Strange matter and kaon to pion ratio in SU(3) PNJL model

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Observables of Hadronization and the QCD Phase Diagram in the Cross-over Domain,
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Content

The 'horn': experiments

Firstly, the "horn" was described by the NA49 Collaboration (NA49 Collaboration, PRC 66, 054902,2002) and then it was shown that data can be placed on the same curve (STAR, AGS)

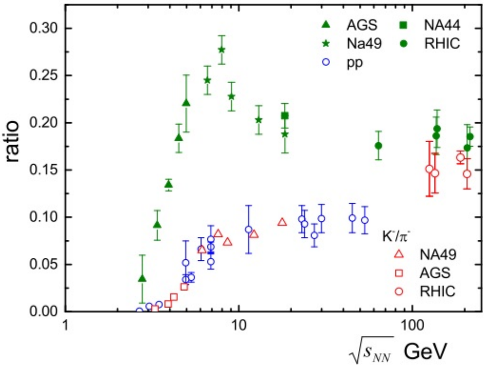
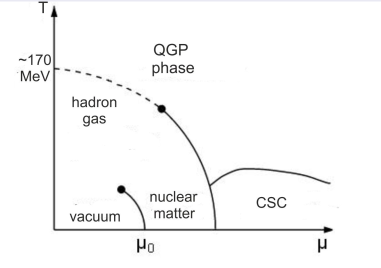


Figure 1: The K^+/π^+ and K^-/π^- ratio as the $\sqrt{s_{NN}}$

The 'horn': theoretical overview

- the statistical model: hadron resonances + σ - meson (implies the existence of the critical temperature for hadrons, the hadron phase transition) $\Rightarrow$ the qualitative reproduction of the peak (Andronic PLB 673, 142 (2009)).
- the SMES (the Statistical Model of Early Stages): a jump in the ratio is a result of the deconfinement transition: when deconfinement transition occurs the strangeness yield becomes independent of energy in the QGP ($m_s \rightarrow m_{s0}$) (M. Gazdzicki, M.I. Gorenstein, Acta Phys. Pol. B 30, 2705 (1999)).
- the microscopic transport model: with the hadron phase only can not reproduce experimental data (W. Ehehalt and W. Cassing, NPA602, 449 (1996); W. Cassing, E. L. Bratkovskaya, PR 308, 65 (1999); H. Petersen et al arXiv:0805.0567 [hep-ph]....)
- the microscopic transport model + the partial restoration of chiral symmetry (A. Palmese, et al. PRC 94, 044912 (2016): the quick increase in the K^+/π^+ appears as a result of the partial chiral symmetry restoration; the decrease is a result of QGP formation.

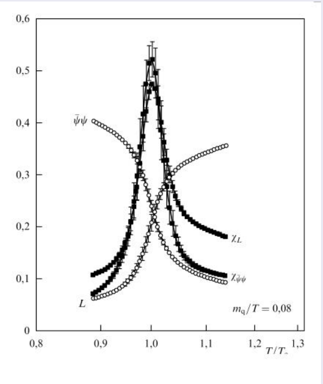
The QCD phase diagram



- chiral symmetry restoration (constituent quarks → current quarks);
- deconfinement;

Lattice QCD

Hands S. Contemp. Phys. 42, 209 [2001]



A.Bazavov et al. PRD85, 054503 [2012]

$T_c = 0.154(9)$ GeV

Propose of the work: use the SU(3) PNJL model

The Lagrangian (P. Costa et al. PRD79, 116003 (2009); E. Blanquier J. Phys. G: NPP 38, 105003 (2011)):

$$\begin{aligned} \mathcal{L} &= \bar{q}\,(\,i\,\gamma^{\mu}\,D_{\mu}\,-\,\hat{m}-\gamma_0\mu)\,q+\frac{1}{2}\,G_s\,\sum_{a=0}^{8}[\,(\,\bar{q}\,\lambda^{a}\,q\,)^2\,+\,(\,\bar{q}\,i\,\gamma_5\,\lambda^{a}\,q\,)^2\,] \\ &+ K\,\{\det[\bar{q}\,(\,1\,+\,\gamma_5\,)\,q]+\det[\bar{q}\,(\,1\,-\,\gamma_5\,)\,q]\,\}-\mathcal{U}(\Phi,\bar{\Phi};T) \end{aligned}$$

