Stellar weak interaction rates for exotic proton-rich nuclei in the beyond-mean-field approach

A. PETROVICI

Horia Hulubei National Institute for Physics and Nuclear Engineering, Bucharest, Romania

Outline

- *complex* **EXCITED VAMPIR beyond-mean-field model**

- **shape-coexistence effects on terrestrial and stellar weak interaction rates for**

  - ***^{68}Se and ^{72}Kr - rp-process waiting points***

  - ***^{70}Kr and ^{74}Sr - Z=N+2 isotopes***

- **possible effects of EXVAM stellar weak interaction rates on the rp process in type I x-ray bursts**

A~70 proton-rich nuclei manifest exotic structure and dynamics *generated by*

- ***shape coexistence and shape mixing***
- ***competing T=0 and T=1 pairing correlations***
- ***isospin-symmetry-breaking interactions***

responsible for

drastic changes in structure with number of nucleons, spin, and excitation energy

Challenges for theory

- ***realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods***
- ***comprehensive understanding of structure phenomena and β-decay properties***
- ***robust predictions on stellar weak interaction rates***

based on

self-consistent description of experimentally accessible properties

complex VAMPIR model family

- **the model space is defined by a finite dimensional set of spherical single particle states**

- **the effective many-body Hamiltonian is represented as a sum of one- and two-body terms**

- **the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua**

- **the HFB transformations are essentially *complex* and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry**

(T=1 and T=0 neutron-proton pairing correlations already included at the mean-field level)

- **the broken symmetries (s=N, Z, I, p) are restored by projection before variation**

** The models allow to use rather large model spaces and realistic effective interactions*

Beyond-mean-field variational procedure: complex EXCITED VAMPIR model

Vampir

$$E^s[F_1^s] = \frac{\langle F_1^s|\hat{H}\hat{\Theta}_{00}^s|F_1^s\rangle}{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}$$

$\Theta^s{}_{00}$ - symmetry projector

$|F^s{}_1>$ - HFB vacuum

$$|\psi(F_1^s); sM\rangle = \frac{\Theta_{M0}^s|F_1^s\rangle}{\sqrt{\langle F_1^s|\hat{\Theta}_{00}^s|F_1^s\rangle}}$$

Excited Vampir

$$|\psi(F_2^s); sM\rangle = \hat{\Theta}_{M0}^s\left\{|F_1^s\rangle\alpha_1^2 + |F_2^s\rangle\alpha_2^2\right\}$$

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^i\,|\phi(F_j^s)\rangle\,\alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \Theta_{M0}^s|F_i^s\rangle$$

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1}\,|\phi(F_j^s)\rangle\,\alpha_j^n + |\phi(F_n^s)\rangle\,\alpha_n^n$$

$$(H - E^{(n)}N)f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM> = \sum_{i=1}^{n}|\psi_i; sM> f_{i\alpha}^{(n)}, \qquad \alpha = 1, \ldots, n$$

A ~ 70 mass region

^{40}Ca - core

model space for protons and neutrons

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $1d_{5/2}$ $0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $2s_{1/2}$ $1d_{3/2}$ $1d_{5/2}$ $0g_{7/2}$ $0g_{9/2}$ $0h_{11/2}$ ***(ext-model space)***

renormalized G-matrix (OBEP- Bonn A/ CD)

- ***pairing properties enhanced by short range Gaussians for:***

  T = 1 : pp *(-35 MeV)*, np *(-20 MeV)*, nn *(-35 MeV)*

  T = 0: np *(-35 MeV)*

- ***onset of deformation influenced by monopole shifts:***

  $\langle 0g_{9/2}\, 0f; T=0\, |G|\, 0g_{9/2}\, 0f; T=0 \rangle$ *($0f_{5/2}$, $0f_{7/2}$)*

  $\langle 1d_{5/2}\, 1p; T=0\, |G|\, 1d_{5/2}\, 1p; T=0 \rangle$ *($1p_{1/2}$, $1p_{3/2}$)*

- ***Coulomb interaction between valence protons added***

Self-consistent terrestrial and stellar weak interaction rates

Fermi transition probabilities

$$B_{if}(F) = \frac{1}{2J_i + 1} \frac{g_V^2}{4\pi} |M_F|^2$$

$$M_F \equiv (\xi_f J_f || \hat{1} || \xi_i J_i)$$
$$= \delta_{J_i J_f} \sum_{ab} M_F(ab) (\xi_f J_f || [c_a^\dagger \tilde{c}_b]_0 || \xi_i J_i)$$

$$M_F(ab) = (a || \hat{1} || b)$$

Gamow-Teller transition probabilities

$$B_{if}(GT) = \frac{1}{2J_i + 1} \frac{g_A^2}{4\pi} |M_{GT}|^2$$

$$M_{GT} \equiv (\xi_f J_f || \hat{\sigma} || \xi_i J_i)$$
$$= \sum_{ab} M_{GT}(ab) (\xi_f J_f || [c_a^\dagger \tilde{c}_b]_1 || \xi_i J_i)$$

$$M_{GT}(ab) = 1/\sqrt{3} (a || \hat{\sigma} || b)$$

Independent chains of variational calculations for all parent and daughter states

Weak interaction rates in X-ray burst astrophysical environment

A. Petrovici and O. Andrei, Eur. Phys. J. A51, 133 (2015)
Phys. Rev. C92, 064305 (2015)

In the X-ray burst stellar environment at densities (~10^6 g/cm^3) and temperatures (~10^9K) typical for the rp-process the contribution of thermally populated low-lying 0^+ and 2^+ states may be relevant.

$$\lambda^{\alpha} = \frac{ln2}{K} \sum_i \frac{(2J_i+1)e^{-E_i/(kT)}}{G(Z,A,T)} \sum_j B_{ij}\phi_{ij}^{\alpha}$$

$$G(Z,A,T) = \sum_i (2J_i+1) exp(-E_i/(kT))$$

$$B_{ij} = B_{ij}(F) + B_{ij}(GT)$$

$$\phi_{ij}^{ec} = \int_{w_l}^{\infty} wp(Q_{ij}+w)^2 F(Z,w) S_e(w)(1-S_\nu(Q_{ij}+w))dw$$

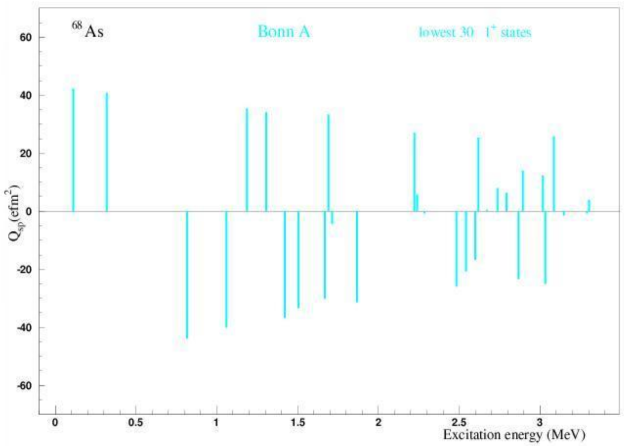
$$\phi_{ij}^{\beta^+} = \int_1^{Q_{ij}} wp(Q_{ij}-w)^2 F(-Z+1,w)(1-S_p(w))(1-S_\nu(Q_{ij}-w))dw$$

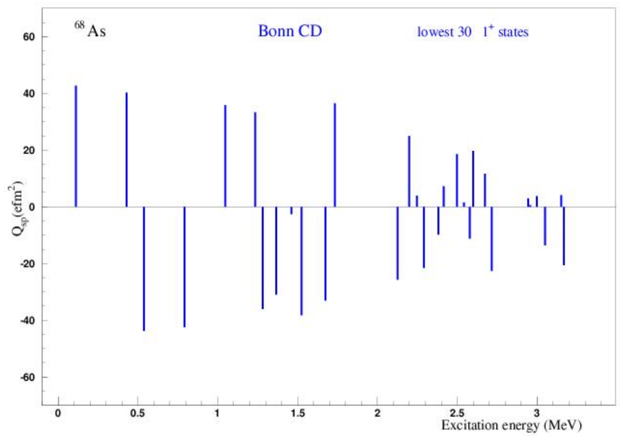
Shape coexistence and weak interaction rates for ^{68}Se rp-process waiting point

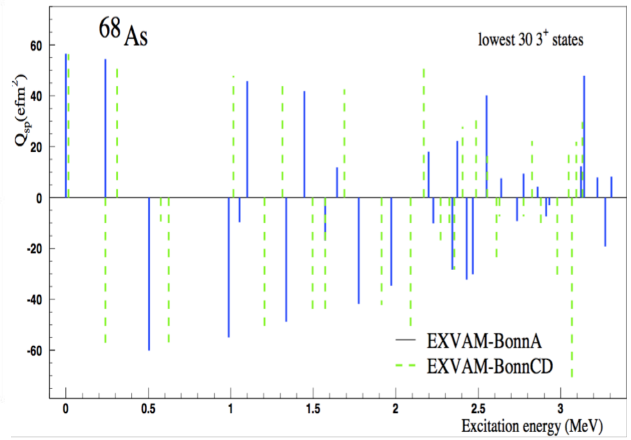
A. Petrovici and O. Andrei, Eur. Phys. J. A51, 133 (2015)

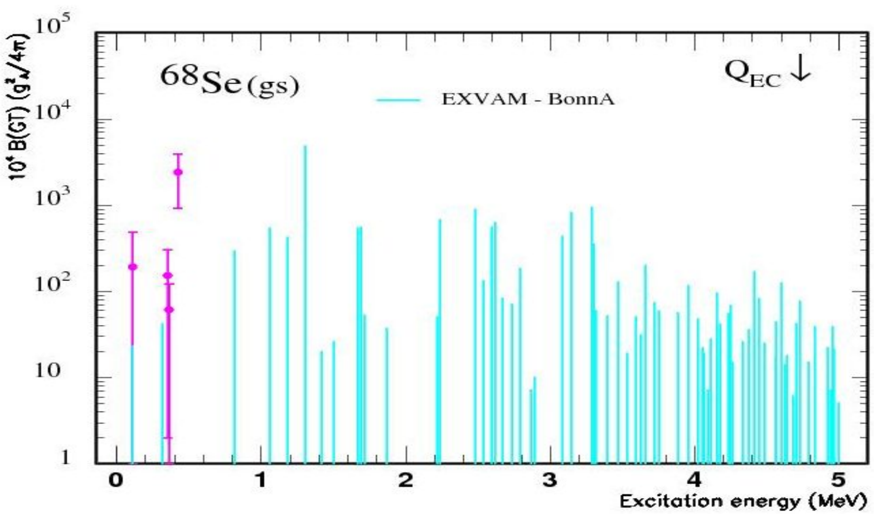
$E_{0^+_{gs}}$ **= 0.0 MeV** [*62/55(%) – oblate (*BonnA/BonnCD)] $E_{2^+_{yr}}$ **= 0.854 MeV** [*60/41(%) – oblate (*BonnA/BonnCD)]

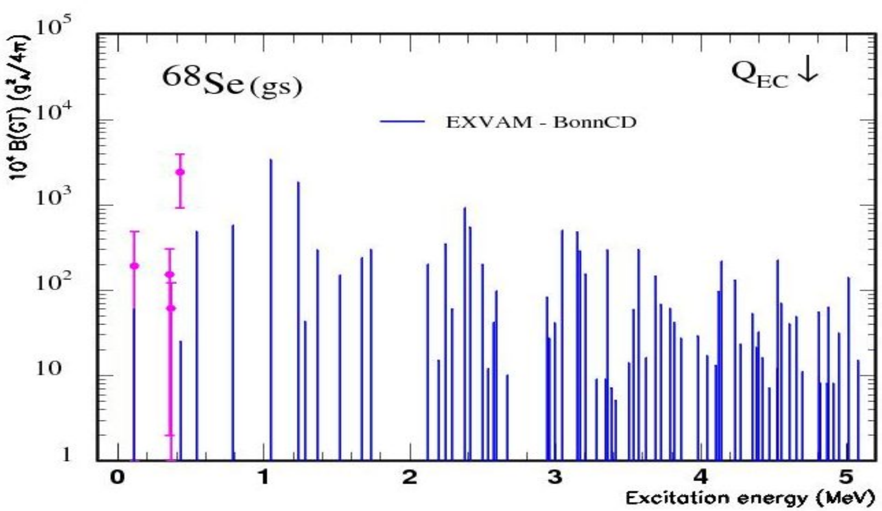
$Q^{sp}_{2^+_{yr}}$ = 3.5 efm^2(A) / -7.1 efm^2 (CD) **B(E2;2$^+$-> 0$^+$) =521/503** e^{2}fm^4 *(*BonnA/BonnCD) **Exp.: 430(60) e^{2}fm^4**

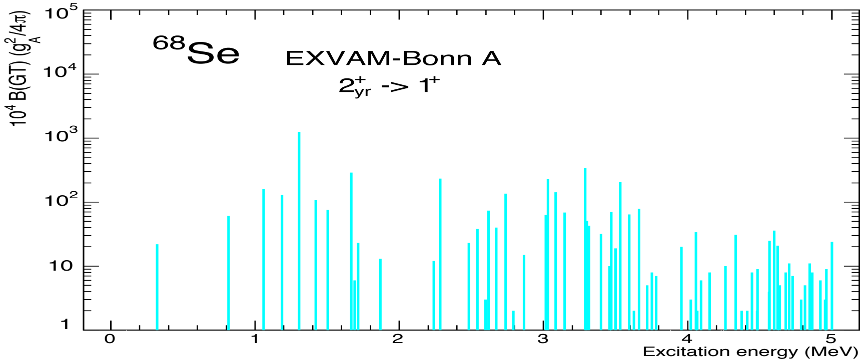


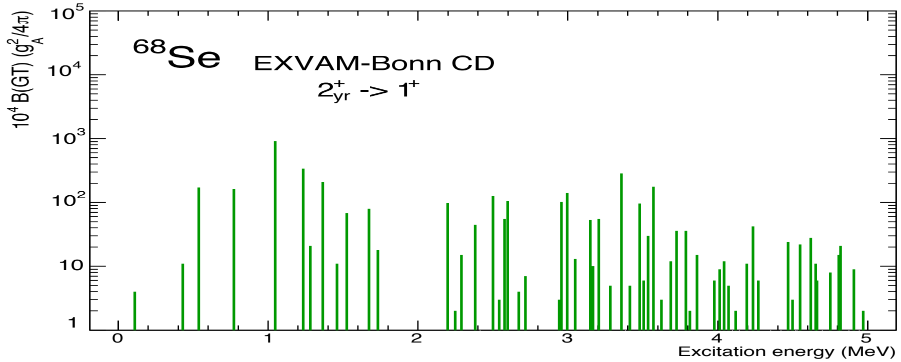


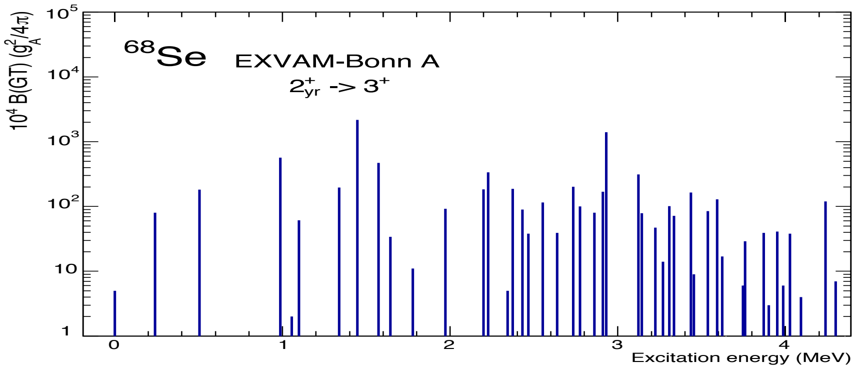


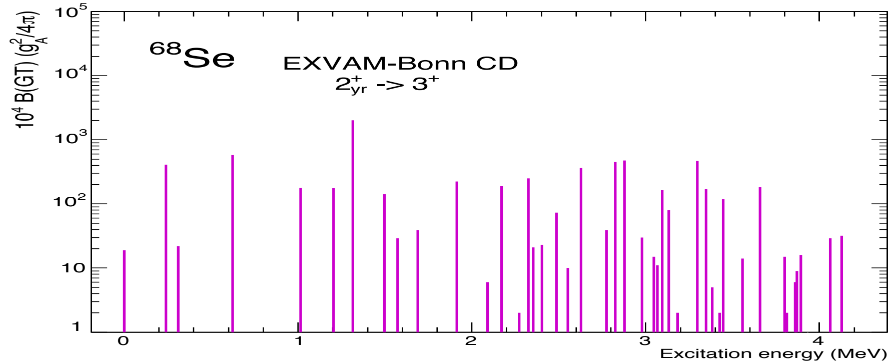






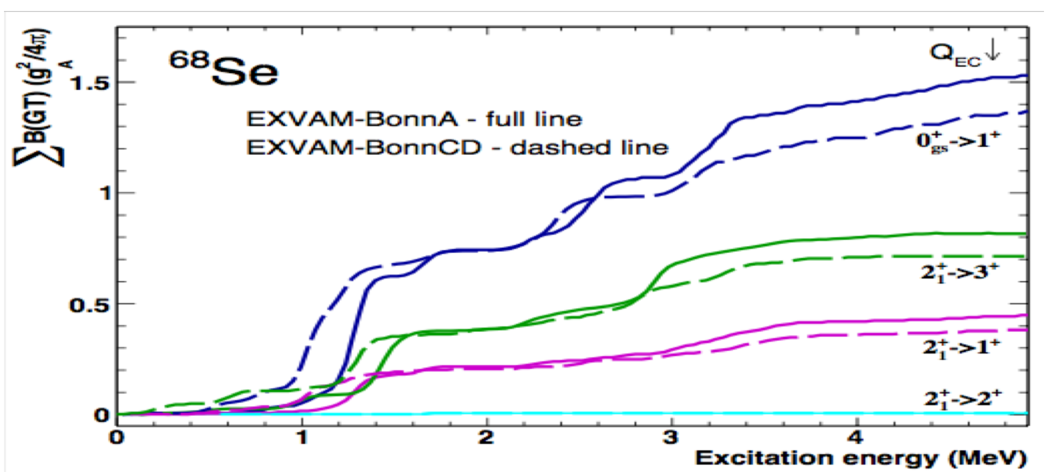






Contributions: - $p^{\nu(\pi)}_{1/2}p^{\pi(\nu)}_{3/2}$, $p^{\nu}_{3/2}p^{\pi}_{3/2}$, $f^{\nu}_{5/2}f^{\pi}_{5/2}$, $f^{\nu(\pi)}_{5/2}f^{\pi(\nu)}_{7/2}$, $g^{\nu}_{9/2}g^{\pi}_{9/2}$ matrix elements (decay to 1^+ states)

