Thermal Photon Rates from the Hadron Gas and Emission from the Late Dilute Stages

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Helmholtz Graduate School for Hadron and Ion Research

Why Photons from the Hadron Gas?

Problem

Theoretical models fail to fully explain experimental photon results

-> Yield and v_2 cannot be described simultaneously

-> Hydrodynamic description with hadronic EoS for late and dilute stages is not successful

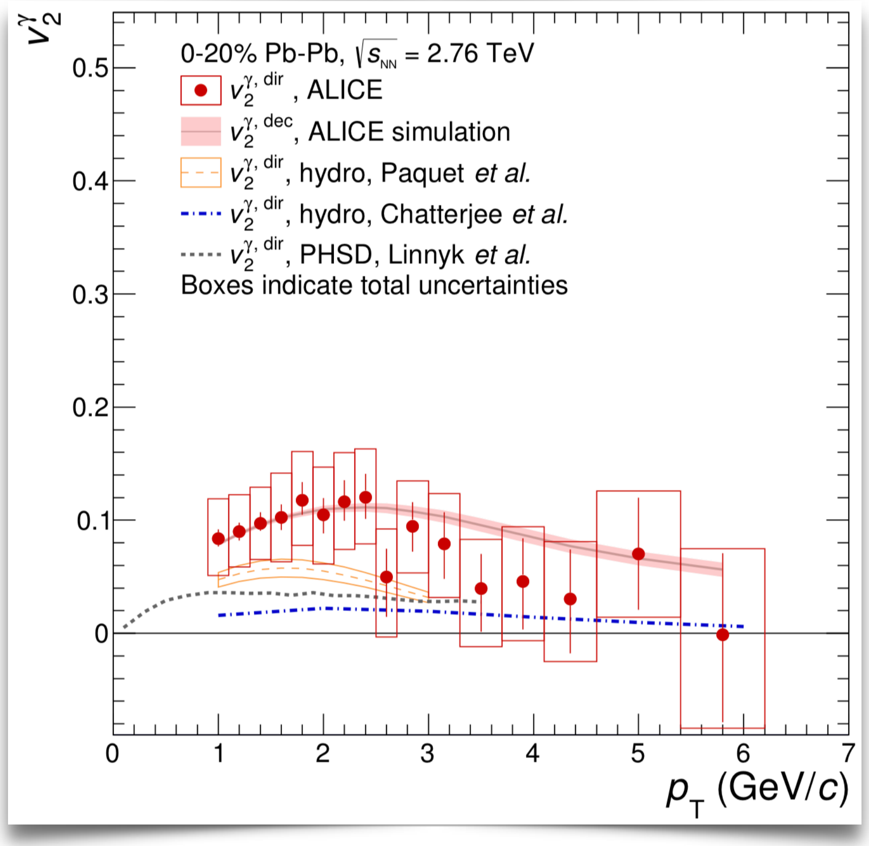
Idea

Photons from non-equilibrium hadronic phase might contribute significantly?

Solution?

Apply hybrid models (hydrodynamics + hadronic afterburner)

-> Assess importance of photons originating from non-equilibrium hadronic phase



ALICE collaboration, arXiv: 1805.04403

Photons from the Hadron Gas

Hadronic Scatterings
$2 \rightarrow 2$

Brems-
strahlung

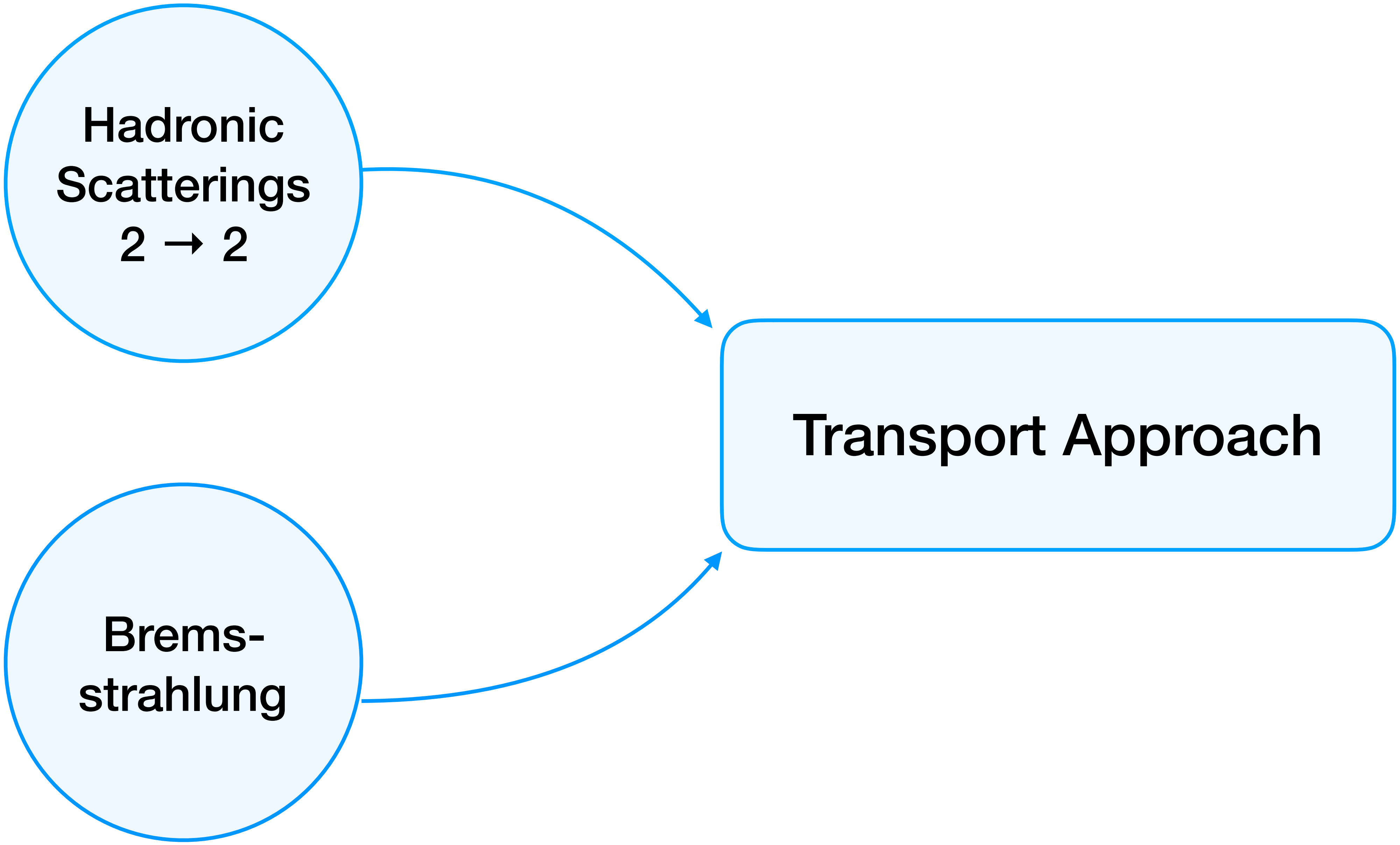
Photons from the Hadron Gas

Hadronic Scatterings
2 → 2

Brems-
strahlung

Transport Approach

Photons from the Hadron Gas



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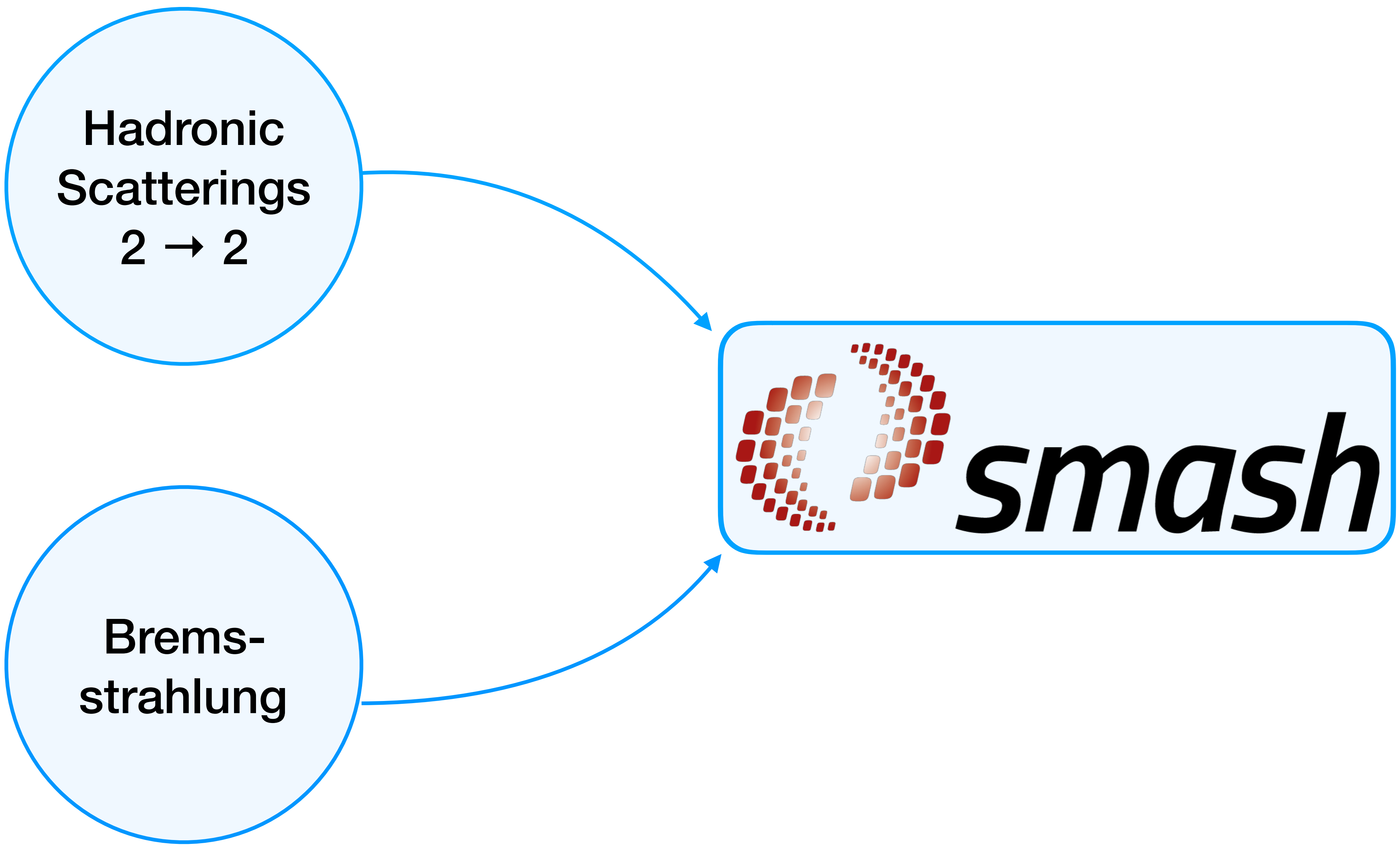

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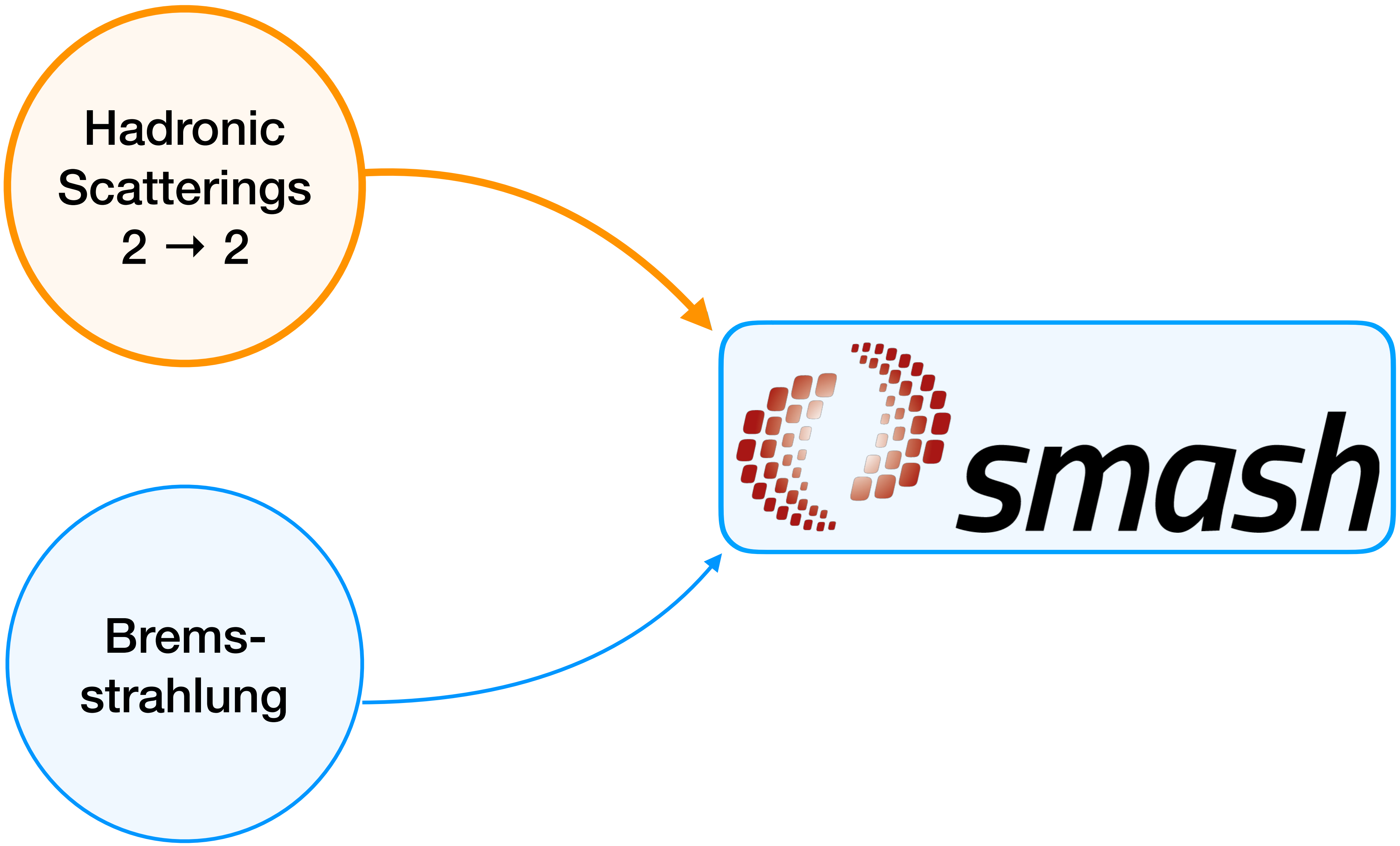

The Hadronic Transport Approach

smash

See also talk by Jan Staudenmaier

What is SMASH?

SMASH =
Simulating Many Accelerated Strongly-Interacting Hadrons

C++ based hadronic transport model

Use of modern tools for development:
git, doxygen, github, …

Features?

