Fluctuation and the QCD phase diagram

Searching for the QCD phase transition with statistics friendly distributions

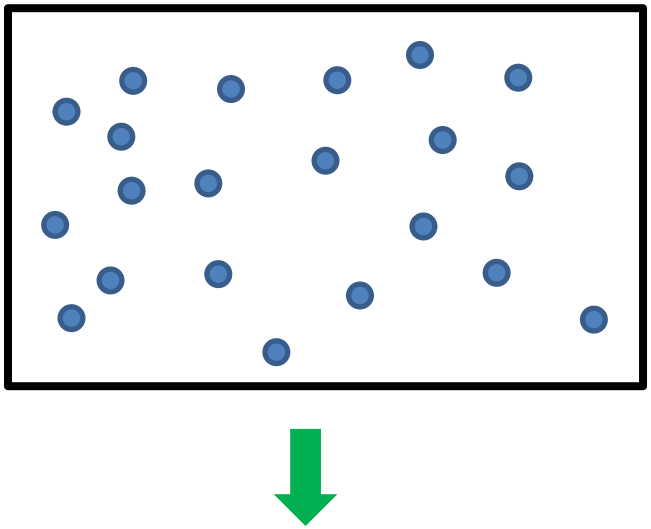
Adam Bzdak

AGH University of Science and Technology, Kraków

Outline

- correlation, interaction
- factorial cumulants, cumulants
- STAR data
- two event classes
- statistics hungry and statistics friendly distributions
- summary

Poisson distribution



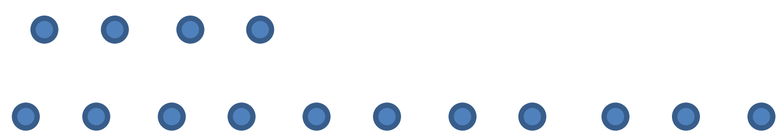
$$N = 10^{10}$$

$$p = 10^{-9}$$

$$\langle n \rangle = Np = 10$$

event # 1

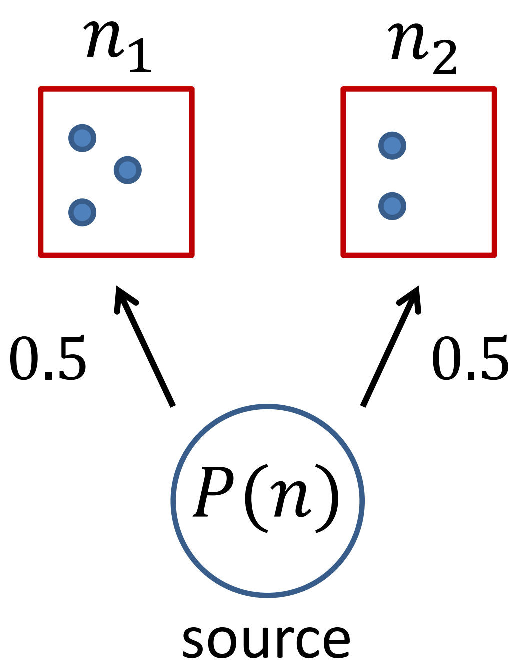
event # 2



$$P(n) = \text{Poisson if } N \to \infty,\ p \to 0,\ \ Np = \langle n \rangle$$

Such source (multiplicity distribution) is characterized by
All **factorial cumulants** $\boldsymbol{C_n = 0}$, $n = 2,3, \ldots$ (“no correlations”)

In what sense “no correlations”?



$$P(n_1, n_2) \stackrel{?}{=} P(n_1)P(n_2)$$

It is true for $P(n)$ = Poisson only

fixed N

finite N

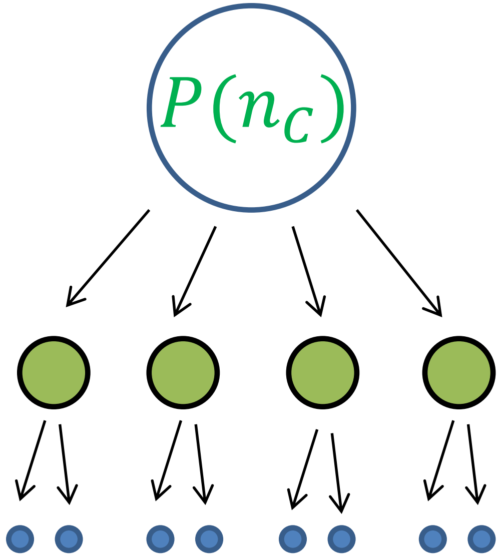
resonances

volume fluctuation

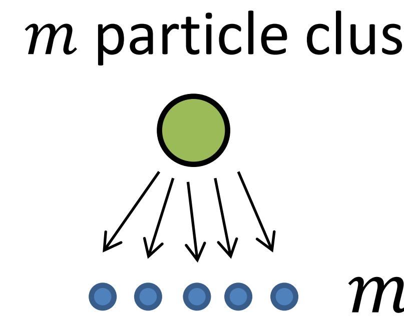
$$P(n_1, n_2) = P(n)\frac{n!}{n_1!\, n_2!}\left(\frac{1}{2}\right)^{n_1}\left(\frac{1}{2}\right)^{n_2}$$

$$n = n_1 + n_2$$

Multiparticle correlations - **factorial cumulants**



Poisson



$$\boldsymbol{C_2} \neq 0$$

$$\boldsymbol{C_k} = 0, k > 2$$

$$\boldsymbol{C_{2,3,\ldots,m}} \neq 0$$

$$\boldsymbol{C_k} = 0, k > m$$

factorial cumulants

$$\boldsymbol{C_k} = \frac{d^k}{dz^k} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n-1)\rangle = \langle n\rangle^2 + \boldsymbol{C_2}$$

$$\langle n(n-1)\rangle = \int \rho_2(y_1, y_2)dy_1dy_2$$

$$\langle n\rangle = \int \rho(y)dy$$

factorial cumulant (integrated correlation function)

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2)dy_1dy_2$$

Interaction

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2$$

factorial cumulant (integrated correlation function)

For Poisson $\boldsymbol{C_2} = 0$ but $\boldsymbol{C_2}(y_1, y_2)$ can have a non-trivial shape due to, e.g., interactions

For example (elliptic flow):

$$\boldsymbol{C_2}(\phi_1, \phi_2) \sim \cos(2\Delta\phi), \quad \Delta\phi = \phi_1 - \phi_2$$

Factorial cumulants vs cumulants

factorial cumulant

$$\boldsymbol{C_i} = \frac{d^i}{dz^i} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

cumulant

$$K_{\boldsymbol{i}} = \frac{d^i}{dt^i} \ln\left(\sum_n P(n) e^{tn}\right)\Big|_{t=0}$$

cumulants naturally appear in statistical physics

$$Z = \sum_i e^{-\beta(E_i - \mu N_i)}$$

Poisson:

$C_i = 0$

$K_i = \langle n \rangle$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix integ. correlation functions of different orders

$$K_5 = \langle N \rangle + 15C_2 + 25C_3 + 10C_4 + C_5$$

$$K_6 = \langle N \rangle + 31C_2 + 90C_3 + 65C_4 + 15C_5 + C_6$$

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2) dy_1 dy_2$$

factorial cumulant

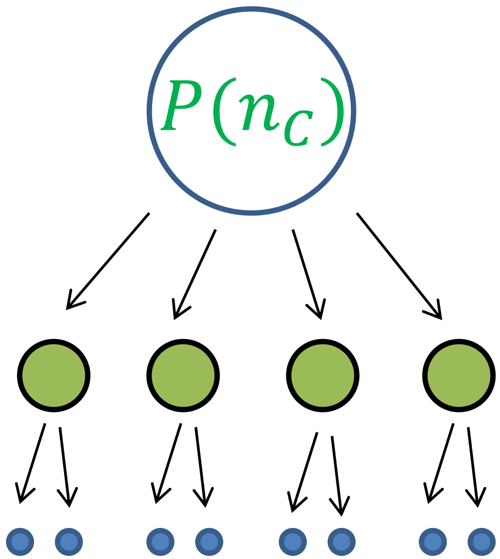
See, e.g.,

B. Ling, M. Stephanov, PRC 93 (2016) 034915

AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906

AB, V.Koch, D.Oliinychenko, J.Steinheimer, 1804.04463 (K_5 and K_6)

Suppose we have a system with two-particle clusters only



Poisson

In this case all information is contained in $\langle n \rangle$ and K_2. No point to measure $K_{3,4,\ldots}$

$$\boldsymbol{C_2} = 2\langle n_C \rangle \qquad \boldsymbol{C_{3,4,\ldots}} = 0$$

$$K_i = 2^i \langle n_C \rangle$$

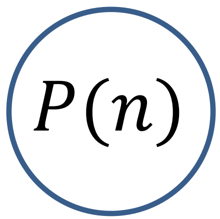
and for example: $\dfrac{K_4}{K_2} = 4$

looks nontrivial but no new information

So are factorial cumulants “easy”?

