Foundations of the Trojan-Horse Method

Stefan Typel

TECHNISCHE UNIVERSITÄT DARMSTADT

Indirect Methods in Nuclear Astrophysics

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Outline

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Introduction I

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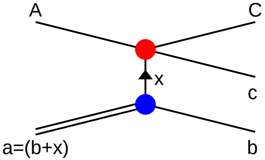
- **basic idea**
  - study breakup reaction

    $$A + a \to C + c + b$$

    to extract cross section of astrophysical charged-particle reaction

    $$A + x \to C + c$$

    with Trojan horse $a = b + x$ and spectator b



  - establish relation of cross sections with help of direct reaction theory

Introduction II

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- method introduced by **Gerhard Baur**:

  Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics
  Physics Letters B 178, 135 (1986)

  suggested already in his invited talk:
  Breakup processes in nuclear reactions
  1985 Varna Int. Summer School on Nuclear Physics
  (Sept. 22 – Oct. 1), Nuclear Energy 25, 183 (1987)

- specific features
  - Fermi motion of x inside Trojan horse a compensates relative motion with respect to A
    $\Rightarrow$ small relative energies in $A + x$ system accessible
    (superseded by interpretation of Claudio Spitaleri)
  - surface dominated reaction $\Rightarrow$ reduction of suppression by Coulomb barrier
  - 'high' relative energy in $A + a$ system $\Rightarrow$ no electron screening

Theory of Direct Reactions I

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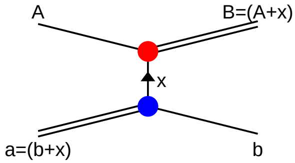
- **transfer reaction to bound state:**
  $A + a \to B + b$ with $a = b + x$ and $B = A + x$
  - general cross section:

    $$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}} \frac{d^3 p_{Bb}}{(2\pi\hbar)^3} |T_{fi}|^2 \, \delta(E_{Bb} - E_{Aa} - Q)$$

    with $Q = (m_A + m_a - m_B - m_b)c^2$
  - T matrix element in post formulation:

    $$T_{fi} = \langle \phi_B \phi_b \exp\left(i\vec{p}_{Bb} \cdot \vec{r}_{Bb} / \hbar \right) | V_{Bb} | \Psi^{(+)}_{Aa} \rangle$$

    with full scattering wave function $\Psi^{(+)}_{Aa}$



  - extracted information:
    spectroscopic factors, asymptotic normalisation coefficients (ANCs)

Theory of Direct Reactions II

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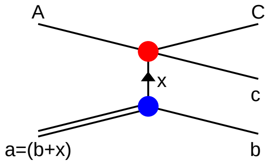
- **transfer reaction to continuum state:**
  $A + a \to C + c + b$ with $a = b + x$ and $B = C + c$
  - general cross section:

    $$d\sigma = \frac{2\pi}{\hbar} \frac{\mu_{Aa}}{p_{Aa}} \frac{d^3 p_{Bb}}{(2\pi\hbar)^3} \frac{d^3 p_{Cc}}{(2\pi\hbar)^3} \left| T_{fi} \right|^2 \delta(E_{Bb} + E_{Cc} - E_{Aa} - Q)$$

    with $Q = (m_A + m_a - m_C - m_c - m_b)c^2$



  - T matrix element in post formulation:

    $$T_{fi} = \langle \Psi^{(-)}_{Cc} \phi_b \exp\left(i\vec{p}_{Bb} \cdot \vec{r}_{Bb}/\hbar \right) \left| V_{Bb} \right| \Psi^{(+)}_{Aa} \rangle$$

    with full scattering wave function $\Psi^{(+)}_{Aa}$
  - extracted information:
    scattering matrix elements of reaction $A + x \to C + c$

Theory of Trojan-Horse Method I

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- **transformation of T matrix element**
  - introduction of distorted waves with optical potentials
  - Gell-Mann Goldberger relation

$$\Rightarrow \quad T_{fi} = \langle \Psi^{(-)}_{Cc} \phi_b \chi^{(-)}_{Bb} | V_{Bb} - U_{Bb} | \Psi^{(+)}_{Aa} \rangle$$

