Fluctuations
Lattice meets experiment

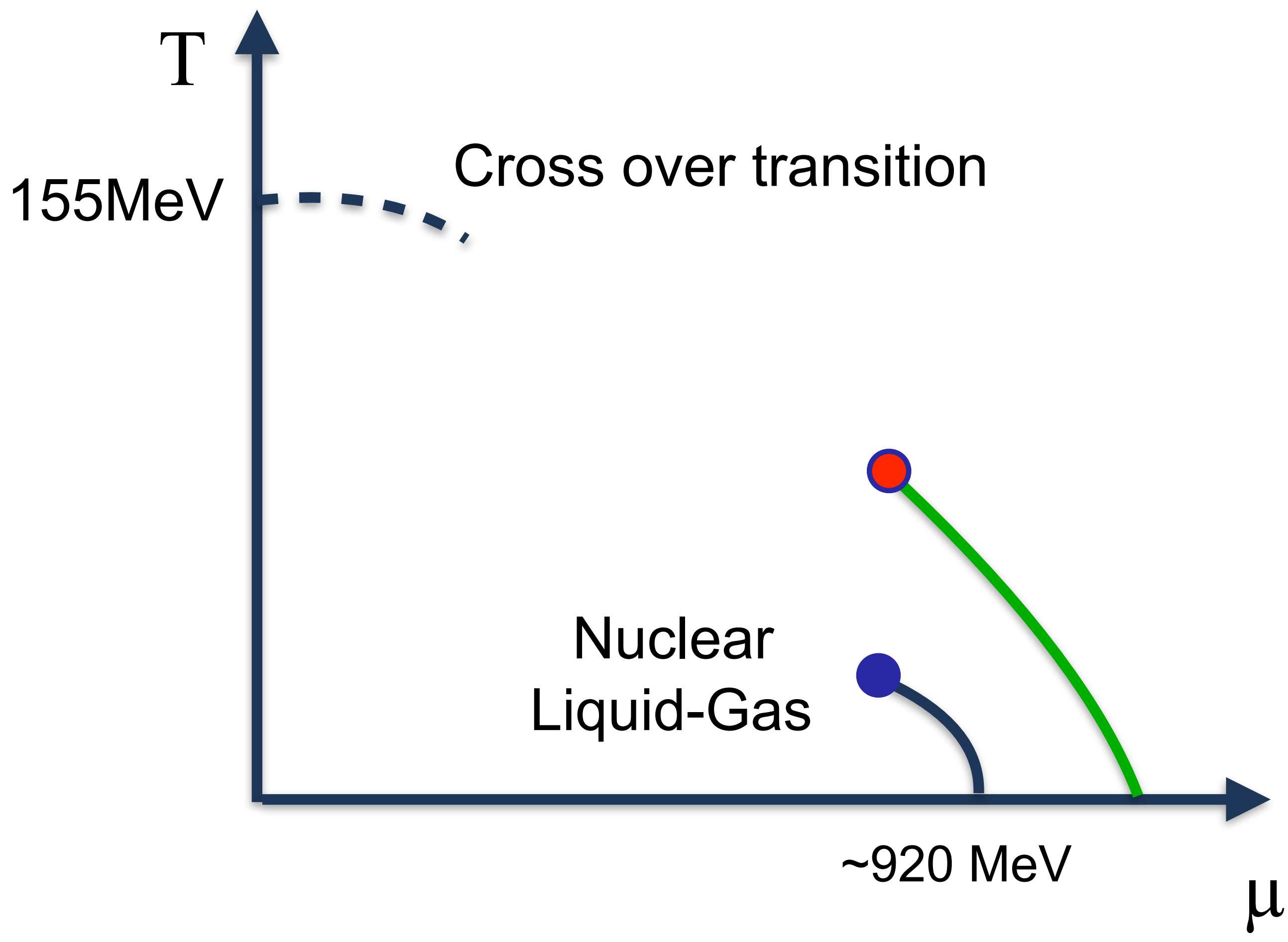
A. Bzdak, VK, N. Strodthoff: arXiv:1607.07375

A. Bzdak, VK: arXiv:1707.02640

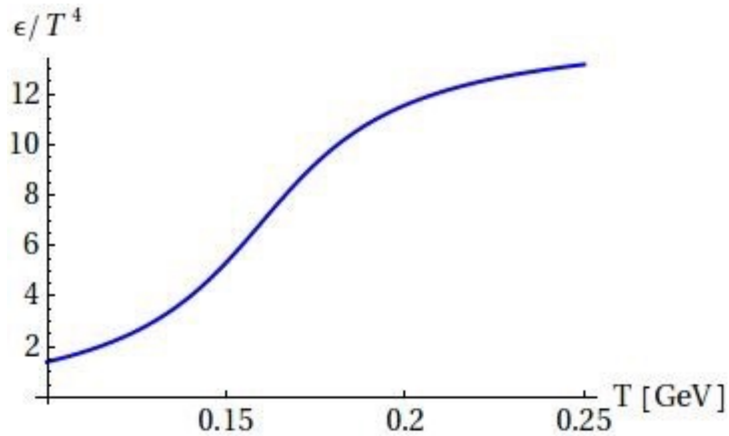
A. Bzdak, VK: arXiv:1810.01913

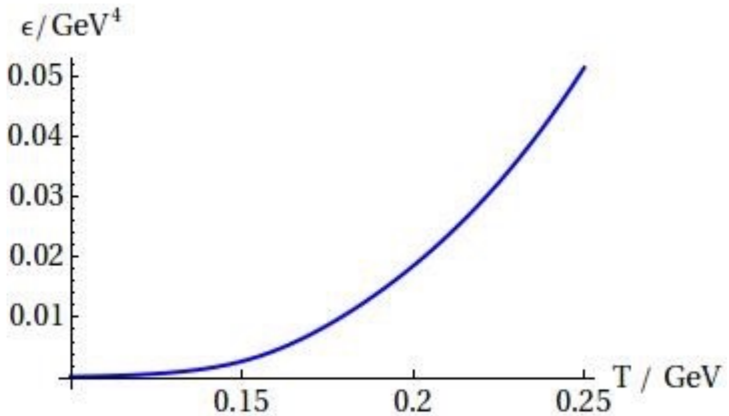
BEST COLLABORATION

Phase diagram



Cumulants and phase structure



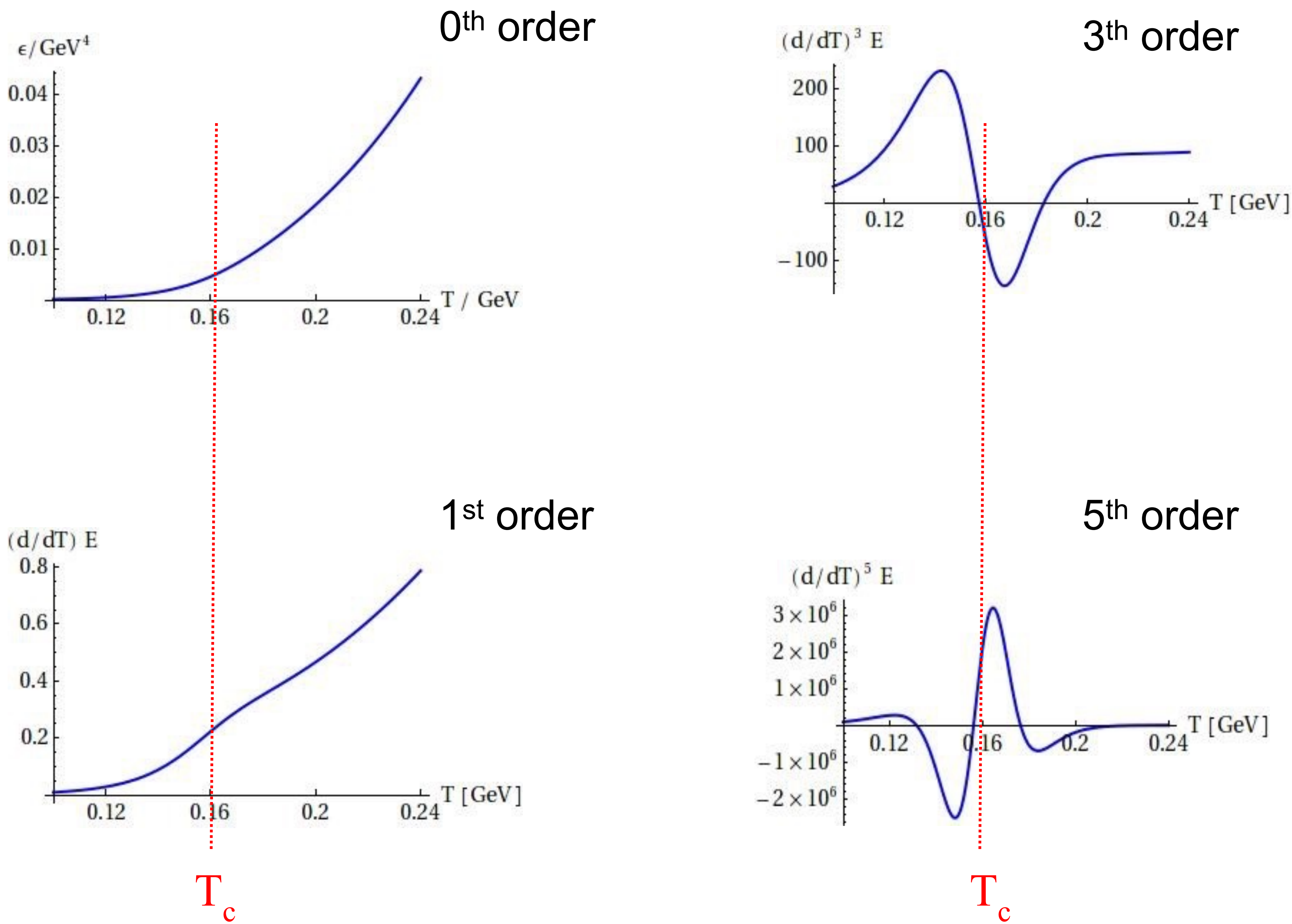


What we always see....

