Light and Heavy Hybrids

Alessandro Pilloni

Spectroscopy at EIC, December 19th, 2018



From data to the spectrum



The connection between data and resonances is not straighforward Using unconstrained functions, one risks to get random results

From data to the spectrum



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Two light 1^{-+} hybrid states?



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$\pi_1(1400)$)) $I^G(J$	$(P^{PC}) = 1^{-}(1^{-+})$
$\pi_1(1400$) MASS	1354 ± 25 MeV (S = 1.8)
$\pi_1(1400$) WIDTH	330 ± 35 MeV
Decay N	lodes	
Mode		Fraction (Γ_i / Γ)
Γ_1	$\eta \pi^0$	seen
Γ_2	$\eta\pi^-$	seen
Γ_3	$\eta'\pi$	
$\pi_1(1600)$)) $I^G(J)$	PC) = 1 ⁻ (1 ⁻⁺)
(1700	MARC	1662+8 MeV
$\pi_1(1600)$	WIDTH	$241 \pm 40 \text{ MeV} (\text{S} = 1.4)$
Decay N	lodes	Fraction (E (E)
Mode		
Г ₁	πππ	seen
Г ₂	$\rho^{\circ}\pi$	not seen
Г3 Г.	$J_2(1270)\pi$	seen
Г4 Г-	u'(058)==	seen
Г 5 Г.	η (958)π f (1285)π	seen
Γ_6	$f_1(1285)\pi$	seen

Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the *N/D* method A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472

A. Rodas, AP et al. (JPAC), 1810.04171





 $\lim_{\mathbb{P}} \int_{\pi^{-}} \int_{\pi^{-}} \int_{\mathbb{P}} \int_{\pi^{-}} \int_{\pi^{$

Production amplitude a(s)

The D(s) has only right hand cuts; it contains all the Final State Interactions constrained by unitarity \rightarrow universal

The n(s), N(s) have left hand cuts only, they depend on the exchanges \rightarrow process-dependent, smooth

Fit to P- and D-wave

A. Rodas, AP et al. (JPAC), 1810.04171



 $\chi^2/dof \sim 1.3$, statistical error estimated via 50k bootstraps

Complex plane



For the best fit solution, we look at the closest Riemann sheet in the complex plane We see a limited amount of poles in the relevant region

We also see some close-to-threshold poles, which are artifacts of the model for N(s)

How to distinguish the two?

Bootstrap



We can identify the poles in the region $m \in [1.2, 2]$ GeV, $\Gamma \in [0, 1]$ GeV

Two stable isolated poles are indentifiable in the *D*-wave Only one is stable in the *P*-wave

Final results







For example, photon collision available at EIC, C = + states available

Complementary to pion + Pomeron at COMPASS

Access to Q^2 dependence, hints on the nature

Heavy hybrids



HadSpec, JHEP 1612 (2016), 089 P. Guo *et al.*, PRD78 (2008), 056003 M. Berwein *et al.*, PRD92 (2015), 114019

Charmonium Hybrid spectrum for $\kappa = 1^{+-}$

M. Berwein, N. Brambilla, JTC, A. Vairo. Phys.Rev.D92 (2015)



- 1. Solid blue bars: Neutral exotic charmonium states (Belle, CDF, BESIII, Babar, LHCb).
- 2. Bands: Predicted masses for hybrid spin-symmetry multiplets \pm uncertainty of $\Lambda_{1^{+-}}$.

Spin-symmetry multiplets

	1	J ^{PC}	
H_1	1	$\{1^{}, (0, 1, 2)^{-+}\}$	Σ_u^- , Π_u
H_2	1	$\{1^{++}, (0, 1, 2)^{+-}\}$	Πμ
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_{u}^{-}
H_4	2	$\{2^{++}, (1, 2, 3)^{+-}\}$	Σ_u^- , Π_u

Vector Y states



Lots of unexpected $J^{PC} = 1^{--}$ states found in ISR/direct production (and nowhere else!) Seen in few final states, mostly $J/\psi \pi\pi$ and $\psi(2S) \pi\pi$

Not seen decaying into open charm pairs (...?) Large HQSS violation



Search for vectors

Vector meson photoproduction is well known, e.g. at HERA Same mechanism to be applied for *Y* states too

$$\begin{array}{c}
\gamma \\
IP \\
A \\
 A
\end{array}$$



Hybrid production cross section can be estimated using NRQCD + some modeling:

- G. Chiladze et al., PRD58 (1998), 034013 for B decays
- No calculation available yet for prompt production

BACKUP



Hadron Spectroscopy

 $\rho(770)$

 $I^{G}(J^{PC}) = 1^{+}(1^{--})$

Review:

The ho(770)

