THE CRITICALITY IN HOT QCD MATTER PROBED WITH FLUCTUATIONS

CONFRONTING EXPERIMENTAL RESULTS WITH THEORY

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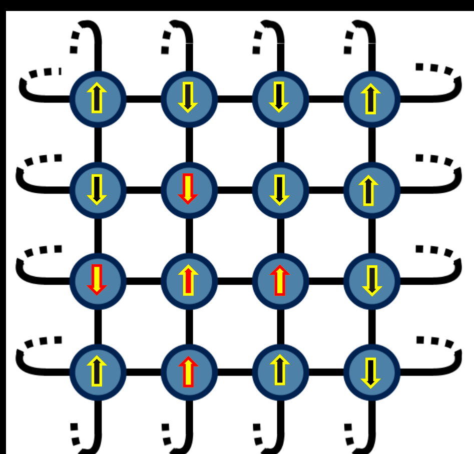
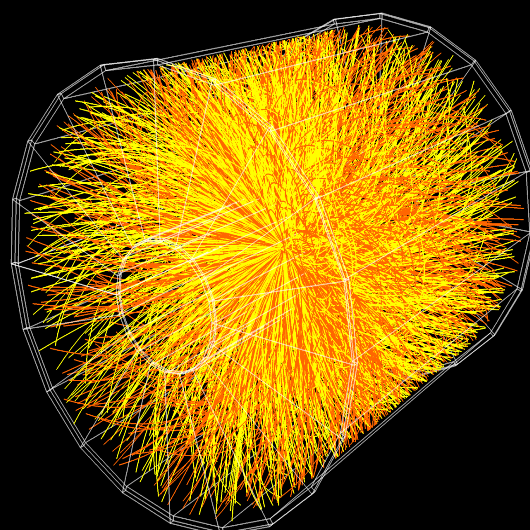
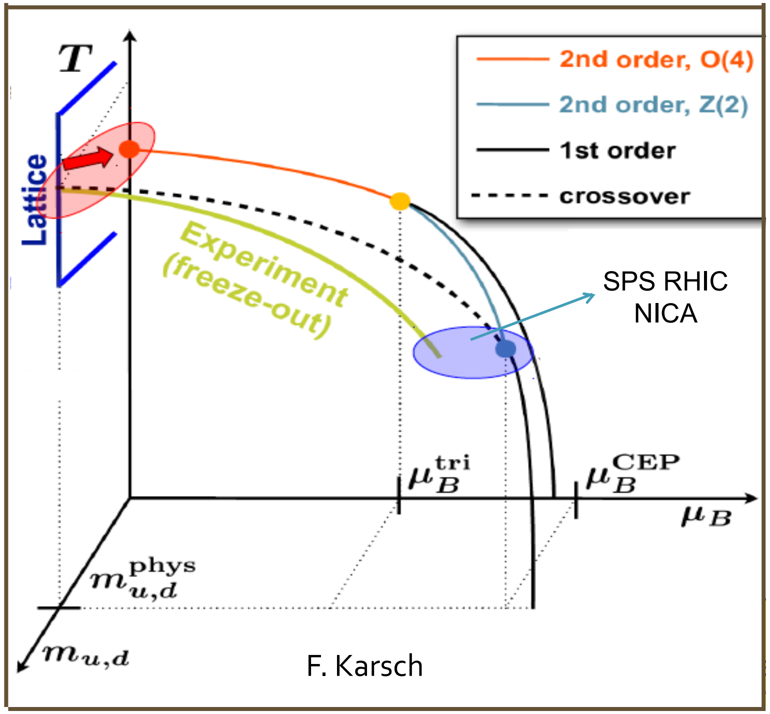
ECT* 2018





- E-by-E Fluctuations
- Criticality
- Acceptance selection
- Volume fluctuations
- Confronting with experiments

Why Fluctuations?



- To probe the structure of strongly interacting matter
  - Locate phase boundaries
  - Search for critical phenomena
  - ...

E-by-E fluctuations are predicted within Grand Canonical Ensemble

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{T\chi}{V}, \quad \chi = -\frac{1}{V}\frac{\partial V}{\partial P}$$

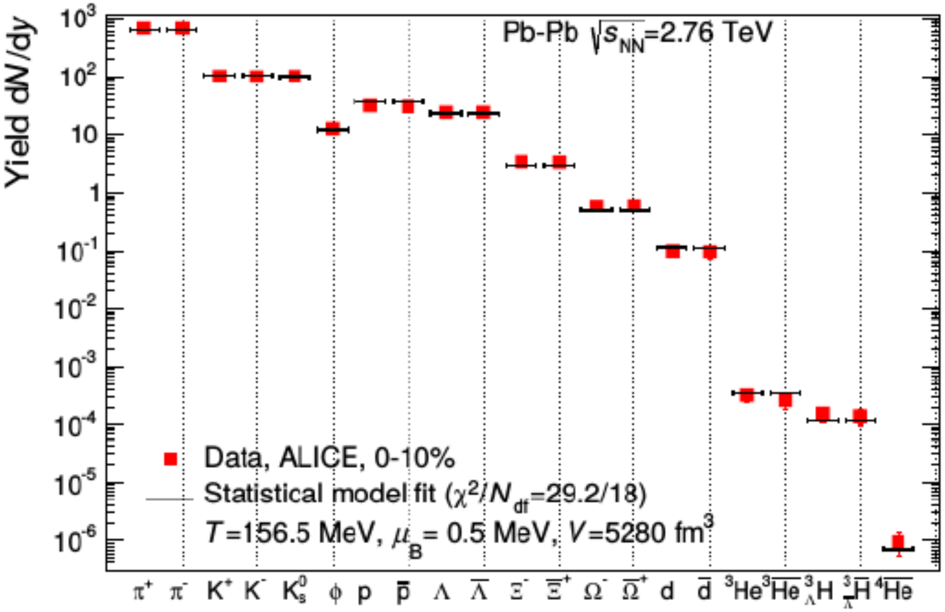
- **fingerprints of criticality for $m_{u,d} = 0$ survive at crossover with $m_{u,d} \neq 0$**

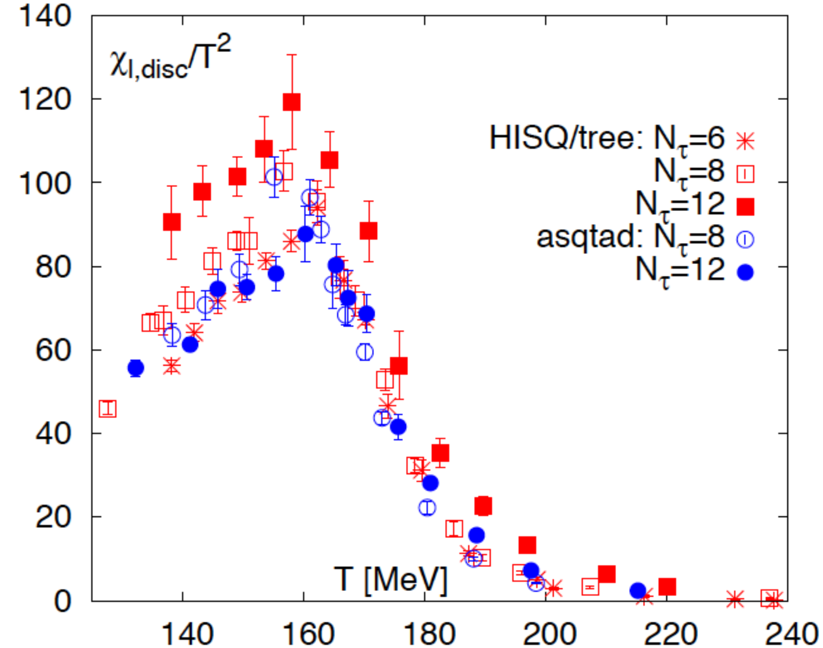
A. Bazavov et al., Phys.Rev. D85 (2012) 054503

😱 **direct link to the EoS**

probing the response of the system to external perturbations

Criticality at crossover





$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i\right)/T\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

freeze-out at the phase boundary

$$T_c^{LQCD} = 156.5 \pm 1.5\ MeV$$

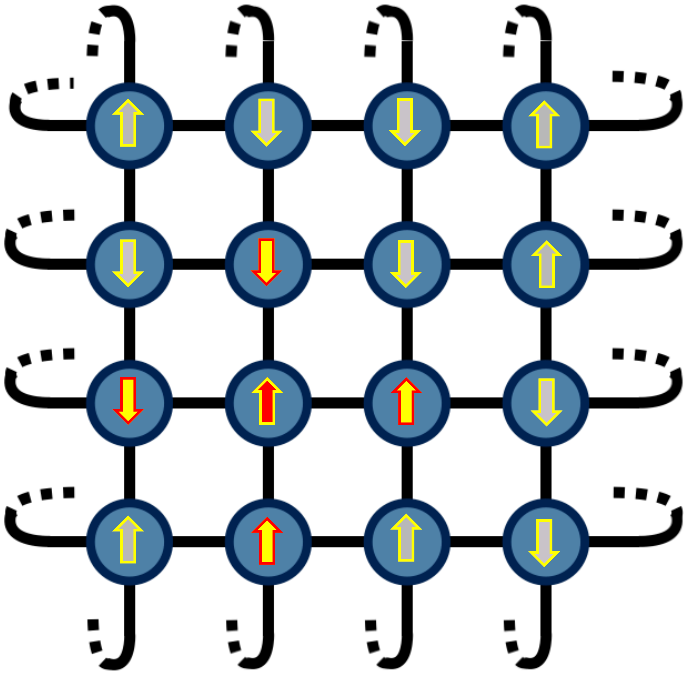
$$T_{fo}^{ALICE} = 156.5 \pm 1.5\text{MeV} \pm 3\text{ MeV(sys)}$$

ALICE, PLB 726 (2013) 610
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)