- $p^{\nu}_{3/2}p^{\pi}_{1/2}$, $p^{\nu}_{3/2}p^{\pi}_{3/2}$, $f^{\nu}_{5/2}f^{\pi}_{7/2}$ matrix elements (decay to 3^+ states)



$T_{1/2}^{exp} = 35.5(7)$ s

$T_{1/2}^{EXVAM} = 48.8 / 33.5$ s (BonnA/BonnCD)

Shape coexistence and weak interaction rates for ^{72}Kr rp-process waiting point

A. Petrovici and O. Andrei, Eur. Phys. J. A51, 133 (2015)

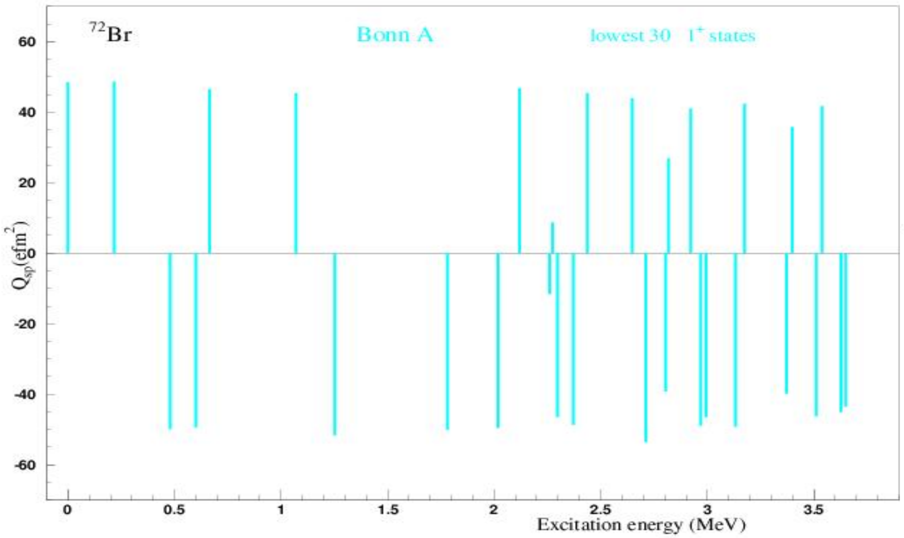
$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $1d_{5/2}$ $0g_{9/2}$ (standard-model space)

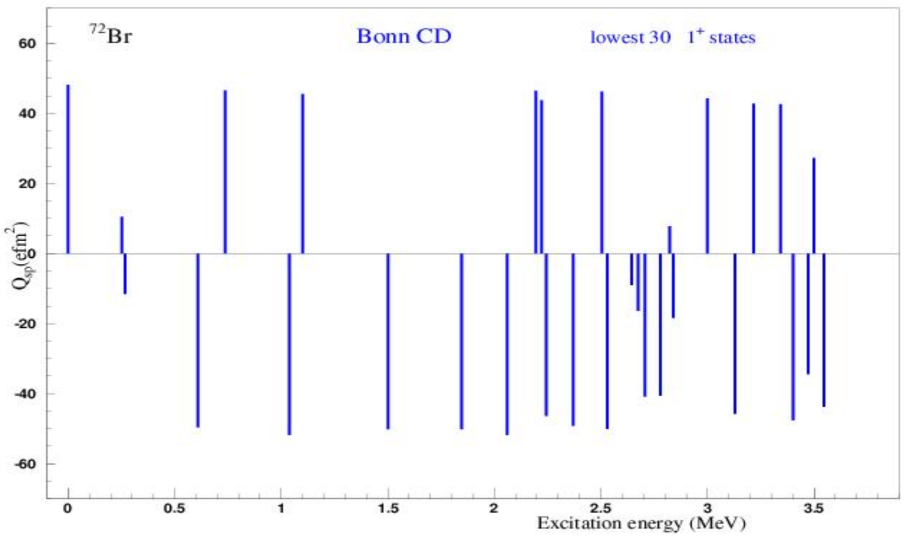
$1p_{1/2}$ $1p_{3/2}$ $0f_{5/2}$ $0f_{7/2}$ $2s_{1/2}$ $1d_{3/2}$ $1d_{5/2}$ $0g_{7/2}$ $0g_{9/2}$ $0h_{11/2}$ (ext-model space)

$E_{0^+_{gs}}$ = 0.0 MeV p/o mixing *60/40%* (BonnA-ext-space) *34/66%* (BonnA-standard space)
$E_{0^+_{exc}}$ = 0.671 MeV p/o mixing *38/62%* (BonnA-ext-space) *63/37%* (BonnA-standard space)
$E_{2^+_{yrast}}$ = 0.710 MeV p/o mixing *41/59%* (BonnA-ext-space)) *7/93%* (BonnA-standard space)

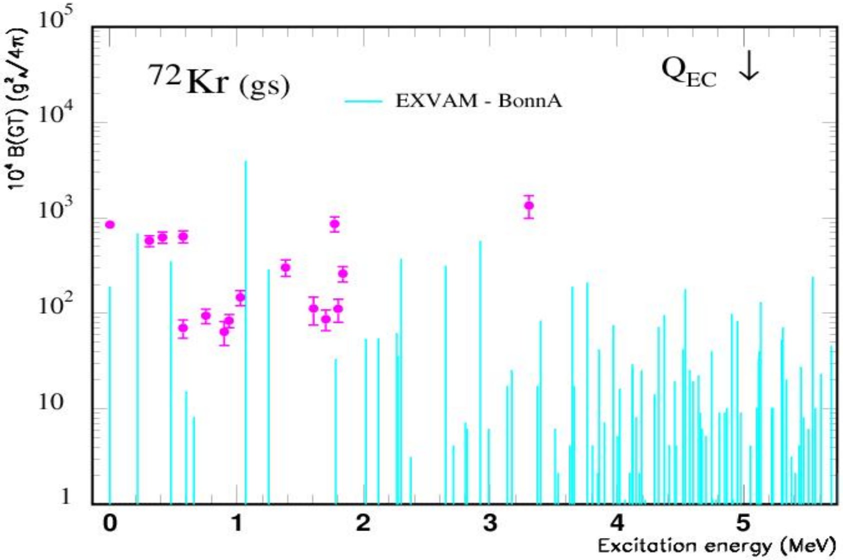
B(E2;$2^+ \to 0^+$) = **853/670** e^2fm^4 (BonnA-extended space/BonnA-standard space) Exp.: **810 (150)** e^2fm^4

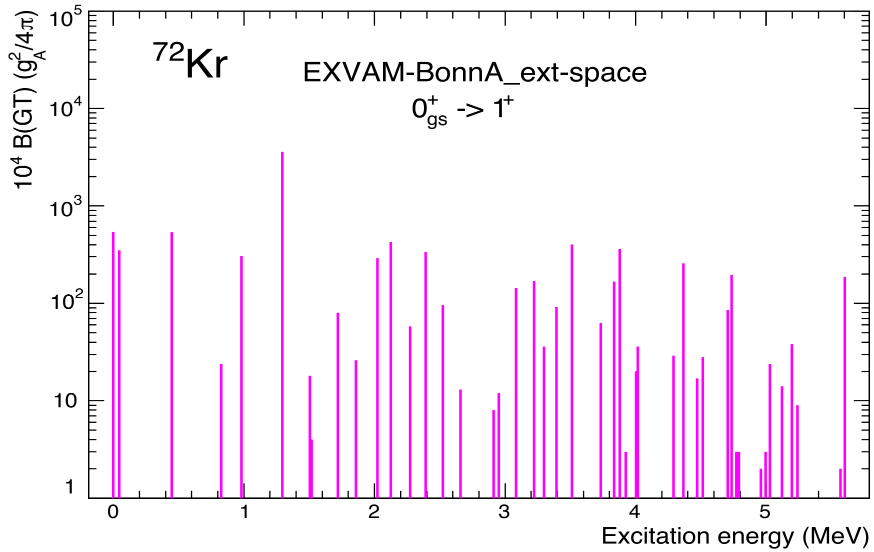
Shape coexistence and mixing in parent and daughter nucleus