$\rho(770)\,\rm MASS$

NEUTRAL ONLY, e^+e^-	
CHARGED ONLY, $ au$ DECAYS and e^+e^-	
MIXED CHARGES, OTHER REACTIONS	
Mass <i>m</i>	
CHARGED ONLY, HADROPRODUCED	
NEUTRAL ONLY, PHOTOPRODUCED	
NEUTRAL ONLY, OTHER REACTIONS	
$m_{\rho(770)^0} - m_{\rho(770)^{\pm}}$	
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	
<i>р</i> (770) WIDTH	
p(770) width NEUTRAL ONLY, e^+e^-	
p(770) WIDTH NEUTRAL ONLY, e^+e^- CHARGED ONLY, τ DECAYS and e^+e^-	
p(770) WIDTH NEUTRAL ONLY, e^+e^- CHARGED ONLY, τ DECAYS and e^+e^- MIXED CHARGES, OTHER REACTIONS	
p(770) WIDTH NEUTRAL ONLY, e^+e^- CHARGED ONLY, τ DECAYS and e^+e^- MIXED CHARGES, OTHER REACTIONS CHARGED ONLY, HADROPRODUCED	
p(770) WIDTH NEUTRAL ONLY, e^+e^- CHARGED ONLY, τ DECAYS and e^+e^- MIXED CHARGES, OTHER REACTIONS CHARGED ONLY, HADROPRODUCED NEUTRAL ONLY, PHOTOPRODUCED	
p(770) WIDTH NEUTRAL ONLY, e^+e^- CHARGED ONLY, τ DECAYS and e^+e^- MIXED CHARGES, OTHER REACTIONS CHARGED ONLY, HADROPRODUCED NEUTRAL ONLY, PHOTOPRODUCED NEUTRAL ONLY, OTHER REACTIONS	
$\rho(770)$ WIDTH NEUTRAL ONLY, e^+e^- CHARGED ONLY, τ DECAYS and e^+e^- MIXED CHARGES, OTHER REACTIONS CHARGED ONLY, HADROPRODUCED NEUTRAL ONLY, PHOTOPRODUCED NEUTRAL ONLY, OTHER REACTIONS $\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^{\pm}}$	

775.26 \pm 0.25 MeV 775.11 \pm 0.34 MeV 763.0 \pm 1.2 MeV

 $766.5 \pm 1.1 \text{ MeV}$ $769.0 \pm 1.0 \text{ MeV}$ $769.0 \pm 0.9 \text{ MeV} (\text{S} = 1.4)$ $-0.7 \pm 0.8 \text{ MeV} (\text{S} = 1.5)$

 $5.3^{+0.9}_{-0.7}~{\rm GeV}^{-1}$

 $147.8 \pm 0.9 \text{ MeV} (\text{S} = 2.0)$ $149.1 \pm 0.8 \text{ MeV}$ $149.5 \pm 1.3 \text{ MeV}$ $150.2 \pm 2.4 \text{ MeV}$ $151.7 \pm 2.6 \text{ MeV}$ $150.9 \pm 1.7 \text{ MeV} (\text{S} = 1.1)$ $0.3 \pm 1.3 \text{ (S} = 1.4)$ 1.8 ± 2.1



Bound states on the real axis 1st sheet Not-so-bound (virtual) states on the real axis 2nd sheet



Light spectrum (1-particle correlators)



Formalism

- Process is at fixed s_{tot}, and integrated t. Interested in resonances in s
- Recoil proton kinematically decouples from final state $\eta\pi$



Expand amplitude into partial waves

$$egin{aligned} & \mathsf{A}_{\mu'\mu}(s_{tot},s,t,s_1,t_1) = \sum_{\mathsf{LM}\epsilon} \mathsf{a}^\epsilon_{\mathsf{LM},\mu'\mu}(s_{tot},t,s) Y^\epsilon_{\mathsf{LM}}(heta,\phi) \end{aligned}$$

Formalism

• The differential cross section is

$$\begin{split} \frac{d\sigma}{ds} &= \frac{1}{2(4\pi)^4 \sqrt{s}} \left(\frac{\hbar c}{m_N P_{lab}}\right)^2 \frac{1}{2} \sum_{LM\epsilon} \int_{t_-}^{t_+} dt \, |\mathbf{p}| \sum_{\mu\mu'} |a_{LM,\mu'\mu}^{\epsilon}(s_{tot},t,s)|^2 \\ &\equiv \frac{N}{\sqrt{s}} \sum_{LM\epsilon} \mathcal{I}_{LM}^{\epsilon}(s_{tot},s) \end{split}$$

where the intensity distribution is defined

$$\mathcal{I}^{\epsilon}_{LM}(s_{tot},s) = \int_{t_{-}}^{t_{+}} dt \left| \mathbf{p} \right| \sum_{\mu\mu'} |a^{\epsilon}_{LM,\mu'\mu}(s_{tot},t,s)|^2$$