Effective solution of relativistic Boltzmann equation:

$$p^\mu \partial_\mu \, f + m \, \partial_{p_\mu} (F_\mu \, f) = C(f)$$

Geometric collision criterion:

$$d_{\text{coll}} < \sqrt{\frac{\sigma_{\text{tot}}}{\pi}}$$

Description of low-energy HIC, hadron gases and late, dilute stages of HIC

Dileptons and bulk properties already studied successfully

Weil et al.: Phys. Rev. C 94 (2016), no.5
Steinberg et al.: arXiv arXiv:1809.03828
Oliinychenko et al: J. Phys. G: 44 034001 (2017)
Staudenmaier et al.: Phys. Rev. C **98** (2018), no.5
Tindall et al.: Phys. Lett. B770
Rose et al.: arXiv 1709.03826

https://smash-transport.github.io

Photons in SMASH

- Photon production in binary elastic and inelastic scattering processes
- Perturbative Treatment
  - EM-coupling is small compared to strong coupling
  - Assumption: photons do not interact with medium after production

  => Photons are not actually produced, they are directly printed to a separate photon output

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Set I

- Cross sections from Kapusta et al.
  Kapusta et al. : Nucl.Phys. A544 (1992)
- Limited number of photon production channels, degrees of freedom: π, ρ, η, γ

Set II

- Cross sections based on Turbide et al. framework
  Turbide et al.: Int.J.Mod.Phys. A19 (2004)
- More evolved effective field theory

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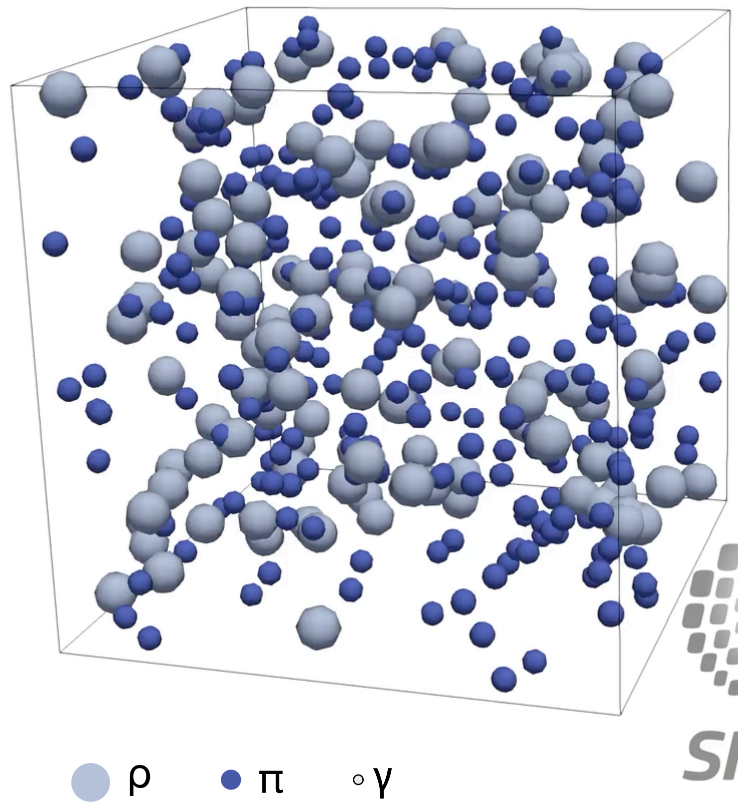
Implemented in SMASH

Photons in SMASH

Challenges

- Direct photons produced rarely throughout the evolution of heavy ion collision
- High statistics required to yield useful results



smash
By J. Mohs

Photons in SMASH

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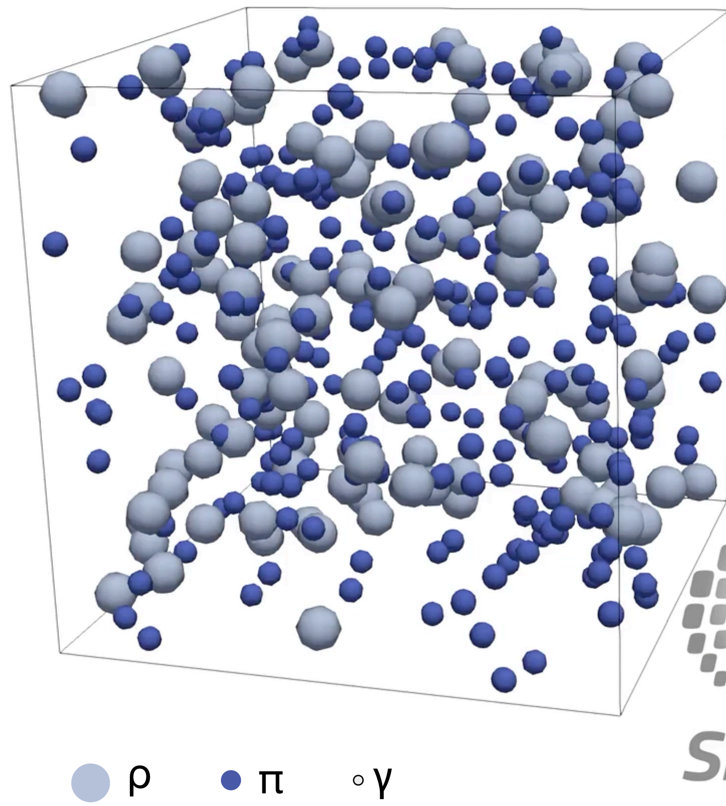
Resolution

- Produce photons perturbatively in each hadronic interaction that can potentially produce photons
- Fractional photons help to cover more phase space
- Introduce weighting factor: $W := \dfrac{\frac{d\sigma_\gamma}{dt}(t_2 - t_1)}{N_{\text{frac}}\ \sigma_{\text{had}}}$



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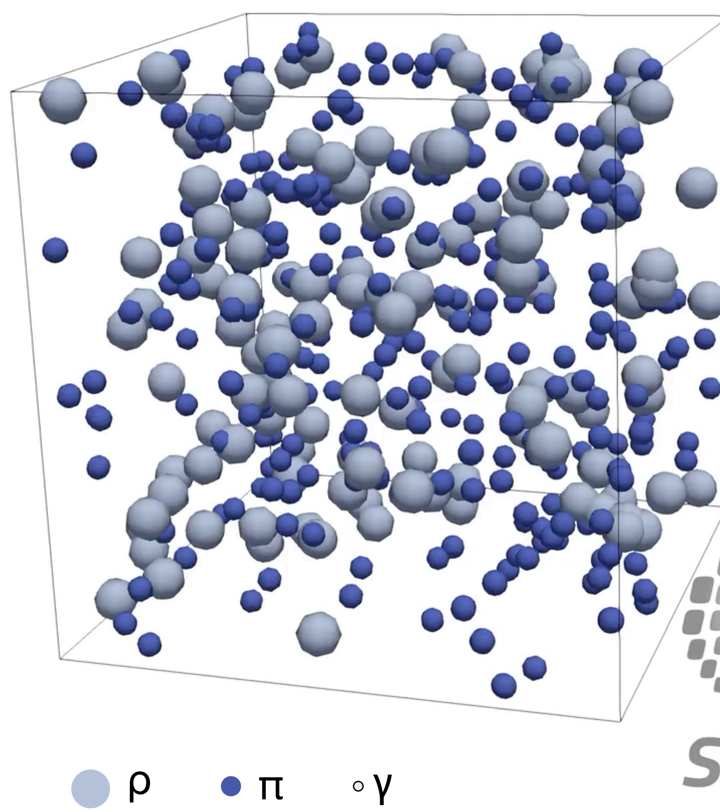

smash

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Turbide et al. Framework

- Chiral effective field theory with mesonic degrees of freedom:
  - Pseudoscalar mesons
  - Vector mesons
  - Axial vector mesons
  - Photon

Turbide et al.: Int.J.Mod.Phys. A19 (2004)

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Turbide et al.: Int.J.Mod.Phys. A19 (2004)

- More realistic description than Kapusta et al. theory
- Cross sections not yet published or determined
  => Proper calculation necessary
- Used in state-of-the-art hydrodynamics code (MUSIC)
  => Hybrid model made up of SMASH and MUSIC
- Further extensions possible, e.g.
  Heffernan, Hohler, Rapp: Phys. Rev. C 91 (2015)
  Holt, Hohler, Rapp: Nuc. Phys. A 945 (2016)

Turbide et al. Lagrangian

$$\mathcal{L} = \frac{1}{8} F_\pi^2 \, Tr\left(D_\mu U D^\mu U^\dagger\right) + \frac{1}{8} F_\pi^2 \, Tr\left(M\left(U + U^\dagger - 2\right)\right) - \frac{1}{2} Tr\left(F^L_{\mu\nu} F^{L\mu\nu} + F^R_{\mu\nu} F^{R\mu\nu}\right)$$

kinetic term & interaction PS — mass term PS — kinetic term & interaction of V and AV

$$+ m_0^2 \, Tr\left(A^L_\mu A^{L\mu} + A^R_\mu A^{R\mu}\right) + \gamma \; Tr\left(F^L_{\mu\nu} U F^{R\mu\nu} U^\dagger\right) - i\xi \, Tr\left(D_\mu U D_\nu U^\dagger F^{L\mu\nu} + D_\mu U^\dagger D_\nu U F^{R\mu\nu}\right)$$

mass term V and AV — higher order interaction of PS, V and AV

$$- \frac{2em_0^2}{g_0} B^\mu \, Tr\Big(Q\left(A^L_\mu + A^R_\mu\right)\Big) - \frac{1}{4}\left(\partial_\mu B^\nu - \partial_\nu B^\mu\right)^2 + \frac{2e^2 m_0^2}{g_0^2} B_\mu B^\mu Tr\left(Q^2\right)$$

electromagnetic interaction of PS, V, AV and γ

$$+ g_{VV\psi} \, \varepsilon_{\mu\nu\alpha\beta} \, Tr\Big[\partial^\mu V^\nu \partial^\alpha V^\beta \psi\Big]$$

Wess-Zumino Term for V-V-PS-vertex

PS = pseudoscalar mesons
V = vector mesons
AV = axial vector mesons

Turbide et al.: Int.J.Mod.Phys. A19 (2004)

Photon Production Channels

Exchange via (π,ρ,a_1)

Exchange via (ω)

Turbide et al.: Int.J.Mod.Phys. A19 (2004)

Photon Production Channels

Exchange via (π,ρ,a_1)

- $\pi^\pm + \rho^0 \rightarrow \pi^\pm + \gamma$
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Photon Production Channels

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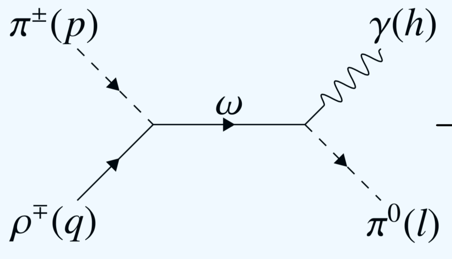
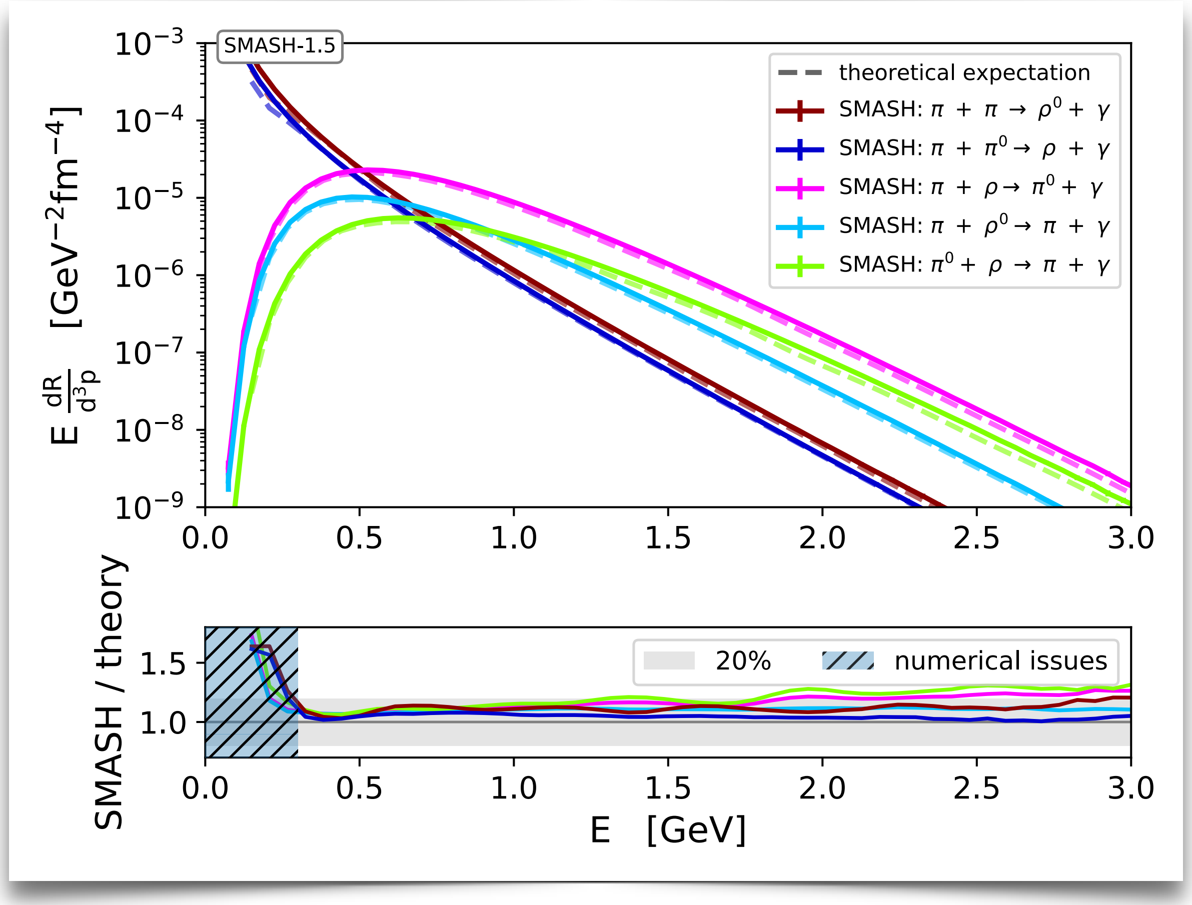
Example:

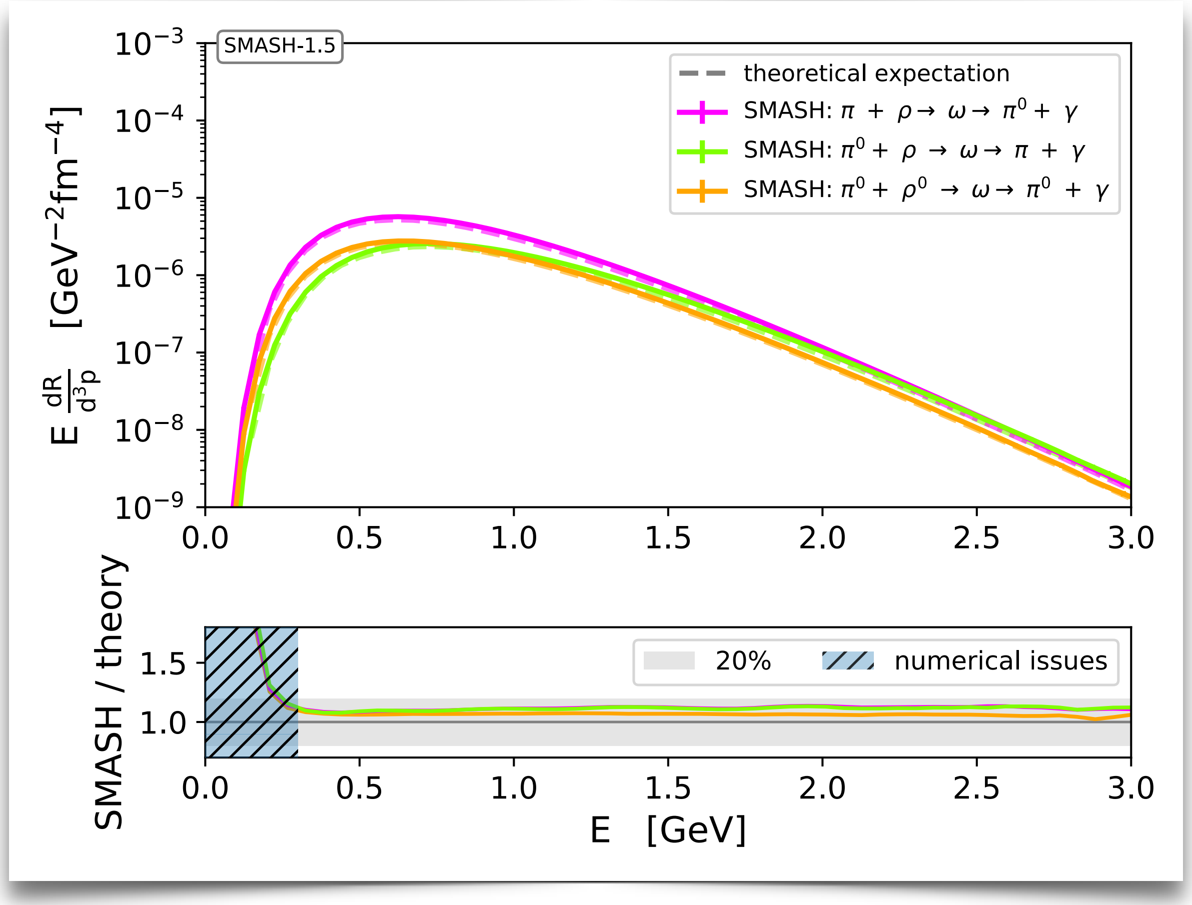


$$\rightarrow |\mathcal{M}|^2 \rightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}t} \rightarrow \sigma$$