A. Bazavov et al., Phys.Rev. D85 (2012) 054503

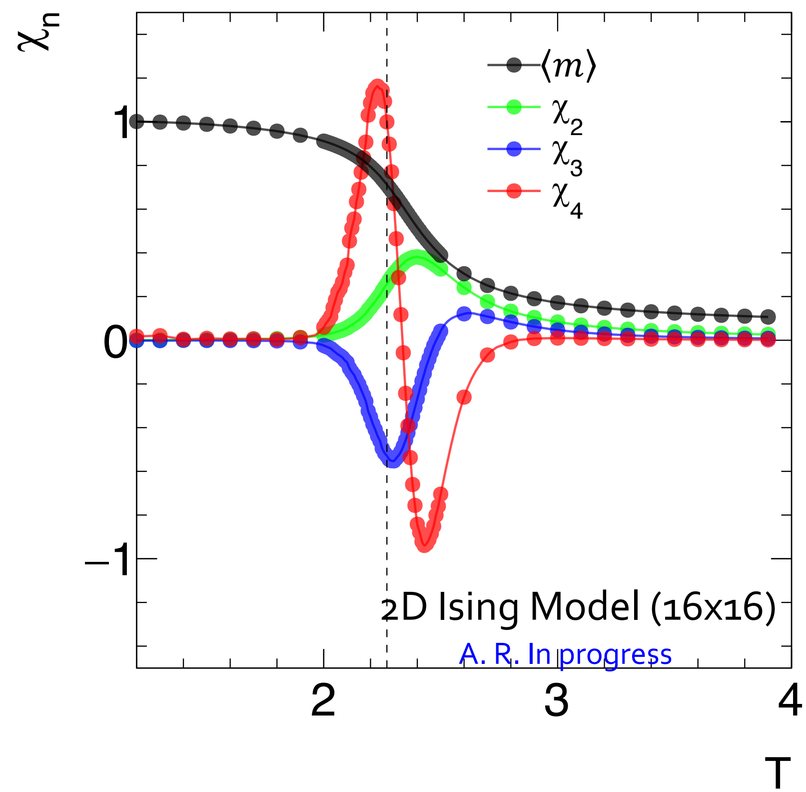
y axis: 9 orders of magnitude; works in the energy range spanning by 3 orders of magnitude

2D Ising Model



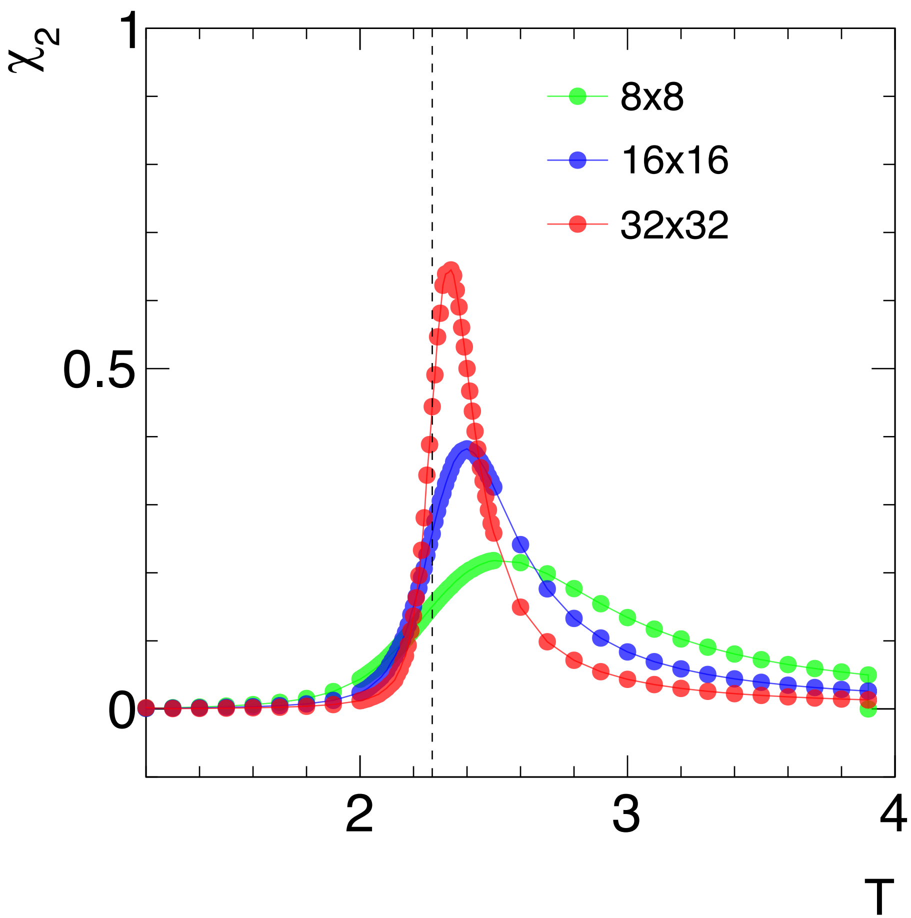
$$\mathcal{H}(s_1, \dots, s_N) = -J \sum_{\langle i,j \rangle} s_i s_j - H \sum_i s_i$$

- competition between ordering agent ***J*** and disordering agent ***T***



$$\langle m \rangle = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \frac{1}{L^2} \left| \sum_{j=1}^{L^2} s_j \right|$$

Finite size effects



magnetic susceptibility

$$\chi_2 = \frac{N}{T}\left[\langle m^2 \rangle - \langle m \rangle^2\right]$$

critical temperature

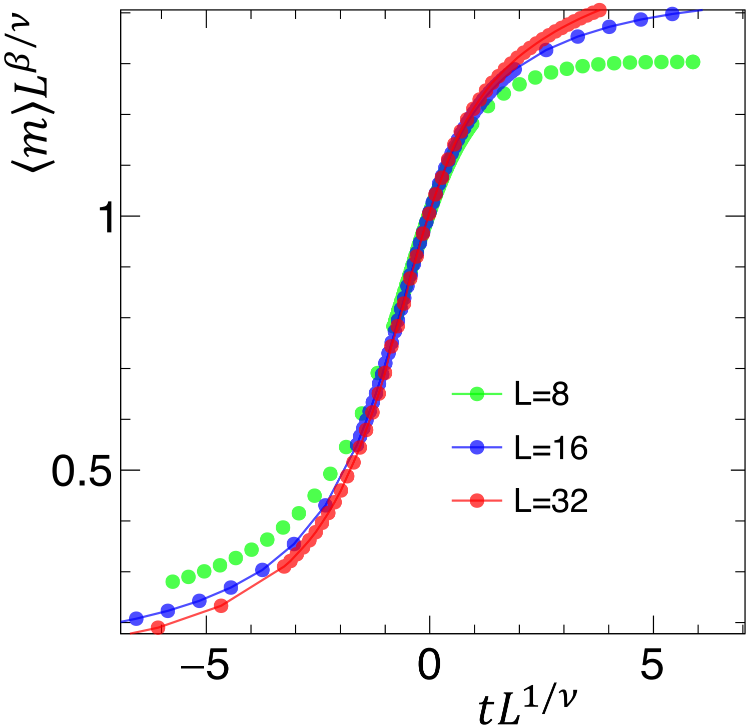
$$\frac{T_c}{J} = \frac{2}{ln\left(1+\sqrt{2}\right)} \approx 2.269$$

The signal depends on system size

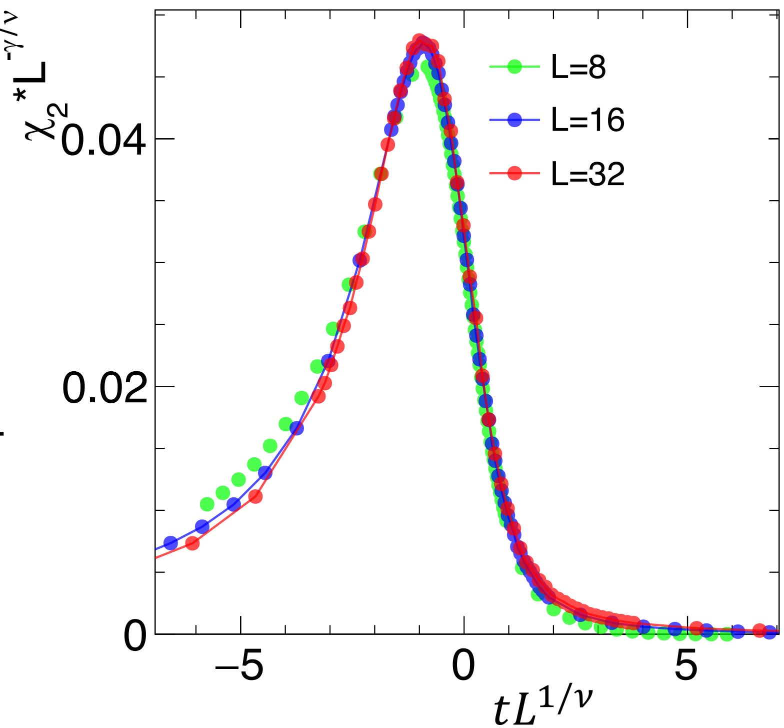
- Change in the peak position
- Change in the signal amplitude

system size dependent analysis is necessary

Scaling and universality



$$t = \frac{T - T_c}{T}$$



$$\widetilde{\langle m\rangle}\left(tL^{1/\nu}\right) = \langle m\rangle(t,L)L^{\beta/\nu}$$

