

Thank you

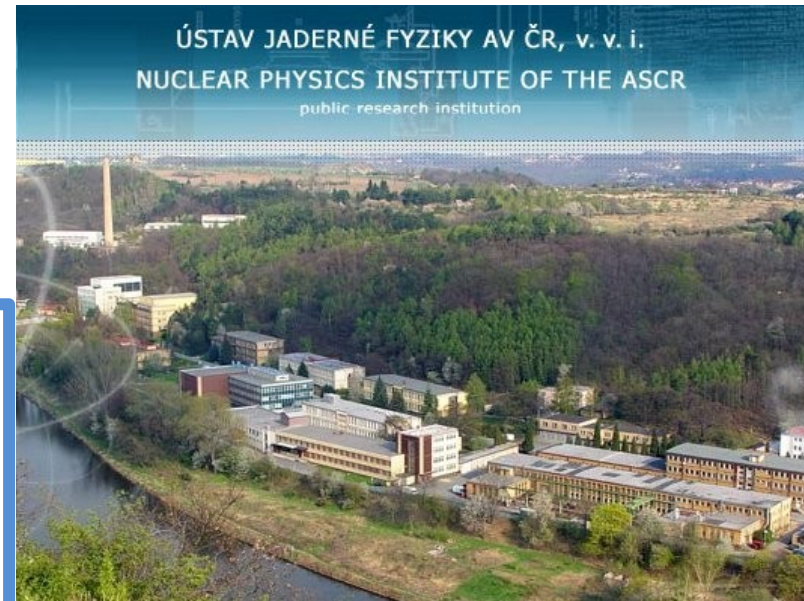
- for the invitation
- opportunity to join and listen

question from the point of view NPI:

- what is the future directions of ANC ?



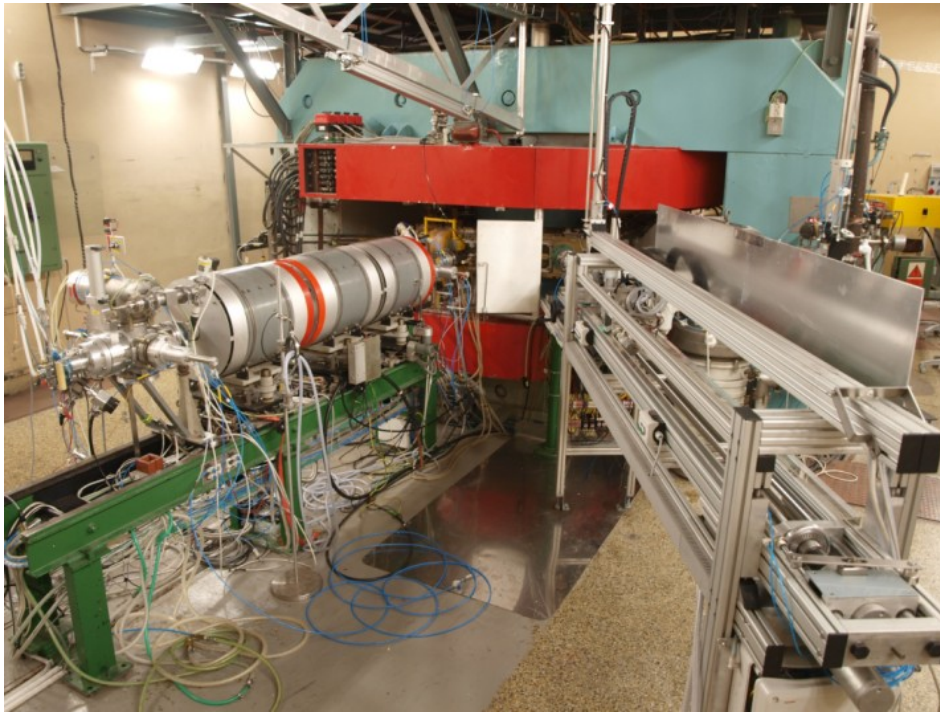
- what is the presence of ANC ?



EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

MEŠMT
MINISTRY OF EDUCATION,
YOUTH AND SPORTS

This work was supported by OP RDE, MEYS, Czech Republic under the project SPIRAL2-CZ, CZ.02.1.01/0.0/0.0/16_003/0001679



- **Compact cyclotron TR24**
- Beams:
 - **p** 18-24 MeV **300uA**

- **Isochronous cyclotron**

U120

- Beams:

- **p** 10-25 MeV 5uA
- **d** 10-20 MeV 5uA
- **³He** 17-53 MeV 2uA
- **alpha** 20-40 MeV 5uA

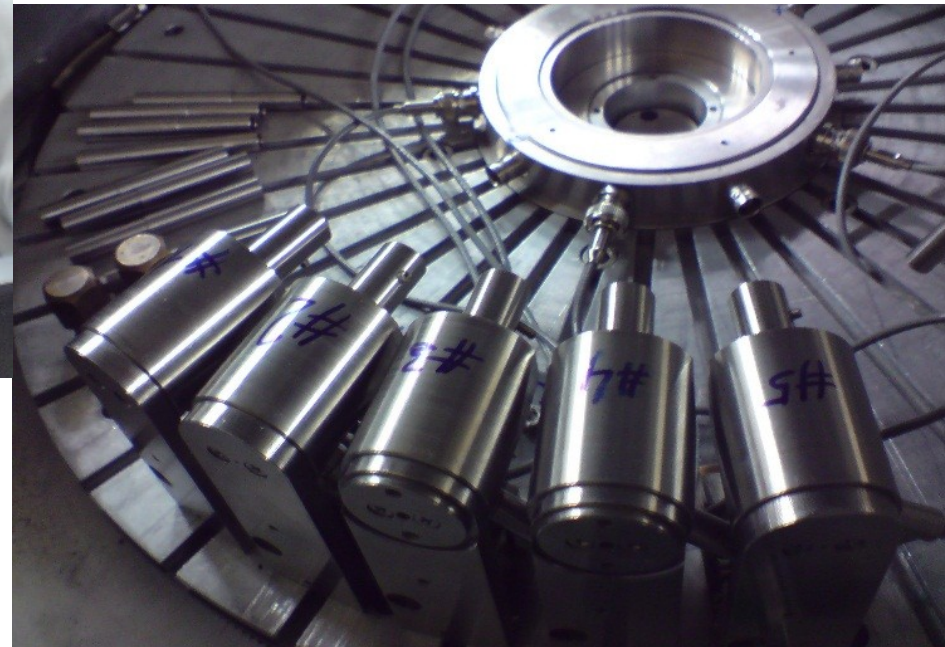
beam



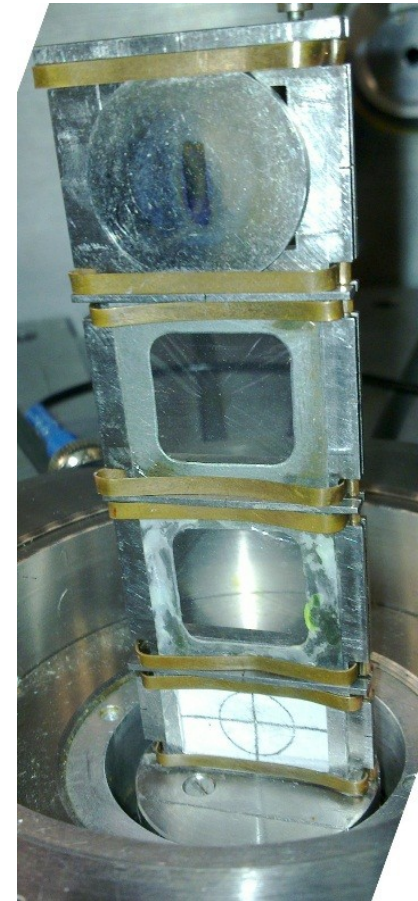
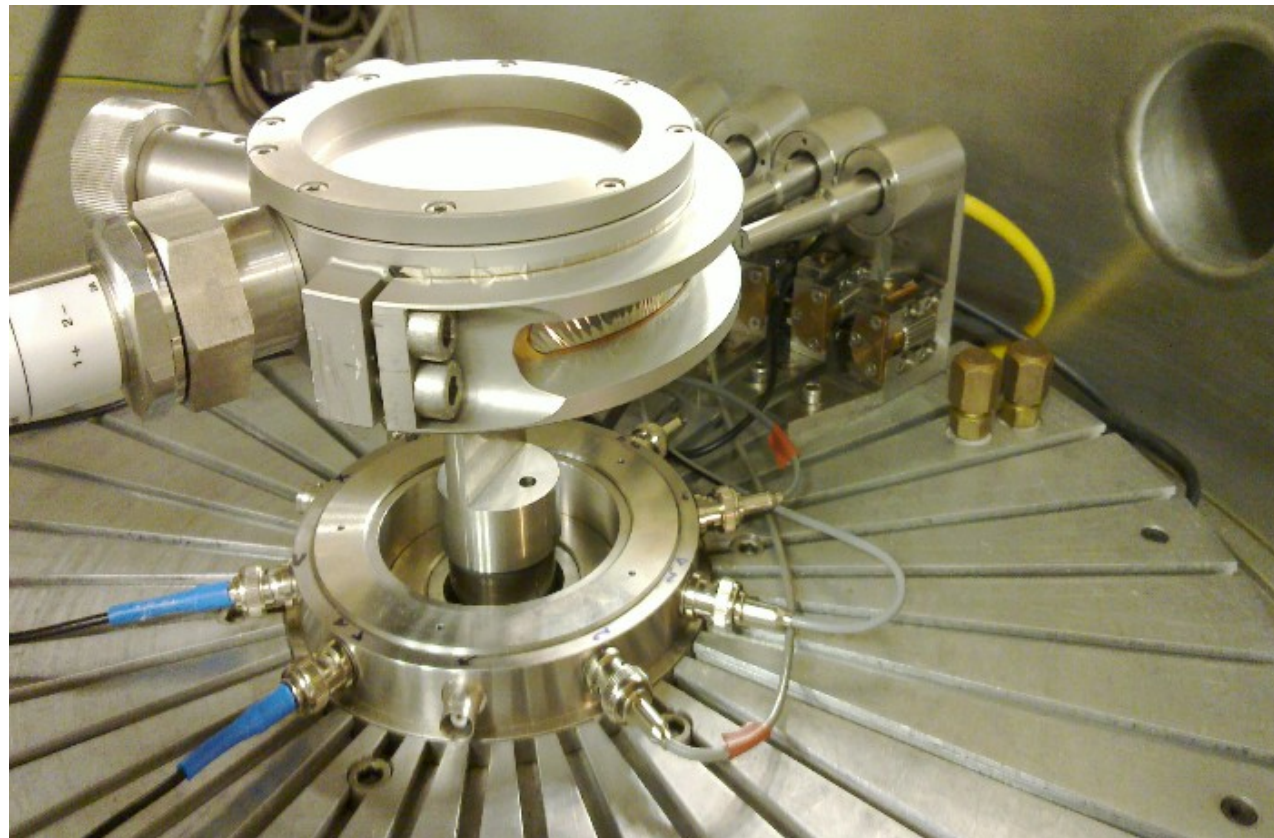
BEAM in experimental hall:

- ~ 10-20 nA of **^3He**
for (p,gamma)
- ~ 10-20nA **deuteron** beam
for (n,gamma)
and mirror studies

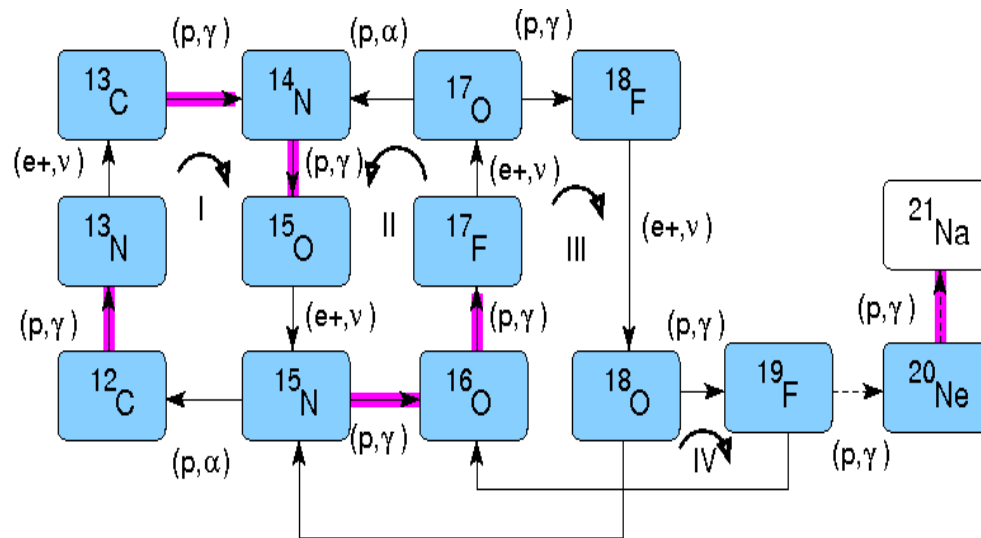
Detectors setup



Gas and solid target setup



- ${}^9\text{Be} + \text{p} \leftrightarrow {}^{10}\text{B}$
- ${}^{13}\text{C} + \text{p} \leftrightarrow {}^{14}\text{N}$
- ${}^{14}\text{N} + \text{p} \leftrightarrow {}^{15}\text{O}$
- ${}^{16}\text{O} + \text{p} \leftrightarrow {}^{17}\text{F}$
- ${}^{15}\text{N} + \text{p} \leftrightarrow {}^{16}\text{O}$



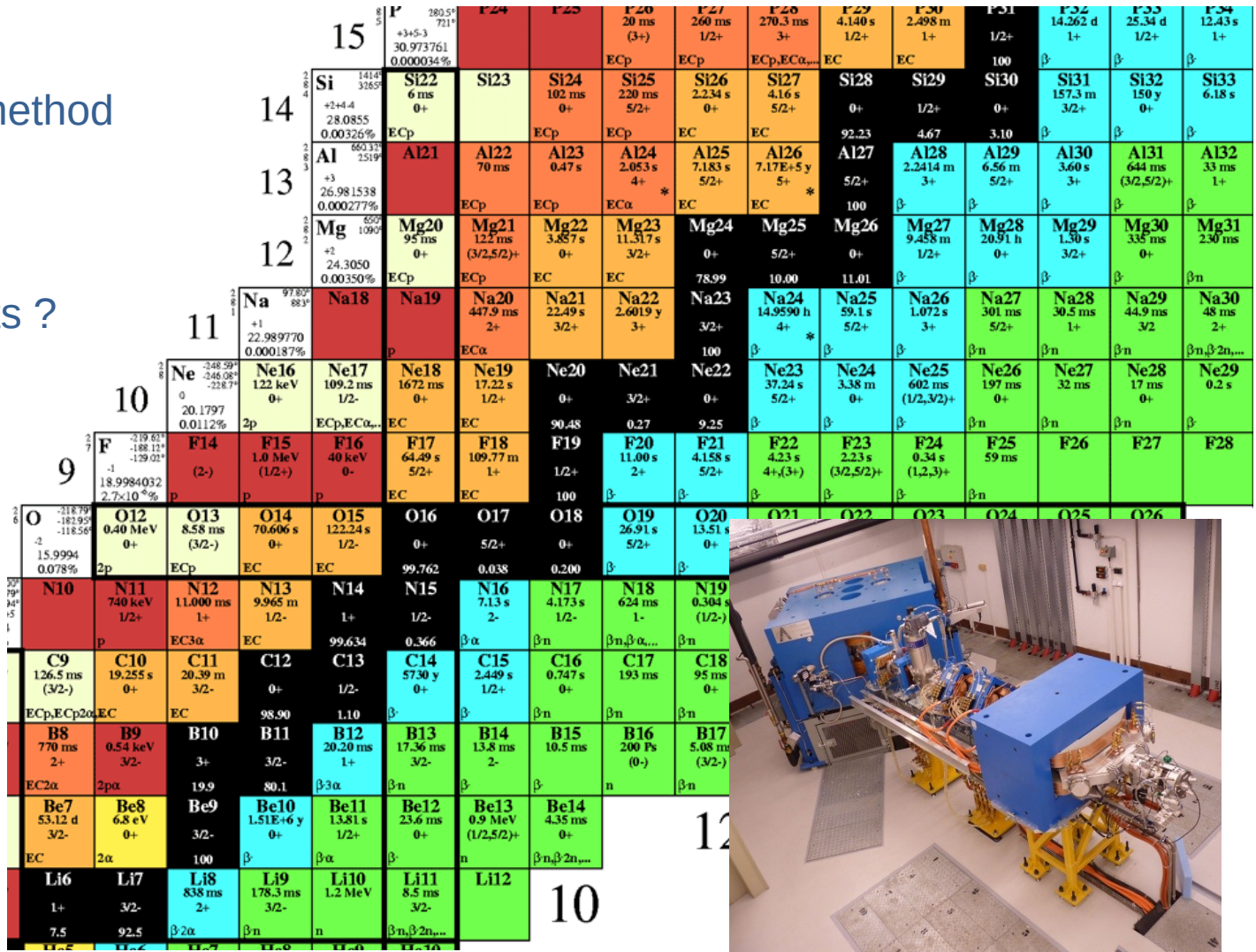
- ${}^{14}\text{C} + \text{n} \leftrightarrow {}^{15}\text{C}$
- ${}^{13}\text{C} + \text{n} \leftrightarrow {}^{14}\text{C}$

${}^{20}\text{Ne} + \text{p} \leftrightarrow {}^{21}\text{Na}$
 V.Kroha, S.Piskor, Z.Hons,
 retired

V.Burjan, JM, + T A&MU (A.Mukhamedzhanov)...

appearing postdocs, students
 connection to GANIL/SPIRAL2 with SPIRAL2-CZ project

- Limits due to the method
- stable targets
- nuclear scheme complexity
- (dp) reaction limits ?