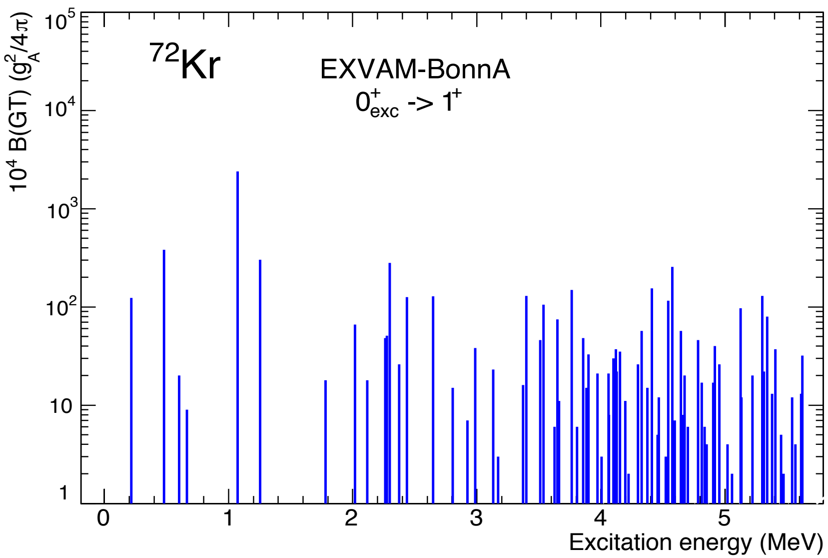


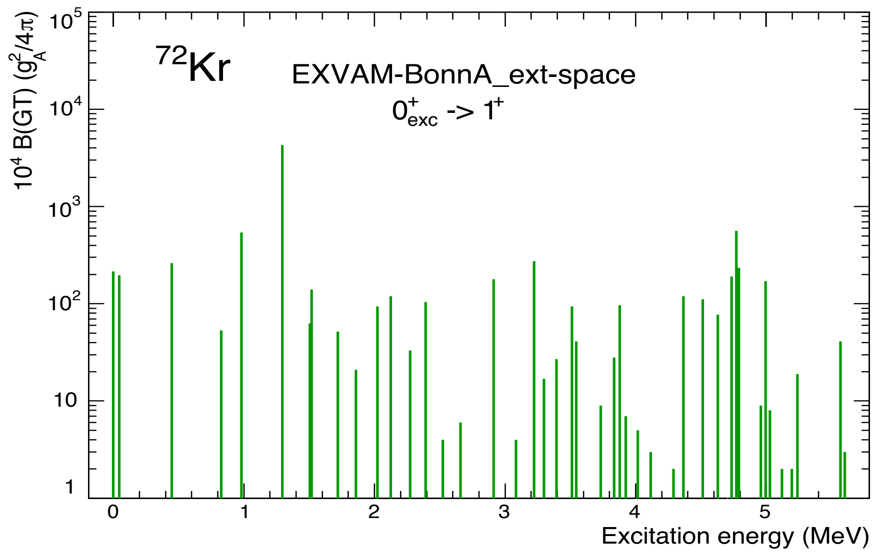


Gamow-Teller strength distributions depend strongly on parent structure

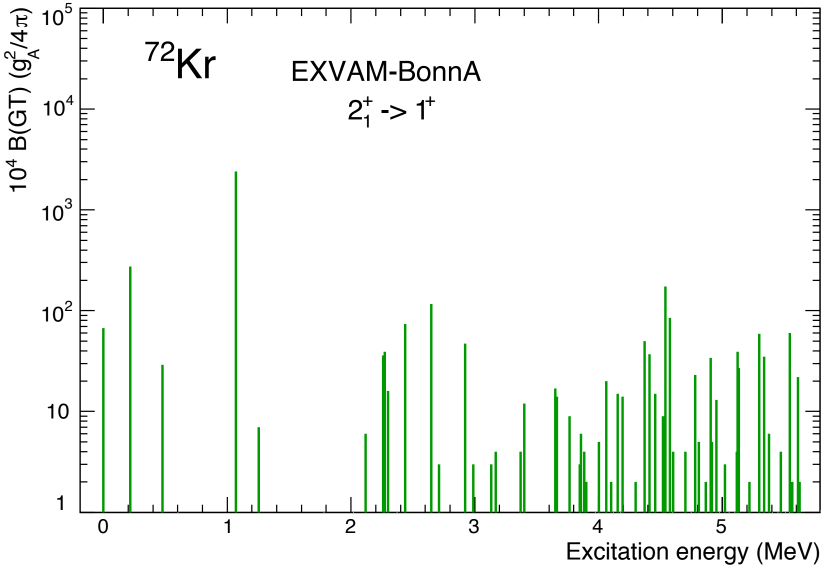


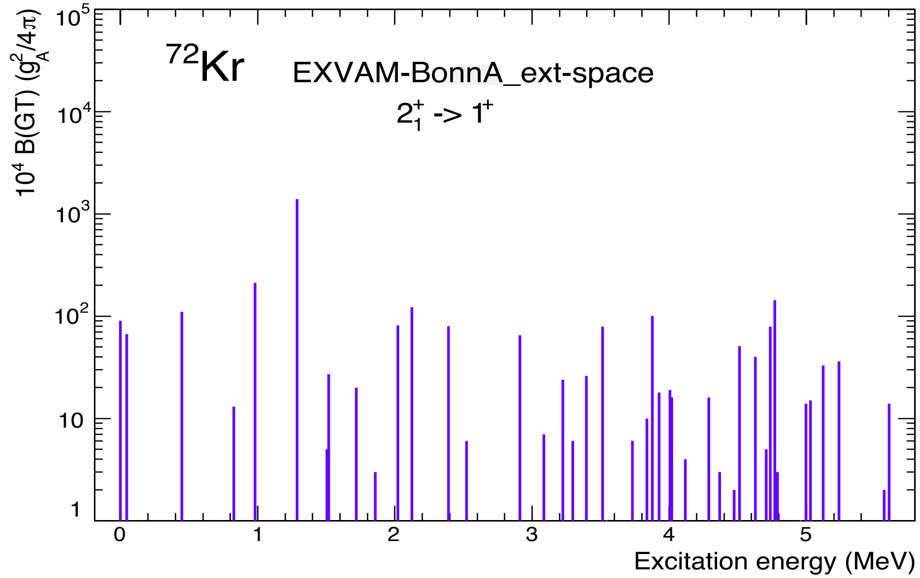


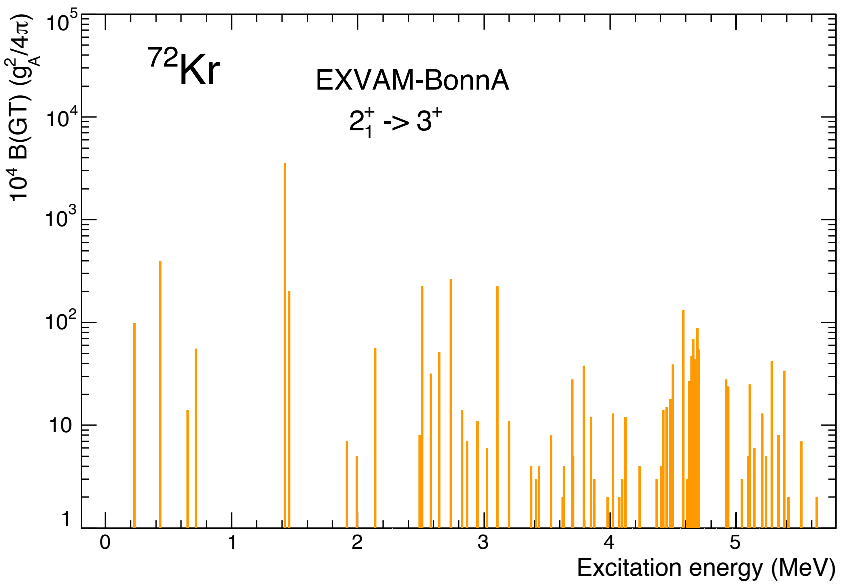


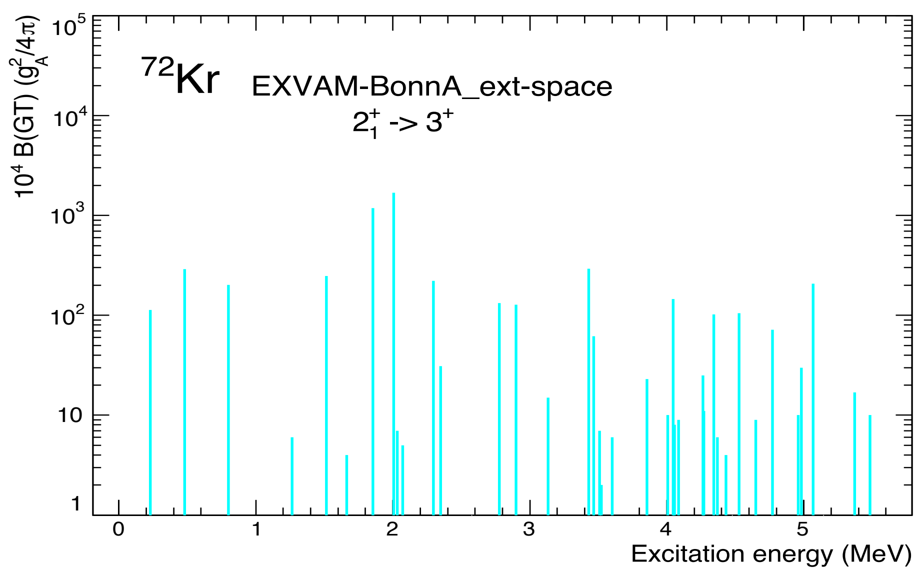


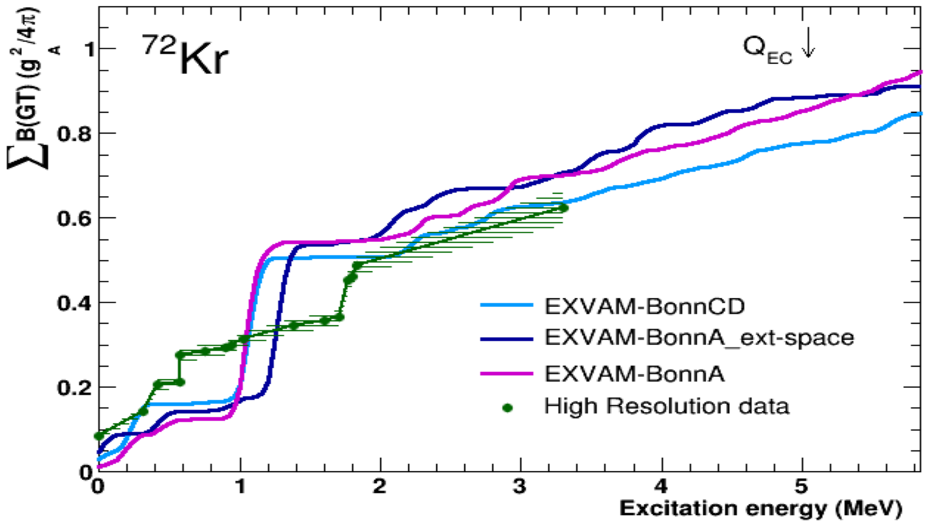
Gamow-Teller strength distributions depend strongly on daughter structure











$T_{1/2}^{EXVAM}$ = 18.9 s (BonnCD)

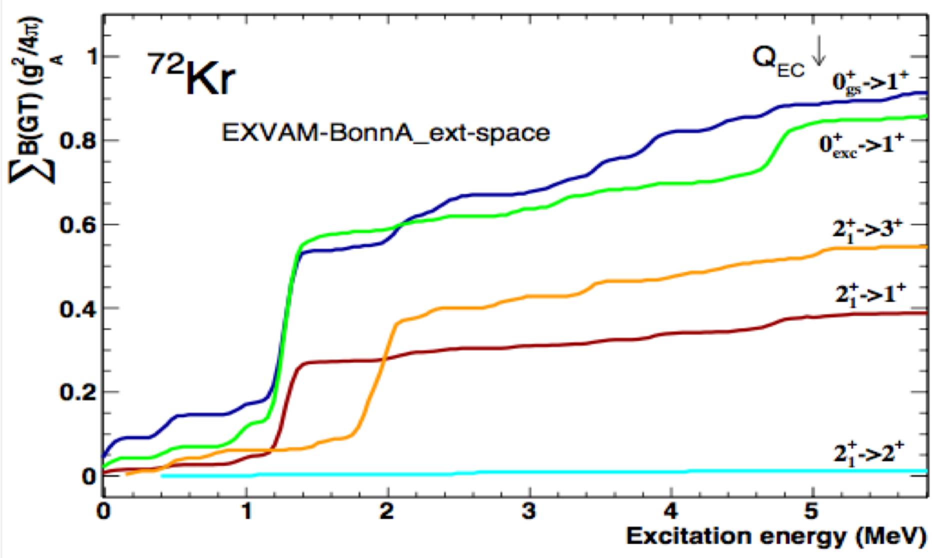
$T_{1/2}^{EXVAM}$ = 20.8 s (BonnA)

$T_{1/2}^{EXVAM}$ = 20.7 s (BonnA-ext-space)

$T_{1/2}^{exp}$ = 17.1(2) s

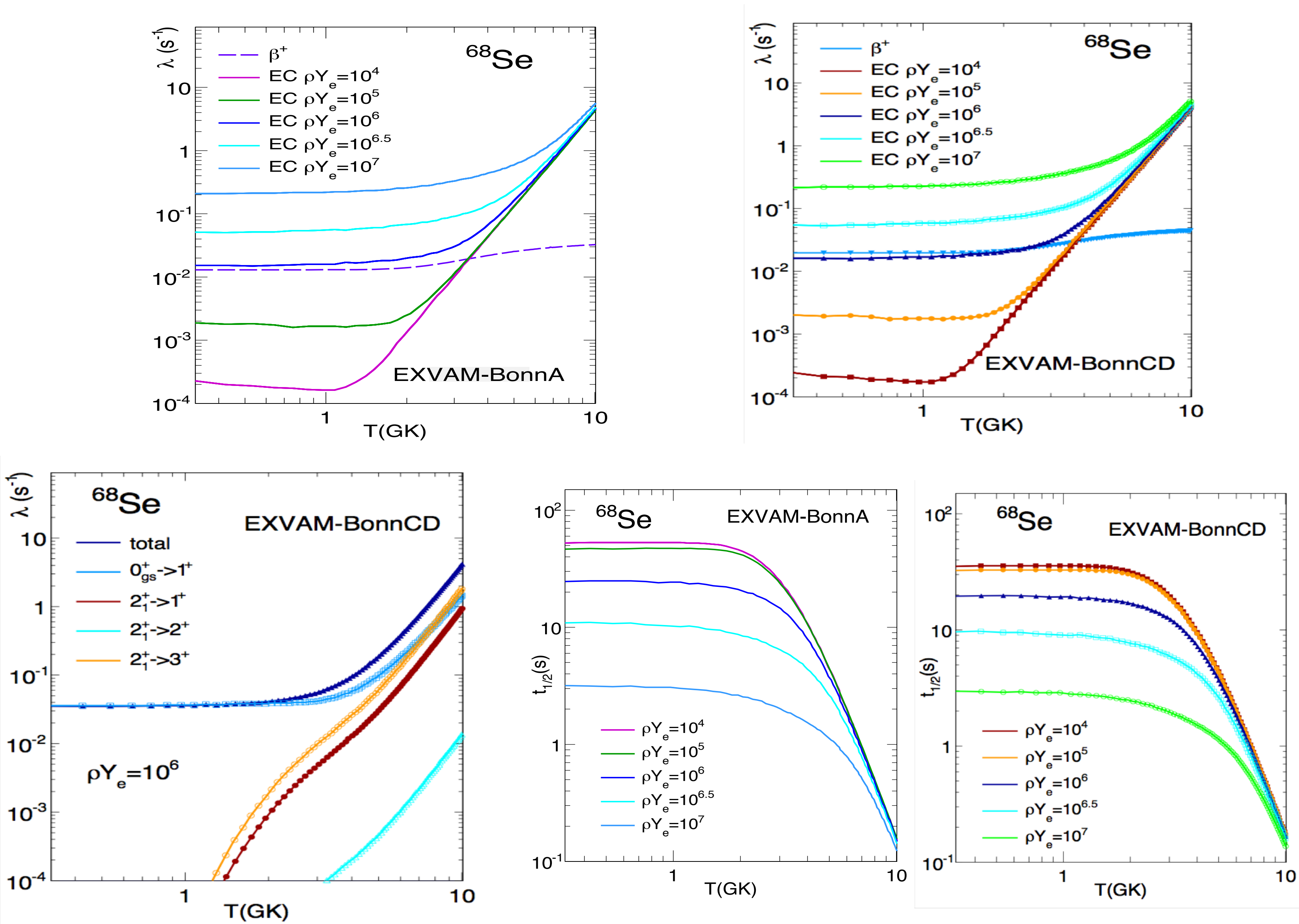
***Contributions:** - $p^{\nu(\pi)}_{1/2}p^{\pi(\nu)}_{3/2}$, $p^{\nu}_{3/2}p^{\pi}_{3/2}$, $f^{\nu}_{5/2}f^{\pi}_{5/2}$, $f^{\nu(\pi)}_{5/2}f^{\pi(\nu)}_{7/2}$, $g^{\nu}_{9/2}g^{\pi}_{9/2}$ matrix elements (decay to 1^+ states)*

- $p^{\nu}_{3/2}p^{\pi}_{1/2}$, $p^{\nu}_{3/2}p^{\pi}_{3/2}$, $f^{\nu}_{5/2}f^{\pi}_{7/2}$ matrix elements (decay to 3^+ states)



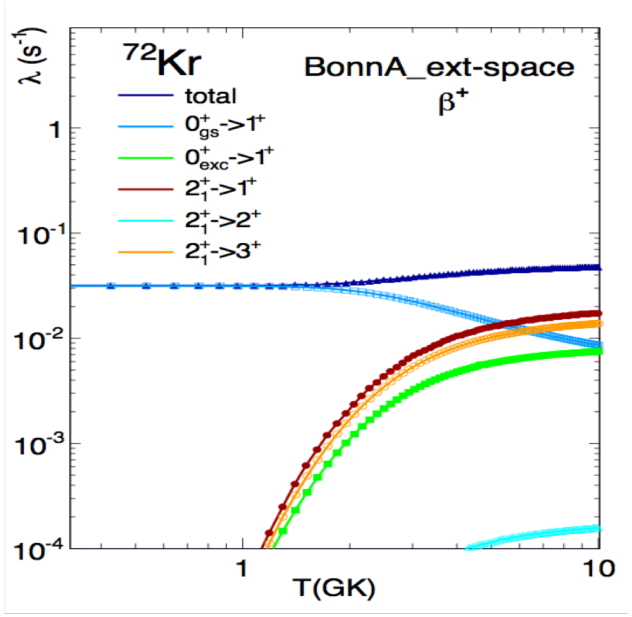
Stellar rates for ^{68}Se : β^+ and continuum electron capture

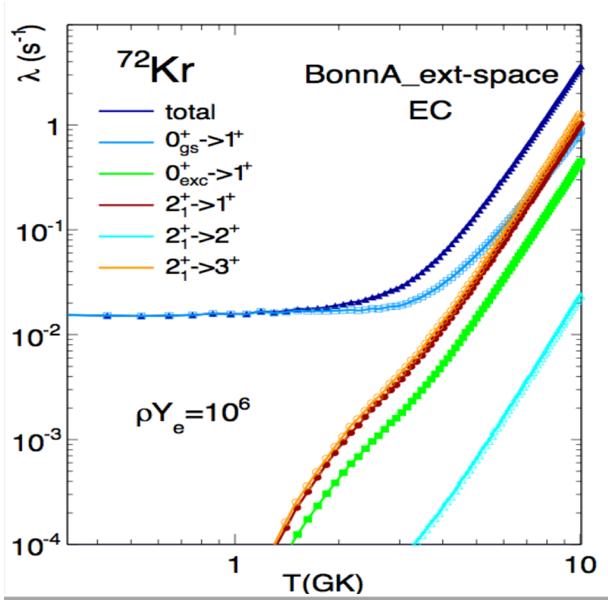
Significant continuum electron capture contribution

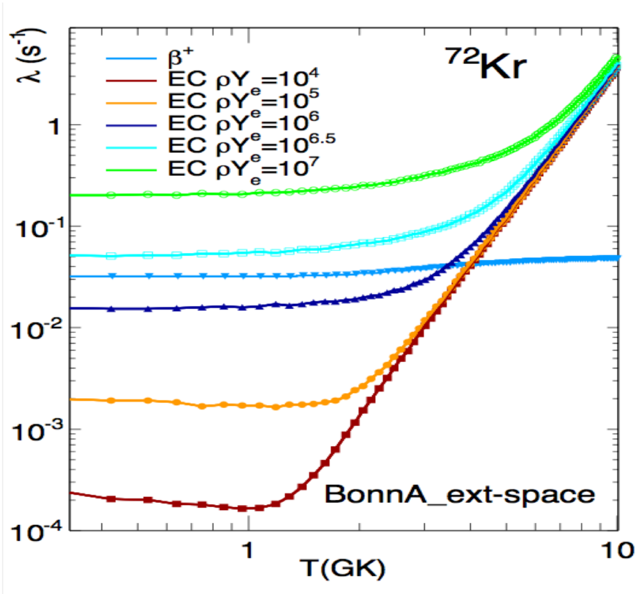


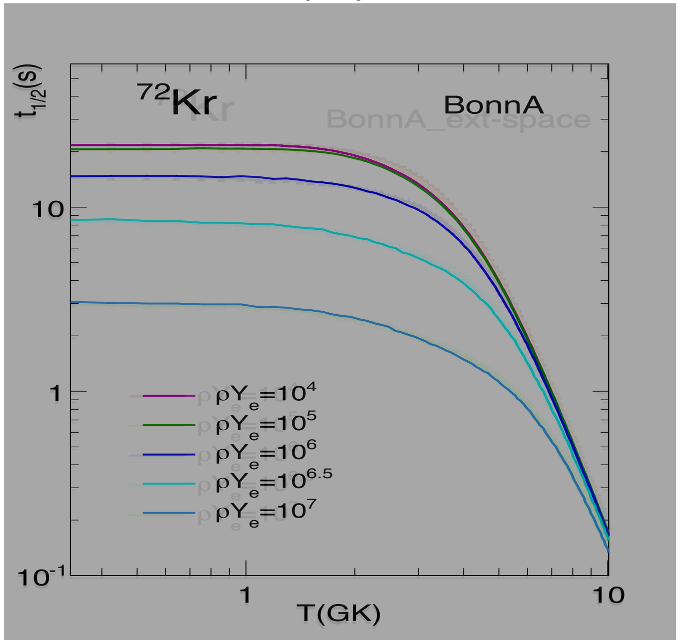
Stellar rates for ^{72}Kr : β^+ and continuum electron capture

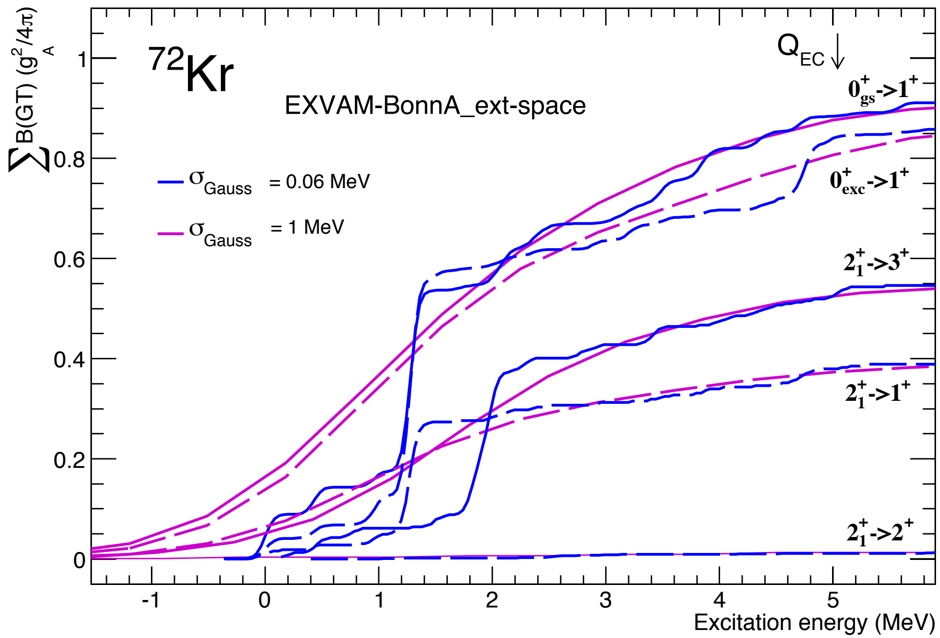
Significant continuum electron capture contribution



