Critical point and nucleus-nucleus collisions

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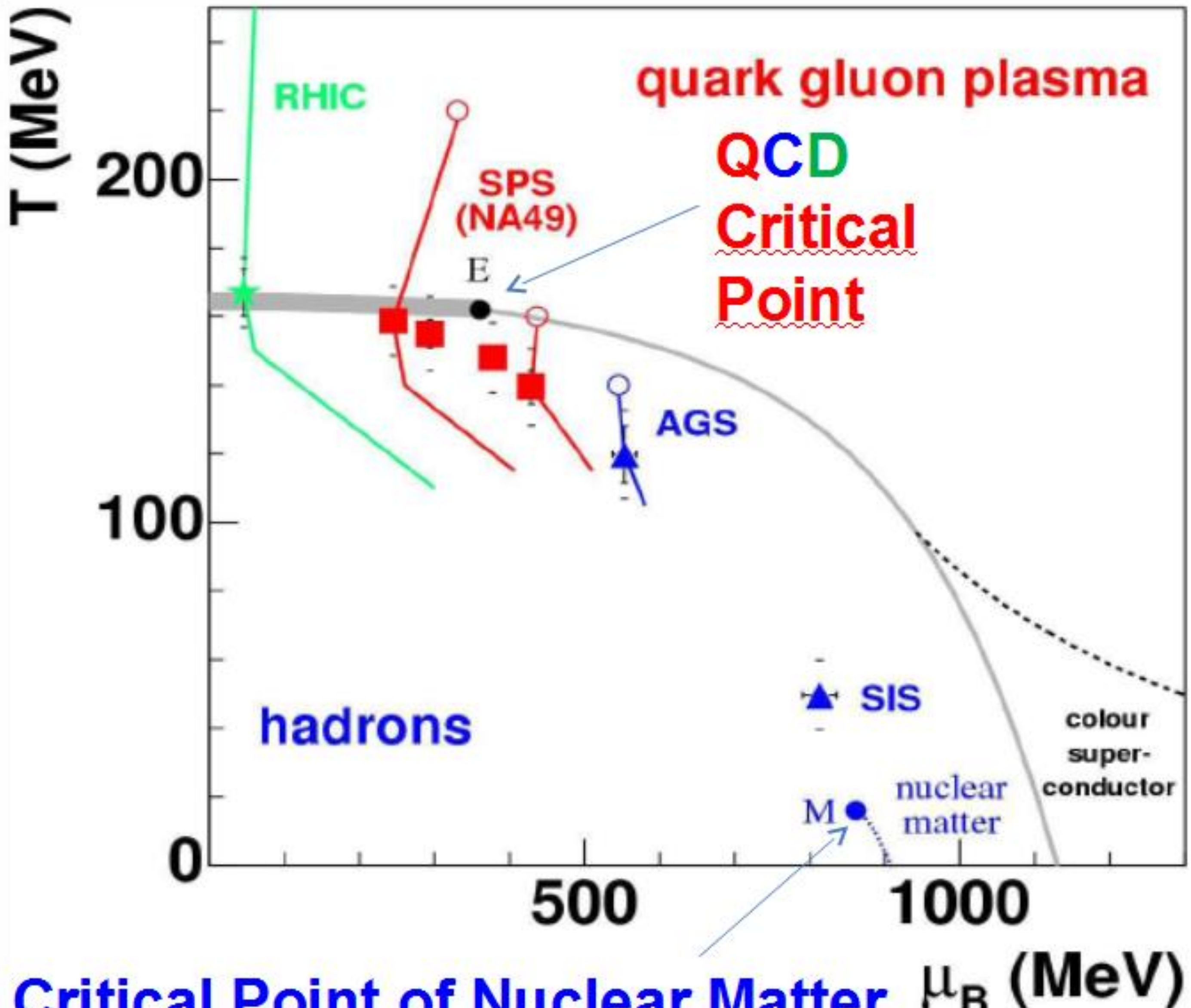
I. Quantum van der Waals Model

II. Critical Point and fluctuations

III. van der Waals interactions between baryons and between antibaryons in the hadron resonance gas

III. STAR data for net proton fluctuations in Pb+Pb collisions

IV. Electric charge fluctuations in the Critical Point



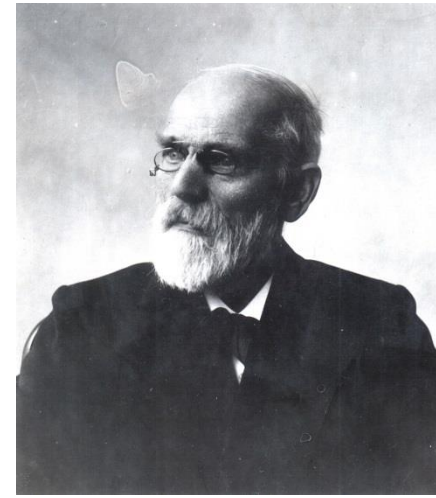
NA61/SHINE
CERN, SPS

STAR
BNL, RHIC

Critical Point of Nuclear Matter

I. Quantum van der Waals Equation of State

van der Waals



1837-1923

1873, Ph. D. Thesis

1910, Nobel Prize in Physics

$$p(V,T,N) = \frac{NT}{V-bN} - a\frac{N^2}{V^2} = \frac{nT}{1-bn} - an^2,$$

$$\frac{\partial p(T,n)}{\partial n} = 0\,, \qquad \frac{\partial p^2(T,n)}{\partial n^2} = 0$$