Factorial cumulants measure deviations from Poisson

Consider a source giving always **one particle**



$$P(n) = 1 \quad \text{for} \quad n = 1$$
$$\quad\;\; = 0 \quad \text{for} \quad n > 1$$

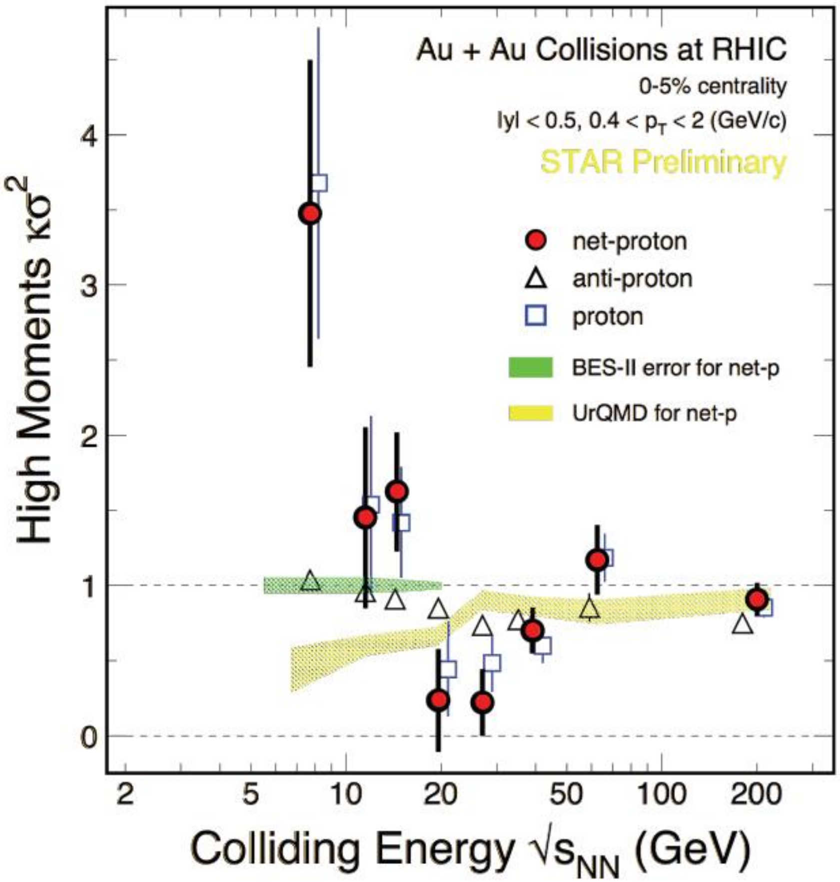
$$\boldsymbol{C_k} = \frac{d^k}{dz^k} \ln(z)\Big|_{z=1}$$

$$\boldsymbol{C_2} = -1,\ \boldsymbol{C_3} = 2,\ \boldsymbol{C_4} = -6, \dots,\ \boldsymbol{C_9} = 40320$$

$$\boldsymbol{C_k} = (-1)^{k-1}(k-1)!$$

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

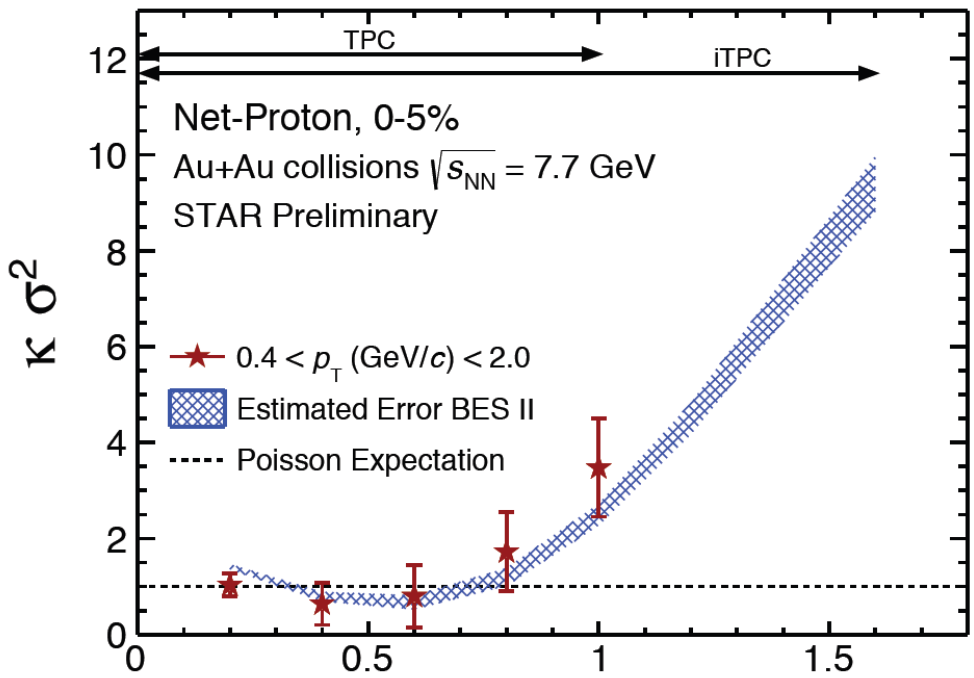
$$K_4/K_2$$

Is proton signal at 7.7 GeV large?

Is physics changing between 7 and 19 GeV?

Preliminary STAR data at 7.7 GeV

X.Luo, N.Xu, 1701.02105



Δy (rapidity window size)

"Poissonizer" ?
(V.Koch)

$$-(\Delta y)/2 < y < (\Delta y)/2$$

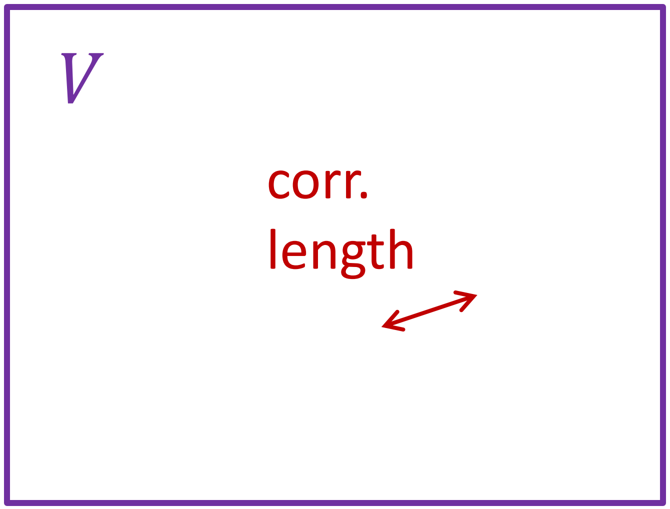
Is this dependence expected?
Is it somehow related to the QCD phase diagram?

General remarks:

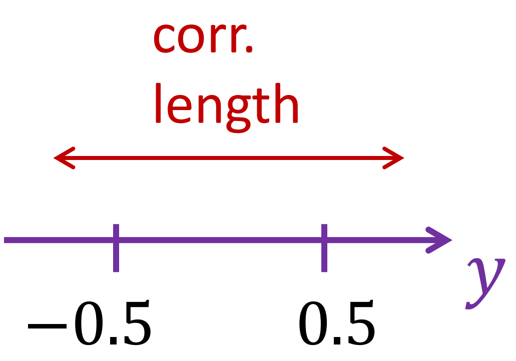
"Cumulant ratios do not depend on volume" but depend on volume fluctuation

It is true if a correlation length is much smaller than the system size

real coordinate space



momentum rapidity space



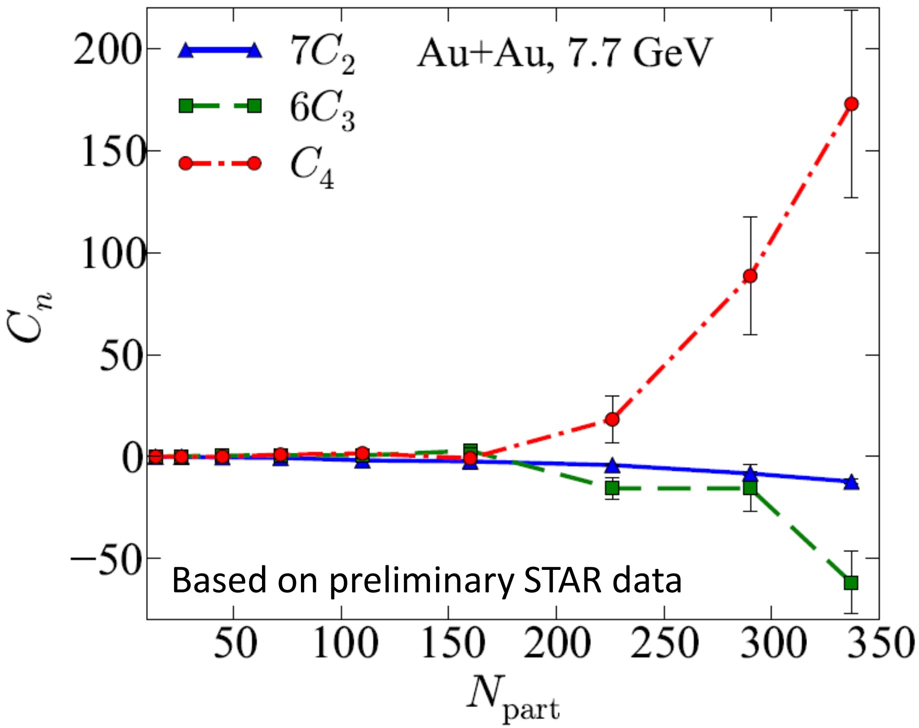
Here this condition is satisfied

Correlation length is usually larger than one unit of rapidity.