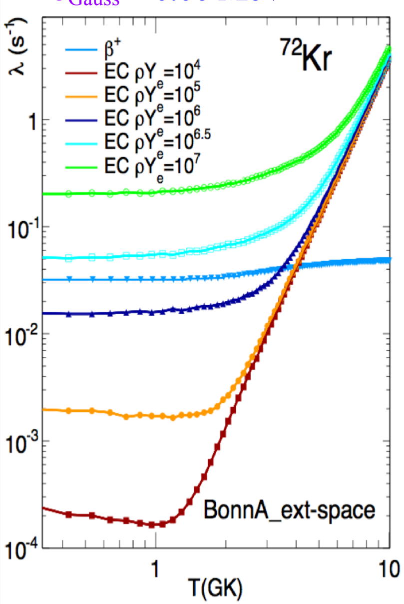




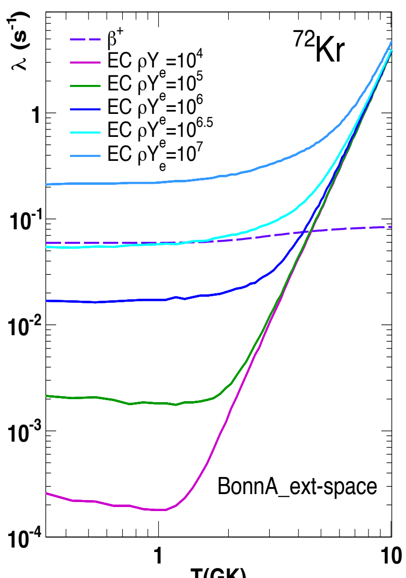




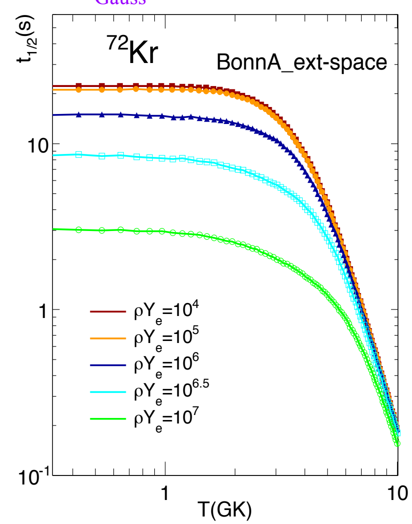
σ_{Gauss} = **0.06 MeV**



σ_{Gauss} = **1 MeV**

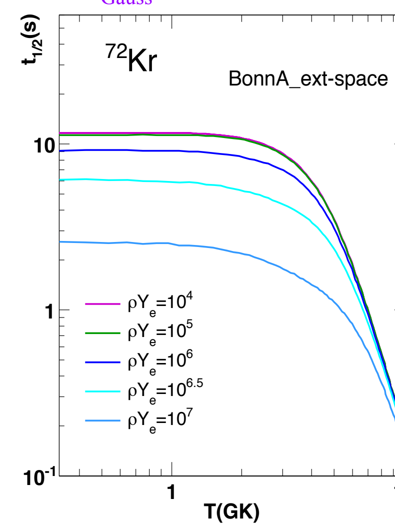


σ_{Gauss} = **0.06 MeV**



$T_{1/2}(T=3GK, \rho Y_e=10^6)$= **10.4** s

σ_{Gauss} = **1 MeV**

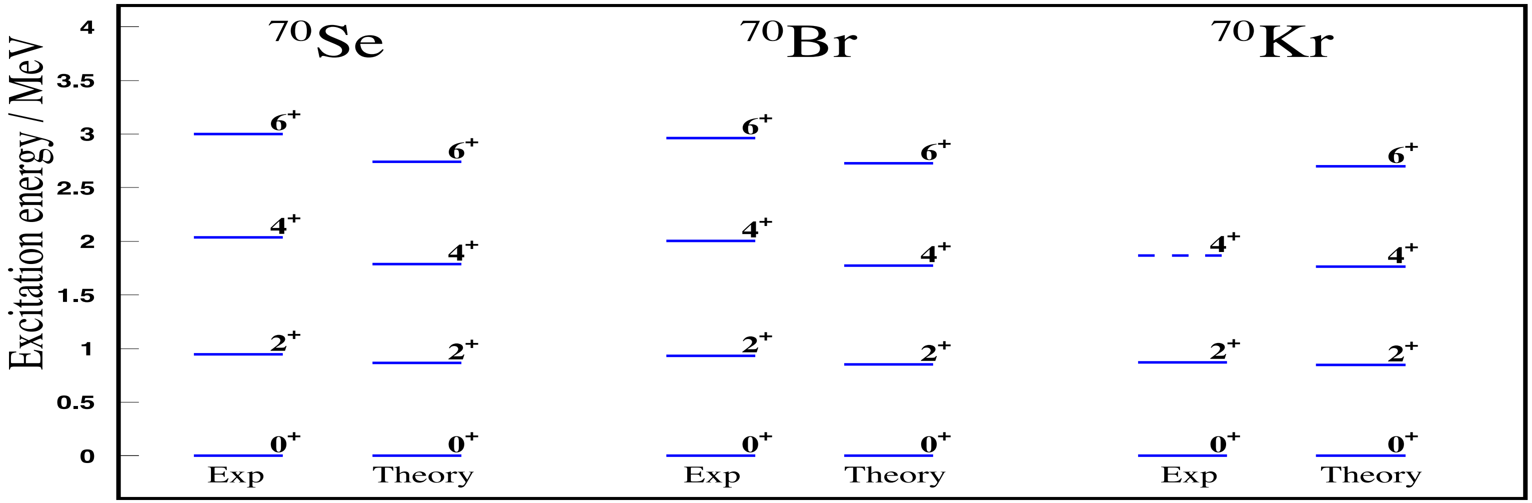


$T_{1/2}(T=3GK, \rho Y_e=10^6)$= **6.9** s

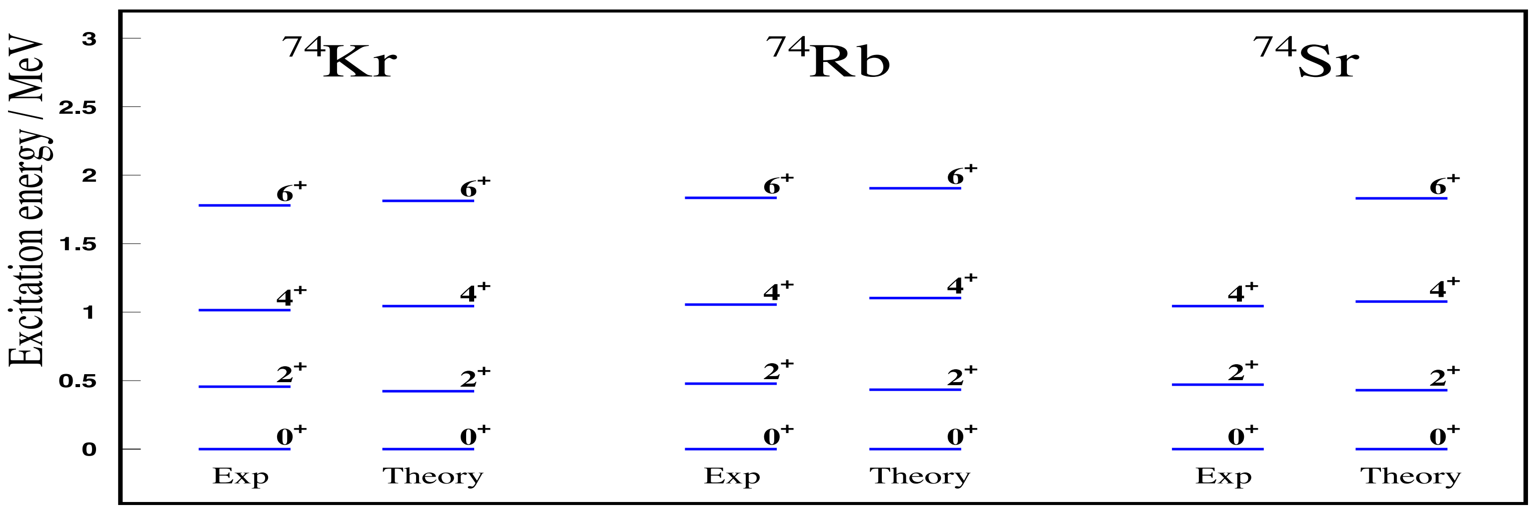
Shape coexistence and isospin symmetry breaking in analog spectra of isovector triplets

A. Petrovici, Phys. Rev. C 91, 014302 (2015), Phys. Scr. 92, 064003 (2017)
A. Petrovici, O. Andrei , A. Chilug, Phys. Scr. 93, 114001 (2018)

${}_{34}Se_{36} – {}_{35}Br_{35} – {}_{36}Kr_{34}$



${}_{36}Kr_{38} – {}_{37}Rb_{37} – {}_{38}Sr_{36}$



Shape mixing in the wave functions of analog states

A = 70 triplet: $_{34}Se_{36} - {}_{35}Br_{35} - {}_{36}Kr_{34}$

Spectroscopic quadrupole moments

$I(\hbar)$	^{70}Se	^{70}Br	^{70}Kr
2_1^+	-7	-18	-25
2_2^+	4	16	18
4_1^+	-7	-30	-42
4_2^+	0	25	33
6_1^+	-49	-59	-65
6_2^+	38	51	53

^{70}Se

$I(\hbar)$	Prolate mixing	Oblate mixing
0^+	41(4)(1)(1) %	51(1) %
2^+	56(2) %	39(2) %
4^+	52(2) %	43(2) %
6^+	76(3)(1)(1) %	17(1) %

- *ground state > dominated by oblate components in ^{70}Se, but prolate ones in ^{70}Br*

^{70}Br

$I(\hbar)$	Prolate mixing	Oblate mixing
0^+	68(1) %	26(2)(1) %
2^+	66(2) %	29(1) %
4^+	68(2)(1) %	26(1) %
6^+	81(4)(2)(1)(1) %	10(1) %

- *similar structure for ^{70}Br and ^{70}Kr*

^{70}Kr

$I(\hbar)$	Prolate mixing	Oblate mixing
0^+	69(3) %	24(3) %
2^+	70(3) %	24(1) %
4^+	75(3) %	19(2) %
6^+	86(3)(2) %	7(2) %

$A = 74$: *wave functions reveal shape mixing*

- *significant oblate-prolate mixing for the yrast states*

^{74}Kr

$I(\hbar)$	Prolate content	Oblate content
0^+	82(1)(1) %	14(1)(1) %
2^+	92(1)(1) %	6 %
4^+	95(1)(1) %	3 %
6^+	97(1) %	1(1) %

Spectroscopic quadrupole moments

$I(\hbar)$	^{74}Kr	Exp	^{74}Rb	^{74}Sr
2_1^+	-54	-53(24)	-57	-50
2_2^+	49	24(21)	53	48
4_1^+	-74	-80(40)	-77	-70
4_2^+	68		72	67
6_1^+	-85	-130(50)	-86	-81
6_2^+	78		81	80

^{74}Rb

$I(\hbar)$	Prolate content	Oblate content
0^+	85(1) %	12(1) %
2^+	94(1) %	4 %
4^+	96(1) %	2 %
6^+	97(1) %	1 %

^{74}Sr

$I(\hbar)$	Prolate content	Oblate content
0^+	77(2) %	19(1) %
2^+	87(1) %	11 %
4^+	90(1) %	8 %
6^+	92(1) %	5(1) %

- *maximum oblate-prolate mixing in ^{74}Sr*

Weak interaction rates and shape coexistence for the Z=N+2 isotopes: ^{70}Kr and ^{74}Sr

Isospin-symmetry-breaking and shape-coexistence effects on superallowed Fermi β-decay

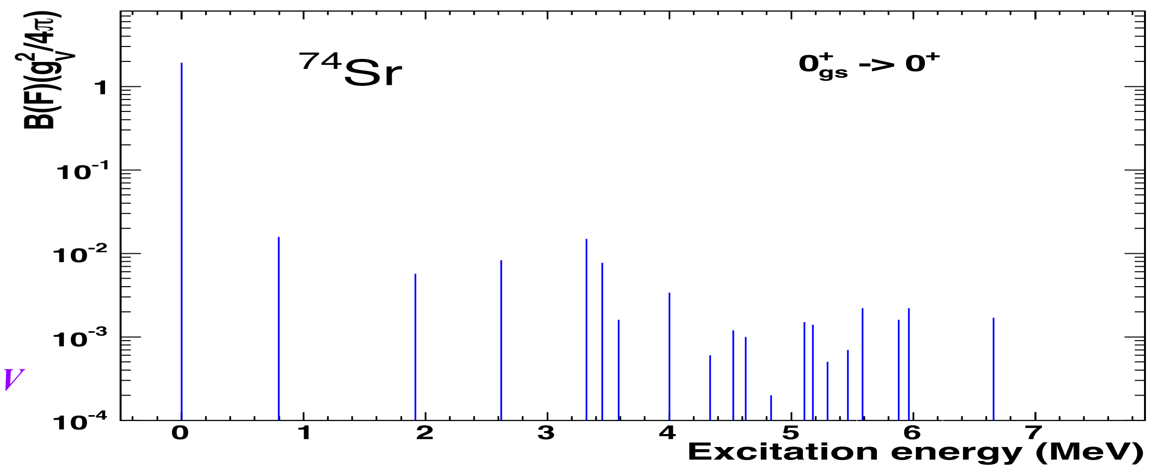
A. Petrovici, J. Phys.: Conf. Series 724, 012038 (2016)
Phys. Scr. 92, 064003 (2017)

$$ft(1+\delta_R)(1-\delta_c) = \frac{K}{2G_v^2(1+\Delta_R^v)}$$

δc – isospin-symmetry-breaking correction

test of the CVC hypothesis

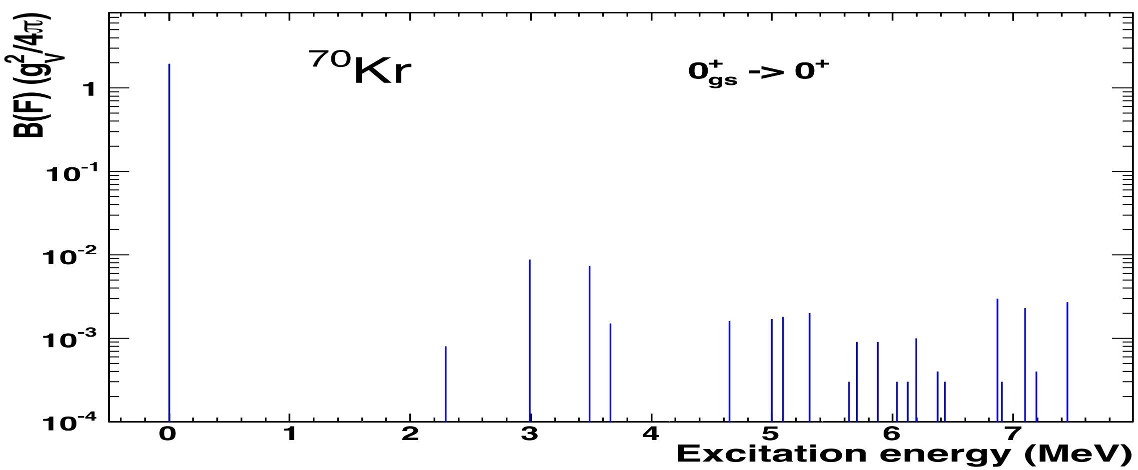
test of the unitarity of CKM matrix



Q_{EC} = 11.090 MeV

$1\% \leq \delta_c \leq 3\%$

Nonanalog branches:
0^+_{II} , 0^+_{VI} $\leq$ 0.8 %



Q_{EC} = 10.480 MeV

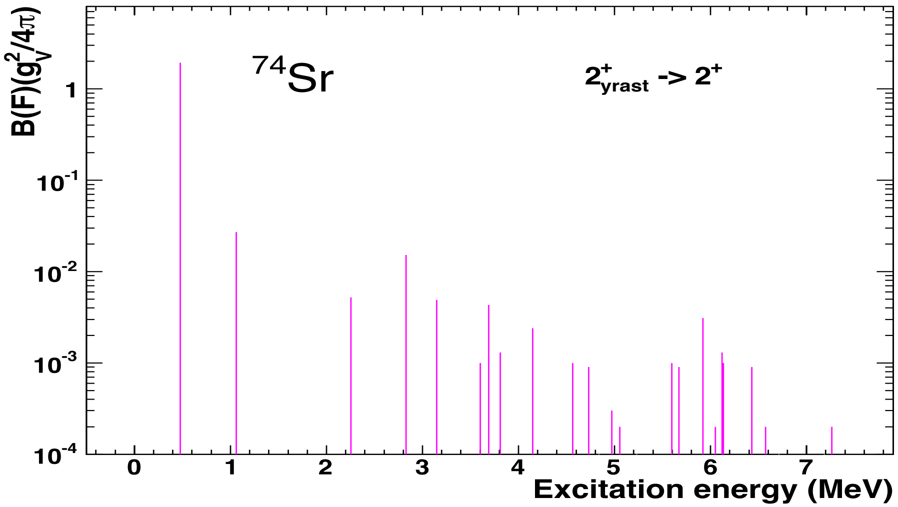
$1\% \leq \delta_c \leq 2\%$

Nonanalog branches:
0^+_{IV} , 0^+_{V} $\leq$ 0.4 %

Fermi β-decay of low-lying 2^+_{yrast}, 0^+_{exc}, 2^+_{sec} states with possible relevance for the rp-process path