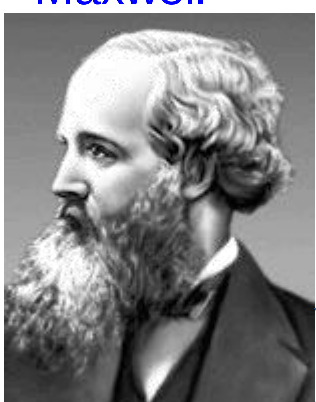
Critical Point

$$T_c = \frac{8a}{27b}, \quad n_c = \frac{1}{3b}, \quad p_c = \frac{a}{27b^2}$$

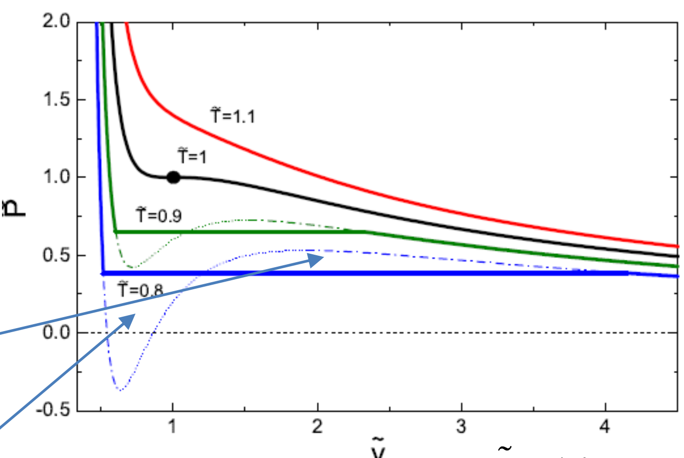
$$\tilde{n} = n/n_c, \quad \tilde{p} = p/p_c, \quad \tilde{T} = T/T_c,$$

$$\tilde{p} = \frac{8\tilde{T}\tilde{n}}{3-\tilde{n}} - 3\tilde{n}^2$$

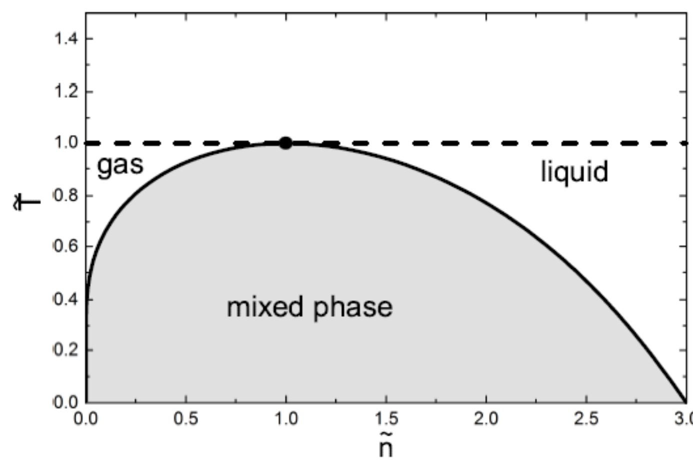
Maxwell



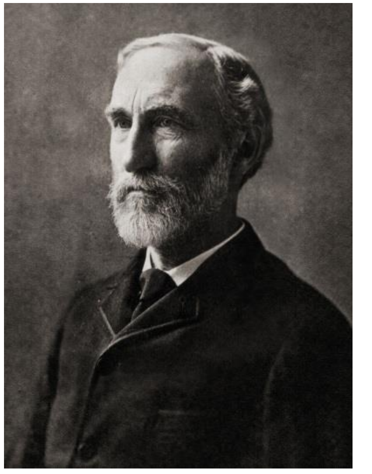
1831-1879



$\tilde{v} \equiv 1/n$



Willadr Gibbs



1839-1903

Chemical potential was invented by Gibbs in 1873

GCE: $p(T,\mu) = p_{\text{id}}\left(T,\mu^*\right) - an^2$

$$\mu^* = \mu - bp(T,\mu) - abn^2 + 2an$$

Vovchenko, Anchishkin, M.I.G., J.Phys. G (2015)

GCE

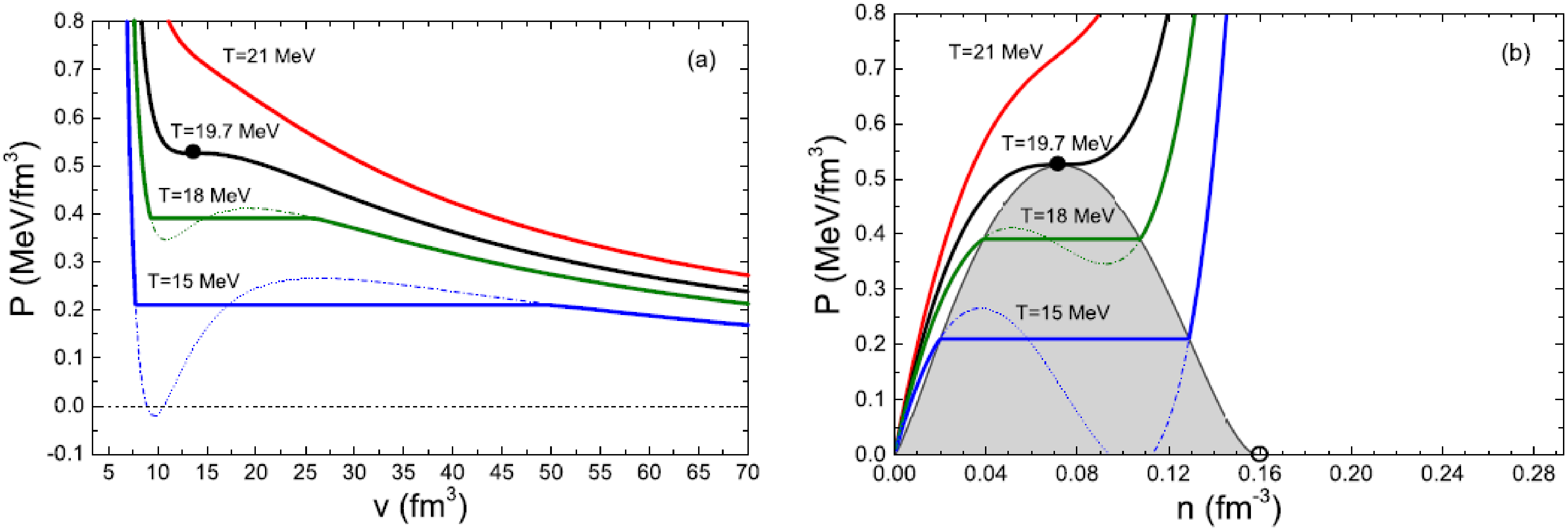
$$p_{\text{id}}\left(T,\mu\right) = \frac{d}{6\pi^2}\int_0^\infty k^2 dk \, \frac{k^2}{\sqrt{k^2+m^2}}\left[\exp\left(\frac{\sqrt{k^2+m^2}}{T} - \frac{\mu}{T}\right) \pm 1\right]^{-1}$$

$$n = \left(\frac{\partial p}{\partial \mu}\right)_T, \quad s = \left(\frac{\partial p}{\partial T}\right)_\mu$$

Vovchenko, Anchishkin, M.I.G., Phys. Rev. C (2015)

Quantum vdW

Nuclear Matter = nucleons with van der Waals EoS



Fermi Statistics, $g = 4$, $m \cong 938$ MeV a, b - ?