Cumulant ratios are expected to depend on acceptance in rapidity

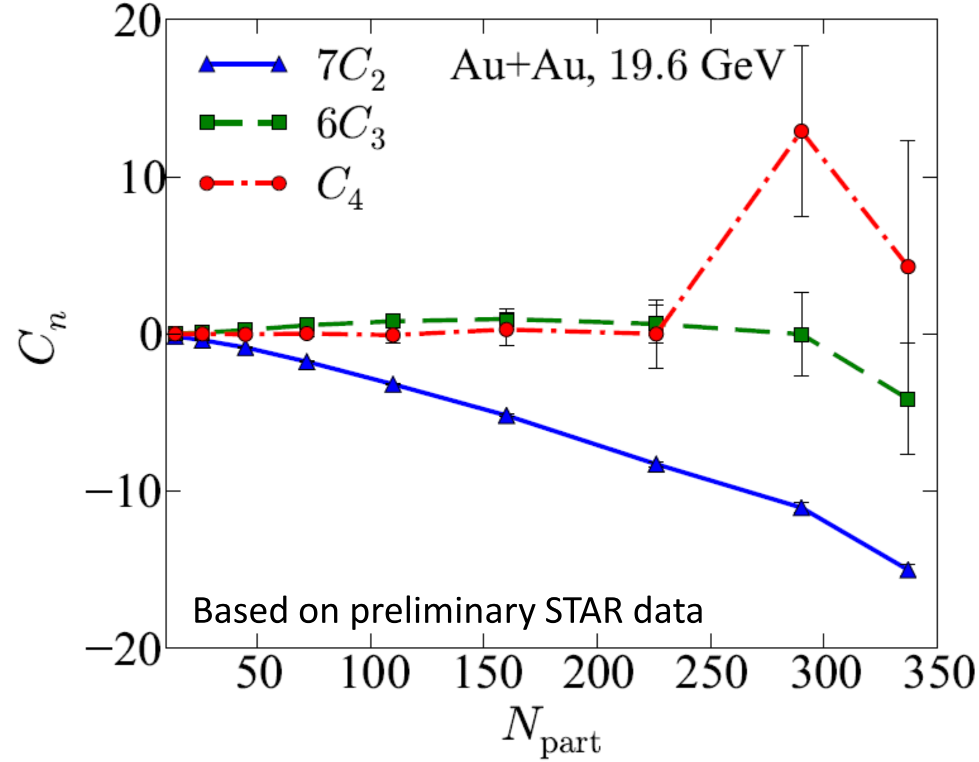
Using preliminary STAR data we obtain C_n

central signal at $\mathbf{7.7}$ GeV is driven by large 4-particle correlations



$$C_4(7.7) \sim \mathbf{170}$$

central signal at $\mathbf{19.6}$ GeV is driven by 2-particle correlations



$\boldsymbol{C_4}$ and $\boldsymbol{6C_3}$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

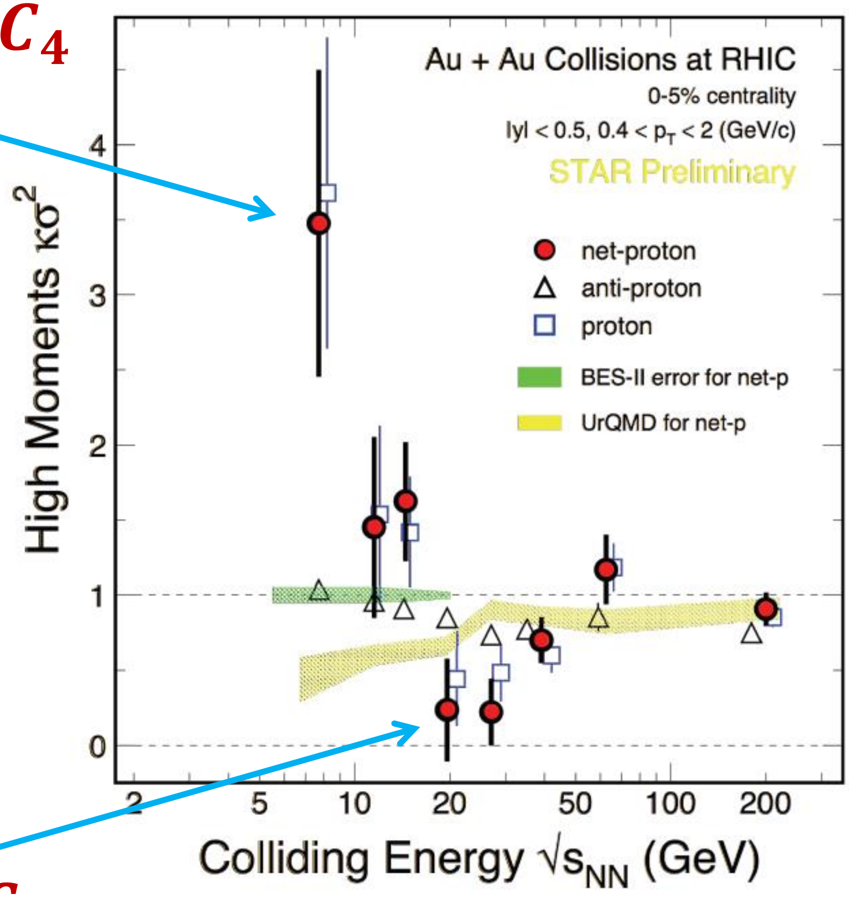
STAR preliminary
X.Luo, N.Xu, 1701.02105

here we see C_4

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$



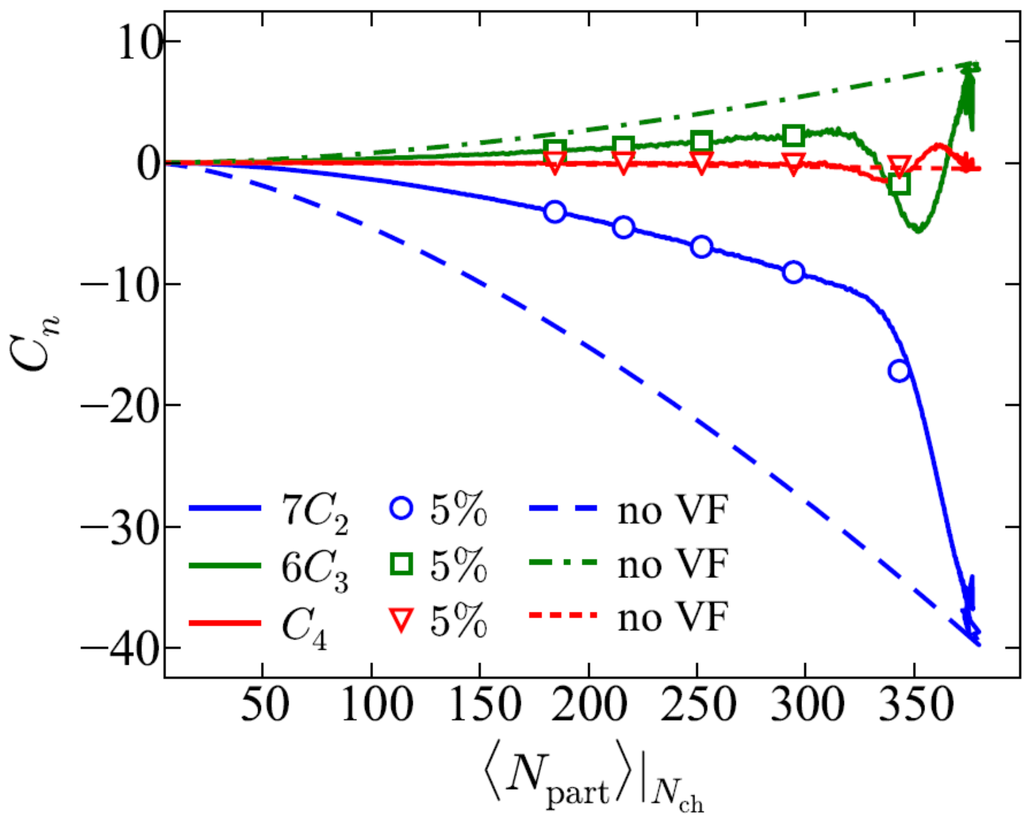
and here C_2

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

Baryon conservation + volume fluctuation (minimal model)

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114

Volume fluctuation + baryon conservation seems to be important for C_2 but irrelevant for C_3 and C_4 (7.7 GeV).

C_4 observed by STAR is larger by almost **three orders of magnitude** than the minimal model.

To explain C_4 we need a strong source of multi-proton correlations.

Let's put the STAR numbers in perspective.

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

Suppose that we have **clusters** (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 4!/(4-k)!$$

↑ mean number of clusters

$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 24$$

for 5-proton clusters:

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 5!/(5-k)!$$

$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 120$$

and $\langle N_{\mathrm{cl}} \rangle \sim 1$

To obtain $\boldsymbol{C_4} \approx 170$ we need $\langle N_{\mathrm{cl}} \rangle \sim 7,$ it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

Can we describe the STAR data at 7.7 GeV with *ordinary* multiplicity distributions?

