys. Rev. Lett.* 120.20, p. 202001. arXiv: 1802.10097 [hep-ph]

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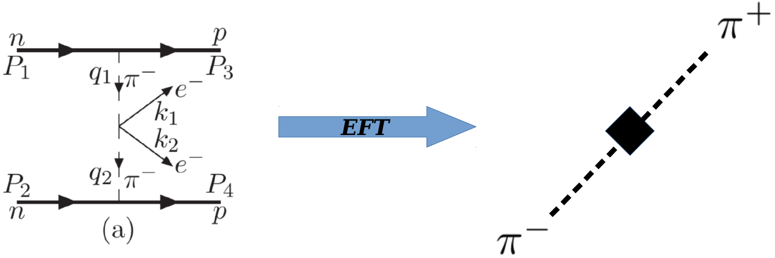
First we focus on the simplest: $\pi^- \to \pi^+$



$\langle f|\, \bar{\mathbf{q}}\boldsymbol{\Gamma}^{\mathbf{1}}\mathbf{q}\bar{\mathbf{q}}\boldsymbol{\Gamma}^{\mathbf{2}}\mathbf{q}\,|i\rangle$ calculation requires the quantity:

$$\mathcal{R}_i(t) \equiv C_i^{3pt}(t, T-t) / \left(C_\pi(t) C_\pi(T-t)\right)$$

LQCD Calculation: Two-point Function

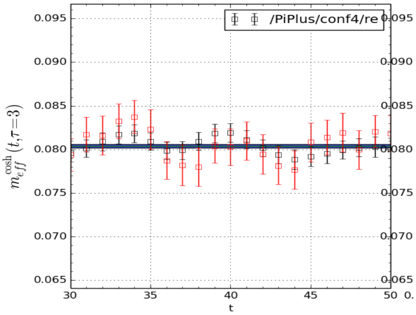
$$C_\pi(t) \equiv \langle 0|\, O_{\pi^+}(t) O^\dagger_{\pi^+}(0)\, |0\rangle = \sum_{\mathbf{x}} e^{ip\cdot x} \langle 0|\, \underbrace{\mathcal{O}(\mathbf{x},t)\mathcal{O}^\dagger(\mathbf{0},0)}_{1=|\Omega\rangle\langle\Omega| + \sum_k |k\rangle\langle k|}\, |0\rangle$$

$$C(t) = \sum_k \langle\Omega|\, O_{\pi^+}\, |k\rangle\langle k|\, O^\dagger_{\pi^+}\, |\Omega\rangle\, e^{-E_k t}$$

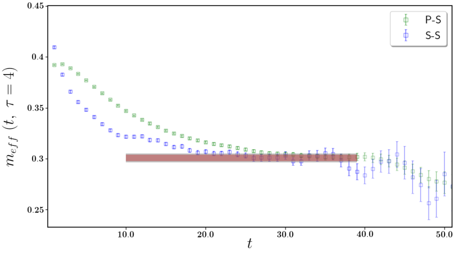
$$C_\pi(t) = \sum_k A_k \left(e^{-E_k t} + e^{-E_k(T-t)}\right) .$$

$$m^{mesons}_{eff}(t,\tau) = \frac{1}{\tau} arcosh\left(\frac{C(t+\tau)+C(t-\tau)}{2C(t)}\right)$$

$$m^{baryons}_{eff}(t,\tau) = \frac{1}{\tau} ln\left(\frac{C(t)}{C(t+\tau)}\right)$$



Ω^- $a_t m_l = -0.0830$ $a_t m_s = -0.0743$ $a_t\delta = 0.0002$



$$C^{3pt}(t_i, t_f) = \sum_{\alpha,\mathbf{x},\mathbf{y}} e^{-E_\alpha T} \langle\alpha| \Pi^+(t_f, \mathbf{x})\mathcal{O}_i(0, \mathbf{0})\Pi^+(t_i, \mathbf{y}) |\alpha\rangle$$

$$1 = |\Omega\rangle\langle\Omega| + \sum_k |k\rangle\langle k|$$

$$C_\pi(t) \approx \left|\frac{Z_0^\pi}{2E_0^\pi}\right| e^{-E_0^\pi t}$$

$$C^{3pt}(ti, tf) \approx \left|\frac{Z_0^\pi}{(2E_0^\pi)^2}\right| e^{-E_0^\pi t_i} e^{-E_0^\pi t_f} \mathcal{O}_{00}$$