  - distorted-wave Born approximation (DWBA) $\Psi^{(+)}_{Aa} = \chi^{(+)}_{Aa} \phi_A \phi_a$
  - approximation of potential $V_{Bb} - U_{Bb} = V_{Cb} + V_{cb} - U_{Bb} = V_{Ab} + V_{xb} - U_{Bb} \approx V_{xb}$

$$\Rightarrow \quad T_{fi} = \langle \Psi^{(-)}_{Cc} \phi_b \chi^{(-)}_{Bb} | V_{xb} | \chi^{(+)}_{Aa} \phi_A \phi_a \rangle$$

  - introduction of momentum distribution $V_{xb}\phi_a = \int \frac{d^3q}{(2\pi)^3} \, W(\vec{q}) \exp\left(i\vec{q} \cdot \vec{r}_{xb}\right) \, \phi_x \phi_b$

$$\Rightarrow \quad W(\vec{q}) = -\left(B_a + \frac{\hbar^2 q^2}{2\mu_{bx}}\right) \Phi_a(\vec{q})$$

with $B_a = (m_x + m_b - m_a)c^2$ and $\Phi_a(\vec{q}) = \langle \exp\left(i\vec{q} \cdot \vec{r}_{xb}\right) \phi_x \phi_b | \phi_a(\vec{r}_{xb}) \rangle$

$$\Rightarrow \quad T_{fi} = \int \frac{d^3q}{(2\pi)^3} \, W(\vec{q}) \, \langle \Psi^{(-)}_{Cc} \phi_b \chi^{(-)}_{Bb} | \exp\left(i\vec{q} \cdot \vec{r}_{xb}\right) \chi^{(+)}_{Aa} \phi_A \phi_x \rangle$$

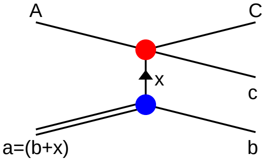
  - surface approximation: use asymptotic form of $\Psi^{(-)}_{Cc}$

Theory of Trojan-Horse Method II

- **simplified interpretation with plane-wave approximation**
  - $\chi^{(-)}_{Bb} \to \exp\left(i\vec{k}_{Bb}\cdot\vec{r}_{Bb}\right)$, $\chi^{(+)}_{Aa} \to \exp\left(i\vec{k}_{Aa}\cdot\vec{r}_{Aa}\right)$
  - integration over $\vec{r}_{Bb}$ and $\vec{q}$

$$\Rightarrow \quad T_{fi} = W(\vec{Q}_{Bb}) \left\langle \Psi^{(-)}_{Cc,\text{asym}} \right| \exp\left(i\vec{Q}_{Aa}\cdot\vec{r}_{Ax}\right) \phi_A\phi_x \rangle$$

with $\vec{Q}_{Bb} = \vec{k}_{Bb} - \frac{m_b}{m_b+m_x}\vec{k}_{Aa}$ and $\vec{Q}_{Aa} = \vec{k}_{Aa} - \frac{m_A}{m_A+m_x}\vec{k}_{Bb}$



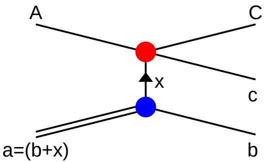
  - factorization of cross section

$$\frac{d^3\sigma}{dE_{Cc}d\Omega_{Cc}d\Omega_{(Cc)b}} = K\left|\Phi_a(\vec{Q}_{Bb})\right|^2 \frac{d\sigma^{HOES}}{d\Omega_{Cc}}(A+x\to C+c)$$

    - kinematic factor K
    - momentum distribution $|\Phi_a(\vec{Q}_{Bb})|^2$ of Trojan-horse ground state (with momentum transfer to spectator $\vec{Q}_{Bb}$)
    - half-off-energy-shell cross section $\frac{d\sigma^{HOES}}{d\Omega_{Cc}}$ of reaction $A+x\to C+c$ $\left(\frac{\hbar^2 Q^2_{Aa}}{2\mu_{Ax}} + m_A + m_x \neq \frac{\hbar^2 k^2_{Cc}}{2\mu_{Cc}} + m_C + m_c\right)$

Conditions for Application

- **dominance of quasifree scattering**
  - small momentum transfer to spectator b
    $\Rightarrow \vec{Q}_{Bb} \approx 0$
  - selection of specific kinematic conditions
    $\Rightarrow$ emission angles of particles C and c correlated
    $\Rightarrow$ full triple differential cross section
    $\frac{d^3\sigma}{dE_{Cc}\,d\Omega_{Cc}\,d\Omega_{(Cc)b}}$ needed in analysis of data