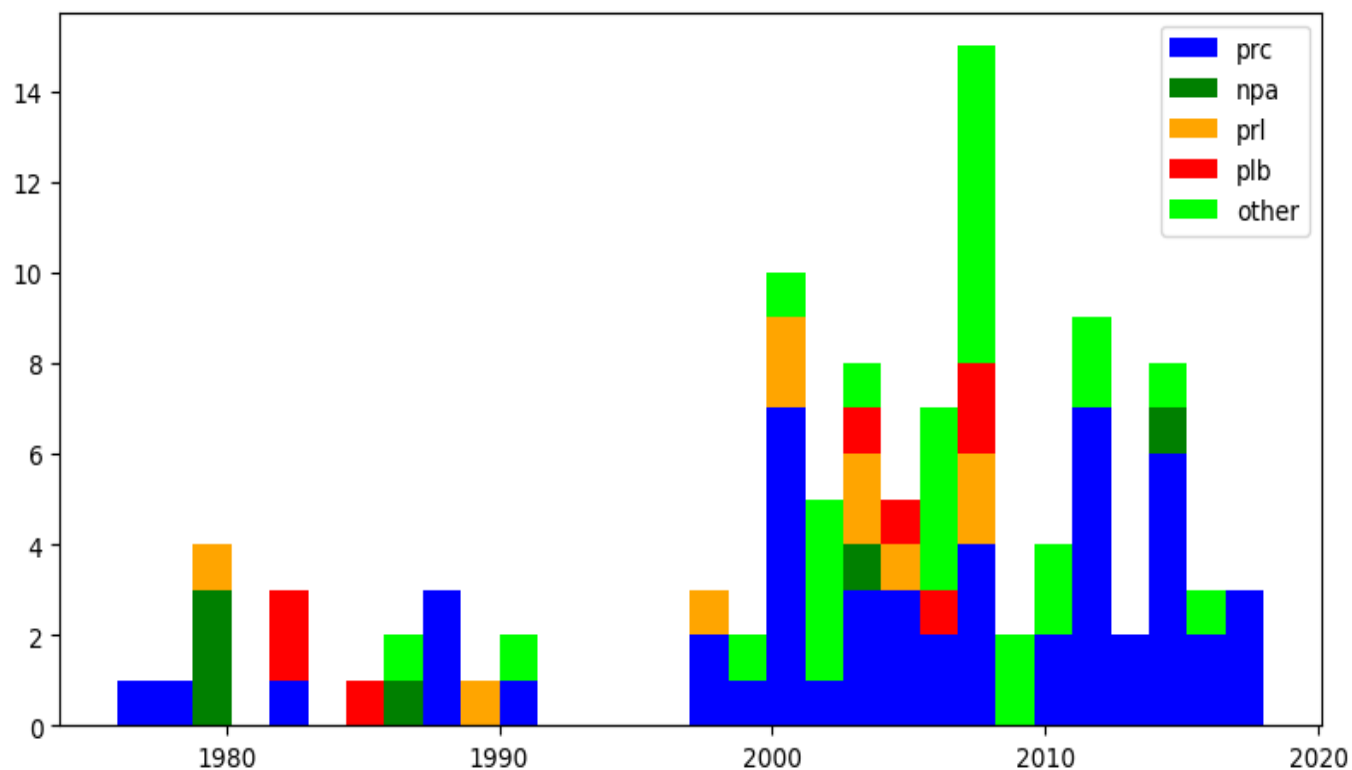
radioactive beams/targets

ANC situation

PLB 6
NPA 10
PRC 52
PRL 8

total 104

Articles with "Asymptotic Normalization" in a title

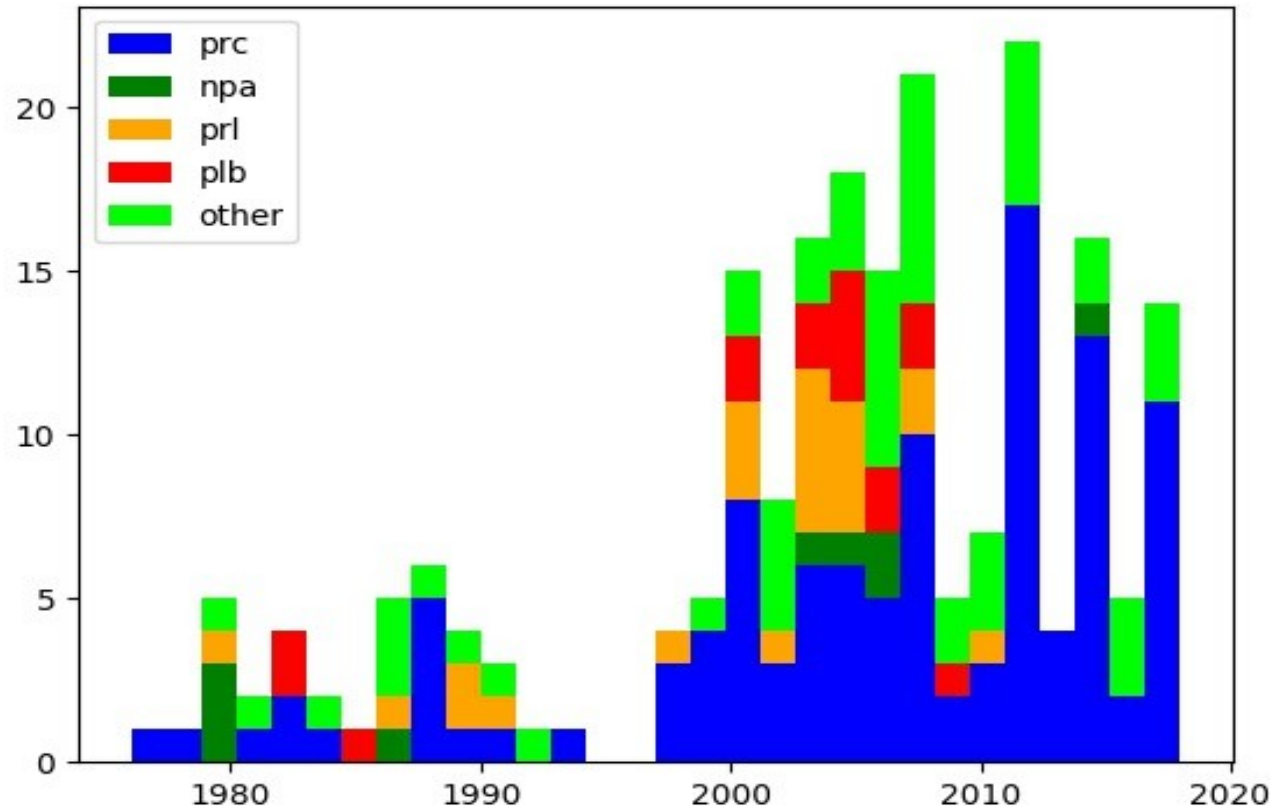


ANC situation

PLB 16
NPA 22
PRC 111
PRL 9
other 53

total 234

+ Articles with "Asymptotic Normalization" keyword



not complete...

1976 D.R.Lehman, B.F.Gibson
Integral relation for the A.N. of the Triton
Phys.Rev.C13, 35 (1976)

3 body problem

1994 H.M.Xu, C.A.Gagliardi, R.E.Tribble, A.M.Mukhamedzhanov, N.K.Timofeyuk
Overall Normalization of the Astrophysical S Factor an the Nuclear Vertex Constant
for $7\text{Be}(p,\gamma)8\text{B}$ Reactions
Phys.Rev.Lett 73 (1994) 2027

theory

$S_{17}(0) : 22.5\text{eVb} \rightarrow 17.6\text{eVb}$

1990 A.M.Mukhamedzhanov, N.K.Timofeyuk
Astrophysical S-factor for the reaction $7\text{Be}+p \rightarrow 8\text{B}+\gamma$
Pisma Zh.Eksp.Teor.Fiz. 51, No.5, 247-249 (1990) ... Uzbekistan
 $S_{17}(0) : 15.5\text{eVb}$

2018 M.L.Avila et al. Phys.Rev.C97, 014313 (2018)
Sub-Coulomb 3He transfer and its use to extract three-particle ANC
 $6\text{Li}(13\text{C},t)16\text{O}$

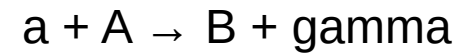
3 body problem

- 15N (p,gamma) 2008
- 12C (p,gamma) 2008
- 16O (p,gamma) 2009
- 13C (p,gamma) 2008
- 15N (p,gamma) 2010
- 22Ne (n,gamma) 2010
- 14C (n,gamma) 2011
- 22Mg (p,gamma) 2011
- 23Al (p,gamma) 2012

- 7Li (n,gamma) 2013 - (8Li,7Li)
- 14C (n,gamma) 2014 SF
- 17O (n,gamma) 2014 AlAbdulah
- 8B (p,gamma) 2015 Fukui, Ender (2013)
- 17F (p,gamma) 2017 Kuvin
- 20Ne (p,gamma) 2017 (low)
- 12N (p,gamma) 2013 Guo

ANC method – Asymptotic Normalization Coefficients

Tool to deduce a **direct** radiative capture



$$\sigma \sim G_{ABa}^2$$

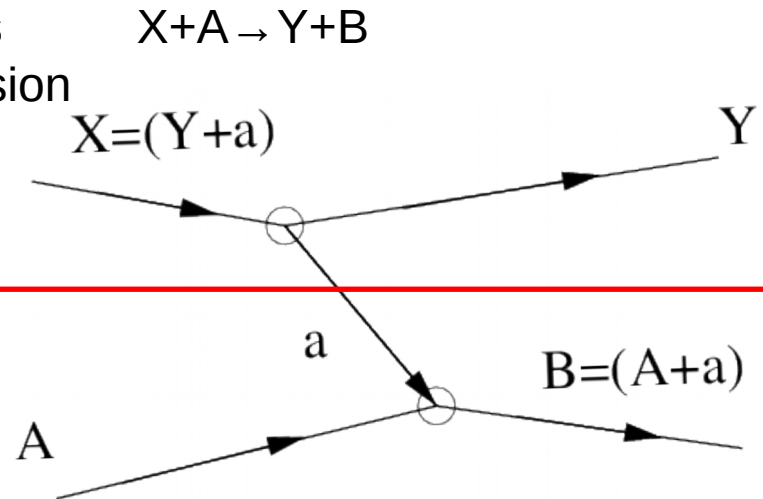
The trick:

- determine the vertex constant(s)/ANCs from binary reaction in peripheral collision

$$\sigma \sim G_{XYa}^2 G_{ABa}^2$$

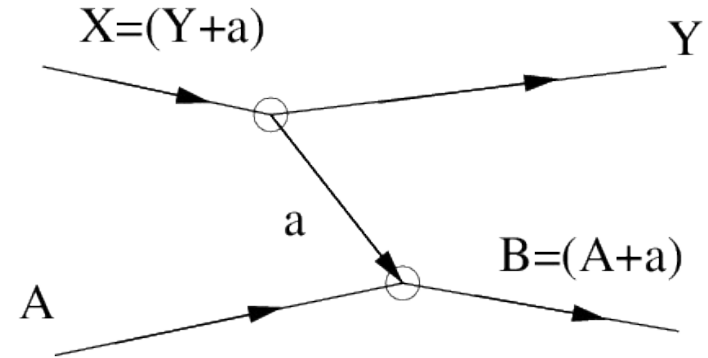
In low energy nuclear reactions

G is nuclear vertex constant NVC



but more than that

DWBA - distorted wave Born approximation



σ ↔

$$M(E_i, \cos\theta) = \sum_{M_a} \langle \chi_f^{(-)} I_{Aa}^B | \Delta V | I_{Ya}^X \chi_i^{(+)} \rangle$$

$$I_{a,b}^{c l_c j_c}(r)$$

being the **radial overlap function**

Approximated by (model) **bound state wave function**

$$\phi_{n_c l_c j_c}(r_{ab})$$

Shell model,
Woods-Saxon well

$$I_{abl_c j_c}^c(r_{ab}) = S_{abl_c j_c}^{1/2} \phi_{n_c l_c j_c}(r_{ab})$$

Spectroscopic Factor

extension to Monday's discussion

What are the conditions for the transfer reaction to study ANC?