Turbide et al.: Int.J.Mod.Phys. A19 (2004)

Proof of Concept





Thermal photon rate: number of photons produced per unit time and volume

-> Allows for comparison to theoretical expectation

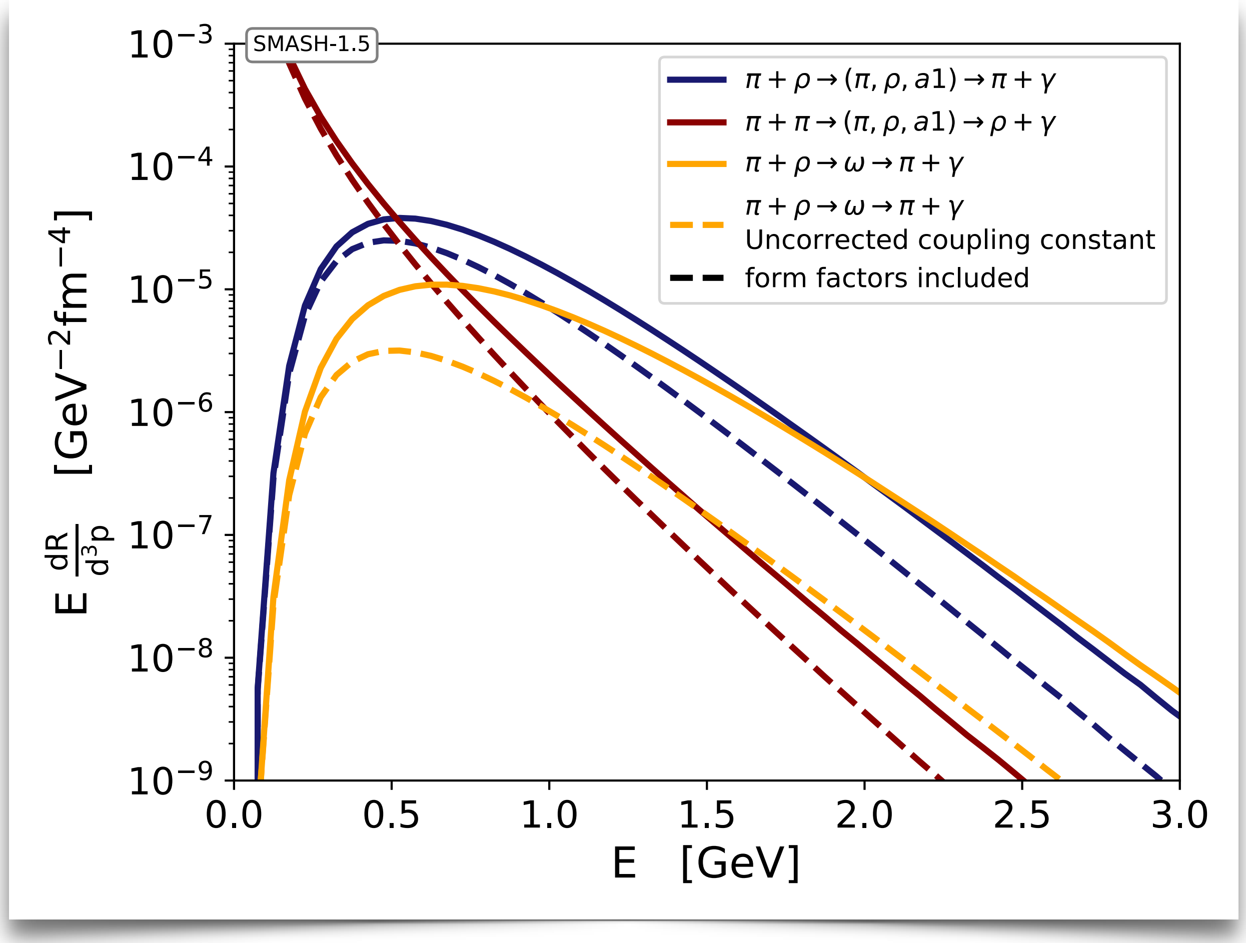
SMASH setup: Infinite matter simulation

-> Hadronic matter in thermal and chemical equilibrium at T = 200 MeV

Form Factors

Form Factor:

- Necessary to account for finite extend of the fields at vertices
- Hadronic dipole form factor (global and photon energy-dependent)
- Different for (π,ρ,a_1) and (ω) mediated processes
- Reduction of photon rate (except for ω channels after correcting coupling constant)



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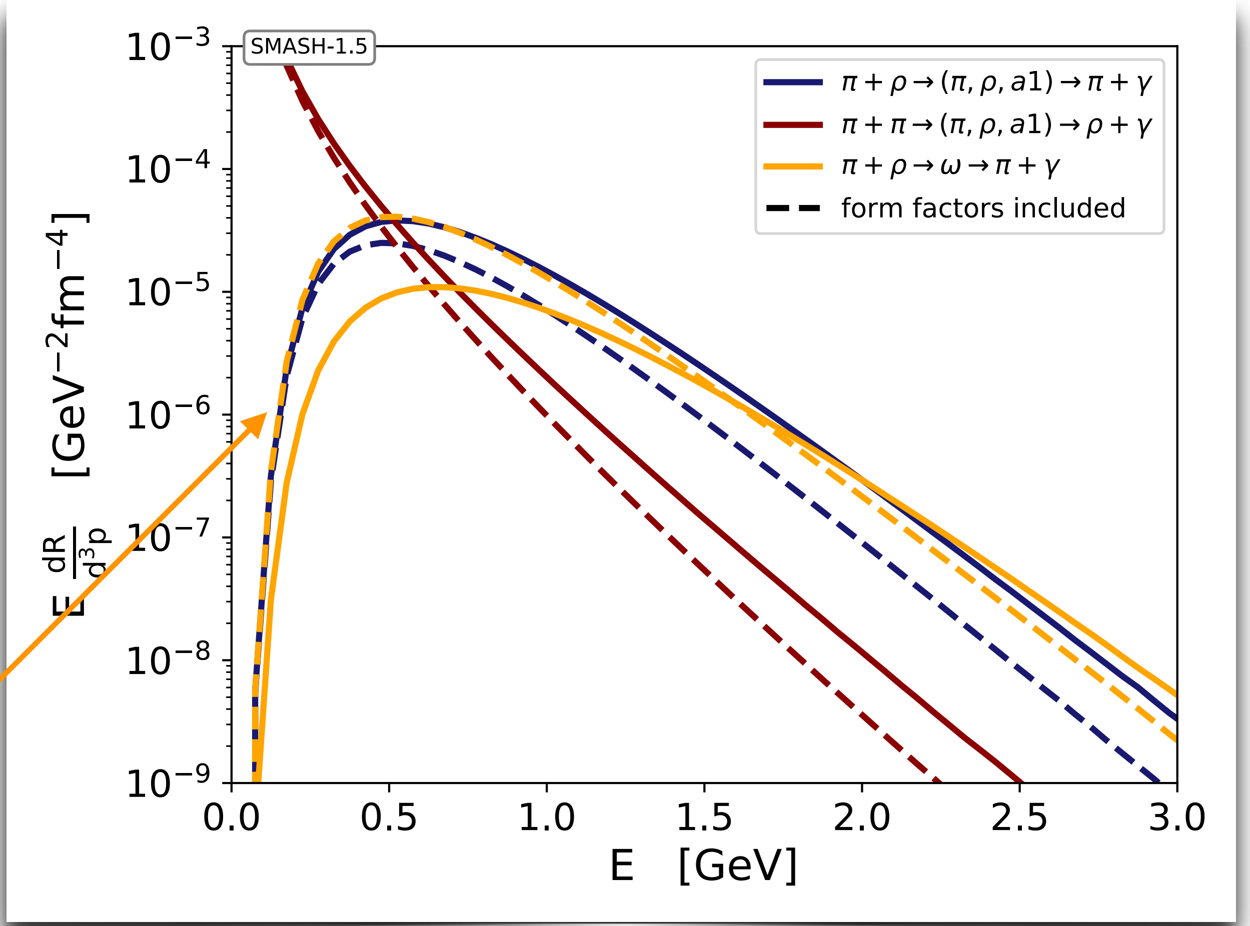

Note:
π-ρ-ω-coupling increases when applying form factors

Photon Rates from SMASH

Contribution of **individual**
$\pi + \pi \to \rho + \gamma$ and
$\pi + \rho \to \pi + \gamma$ channels

Contribution of **combined**
$\pi + \pi \to \rho + \gamma$ and
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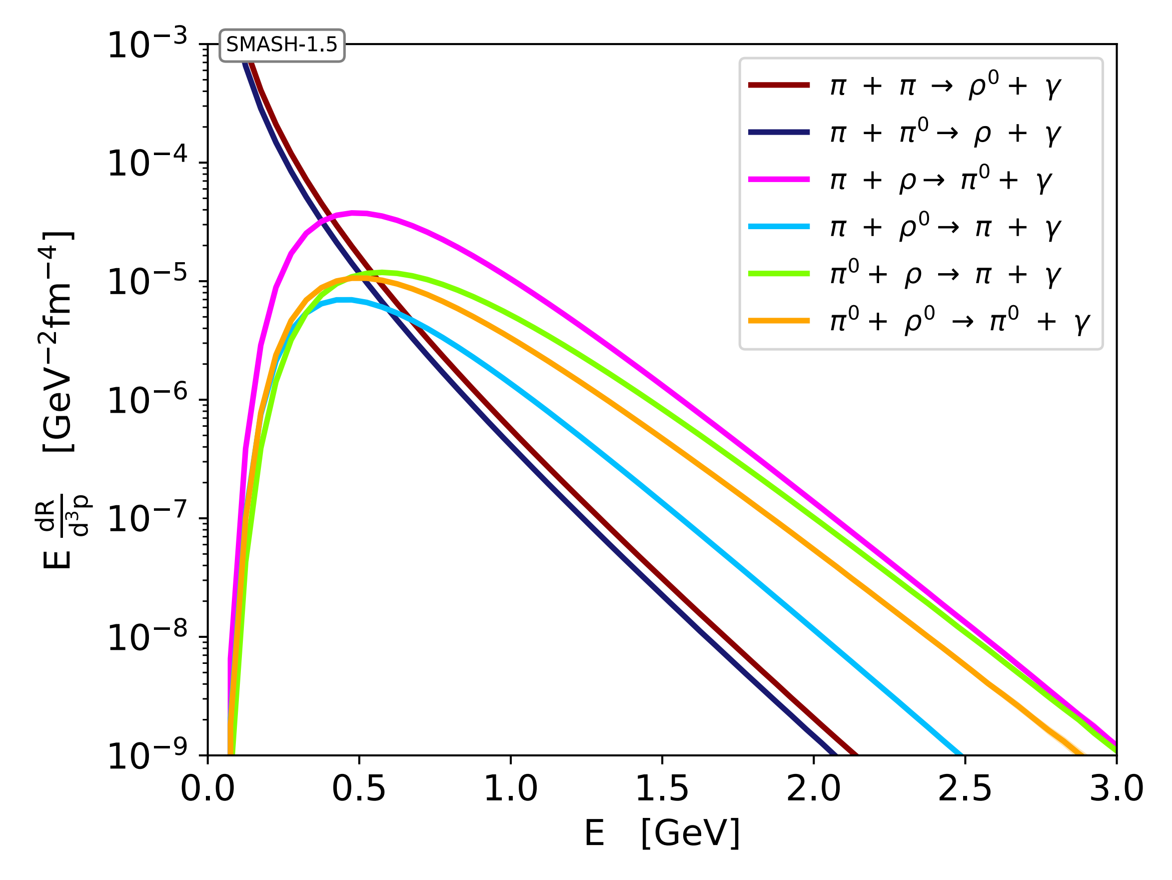
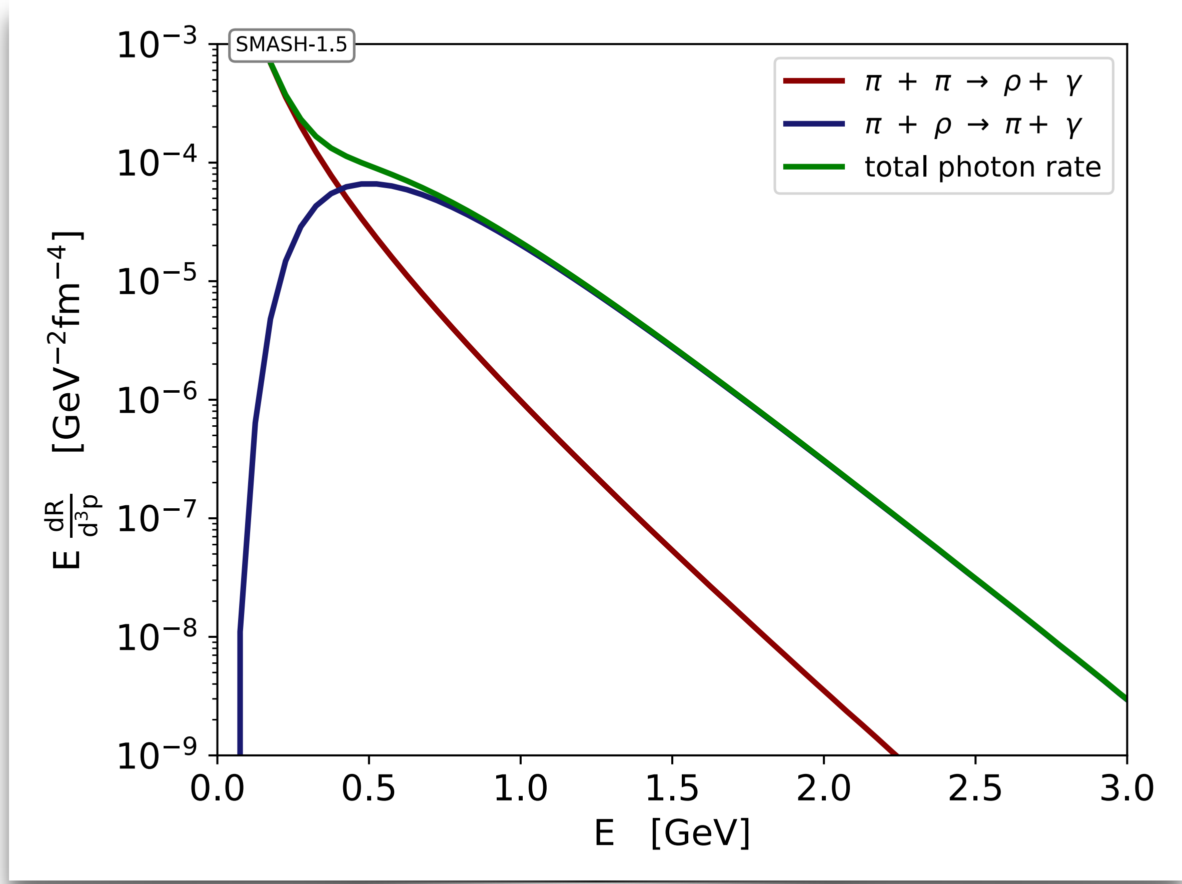
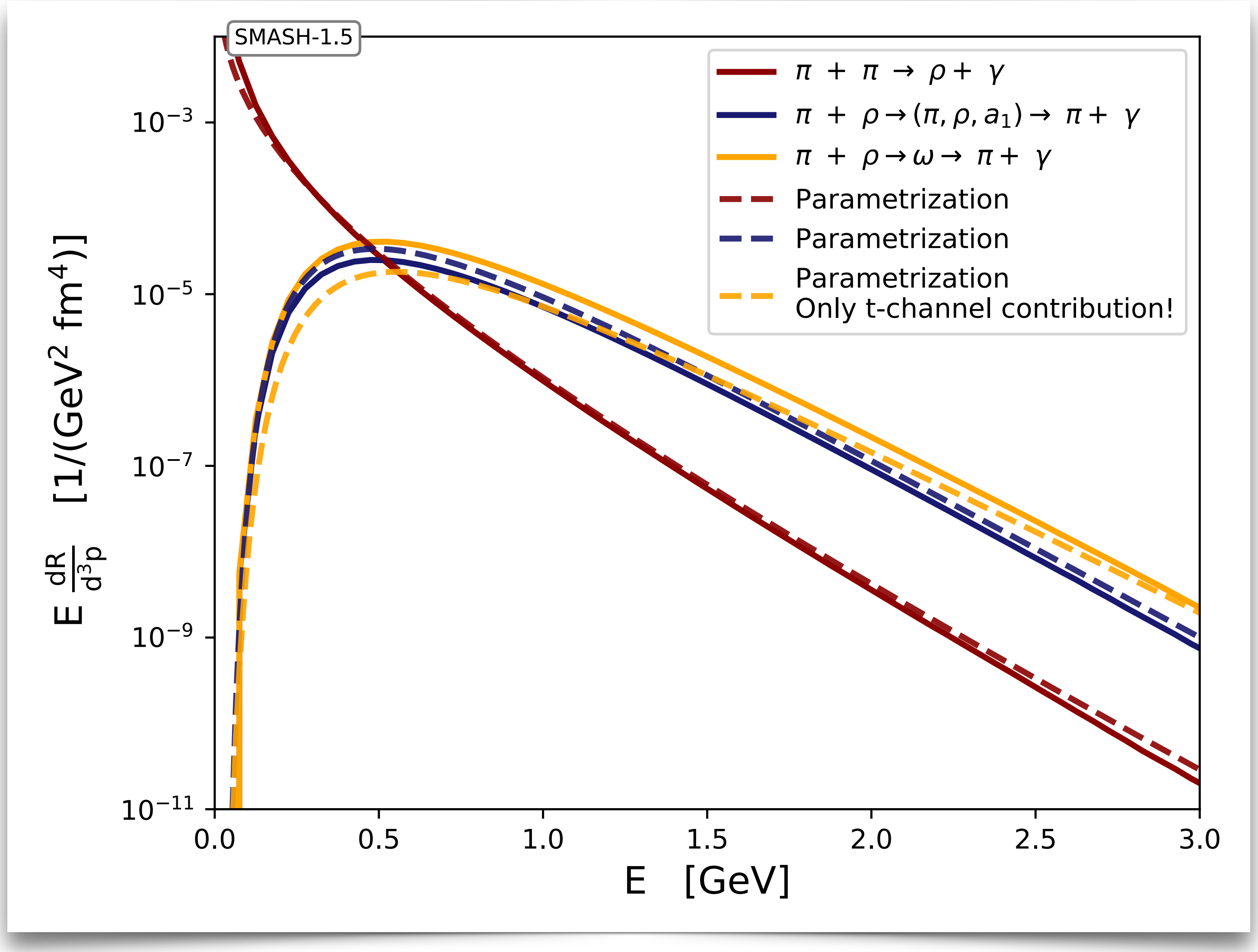