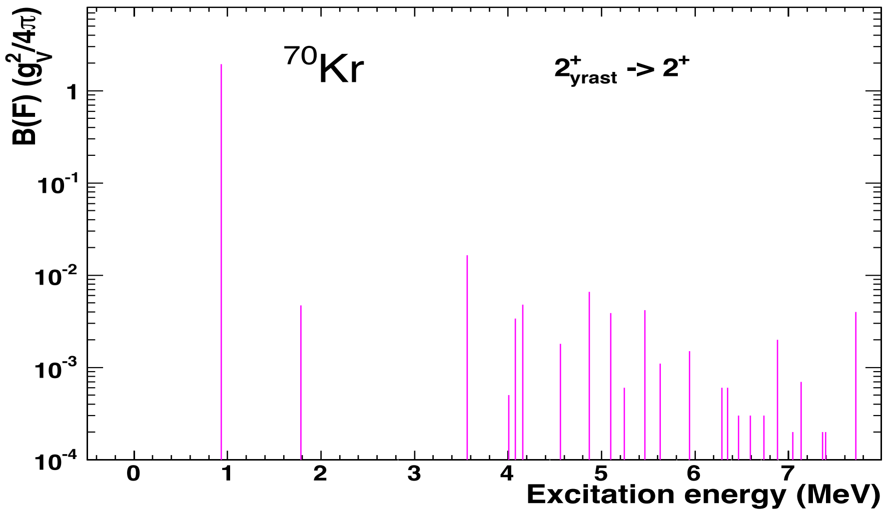
$1\% \leq \delta_c \leq 3.6\ \%$

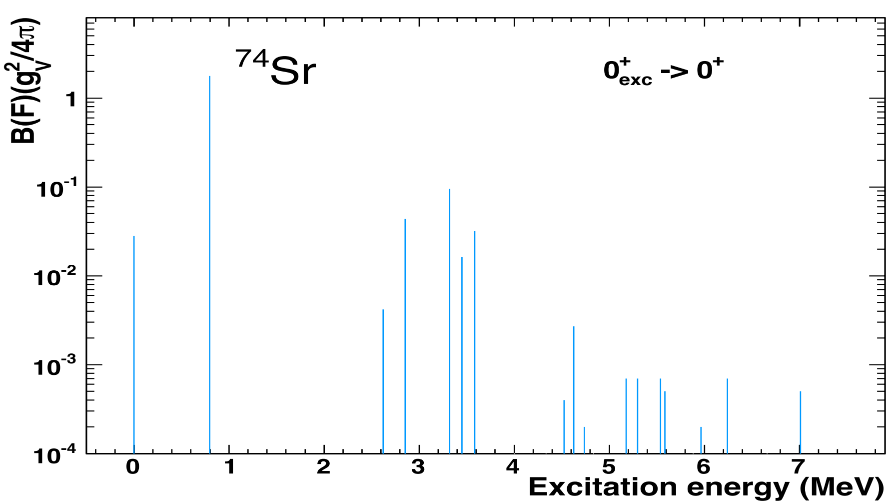
Nonanalog branches:
$2^+_{II} \leq 1.3\ \%$, $2^+_{IV} \leq 0.8\ \%$

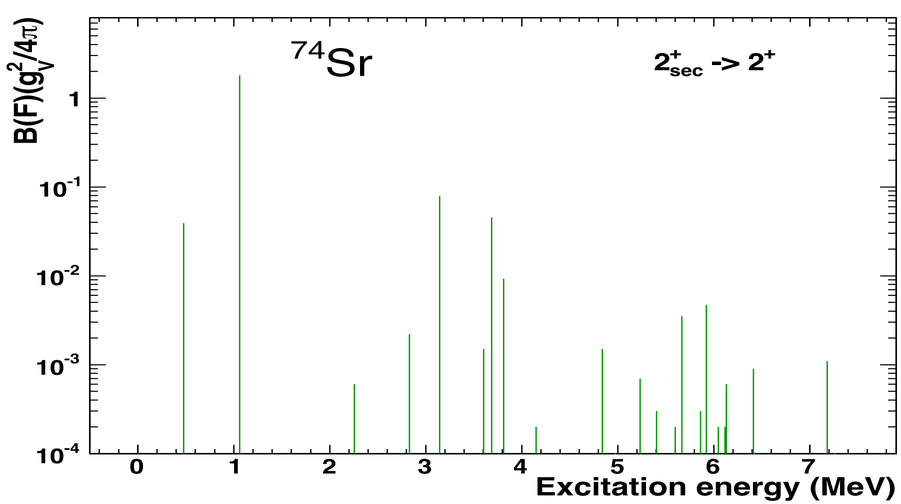


$1\% \leq \delta_c \leq 3\ \%$

Nonanalog branches:
$2^+_{IV} \leq 1.3\ \%$





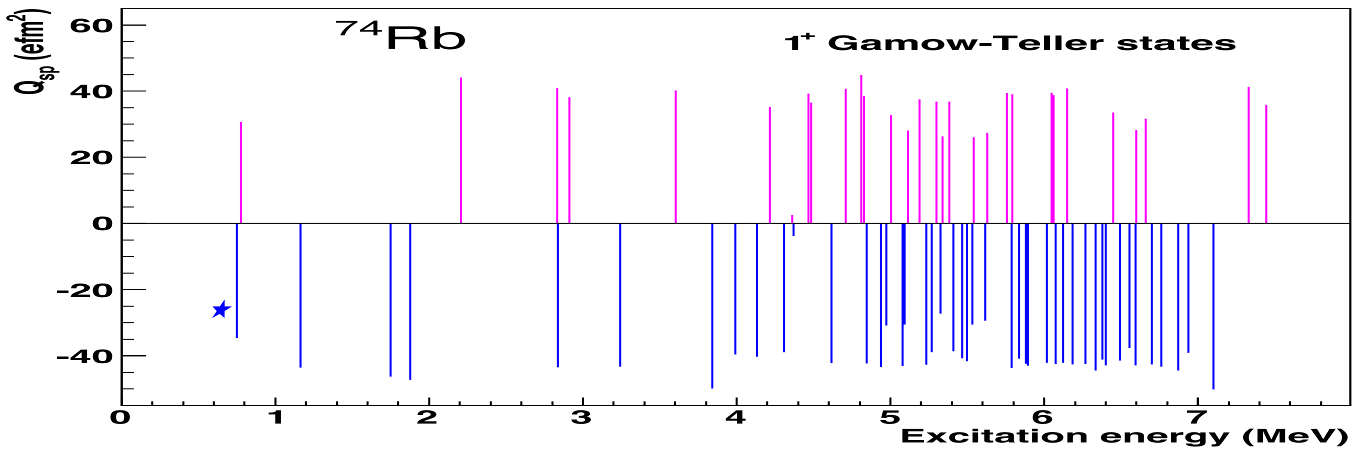


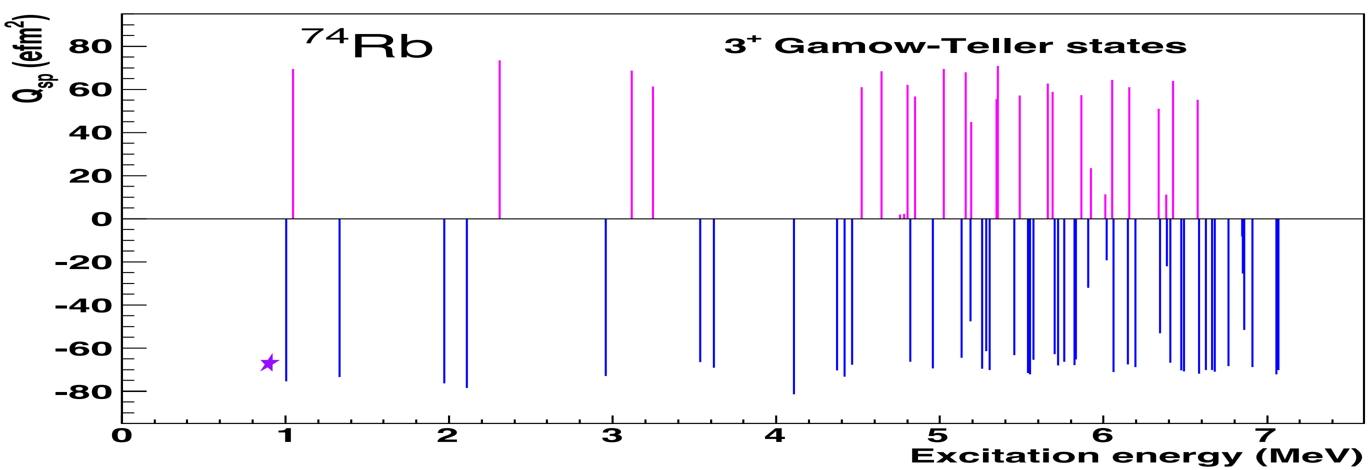
Gamow-Teller β-decay and shape coexistence for ^{74}Sr

A. Petrovici and O. Andrei, Phys. Rev. C92, 064305 (2015)
AIP Conf. Proc. 1852, 030004 (2017)

Independent chains of variational calculations in parent and daughter nuclei

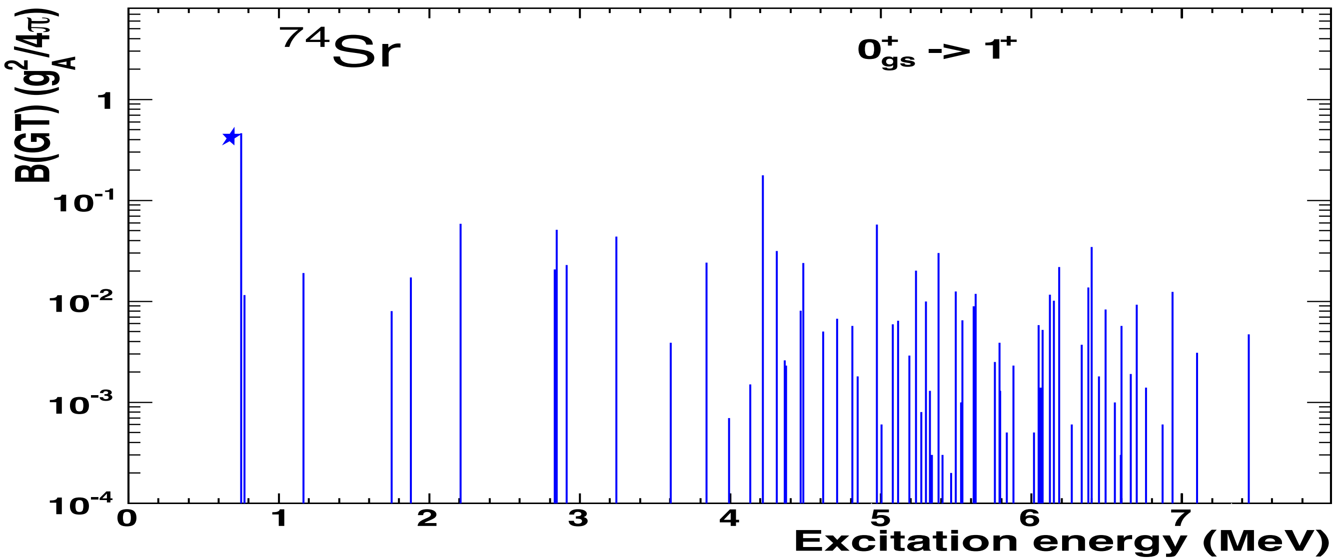
Large variety of deformations for daughter states revealed by spectroscopic quadrupole moments

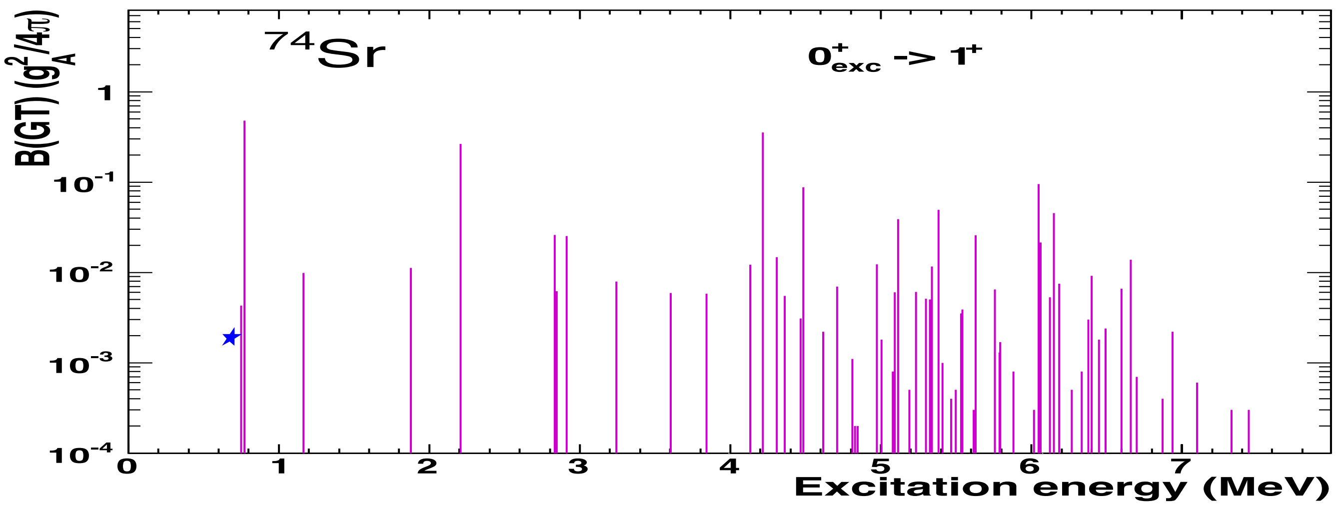




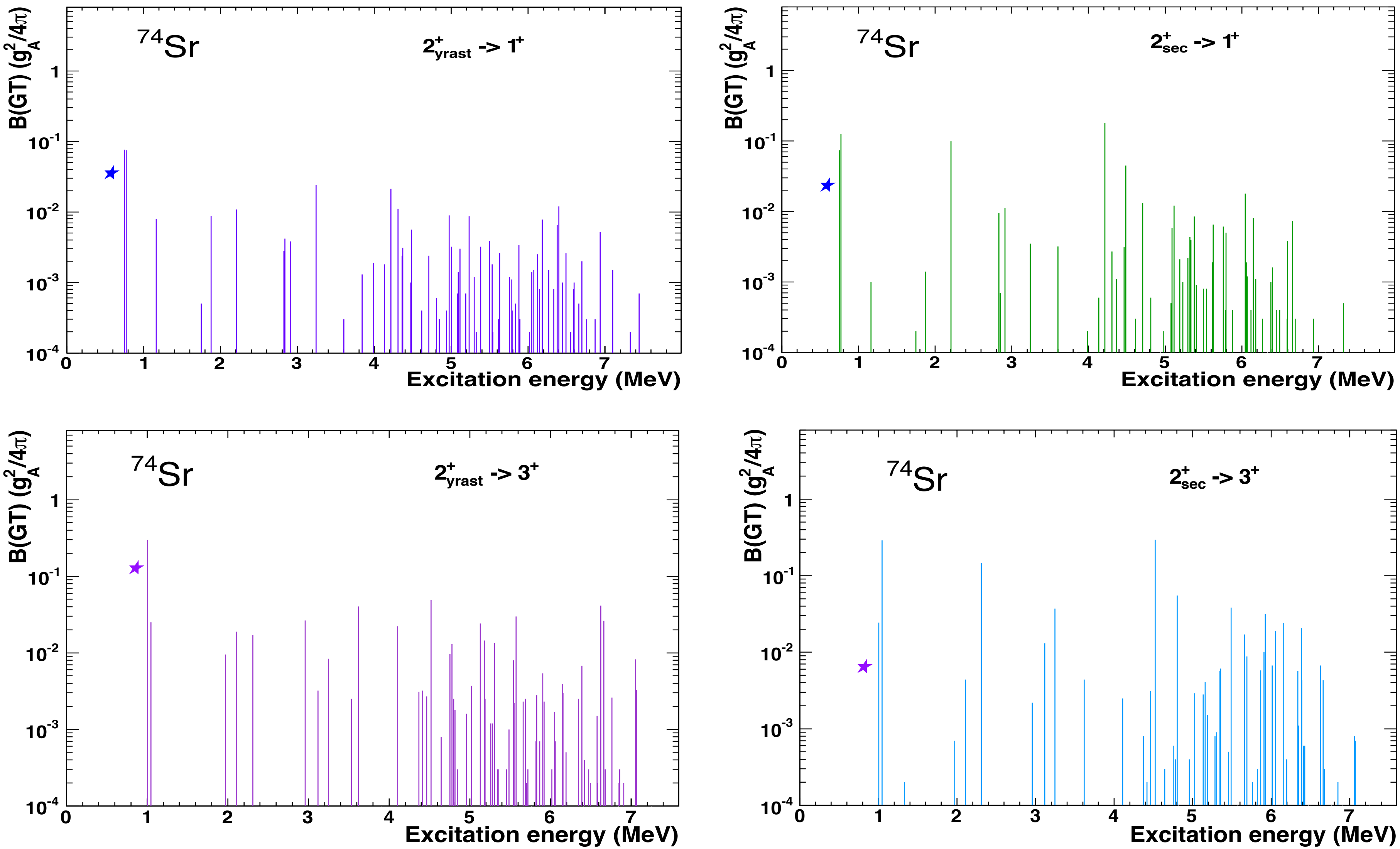
Gamow-Teller strength distributions for the decay of 0^+ and 2^+ states in ^{74}Sr