Experimental

peripherality 7-10 MeV/A for fenom.OP
 for (n,gamma): what with L=0?
 beam / target (light beams+ stable targets limited)
 level complexity (stable sd-shell nuclei)
 state of the art detection systems for inverse kinematics ?

Theoretical

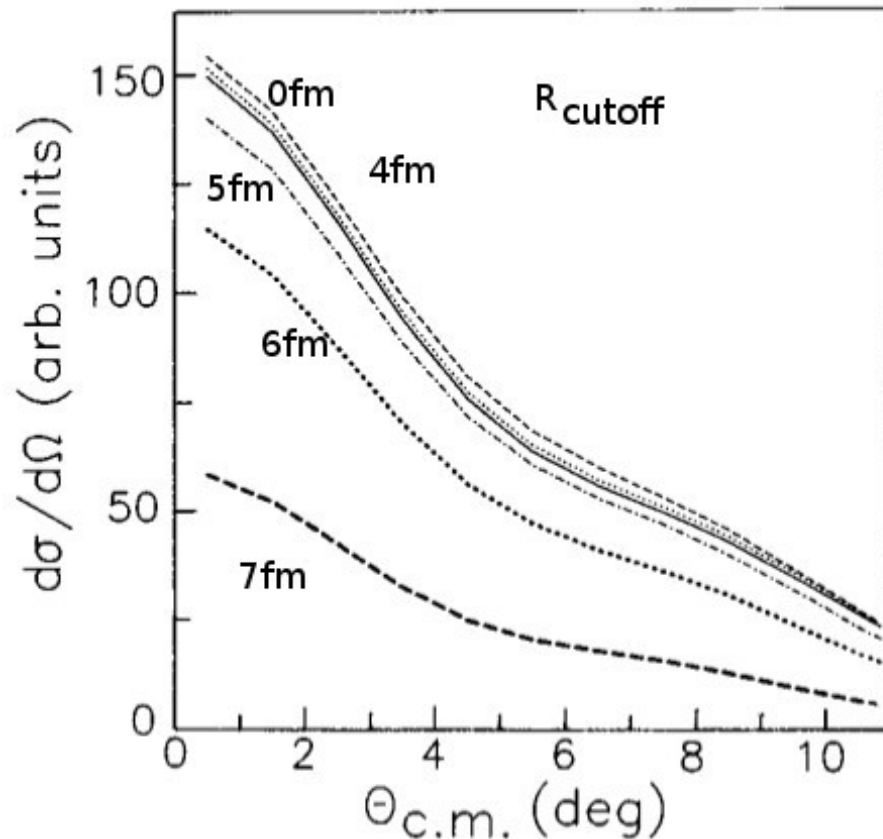
potential selection - (discussion on Monday)
 reaction selection (natural question)
 - uncertainty of the 2nd $|C|^2$?
 - (3He,d) nice - but as a target?
 - (d,p) not so much - (t,d)?
 - (d,n) used nowadays in inv.kin. - comments?

Practical

n-detection, 3-body problem
 Sources for ANC
 impact of ANC on astrophysical S factor
 Using mirror reactions
 Spectroscopic Factor improvements using ANC
 Nuclear radius measurement - (method of 2nd choice?)

To verify a peripherality of the reaction in experimental conditions, several checks are done.

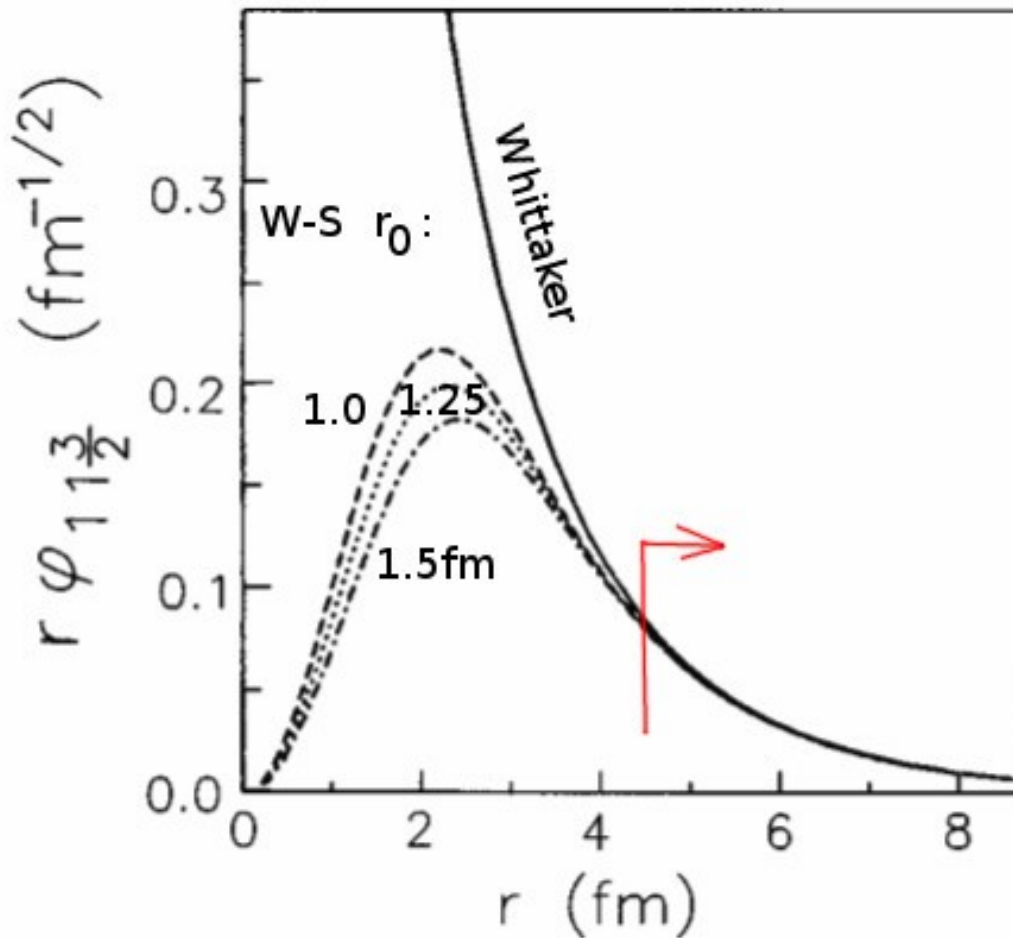
- optical potentials were deduced first from the angular distributions



cross section behavior
without interior (R_{cutoff})

different integration cutoff used
to prove the small dependence
on the interior

A. Mukhamedzhanov et al., PhysRev56,1302



Radial asymptotic (normalized)

Asymptotics of s.p. bound state w.f. for different W-S parameters r_0

correspond to Whittaker f.