- **well clustered Trojan horse nucleus a = b + x**
  - s-wave ground state (e.g. ^{2}H, ^{6}Li)
    $\Rightarrow$ maximum of $|\Phi_a(\vec{Q}_{Bb})|^2$ at $\vec{Q}_{Bb} = 0$, favored case
  - p-wave ground state (e.g. ^{7}Li)
    $\Rightarrow$ maximum of $|\Phi_a(\vec{Q}_{Bb})|^2$ at $\vec{Q}_{Bb} \neq 0$, disfavored case
  - Claudio Spitaleri: change of Gerhard Baur's original idea
    $\Rightarrow$ not tail (Fermi motion), but maximum of momentum distribution relevant

Kinematics

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- **reactions** $A + a \to b + c + C$ $A + x \to c + C$
- **Q values** $Q_3 = (m_A + m_a - m_b - m_c - m_C)c^2$ $Q_2 = (m_A + m_x - m_c - m_C)c^2$

  $\Rightarrow$ binding energy $B_a = (m_b + m_x - m_a)c^2 = Q_2 - Q_3 > 0$
- **energy conservation**

  $$E_{Aa} = E_{Bb} + E_{Cc} - Q_3 \qquad E_{Ax} = E_{Cc} - Q_2$$

  $\Rightarrow E_{Ax} = E_{Aa} - E_{Bb} - B_a$
- **argument of momentum distribution Φ_a**

  $\hbar\vec{Q}_{Bb} = \vec{p}_{Bb} - \frac{m_b}{m_b+m_x}\vec{p}_{Aa}$

  $\Rightarrow$ intervall of accessible energies E_{Ax}
  - central value (for $Q_{Bb} = 0$): $E^c_{Ax} = E_{Aa}\left[1 - \frac{\mu_{Aa}}{\mu_{Bb}}\left(\frac{m_b}{m_b+m_x}\right)^2\right] - B_a$
  - width of energy interval: $E^{\max}_{Ax} - E^{\min}_{Ax} = 2\frac{m_B+m_b}{m_B(m_b+m_x)}\sqrt{2\mu_{Aa}E_{Aa}}\,\hbar Q^{\max}_{Bb}$

  $\Rightarrow$ selection of E_{Aa}

Applications

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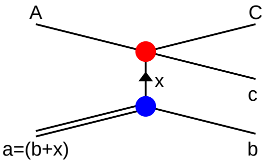

- **extraction of energy dependence of cross section** $\frac{d\sigma}{d\Omega_{Cc}}(A + x \rightarrow C + c)$
  - non-resonant rearrangement reactions
  - theoretical approximations
    $\Rightarrow$ no absolut cross sections
  - normalization to directly measured data at high energies
  - small energies in $C + c$ system reachable with 'large' energies in $A + a$ channel
    $\Rightarrow$ no suppression by Coulomb barrier
  - relation between off-shell and on-shell cross sections ?
  - validity of factorization of THM cross section ?
- **extensions**
  - low-energy elastic scattering
  - study of resonance properties
  - study of electron screening

Beyond the Plane-Wave Approximation

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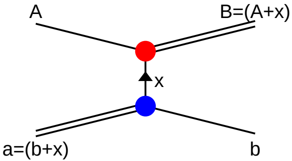
- **some references**
  - **Extraction of astrophysical cross sections in the Trojan-horse method**
    S. Typel, H. H. Wolter, Few-Body Systems 29, 75 (2000)
  - **Theory of the Trojan-horse method**
    S. Typel, G. Baur, Ann. Phys. 305, 228 (2003)
  - **Indirect techniques in nuclear astrophysics: Asymptotic normalization coefficient and Trojan Horse**
    A. M. Mukhamedzhanov et al., Eur. Phys. J. A 27, Suppl. 1, 205 (2006)
  - **Trojan Horse as an indirect technique in nuclear astrophysics. Resonance reactions**
    A. M. Mukhamedzhanov et al., J. Phys. G. 35, 014016 (2008)
  - **The Trojan Horse Method in nuclear astrophysics**
    C. Spitaleri et al., Phys. At. Nucl. 74, 1725 (2011)
  - **Indirect techniques in nuclear astrophysics: a review**
    R. E. Tribble et al., Rep. Prog. Phys. 77, 106901 (2014)