Model will be compared to intensity distributions given by COMPASS

Formalism

- $\pi p \rightarrow \eta \pi p$ is high-energy peripheral process \implies pomeron dominated exchange
- Factorize pomeron-nuclear vertex
- Pomeron has effective mass $\sqrt{-t}$



• Denote $p = |\mathbf{p}|$ the momentum of the $\eta \pi$ system, and $q = |\mathbf{q}|$ the momentum of the $\pi \mathbb{P}$ system

Amplitudes for $\eta^{(\prime)}\pi$

We build the partial wave amplitudes according to the N/D method

a(s) is an effective 2 \rightarrow 2 process, where the Pomeron is treated as a vector quasi-particle with virtuality $t_{\rm eff} = -0.1~{\rm GeV^2}$



Recap: single channel $\eta\pi$

The denominator D(s) contains all the FSI constrained by unitarity \rightarrow universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi} \int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'(s'-s)} ds'$$

$$K(s) = \sum_{R} \frac{g_{R}^{2}}{M_{R}^{2} - s} \quad \text{OR} \quad K^{-1}(s) = c_{0} - c_{1}s + \sum_{i} \frac{c_{i}}{M_{i}^{2} - s}$$

$$\rho(s)N(s) = g \frac{\lambda^{(2l+1)/2} \left(s, m_{\pi}^2, m_{\eta}^2\right)}{\left(s+s_R\right)^7} \qquad n(s) = \sum_n a_n T_n\left(\frac{s}{s+s_0}\right)$$

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Numerator functions know about crossed channel dynamics unconstrained, we use a smooth model

$$\rho(s)N(s) = g \frac{\lambda^{(2l+1)/2} \left(s, m_{\pi}^2, m_{\eta}^2\right)}{\left(s+s_R\right)^7} \qquad n(s) = \sum_n a_n T_n\left(\frac{s}{s+s_0}\right)$$

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Coupled channel: the model

A. Rodas, AP et al. 1810.04171

Two channels, $i, k = \eta \pi, \eta' \pi$ Two waves, J = P, D 37 fit parameters

$$D_{ki}^{J}(s) = \left[K^{J}(s)^{-1}\right]_{ki} - \frac{s}{\pi} \int_{s_{k}}^{\infty} ds' \frac{\rho N_{ki}^{J}(s')}{s'(s'-s-i\epsilon)}$$

$$K_{ki}^{J}(s) = \sum_{R} \frac{g_{k}^{(R)} g_{i}^{(R)}}{m_{R}^{2} - s} + c_{ki}^{J} + d_{ki}^{J} s$$

$$\rho N_{ki}^J(s') = \delta_{ki} \frac{\lambda^{J+1/2} \left(s', m_{\eta^{(J)}}^2, m_{\pi}^2\right)}{\left(s'+s_R\right)^{2J+1+\alpha}} \qquad n_k^J(s) = \sum_{n=0}^3 a_n^{J,k} T_n\left(\frac{s}{s+s_0}\right)$$

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$$K_{ki}^{J}(s) = \sum_{R} \frac{g_{k}^{(R)} g_{i}^{(R)}}{m_{R}^{2} - s} + c_{ki}^{J} + d_{ki}^{J} s$$

1 *K*-matrix pole for the P-wave 2 *K*-matrix poles for the D-wave

$$oN_{ki}^{J}(s') = \delta_{ki} \frac{\lambda^{J+1/2} \left(s', m_{\eta^{(\prime)}}^{2}, m_{\pi}^{2}\right)}{\left(s' + s_{R}\right)^{2J+1+\alpha}} \qquad n_{k}^{J}(s) = \sum_{n=0}^{3} a_{n}^{J,k} T_{n}\left(\frac{s}{s+s_{0}}\right)$$

Left-hand scale (Blatt-Weisskopf radius) $s_{R} = s_{0} = 1 \text{ GeV}^{2}$

 $\alpha = 2$ as in the single channel, 3rd order polynomial for $n_k^J(s)$

Complex plane



For the best fit solution, we look at the closest Riemann sheet in the complex plane We see a limited amount of poles in the relevant region

We also see some close-to-threshold poles, which are artifacts of the model for N(s)

How to distinguish the two?