A.Mukhamedzhanov et al., PhysRev56,1302

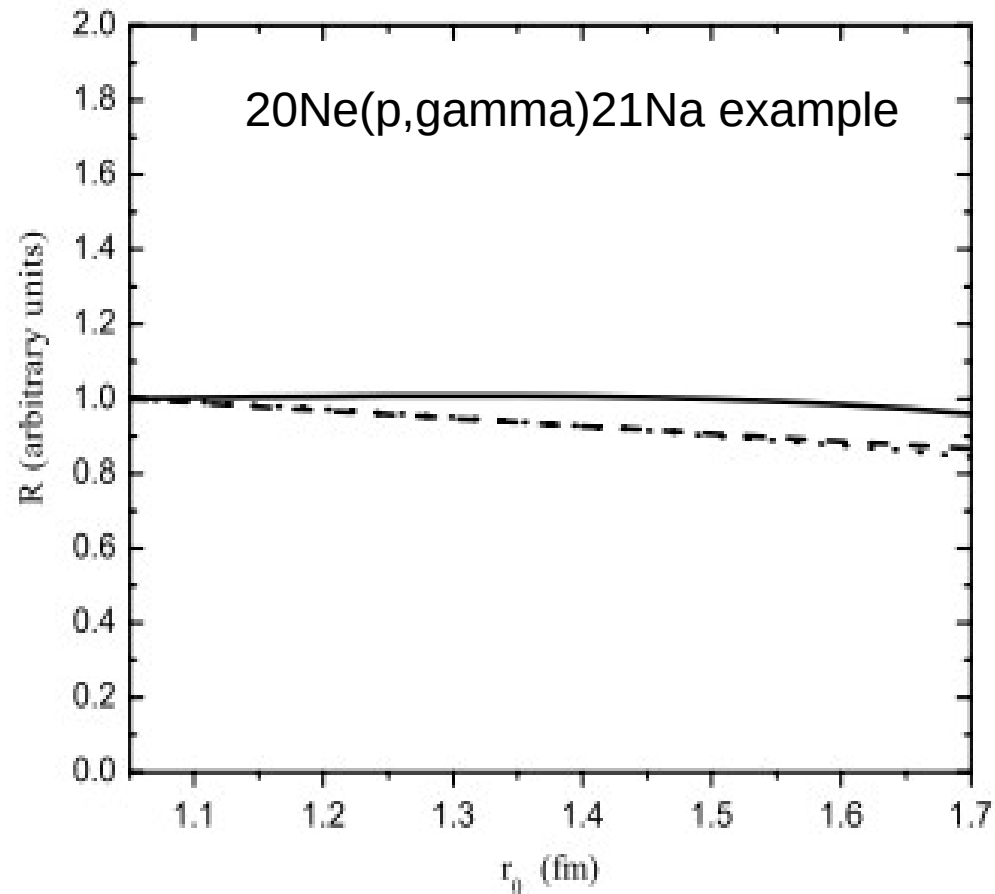
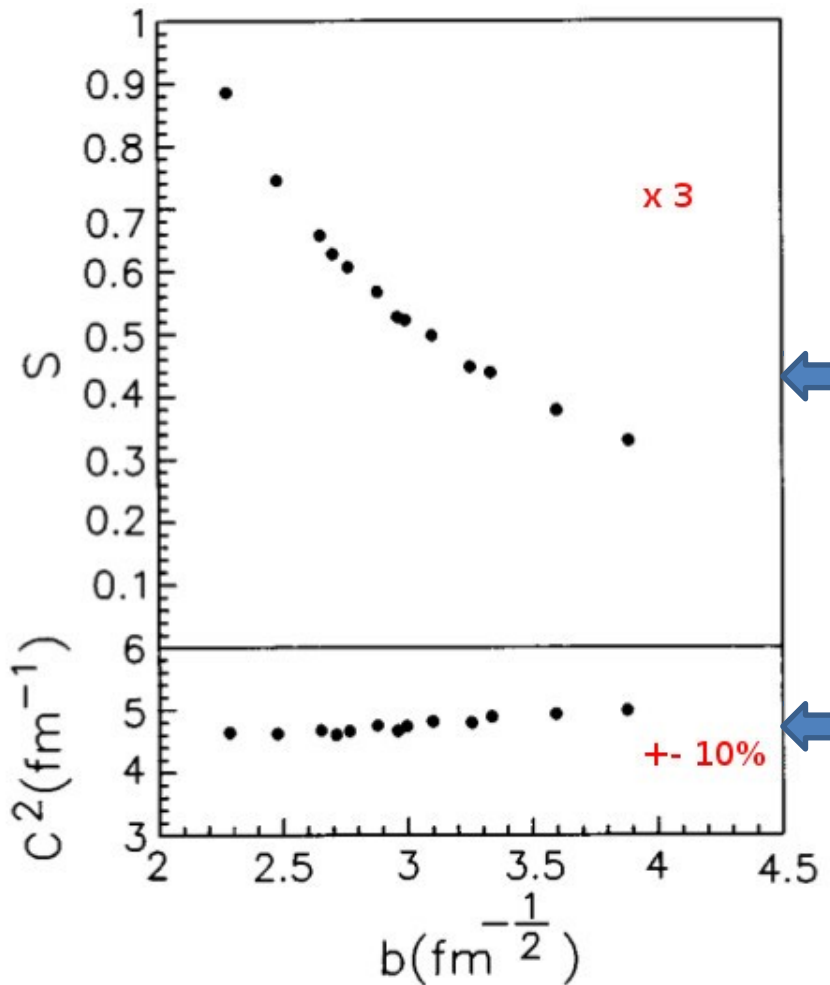


FIG. 5. The ratio R for three transitions from the $^{20}\text{Ne}(^3\text{He}, d)^{21}\text{Na}$ reaction. The solid line and dashed and dotted curves are for the transitions to the subthreshold state, the ground state, and the first excited state, respectively.

(d,p) reaction usage problem

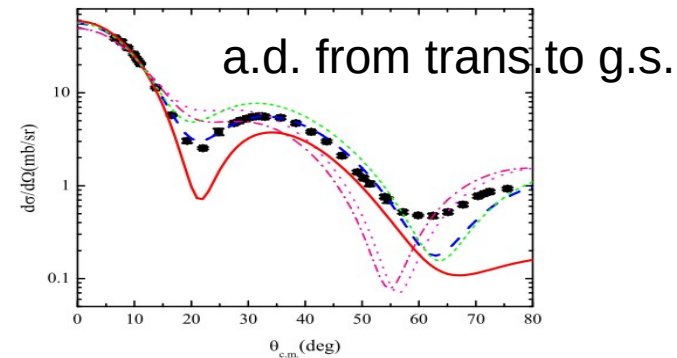
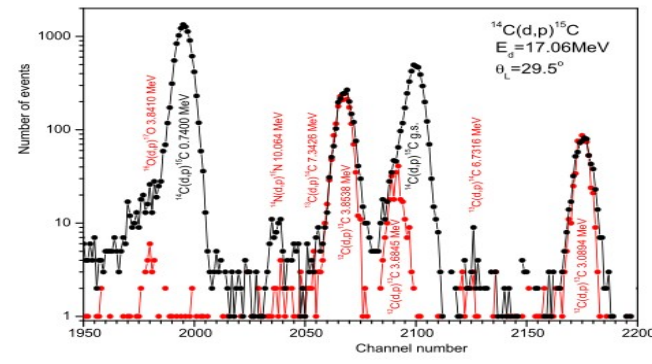
- $^{14}\text{C}(d,p)$
- nice illustration on $^{26}\text{Mg}(n,\gamma)$
- what to use - CDCC ? FR-ADWA ? different reaction ?

$^{14}\text{C}(n,\gamma)$

A.M.Mukhamedzhanov et al., *Phys.Rev.C84, 024616 (2011)*

ANC for the $^{14}\text{C}(n,\gamma)^{15}\text{C}$ from (d,p) reaction

- 'depletes' ^{14}C in inhomogenous big bang models (production $A>20$)



- ANC's determined for g.s. and 1st excited state

$$C^2_{01/2} = 1.64 \pm 0.26 \text{ fm}^{-1} \quad C^2_{25/2} = (3.55 \pm 0.43) \cdot 10^{-3} \text{ fm}^{-1}$$

- **FR-ADWA** approach decreased the errors (24% → 16%)

- older d,p measurement overestimated xs.

By 30% at fw angles

$C^2_{01/2}$	
1.89 ± 0.11	^{15}F mirror
1.48 ± 0.18	prev.exp.
1.64 ± 0.03	Coul.dissoc
1.64 ± 0.26	NPI CAS

JLM $C^2(14\text{C},15\text{C})$ 2.09 ± 0.29 fm⁻¹

ADWA $C^2(\text{d,p})$ 1.77 ± 0.21 fm⁻¹

McCleskey, A.M.Mukhamedzhanov et al., *Phys.Rev.C89, 044605 (2014)*

How unique is the asymptotic normalization coefficient method?

J. C. Fernandes* and R. Crespo[†]

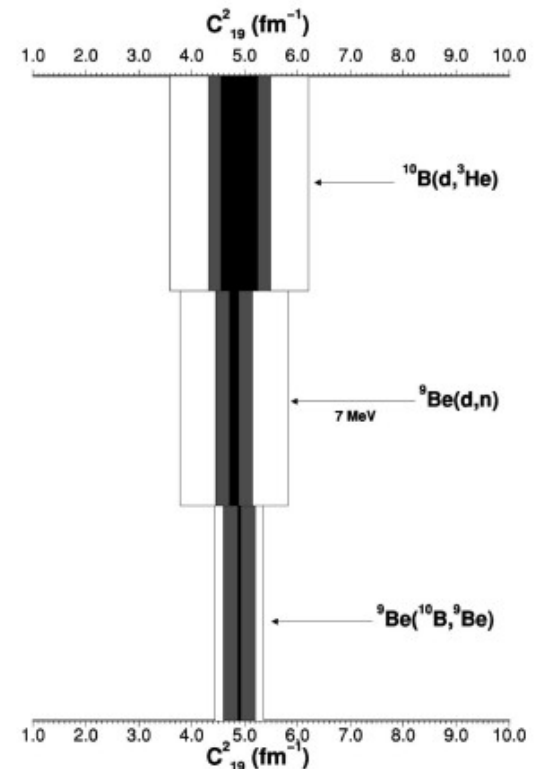
*Departamento de Física, Instituto Superior Técnico, and Centro Multidisciplinar de Astrofísica (CENTRA),
 Av. Rovisco Pais 1096 Lisboa Codex, Portugal*

F. M. Nunes[‡]

*How unique is ANC deduced from
 different transfer reactions or at
 different energies ?*

What is the actual answer?