Form Factors included!

Comparison to Parametrized Rates



Parameterizations taken from:
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Parametrizations
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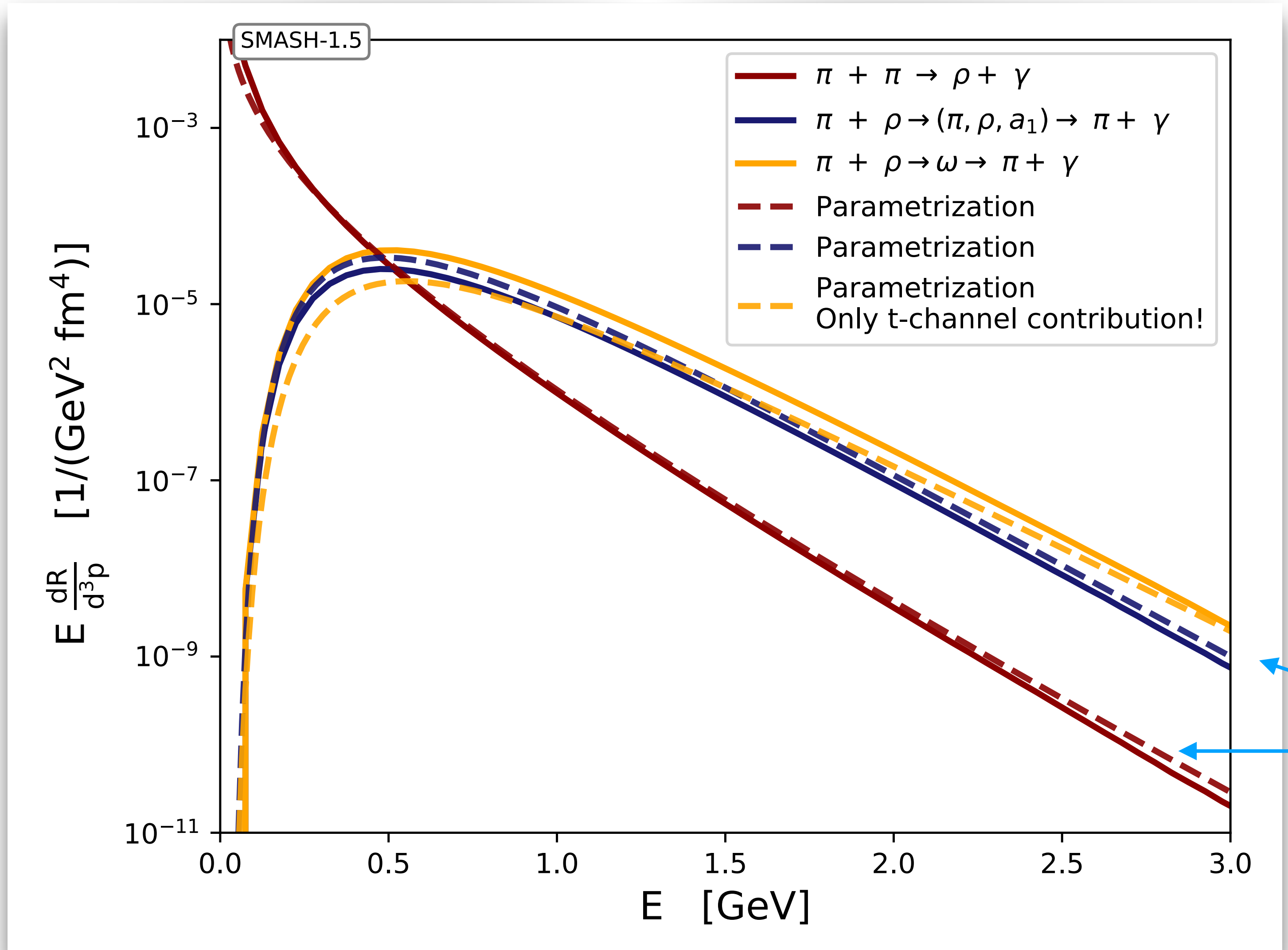

Decent agreement

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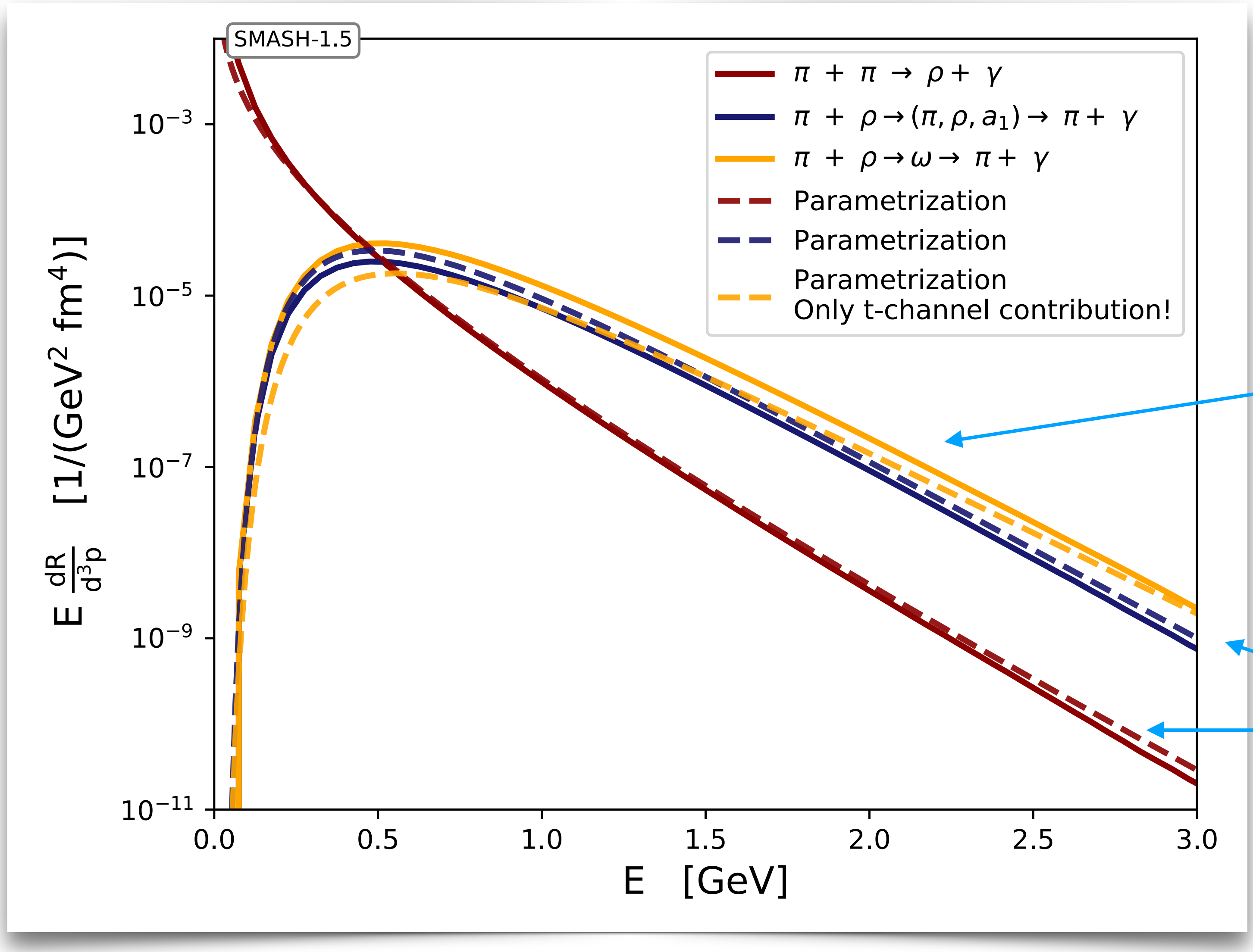

Systematic deviation

-> Parametrization only accounts for t-channel contributions

-> SMASH contains s- and t-channels

Decent agreement

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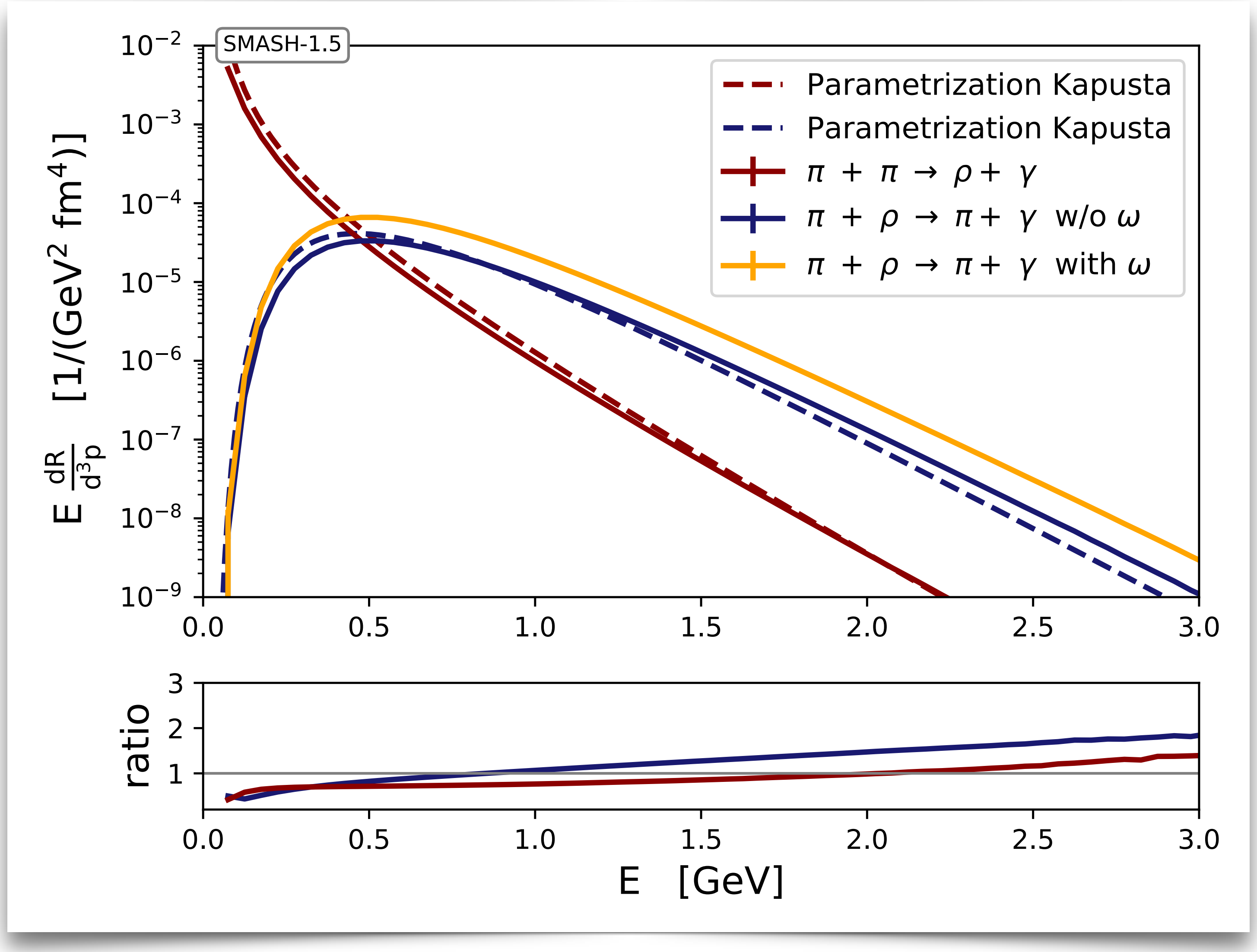
Quantifying the Difference to Kapusta et al.