Strength distributions reveal ***specific shape mixing for the parent states***



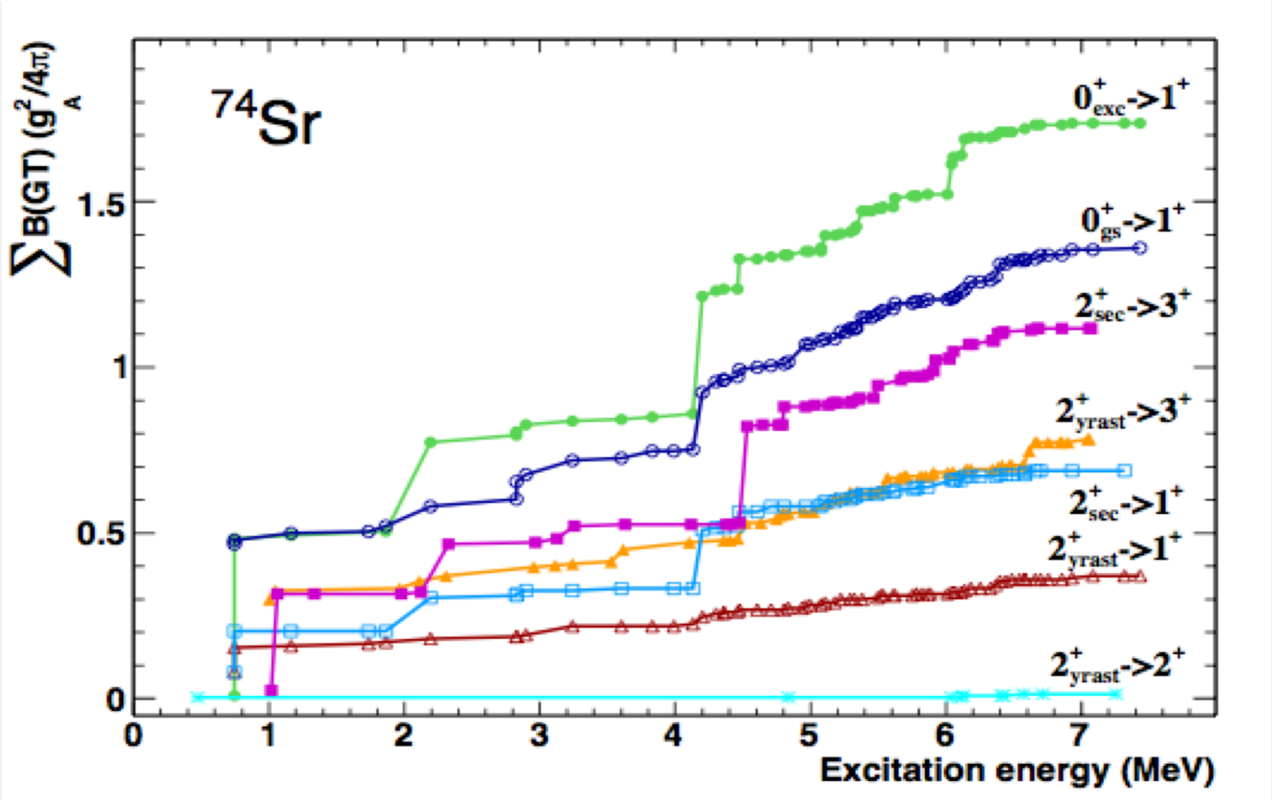


*Influence of **shape mixing in parent and daughter states** on strength distributions*



Contributions from $p^{\nu(\pi)}{}_{1/2}p^{\pi(\nu)}{}_{3/2}$, $p^{\nu}{}_{3/2}p^{\pi}{}_{3/2}$, $f^{\nu}{}_{5/2}f^{\pi}{}_{5/2}$, $f^{\nu(\pi)}{}_{5/2}f^{\pi(\nu)}{}_{7/2}$, $g^{\nu}{}_{9/2}g^{\pi}{}_{9/2}$ ***matrix elements***

(coherent / cancelling effect)



Terrestrial half-lives

$$\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0<E_f<Q_{EC}} f(Z,E_f)[B_{if}(GT) + B_{if}(F)]$$

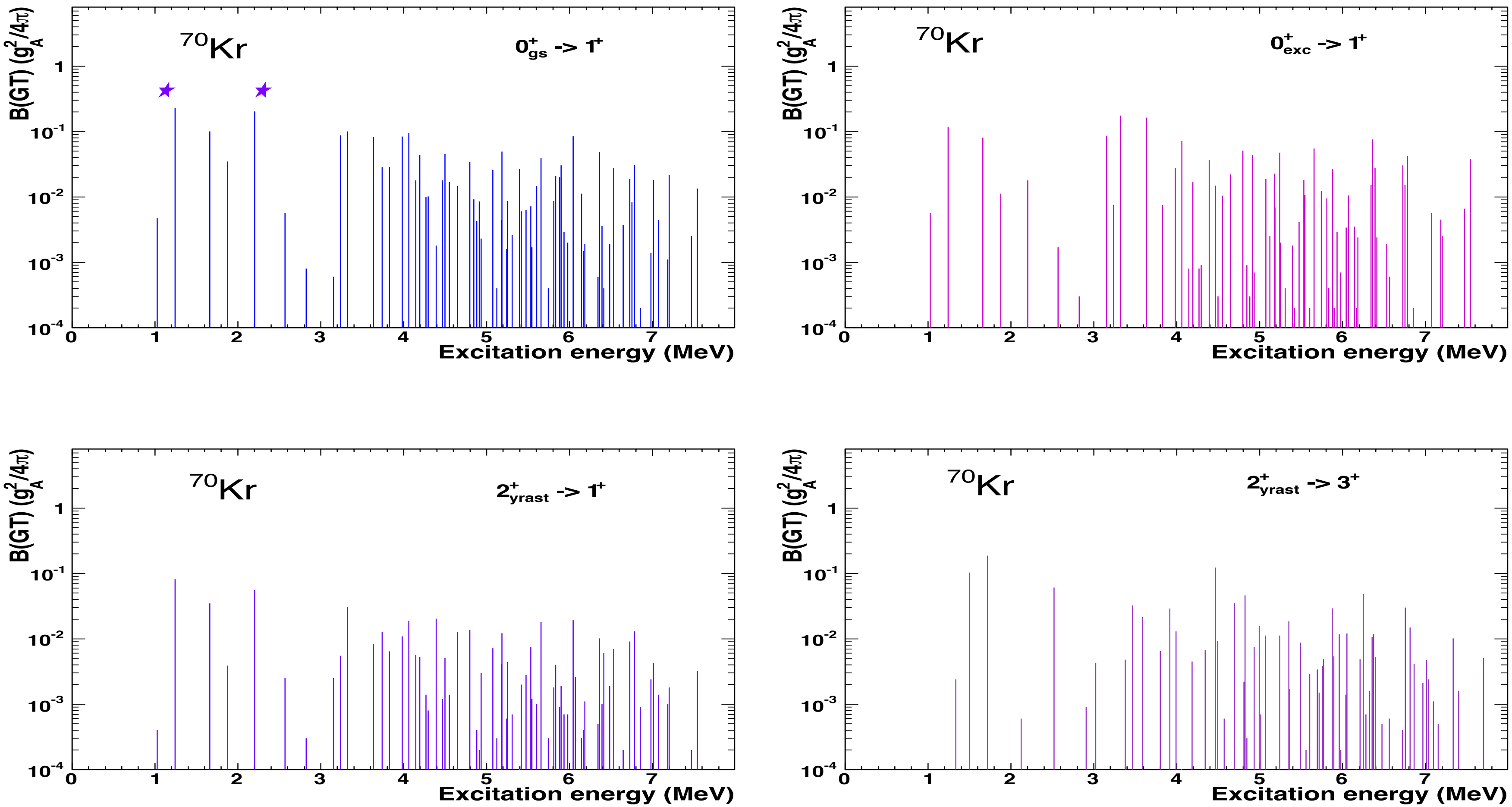
$T_{1/2}^{GT} = 137$ ms $T_{1/2}^{F} = 48$ ms

$T_{1/2}^{EXVAM} = 36$ ms   **$T_{1/2}^{exp} = 27(8)$ ms**

Gamow-Teller strength distributions for the decay of 0^+ and 2^+ states in ^{70}Kr

A. Petrovici and O. Andrei, Phys. Rev. C 92, 064305 (2015)
A. Petrovici , Phys. Scr. 92, 064003 (2017)

Specific shape mixing for each parent and daughter state influences the strength distributions

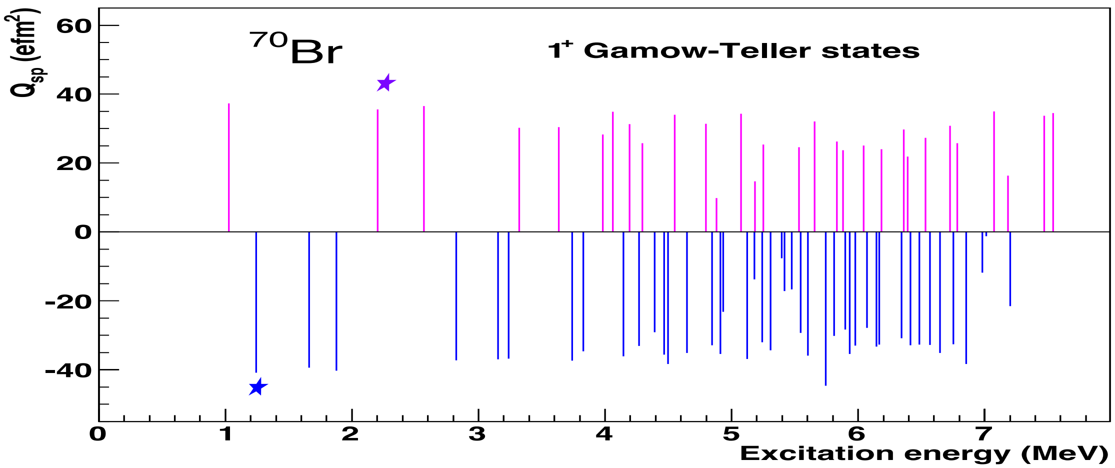


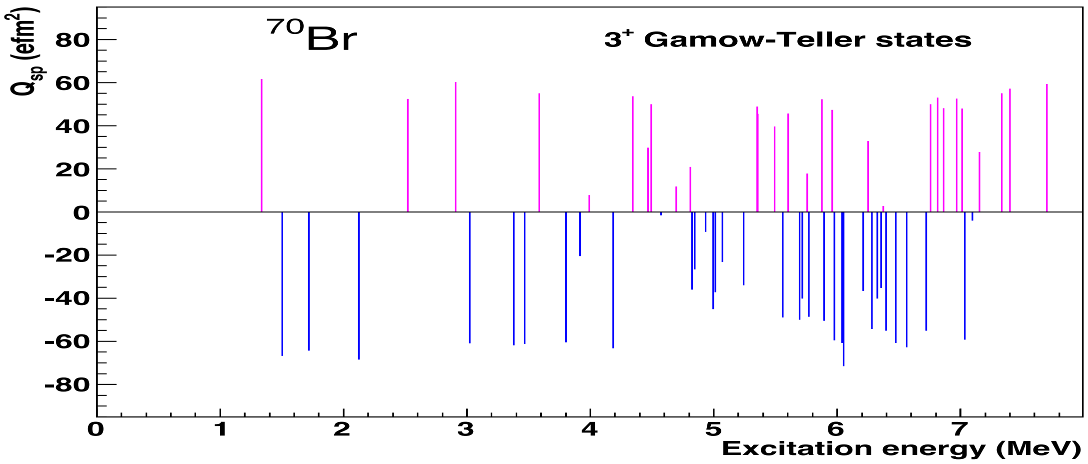
Contributions from $p^{\nu(\pi)}_{1/2}p^{\pi(\nu)}_{3/2}$, $p^{\nu}_{3/2}p^{\pi}_{3/2}$, $f^{\nu}_{5/2}f^{\pi}_{5/2}$, $f^{\nu(\pi)}_{5/2}f^{\pi(\nu)}_{7/2}$, $g^{\nu}_{9/2}g^{\pi}_{9/2}$ ***matrix elements***

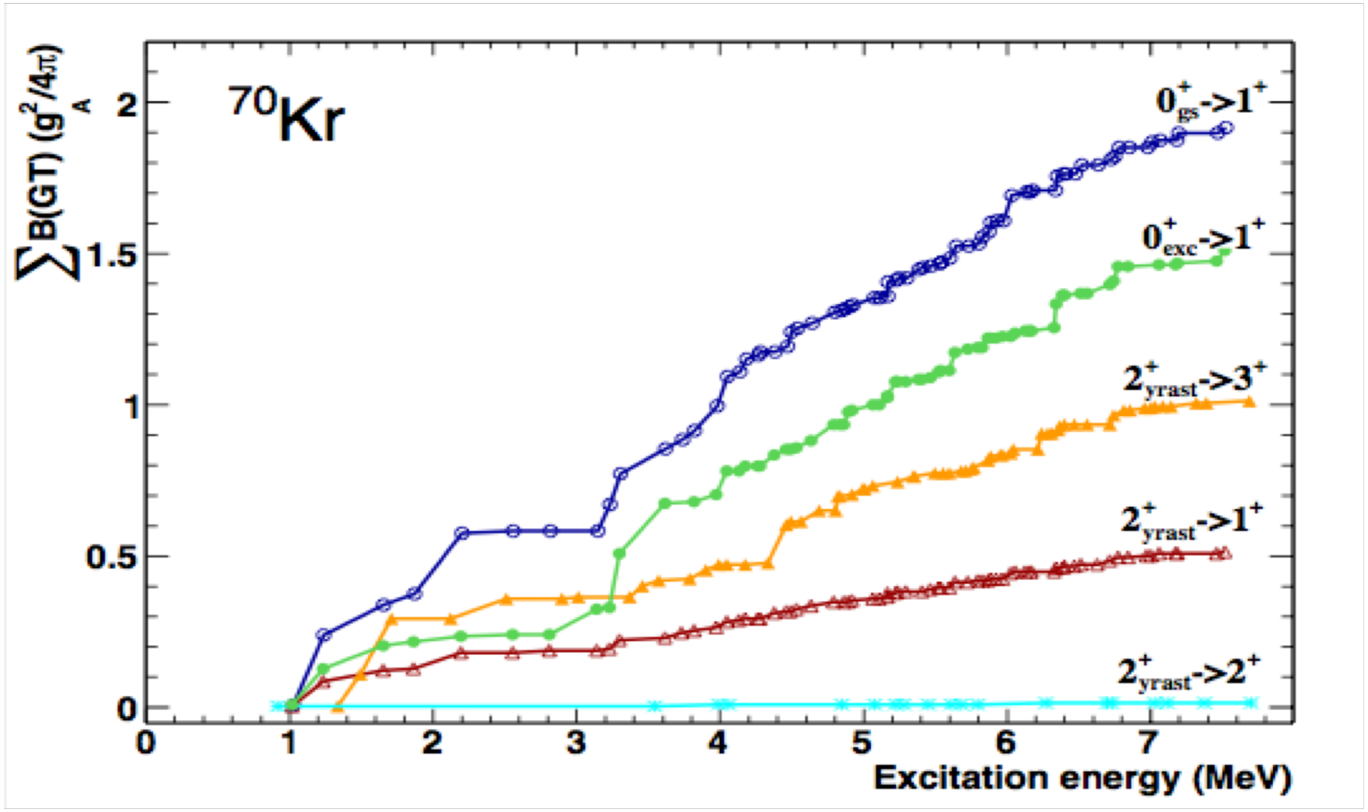
(coherent / cancelling effect)

Shape coexistence displayed by the daughter states

Large variety of deformations in daughter states revealed by spectroscopic quadrupole moments