$D_{\mu}=\partial^{\mu}-iA^{\mu}$, where A^{μ} is the gauge field with $A^0=-iA_4$ and $A^{\mu}(x)=G_sA_a^{\mu}\frac{\lambda_a}{2}$
The effective potential has to reproduce the Lattice calculation in the pure gauge sector:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4}=-\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi-\frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right)+\frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,$$

$$b_2\left(T\right)=a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2+a_3\left(\frac{T_0}{T}\right)^3\ .$$

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$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2+a_3\left(\frac{T_0}{T}\right)^3 .$$

We can:

- explain and describe spontaneous chiral symmetry broken as $m_q = m_0 + < \bar{q}q >$;

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We can:

- explain and describe spontaneous chiral symmetry broken as $m_q=m_0+<\bar{q}q>$;
- simulate the confinement/deconfinement transition
- build the phase diagram with crossover at low chemical potential and 1st order transition at high chemical potential ($m_0\neq 0$),

The mean-field approximation

The grand potential density:

$$\begin{aligned}\Omega &= U(\Phi,\bar{\Phi};T) + g_S \sum_{i=u,d,s} \langle \bar{q}_i q_i \rangle^2 + 4 g_D \langle \bar{q}_u q_u \rangle \langle \bar{q}_d q_d \rangle \langle \bar{q}_s q_s \rangle - 2N_c \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_i - \\ &- 2T \sum_{i=u,d,s} \int \frac{d^3p}{(2\pi)^3} (N_\Phi^+(E_i) + N_\Phi^-(E_i))\end{aligned}$$

with the functions

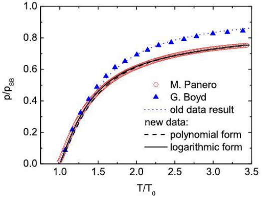
$$\begin{aligned}N_\Phi^+(E_i) &= \mathrm{Tr}_c \left[\ln(1 + L^\dagger e^{-\beta(E_i-\mu_i)})\right] = \left[1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_i^+}\right) e^{-\beta E_i^+} + e^{-3\beta E_i^+}\right], \\ N_\Phi^-(E_i) &= \mathrm{Tr}_c \left[\ln(1 + L e^{-\beta(E_p+\mu_i)})\right] = \left[1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_p^-}\right) e^{-\beta E_p^-} + e^{-3\beta E_p^-}\right] ,\end{aligned}$$

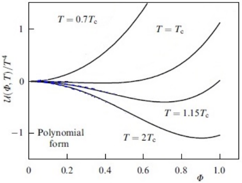
where $E_i^\pm = E_i \mp \mu_i$, $\beta = 1/T$, $E_i = \sqrt{{p_i}^2 + m_i^2}$ is the energy of quarks and $\langle \bar{q}_i q_i \rangle$ is the quark condensate.
The equations of motion

$$\frac{\partial \Omega}{\partial \sigma_f} = 0, \quad \frac{\partial \Omega}{\partial \Phi} = 0, \frac{\partial \Omega}{\partial \bar{\Phi}} = 0.$$

The model properties

The effective potential $\mathcal{U}(\Phi, \bar{\Phi}; \mathrm{T})$ properties

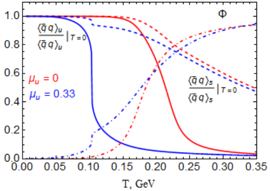


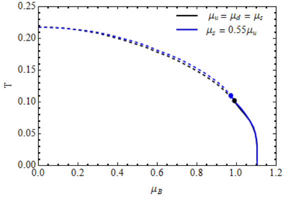


The chiral phase transition properties:

a) $\mu_{\mathrm{u}} = \mu_{\mathrm{d}} = \mu_{\mathrm{s}}\,,$

b) $\mu_{\mathrm{u}} = \mu_{\mathrm{d}}\,; \mu_{\mathrm{s}} = 0.55\mu_{\mathrm{u}}$





Masses

The gap equation for quarks depends on the quark condensates:

$$m_i = m_{0i} - 2g_S \langle \bar{q}_i q_i \rangle - 2g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle,$$

The meson masses are defined by the Bethe-Salpeter equation at $P = 0$

$$1 - P_{ij} \Pi^P_{ij}(P_0 = M, P = 0) = 0 \ ,$$

with

$$P_\pi = G_s + K \langle \bar{q}_s q_s \rangle \ , \quad P_K = G_s + K \langle \bar{q}_u q_u \rangle$$

and the polarization operator:

$$\Pi^P_{ij}(P_0) = 4 \left((I^i_1 + I^j_1) - [P_0^2 - (m_i - m_j)^2] \ I^{ij}_2(P_0) \right) ,$$

When $T > T_{Mott}$ $(P_0 > m_i + m_j)$ $\rightarrow$ the meson $\rightarrow$ the resonance state $\rightarrow$ the solution has to be defined in the form $P_0 = M_M - \frac{1}{2} i \Gamma_M$.