Complex plane



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Result (stat. error only)



The variance of the bootstrapped poles gives the statistical error

Poles	Mass~(MeV)	Width (MeV)
$a_2(1320)$	1306.0 ± 0.8	114.4 ± 1.6
$a_2'(1700)$	1722 ± 15	247 ± 17
π_1	1564 ± 24	492 ± 54

Again into the complex plane



The strength of the pole propagates differently in the two channels

In $\eta\pi$ the strength move to lighter values

Production (numerator) parameters

										4								100
$d^D_{\eta'\pi,\eta'\pi}$	53	-52		-50	-15	7	-17	-4	-38		-37		6	-11	-3	-6		
$d^D_{\eta\pi,\eta'\pi}$	-46	45	-44	41	24	-19	22	1	40	-39		-31	-18	23	-5	9		80
$d^D_{\eta\pi,\eta\pi}$	1	-0	-1	4	-19	32	-21	14	-10	7	-1	-11	34	-30	15	-1	-	60
$d^P_{\eta'\pi,\eta'\pi}$	6	-6	5	-4	-6	4	-4	-0	-17	19	-23	30	4	-6	4	-3	_	40 H
$d^P_{\eta\pi,\eta'\pi}$	-13	13	-13	11	3	-0	-0	4	-4	6	-12	22	1	2	-0	4		20 (
$d^P_{\eta\pi,\eta\pi}$	15	-14	13	-10	-2	9	-19	10	-25	23	-17	4	9	-4	-12	9		SACU
$c^D_{\eta'\pi,\eta'\pi}$	51	-51		-51	-2	-4	-8	-2	-39	40	-41		-2	0	-8	0		» pe
$\mathcal{C}^{D}_{\eta\pi,\eta'\pi}$	-35		-35		8	-5	5	1	37	-38		-39	-5	8	-4	4		-20 IT d
$c^{D}_{\eta\pi,\eta\pi}$	-11	11	-11	11	-2	14	-1	10	2	-2	2	-3	16	-11	13	2		40 Let
$\mathcal{C}^{P}_{\eta'\pi,\eta'\pi}$	-8	8	-6	4	8	-6	6	0	24	-25	28	-30	-1	2	1	1		-60
$\mathcal{C}^{\mathcal{P}}_{\eta\pi,\eta'\pi}$	12	-11	10	-8	-5	3	-1	-3	6	-8	15	-26	3	-6	4	-6		-80
$c^{P}_{\eta\pi,\eta\pi}$	-19	18	-17	14	-2	-5	20	-10	30	-27	21	-8	-5	-0	18	-12		100
	$a_0^{P,\eta\pi}$	$a_1^{P,\eta\pi}$	$a_2^{P,\mathfrak{n}\pi}$	$a_3^{P,\mathfrak{n}\pi}$	$a_0^{D,\eta\pi}$	$a_1^{D,\eta\pi}$	$a_2^{D,\eta\pi}$	$a_3^{D,\eta\pi}$	$a_{_0}^{P,\eta'\pi}$	$a_1^{P,\mathfrak{n}'\pi}$	$a_2^{P,\mathfrak{n}'\pi}$	$a_3^{P,\mathfrak{n}'\pi}$	$a_0^{D,\eta'\pi}$	$a_1^{D,\eta'\pi}$	$a_2^{D,\eta'\pi}$	$a_3^{D,\eta'\pi}$		-100

Denominator parameters not very correlated with the numerator ones ✓

K-matrix «bkg» parameters

							<u> </u>							_ 100
$m^{2}_{D,2}$	-10	5	-12	-77	13	21	12	-4	13	-20	-55	61		100
$g^{\scriptscriptstyle D,2}_{\eta'\pi}$	-20	-6	-21	-30	-85	96	15	2	19	-18	-81	86		80
$g^{\scriptscriptstyle D,2}_{\scriptscriptstyle\eta\pi}$	-6	-3	-11	-98	18	26	4	-1	11	-66	-34	59		
$m^{2}_{D,1}$	-58	27	4	-13	-1	9	45	-26	-15	37	-34	25		
$g^{\scriptscriptstyle D, \scriptscriptstyle 1}_{\eta' \pi}$	-1	-16	-9	-32	-18	31	-0	11	10	-35	-14	29	_	40 AL
$g^{\scriptscriptstyle D, \scriptscriptstyle 1}_{\scriptscriptstyle \eta\pi}$	-54	30	8	-14	-1	9	41	-29	-19	32	-32	24		rix
$m_{P,1}^{2}$	-10	-23	-49	-43	-35	50	7	17		-23	-52	60		²⁰ ~ po
$g^{P,1}_{\eta'\pi}$	-7	-15	18	20	9	-18	8	27	22	12	18	-23		• 01ex
$g^{\scriptscriptstyle P,1}_{\scriptscriptstyle \eta\pi}$	14	42	24	21	6	-17	21	-12	-18	17	7	-15		
Γ_{π_1}	20	68	64	-13	-8	15	-19	-68	-56	-6	-18	21		20 ITar
m_{π_1}	-22	-6		11	14	-18	12	-1	-42	5	21	-22		-40 net
$\Gamma_{a'_{_2}}$	29	-27	-17	14	2	-11	-16	30	25	19	-5	-6		ers
т _{а'2}	37	31	17	7	-12	7	-25	-26	-9	3	-16	12		60
Γ_{a_2}	-12	-11	-7	-1	3	-2	6	8	2	-8	15	-9		-80
т _{а2}	11	-8	-7	5	0	-4	-6	10	9	-6	8	-6		
	$c^P_{\mathfrak{n}_\pi,\mathfrak{n}_\pi}$	$c^P_{\eta\pi,\eta'\pi}$	$c^P_{\eta'\pi,\eta'\pi}$	$c^D_{\eta \pi,\eta \pi}$	$c^D_{\eta\pi,\eta'\pi}$	$c^D_{\eta'\pi,\eta'\pi}$	$d^P_{\eta \pi,\eta \pi}$	$d^P_{\eta\pi,\eta'\pi}$	$d^P_{\eta'\pi,\eta'\pi}$	$d^D_{\eta \pi,\eta \pi}$	$d^D_{\eta\pi,\eta'\pi}$	$d^D_{\eta^{'\pi,\eta^{'\pi}}}$		- 100