$T = 0$, $p = 0$: $\varepsilon / n - m = -16$ MeV, $n = n_0 = 0.16$ fm^{-3}

$a = 329$ MeV fm^3, $b = 3.42$ fm^3

$T_c \cong 19.7$ MeV,

$n_c \cong 0.07$ fm^{-3}

II. Critical Point and Fluctuations

Scaled Variance

Vovchenko, Anchishkin, M.I.G., J.Phys. A (2015)

$$\omega[N] \equiv \frac{< N^2 > - < N >^2}{< N >} = \left[\frac{1}{(1-bn)^2} - \frac{2an}{T} \right]^{-1}$$

$$a = 0, \quad b = 0 \ \rightarrow \ \omega[N] = 1$$

$$a = 0 \rightarrow \ \omega[N] = (1-bn)^2 < 1$$

M.I.G., Hauer, Nikolajenko,
Phys. Rev. C (2007)
Excluded Volume Model, i.e. a=0

$$n \rightarrow 0 \quad \omega[N] = 1$$

Skewness and Kurtosis

Central Moments: $<(\Delta N)^2>,\ <(\Delta N)^3>,\ <(\Delta N)^4>, \ldots$

Scaled Variance: $\omega[N] = \dfrac{<(\Delta N)^2>}{<N>}, \qquad \Delta N = N - <N>$

Skewness: $S\sigma = \dfrac{<(\Delta N)^3>}{<(\Delta N)^2>},$

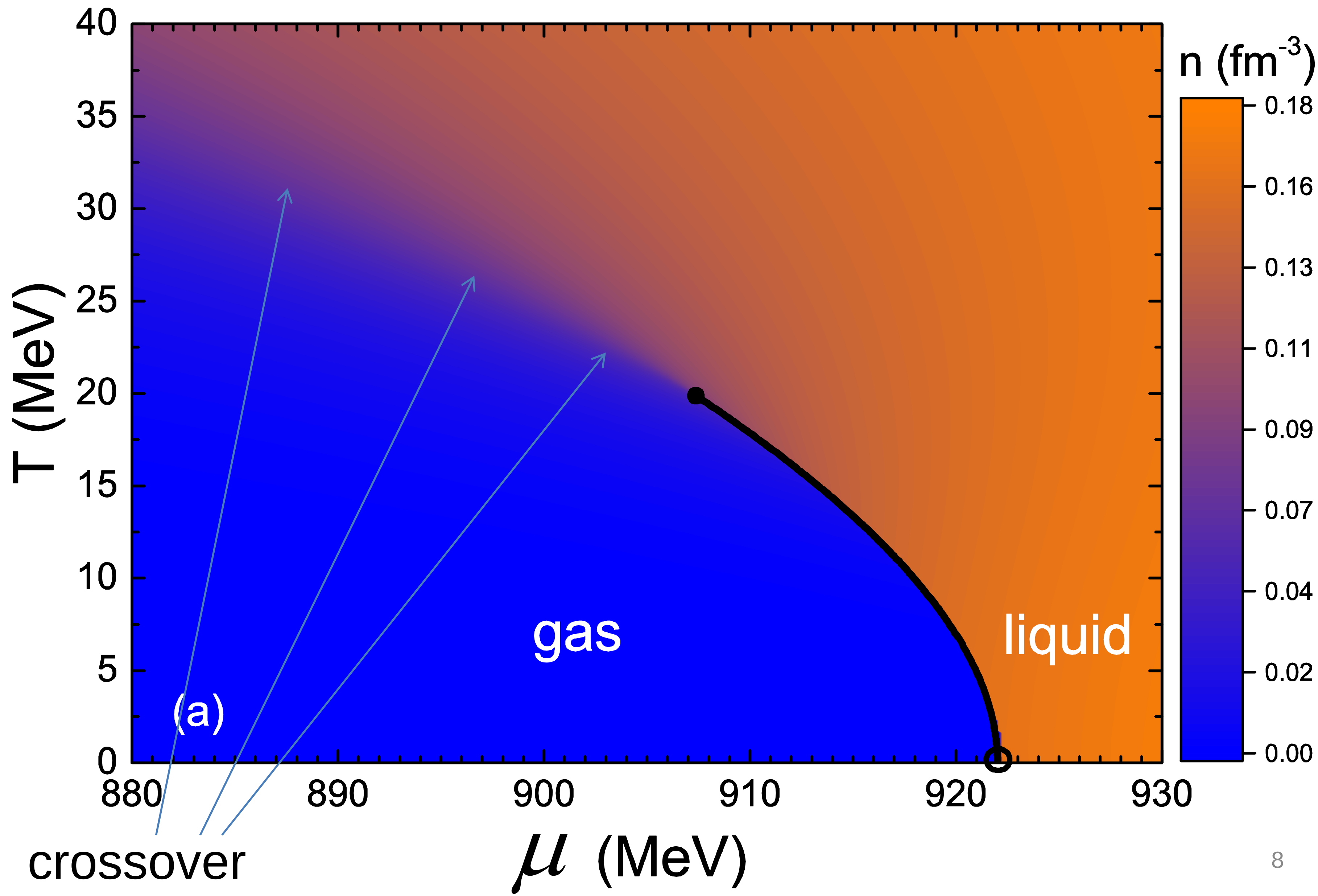
Kurtosis: $\kappa\sigma^2 = \dfrac{<(\Delta N)^4 - 3<(\Delta N)^2>^2}{<(\Delta N)^2>}.$