The mass spectra

$\mu_u = \mu_d = \mu_s$, $\mu_B = 3\mu_u$, $\rho_B = (\rho_u + \rho_d + \rho_s)/3$
To describe the mesons in dense matter, Fermi momentum λ_i, should be related with the chemical potential of quarks $\mu_i = \sqrt{\lambda_i^2 + m_i^2}$.

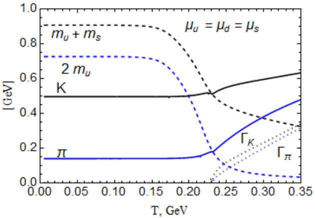
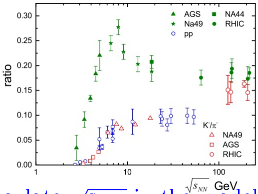
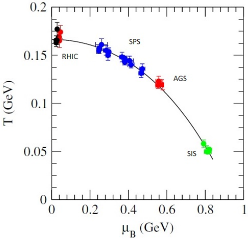


Figure 2: The mass spectra at zero and finite chemical potential (see for discussion P.Costa et al. PRD71 (2005) 116002)

The experimental results



How calculate $\sqrt{s_{NN}}$ in the model?



T-μ variables J. Cleymans et al. PRC73, 034905 (2006)

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \quad \mu_B(\sqrt{s}) = \frac{d}{1 + e\sqrt{s}}$$

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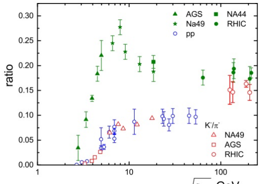

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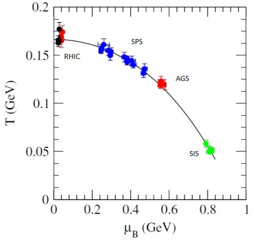

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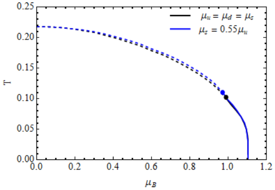

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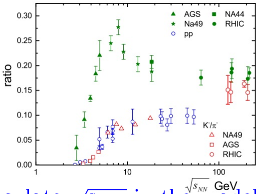

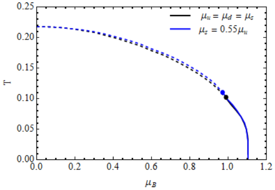

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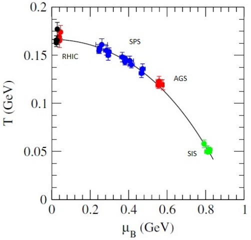
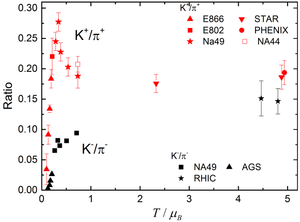
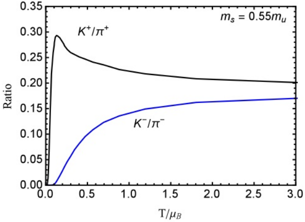


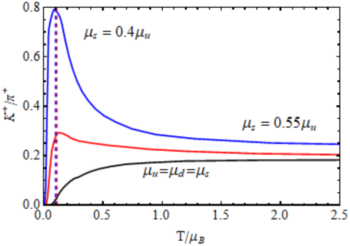


Kaon to pion ratio

$$n_{K^\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{K^\pm}} \mp \mu_{K^\pm})} - 1},$$
$$n_{\pi^\pm} = \int_0^\infty p^2 dp \frac{1}{e^{(\sqrt{p^2+m_{\pi^\pm}} \mp \mu_{\pi^\pm})} - 1}.$$

with parameter $\mu_\pi = 0.135$ (M. Kataja, P.V. Ruuskanen PLB 243, 181 (1990)) and $\mu_K = \mu_u - \mu_s$ (see for example A. Lavagno and D. Pigato, EPJ Web of Conferences 37, 09022 (2012)).