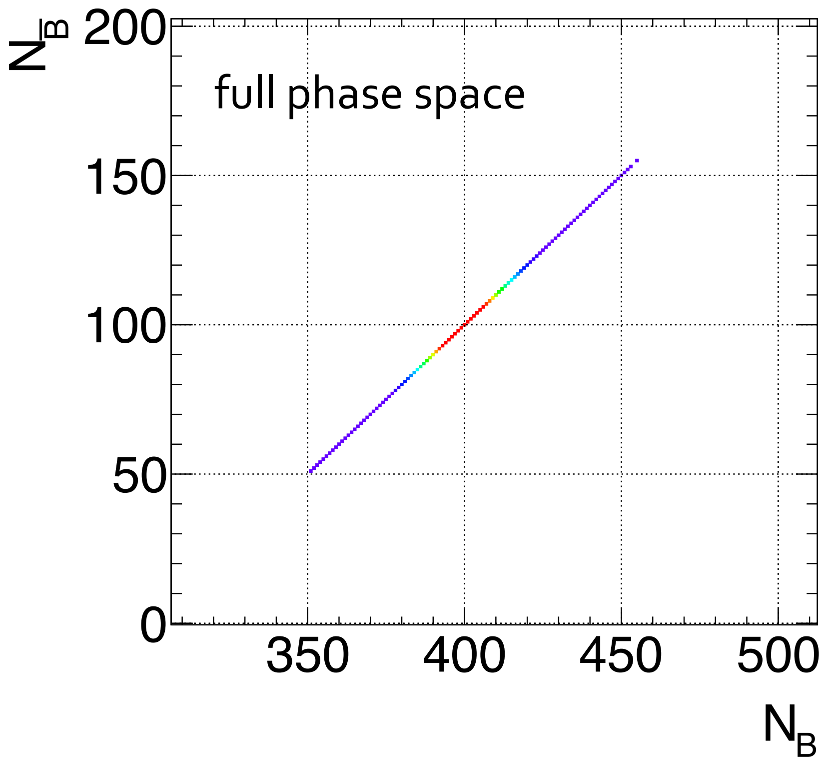
$$\tilde{\chi}_2\left(tL^{1/\nu}\right) = \chi_2(t,L)L^{-\gamma/\nu}$$

extract critical parameters (β, γ, ν, T_c) by function minimization

corresponding exact values for 2D Ising: $\frac{1}{8}, \frac{7}{4}, 1, 2.269$ (used for the plots)

Fluctuations in nuclear reactions

$\langle N_B \rangle = 400, \quad \langle N_{\bar{B}} \rangle = 100$



fluctuations of conserved quantities

e.g., $N_B - N_{\bar{B}}$

conservation of the net-baryon number

P. Braun-Munzinger, A. R., J. Stachel, QM 18

Fluctuations in nuclear reactions

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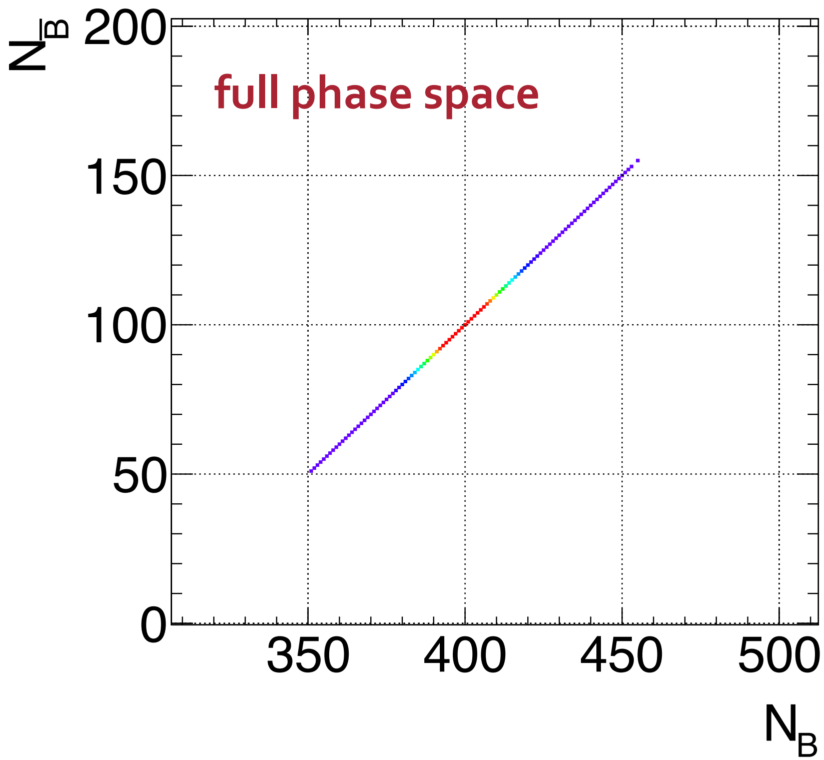
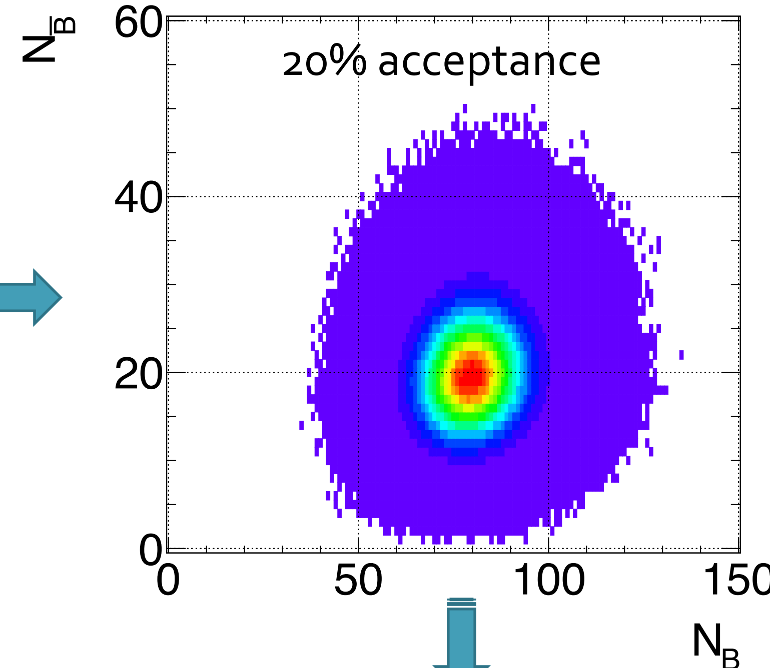
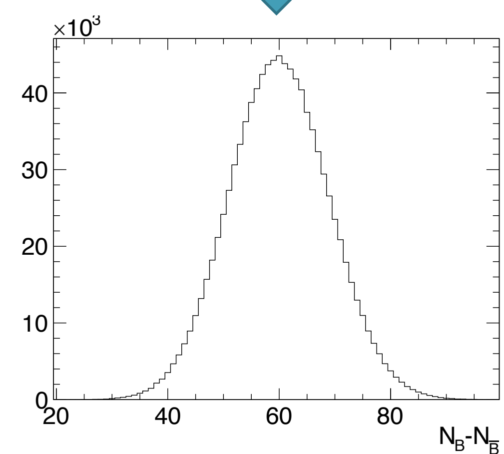






- **fluctuations of net-baryons appear only inside finite acceptance**

P. Braun-Munzinger, A. R., J. Stachel, QM 18

Net-particle cumulants, definitions

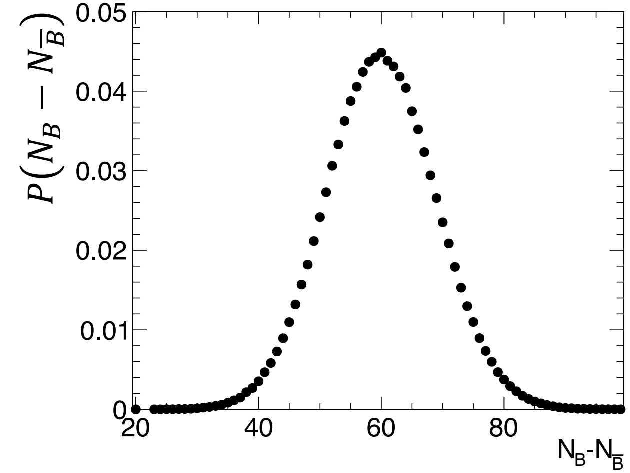
$$X = N_B - N_{\overline{B}}$$

r^{th} central moment:

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

first four cumulants

$$\kappa_1 = \langle X \rangle, \quad \kappa_2 = \mu_2, \quad \kappa_3 = \mu_3, \quad \kappa_4 = \mu_4 - 3\mu_2^2$$



Uncorrelated Poisson limit: $\langle N_B N_{\overline{B}} \rangle = \langle N_B \rangle \langle N_{\overline{B}} \rangle$

Net-Baryons → Skellam

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\overline{B}} \rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \tanh\left(\frac{\mu}{T}\right) = \frac{\langle N_B \rangle - \langle N_{\overline{B}} \rangle}{\langle N_B \rangle + \langle N_{\overline{B}} \rangle}$$

Baselines from LQCD

for a thermal system in a fixed volume V
within the Grand Canonical Ensemble

$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3} \equiv \frac{\kappa_2(\Delta N_B)}{VT^3}$$

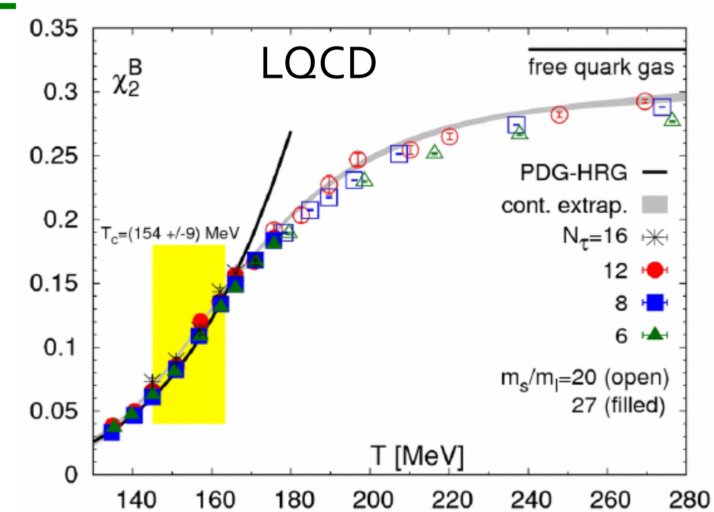
$$\hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial(\mu_N/T)^n} \qquad \frac{P}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu_{B,Q,S})$$