Terrestrial half-lives

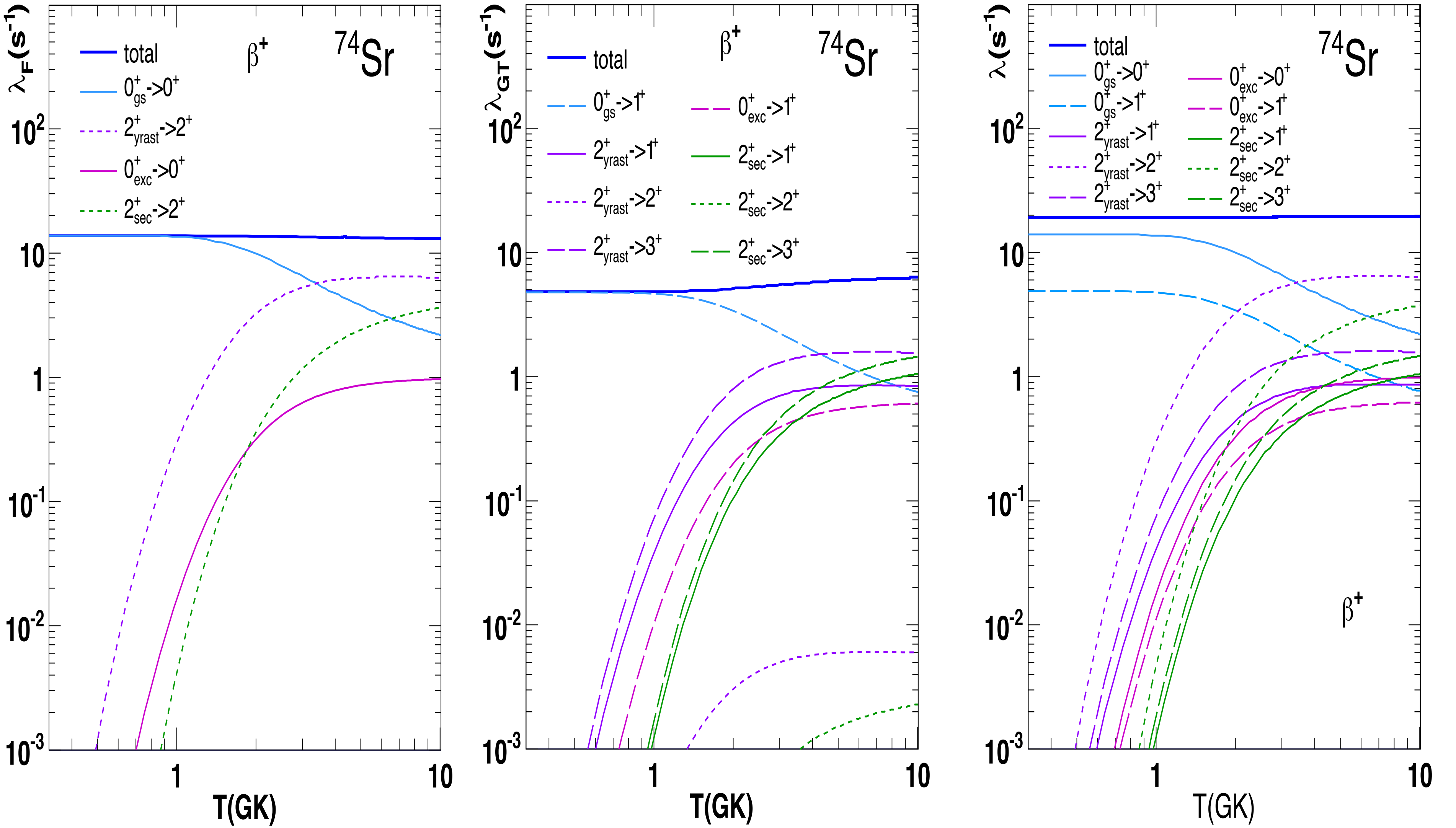
$$\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0<E_f<Q_{EC}} f(Z, E_f)[B_{if}(GT) + B_{if}(F)]$$

$T_{1/2}^{GT} = 258$ ms $\qquad T_{1/2}^{F} = 63$ ms

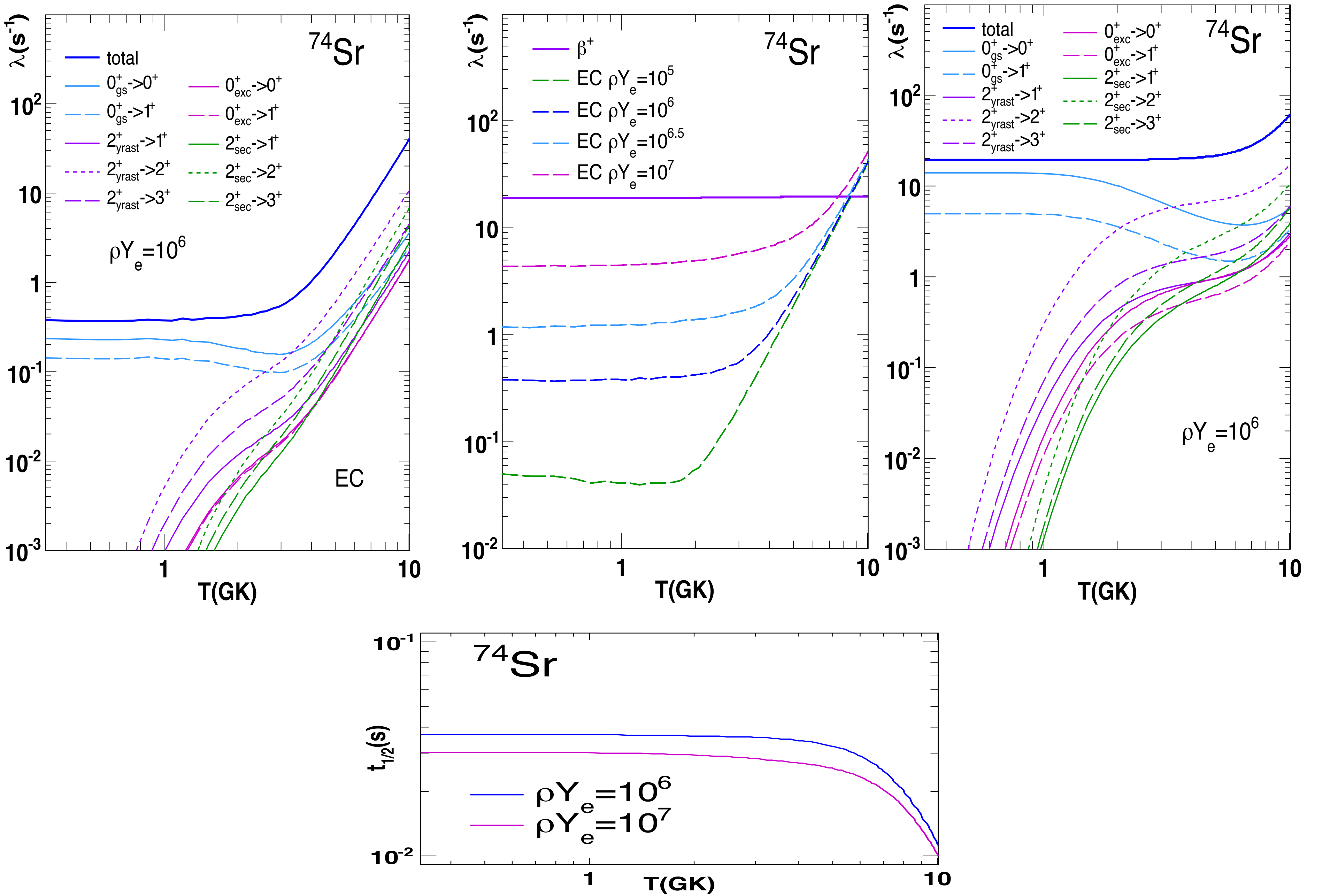
$\mathbf{T_{1/2}^{exp} = 52(17)}$ **ms** $\qquad \mathbf{T_{1/2}^{EXVAM} = 51}$ **ms**

Stellar rates for ^{74}Sr : β^+ - decay

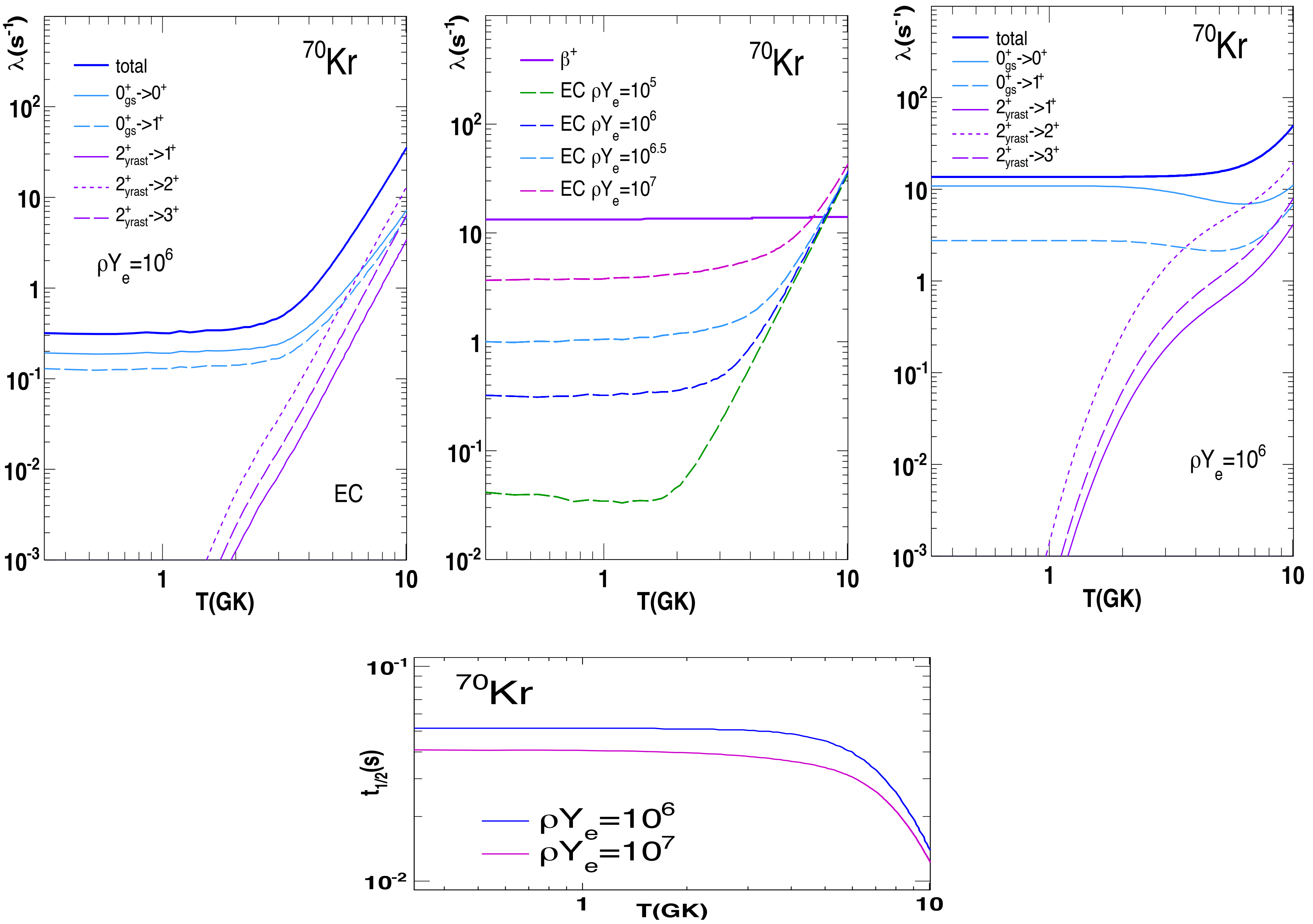
$E^{th}_{0^+_{exc}} = 0.564$ MeV $E_{2^+_{yrast}} = 0.471$ MeV $E^{th}_{2^+_{sec}} = 0.823$ MeV



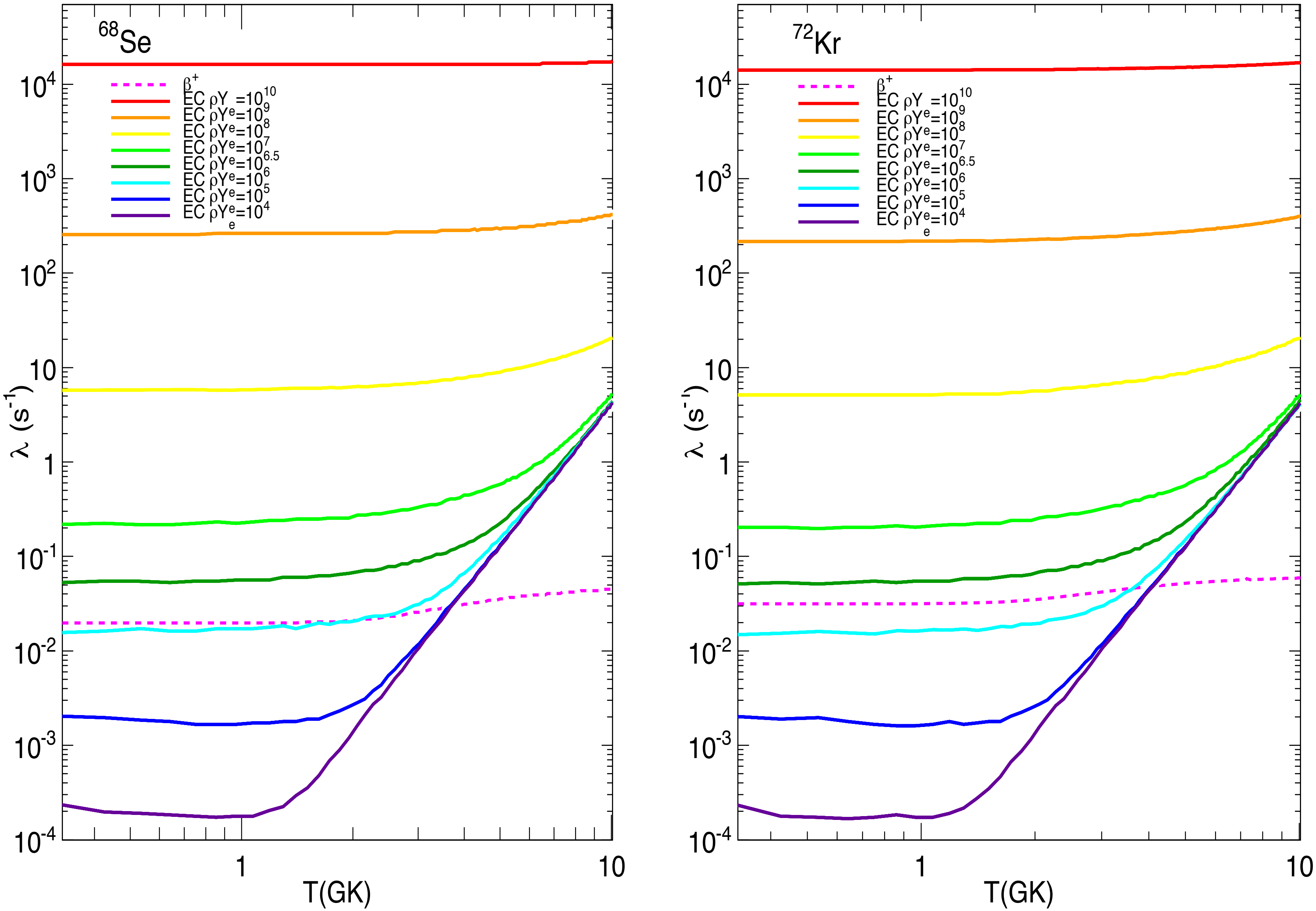
β+ and electron capture rates for ^{74}Sr