**Because we (NPI CAS) have data
 on $^{26}\text{Mg}(d,p)$ @19MeV**



N.K. Timofeyuk et al., *Phys.Rev.Lett* 91, 232501 (2003)

Timofeyuk, Descouvemont, *Phys.Rev.C* 71, 064305 (2005)

$$|C_p/C_n|^2 = \mathcal{R} \approx \mathcal{R}_0 = \left| \frac{F_l(i\kappa_p R_N)}{\kappa_p R_N j_l(i\kappa_n R_N)} \right|^2$$

there exists a link between C_n and Γ_p

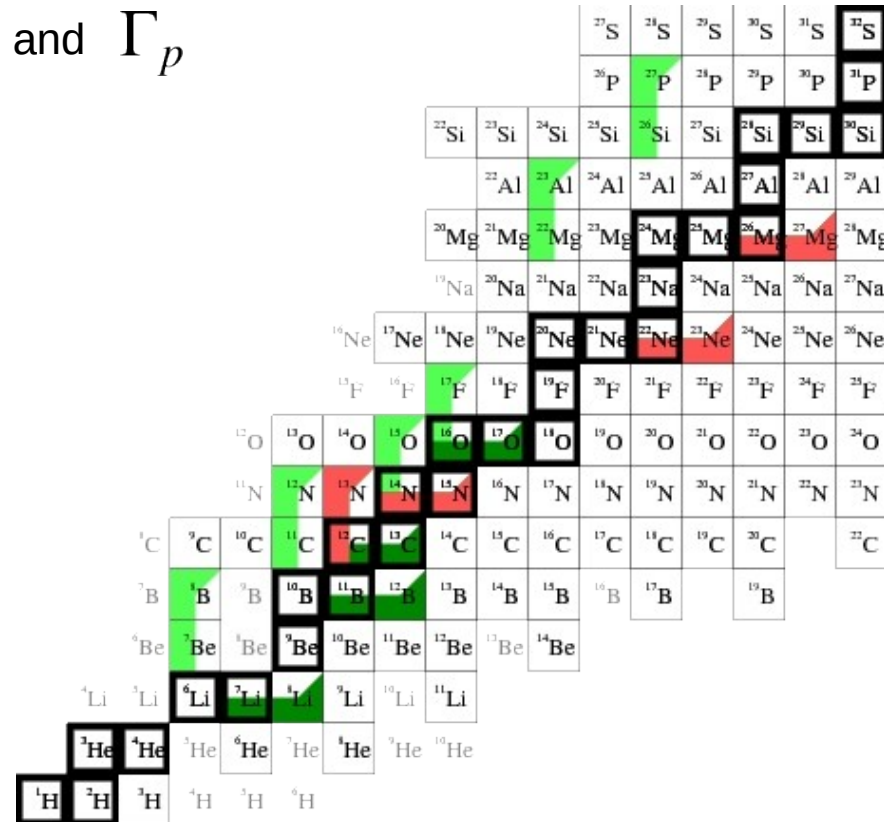
$$\Gamma_p / |C_n|^2 = \mathcal{R}_\Gamma \approx \mathcal{R}_0^{res} = \frac{\kappa_p}{\mu} \left| \frac{F_l(\kappa_p R_N)}{\kappa_p R_N j_l(i\kappa_n R_N)} \right|^2$$

On the sample of mirror cases it was shown, that with few % precision

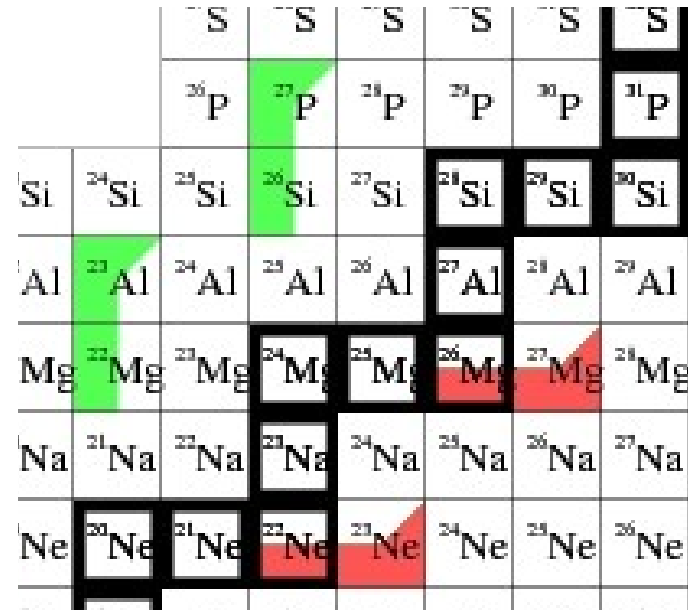
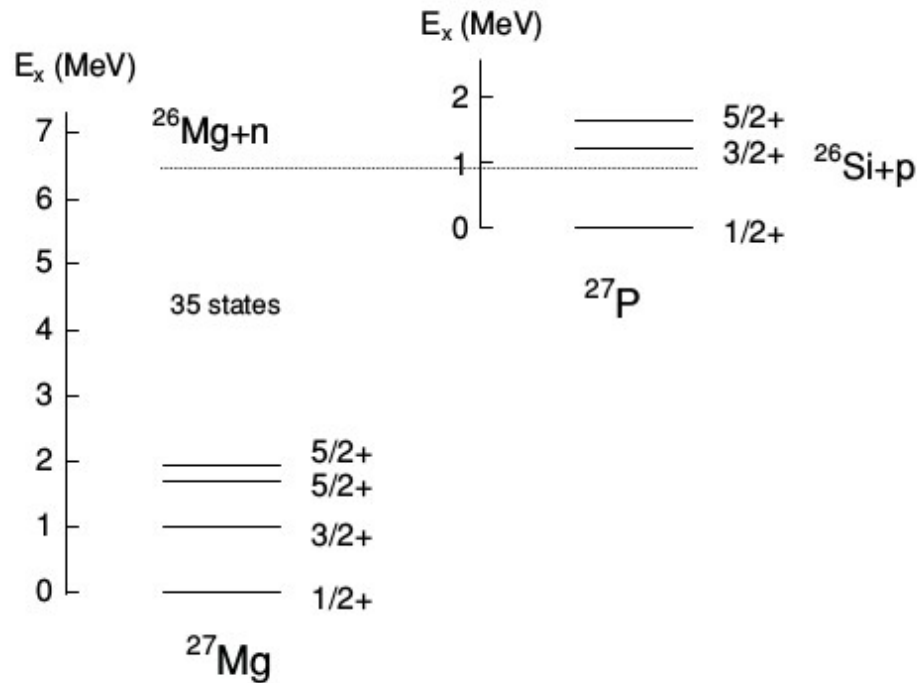
- **microscopic cluster model calculations** should be used to deduce mirror ANC,

- or simultaneous use of the above analytical formulae and single-particle estimate.

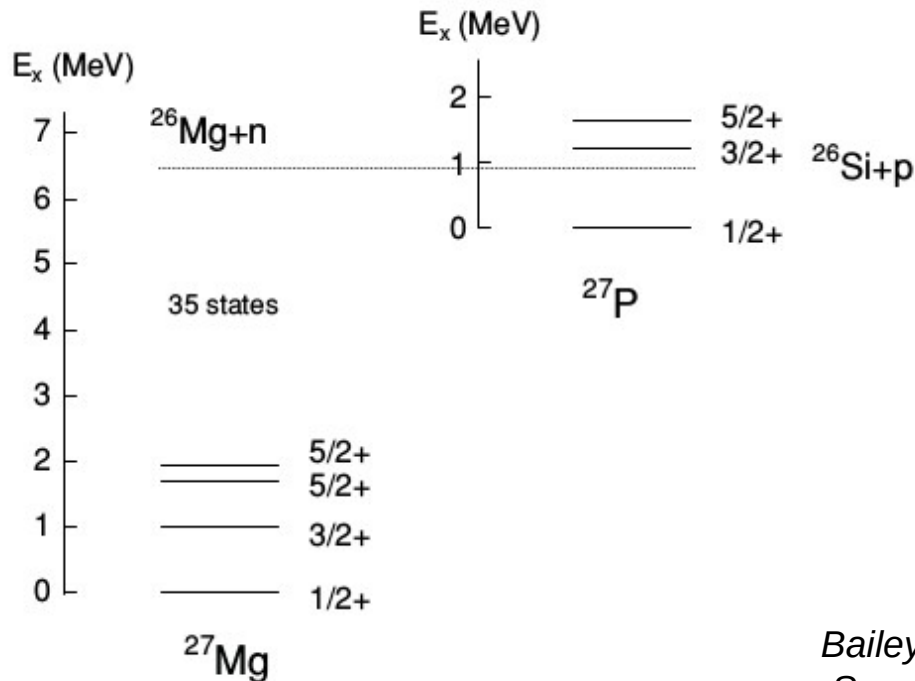
Core polarization effects created 12%.



Case of $^{26}\text{Mg}(d,p)$ for ^{27}P



Case of $^{26}\text{Mg}(d,p)$ for $^{26}\text{Si}(p,g)$



Guo et al. PRC73 (2006)
(d,p) data at 12MeV

Timofeyuk, Descouvemont, Thompson
PRC78 (2008)
reanalysis of (t,d) data
theoretical arguments against (d,p)

Bailey, Timofeyuk, Tostevin, PRL117 162502 (2016)
Sensitivity (d,p) to high n-p momenta, consequences

- x.s. can be very sensitive to n-p interactions in Ad.Approx

Gomez-Ramos, Timofeyuk, PRC98 011601R (2018)
Reduced sensitivity of (dp) x.s to the deuteron model
beyond adiabatic approximation

CDCC - recom. to extend (d,p) analysis beyond Ad.approx.

	Guo	Timofeyuk
$1/2+$	44+-5	24.5 +- 5
$3/2+$	3.4+-0.3	1.1+- 0.15

analysis of elastic scattering

Irvinski et al. PRC29, 349 (1984)

Blokhintsev et al. PRC 48 2390 (1993)

...

Ramirez Suarez PRC96, 034601 (2017)

peripheral transfer reactions

${}^9\text{Be}$ (p, γ) (1997) Mukhamedzhanov et al.

...

...

${}^{17}\text{F}$ (p, γ) (2017) Kuvín et al. PRC96 via (d,n)

theoretical calculations

Mukhamedzhanov Timofeuyk, Sov.J.Nucl.Phys.51, 431 (1990)

...