What it really means....

"T_c" ~ 160 MeV

Derivatives



How to measure derivatives

At $\mu = 0$:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T}\right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T}\right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T}\right)^{n-1} \langle E \rangle$$

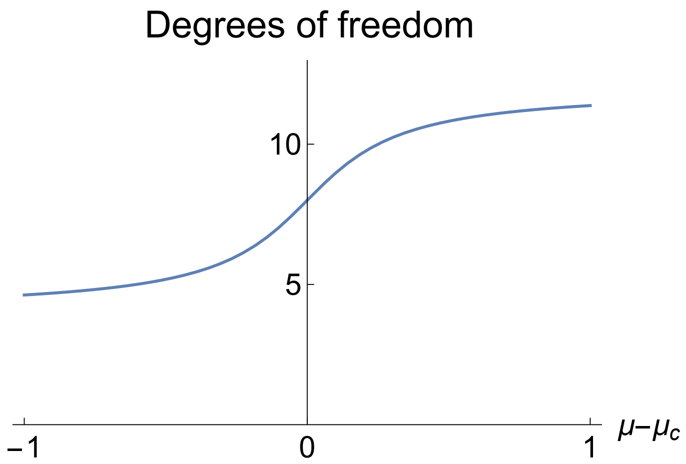
Cumulants of Energy measure the temperature derivatives of the EOS

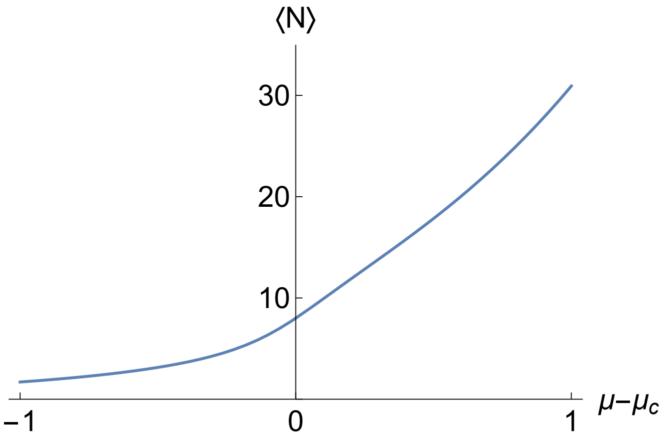
Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

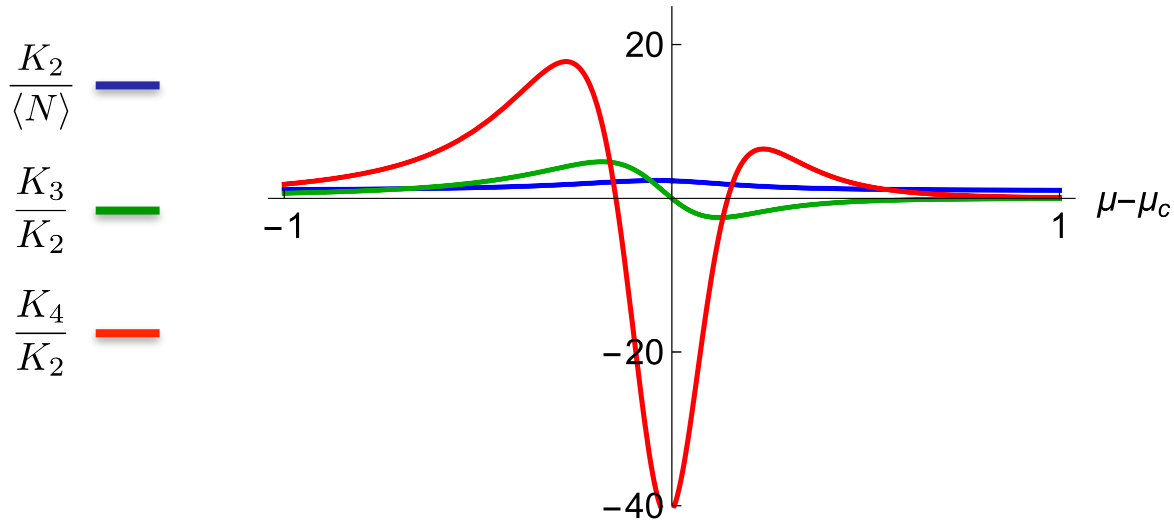
Simple model

Change degrees of freedom at phase transition

$$\langle N \rangle = dof(\mu)\, e^{\mu/T} \int d^3p e^{-E/T}$$



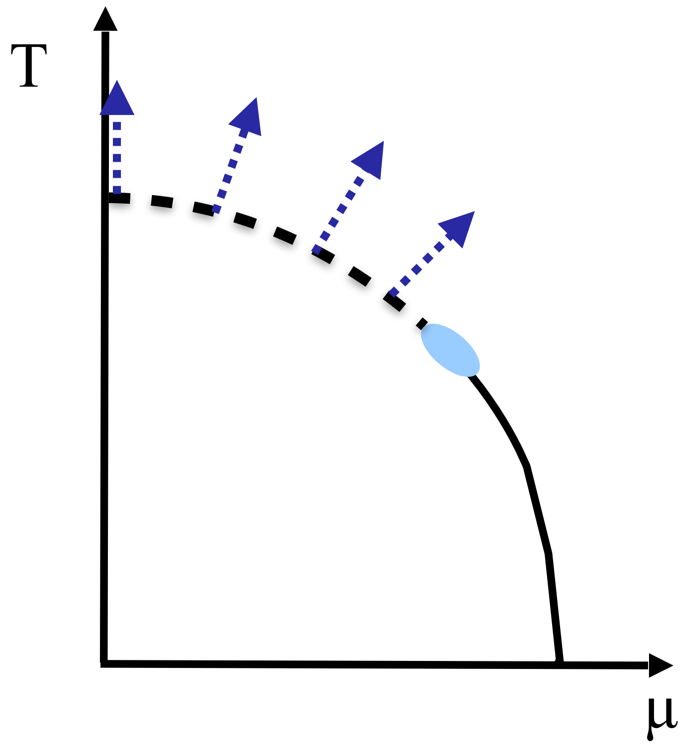




Close to μ=0

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

a ~ curvature of critical line



$$\frac{\partial^2}{\partial\mu^2} F(T,\mu)|_{\mu=0} = \frac{a}{T}\frac{\partial}{\partial T} F(T,\mu=0) \sim \langle E \rangle$$

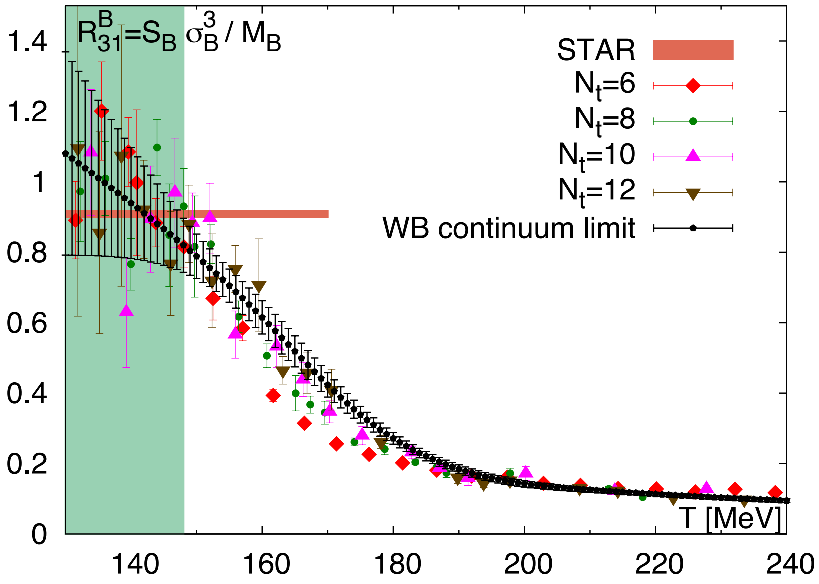
$$\frac{\partial^3}{\partial\mu^3} F(T,\mu)|_{\mu=0} = 3\frac{a^3}{T}\left(T\frac{\partial^2}{\partial T^2} - \frac{\partial}{\partial T}\right) F(T,\mu=0)$$

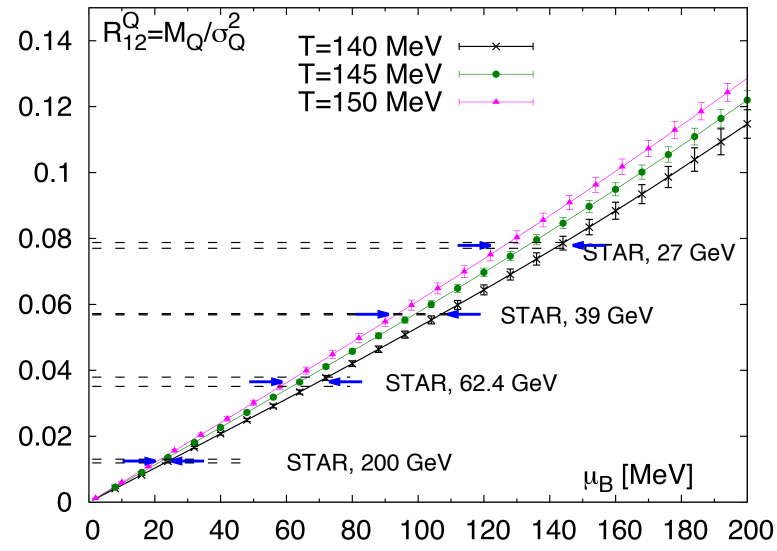
Needs higher order cumulants (derivatives) at μ ~ 0

Compare Data with Lattice QCD

Compare Data with Lattice QCD

- Lattice cannot calculated hadron abundances
- Cumulants are well defined quantities
- Compare cumulants !?