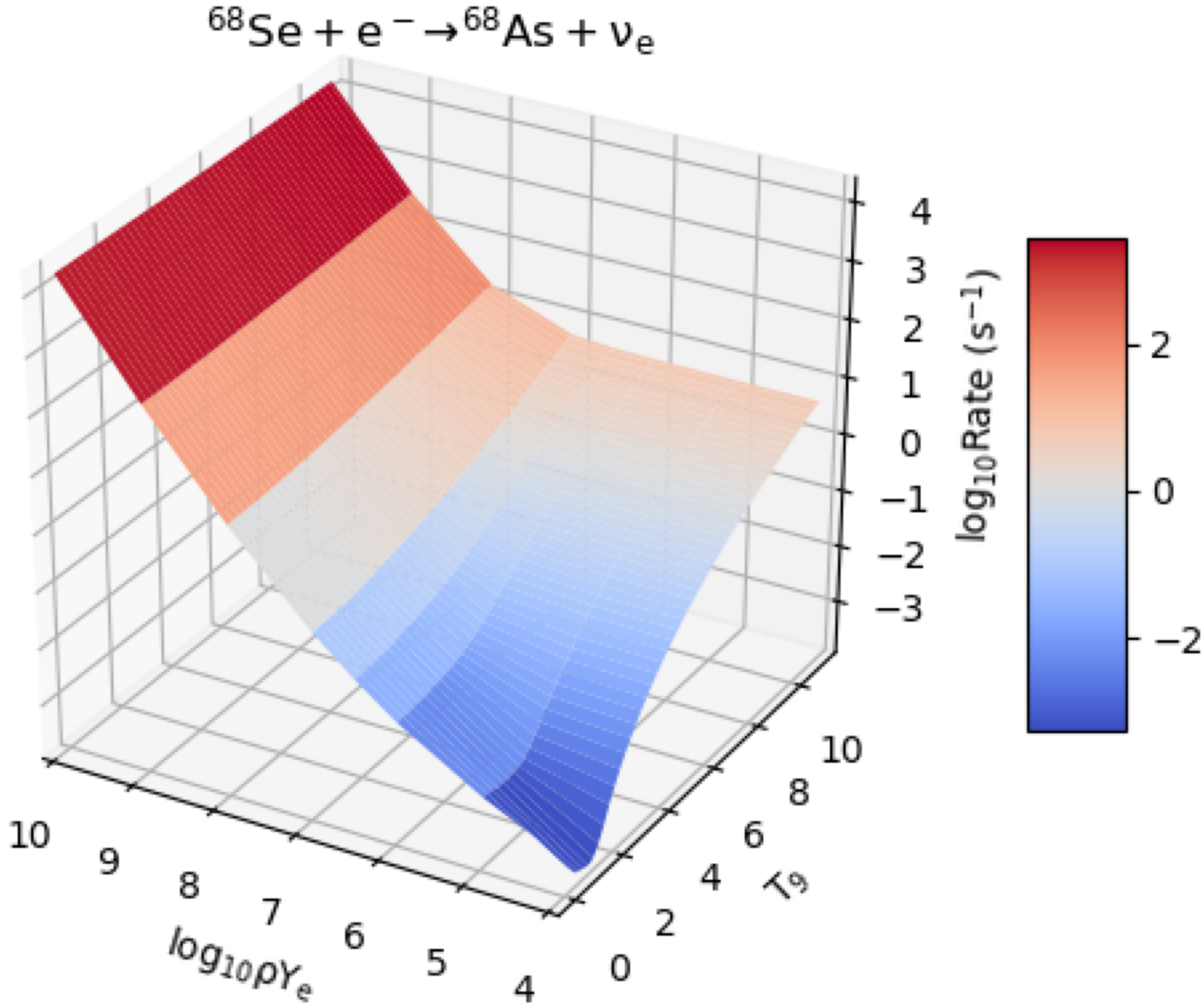


β^+ and electron capture stellar rates for ^{70}Kr



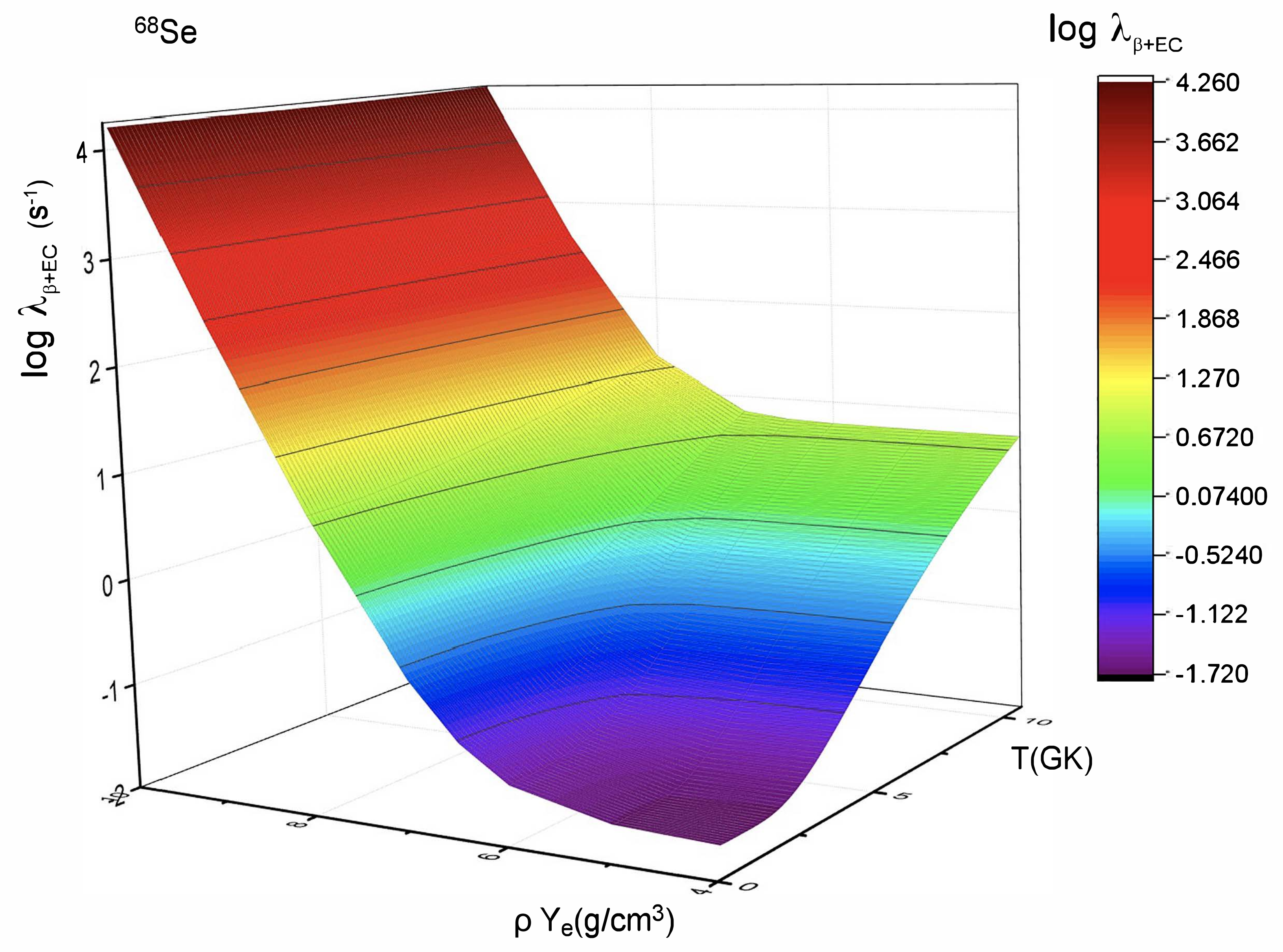
Possible effects of EXVAM stellar rates on the rp process in type I x-ray bursts





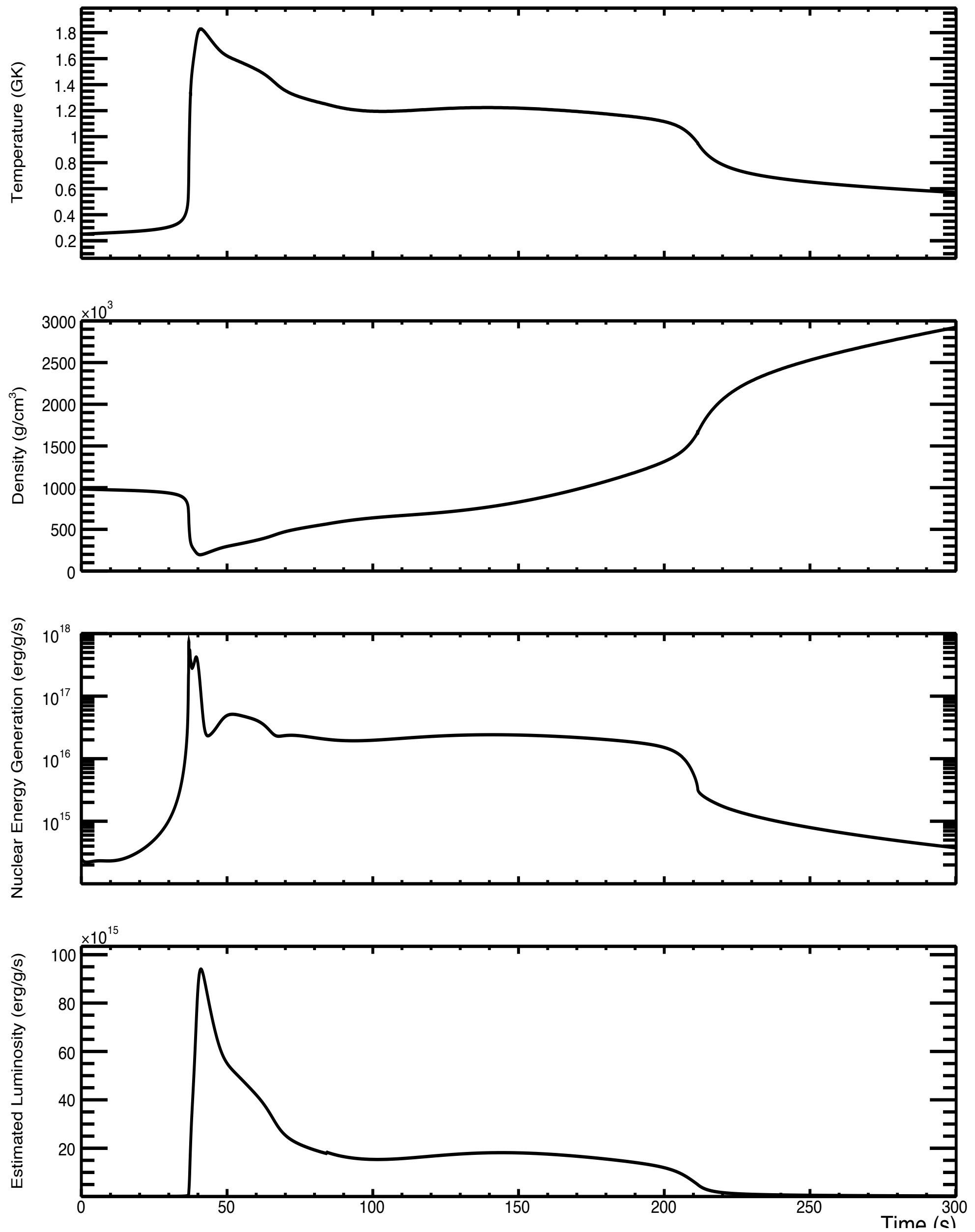
$^{68}Se + e^{-} \rightarrow {}^{68}As + \nu_e$
4
3
2
1
0
−1
−2
−3
$\log_{10}$Rate (s^{-1})
2
0
−2
10
9
8
7
6
5
4
$\log_{10}\rho Y_e$
0
2
4
6
8
10
T_9

λ^{EXVAM} $(^{68}Se_{\beta+EC})$



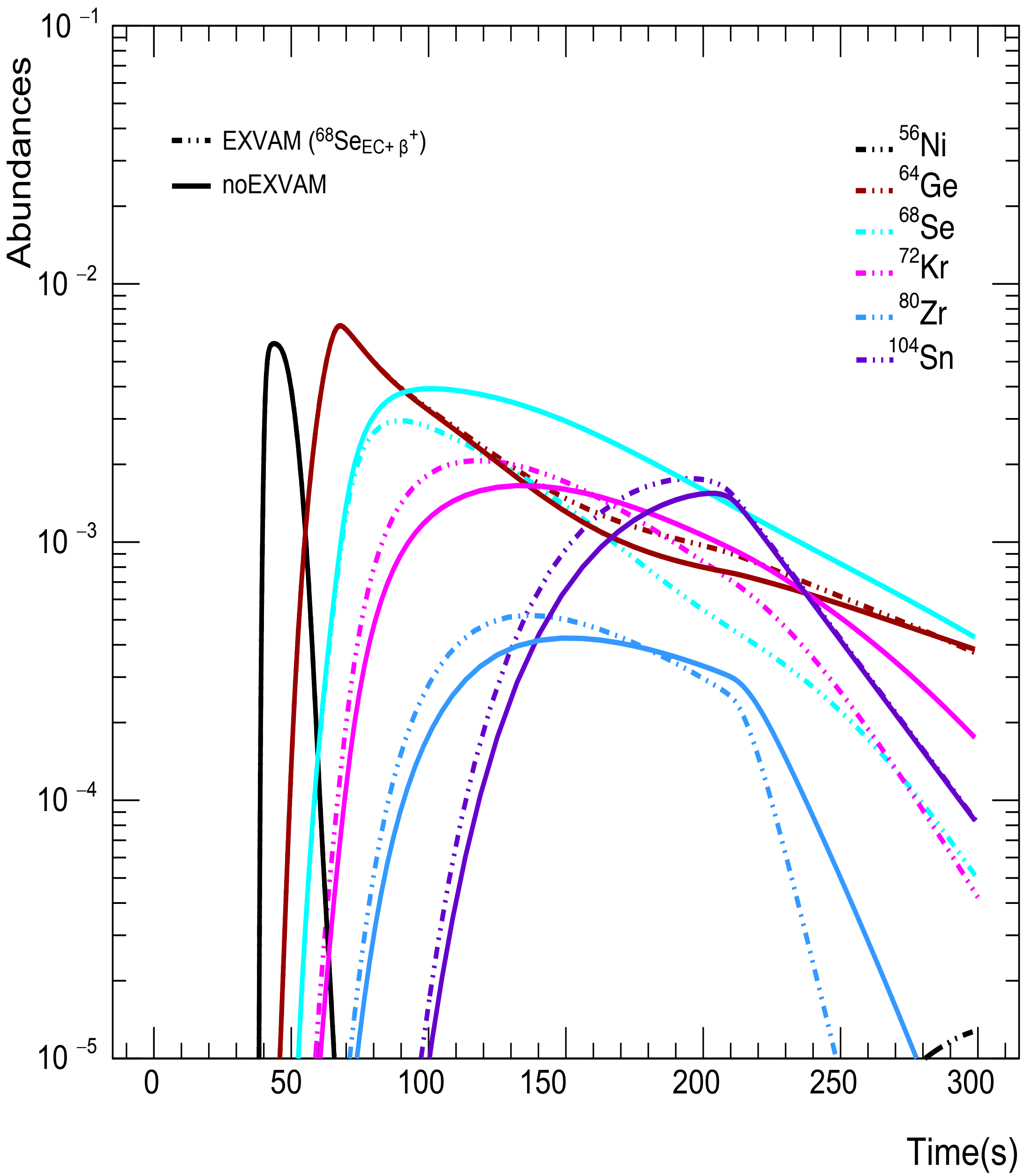
rp process on accreting neutron stars

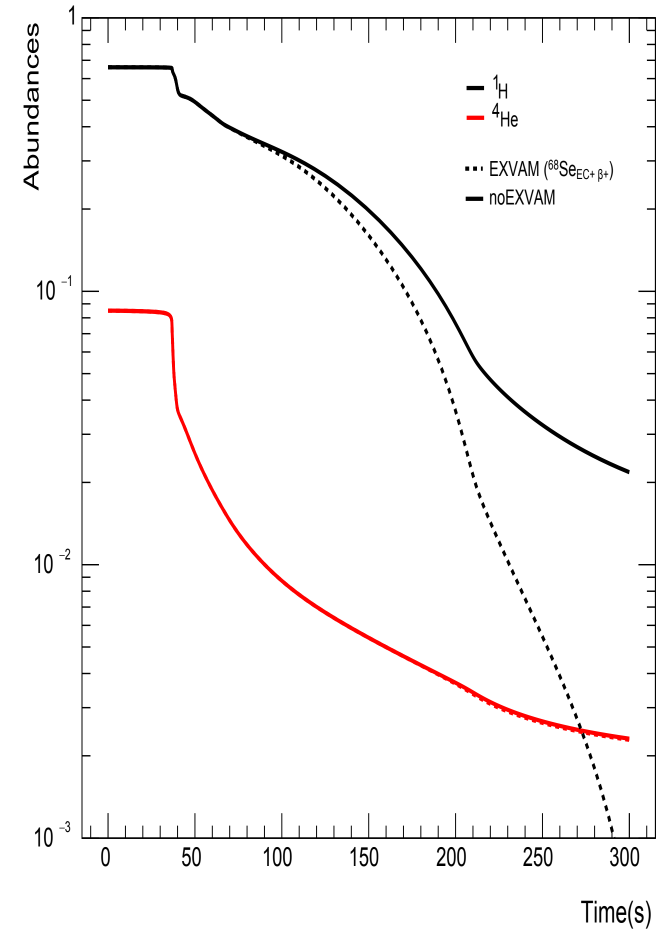
H. Schatz et al, Phys. Rev. Lett. 86, 3471 (2001)



Abundances of waiting point nuclei, H, and He

λ^{EXVAM} (T=1.2 GK; $\rho Y_e=10^6$ g/cm^3)





Summary

complex **EXCITED VAMPIR beyond-mean-field model self-consistently describes**

shape-coexistence effects on

- ***isospin-related phenomena in the A=70 and A=74 isovector triplets:***

  superallowed Fermi β-decay

- ***terrestrial and stellar weak interaction rates for A~70 proton-rich nuclei:***
  - ***Z=N+2: ^{70}Kr and ^{74}Sr***
  - ***rp-process waiting points: ^{68}Se and ^{72}Kr***

- ***possible effects of EXVAM stellar weak interaction rates on the rp process in type I x-ray bursts***

In collaboration with:

O. Andrei
IFIN-HH
A. Mare
IFIN-HH
Bucharest University
B. S. Mayer
Clemson University, South Carolina
H. Schatz
Michigan State University, East Lancing