- **one-step or two-step process ?**
  - two-vertices in diagram:
    explicit propagation of transferred nucleus *x* ?
  - hints from Inclusive Non-Elastic Breakup (INEB) theory
    (M. S. Hussein, C. A. Bertulani, S. Typel, Phys. Lett. B 776 (2018) 217)

Inclusive Non-Elastic Breakup Theory I

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- **cross section of reaction a + A $\rightarrow$ b + c + C** (nucleus A infinitely heavy)

  $$\frac{d^2\sigma^{INEB}}{dE_b d\Omega_b} = \hat{\sigma}_R^x \, \rho_b \quad \text{with density of states} \quad \rho_b = \frac{\mu_b k_b}{(2\pi\hbar)^3}$$

  and reaction cross section

  $$\hat{\sigma}_R^x = -\frac{k_x}{E_x} \langle \hat{\rho}_x(\vec{r}_x) | W_x(\vec{r}_x) | \hat{\rho}_x(\vec{r}_x) \rangle \quad \text{with imaginary part of optical potential} \quad W_x = \mathrm{Im} U_x$$

- **source function**

  $$\hat{\rho}_x(\vec{r}_x) = (\chi_b^{(-)} | \Psi_{3B}^{(+)} \rangle \quad \text{with exact three-body } (x + b + A) \text{ wave function} \quad |\Psi_{3B}^{(+)}\rangle$$

  - Ichimura-Austern-Vincent (post) form
    $|\Psi_{3B}^{(+)}\rangle = (E - K_b - U_b - K_x - U_x + i\epsilon)^{-1} V_{xb} |\chi_a^{(+)} \phi_a\rangle$
  - Udagawa-Tamura (prior) form
    $|\Psi_{3B}^{(+)}\rangle = (E - K_b - U_b - K_x - U_x + i\epsilon)^{-1} (U_x + U_b - U_a) |\chi_a^{(+)} \phi_a\rangle$

Inclusive Non-Elastic Breakup Theory II

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- **relation of source functions**

  $\hat{\rho}_x^{IAV}(\vec{r}_x) = \hat{\rho}_x^{UT}(\vec{r}_x) + \hat{\rho}_x^{HM}(\vec{r}_x)$

  with

  - $\hat{\rho}_x^{IAV}(\vec{r}_x) = G_x^{(+)}(E_x)(\chi_b^{(-)}|V_{xb}|\chi_a^{(+)}\phi_a\rangle$ (Ichimura-Austern Vincent)
  - $\hat{\rho}_x^{UT}(\vec{r}_x) = G_x^{(+)}(E_x)(\chi_b^{(-)}|(U_b + U_x - U_a)|\chi_a^{(+)}\phi_a\rangle$ (Udagawa-Tamura)
  - $\hat{\rho}_x^{HM}(\vec{r}_x) = (\chi_b^{(-)}|\chi_a^{(+)}\phi_a\rangle$ (Hussein-McVoy, non-orthogonality condition)

  and Green's function $G_x^{(+)}(E_x) = (E_x - K_x - U_x + i\epsilon)^{-1}$ of particle x

- **meaning of source functions**
  - UT: elastic breakup followed by capture of x, target remains in ground state
  - IAV: UT + all other processes (target excited or other channels)

  $\Rightarrow$ only IAV and HM relevant for THM reaction

Inclusive Non-Elastic Breakup Theory III

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- **general structure of imaginary part of optical potential** for direct processes

$$-W_x^D = \pi \sum_f \int \frac{d^3 k_f}{(2\pi)^3} \, V_{(0,f)} |\chi_f^{(-)}(\vec{k}_f)\rangle \langle \chi_f^{(-)}(\vec{k}_f)| \, V_{(f,0)} \; \delta(E_x - E_f)$$

  with sum over all intermediate states f, potential $V_{(0,f)} = P_x^{(0)} V P_x^{(D)}$ and projectors $P_x^{(D)}$ on direct, non-elastic $x + A$ channels and $P_x^{(0)}$ on elastic ones