Protons vs Baryons

Kitazawa and Asakawa, PRC85, 021901 (2012)

Possible to relate proton cumulants to baryon cumulants
if fast isospin equilibration

$$\langle N_{\rm B}^{(\rm net)} \rangle = 2\langle N_p^{(\rm net)} \rangle, \tag{9}$$

$$\langle (\delta N_{\rm B}^{(\rm net)})^2 \rangle = 4\langle (\delta N_p^{(\rm net)})^2 \rangle - 2\langle N_p^{(\rm tot)} \rangle, \tag{10}$$

$$\langle (\delta N_{\rm B}^{(\rm net)})^3 \rangle = 8\langle (\delta N_p^{(\rm net)})^3 \rangle - 12\langle \delta N_p^{(\rm net)} \delta N_p^{(\rm tot)} \rangle + 6\langle N_p^{(\rm net)} \rangle, \tag{11}$$

$$\begin{aligned} \langle (\delta N_{\rm B}^{(\rm net)})^4 \rangle_c = {} & 16\langle (\delta N_p^{(\rm net)})^4 \rangle_c - 48\langle (\delta N_p^{(\rm net)})^2 \delta N_p^{(\rm tot)} \rangle \\ & + 48\langle (\delta N_p^{(\rm net)})^2 \rangle + 12\langle (\delta N_p^{(\rm tot)})^2 \rangle - 26\langle N_p^{(\rm tot)} \rangle, \end{aligned} \tag{12}$$

Doable but in general $\dfrac{K_m^{(net-p)}}{K_n^{(net-p)}} \neq \dfrac{K_m^{(net-B)}}{K_n^{(net-B)}}$

Compare Data with Lattice QCD

Example: “Charge” susceptibility

$$\chi_Q = \int d^3x < \rho(x)\rho(0) >= \int d^3p < \tilde{\rho}(p)\tilde{\rho}(0) >$$

Equivalence of *Integrated* coordinate space and momentum space correlation function

Experiment almost never integrates ALL of momentum space!

Lattice (hopefully) does integrate over all coordinate space

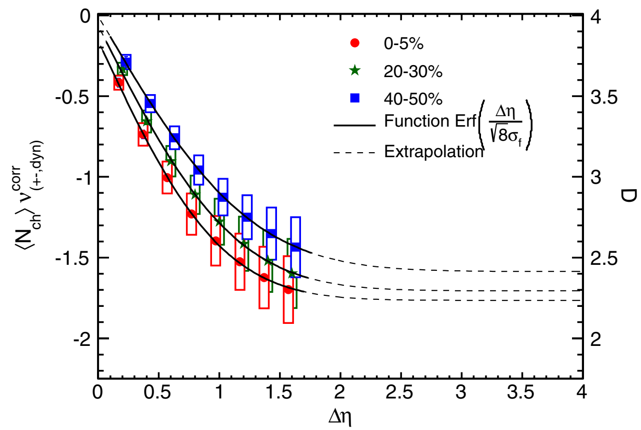
Correlations: Lattice vs Data

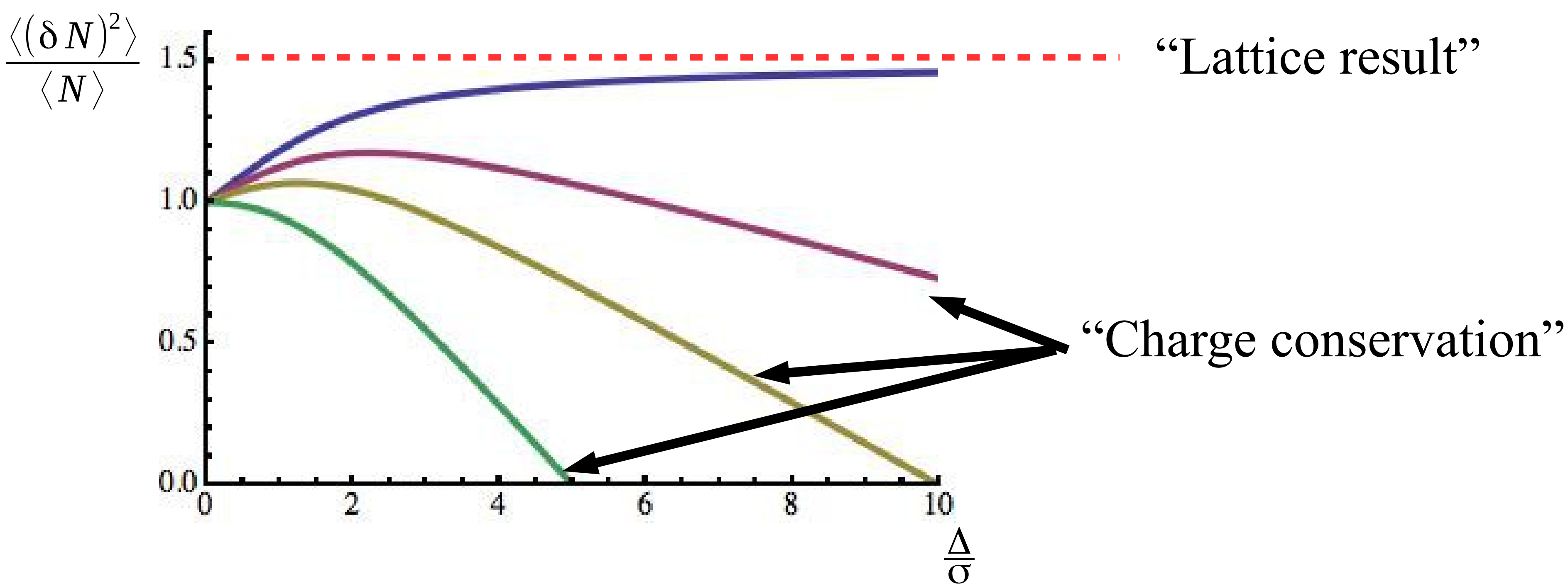
$$\langle n(y_1)(\,n(y_2)-\delta(y_1-y_2)\,)\rangle=\langle n(y_1)\rangle\langle n(y_2)\rangle\left(1+C(y_1,y_2)\right)$$

$$C(y_{1,}y_2)\sim\exp\left(\frac{-(y_1-y_2)^2}{2\sigma^2}\right)$$

$$\frac{\langle(\delta N)^2\rangle}{\langle N\rangle}=1+\langle N\rangle\int_{\Delta/2}^{\Delta/2}C(y1,y2)\,dy1\,dy_2$$

Alice Charge Flucts





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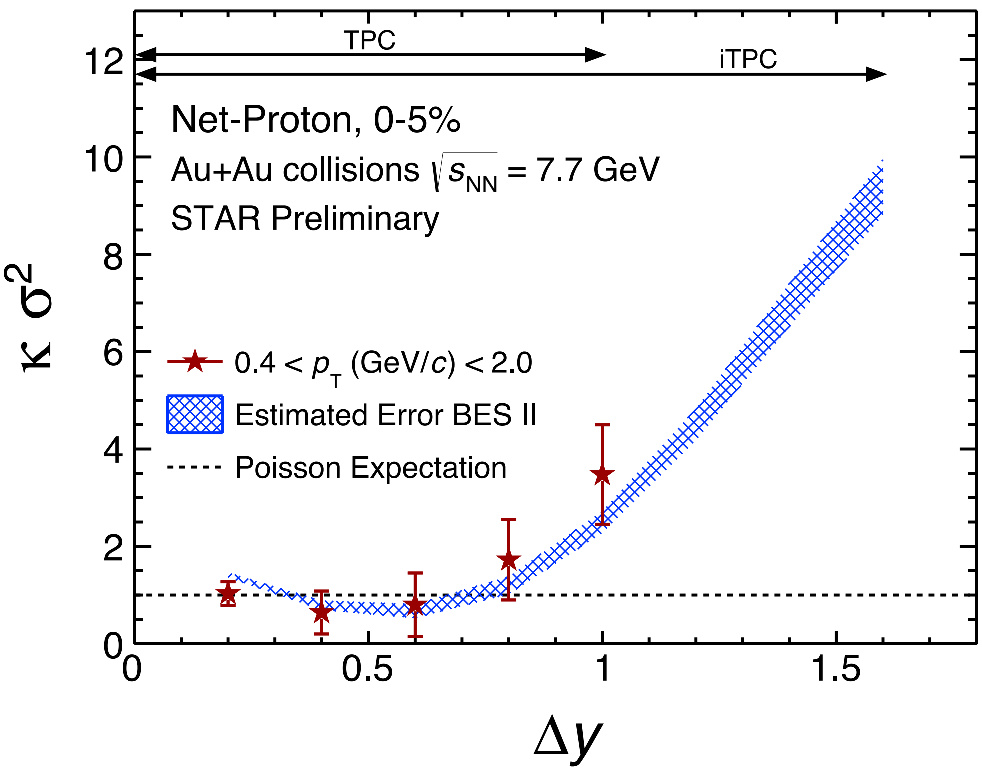
Dependence on Rapidity window