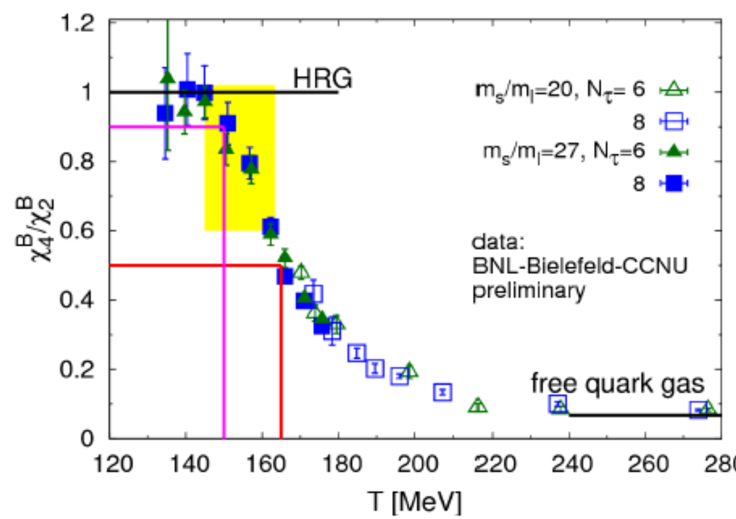
$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B} \qquad \frac{\kappa_3(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_3^B}{\hat{\chi}_2^B}$$

valid only for a fixed system volume

V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911
P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

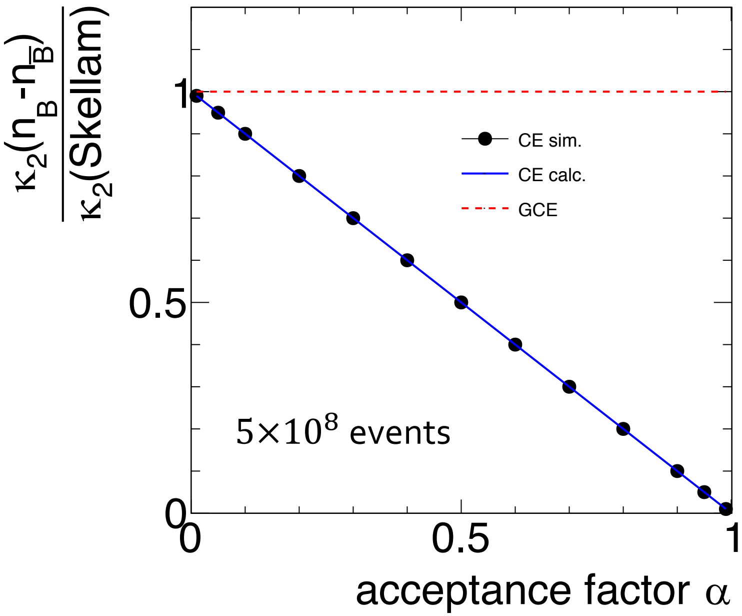
- *In experiments*
  - *Volume (participants) fluctuates from E-to-E*
    - *Centrality selection is crucial*
  - *Global conservation laws are important*
    - *Acceptance selection is crucial*





smaller than in HRG for T > 150 MeV

F. Karsch; QM17, arXiv:1706.01620
O. Kaczmarek; QM17, arXiv:1705.10682



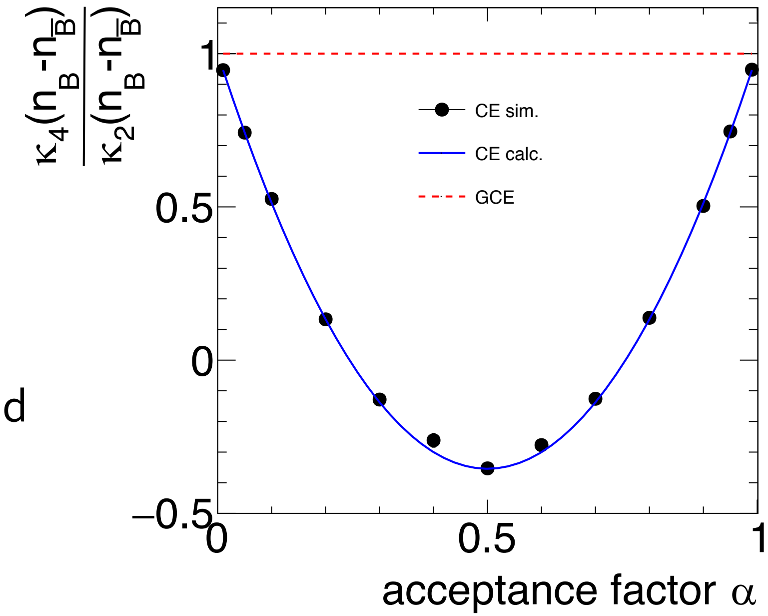
$\langle N_B \rangle = 370$

$\langle N_{\overline{B}} \rangle = 20$

$$\alpha = \frac{\langle n_B \rangle}{\langle N_B \rangle}$$

n_B - measured

N_B - in 4π



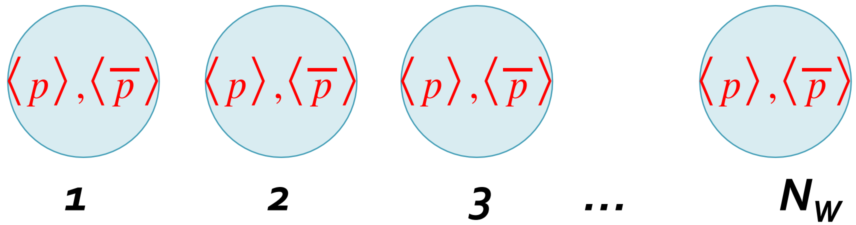
$$\frac{\kappa_2(n_B - n_{\overline{B}})}{\kappa_2(Skellam)} = 1 - \alpha$$

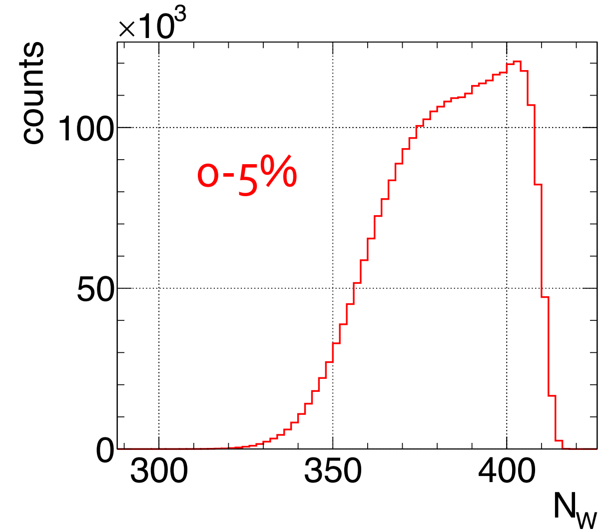
$$\frac{\kappa_3(n_B - n_{\overline{B}})}{\kappa_2(n_B - n_{\overline{B}})} = \frac{\langle n_B - n_{\overline{B}} \rangle_{CE}}{\langle n_B + n_{\overline{B}} \rangle_{CE}} (1 - 2\alpha)$$

$$\frac{\kappa_4(n_B - n_{\overline{B}})}{\kappa_2(n_B - n_{\overline{B}})} = 1 - 6\alpha(1 - \alpha) \left[1 - \frac{2}{\langle N_B + N_{\overline{B}} \rangle_{CE}} \left(\langle N_B \rangle_{GCE} \langle N_{\overline{B}} \rangle_{GCE} - \langle N_B \rangle_{CE} \langle N_{\overline{B}} \rangle_{CE} \right) \right]$$

P. Braun-Munzinger, A. R., J. Stachel, arXiv:1807.08927

Impact of participant fluctuations





- ***Each source is treated Grand Canonically***

$$\langle \Delta n \rangle = \langle p \rangle - \langle \bar{p} \rangle$$

$$\kappa_1(\Delta N) = \langle \boldsymbol{N_w} \rangle \boldsymbol{\kappa_1}(\boldsymbol{\Delta n})$$

$$\kappa_2(\Delta N) = \langle \boldsymbol{N_w} \rangle \boldsymbol{\kappa_2}(\boldsymbol{\Delta n}) + \langle \Delta n \rangle^2 \kappa_2(N_W)$$

$$\kappa_3(\Delta N) = \langle \boldsymbol{N_w} \rangle \boldsymbol{\kappa_3}(\boldsymbol{\Delta n}) + 3\langle \Delta n \rangle \kappa_2(\Delta n) \kappa_2(N_W) + \langle \Delta n \rangle^3 \kappa_3(N_W)$$

$$\kappa_4(\Delta N) = \langle \boldsymbol{N_w} \rangle \boldsymbol{\kappa_4}(\boldsymbol{\Delta n}) + 3\kappa_2^2(\Delta n) \kappa_2(N_W) + 4\langle \Delta n \rangle \kappa_3(\Delta n) \kappa_2(N_W) + 6\langle \Delta n \rangle^2 \kappa_2(\Delta n) \kappa_3(N_W) + \langle \Delta n \rangle^4 \kappa_4(N_W)$$