- **direct part of INEB cross section**

$$\frac{d^2\sigma^{INEB,D}}{dE_b d\Omega_b} = \pi \rho_b(E_b) \frac{k_x}{E_x} \sum_f \int \frac{d^3 k_f}{(2\pi)^3} \; \delta(E_x - E_f) \; \left| \langle \chi_f^{(-)}(\vec{k}_f) | V_{(0,f)} | \hat{\rho}_x \rangle \right|^2$$

  with $\hat{\rho}_x = \hat{\rho}_x^{IAV}$ or $\hat{\rho}_x = \hat{\rho}_x^{HM}$

Inclusive Non-Elastic Breakup Theory IV

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- **selection of particular channel, e.g., c + C**

$$\frac{d^4\sigma^{INEB,D}}{dE_b d\Omega_b dE_c d\Omega_c} = \pi\rho_b(E_b)\rho_c(E_c)\frac{k_x}{E_x}\left|A_{x,c}\right|^2$$

  with amplitude $A_{x,c} = \langle\chi_c^{(-)}(\vec{k}_c)|V_{(x,c)}|\hat{\rho}_x\rangle$

- **eikonal/Glauber-type approximation** $\chi_a^{(+)}(\vec{r}_b,\vec{r}_x) = \chi_b^{(+)}(\vec{r}_b)\chi_x^{(+)}(\vec{r}_x)$
  $\Rightarrow$ amplitudes
  - IAV source function: $A_{x,c} = \langle\chi_c^{(-)}(\vec{k}_c)|V_{(x,c)}G_x^{(+)}(E_x)V_{\text{ebu}}|\chi_x^{(+)}(\vec{k}_x)\rangle$

    with elastic breakup potential $V_{\text{ebu}} = \langle\chi_b^{(-)}|V_{xb}|\chi_b^{(+)}\phi_a\rangle \Rightarrow$ two-step process

  - HM source function: $A_{x,c} = \langle\chi_c^{(-)}(\vec{k}_c)|V_{(x,c)}\hat{S}_b(\vec{r}_x)|\chi_x^{(+)}(\vec{k}_x)\rangle$

    with modified *b*-fragment elastic S-matrix element
    $\hat{S}_b(\vec{r}_x) = \langle\chi_b^{(-)}(\vec{k}'_b,\vec{r}_b)|\phi_a(\vec{r}_b,\vec{r}_x)\chi_b^{(+)}(\vec{k}_b,\vec{r}_b)\rangle \Rightarrow$ one-step process

    – with $\hat{S}_b(\vec{r}_x) \approx \phi_a(\vec{k}'_b - \vec{k}_b) \Rightarrow$ THM form of amplitude

Three-Body Trojan-Horse Reaction Theory

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- **T matrix element in post-form DWBA**

  $T_{THM} = \langle \Psi_{cC}^{(-)} \phi_b \chi_{bB}^{(-)} | V_{xb} | \Psi_{Aa}^{(+)} \rangle$ with full three-body wave function $|\Psi_{Aa}^{(+)}\rangle$

  - standard THM DWBA $|\Psi_{Aa}^{(+)}\rangle \approx \chi_{Aa}^{(+)} \phi_A \phi_a$

    $\Rightarrow T_{\mathrm{THM}} = \langle \Psi_{cC}^{(-)} \phi_b \chi_{bB}^{(-)} | V_{xb} | \chi_{Aa}^{(+)} \phi_A \phi_a \rangle$

    $\Rightarrow$ one-step process

  - Faddeev decomposition $\Psi_{Aa}^{(+)} = \Psi_{xb}^{(+)} + \Psi_{xA}^{(+)} + \Psi_{bA}^{(+)}$

    with dominant component $\Psi_{xb}^{(+)} = G_{x,b}^{(+)} V_{xb} \Psi_{Aa}^{(+)}$

    and Green's function $G_{x,b}^{(+)} = (E - K_b - U_b - K_x - U_x + i\epsilon)^{-1}$

    $\Rightarrow T_{THM} = \langle \Psi_{cC}^{(-)} \phi_b \chi_{bB}^{(-)} | V_{xb} G_{x,b}^{(+)} V_{xb} | \chi_{Aa}^{(+)} \phi_A \phi_a \rangle$ in DWBA

    $\Rightarrow$ two-step process

  to be explored further

Conclusions

- Trojan-horse method is indirect method to study low-energy rearrangement reactions
- energy dependence of astrophysical two-body reactions can be extracted from three-body reactions
- basic reaction theory well developed
- main features understood
- detailed comparison of different approximations needed
- more rigorous theoretical treatment necessary