X. Luo, EMMI Workshop, Nov. 2015

- Kurtosis depends strongly on Rapidity window
- Comparison with Lattice:
  - Lattice catches the full correlation length
  - need to expand rapidity window until signal saturates

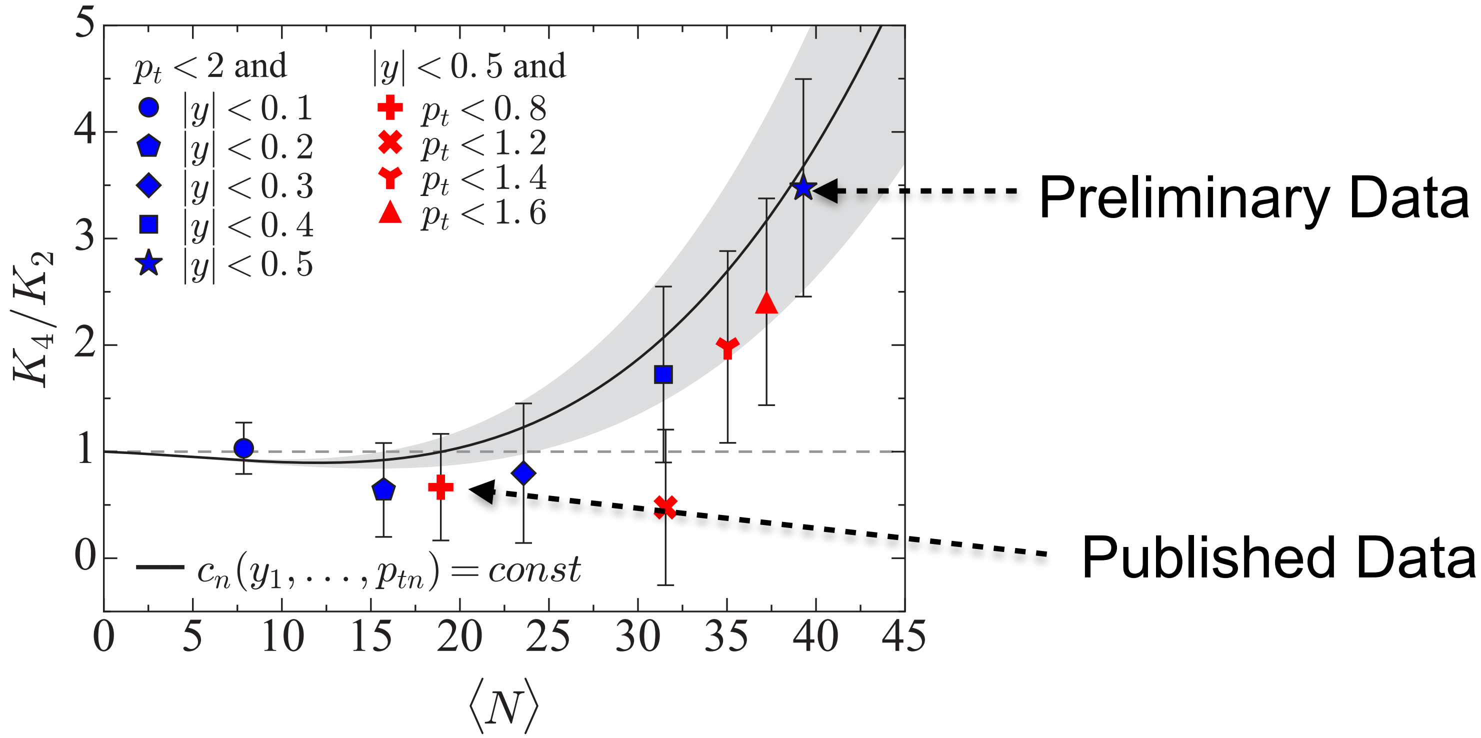


Any comparison of Lattice to Data needs to assure that cumulants reach asymptotic value in experiment.

So far this has NOT ben established for proton cumulants

Long range correlations

Large correlation length (σ_Y >> ΔY=1): K_n=K_n(<N>)



STAR data at 7.7 GeV consistent with σ_Y >> 1

Net-baryon multiplicity distribution

Utilize of cluster expansion model of Vovchenko et al arXiv:1711.01261

Virial expansion: $$\frac{P}{T^4} = \frac{1}{VT^3}\ln(Z) = \sum_{k=0}^{\infty} p_k(T)\cosh\left(k\hat{\mu}_B\right)$$

Cluster model: $$p_k = f(p_1, p_2);\ k > 2$$

Lattice QCD: p_1, p_2 Vovchenko et al, arXiv:1708.02852

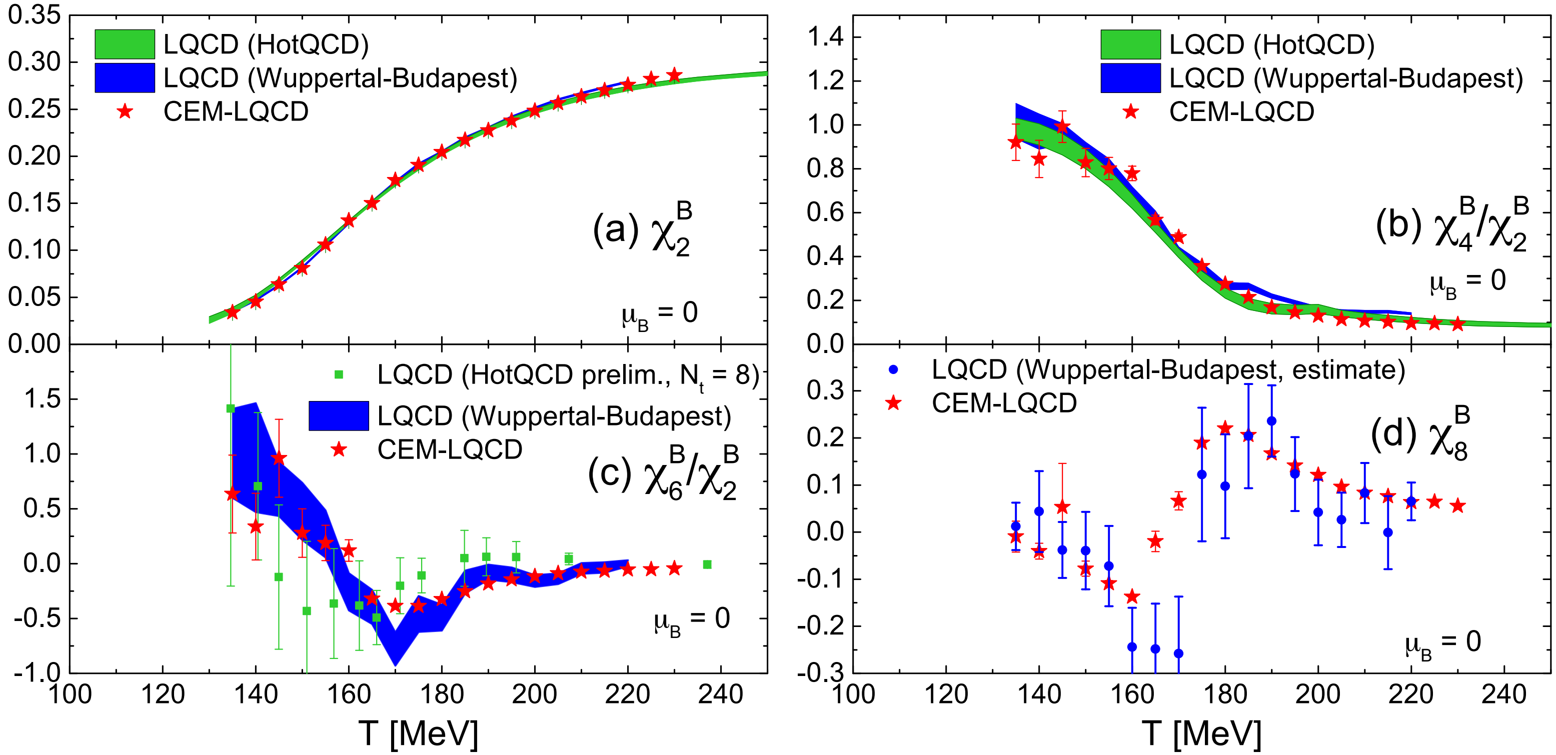


Figure from: arXiv:1807.06472

Net-baryon multiplicity distribution

Virial expansion:

$$\frac{P}{T^4} = \frac{1}{VT^3}\ln(Z) = \sum_{k=0}^{\infty} p_k(T)\cosh\left(k\hat{\mu}_B\right)$$

$$Z = \exp\left[VT^3 \sum_{k=0}^{\infty} p_k(T)\cosh(k\hat{\mu}_B)\right] = z_0 + 2\sum_{\mathcal{N}=1}^{\infty} z_{\mathcal{N}}\cosh(\mathcal{N}\hat{\mu}_B)$$