Denominator parameters uncorrelated between *P*- and *D*-wave ✓

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a a																			100
a ₂ ^{0,n} 10 10 70 35 32 57 11 10 10 68 42 682 100 655 a ₁ ^{0,n} 4 44 44 3 73 67 24 55 55 66 66 66 65 100 682 426 88 a ₀ ^{0,n} 77 77 76 64 66	$a_{3}^{D,\eta'\pi}$	6	-7	7	-8	67	1	-67	89	-2	2	-3	4	8	46	-85	100		100
$a_1^{\alpha_1\pi}$.4 .4 .4 .3 .73 .67 .24 .5 .6 .6 .65 .100 .82 .46 $a_0^{\alpha_1\pi}$.7 .7 .6 .42 .76 .66	$a_2^{D,\eta'\pi}$	-10	10	-10	10	-79	35	32	-57	11	-10	10	-8	42	-82	100	-85	_	80
$a_0^{0,n\pi}$ 7 7 6 42 76 66 46 6 6 6 100 65 42 8 $a_3^{P,n\pi}$ 31 31 32 33 66 6 90 90 90 66 6 8 4 $a_2^{P,n\pi}$ 35 35 34 9 60 90 66 66 100 66 66 100 66 66 100 100 100 90 66 66 100 100 100 90 66 66 100 <	$a_1^{D,\eta'\pi}$	-4	4	-4	3	73	-67	24	5	-5	6	-6	6	-85	100	-82	46		
$a_3^{P,\eta^*\pi}$ 31 31 32 33 -6 6 -9 3 -87 90 -96 100 -6 6 -8 4 $a_2^{P,\eta^*\pi}$ 35 35 35 34 9 -8 10 -1 98 -99 100 -96 66 -66 10 -3 $a_1^{P,\eta^*\pi}$ 37 -36 36 -34 10 9 -11 1 -100 100 -99 90 -66 66 -10 2 $a_0^{P,\eta^*\pi}$ 37 37 36 34 10 -9 11 -0 100 100 99 90 -66 66 10 -2 10 -2 -2 -67 89 -67 89 -67 89 -67 35 1 -2 -67 89 -67 55 10 -67 10 -67 10 -67 10 -67 10 -67 10 -67 10 -67 10 -67 10 -67 10 -67	$a_{0}^{D,\eta'\pi}$	7	-7	7	-6	-42	76	-66	46	6	-6	6	-6	100	-85	42	8		60
$a_2^{P,n'\pi}$ $\cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot \cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot \cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot \cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$ $\cdot \cdot \cdot$ $\cdot \cdot \cdot$ $\cdot \cdot \cdot$ $\cdot \cdot \cdot$ $\cdot \cdot $	$a_{3}^{P,\eta'\pi}$	31	-31	32	-33	-6	6	-9	3	-87	90	-96	100	-6	6	-8	4		40
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$a_2^{P,\eta'\pi}$	-35		-35	34	9	-8	10	-1	98	-99	100	-96	6	-6	10	-3		
$A_{0}^{P,\eta,\eta}$ $A_{3}^{P,\eta,\eta}$ A_{3}^{P,η,η	a ^{P,η'π}	37	-36		-34	-10	9	-11	1	-100	100	-99	90	-6	6	-10	2	-	20
$a_3^{0,\eta\pi}$ 8-88-93640-90100-01-1133465-5789 $a_2^{0,\eta\pi}$ -2020-20193-70100-9011-1110-9-662432-67 $a_1^{0,\eta\pi}$ 8-88-7-71100-7040-99-86676-67351 $a_0^{0,\eta\pi}$ -554100-71333610-109-662473-7967 $a_3^{0,\eta\pi}$ -554100-71333610-109-66-4273-7967 $a_3^{0,\eta\pi}$ -9999-99-9934-3434-33-66310-8 $a_3^{0,\eta\pi}$ -9999-99-9934-3434-33-66310-8 $a_2^{0,\eta\pi}$ -9999-99-9934-34-3434-33-66310-7 $a_3^{0,\eta\pi}$ -9999-99-99-9934-35-35-35<	a ^{P,η'π}	-37	37	-36	34	10	-9	11	-0	100	-100	98	-87	6	-5	11	-2		0