Model with **two event classes**

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

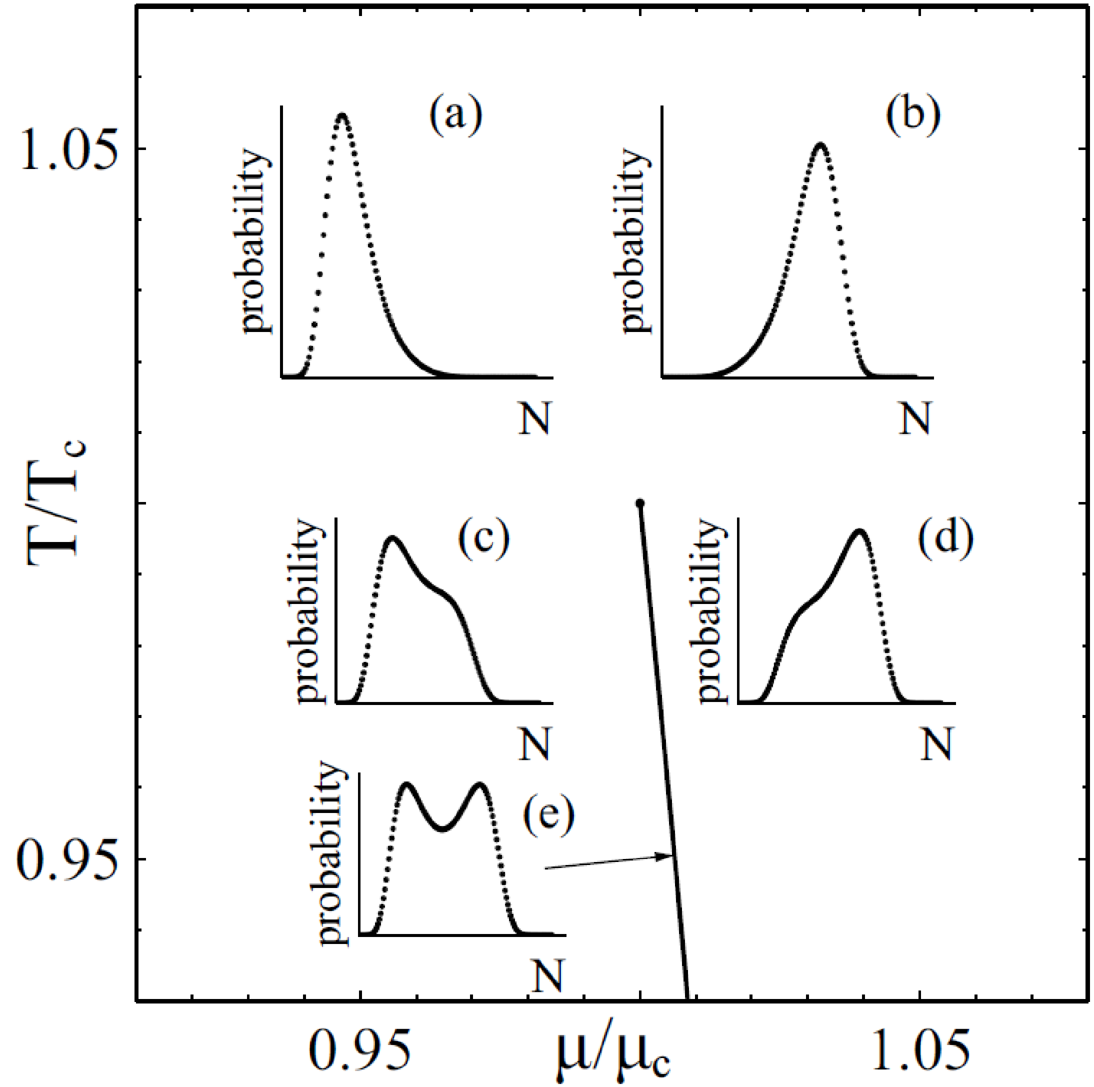
Poisson, binomial, etc..

Poisson, binomial, etc.

That is, with probability $1-\alpha$ we have $P_{(a)}(N)$ and with probability α we have $P_{(b)}(N)$

AB, V. Koch, D. Oliinychenko, J. Steinheimer, 1804.04463

A finite volume van der Walls model



AB, V. Koch, D. Oliinychenko, J. Steinheimer, 1804.04463

$$C_2 = \alpha(1-\alpha)\overline{N}^2 \approx \alpha\overline{N}^2,$$

$$C_3 = -\alpha(1-\alpha)(1-2\alpha)\overline{N}^3 \approx -\alpha\overline{N}^3,$$

$$C_4 = \alpha(1-\alpha)(1-6\alpha+6\alpha^2)\overline{N}^4 \approx \alpha\overline{N}^4,$$

$$\overline{N} = \langle N_{(a)}\rangle - \langle N_{(b)}\rangle$$

$$\frac{C_6}{C_5} \approx \frac{C_5}{C_4} \approx \frac{C_4}{C_3} = -17 \pm 6$$

parameter-free prediction at 7.7 GeV ($\alpha \ll 1$)

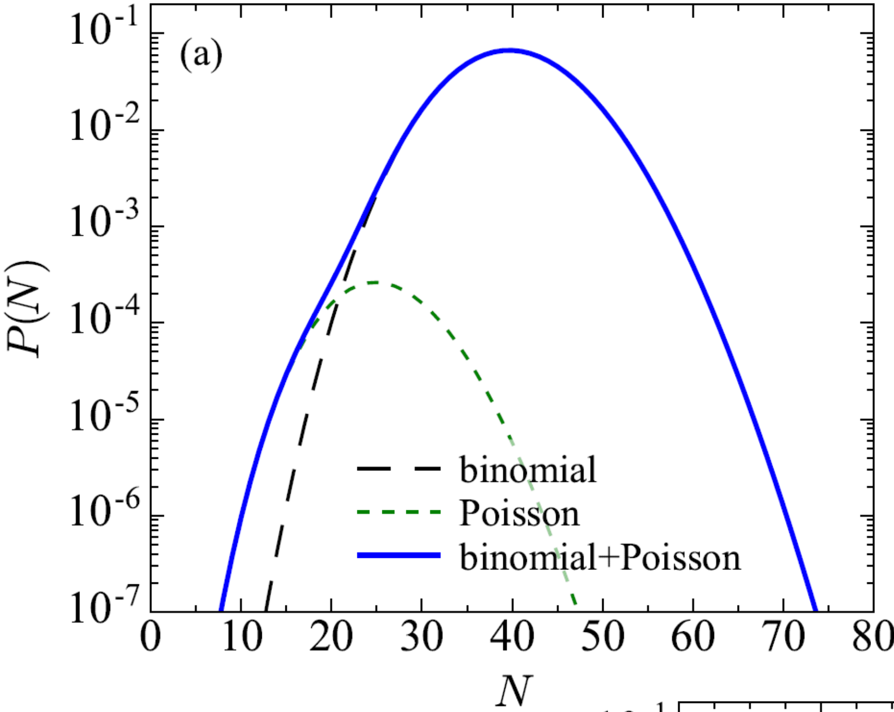
$$C_5 \approx -2645,$$
$$C_6 \approx 40900,$$

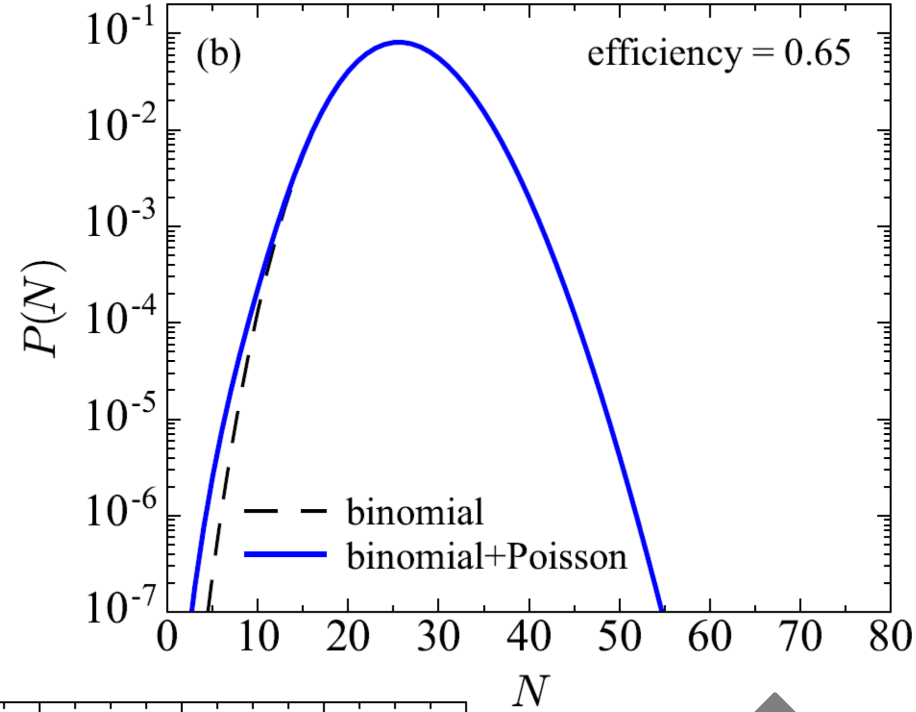
assuming $C_4 = 170$

We can describe the data with $\alpha \approx 0.0033$

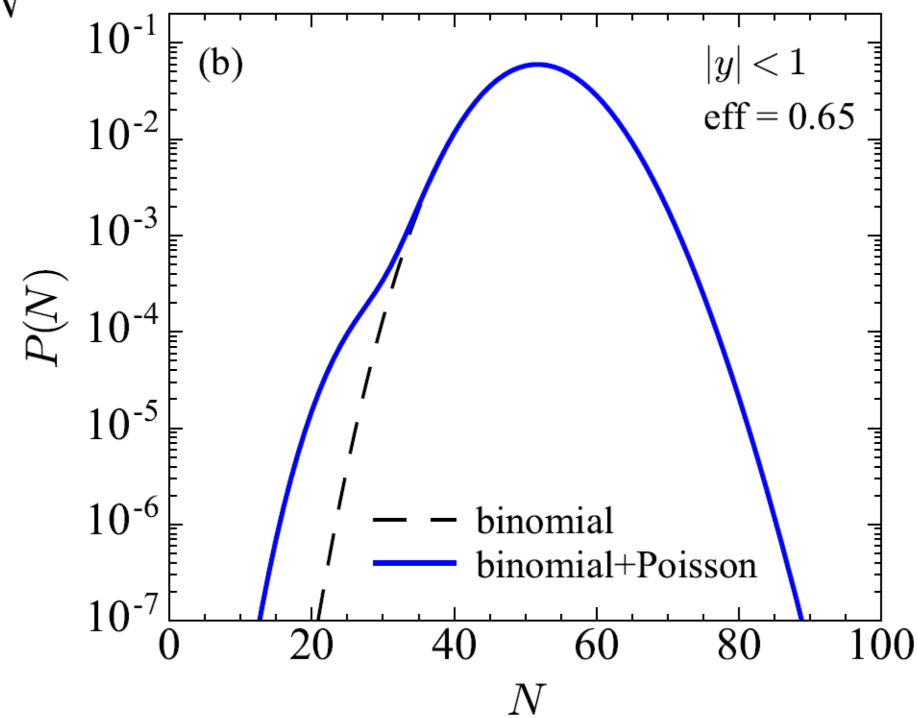
$$\langle N_{(a)}\rangle \approx 40,\ \langle N_{(b)}\rangle \approx 25$$