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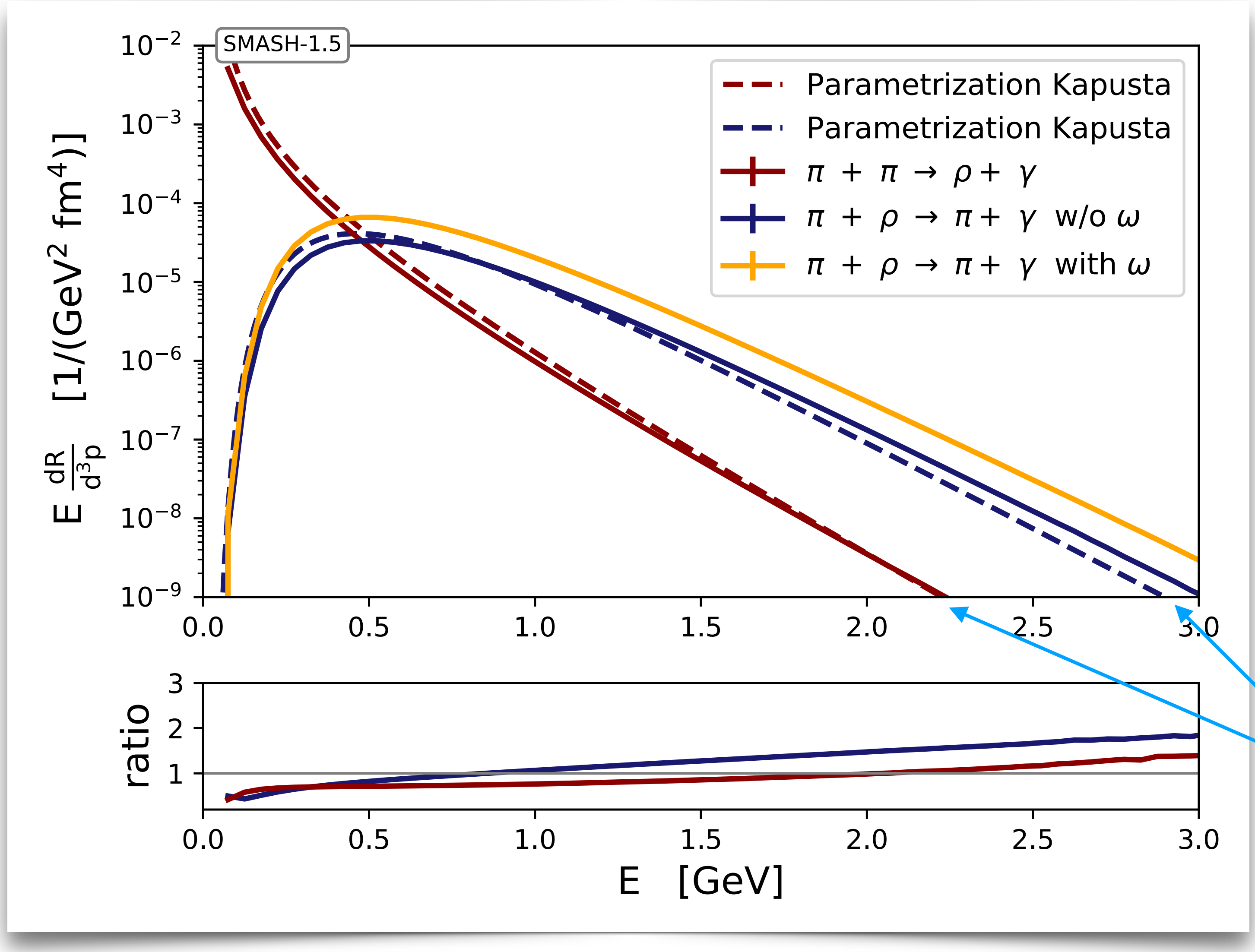

Kapusta > SMASH for small E
Kapusta < SMASH for large E

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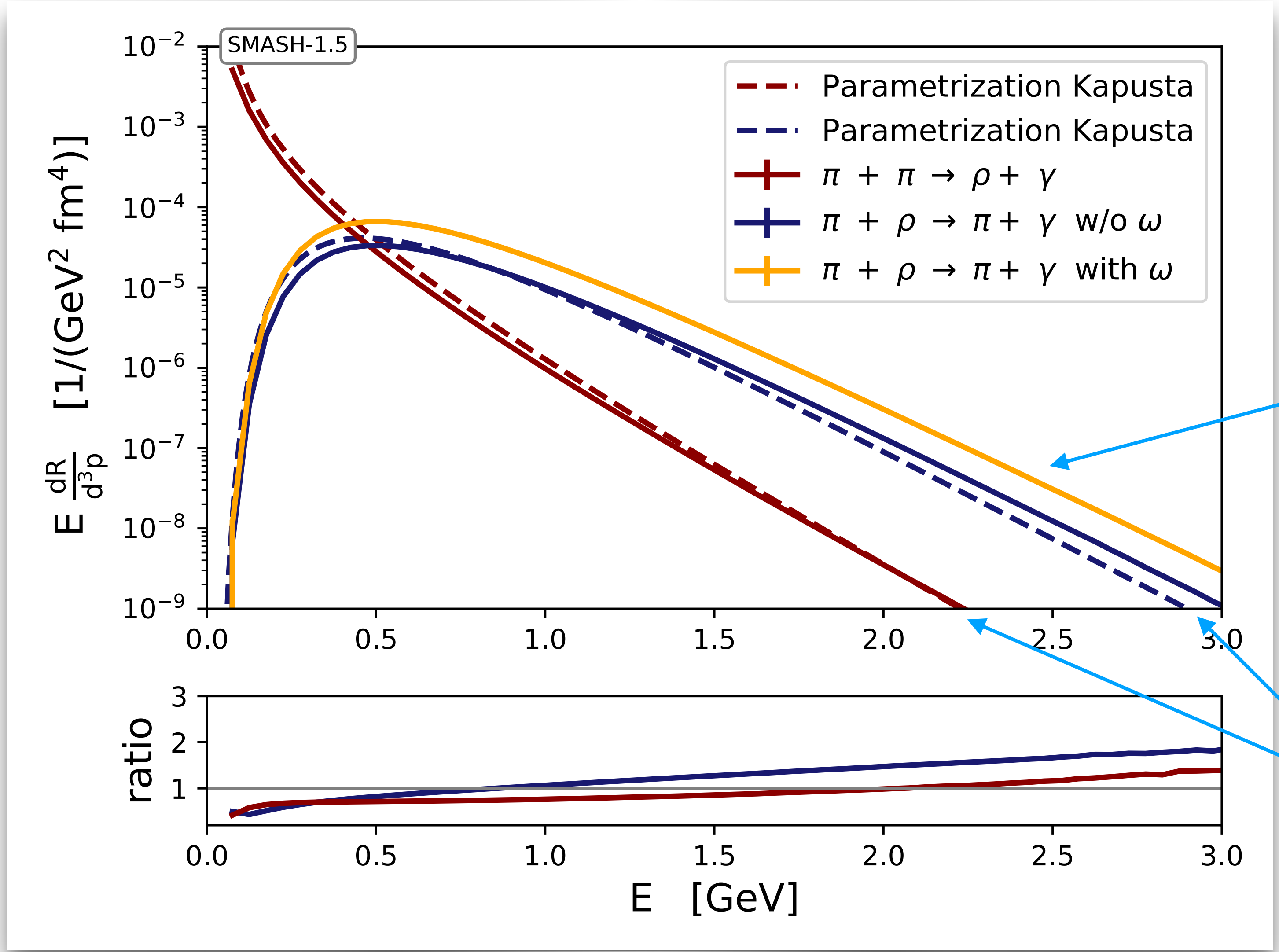

ω channels not included in Kapusta et al. framework:

-> Enhanced photon production in π + ρ -> π + γ channels within Turbide et al. framework

=> ω mediated processes contribute significantly

Also identified by: Holt, Hohler, Rapp; Nuc. Phys. A 945 (2016)

Kapusta > SMASH for small E
Kapusta < SMASH for large E

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Going One Step Further: Finite ρ Width

Up to now: ρ mesons considered stable particles

But: ρ mesons are actually broad particles, Γ_ρ = 0.149 GeV

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- Masses of in/outgoing rho mesons and intermediate rho mesons are considered equal
  => Ensure gauge invariance

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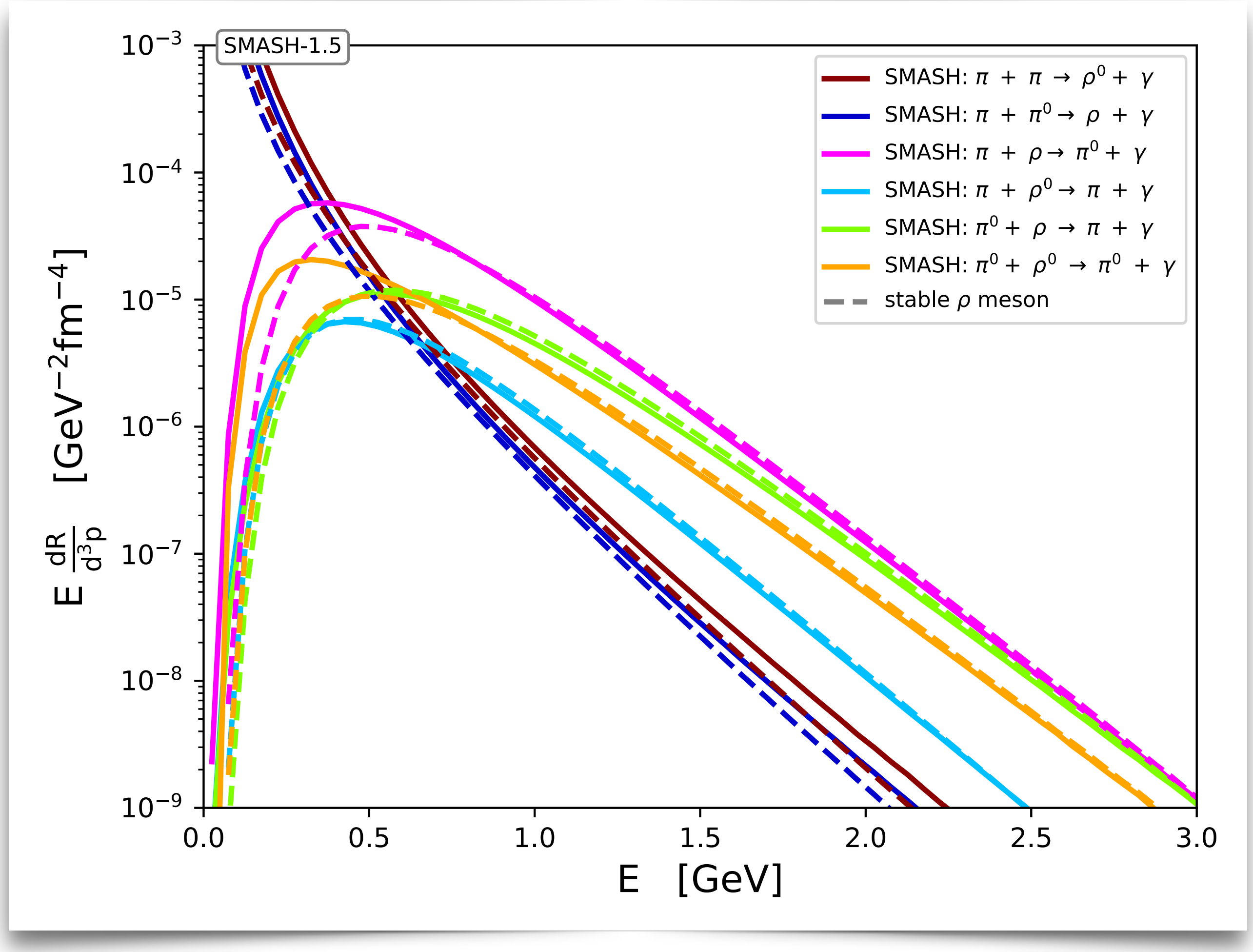

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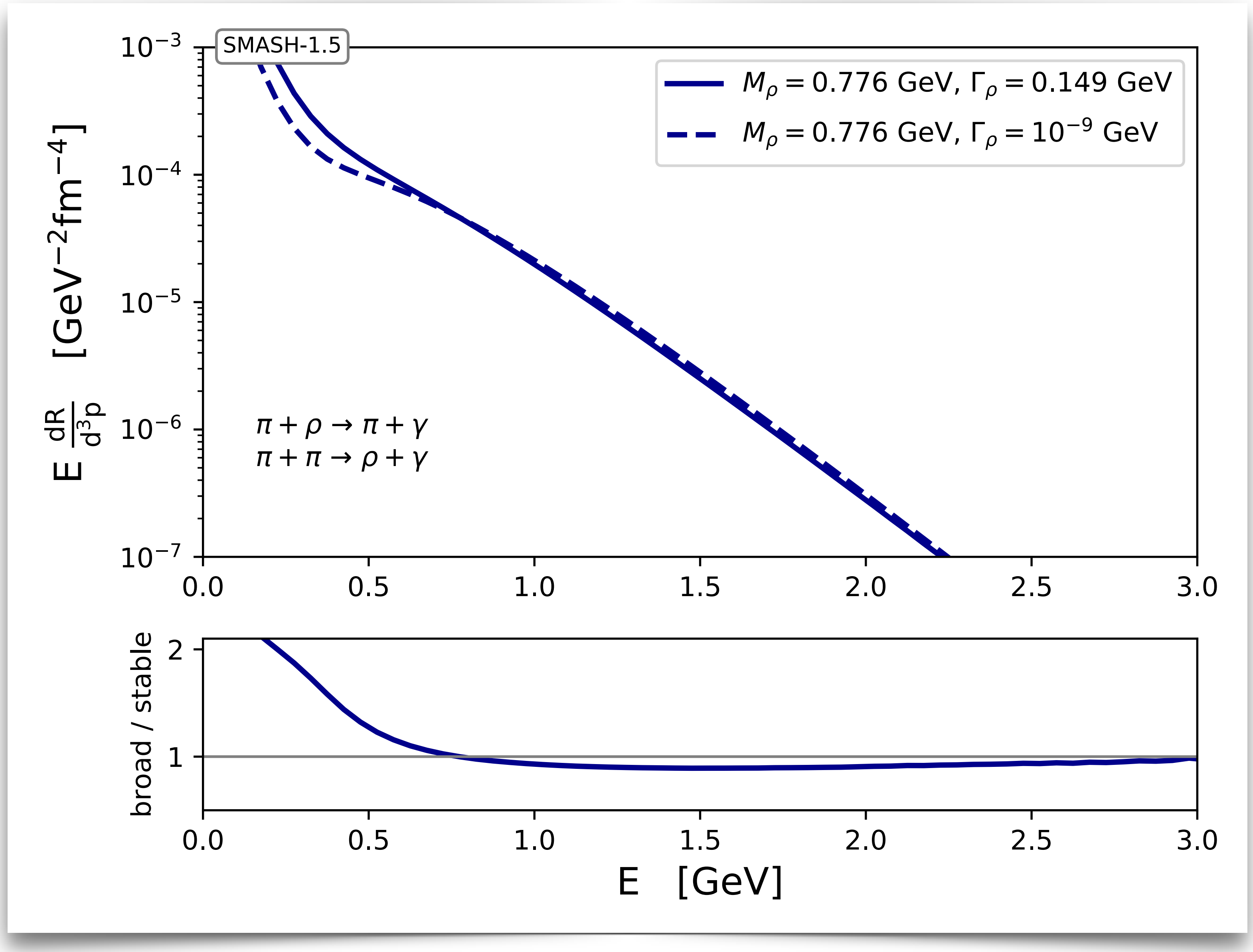