Multiplicity distribution:

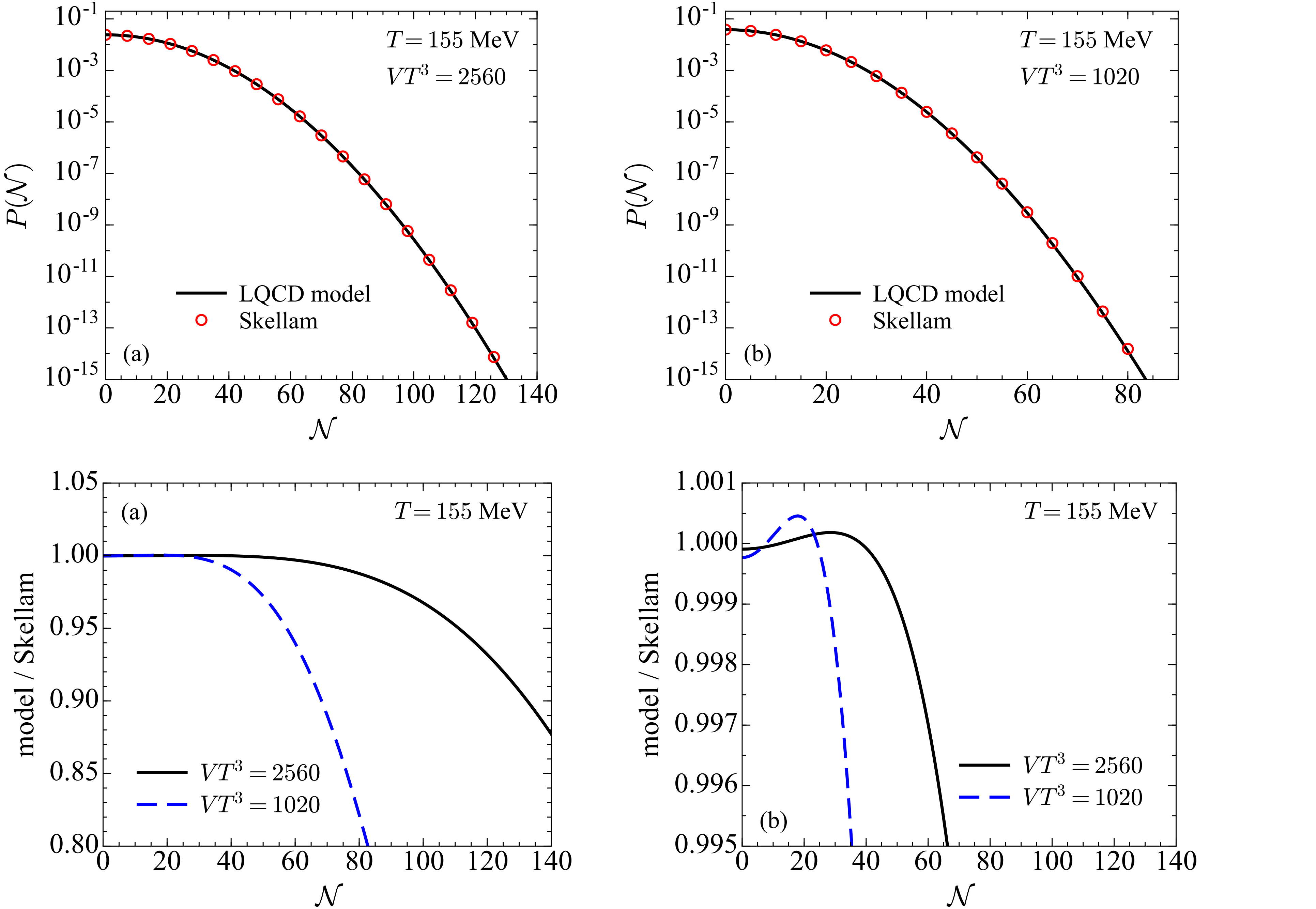
$$P(\mathcal{N}) = \frac{z_{\mathcal{N}} e^{\hat{\mu}_B \mathcal{N}}}{Z}$$

$$\hat{\mu}_B \to i\bar{\mu}_B$$

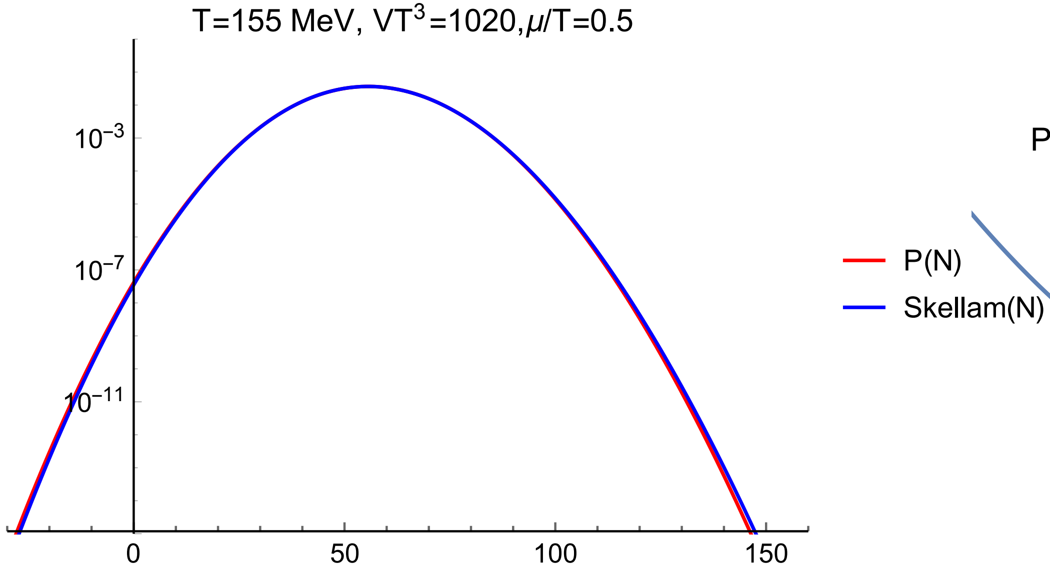
$$\boxed{P(\mathcal{N}) = \frac{1}{\pi}\int_0^{\pi} d\bar{\mu}_B \, \cos(\mathcal{N}\bar{\mu}_B)\, \frac{\exp\left[VT^3 \sum_{k=1}^{\infty} p_k(T)\cos\left(k\bar{\mu}_B\right)\right]}{\exp\left[VT^3 \sum_{k=1}^{\infty} p_k(T)\right]}}$$

$$\hat{\mu}_B = 0$$

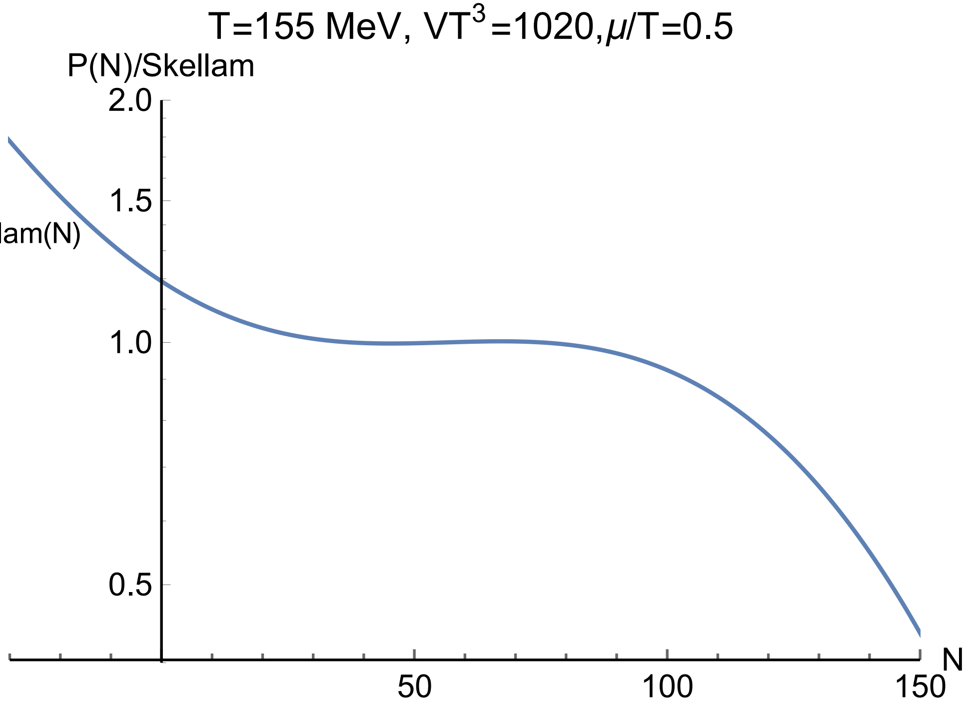
LHC　　　Top RHIC



$T = 155$ MeV
$VT^3 = 2560$
$P(\mathcal{N})$
LQCD model
Skellam
(a)
$\mathcal{N}$
$T = 155$ MeV
$VT^3 = 1020$
$P(\mathcal{N})$
LQCD model
Skellam
(b)
$\mathcal{N}$
(a)
$T = 155$ MeV
model / Skellam
$VT^3 = 2560$
$VT^3 = 1020$
$\mathcal{N}$
$T = 155$ MeV
model / Skellam
$VT^3 = 2560$
$VT^3 = 1020$
(b)
$\mathcal{N}$