Mukhamedzhanov PRC86, 044615 (2012)

Coulomb renormalization ANC for mirror

Timofeyuk, Phys.Rev. C 88, 044315 (2013)

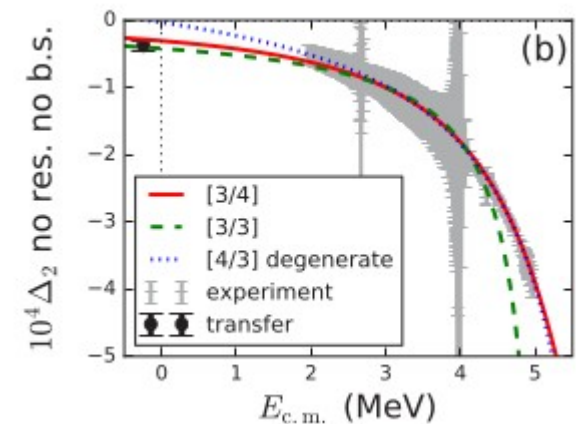
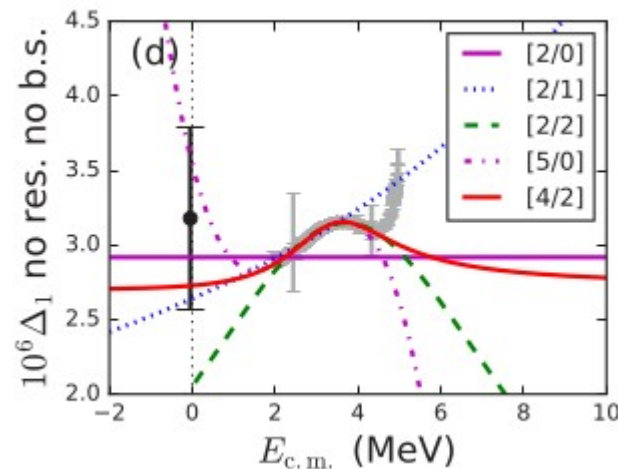
SF and ANC for $0p$ shell nuclei

analysis of elastic scattering

Suarez-Ramirez, Sparenberg PRC 96 034601 (2017)

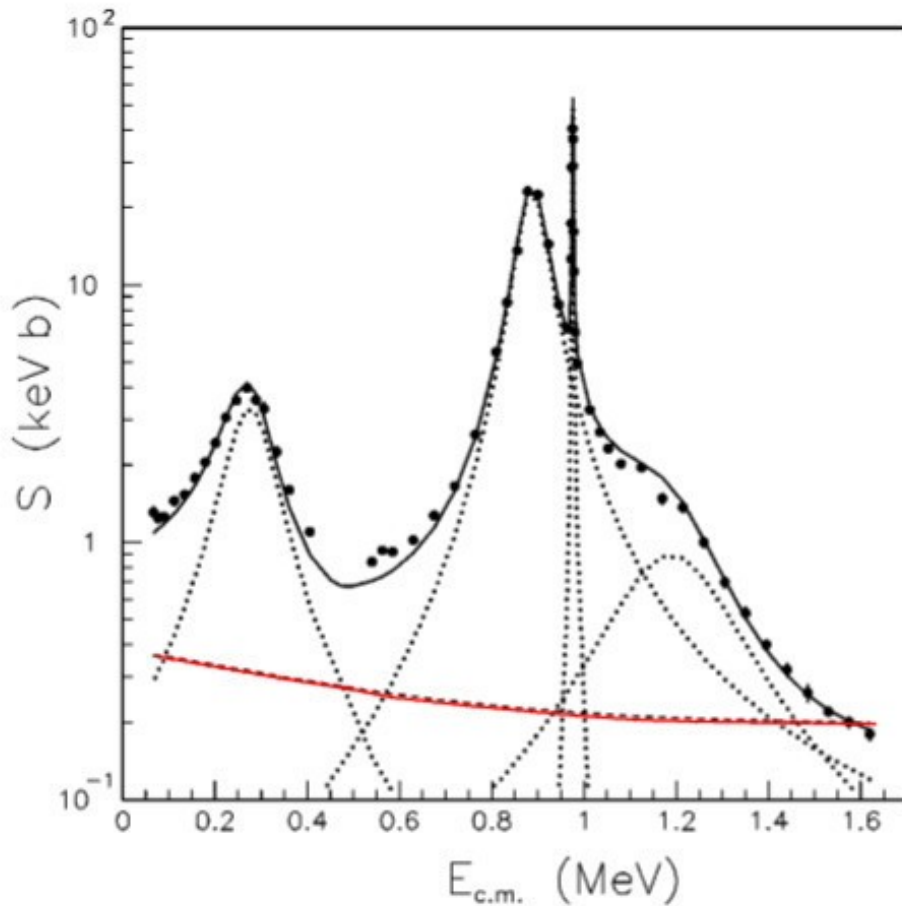
Instead of effective range function

$$\Delta_l = \frac{2\pi}{a_N} \frac{\cot \delta_l}{e^{2\pi\eta} - 1}$$



extrapolation to subthreshold state of alpha+12C

Impact of ANC



ANC values for 4 states
from ${}^9\text{Be}({}^{10}\text{B}, {}^9\text{Be})$ **agree**
to those from ${}^9\text{Be}({}^3\text{He}, \text{d})$ reaction

Later - **R-matrix** fit –

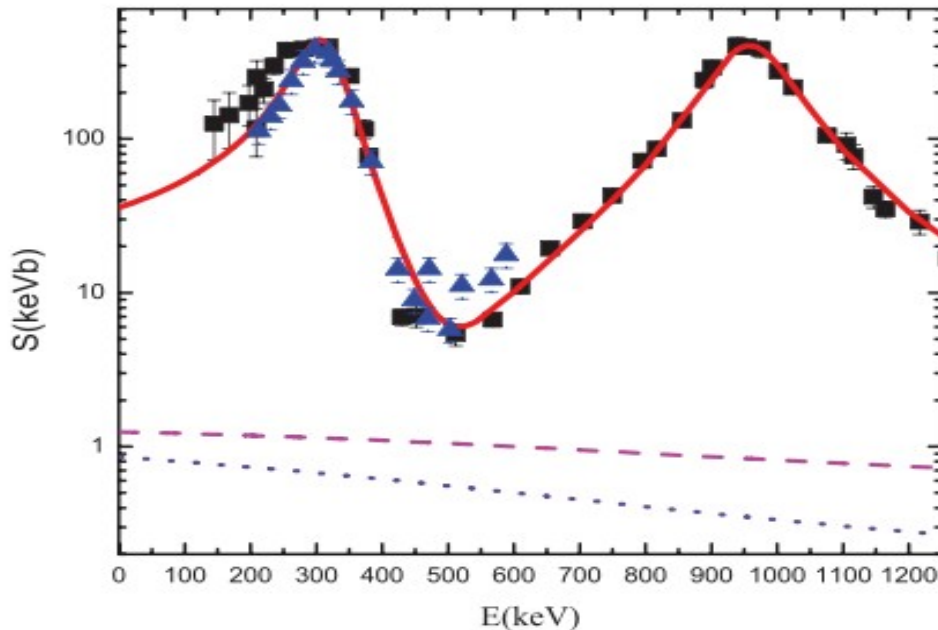
Sattarov et al. Phys.Rev C60, 035801 (TAMU)

S=3.98±0.12 keV b @ 269 keV

← **~30% of S**

deduced **ANC $C^2=192 \pm 26 \text{ fm}^{-1}$** for the g.s.

- the dominated by resonant capture to g.s. through two $J=1^-$ resonances



$S(0)=36.0 (6.0) \text{ keV b}$

... below the previous value NPA235 (1974)

rate of leak from CN cycle -

- one in every 2200 (300) (previously 1200)