Now we can plot $P(N)$

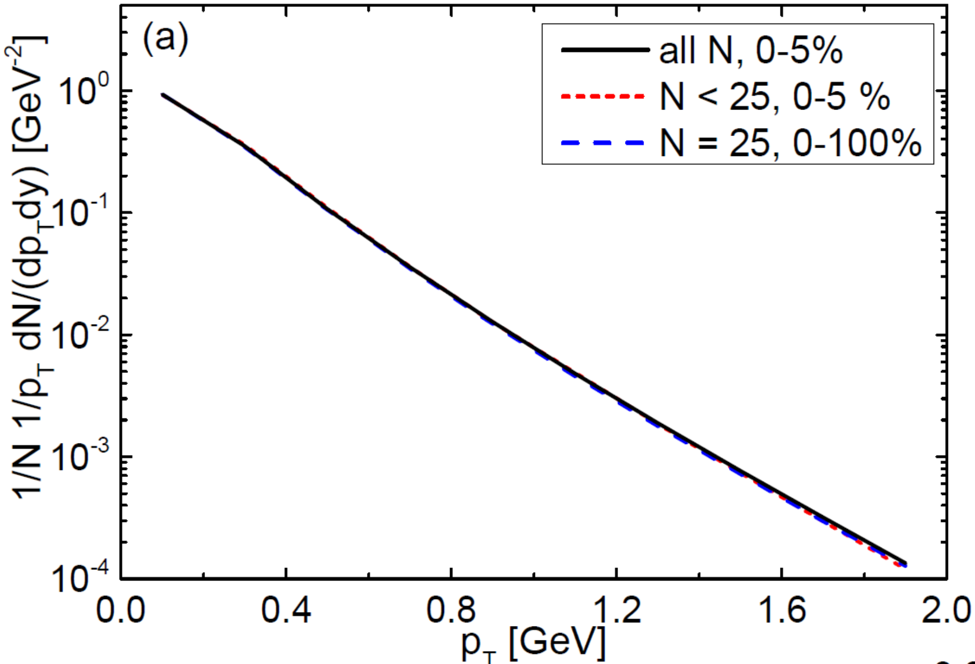

(a)
binomial
Poisson
binomial+Poisson
P(N)
N


(b)
efficiency = 0.65
binomial
binomial+Poisson
P(N)
N

$|y| < 0.5$

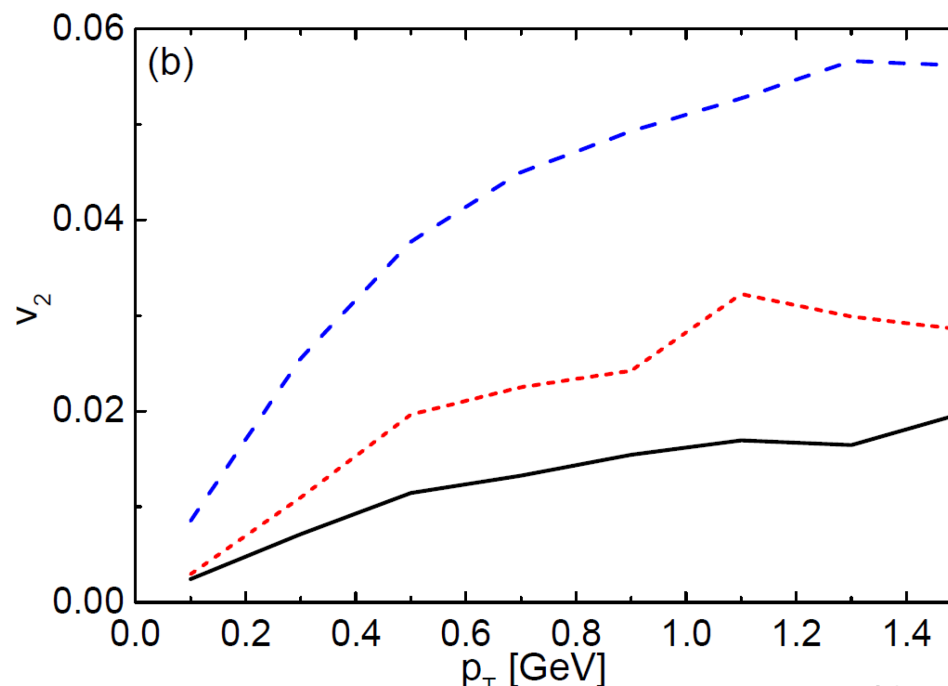

(b)
|y| < 1
eff = 0.65
binomial
binomial+Poisson
P(N)
N

$|y| < 1$



Cuts in the number of protons for central collisions

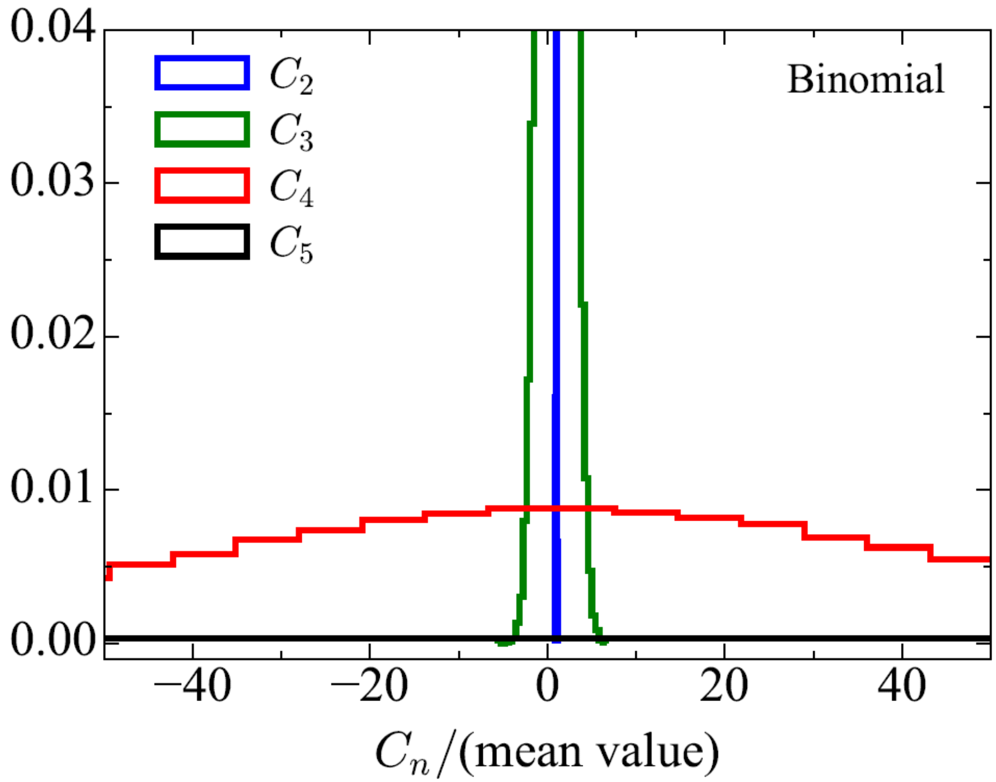
UrQMD



AB, V. Koch, D. Oliinychenko, J. Steinheimer, 1804.04463

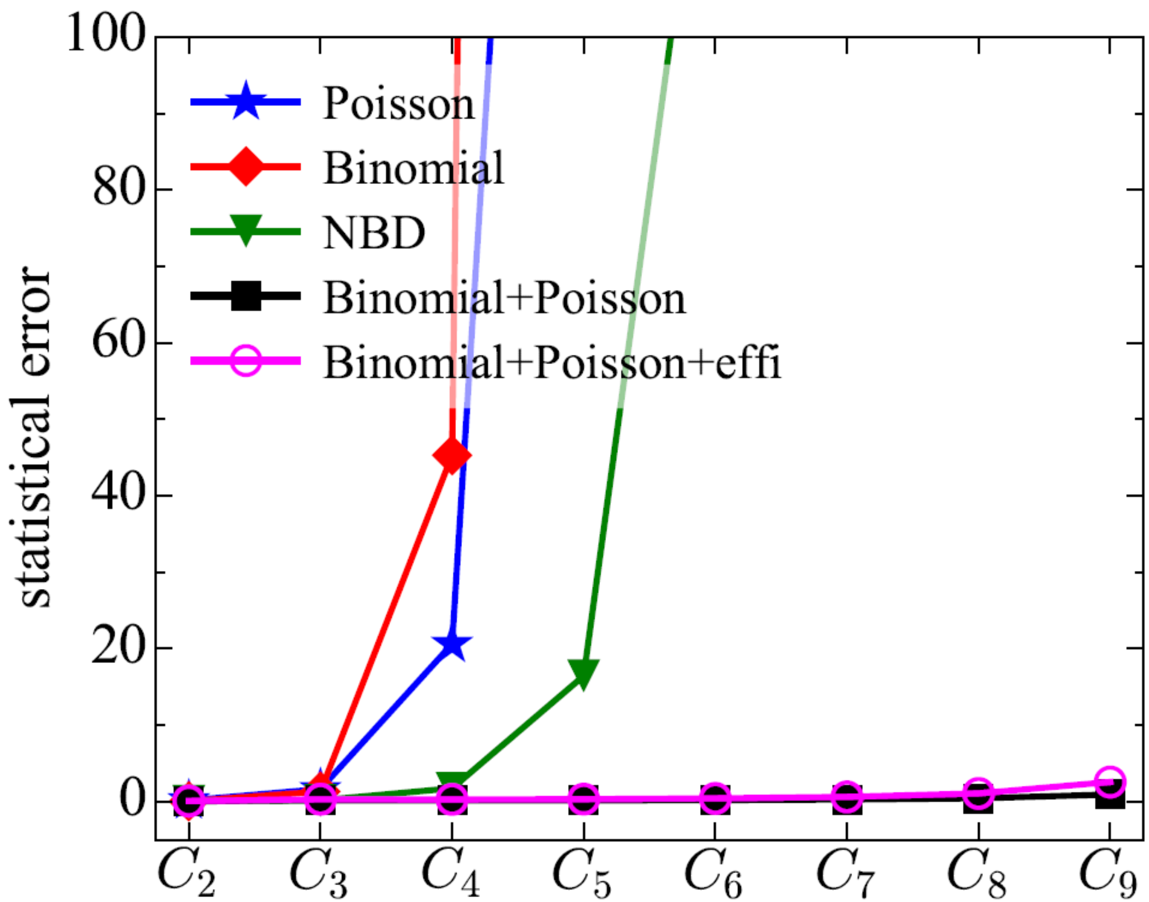
Can we verify the model (C_5 and C_6) with 144393 events available at STAR?