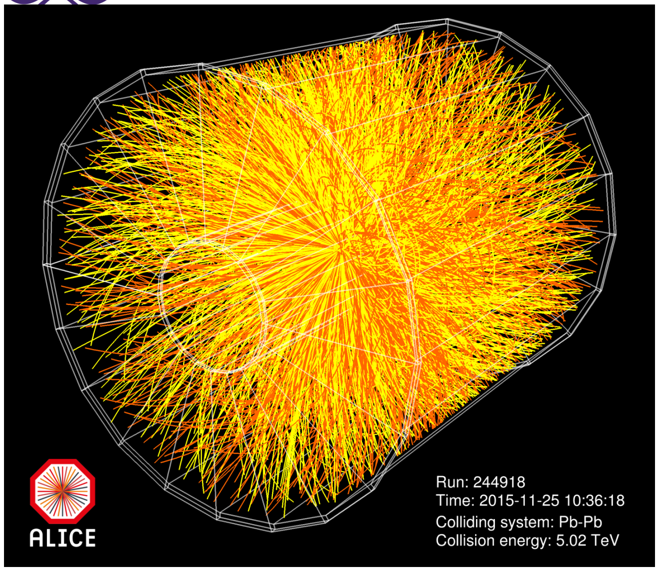
measured

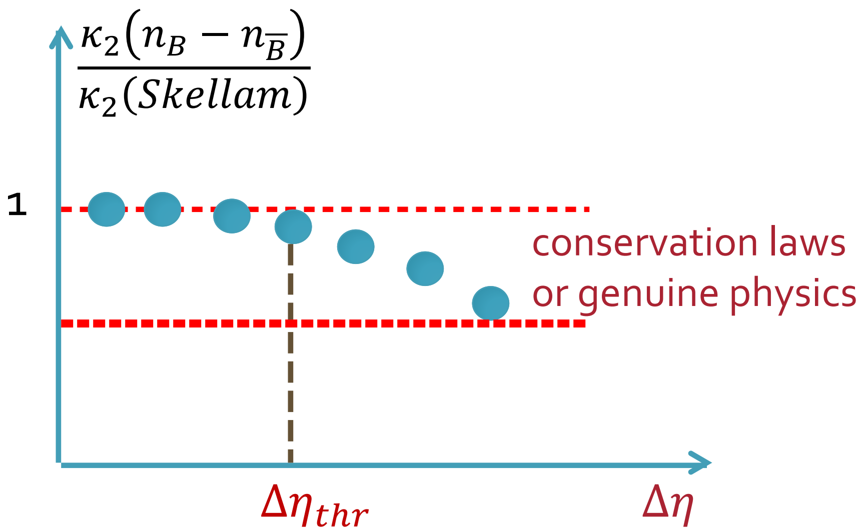
- In ALICE $\langle \Delta n \rangle = 0$ at midrapidity
- At lower energies volume fluctuations are more important

Detailed recipe for subtracting volume (participant) fluctuations:

P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

Experimental approach





$\Delta\eta > \Delta\eta_{thr}$: conservations dominate

$\Delta\eta < \Delta\eta_{thr}$: dynamical fluctuations may disappear

- The strategy
  - Perform analysis for $\Delta\eta > \Delta\eta_{thr}$
  - Correct for non-dynamical contributions
    - Conservation laws
    - Volume fluctuations
    - etc.
  - Compare to theory

P. Braun-Munzinger, A. R., J. Stachel, NPA 960 (2017) 114

Contribution from global baryon number conservation

$$\frac{\kappa_2(p-\bar{p})}{\kappa_2(Skellam)} = 1-\alpha \qquad \alpha = \frac{\langle p \rangle^{measured}}{\langle B \rangle^{4\pi}}$$

P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

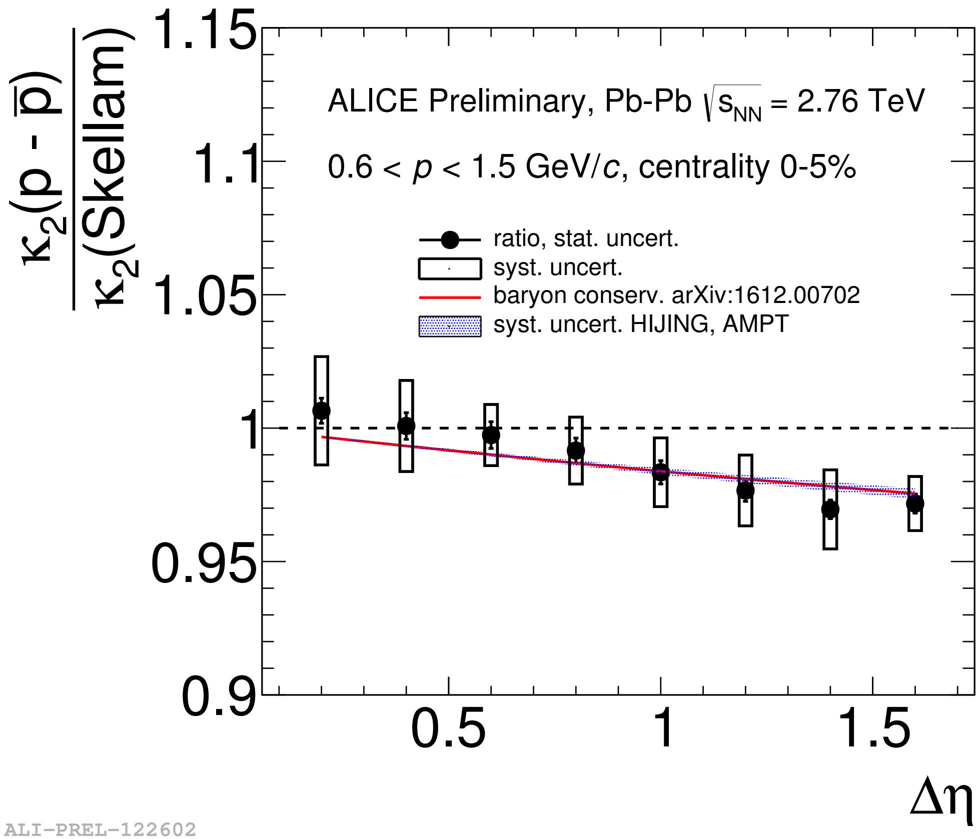
Inputs for $\langle B \rangle^{acc}$ from:

Phys. Lett. B 747, 292 (2015)
P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel

extrapolation from $\langle B \rangle^{acc}$ to $\langle B \rangle^{4\pi}$

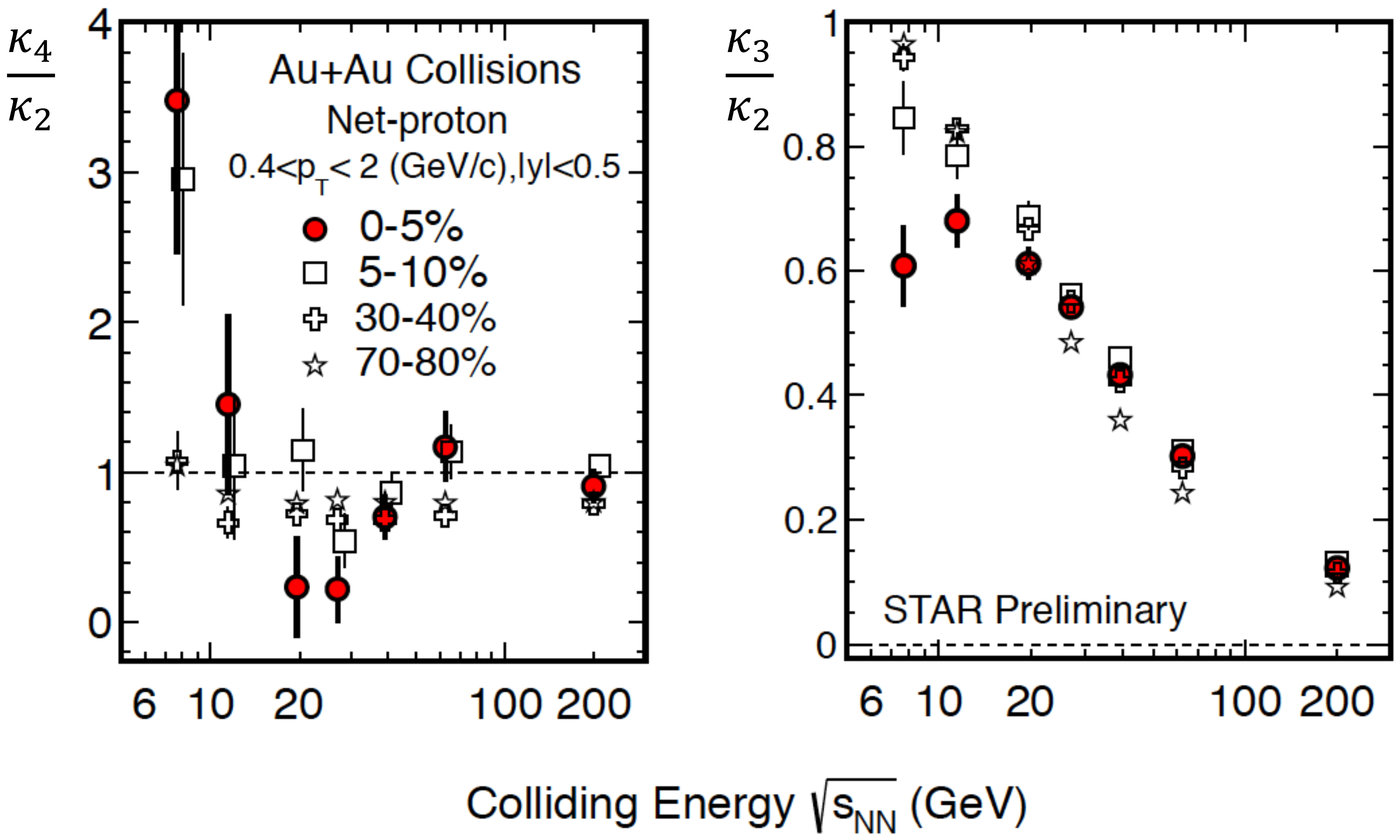
using HIJING and AMPT models

A. R., QM2017, arXiv:1704.05329



The deviation from Skellam is due to the global baryon number conservation.