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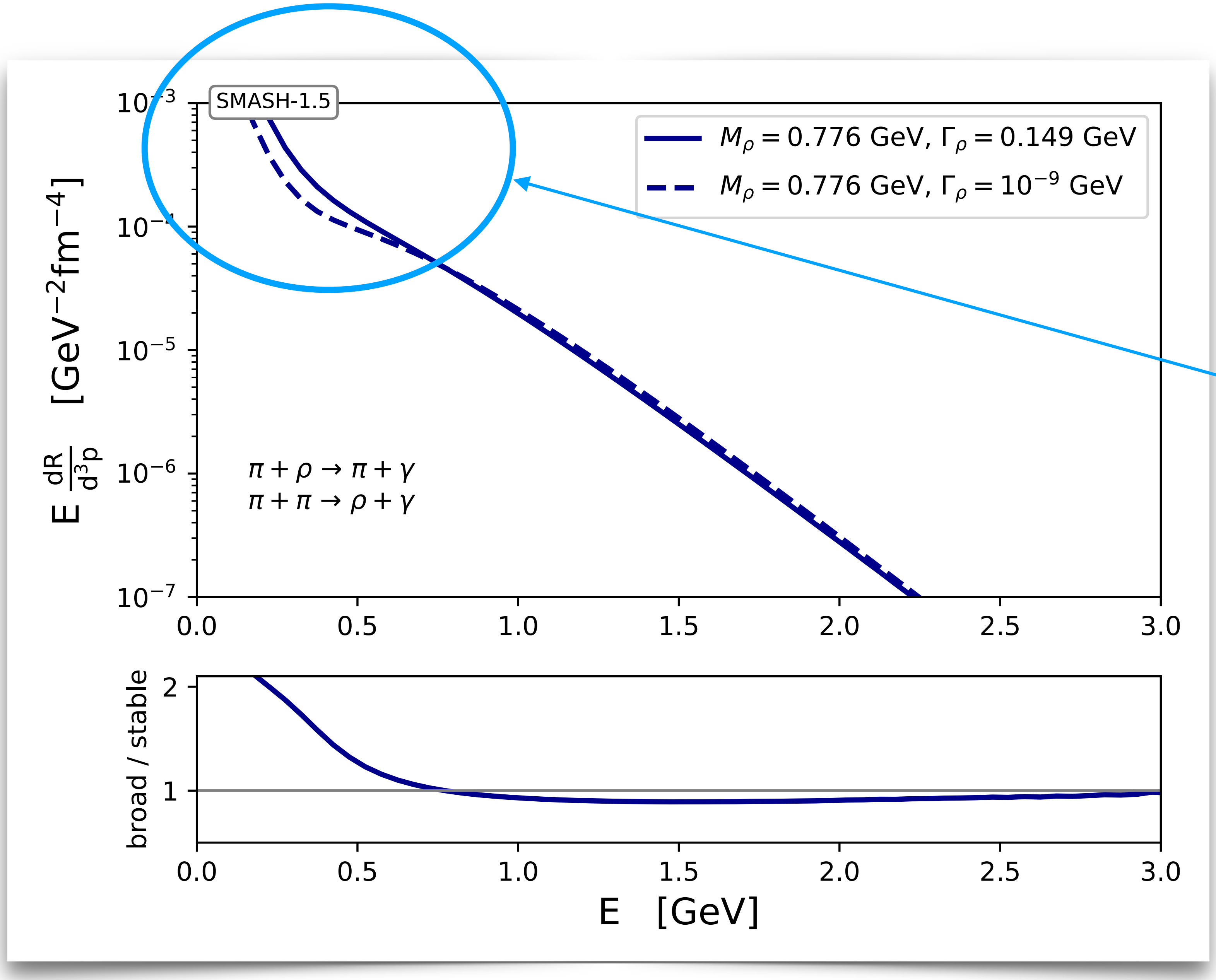
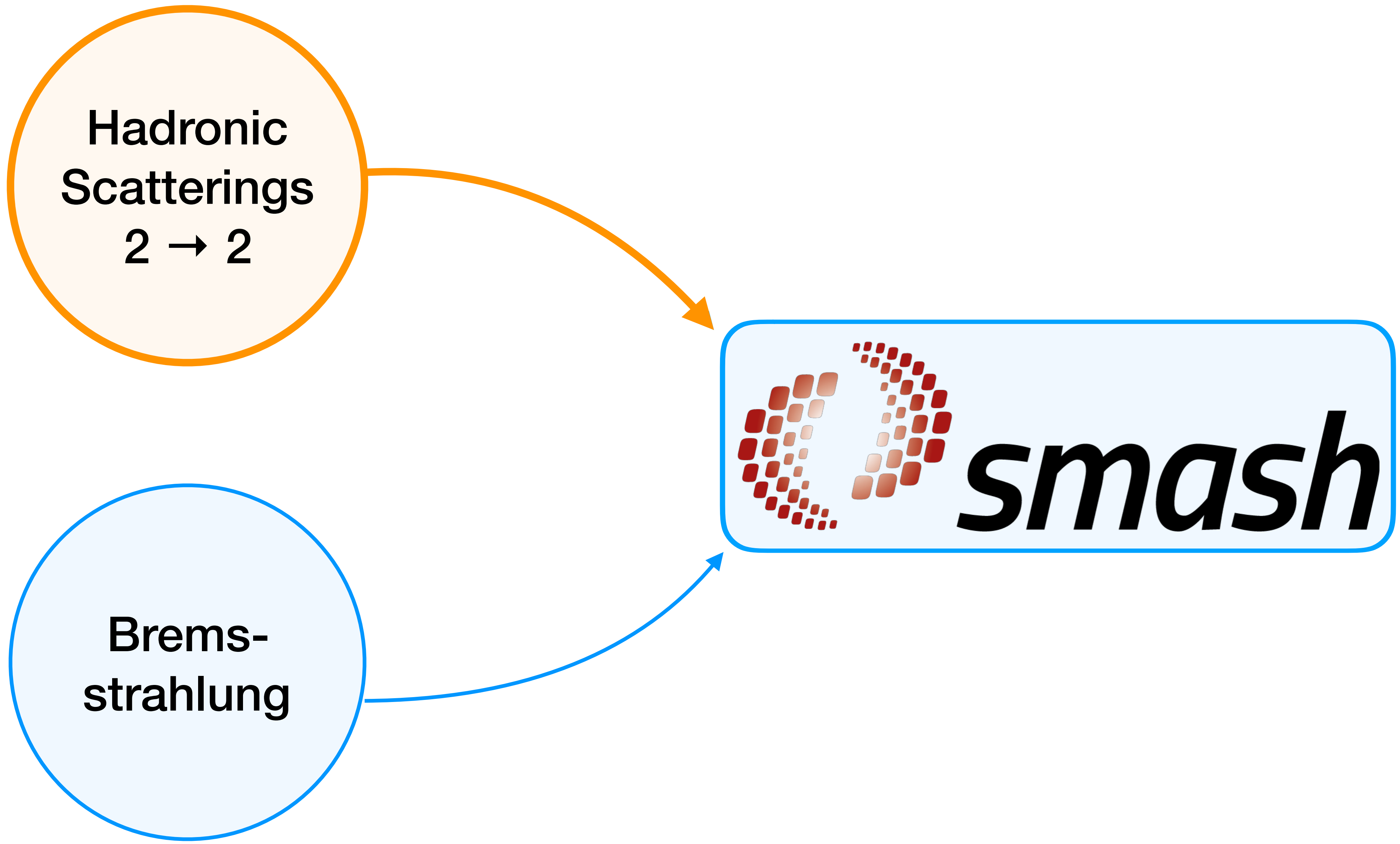


Finite ρ width affects photon rate significantly

-> Especially in low-energy region

Photons from the Hadron Gas



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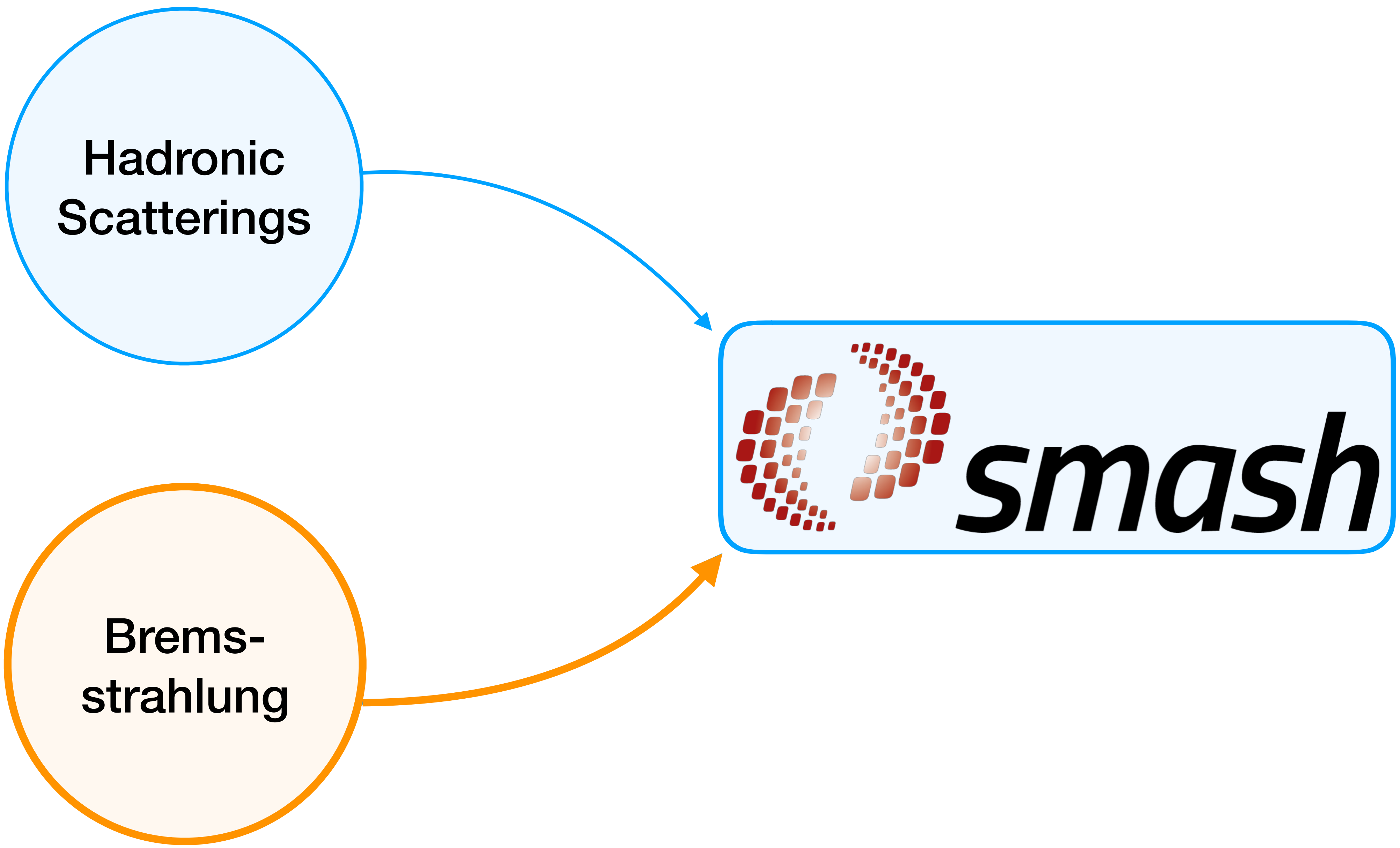
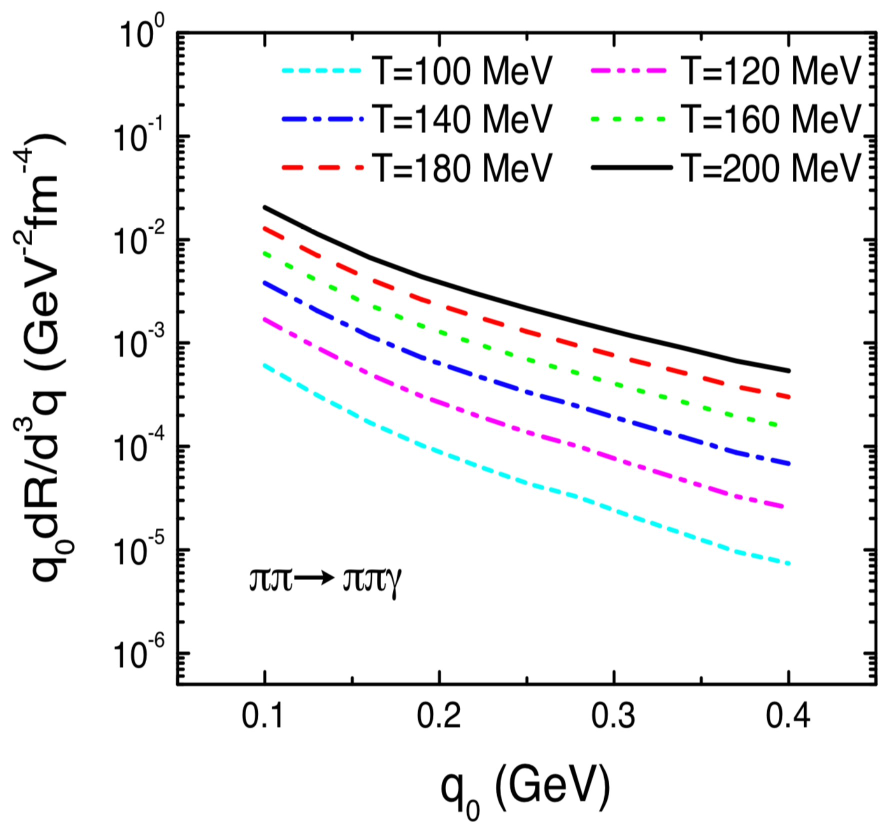


Why also Bremsstrahlung?