$A_2^{D,\eta\pi}$ 20 20 20 19 3 70 100 90 11 1 11 10 90 66 24 32 67 40 $A_1^{D,\eta\pi}$ 8 8 8 77 71 100 70 40 99 98 86 76 76 73 35 1 $A_0^{D,\eta\pi}$ 55 5 4 4 100 71 3 36 10 10 99 66 42 73 79 67 $A_3^{P,\eta\pi}$ 99 99 99 99 100 4 77 19 93 34 34 34 34 63 6. 35 30 10 79 67 $A_2^{P,\eta\pi}$ 100 100 99 5 8 8 20 8 36 36 36 35 32 7. 4 10 7. $A_1^{P,\eta\pi}$ 100 100 99 5 8 8 20 8 37 36 31 7. 4 10 7. $A_1^{P,\eta\pi}$ 100 100 100 99 5. 8 20 8 37 37 37 37 5. 31 7. 4 10 7.	$a_3^{D,\eta\pi}$	8	-8	8	-9		40	-90	100	-0	1	-1	3	46	5	-57	89		0
$a_1^{D,\eta\pi}$ 8-88-7-71100-7040-99-86676-67351 $a_0^{D,\eta\pi}$ -55-54100-7133610-109-6-42737967 $a_3^{P,\eta\pi}$ 999999-991004-7719-934343434-33-63310-8 $a_2^{P,\eta\pi}$ 100-100100-99-558-208-3636353277410070 $a_1^{P,\eta\pi}$ 100-1009955820837363531-774100-77 $a_1^{P,\eta\pi}$ 100-1009955820837363531-774100-77 $a_1^{P,\eta\pi}$ 100-1009955820837363531-774100-77 $a_1^{P,\eta\pi}$ 100-1009955820837373531774100-77 $a_1^{P,\eta\pi}$ 100-10099-558-20837373531774100-77 $a_1^{P,\eta\pi}$ 100-10099-558-2083737353177410061 <th>$a_2^{D,\eta\pi}$</th> <th>-20</th> <th>20</th> <th>-20</th> <th>19</th> <th>3</th> <th>-70</th> <th>100</th> <th>-90</th> <th>11</th> <th>-11</th> <th>10</th> <th>-9</th> <th>-66</th> <th>24</th> <th>32</th> <th>-67</th> <th>_</th> <th>-20</th>	$a_2^{D,\eta\pi}$	-20	20	-20	19	3	-70	100	-90	11	-11	10	-9	-66	24	32	-67	_	-20
$A_{0}^{0,\eta\pi}$ [-5] [5] [5] [-5] [4] [100 [-71] [3] [36] [10] [-10] [9] [-6] [-42] [73] [-79] [67] [47] [47] [47] [47] [47] [47] [47] [4	$a_1^{D,\eta\pi}$	8	-8	8	-7	-71	100	-70	40	-9	9	-8	6	76	-67	35	1		
$a_3^{P,\eta\pi}$ $\begin{array}{cccccccccccccccccccccccccccccccccccc$	$a_0^{D,\eta\pi}$	-5	5	-5	4	100	-71	3	36	10	-10	9	-6	-42	73	-79	67		-40
$A_2^{P,\eta\pi}$ 100 -100 100 -99 -5 8 -20 8 -36 36 -35 32 7 -4 -10 7 $A_1^{P,\eta\pi}$ -100 100 -100 99 5 -8 20 -8 37 -36 35 -31 -7 4 10 -7 $A_0^{P,\eta\pi}$ 100 -100 100 -99 -5 8 -20 8 -37 37 -35 31 7 -4 10 6	$a_3^{P,\eta\pi}$	-99	99	-99	100	4	-7	19	-9	34	-34	34	-33	-6	3	10	-8		-60
a ^{P,ηπ} -100 100 -100 99 5 -8 20 -8 37 -36 35 -31 -7 4 10 -7 a ^{P,ηπ} 100 -100 100 -99 -5 8 -20 8 -37 37 -35 31 7 -4 -10 6 Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε Ε	$a_2^{P,\eta\pi}$	100	-100	100	-99	-5	8	-20	8	-36		-35	32	7	-4	-10	7		
a ^{P,ηπ} 100 -100 100 -99 -5 8 -20 8 -37 37 -35 31 7 -4 -10 6	$a_1^{P,\eta\pi}$	-100	100	-100	99	5	-8	20	-8	37	-36		-31	-7	4	10	-7	_	-80
	$a_0^{P,\eta\pi}$	100	-100	100	-99	-5	8	-20	8	-37	37	-35	31	7	-4	-10	6		
		a ₀ P,ηπ	a ^{P,ηπ}	a ^{P,} դπ 22	a [,] P,nπ	դ ^D ,դπ	դ ^{D,11} π 1	դ ^{D,} դπ 2	$\mathbf{I}_3^{D,\eta\pi}$	ρ ^{, μ} , η'π	ρ [,] ,η'π 1	P,η'π 12	l ^{, P,η'π} 13	<i>D</i> ,ŋ'π 0	D,η'π 1	D,η'π 2	_D,η'π 3		'–100