Cummulants: $\mathrm{k}_n = \dfrac{\partial^n (p/T^4)}{\partial(\mu/T)^n}, \quad n = 1, 2, \ldots$

$$\omega[N] = \frac{\mathrm{k}_2}{\mathrm{k}_1}, \quad S\sigma = \frac{\mathrm{k}_3}{\mathrm{k}_2}, \quad \kappa\sigma^2 = \frac{\mathrm{k}_4}{\mathrm{k}_2}.$$

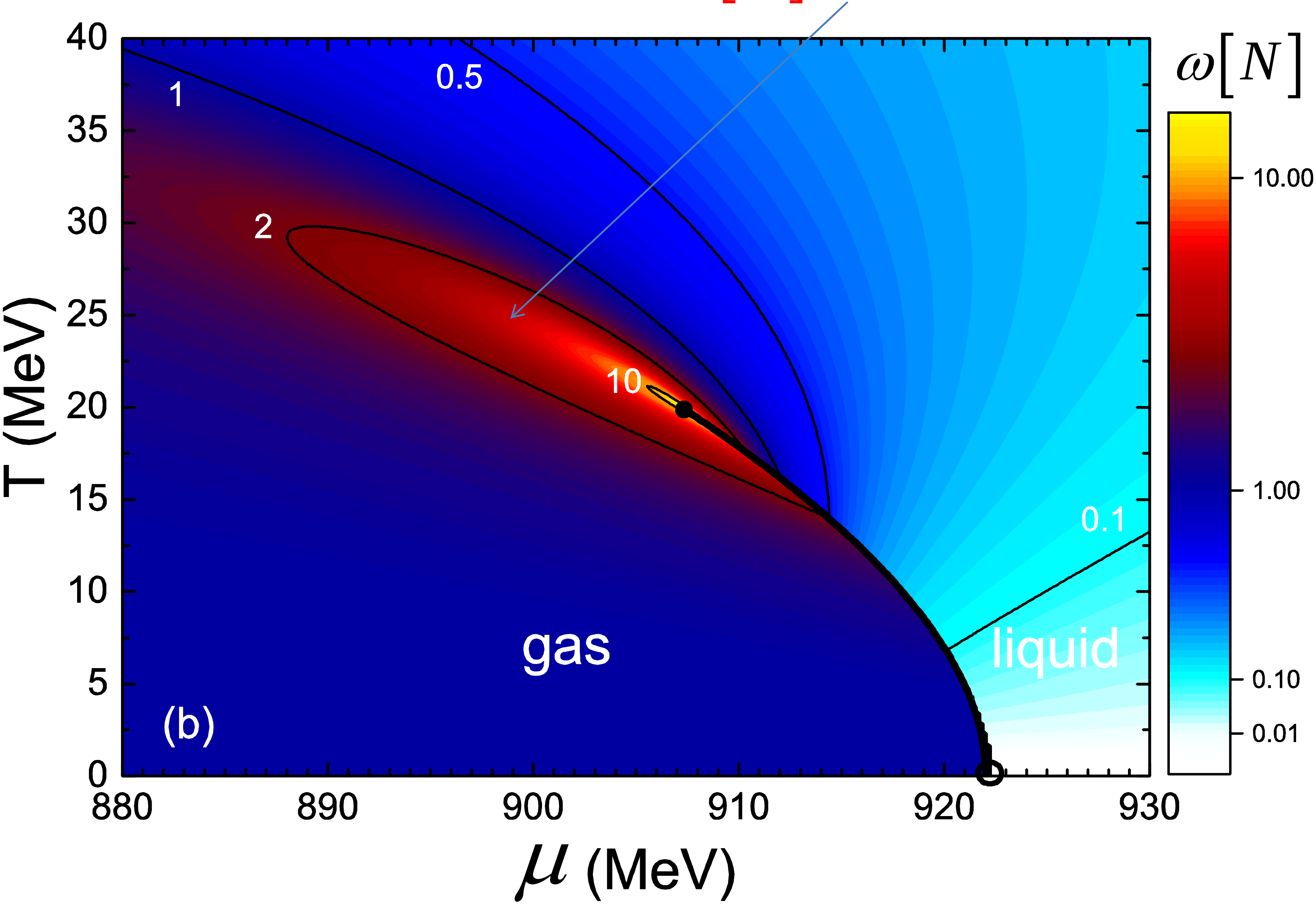
Vovchenko, Anchishkin, M.I.G., Poberezhnjuk, Phys. Rev. C (2015)