T=155 MeV, VT^3=1020,μ/T=0.5
10^{-3}
10^{-7}
10^{-11}
0
50
100
150
P(N)
Skellam(N)

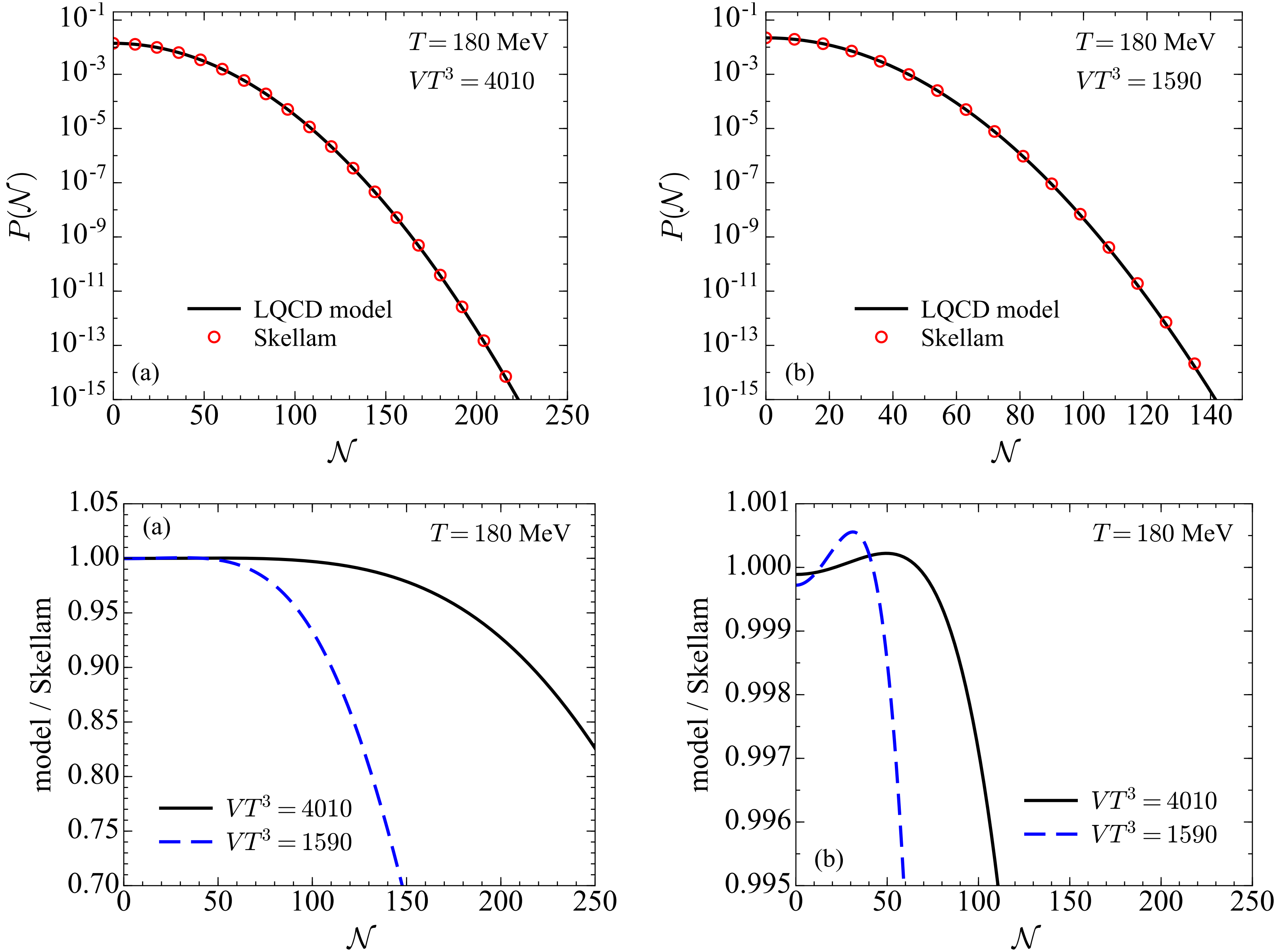


T=155 MeV, VT^3=1020,μ/T=0.5
P(N)/Skellam
2.0
1.5
1.0
0.5
N
50
100
150

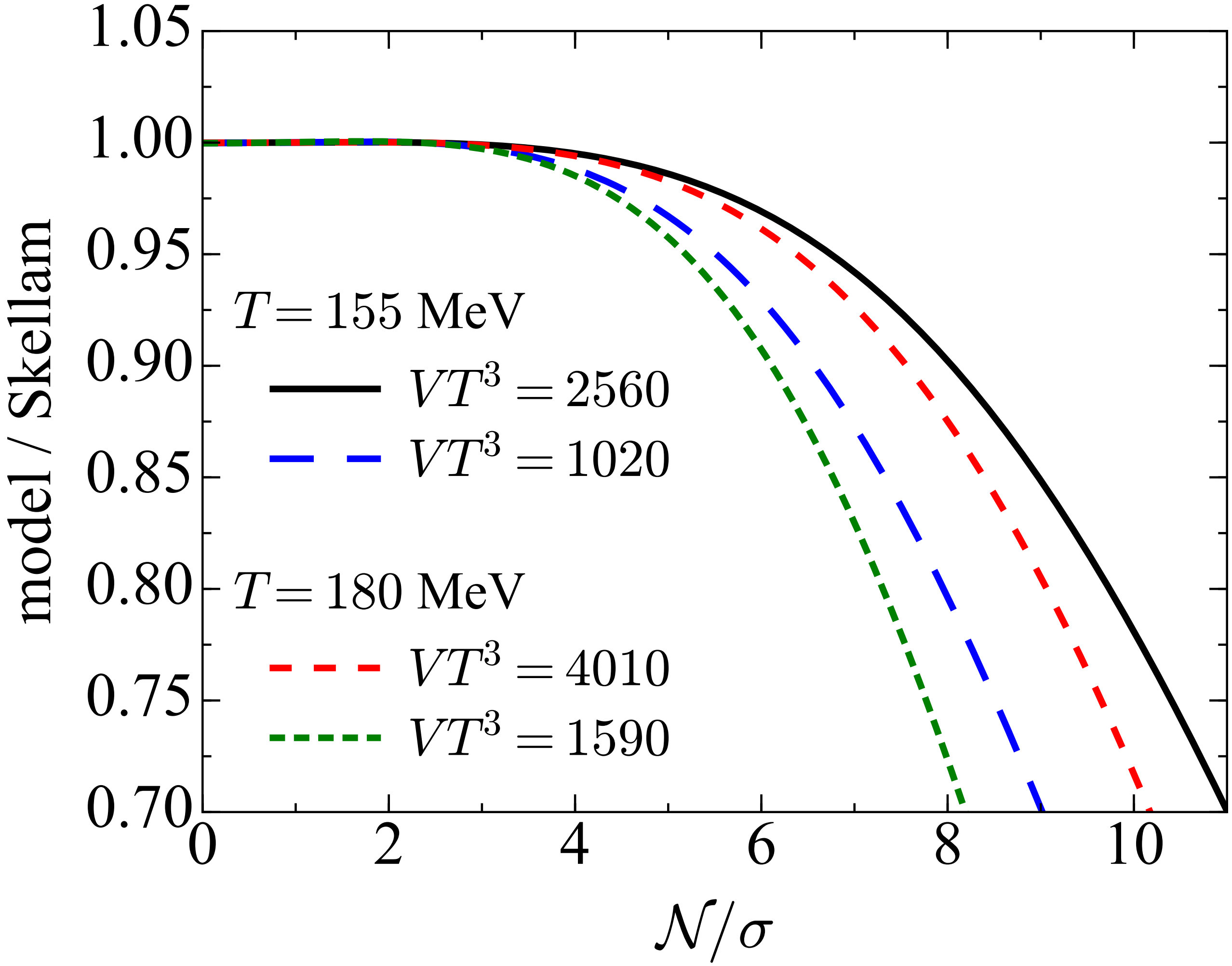
Summary

- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Cumulants contain information about correlations
- Comparison with Lattice require some care
- Net-baryon number distribution consistent with lattice
  - Deviation from Skellam is very small!
  - Measuring chiral criticality likely difficult

Thank You



$T = 180$ MeV
$VT^3 = 4010$
$VT^3 = 1590$
LQCD model
Skellam
(a)
(b)
$P(\mathcal{N})$
$\mathcal{N}$
model / Skellam