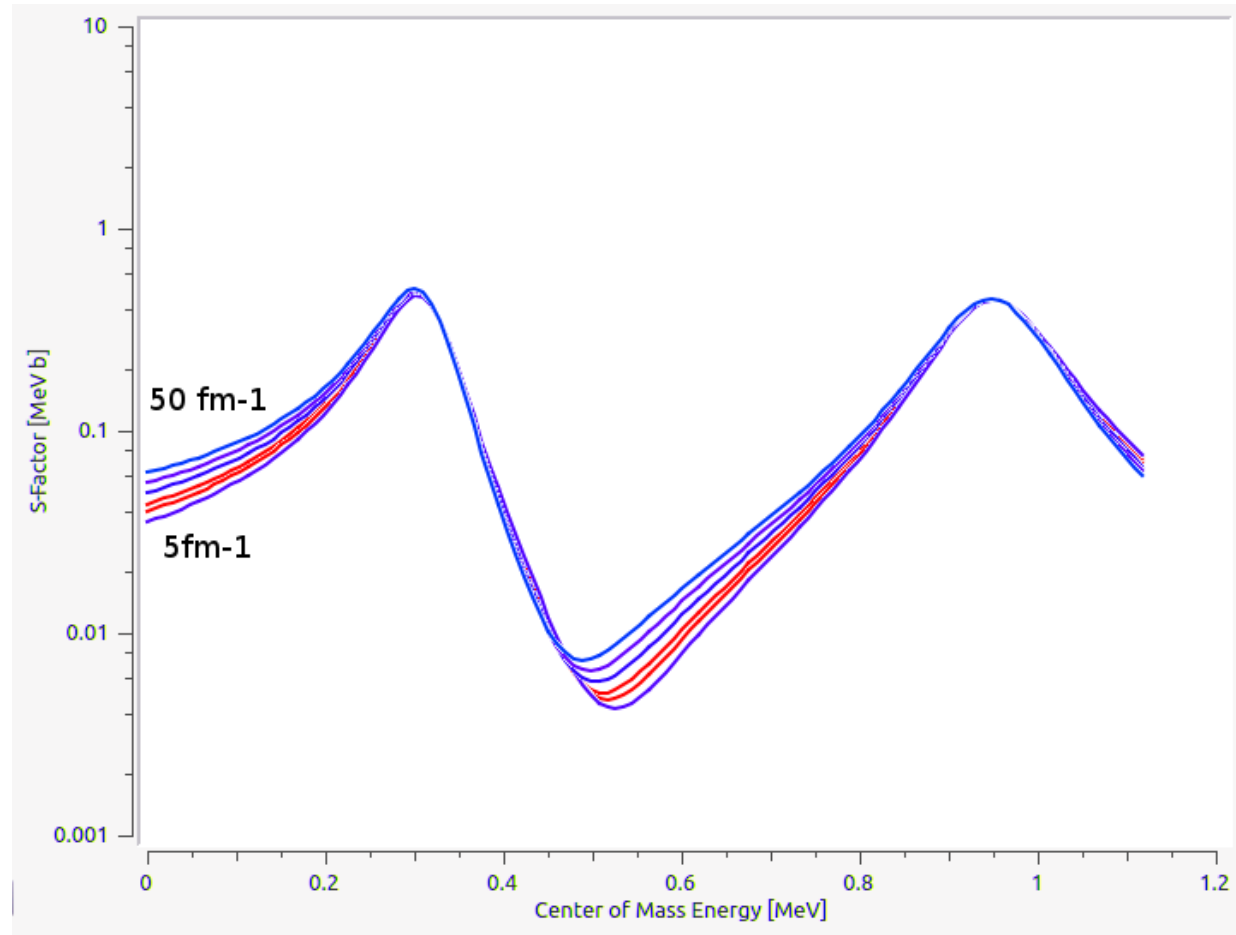
~3% of S

A.M.Mukhamedzhanov et al., PRC78, 015804 (2008)

Values of $|C|$ for g.s. ^{15}N
 (used for the ground state)

5 fm ^{-1/2}	$ C ^2$ 25
13.8 fm^{-1/2}	$ C ^2$ 198
20 fm ^{-1/2}	
30 fm ^{-1/2}	
40 fm ^{-1/2}	
50 fm ^{-1/2}	$ C ^2$ 2500

15% unc. in ANC
 induces 2% inc. in $S(0)$
 (if all parameters fixed)



SF – rely on internal wave functions

SF are not observables, they are not invariant under finite range unitary transformations of the phase equivalent N-N potentials

Mukhamedzhanov et al PRC82 051601 (2010)

Are occupation numbers observable?, *Furnstahl,-W. Hammer, PLB531 (2002)*

Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors,

A.M. Mukhamedzhanov and A.S. Kadyrov, (2010)

SF are meaningful within the context of the model used.

Non-observability of spectroscopic factors, B.K. Jennings, (2011)

Akram Mukhamedzhanov came with idea -

DWBA transition amplitude

$$M(E_i, \cos\theta) = \sum_{M_a} \langle \chi_f^{(-)} I_{Aa}^B | \Delta V | I_{Ya}^X \chi_i^{(+)} \rangle$$

$$\phi_{n_\alpha l_\alpha j_\alpha}(r_{\beta\gamma}) \xrightarrow{r_{\beta\gamma} > R_N} b_{\beta\gamma l_\alpha j_\alpha} \frac{W_{-\eta_\alpha, l_\alpha + 1/2}(2\kappa_{\beta\gamma} r_{\beta\gamma})}{r_{\beta\gamma}}$$

$$I_{\beta\gamma l_\alpha j_\alpha}^\alpha(r_{\beta\gamma}) = S_{\beta\gamma l_\alpha j_\alpha}^{1/2} \phi_{n_\alpha l_\alpha j_\alpha}(r_{\beta\gamma})$$

A. Mukhamedzhanov, F. Nunez, PRC72, 017602 (2005)

Akram Mukhamedzhanov came with idea -

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$$M(E_i, \cos\theta) = \sum_{M_a} \langle \chi_f^{(-)} I_{Aa}^B | \Delta V | I_{Ya}^X \chi_i^{(+)} \rangle$$

$$\phi_{n_\alpha l_\alpha j_\alpha}(r_{\beta\gamma}) \xrightarrow{r_{\beta\gamma} > R_N} b_{\beta\gamma l_\alpha j_\alpha} \frac{W_{-\eta_\alpha, l_\alpha + 1/2}(2\kappa_{\beta\gamma} r_{\beta\gamma})}{r_{\beta\gamma}}$$

$$I_{\beta\gamma l_\alpha j_\alpha}^\alpha(r_{\beta\gamma}) = S_{\beta\gamma l_\alpha j_\alpha}^{1/2} \phi_{n_\alpha l_\alpha j_\alpha}(r_{\beta\gamma})$$

to split inner part and outer part of the DWBA amplitude

depends on b **fixed by ANC in a peripheral reaction**

$$M = K_{n,l,j} M_{int}[b] + K_{n,l,j} b_{n,l,j} M_{ext}$$

K has a sense of S

$$\sum_{M_a} \langle \chi_f^{(-)} I_{Aa}^B | \Delta V | I_{Ya}^X \chi_i^{(+)} \rangle_{r < R}$$

$$\sum_{M_a} \langle \chi_f^{(-)} I_{Aa}^B | \Delta V | I_{Ya}^X \chi_i^{(+)} \rangle_{r > R}$$

M is divided by ANC and ...

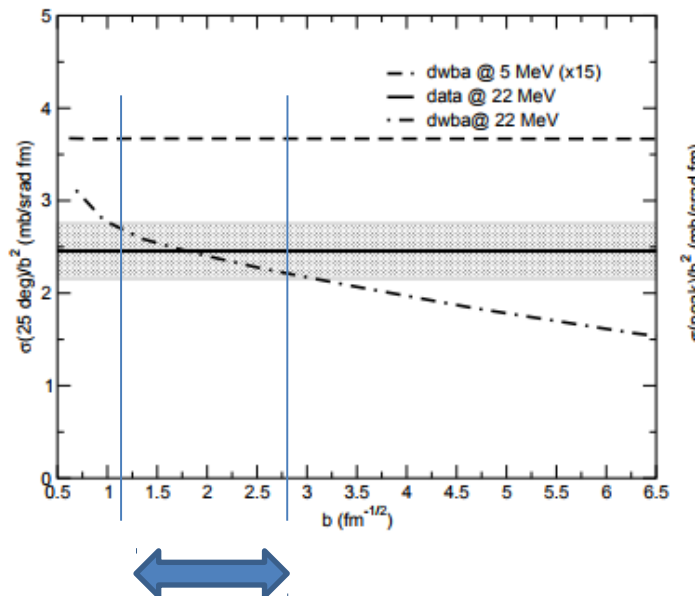
A. Mukhamedzhanov, F. Nunez, PRC72, 017602 (2005)

The idea can be written as two new functions – theoretical and experimental

$$R^{DW}(b_{n,lj}) = \left| \frac{\tilde{M}_{int}[b]}{b_{n,lj}} + \tilde{M}_{ext} \right|^2 = R^{exp} = \frac{d\sigma^{exp}}{d\Omega} / C_{lj}^2$$

imposing **equality** between the two will provide a correct b_{nlj} and thus a correct $S=C/b$

the procedure to find **SF** loses an artificial degree of freedom - *calculation on ^{208}Pb*



DWBA prediction
adiabatic

DWBA prediction

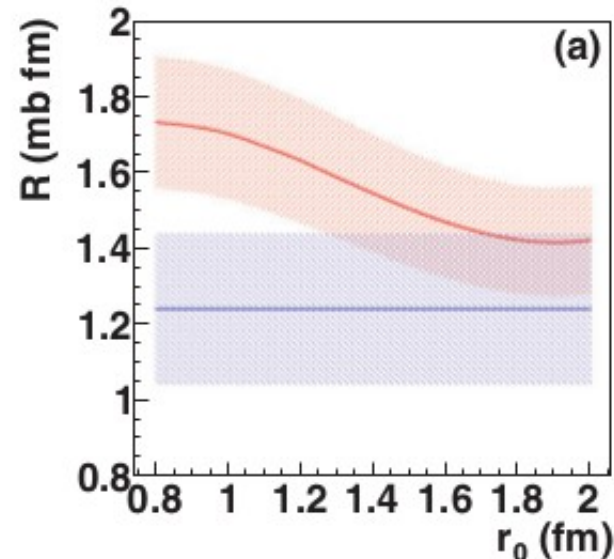
$^{14}\text{C}(n,\gamma)$ – neutron capture $^{13}\text{C}(^{14}\text{C},^{15}\text{C})^{12}\text{C}$ and on deuterium target

$C^2(\text{g.s.}) 1.88 \pm 0.18 \text{ fm}^{-1}$

1st experiment trying to complement SF with ANC deuterons 60MeV

reaction with 60MeV d
was expected
to be nonperipheral

analysis gives a range for SF 1.62 - 1.18
BUT other methods (mirror) ≤ 0.95 !
... bad momentum matching



McCleskey, A.M. Mukhamedzhanov et al., *Phys.Rev.C*89, 044605 (2014)

ANC is observable - are there other applications of ANC?

nuclear radii measurements:

^{16}N	Li, Guo
^{13}C , ^{11}Be	Belyaeva 2014
^8Li	Howel
^9C	Guo 2005
^8B	2001
^{12}B , ^{13}C	2001

ANC under breakup energy - can they help enlighten the situation with (d,p) ?

Thank you

Can new theories improve the situation or CDCC is the final step?

Not only Yarmukhamedzhanov but also Mukhamedzhanov creates a **new reaction theory**:

A.M. Mukhamedzhanov, PRC 84, 044616 (2011)

Theory of deuteron stripping: From surface integrals to a generalized R-matrix approach