Particle Number Density $n(T,\mu)$ $\mu_c \cong 908$ MeV

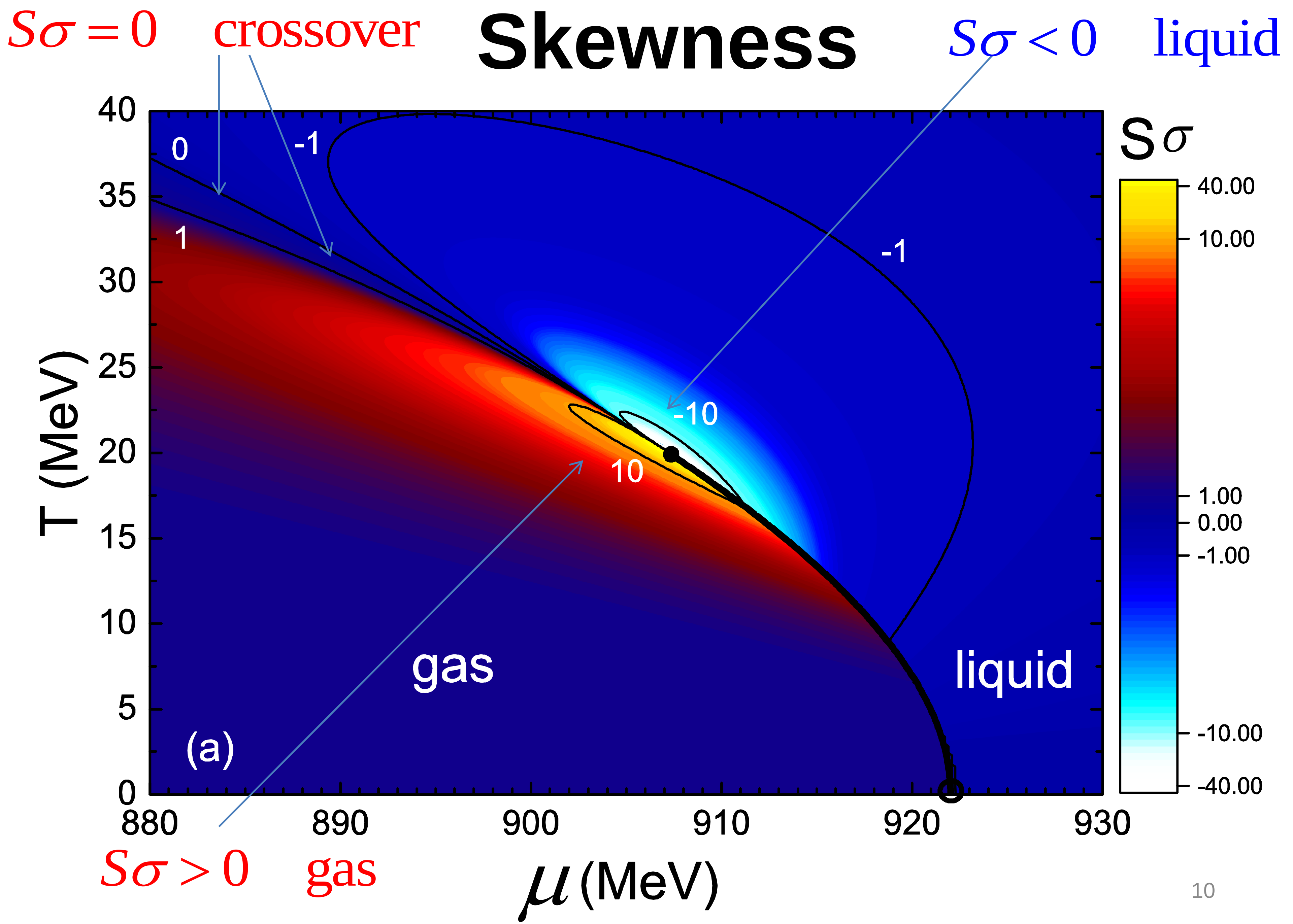


Scaled Variance

$\omega[N]$ >>1 along crossover

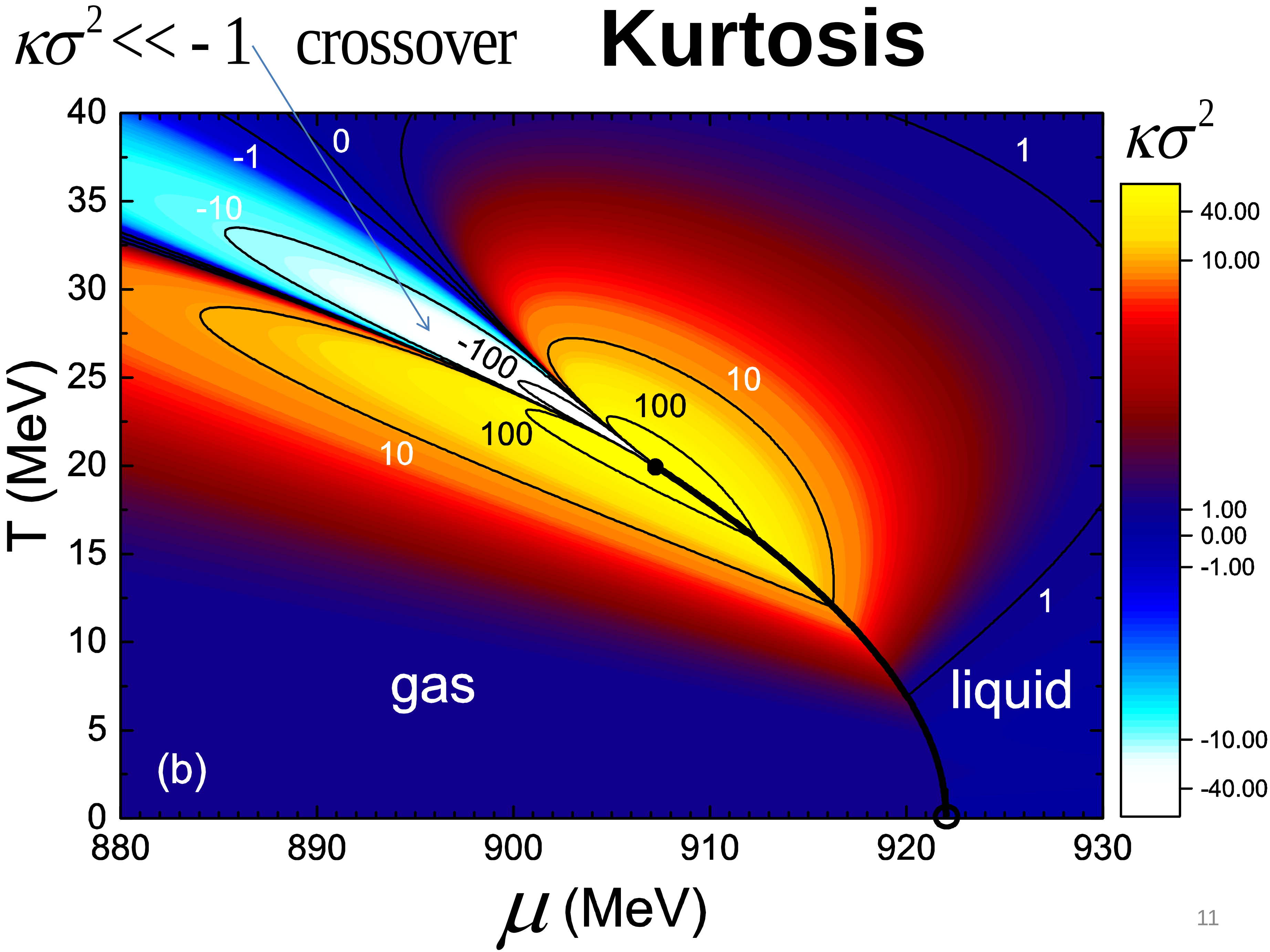


Skewness



$S\sigma = 0$ crossover
$S\sigma < 0$ liquid
$S\sigma > 0$ gas
$S\sigma$
40.00
10.00
1.00
0.00
-1.00
-10.00
-40.00
0
1
-1
-1
-10
10
gas
liquid
(a)
T (MeV)
μ (MeV)
40
35
30
25
20
15
10
5
0
880
890
900
910
920
930