$\Rightarrow$

$$\mathcal{R}_i(t) \equiv C^{3pt}(t, T-t)/\left(C_\pi(t)C_\pi(T-t)\right) = \frac{a^4 \langle\pi| \mathcal{O}_{i+}^{++} |\pi\rangle}{(a^2 Z_0^\pi)^2} + \mathcal{R}_{e.s.}(t)$$

LQCD Calculation: Three-point Function

$$C^{3pt}(t_i, t_f) = \sum_{\alpha,\mathbf{x},\mathbf{y}} e^{-E_\alpha T} \langle\alpha| \Pi^+(t_f, \mathbf{x})\mathcal{O}_i(0, \mathbf{0})\Pi^+(t_i, \mathbf{y}) |\alpha\rangle$$

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$$\mathcal{R}_i(t) \equiv C^{3pt}(t, T - t)/\left(C_\pi(t)C_\pi(T - t)\right)$$

$$= \frac{a^4 \langle\pi| \mathcal{O}_{i+}^{++} |\pi\rangle}{(a^2 Z_0^\pi)^2} + \mathcal{R}_{e.s.}(t)$$

LQCD Calculation

	$m_\pi \sim 310$ MeV		$m_\pi \sim 220$ MeV		$m_\pi \sim 130$ MeV	
a(fm)	V	$m_\pi L$	V	$m_\pi L$	V	$m_\pi L$
0.15	$16^3 \times 48$	3.78	$24^3 \times 48$	3.99		
0.12			$24^3 \times 64$	3.22		
0.12	$24^3 \times 64$	4.54	$32^3 \times 64$	4.29	$48^3 \times 64$	3.91
0.12			$40^3 \times 64$	5.36		
0.09	$32^3 \times 96$	4.50	$48^3 \times 96$	4.73		

Table: List of HISQ ensembles used for the calculation.

The corresponding 3-point correlation functions are computed as follows:

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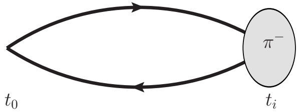

$\bar{d}\gamma^5 u \rangle$

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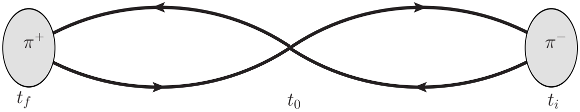

$$\langle \bar{d}\gamma^5 u \qquad \bar{d}\gamma^5 u \rangle$$

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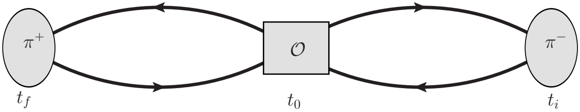
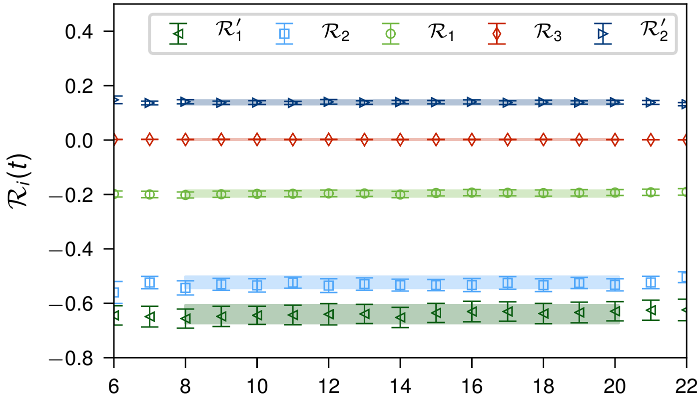


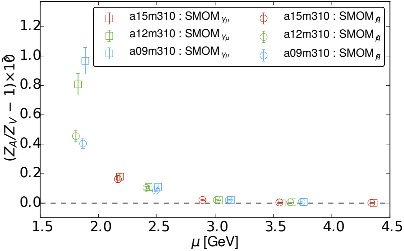
$$\langle \bar{d}\gamma^5 u \quad |\bar{u}\;\Gamma^1\; d\;\; \bar{u}\;\Gamma^2\; d| \quad \bar{d}\gamma^5 u \rangle$$

Ratio Results



$$\mathcal{R}_i(t) = \frac{a^4 \langle \pi | \mathcal{O}_{i+}^{++} | \pi \rangle}{(a^2 Z_0^\pi)^2} + \mathcal{R}_{\text{e.s.}}(t)$$