How we can improve the PNJL model? I

- introduce a phenomenological dependence of $G_s(\Phi)$ (Y. Sakai et al PRD 82, 076003 (2010), P. de Forcrand, O. Philipsen NPB 642, 290(2002), A. Friesen et al. IJMPA30, 1550089 (2015).)

$$\tilde{G}_s(\Phi) = G_s[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)]$$

with $\alpha_1 = \alpha_2 = 0.2$.

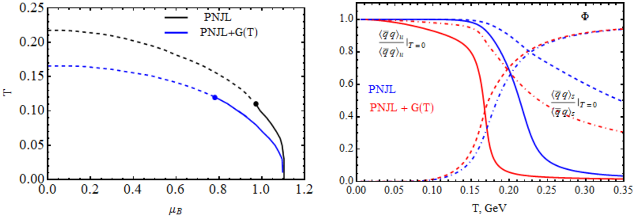


Figure 3: $\mu_s = 0.5\mu_u$

How we can improve the PNJL model? II

- the effect of axial symmetry and the coupling $K = K_0 \exp(-(\rho/\rho_0)^2)$ on the dense states see for discussion P. Costa et al AIP Conf.Proc. 775 (2005) 173; arXiv::0503258
- and introduce $G_s(\Phi)$ with $\alpha_1 = \alpha_2 = 0.2$.

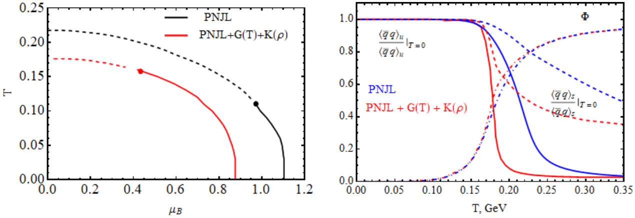


Figure 4: $\mu_s = 0.5\mu_u$

Results

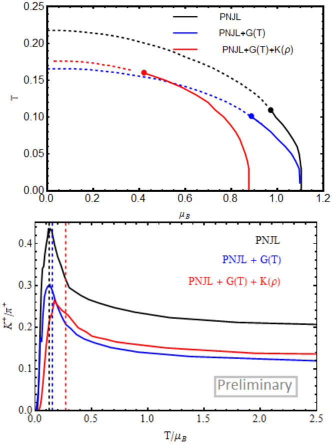
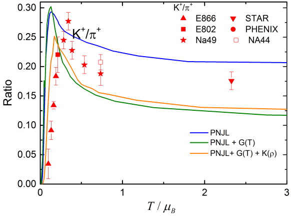


Figure 5: $\mu_s = 0.5\mu_u$

Theory + experiment



Results and outlooks

- splitting of kaons masses at high densities ⇒ the difference in the behavior of the K/π at low energies.
- the hight of the peak in the model depends on the properties of the matter (strange chemical potential, T and μ_B) - it looks like we need more realistic description of the media.
- the position of the peak pretends to be depend on curvature of phase diagram/CEP position. This statement can be checked more carefully, for example in he PNJL model with vector interaction, where at critical value of G_v first order transition can disappear.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Tack
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Asante
ευχαριστώ
Obrigado
Спасибі
dank u
kiitos
Danke
terima kasih
Grazie
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Tak
Thank you
Dankie
谢谢
хвала
Gracias
Спасибо
teşekkür ederim
paldies
Merci
Благодаря
धन्यवाद
ďakujem
تشکر از شما
dziękuję
Sağol
Hvala
Děkuji
Дзякуй
salamat

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  with $\alpha_1 = \alpha_2 = 0.2$.
- introduce a $K = K_0 \exp(-(\rho/\rho_0)^2)$ (see for discussion P. Costa et al AIP Conf.Proc. 775 (2005) 173)

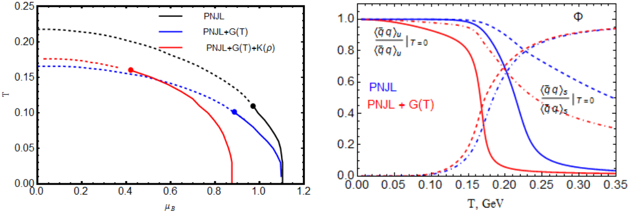

Figure 6: $\mu_s = 0.5\mu_u$