Polynomial parameters uncorrelated between *P*- and *D*-wave ✓

																100
$m^{2}_{D,2}$	0	-19	28	27	-22	21	-2	-17		7	22	10	84	23	100	100
$g^{\scriptscriptstyle D, \mathtt{2}}_{\mathfrak{\eta}' \pi}$	-5	-3	5	-14	-18	14	-17	-20		14	30	13	30	100	23	- 80
$g^{\scriptscriptstyle D,2}_{\scriptscriptstyle \eta\pi}$	-1	-5	5	-0	-16	16	-17	-20		2	34	1	100	30	84	
$m^{2}_{D,1}$	-22	14	-24	-67	17	14	-10	-3	-6	99	-23	100	1	13	10	60
$g^{\scriptscriptstyle D,1}_{\eta^{\prime}\pi}$	14	-6	-0	9	-11	6	-12	-16	24	-20	100	-23	34	30	22	- 40
$g^{^{D,1}}_{_{\eta\pi}}$	-33	13	-19	-74	19	18	-10	-2	-7	100	-20	99	2	14	7	
$m_{P,1}^{2}$	5	-9	21	16	-38	0	-26	-51	100	-7	24	-6	49	54	51	20
$g^{P,1}_{\eta'\pi}$	-2	-2	4	-1	11	1	16	100	-51	-2	-16	-3	-20	-20	-17	0
$g^{\scriptscriptstyle P,1}_{\scriptscriptstyle \eta\pi}$	5	-16	34	13	-22	14	100	16	-26	-10	-12	-10	-17	-17	-2	
Γ_{π_1}	-8	-17		-20	8	100	14	1	0	18	6	14	16	14	21	-20
m_{π_1}	-13	5	-13	-25	100	8	-22	11	-38	19	-11	17	-16	-18	-22	-40
Γ _{a'2}	30	-14	16	100	-25	-20	13	-1	16	-74	9	-67	-0	-14	27	
т _{а'2}	2	-41	100	16	-13		34	4	21	-19	-0	-24	5	5	28	-60
Γ_{a_2}	0	100	-41	-14	5	-17	-16	-2	-9	13	-6	14	-5	-3	-19	
т _{а2}	100	0	2	30	-13	-8	5	-2	5	-33	14	-22	-1	-5	0	
	m _{a2}	Γ_{a_2}	т _{а'}	$\Gamma_{a'}^{a'}$	m_{π_1}	Γ_{π_1}	$g^{P,1}_{\eta\pi}$	$g^{P,1}_{\eta'\pi}$	$m_{P,1}^{2}$	$g^{D,1}_{\eta\pi}$	$g_{\mathfrak{n}'\pi}^{D,1}$	$m_{D,1}^{2}$	$g^{D,2}_{\eta\pi}$	$g^{D,2}_{\mathfrak{n}^{'\pi}}$	n _{D,2}	-100

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Change of functional form and parameters in the denominator

$$\rho N_{ki}^{J}(s') = g \,\delta_{ki} \,\frac{\lambda^{J+1/2} \left(s', m_{\eta^{(\prime)}}^2, m_{\pi}^2\right)}{\left(s'+s_R\right)^{2J+1+\alpha}}$$

- Default: $s_R = 1 \text{ GeV}^2$. We try $s_R = 0.8$, 1.8 GeV² •
- Default: $\alpha = 2$. We try $\alpha = 1$
- We also try a different function: $\rho N_{ki}^J(s') = g \,\delta_{ki} \, \frac{Q_J(z_{s'})}{s'^{\alpha} \lambda^{1/2}(s', m_{n'}), m_{\pi})}$ with $\alpha = 2, 1.5, 1$
- Change of parameters in the numerator
 - Default: $t_{eff} = -0.1 \text{ GeV}^2$. We try $t_{eff} = -0.5 \text{ GeV}^2$
 - Default: 3rd order polynomial. We try 4th