A. Nicholson et al. (2018). In: arXiv: 1805.02634 [nucl-th]
C. C. Chang et al. (2018). In: *Nature* 558.7708, pp. 91–94. arXiv: 1805.12130 [hep-lat]



$$Z^{a09}(\mu = 3 \text{ GeV})$$

$$\begin{pmatrix} 0.9483(44) & -0.0269(17) & 0 & 0 & 0 \\ -0.0236(29) & 0.9369(54) & 0 & 0 & 0 \\ 0 & 0 & 0.9209(91) & -0.0224(49) & 0 \\ 0 & 0 & -0.0230(28) & 0.9332(47) & 0 \\ 0 & 0 & 0 & 0 & 0.9017(39) \end{pmatrix}$$

Method RI-SMOM[1]:

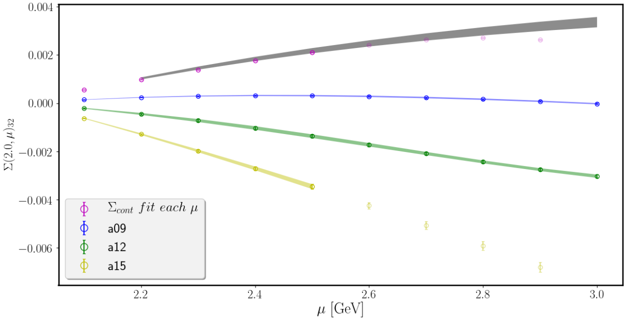
Three Lattice spacings:0.09,0.12,0.15fm

Projectors: γ and $\not{q}$ show agreement after $\overline{MS}$ conversion

Step scaling functions are used to handle reduced renormalization windows (0.15)

[1] C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, C. T. C. Sachrajda, and A. Soni (2009). In: *Phys. Rev.* D80, p. 014501. arXiv: 0901.2599 [hep-ph]

Renormalization Constants Running



Renormalization Group $\Rightarrow$ cont. running $\Sigma(\mu_1,\mu_2) = Z(\mu_1)Z(\mu_2)^{-1}$

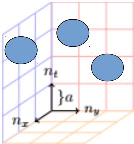
In the Lattice: $\Sigma(\mu_1,\mu_2,a) = \Sigma(\mu_1,\mu_2)_{cont} + \Delta a^2$

Fit assuming smooth μ dependence to obtain $\Sigma(\mu_1,\mu_2)_{cont}$

R. Arthur and P. A. Boyle (2011). In: *Phys. Rev.* D83, p. 114511. arXiv: 1006.0422 [hep-lat]

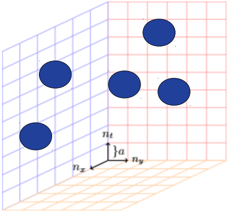
Physical Limit

- Continuum and Infinite Volume extrapolation



$\mathbf{V} \to \infty$

$\mathbf{a} \to \mathbf{0}$



- Physical pion mass limit

χPT Expansion

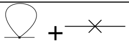
Pion Fields are included through Σ:

$$\Sigma = e^{\sqrt{2}i\phi/F} \qquad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}$$

Σ expansions tell us the relevant terms at a given order:

$$\Sigma = 1 + \frac{\sqrt{2}i\phi/F}{1!} + \frac{\left(\sqrt{2}i\phi/F\right)^2}{2!} + \frac{\left(\sqrt{2}i\phi/F\right)^3}{3!} + \cdots$$

$$\mathcal{O}^{\chi}_{1+} = Tr\left(\Sigma^\dagger \tau_L^+ \Sigma \tau_R^+\right) \qquad \mathcal{O}^{nlo}_{1+} = \frac{Tr\left(\partial_\mu \Sigma^\dagger \tau_L^+ \partial^\mu \Sigma \tau_R^+\right)}{\Lambda^2_{\chi 0}}$$

Ops	LO	NLO
$\Sigma^\dagger \tau_L^+ \Sigma \tau_R^+$ $\partial_\mu \Sigma^\dagger \tau_L^+ \partial^\mu \Sigma \tau_R^+$		

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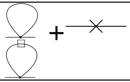
Σ expansions tell us the relevant terms at a given order:

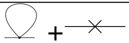
$$\Sigma = 1 + \frac{\sqrt{2} i\phi/F}{1!} + \frac{\left(\sqrt{2} i\phi/F\right)^2}{2!} + \frac{\left(\sqrt{2} i\phi/F\right)^3}{3!} + \cdots$$

$$\mathcal{O}^{\chi}_{1+} = Tr\left(\Sigma^\dagger \tau_L^+ \Sigma \tau_R^+\right) \qquad \mathcal{O}^{nlo}_{1+} = \frac{Tr\left(\partial_\mu \Sigma^\dagger \tau_L^+ \partial^\mu \Sigma \tau_R^+\right)}{\Lambda^2_{\chi 0}}$$

Ops	LO	NLO
$\Sigma^\dagger \tau_L^+ \Sigma \tau_R^+$ $\partial_\mu \Sigma^\dagger \tau_L^+ \partial^\mu \Sigma \tau_R^+$		

χPT Expansion

$$\mathcal{O}^{\chi}_{1+} = Tr\left(\Sigma^{\dagger}\tau^{+}_{L}\Sigma\tau^{+}_{R}\right) \qquad \mathcal{O}^{nlo}_{1+} = \frac{Tr\left(\partial_{\mu}\Sigma^{\dagger}\tau^{+}_{L}\partial^{\mu}\Sigma\tau^{+}_{R}\right)}{\Lambda^{2}_{\chi_0}}$$

Ops	LO	NLO
$\Sigma^{\dagger}\tau^{+}_{L}\Sigma\tau^{+}_{R}$		
$\partial_{\mu}\Sigma^{\dagger}\tau^{+}_{L}\partial^{\mu}\Sigma\tau^{+}_{R}$		

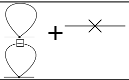
$$O_1 = \frac{\beta_1\Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2\ln(\epsilon_\pi^2) + c_1\epsilon_\pi^2 + \alpha_1\epsilon_a^2 + \alpha_1^{(4)}\epsilon_a^4 + c_1^{(4)}\epsilon_\pi^4 + m_1\epsilon_a^2\epsilon_\pi^2\right]$$

$$O_2 = \frac{\beta_2\Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + \alpha_2\epsilon_a^2 + \alpha_2^{(4)}\epsilon_a^4 + c_2^{(4)}\epsilon_\pi^4 + m_2\epsilon_a^2\epsilon_\pi^2\right]$$

$$\frac{O_3}{\epsilon_\pi^2} = \frac{\beta_3\Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{4}{3}\epsilon_\pi^2\ln(\epsilon_\pi^2) + c_3\epsilon_\pi^2 + \alpha_3\epsilon_a^2 + \alpha_3^{(4)}\epsilon_a^4 + c_3^{(4)}\epsilon_\pi^4 + m_3\epsilon_a^2\epsilon_\pi^2\right]$$

χPT Expansion

$$\mathcal{O}_{1+}^{\chi} = Tr\left(\Sigma^{\dagger}\tau_L^{+}\Sigma\tau_R^{+}\right) \qquad \mathcal{O}_{1+}^{nlo} = \frac{Tr\left(\partial_\mu\Sigma^{\dagger}\tau_L^{+}\partial^{\mu}\Sigma\tau_R^{+}\right)}{\Lambda_{\chi_0}^2}$$

Ops	LO	NLO
$\Sigma^{\dagger}\tau_L^{+}\Sigma\tau_R^{+}$ $\partial_\mu\Sigma^{\dagger}\tau_L^{+}\partial^{\mu}\Sigma\tau_R^{+}$		

$$O_1 = \frac{\beta_1\Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2\ln(\epsilon_\pi^2) + c_1\epsilon_\pi^2 + \alpha_1\epsilon_a^2 + \alpha_1^{(4)}\epsilon_a^4 + c_1^{(4)}\epsilon_\pi^4 + m_1\epsilon_a^2\epsilon_\pi^2\right] +$$

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$$\frac{O_3}{\epsilon_\pi^2} = \frac{\beta_3\Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{4}{3}\epsilon_\pi^2\ln(\epsilon_\pi^2) + c_3\epsilon_\pi^2 + \alpha_3\epsilon_a^2 + \alpha_3^{(4)}\epsilon_a^4 + c_3^{(4)}\epsilon_\pi^4 + m_3\epsilon_a^2\epsilon_\pi^2\right] +$$

Finite volume corrections are easy to add as well

Physical Results

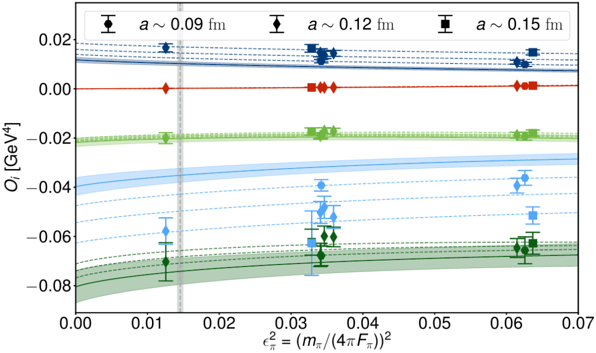

TABLE II. Resulting matrix elements extrapolated to the physical point, renormalized in RI/SMOM and $\overline{\text{MS}}$, both at $\mu = 3$ GeV.