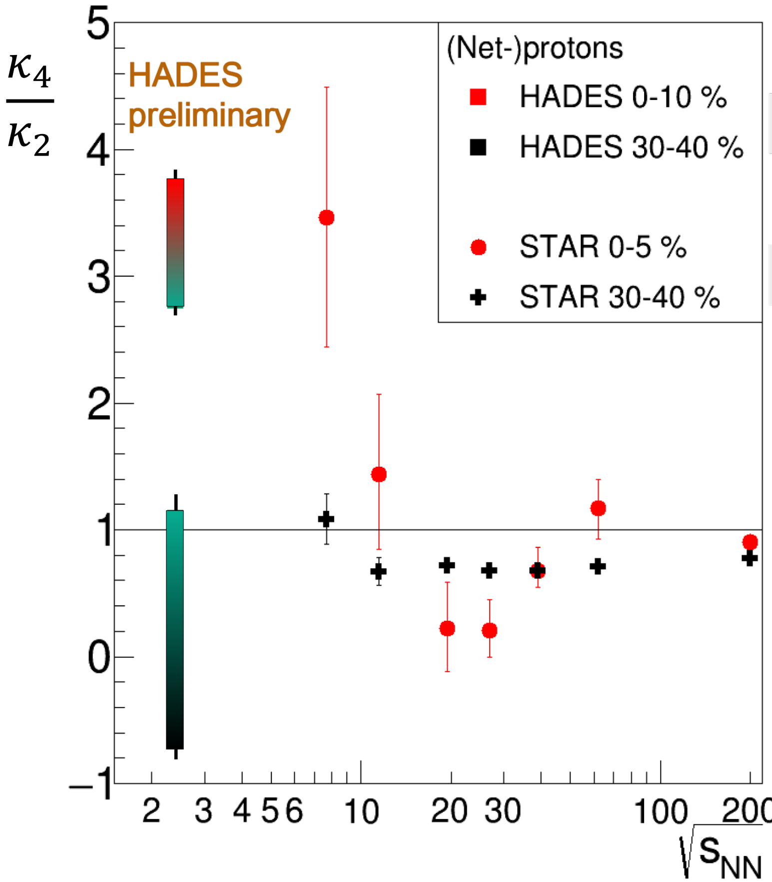
Results from STAR



- Approach to unity for the highest energies
- Non-monotonic behavior below 39 GeV
- Drop at 7.7 GeV for central events

X. Luo, PoS CPOD2014, 019 (2015)
STAR: PRL 112, 032302 (2014)

Results from HADES



Δy=0.2

Δy=0.5

Unfolding + vol. fluct. corr.

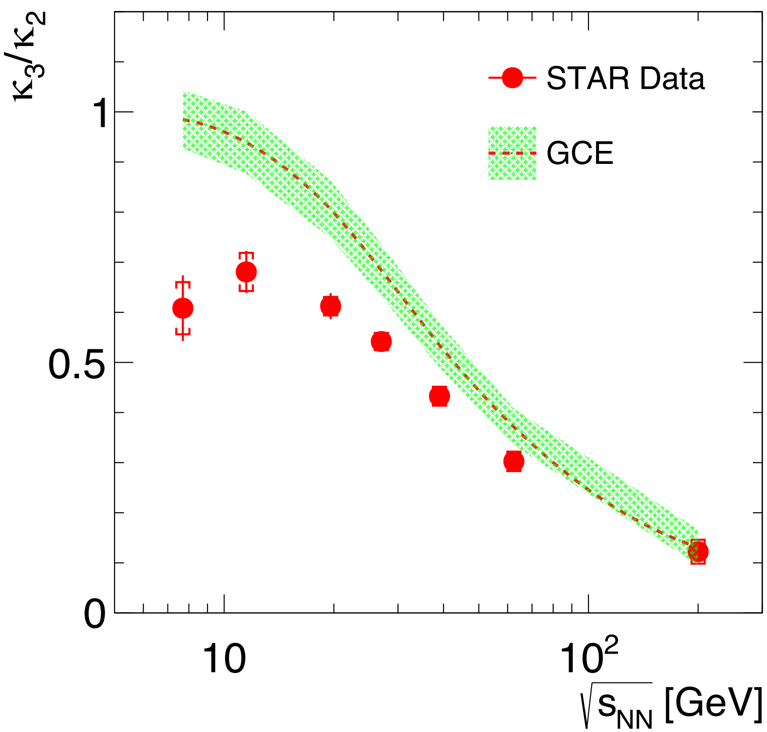
E-by-E corr. of factorial moments + vol. fluct. corr.

Under investigation:

- contamination of peripheral events in centrality selection procedure

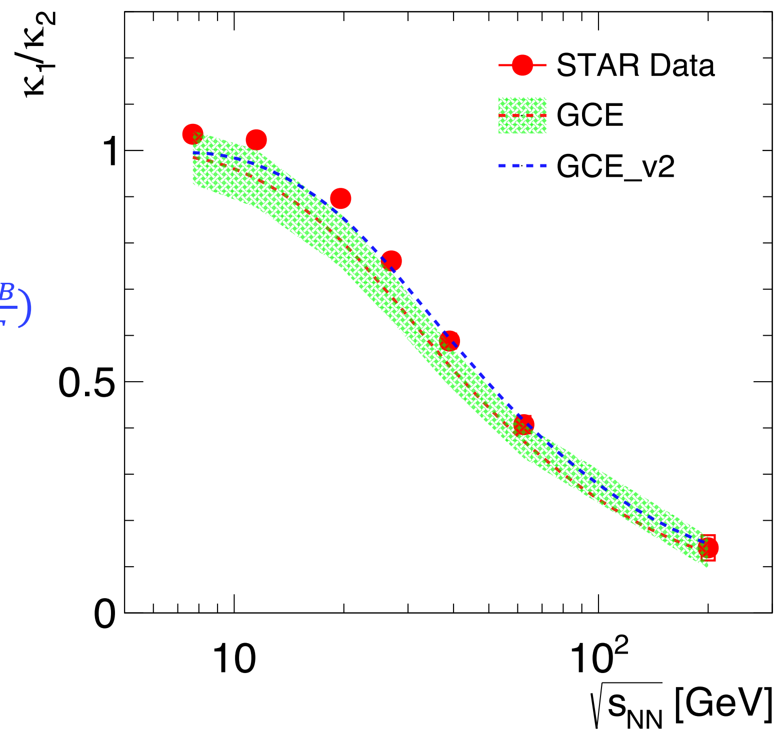
T. Galatyuk, CPOD 2018

Results from STAR



GCE: $\frac{\langle n_p \rangle - \langle n_{\bar{p}} \rangle}{\langle n_p \rangle + \langle n_{\bar{p}} \rangle}$

GCE_v2 : $tanh(\frac{\mu_B}{T})$

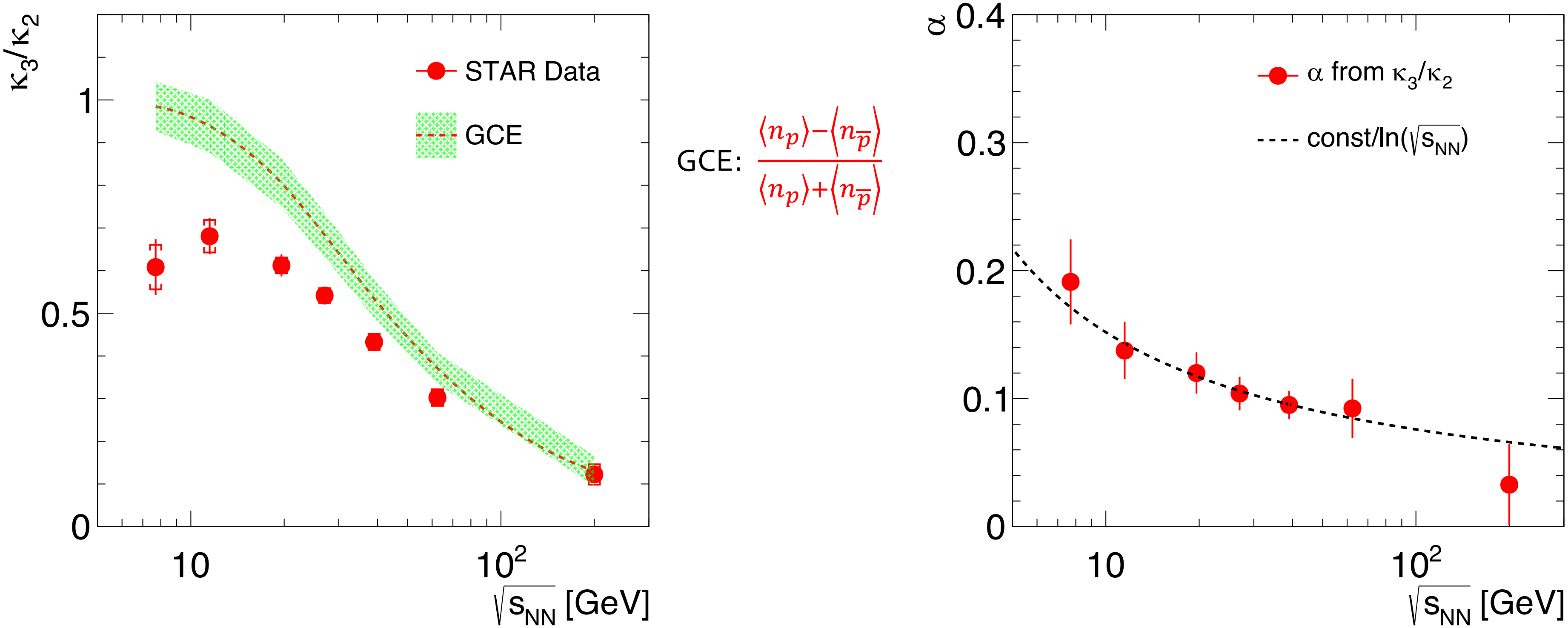


- why do κ_3/κ_2 and κ_1/κ_2 look so different?
- critical phenomenon or non-dynamical effects?