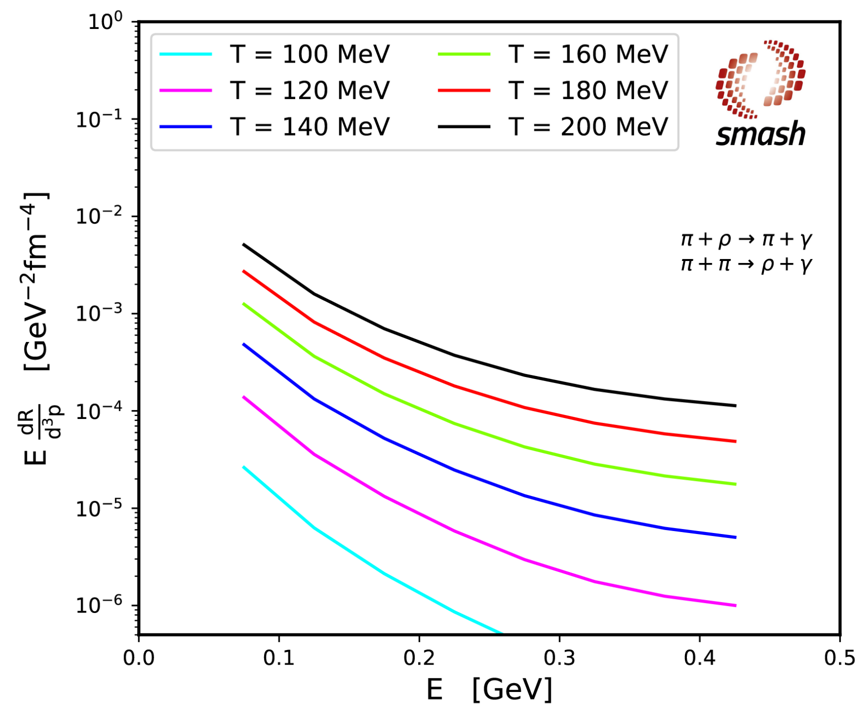
Bremsstrahlung contribution

from W.Liu, R.Rapp Nucl.Phys.A, 796, 2007



2 → 2 contribution:

π + π -> ρ + γ and π + ρ -> π + γ



=> Contribution of Bremsstrahlung photons is even higher than that of photons originating from hadronic scattering processes

Bremsstrahlung: Theoretical Background

- Leading contribution:
  - -> Photon emission from external pion legs in elastic $\pi + \pi \rightarrow \pi + \pi$ scatterings
  - -> Photons originating from internal lines or vertices (contact term)

- Underlying Theory: one boson exchange (OBE) model
  - -> Microscopic effective field theory
  - -> Interactions through exchange of σ, ρ and $f_2(1270)$ resonances
  - -> Interaction Lagrangian:

$$\mathscr{L}_{\text{int}} = g_\sigma \, \sigma \, \partial_\mu \pi \partial^\mu \pi + g_\rho \, \rho^\mu \, (\pi \times \partial_\mu \pi) + g_f \, f_{\mu\nu} \, \partial^\mu \pi \partial^\nu \pi$$

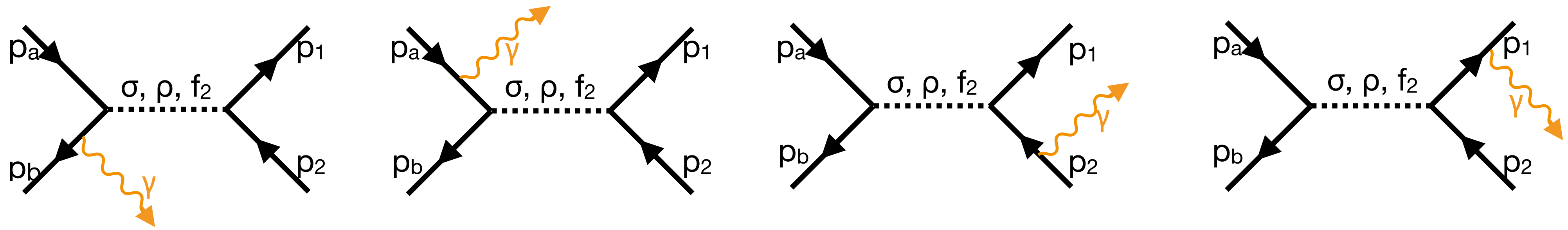
  - -> Form factors incorporated in the vertices in t- and u-channels

- Attempt to go beyond the soft photon approximation (SPA)
  - -> Dependency on photon momentum neglected in elastic scattering matrix element
  - -> Only valid for production of low-energy photons: $E_\gamma / E_{\text{hadron}} << 1$
  - -> Bremsstrahlung beyond SPA also implemented in PHSD

Eggers et al. Phys. Rev. D53, 1996 Lynnik et al. Phys. Rev. C92, 2015 Haglin et al. Phys. Rev. D47, 1993

Bremsstrahlung: Theoretical Background

- Emission from external pion legs:



+ corresponding t and u channels
+ contact term for internal line and vertex emission

- Cross section: $$d\sigma^{\gamma} = \frac{1}{2\sqrt{s\,(s-4m_{\pi}^2)}}\,|\mathcal{M}(\gamma)|^2\,dR_3$$ where

$$|\mathcal{M}(\gamma)|^2 = \Big|\sum_{i=1,2,a,b} e\,J_i^{\mu}[\sum_{k=s,t,u}(\sum_{j=\sigma,\rho,f_2}\mathcal{M}_{\rm el}^{j}(p_a,p_b,p_1,p_2,q)\,)^k]_i + \mathcal{M}_c\Big|^2$$

Eggers et al. Phys. Rev. D53, 1996 Lynnik et al. Phys. Rev. C92, 2015 Haglin et al. Phys. Rev. D47, 1993

Status Update: Bremsstrahlung

WORK IN PROGRESS

- Cross Section determination requires challenging integration
  => Result not yet satisfactory

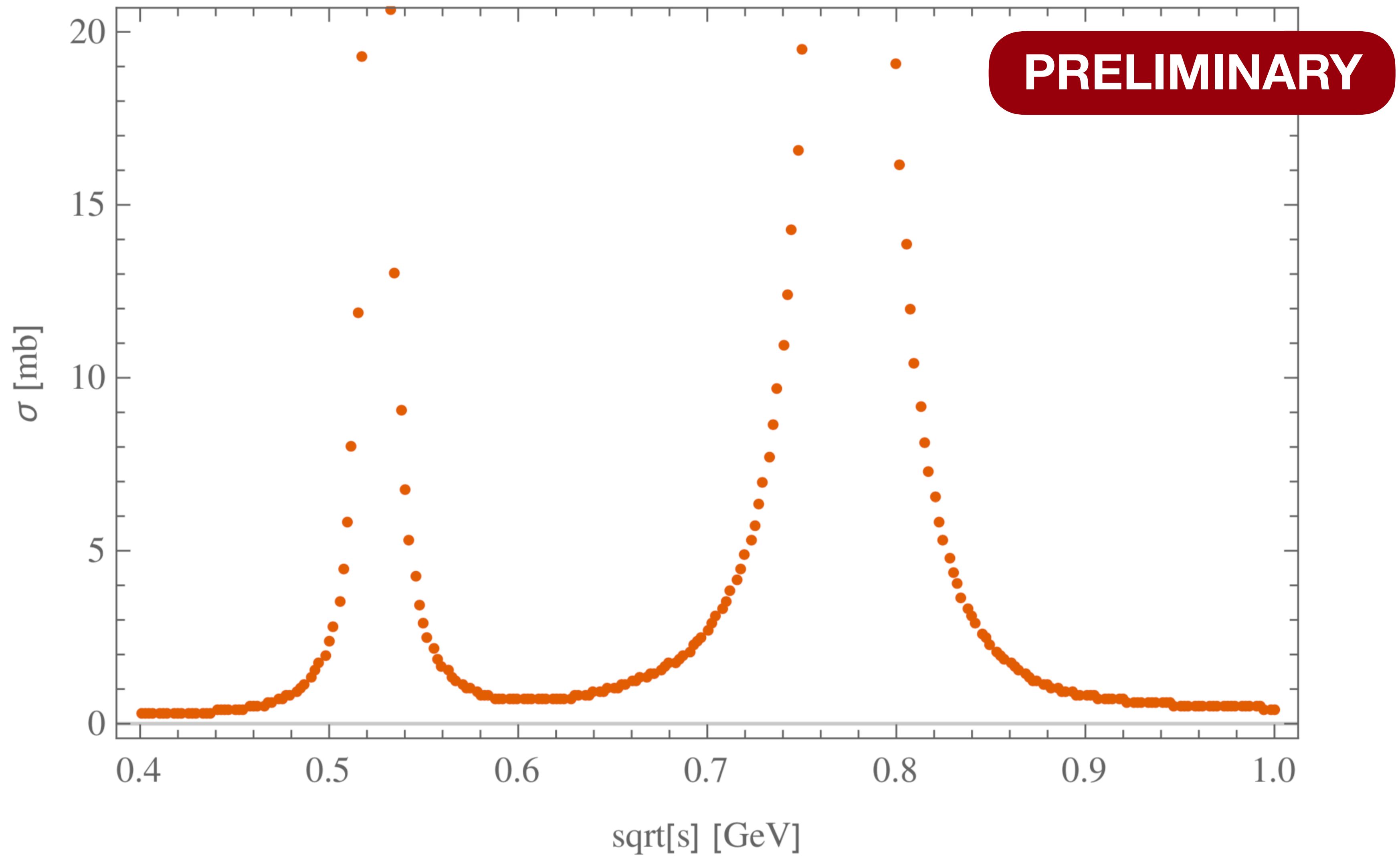
Proposed Treatment

- Express matrix element in terms of 5 invariants of 2 -> 3 scattering process: s_1, s_2, t_1, t_2, s
- Perform 4D Monte-Carlo Integration according to VEGAS algorithm to yield total cross section
- Revise treatment of fractional photons (previous implementation not valid for 2 -> 3 scattering processes):
  -> Sample N fractional photons at each interaction by means of adaptive rejection sampling / Metropolis sampling
  -> Adjust weighting factor: $W = \frac{1}{N_{\mathrm{frac}}\ \sigma_{\mathrm{had}}}$

Eggers et al. Phys. Rev. D53, 1996 Liu et al. Nucl. Phys. A796, 2007 Lepage J. Comp. Phys. 27, 1987

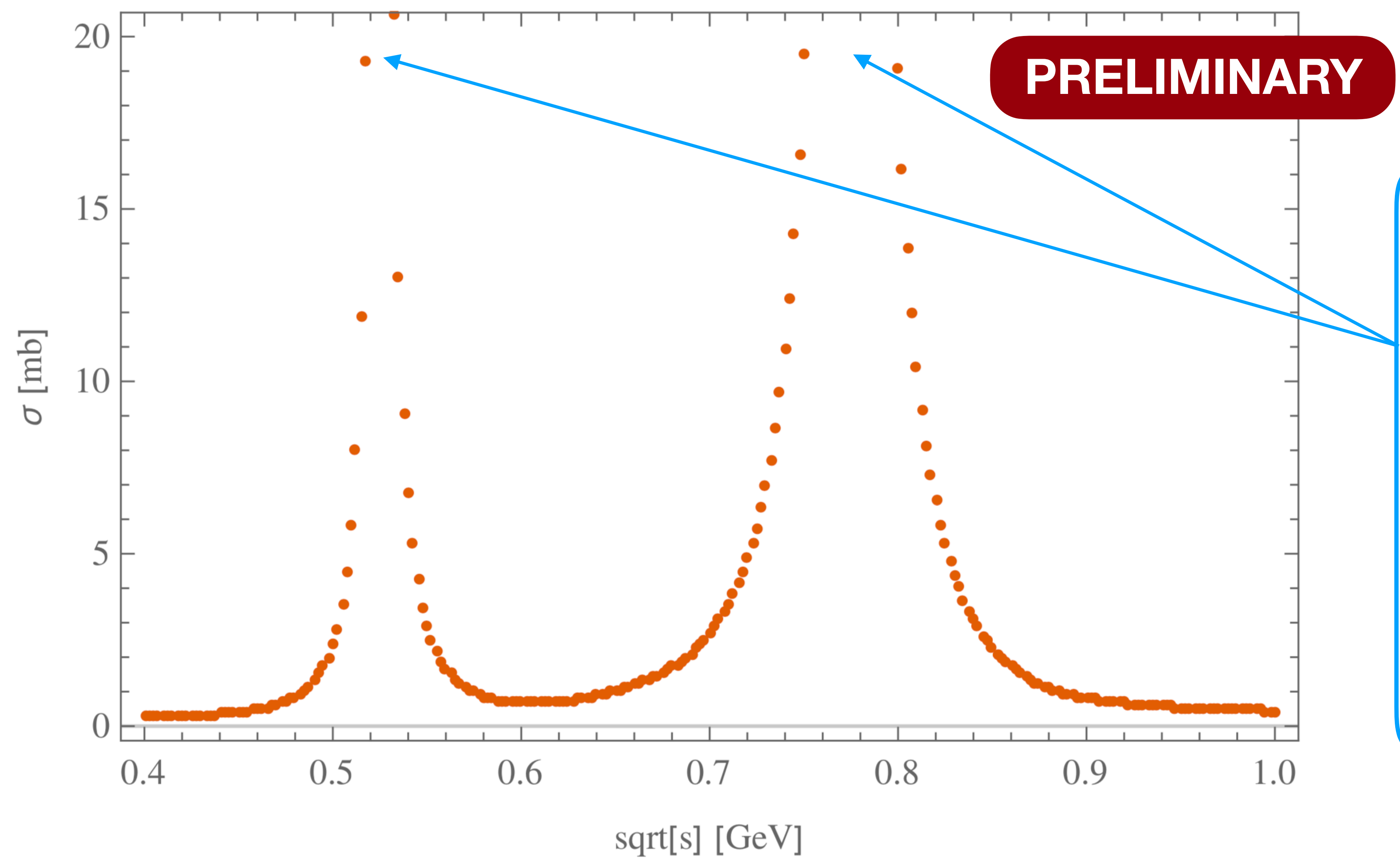
Status Update: Bremsstrahlung

WORK IN PROGRESS



Status Update: Bremsstrahlung

WORK IN PROGRESS



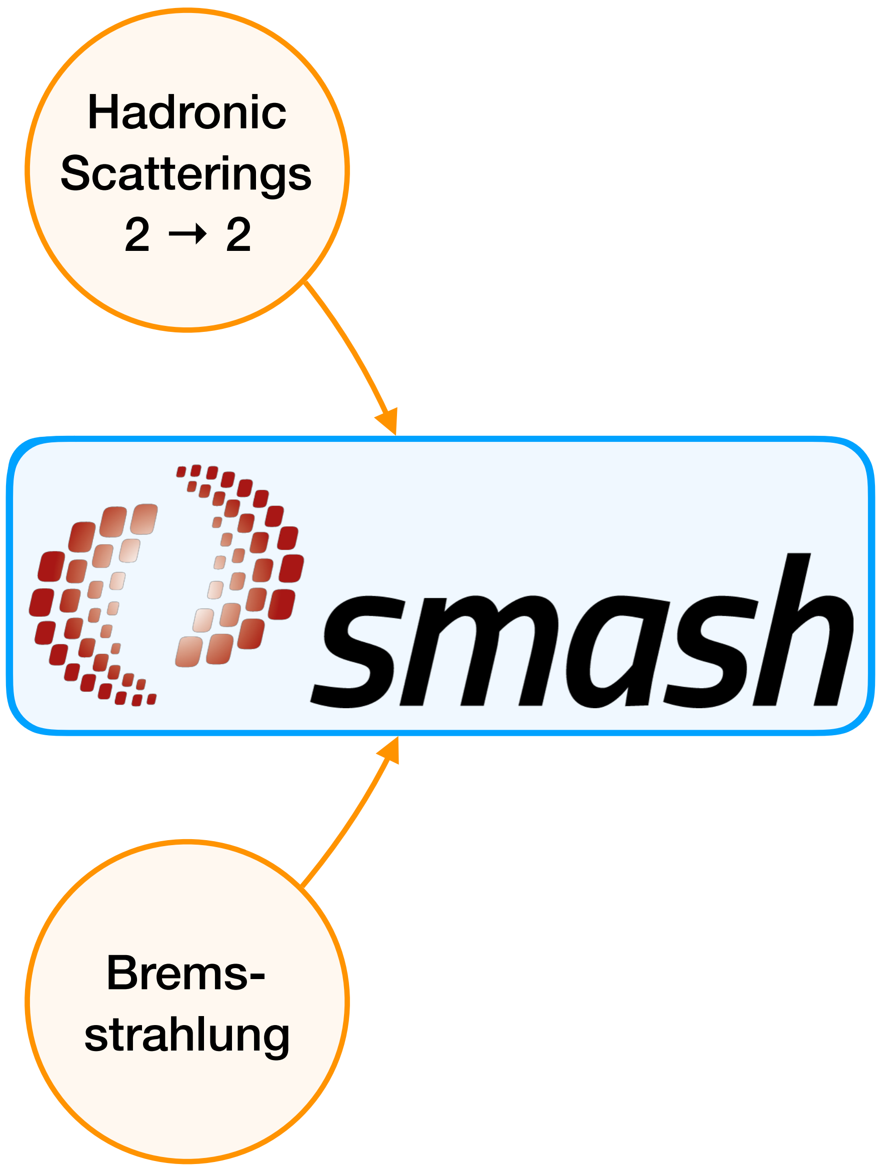
Cross section not satisfactory:

-> Poles instead of peaks at the σ and ρ pole masses

-> Probably related to problems in the integration algorithm

-> Currently under investigation

What Next?