For each class, the maximum deviation of mass and width is taken as a systematic error Deviation smaller than the statistical error are neglected Systematic of different classes are summed in quadrature

Systematic	Poles	Mass (MeV)	Deviation (MeV)	Width (MeV)	Deviation (MeV)
		Variation of the	he function $\rho N(s')$		
	$a_2(1320)$	1306.4	0.4	115.0	0.6
$s_R = 0.8 { m GeV}^2$	$a_2'(1700)$	1720	-3	272	26
	π_1	1532	-33	484	-8
	$a_2(1320)$	1305.6	-0.4	113.2	-1.2
$s_R = 1.8 \mathrm{GeV}^2$	$a_2'(1700)$	1743	21	254	7
	π_1	1528	-36	410	-82
	$a_2(1320)$		0.0		0.0
Systematic assigned	$a_2'(1700)$		21		26
	π_1		36		82
	$a_2(1320)$	1305.9	-0.1	114.7	0.3
$\alpha = 1$	$a_2'(1700)$	1685	-37	299	52
	π_1	1506	-58	552	60
	$a_2(1320)$		0.0		0.0
Systematic assigned	$a_2'(1700)$		37		52
	π_1		58		60
	$a_2(1320)$	1304.9	-1.1	114.2	-0.2
$Q_J, \alpha = 1$	$a_2'(1700)$	1670	-52	269	22
	π_1	1511	-53	528	36
	$a_2(1320)$	1306.0	0.1	115.0	0.6
$Q_J, \alpha = 1.5$	$a_2'(1700)$	1717	-5	272	25
	π_1	1578	14	530	39
Construction of the second	$a_2(1320)$	1306.2	0.2	114.7	0.3
$Q_J, \alpha = 2$	$a_2'(1700)$	1723	1	261	15
	π_1	1570	6	508	16
	$a_2(1320)$		1.1		0.0
Systematic assigned	$a_2'(1700)$		52		25
	π_1		53		0

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		Variation of the nu	merator function $n($	s)	
	$a_2(1320)$	1305.9	-0.1	114.7	0.3
Polynomial expansion	$a_2'(1700)$	1723	1	249	2
	π_1	1563	-1	479	-13
	$a_2(1320)$		0.0	and the second second	0.0
Systematic assigned	$a_2'(1700)$		0		0
	π_1		0		0
	$a_2(1320)$	1306.8	0.8	114.1	-0.3
$t_{\rm eff} = -0.5 { m GeV}^2$	$a_2'(1700)$	1730	8	259	13
	π_1	1546	-18	443	-49
	$a_2(1320)$		0.8		0.0
Systematic assigned	$a_2'(1700)$		0		0
	π_1		0		0

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Bootstrap for $s_R = 1.8 \text{ GeV}^2$



Our skepticism about a second pole in the relevant region is confirmed: It is unstable and not trustable

$\pi_1(1600) \to \rho\pi \to \pi\pi\pi$

The strength of the Deck effect depends on the momentum transferred t, but the precise estimates rely on the model for the Deck amplitude





More complicated structure when more thresholds arise: two sheets for each new threshold

> III sheet: usual resonances IV sheet: cusps (virtual states)



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P_c photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction.



Vector meson dominance relates the radiative width to the hadronic width

$$\langle \lambda_{\psi} \lambda_{p'} | T_r | \lambda_{\gamma} \lambda_p \rangle = \frac{ \langle \lambda_{\psi} \lambda_{p'} | T_{\text{dec}} | \lambda_R \rangle}{M_r^2 - W^2 - \mathrm{i}\Gamma_r M_r} \frac{ \langle \lambda_{\mu} \lambda_p \rangle}{M_r^2 - W^2 - \mathrm{i}\Gamma_r M_r}$$

Hadronic part

- 3 independent helicity couplings,
 - \rightarrow approx. equal, $g_{\lambda_{\psi},\lambda_{p'}} \sim g$
- g extracted from total width and (unknown) branching ratio

$$\Gamma_{\gamma} = 4\pi\alpha\,\Gamma_{\psi p} \left(\frac{f_{\psi}}{M_{\psi}}\right)^2 \left(\frac{\bar{p}_i}{\bar{p}_f}\right)^{2\ell+1} \times \frac{4}{6}$$

Hiller Blin, AP et al. (JPAC), PRD94, 034002

Pentaquark photoproduction



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Interaction with nuclear media



Esposito, Guerrieri, Maiani, Piccinini, AP, Polosa, Riquer, PRD92, 034028



The interaction of (exotic) resonances with nuclear media is of primary interest to identify their nature

Also study the lineshape dependence