	κ_3/κ_2	κ_1/κ_2
vol. fluct	↗	↘
conserv. laws	↘	↗

conservation laws dominate!

The α parameter

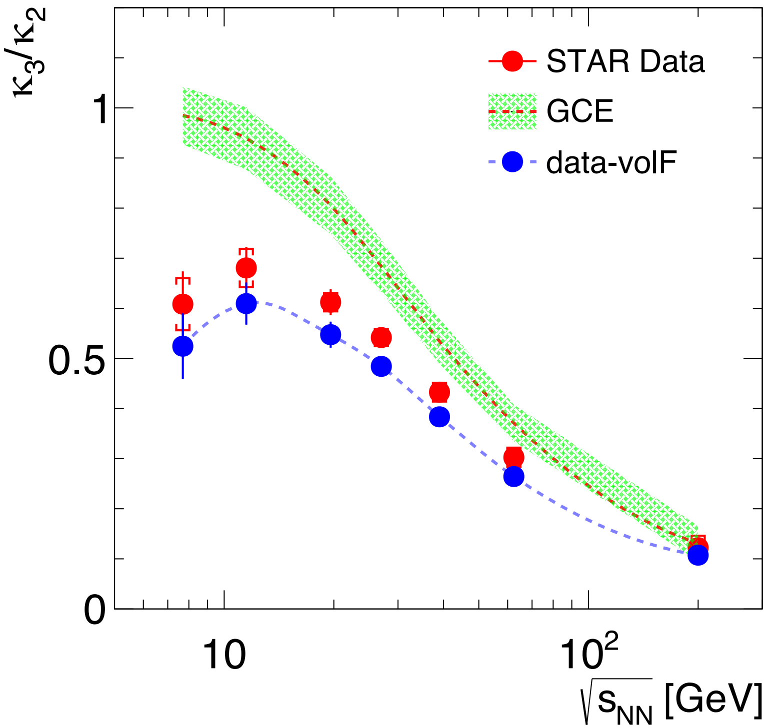


GCE: $\frac{\langle n_p \rangle - \langle n_{\bar{p}} \rangle}{\langle n_p \rangle + \langle n_{\bar{p}} \rangle}$

$$\frac{\kappa_3(n_p - n_{\bar{p}})}{\kappa_2(n_p - n_{\bar{p}})} = \frac{\langle n_p - n_{\bar{p}} \rangle_{CE}}{\langle n_p + n_{\bar{p}} \rangle_{CE}} (1 - 2\alpha)$$

$\langle n_p \rangle, \langle n_{\bar{p}} \rangle$ - also taken from STAR data

The α parameter



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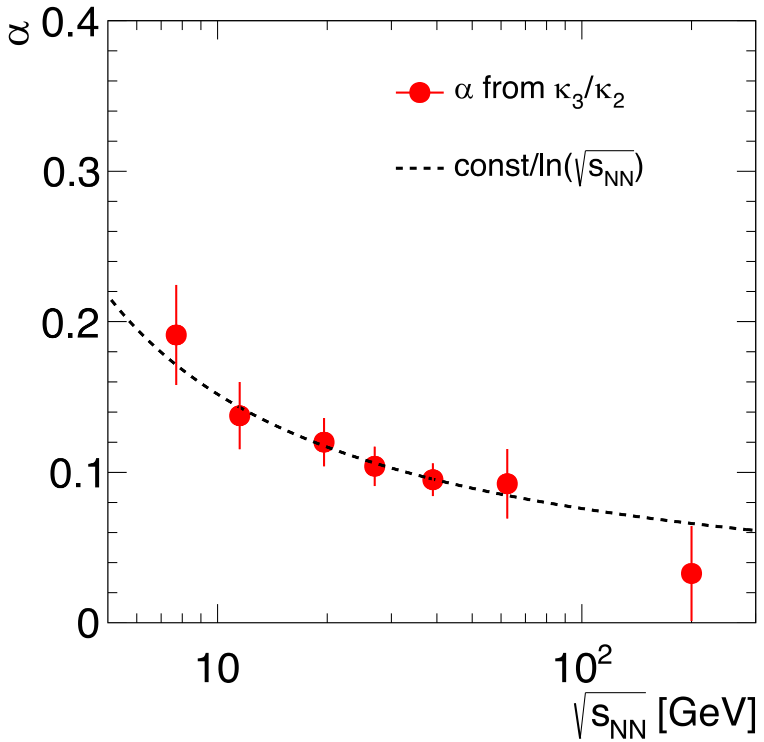
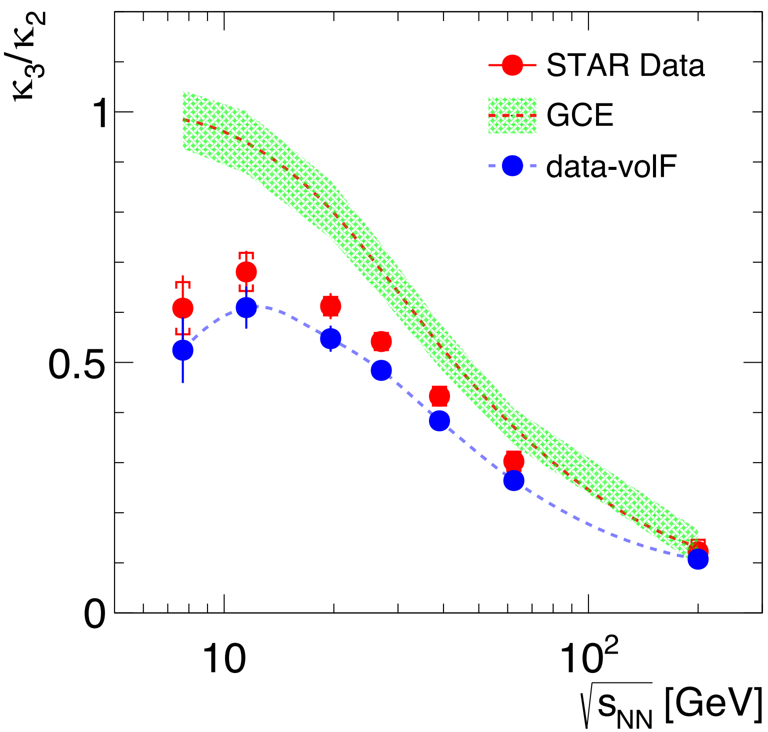


$$\frac{\kappa_3(n_p - n_{\bar{p}})}{\kappa_2(n_p - n_{\bar{p}})} = \frac{\langle n_p - n_{\bar{p}} \rangle_{CE}}{\langle n_p + n_{\bar{p}} \rangle_{CE}} (1 - 2\alpha)$$

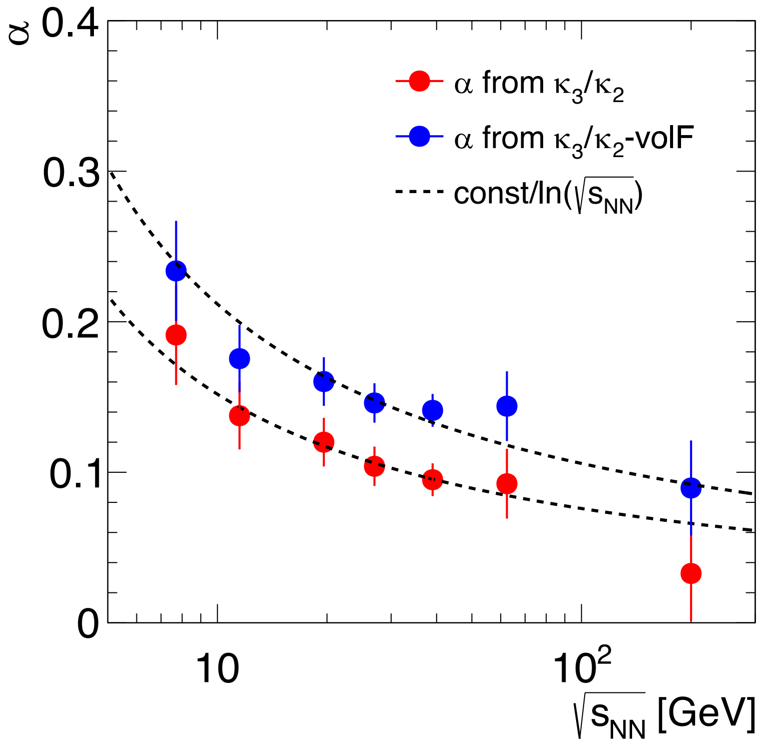
P. Braun-Munzinger, A. R., J. Stachel, arXiv:1807.08927

Volume fluctuations: P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

The α parameter



GCE: $\frac{\langle n_p \rangle - \langle n_{\bar{p}} \rangle}{\langle n_p \rangle + \langle n_{\bar{p}} \rangle}$



$$\frac{\kappa_3(n_p - n_{\bar{p}})}{\kappa_2(n_p - n_{\bar{p}})} = \frac{\langle n_p - n_{\bar{p}} \rangle_{CE}}{\langle n_p + n_{\bar{p}} \rangle_{CE}}(1 - 2\alpha)$$