1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
model / Skellam
0
2
4
6
8
10
$\mathcal{N}/\sigma$
$T = 155$ MeV
$VT^3 = 2560$
$VT^3 = 1020$
$T = 180$ MeV
$VT^3 = 4010$
$VT^3 = 1590$

Cumulants of (Baryon) Number

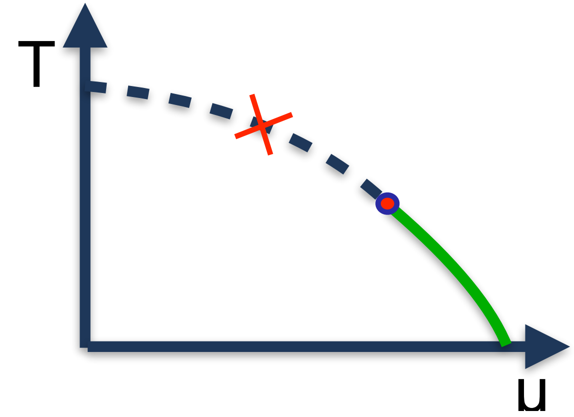
$$K_n = \frac{\partial^n}{\partial(\mu/T)^n} \ln Z = \frac{\partial^{n-1}}{\partial(\mu/T)^{n-1}} \langle N \rangle$$

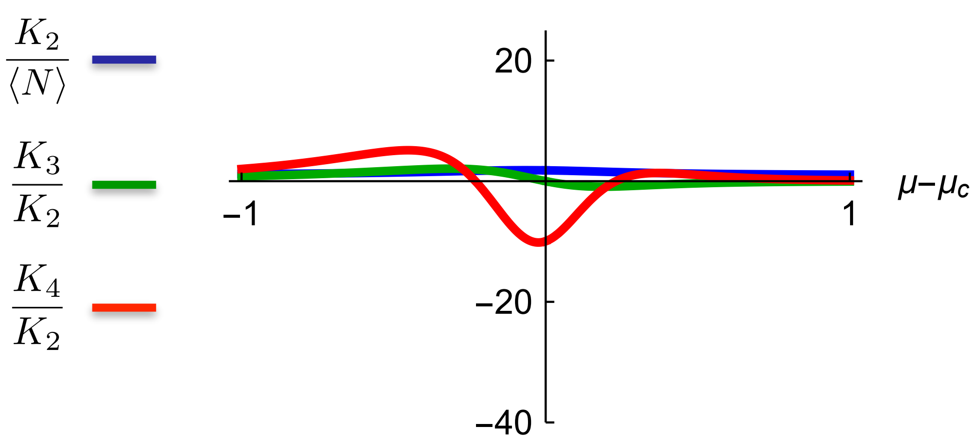
$$K_1 = \langle N \rangle \,, \;\; K_2 = \langle N - \langle N \rangle \rangle^2 \,, \;\; K_3 = \langle N - \langle N \rangle \rangle^3$$

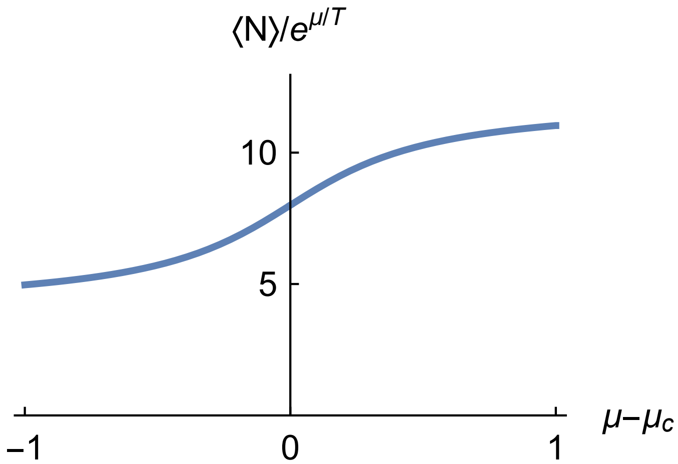
Cumulants scale with volume (extensive): $K_n \sim V$

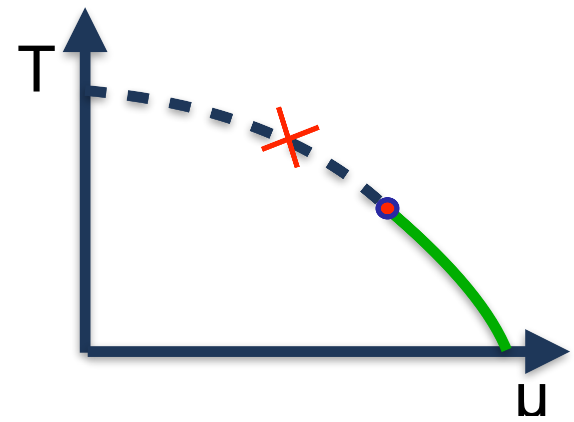
Volume not well controlled in heavy ion collisions

Cumulant Ratios: $\frac{K_2}{\langle N \rangle}, \;\; \frac{K_3}{K_2}, \;\; \frac{K_4}{K_2}$