	RI/SMOM	$\overline{\text{MS}}$
O_i[GeV]^4	$\mu = 3$ GeV	$\mu = 3$ GeV
O_1	$-1.96(14) \times 10^{-2}$	$-1.94(14) \times 10^{-2}$
O_1'	$-7.21(53) \times 10^{-2}$	$-7.81(57) \times 10^{-2}$
O_2	$-3.60(30) \times 10^{-2}$	$-3.69(31) \times 10^{-2}$
O_2'	$1.05(09) \times 10^{-2}$	$1.12(10) \times 10^{-2}$
O_3	$1.89(09) \times 10^{-4}$	$1.90(09) \times 10^{-4}$

TABLE IV. Contributions to the total uncertainties in our final results (Table II) coming from statistics, pion mass, continuum, and infinite volume extrapolations, and uncertainties in the PDG values used for $m_\pi^{\text{phys.}}$ & $F_\pi^{\text{phys.}}$ [117].

O_i	stat (%)	chiral (%)	cont. (%)	$m_\pi^{\text{phys.}}$ & $F_\pi^{\text{phys.}}$ (%)	finite vol. (%)	total (%)
O_1	5.347	2.833	3.989	5.352	0.481	9.022
O_1'	5.404	2.742	3.536	5.395	1.010	8.908
O_2	6.412	2.842	4.785	5.483	1.449	10.210
O_2'	7.027	3.309	4.868	5.602	1.661	10.870
O_3	3.442	2.507	3.396	2.245	0.310	5.899

A. Nicholson et al. (2018). In: *Phys. Rev. Lett.* 121.17, p. 172501. DOI: 10.1103/PhysRevLett.121.172501. arXiv: 1805.02634 [nucl-th]

Notebook → https://zenodo.org/record/1243313#.XTF-Ut-YVZY

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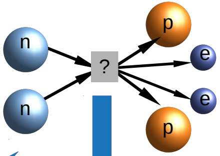
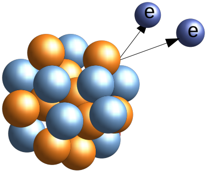


$$\langle {}^{76}\mathbf{Se}|\ \mathbf{V_{nn\to pp}}\ |{}^{76}\mathbf{Ge}\rangle$$



$$V_i^{nn\to pp}(|\mathbf{q}|) = -O_i \frac{g_A^2}{4F_\pi^2}\tau_1^+\tau_2^+ \frac{\sigma_1 \cdot \mathbf{q}\sigma_2 \cdot \mathbf{q}}{(|\mathbf{q}|^2 + m_\pi^2)^2}$$

$nn \longrightarrow pp$

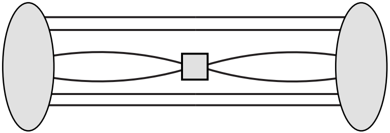
Methods are well known for current insertions between meson states:

$$\langle A| \, \bar{q}\Gamma^1 q\bar{q}\Gamma^2 q \, |B\rangle \qquad \langle A| \, \bar{q}\Gamma^1 q \, |B\rangle$$

Bilinear current insertions between nucleon are known but more complex:

$$\langle NN| \, \bar{q}\Gamma^1 q \, |NN\rangle$$

Four quark current insertions between nucleons even more challenging:



$$\sum_{x,y} \underbrace{\langle NN(y)| \, \bar{q}(x)\Gamma^1 q(x)\bar{q}(x)\Gamma^2 q(x) \, |NN(0)\rangle}_{\text{Momentum projection}}$$

all x-to-all y propagators required

Four-quark Feynman-Hellman Method: $\pi^- \to \pi^+$

Analog of method implemented for baryons and bilinear currents [2]