Statistics hungry distributions (SHD): binomial, Poisson, NBD,…



Histogram of C_n/(mean value) based on 144393 events.

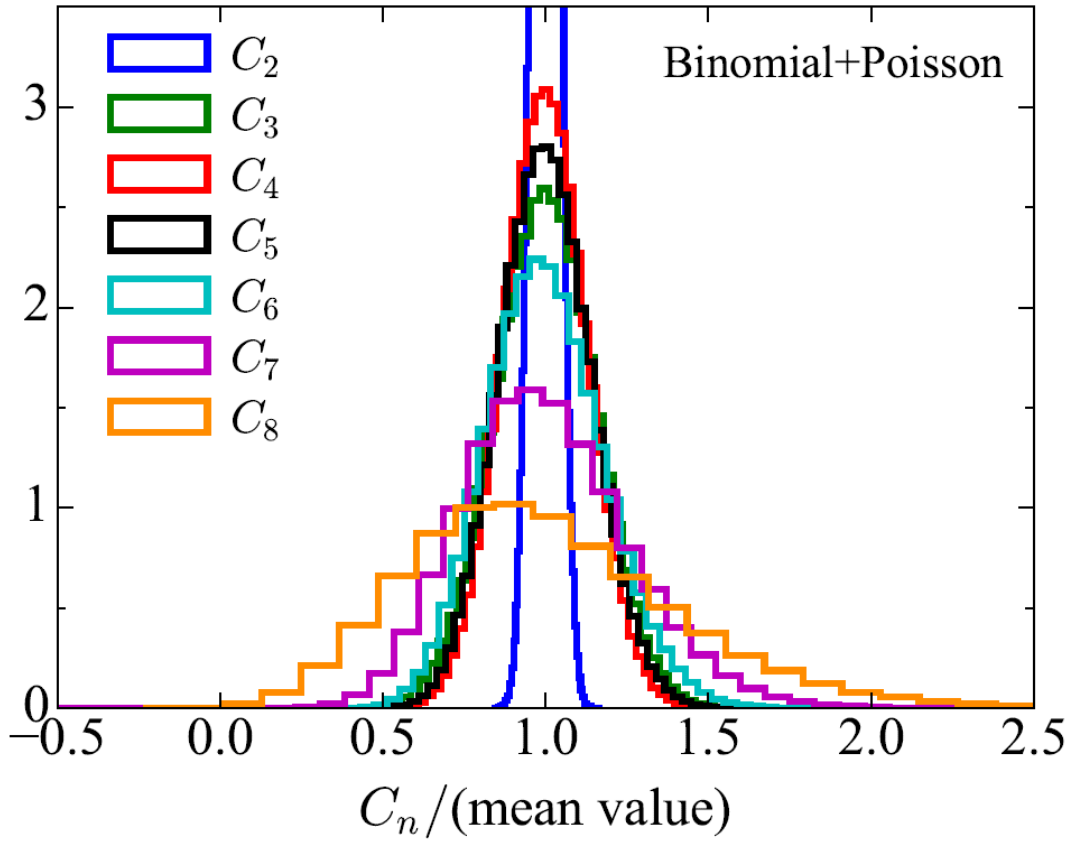
Are there **statistics friendly distributions** (SFD)? Yes.



based on 144393 events

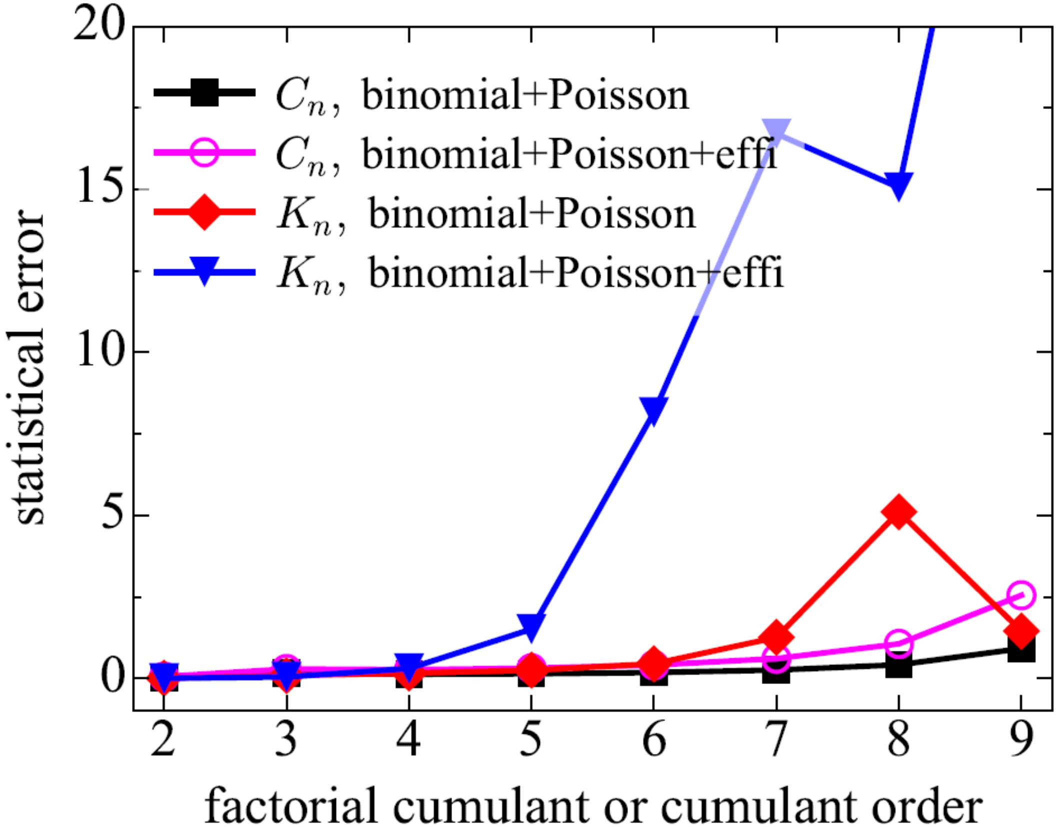
AB, V.Koch, in progress (to appear soon)

Histogram of C_n/(mean value) based on 144393 events.



Here I take $\langle N \rangle = 40$
It survives when hit with efficiency of 0.65

Regular cumulants are not as friendly



All of this can be understood

AB, V.Koch, in progress (to appear soon)

Our request to STAR.

Be brave and measure C_5, C_6, C_7, C_8 (**factorial cumulants**) and you should see:

$$C_5 \sim -2650$$

$$C_6 \sim +40900$$

$$C_7 \sim -615000$$

$$C_8 \sim +8520000$$

Plese remember that for Poisson $C_n = 0$

More details soon…

Conclusions:

Four-proton factorial cumulant (int. correlation function) at 7.7 GeV is surprisingly large. Three orders of magnitude larger than the minimal model.

Volume fluctuation and baryon conservation seem to be irrelevant for C_3 and C_4. C_2 (and K_2) is likely contaminated by background.

Proton clusters?

Two event classes? Bumpy structure of $P(N)$.
Parameter-free predictions.

Statistics friendly distributions close to the phase transition?