What Next?

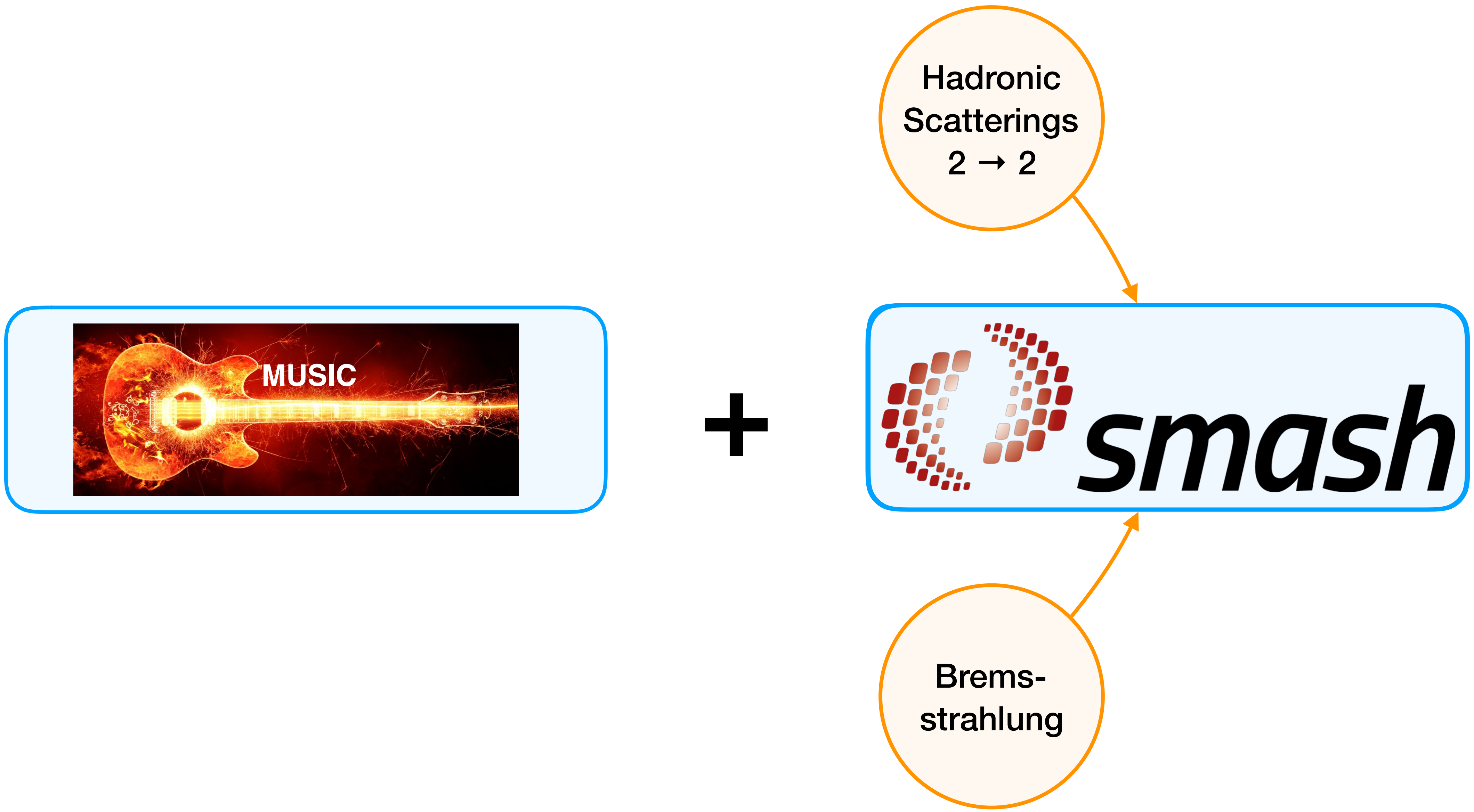
+

smash

Hadronic Scatterings 2 → 2

Brems-strahlung

What Next?

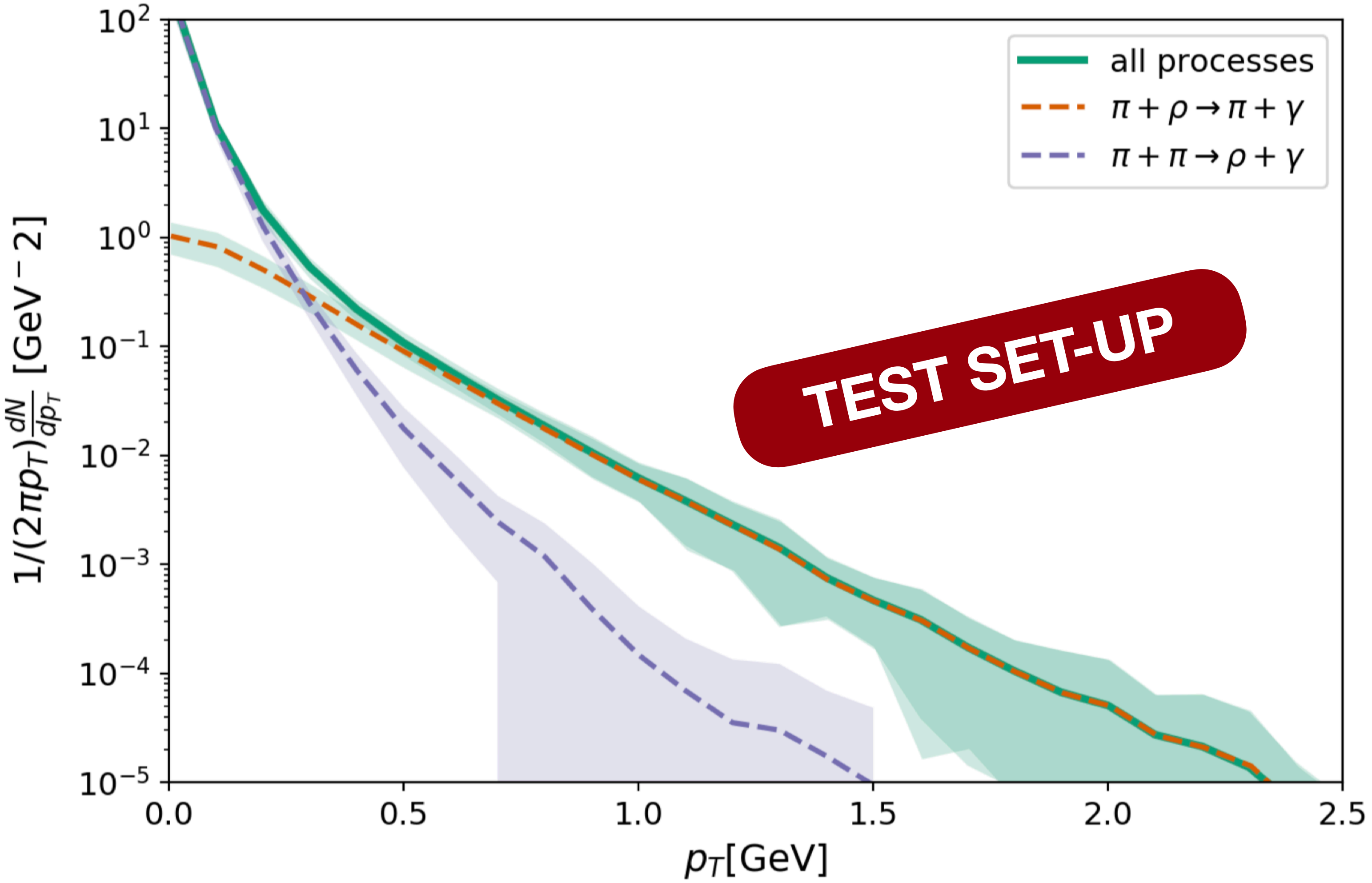


Final goal: Hadronic Afterburner simulation coupling MUSIC and SMASH

-> Advantage: MUSIC and SMASH use the same framework to produce photons from hadronic scatterings

Coupling MUSIC and SMASH

See also talk by Jan Staudenmaier



Goal:
-> Build basic framework for hybrid approach

Test set-up:
-> Run Cooper-Frye sampler + SMASH afterburner on MUSIC hypersurface

- Framework to couple MUSIC and SMASH by means of Cooper-Frye sampling has been developed
  -> Missing Bremsstrahlung processes prevent us from running realistic scenarios

Ryu et al. PRL 115, 132301 2015

Summary & Outlook

What has been done?

- Hadronic photon production in 2 -> 2 processes implemented and applied in SMASH

  -> Results in good agreement with Turbide et al. rates

  -> (ω) mediated processes contribute significantly

  -> Photon rate increases once finite ρ meson width is taken into account

- Implementation of Bremsstrahlung processes in preparation
- Afterburner framework to couple MUSIC + SMASH set up

Future Plans

- Finish Bremsstrahlung implementation
- Study photon production in heavy ion collisions by means of hybrid model

Backup

Thermal Photon Rate

Analytic formula:

$$E\frac{dR}{d^3p} = N \int (2\pi)^4 \; \delta^4(p_A + p_B - p_C - p) \; |\mathcal{M}|^2 \; f(E_A) \; f(E_B) \; (1 + \varepsilon f(E_C))$$
$$\times \frac{1}{2(2\pi)^3} \; \frac{d^3p_A}{2E_A(2\pi)^3} \; \frac{d^3p_B}{2E_B(2\pi)^3} \; \frac{d^3p_C}{2E_C(2\pi)^3}$$

In the case of Boltzmann statistics:

$$E\frac{dR}{d^3p} = N \int \delta^4(p_A + p_B - p_C - p) \; |\mathcal{M}|^2 \; f(E_A) \; f(E_B) \; f(E_C) \; \frac{d^3p_A \; d^3p_B \; d^3p_C}{16(2\pi)^8}$$

Photon rate from SMASH:

$$E \; \frac{dR}{d^3p} = E\frac{dN}{4\pi \; p^2 dp \; \Delta t \; V} = \frac{1}{4\pi \; E \; \Delta t \; V} \; \frac{dN}{dE}$$

Form Factors

- Fourier Transform of charge distribution inside hadrons:

$$\hat{F}(q) = \int e^{-iqx} \rho(x)\ d^3x$$

- Parametrization as Hadronic Dipole Form Factor:

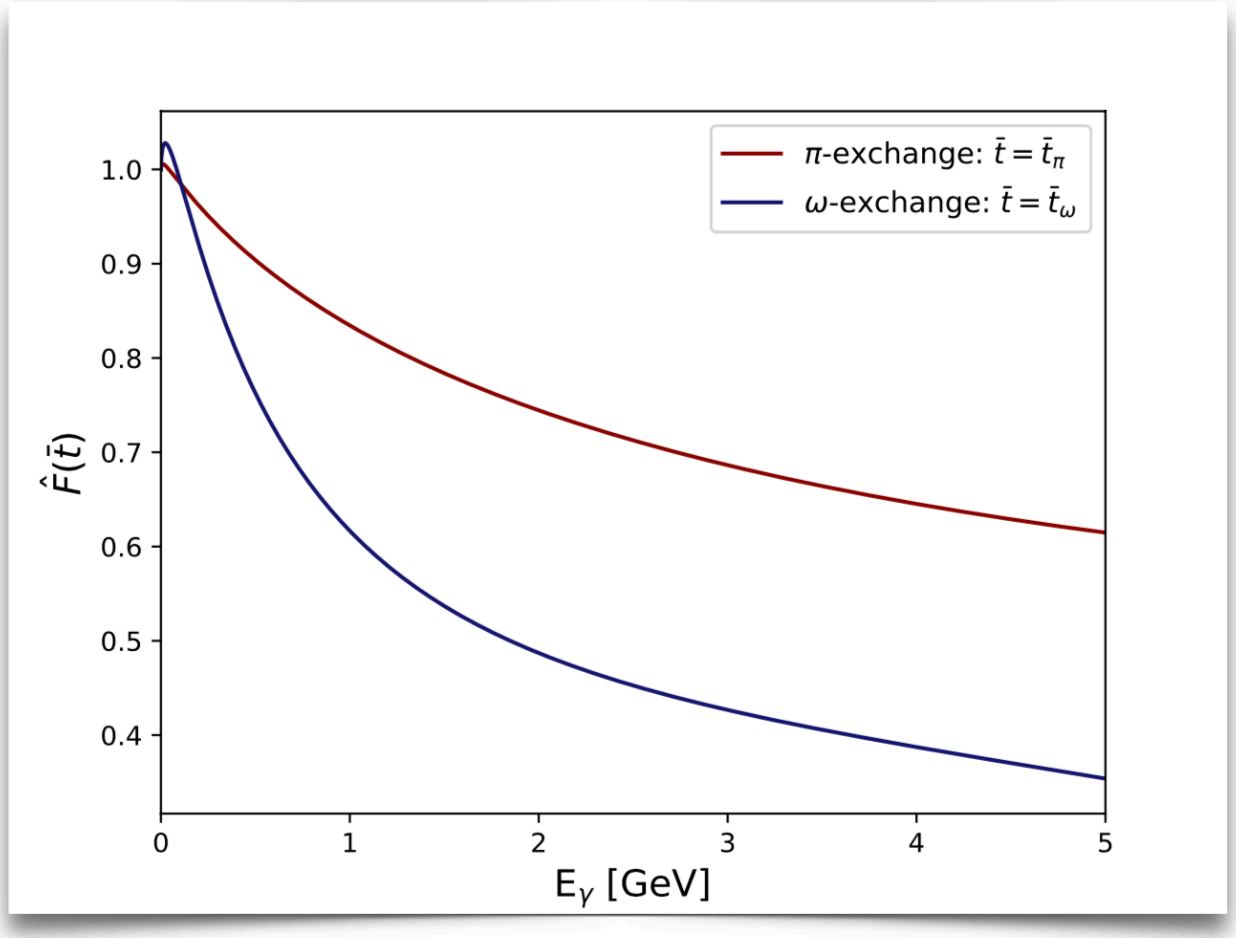
$$\hat{F}(E_\gamma) = \frac{2\Lambda^2}{2\Lambda^2 - \bar{t}(E_\gamma)}$$

- Application in SMASH:

$$\sigma_\gamma \ \rightarrow\ \hat{F}^4(E_\gamma) \cdot \sigma_\gamma$$

- Combination of channels:

$$\sigma_\gamma = \hat{F}^4_{(\pi\rho a_1)}\ \sigma_{(\pi\rho a_1)} + \hat{F}^4_{(\omega)}\ \sigma_{(\omega)}$$



$$\Gamma_{\omega \to \pi\gamma}\ \propto\ |\mathcal{M}|^2\ \hat{F}^2$$

From experiment: 0.7174 MeV