Kurtosis



$\kappa\sigma^2 << -1$ crossover
40
35
30
25
20
15
10
5
0
T (MeV)
880
890
900
910
920
930
μ (MeV)
-1
0
-10
-100
10
100
100
10
1
1
gas
liquid
(b)
$\kappa\sigma^2$
40.00
10.00
1.00
0.00
-1.00
-10.00
-40.00

III. QvdW equation for the HRG

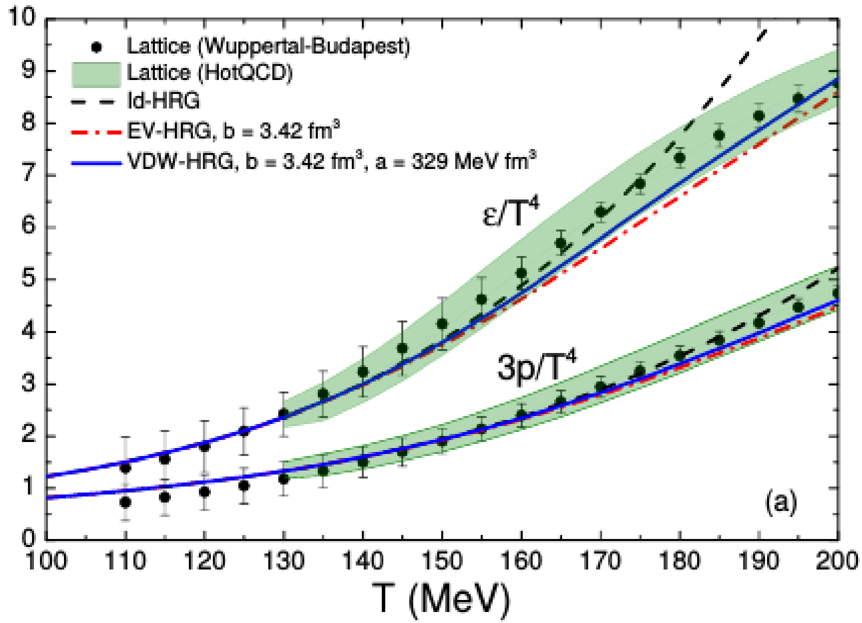
$$p(T,\mu) = p_M + p_B + p_{\bar{B}}, \quad p_M = \sum_{i\in M} p_i^{\mathrm{id}}(T,\mu_i),$$

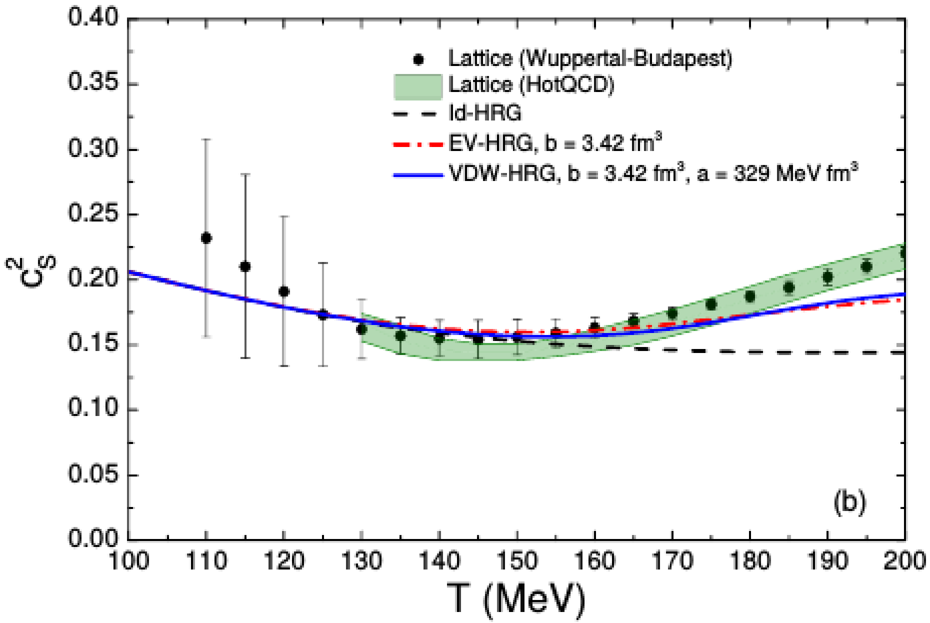
Vovchenko, M.I.G.,Stoecker, Phys. Rev. Lett. (2017)

$$p_B = \sum_{i\in B} p_i^{\mathrm{id}}(T,\mu_i^*) - a n_B^2, \quad p_{\bar{B}} = \sum_{i\in B} p_i^{\mathrm{id}}(T,\mu_i^*) - a n_{\bar{B}}^2,$$