P. Braun-Munzinger, A. R., J. Stachel, arXiv:1807.08927

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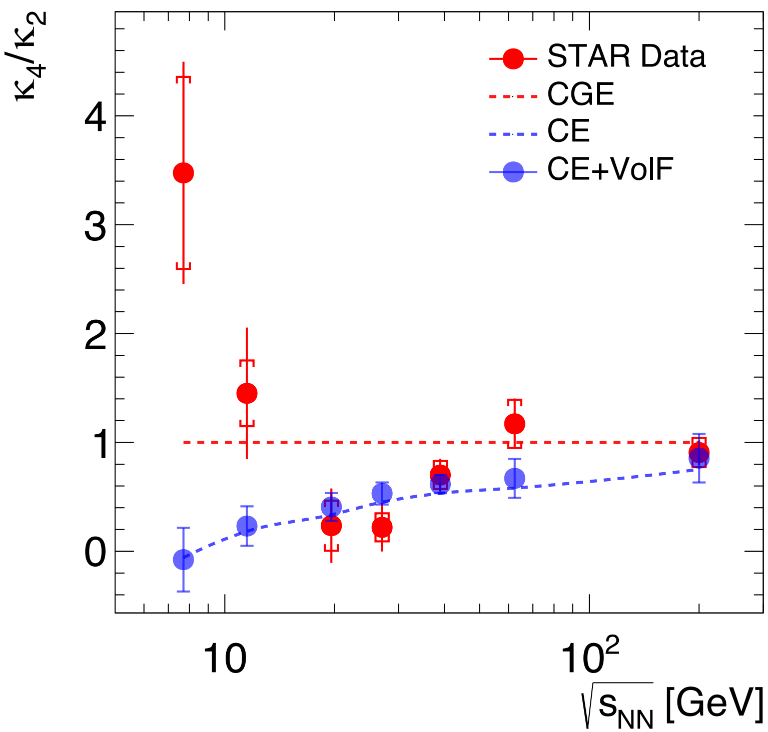
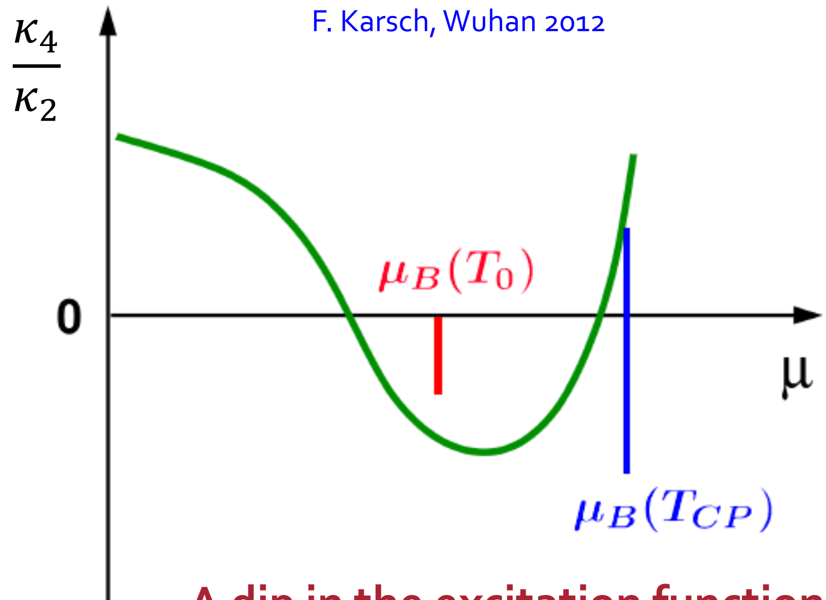
Predictions for κ_4/κ_2



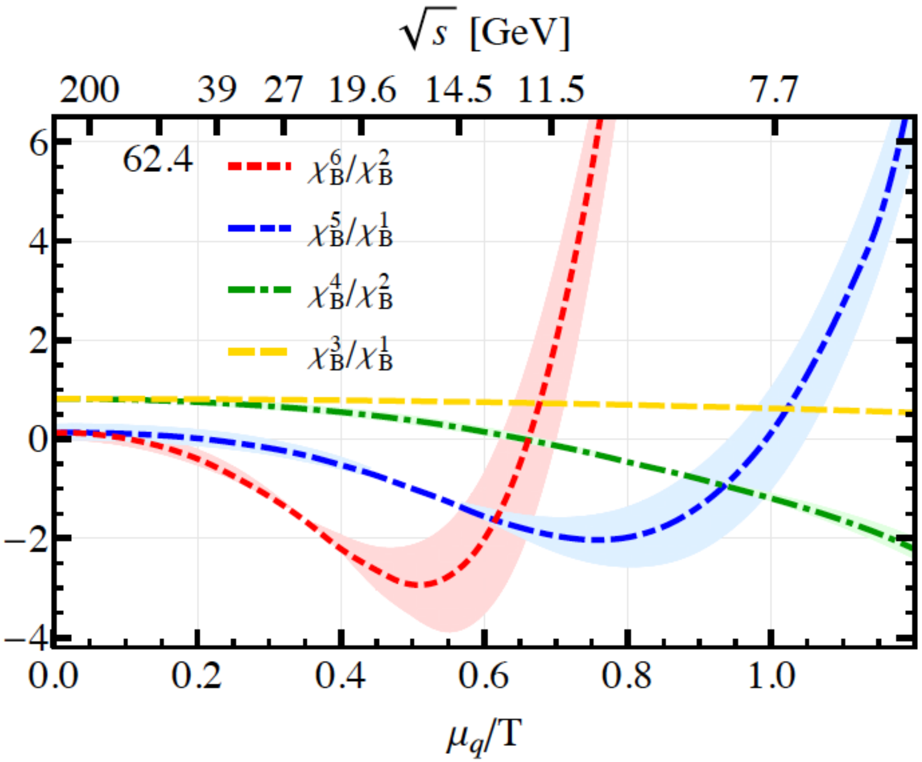


A dip in the excitation function seems to be generic

$$\frac{\kappa_4(n_B - n_{\overline{B}})}{\kappa_2(n_B - n_{\overline{B}})} = 1 - 6\alpha(1-\alpha)\left[1 - \frac{2}{\langle N_B + N_{\overline{B}}\rangle_{CE}}\left(\langle N_B\rangle_{GCE}\langle N_{\overline{B}}\rangle_{GCE} - \langle N_B\rangle_{CE}\langle N_{\overline{B}}\rangle_{CE}\right)\right]$$

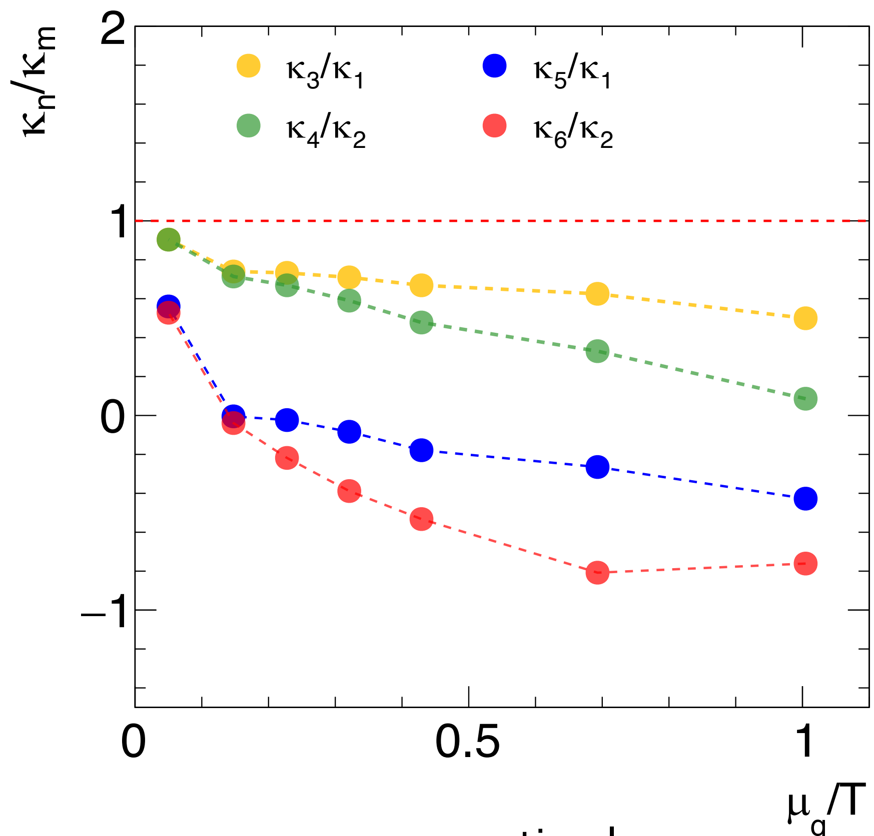
above 11.5 GeV CE suppression describes the data

Comparison to critical fluctuations



critical fluctuations

G. A. Almasi, B. Friman, K. Redlich, P.R.D96 (2017) 1, 014027.



conservation laws

- **Data have to be corrected for conservation laws and volume fluctuations**
- **Qualitative differences emerge above 4th order cumulants!**

Conclusions

- The measured second cumulants of net-protons at ALICE are, after accounting for baryon number conservation, in agreement with the corresponding second cumulants of the Skellam distribution.
- All net-proton cumulants from STAR show deviations from the Skellam baseline.
- Above 11.5 GeV these deviations can be consistently described with the global baryon number conservation + unavoidable fluctuations of participating nucleons

Before making any quantitative statements the data on cumulants have to be corrected for conservation laws and volume fluctuations

Implementing conservations

Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh(\mu/T)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

z – single baryon partition function

Uncorrelated Poisson limit: $\langle N_B N_{\bar{B}}\rangle = \langle N_B\rangle\langle N_{\bar{B}}\rangle$

Net-Baryons → Skellam

$$\kappa_n = \langle N_B\rangle + (-1)^n\langle N_{\bar{B}}\rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \tanh\left(\frac{\mu}{T}\right) = \frac{\langle N_B\rangle - \langle N_{\bar{B}}\rangle}{\langle N_B\rangle + \langle N_{\bar{B}}\rangle}$$

$$Z_{CE}(V,T,B)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B-N_{\bar{B}}-B)=I_B(2z)\Big|_{\lambda_B=\lambda_{\bar{B}}=1}$$

- Non-Poisson single particles → **Canonical Suppression**
- Strong correlations $\langle N_B N_{\bar{B}}\rangle \neq \langle N_B\rangle\langle N_{\bar{B}}\rangle$
- **Net-Baryons do not fluctuate!**

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