Backup

Genuine three-particle correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)\boldsymbol{C_2}(y_2, y_3) + \cdots$$

three possibilities

$$+ \boldsymbol{C_3}(y_1, y_2, y_3)$$

Integrating both sides

$$\langle n(n-1)(n-2)\rangle = \langle n\rangle^3 + 3\langle n\rangle \boldsymbol{C_2} + \boldsymbol{C_3}$$

factorial cumulant (integrated correlation function)

$$\boldsymbol{C_3} = \int \boldsymbol{C_3}(y_1, y_2, y_3)\, dy_1\, dy_2\, dy_3$$

and analogously for higher-order correlation functions

Cumulants are not optimal

$$K_2 = \langle(\delta N)^2\rangle \qquad \delta N = N - \langle N\rangle$$

N – number of protons

we neglect anti-protons, good at low energies

$$K_3 = \langle(\delta N)^3\rangle$$

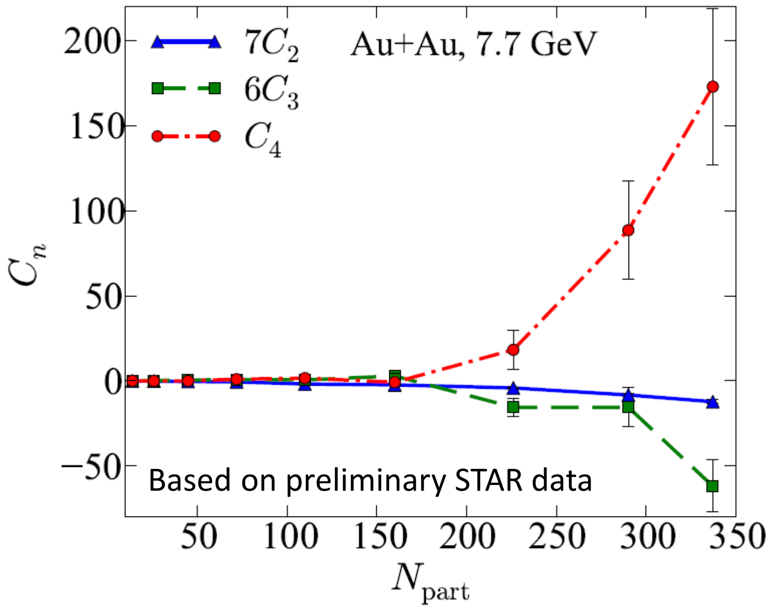
$$K_4 = \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2$$

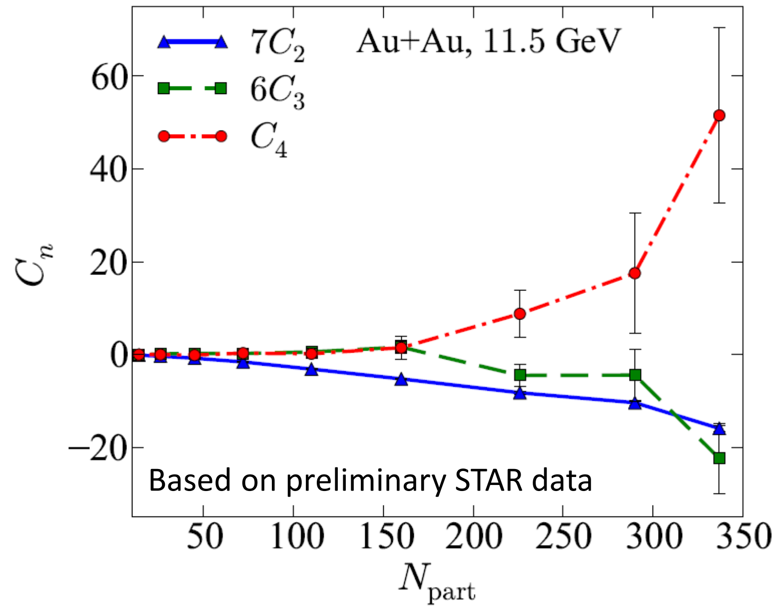
$$K_i = \langle N\rangle + physics[2, \ldots, i]$$

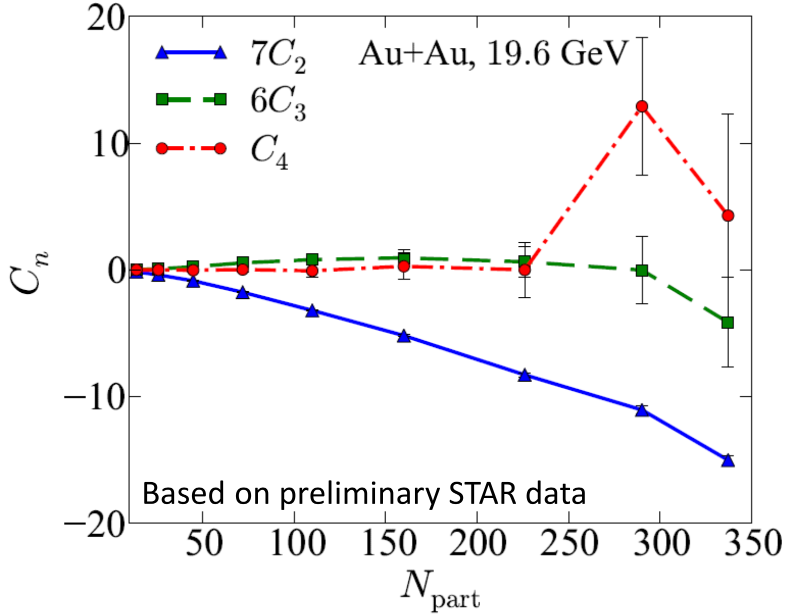
$physics$ = two-, three-, *n*-particle factorial cumulants

for Poisson distribution $K_i = \langle N\rangle$, $(physics = 0)$

Comparison of 7.7, 11.5 and 19.6 GeV



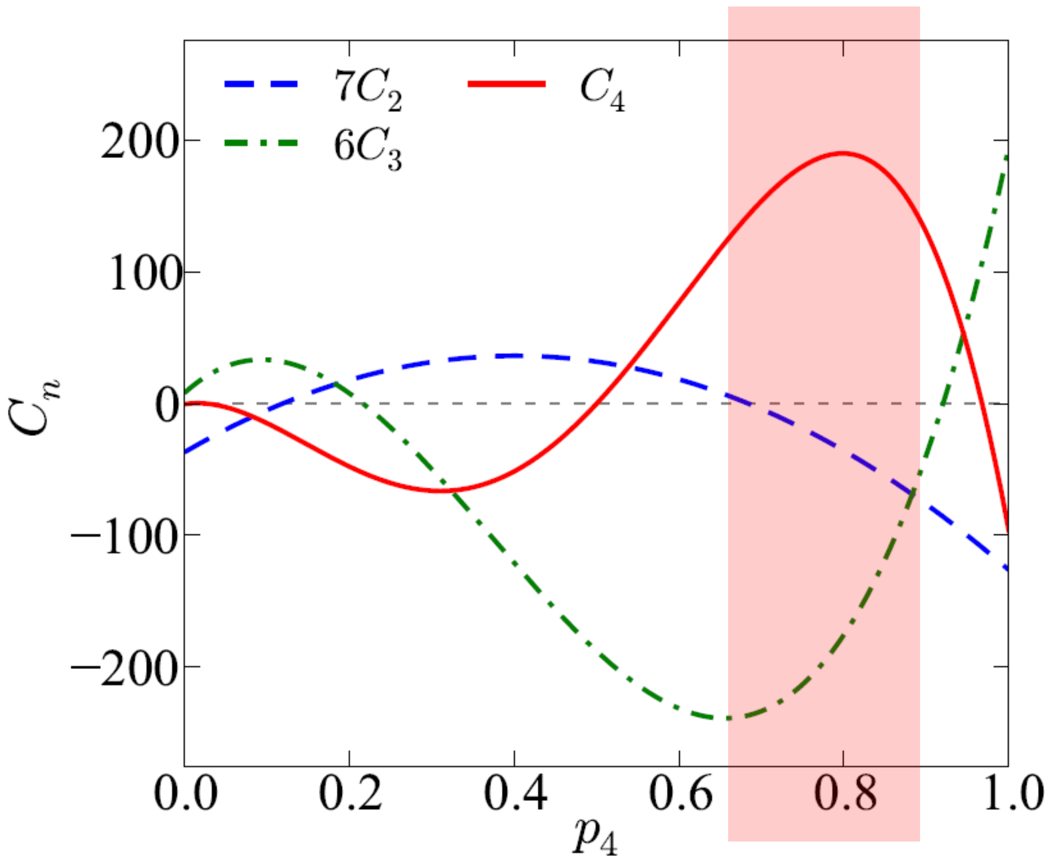




Toy model:

AB, V. Koch, V. Skokov,
EPJC 77 (2017) 288

- 16 protons stop in quartets with probability p_4
- remaining protons stop independently with some small probability $p_1 \sim 0.1$



qualitatively consistent with STAR

STAR

$C_4 \sim 170$

$6C_3 \sim -60$

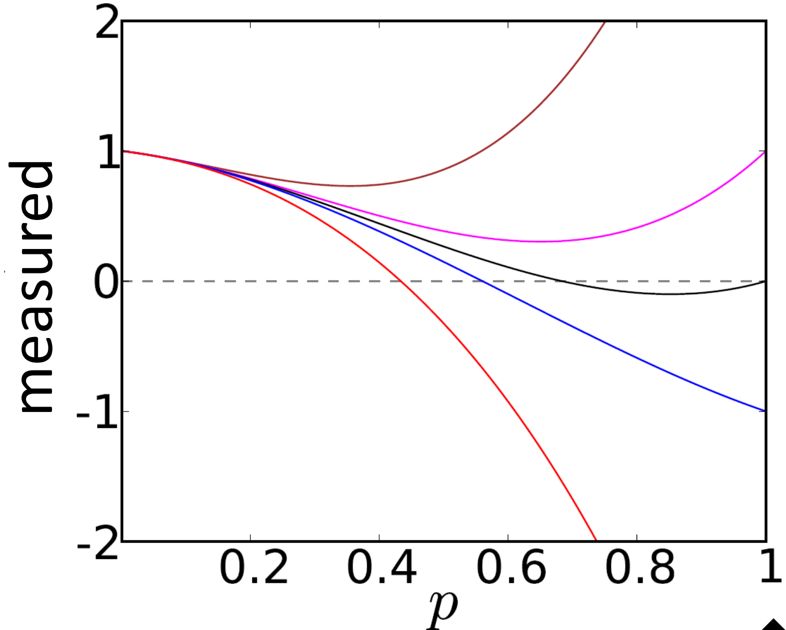
$7C_2 \sim -15$

We obviously need more serious cluster model.
See, e.g., E.Shuryak, J.M. Torres-Rincon, 1805.04444