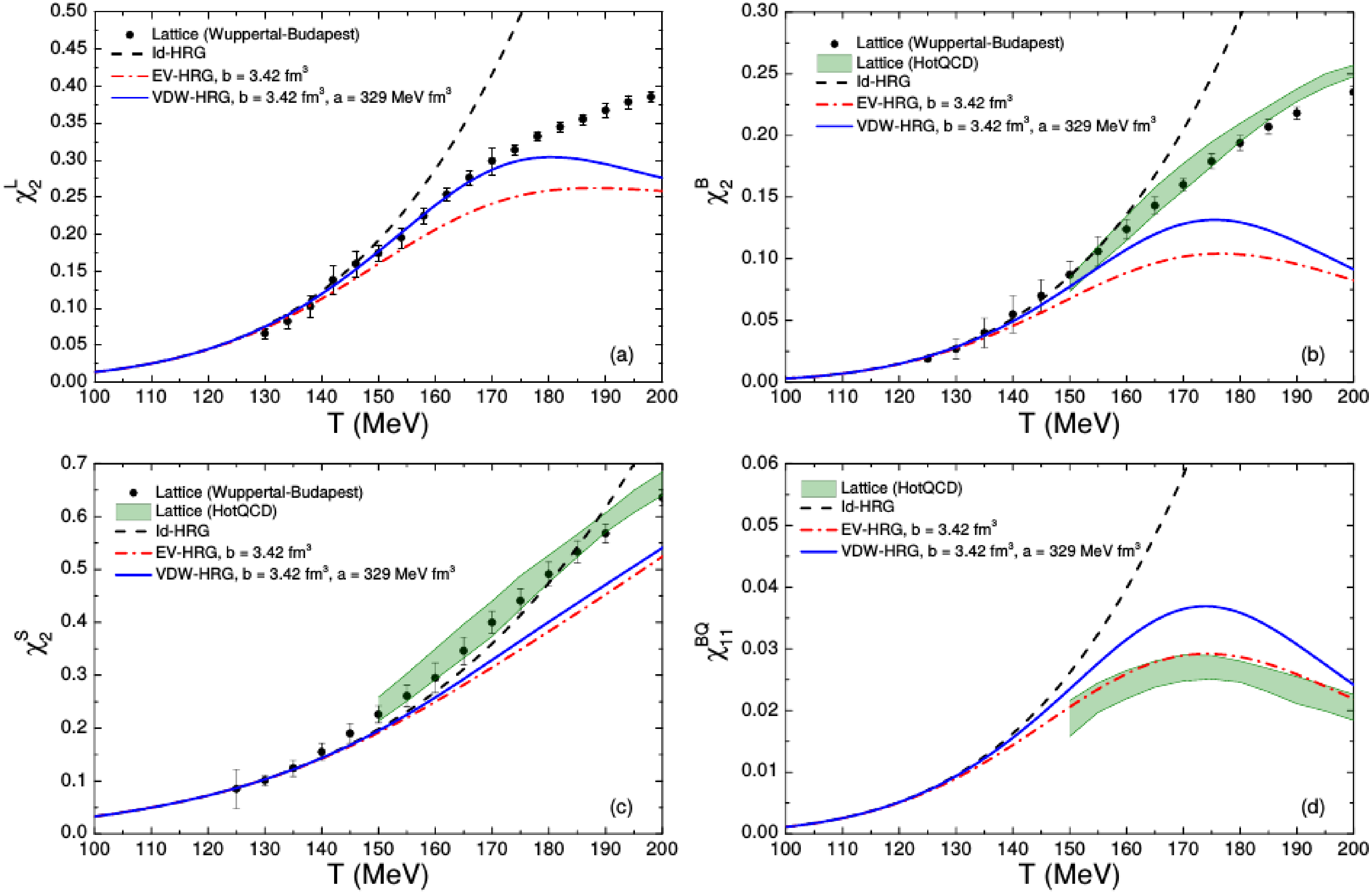
$$\mu_i^{B(\bar{B})*} = \mu_i - b p_{B(\bar{B})} - a b n_{B(\bar{B})}^2 + 2 a n_{B(\bar{B})},$$

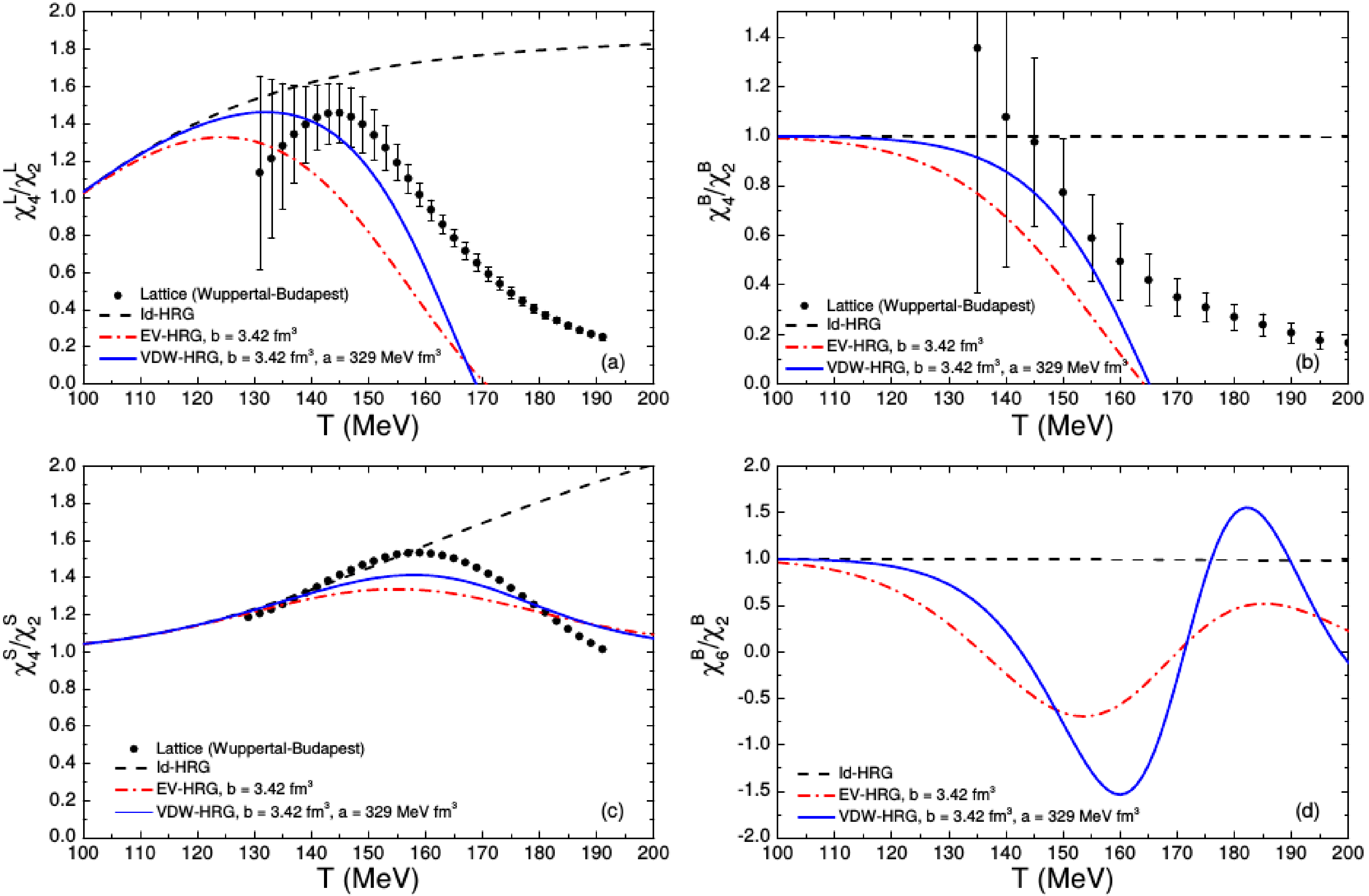
$$\mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$$