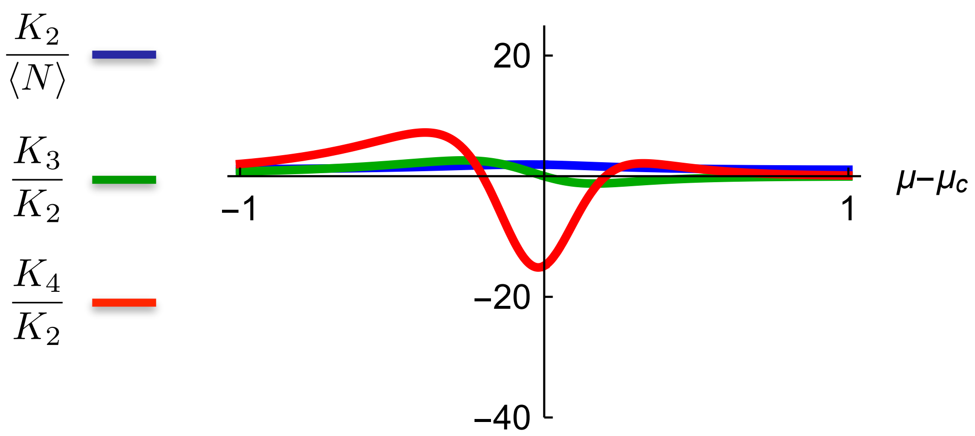

T
u


$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
μ−μc

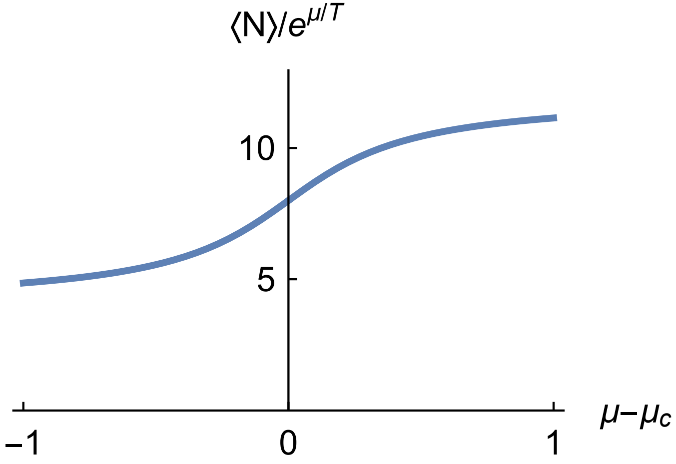

⟨N⟩/e^{μ/T}
10
5
−1
0
1
μ−μc



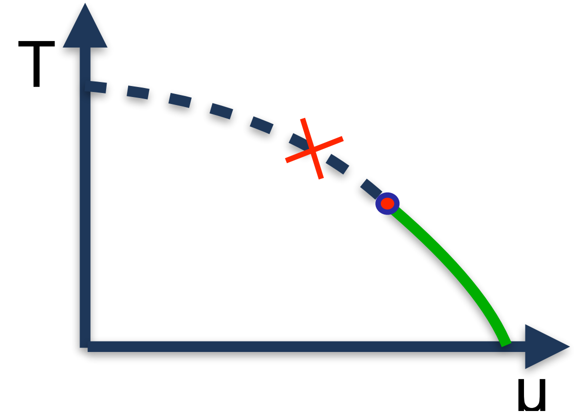
T
u

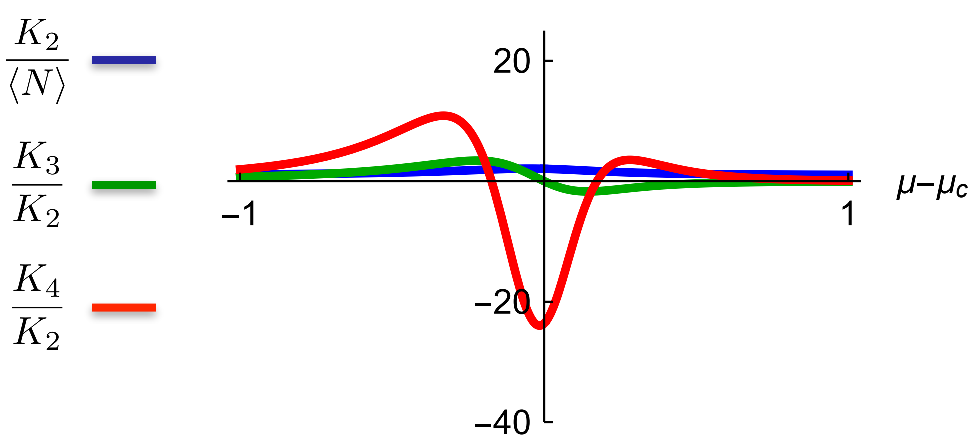


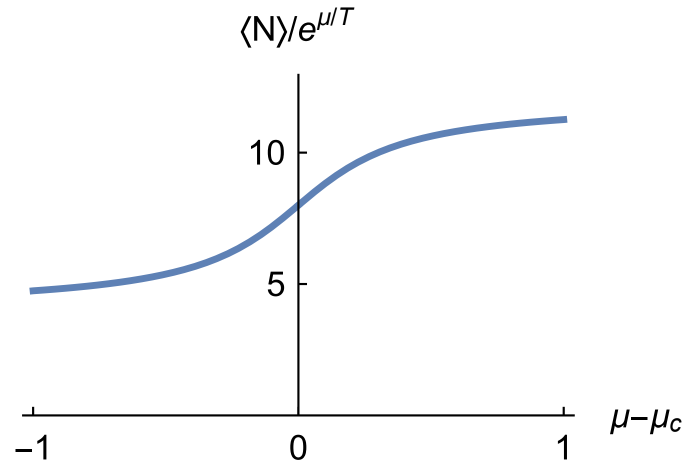
$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
$\mu-\mu_c$

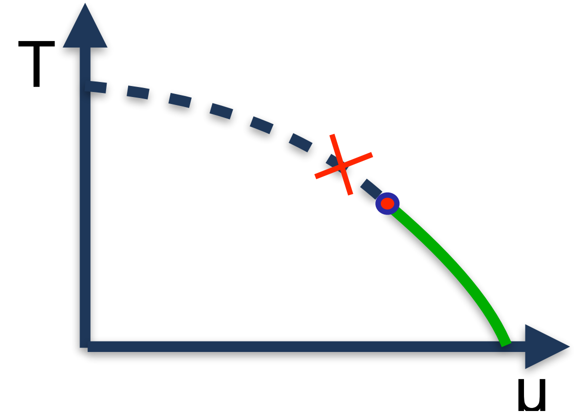


$\langle N \rangle / e^{\mu/T}$
10
5
−1
0
1
$\mu-\mu_c$

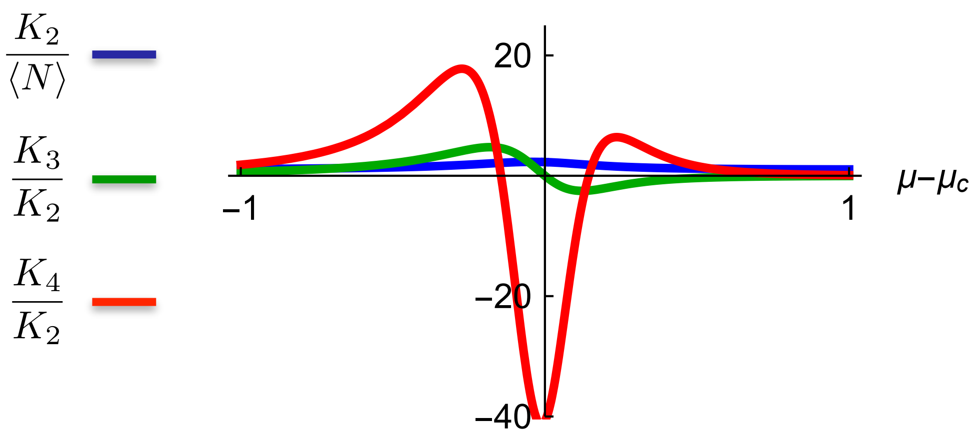

T
u


$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
$\mu - \mu_c$

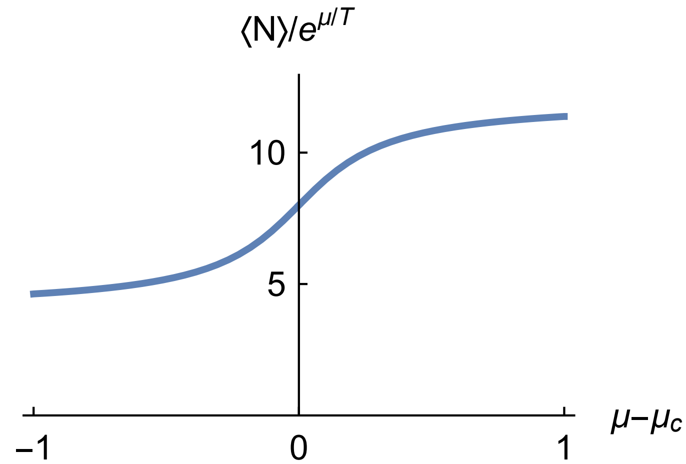

$\langle N \rangle / e^{\mu/T}$
10
5
−1
0
1
$\mu - \mu_c$



T
μ

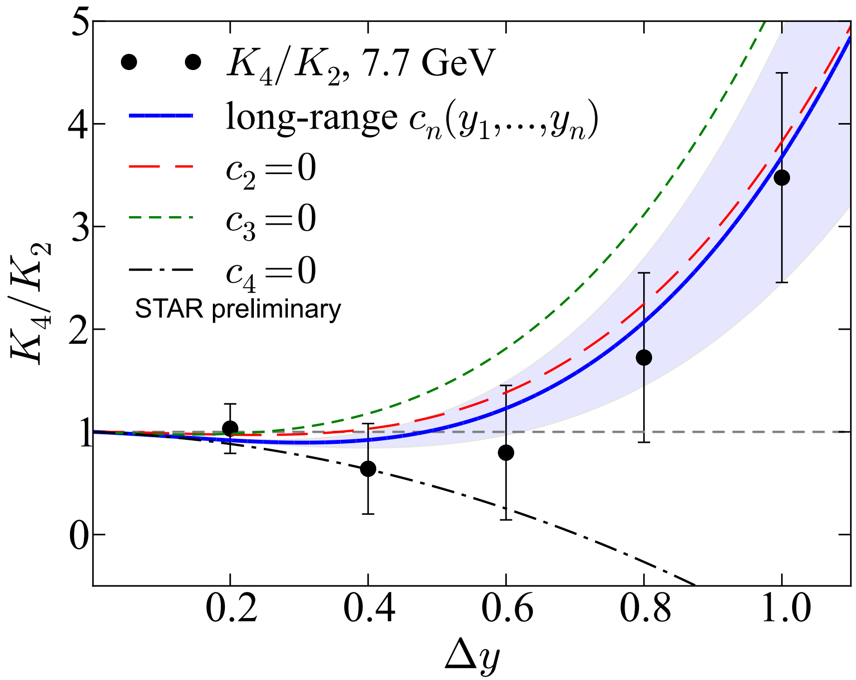


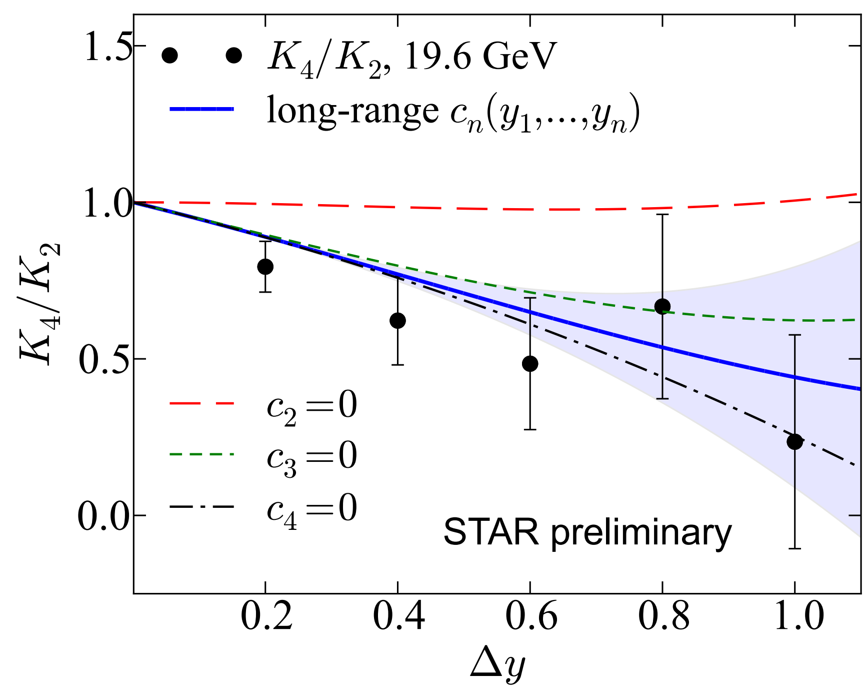
$\frac{K_2}{\langle N \rangle}$
$\frac{K_3}{K_2}$
$\frac{K_4}{K_2}$
20
−20
−40
−1
1
μ−μc



⟨N⟩/$e^{\mu/T}$
10
5
−1
0
1
μ−μc

Preliminary Star data are consistent with long range correlations





7.7 GeV
central

19.6 GeV
central