$\partial_\lambda E_\lambda = \langle n| H_\lambda |n\rangle \qquad S_\lambda = \lambda \int d^4x \bar{\psi}\Gamma^1\psi\bar{\psi}\Gamma^2\psi = \lambda \int d^4x \mathcal{J}(x)$

$\partial_\lambda E_\lambda$

For a meson effective mass:

$$\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} = -\frac{\partial_\lambda C(t+\tau) + \partial_\lambda C(t-\tau) - 2cosh(m_{eff}\tau)\partial_\lambda C(t)}{2\tau C(t) sinh(m_{eff}\tau)}$$

For long enough t $\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} \approx \frac{\mathcal{J}_{00}}{2E_0^2}$

$\partial_\lambda C(t)$

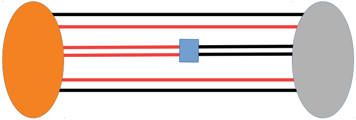
Matrix element is pulled down with ∂_λ

$$N(t) = \int d^4x \left\langle \Omega | T\mathcal{O}(t)\mathcal{J}(x)\mathcal{O}^\dagger(0)|\Omega \right\rangle$$

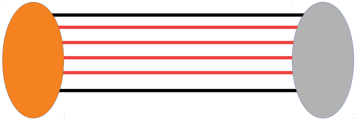
[2] C. Bouchard, C. C. Chang, T. Kurth, K. Orginos, and A. Walker-Loud (2017). In: *Phys. Rev.* D96.1, p. 014504. arXiv: 1612.06963 [hep-lat]

$$\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} = -\frac{\partial_\lambda C(t+\tau) + \partial_\lambda C(t-\tau) - 2cosh(m_{eff}\tau)\partial_\lambda C(t)}{2\tau C(t) sinh(m_{eff}\tau)}$$

$\partial_\lambda C_\lambda(t)$ →



$C(t)$ →

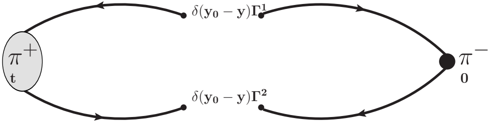


→ $\int dx^4 \mathcal{J}(x)$

C. Bouchard, C. C. Chang, T. Kurth, K. Orginos, and A. Walker-Loud (2017). In: *Phys. Rev.* D96.1, p. 014504. arXiv: 1612.06963 [hep-lat]

Brute force calculation on small Lattice:

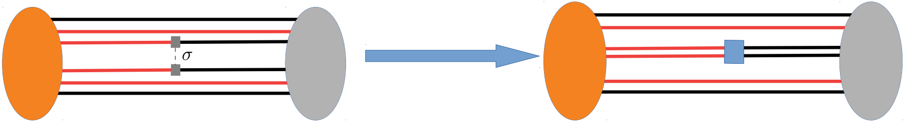
$$\int \mathbf{d^4x} \left\langle \mathbf{\Omega} | \mathbf{T} \mathcal{O}(\mathbf{t}) \mathcal{J}(\mathbf{x}) \mathcal{O}^{\dagger}(\mathbf{0}) | \mathbf{\Omega} \right\rangle = \sum_{\mathbf{y_0} \in \mathbf{V}}$$



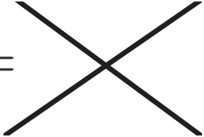
Hubbard-Stratanovich Transformation:

$$e^{-\lambda^2 \int d^4x (\overline{\psi}\Gamma\psi)^2} = \alpha \int_{-\infty}^{\infty} d\sigma e^{-\int d^4x \{ \frac{\sigma^2}{4} + \lambda i \sigma (\overline{\psi}\Gamma\psi) \}}$$

D. J. Gross and A. Neveu (1974). In: *Phys. Rev.* D10, p. 3235
R. L. Stratonovich (1957). In: *Doklady Akad. Nauk S.S.S.R.* 115, p. 1097,J. Hubbard (1959). In: *Phys. Rev. Lett.* 3, pp. 77–80



$$\int \mathbf{d}\sigma =$$



$$= \int d^4x \;\text{———}\; \mathcal{J}(x) \;\text{———}$$

Summary:

Contribution to $\pi^- \to \pi^+$ was presented

A new method is proposed to compute contributions from four-quark operators.

Next steps:

- Reproduce $\pi^- \to \pi^+$ calculation with the new method
- Implement calculation using the Hubbard-Stratanovich transformation
- Apply method to $nn \to pp$ calculation

Thanks!