Efficiency

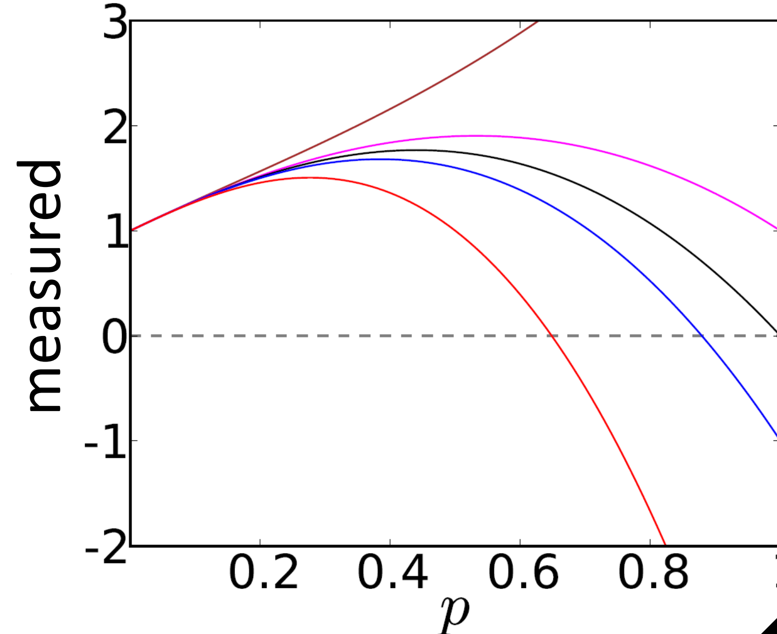
AB, V.Koch, PRC 86 (2012) 044904

multiplicity distr. narrower than Poisson



K_4/K_2

multiplicity distr. broader than Poisson

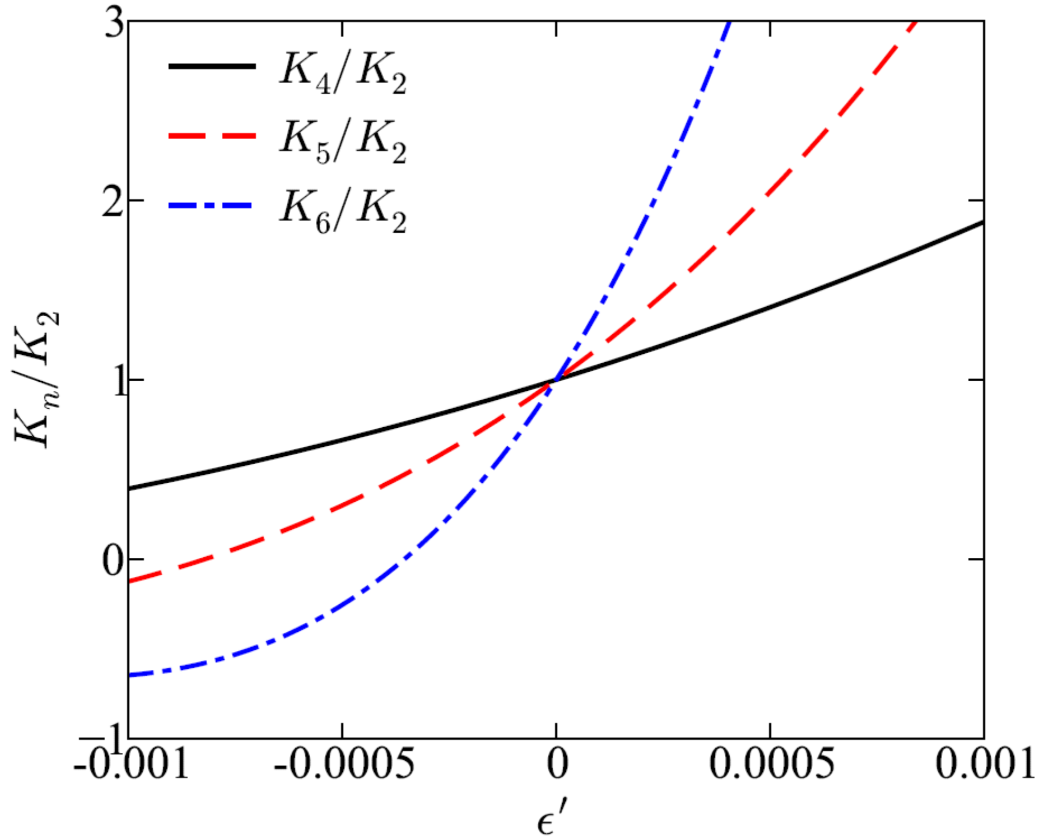


K_4/K_2

$$\frac{K_4}{K_2} = 5, 1, 0, -1, -5$$

Multiplicity dependent efficiency

AB, R.Holzmann, V.Koch
PRC 94 (2016) 064907



$$P(N) = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle},$$

$$\epsilon(N) = \epsilon_0 + \epsilon'(N - \langle N \rangle)$$

Large corrections for small ϵ'

Mixed integrated correlation functions

$$C_2^{(2,0)} = -F_{1,0}^2 + F_{2,0}$$

$$C_2^{(1,1)} = -F_{0,1}F_{1,0} + F_{1,1}$$

$$C_3^{(3,0)} = 2F_{1,0}^3 - 3F_{1,0}F_{2,0} + F_{3,0}$$

$$C_3^{(2,1)} = 2F_{0,1}F_{1,0}^2 - 2F_{1,0}F_{1,1} - F_{0,1}F_{2,0} + F_{2,1}$$

$$F_{i,k} \equiv \left\langle \frac{N!}{(N-i)!}\frac{\bar{N}!}{(\bar{N}-k)!} \right\rangle$$

Cumulants

$$K_2 = \langle N \rangle + \langle \bar{N} \rangle + C_2^{(2,0)} + C_2^{(0,2)} - 2C_2^{(1,1)}$$

$$K_3 = \langle N \rangle - \langle \bar{N} \rangle + 3C_2^{(2,0)} - 3C_2^{(0,2)} + C_3^{(3,0)} - C_3^{(0,3)} - 3C_3^{(2,1)} + 3C_3^{(1,2)}$$

For $C_4^{(i,k)}$ and K_4 see the appendix of

AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906

First model (AMPT) calculations by

Yufu Lin, Lizhu Chen, Zhiming Li, PRC 96 (2017) 044906

Mixed correlation functions and cumulants

$$C_2^{(2,0)} = -F_{1,0}^2 + F_{2,0}$$
$$C_2^{(1,1)} = -F_{0,1}F_{1,0} + F_{1,1}$$
$$C_3^{(3,0)} = 2F_{1,0}^3 - 3F_{1,0}F_{2,0} + F_{3,0}$$
$$C_3^{(2,1)} = 2F_{0,1}F_{1,0}^2 - 2F_{1,0}F_{1,1} - F_{0,1}F_{2,0} + F_{2,1}$$
$$C_4^{(4,0)} = -6F_{1,0}^4 + 12F_{1,0}^2F_{2,0} - 3F_{2,0}^2 - 4F_{1,0}F_{3,0} + F_{4,0}$$
$$C_4^{(3,1)} = -6F_{0,1}F_{1,0}^3 + 6F_{1,0}^2F_{1,1} + 6F_{0,1}F_{1,0}F_{2,0} - 3F_{1,1}F_{2,0} - 3F_{1,0}F_{2,1} - F_{0,1}F_{3,0} + F_{3,1}$$
$$C_4^{(2,2)} = (-6F_{0,1}^2 + 2F_{0,2})F_{1,0}^2 + 8F_{0,1}F_{1,0}F_{1,1} - 2F_{1,1}^2 - 2F_{1,0}F_{1,2} + (2F_{0,1}^2 - F_{0,2})F_{2,0} - 2F_{0,1}F_{2,1} + F_{2,2}$$

$$K_2 = \langle N \rangle + \langle \bar{N} \rangle + C_2^{(2,0)} + C_2^{(0,2)} - 2C_2^{(1,1)}$$
$$K_3 = \langle N \rangle - \langle \bar{N} \rangle + 3C_2^{(2,0)} - 3C_2^{(0,2)} + C_3^{(3,0)} - C_3^{(0,3)} - 3C_3^{(2,1)} + 3C_3^{(1,2)}$$
$$K_4 = \langle N \rangle + \langle \bar{N} \rangle + 7C_2^{(2,0)} + 7C_2^{(0,2)} - 2C_2^{(1,1)} + 6C_3^{(3,0)} + 6C_3^{(0,3)} - 6C_3^{(2,1)} - 6C_3^{(1,2)} + C_4^{(4,0)} + C_4^{(0,4)} - 4C_4^{(3,1)} - 4C_4^{(1,3)} + 6C_4^{(2,2)}$$

$$c_{n+m}^{(n,m)} = \frac{C_{n+m}^{(n,m)}}{\langle N \rangle^n \langle \bar{N} \rangle^m}$$

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Full acceptance

$$N_{(b)}$$

$$\boxed{N_{(a)}}$$

$$N_{(a)} + N_{(b)} = B = const.$$

baryon conservation

$$K_{2,(a)} = K_{2,(b)} \qquad K_{3,(a)} = -K_{3,(b)}$$

$$K_{4,(a)} = K_{4,(b)} \qquad K_{5,(a)} = -K_{5,(b)}$$

$$\frac{K_4}{K_2} \rightarrow 1, \quad \frac{K_3}{K_2} \rightarrow -1$$

for full acceptance