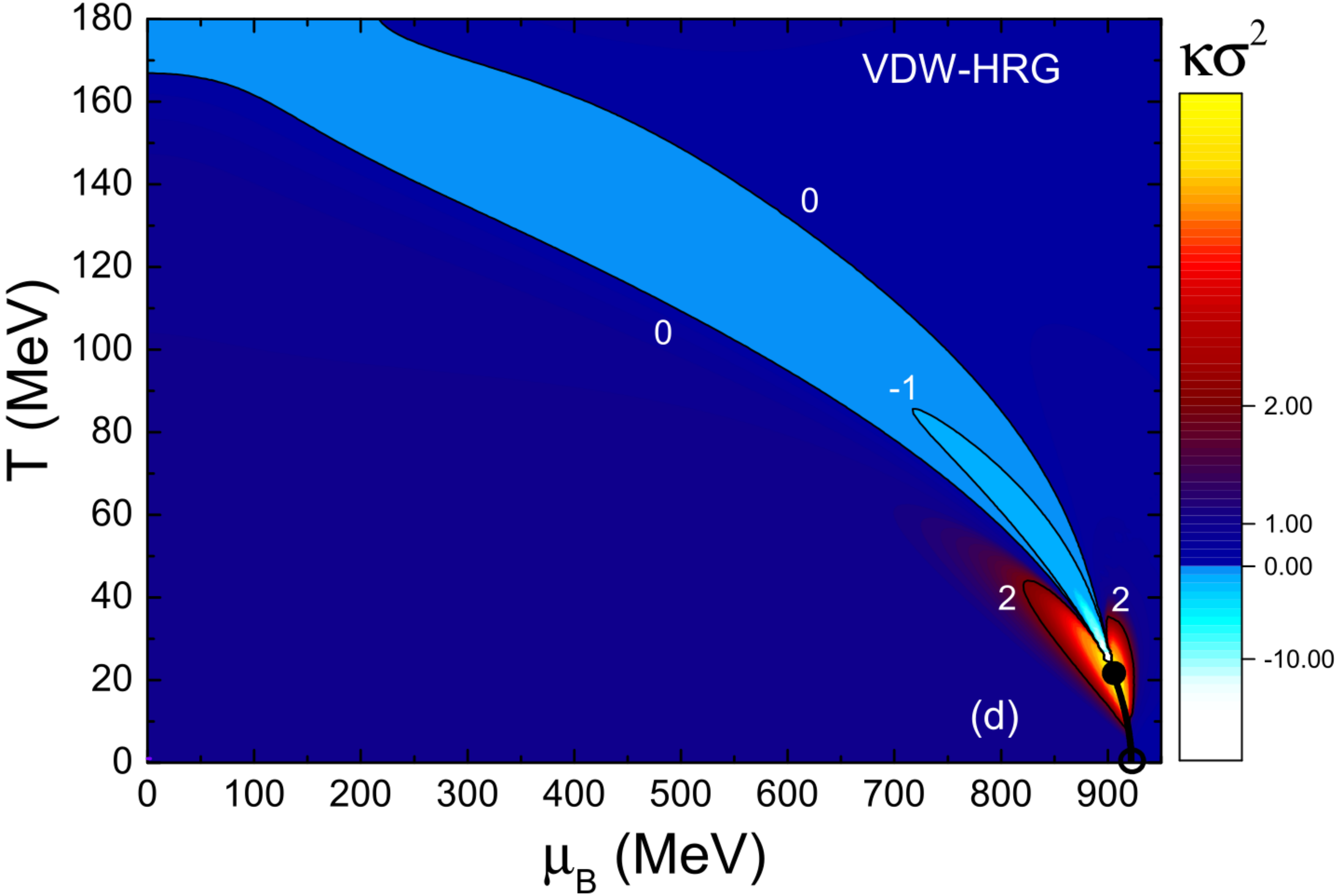


$$\chi_{lmn}^{BQS} = \frac{\partial^{lmn}(p/T^4)}{\partial(\mu_B/T)^l\,\partial(\mu_Q/T)^m\,\partial(\mu_S/T)^n}$$





(a)
χ_4^L/χ_2^L
T (MeV)
Lattice (Wuppertal-Budapest)
Id-HRG
EV-HRG, b = 3.42 fm^3
VDW-HRG, b = 3.42 fm^3, a = 329 MeV fm^3
(b)
χ_4^B/χ_2^B
T (MeV)
Lattice (Wuppertal-Budapest)
Id-HRG
EV-HRG, b = 3.42 fm^3
VDW-HRG, b = 3.42 fm^3, a = 329 MeV fm^3
(c)
χ_4^S/χ_2^S
T (MeV)
Lattice (Wuppertal-Budapest)
Id-HRG
EV-HRG, b = 3.42 fm^3
VDW-HRG, b = 3.42 fm^3, a = 329 MeV fm^3
(d)
χ_6^B/χ_2^B
T (MeV)
Id-HRG
EV-HRG, b = 3.42 fm^3
VDW-HRG, b = 3.42 fm^3, a = 329 MeV fm^3



VDW-HRG
κσ²
T (MeV)
μB (MeV)
0
0
-1
2
2
(d)
180
160
140
120
100
80
60
40
20
0
0
100
200
300
400
500
600
700
800
900
2.00
1.00
0.00
-10.00

IV. STAR data for net proton fluctuations in Pb+Pb collisions

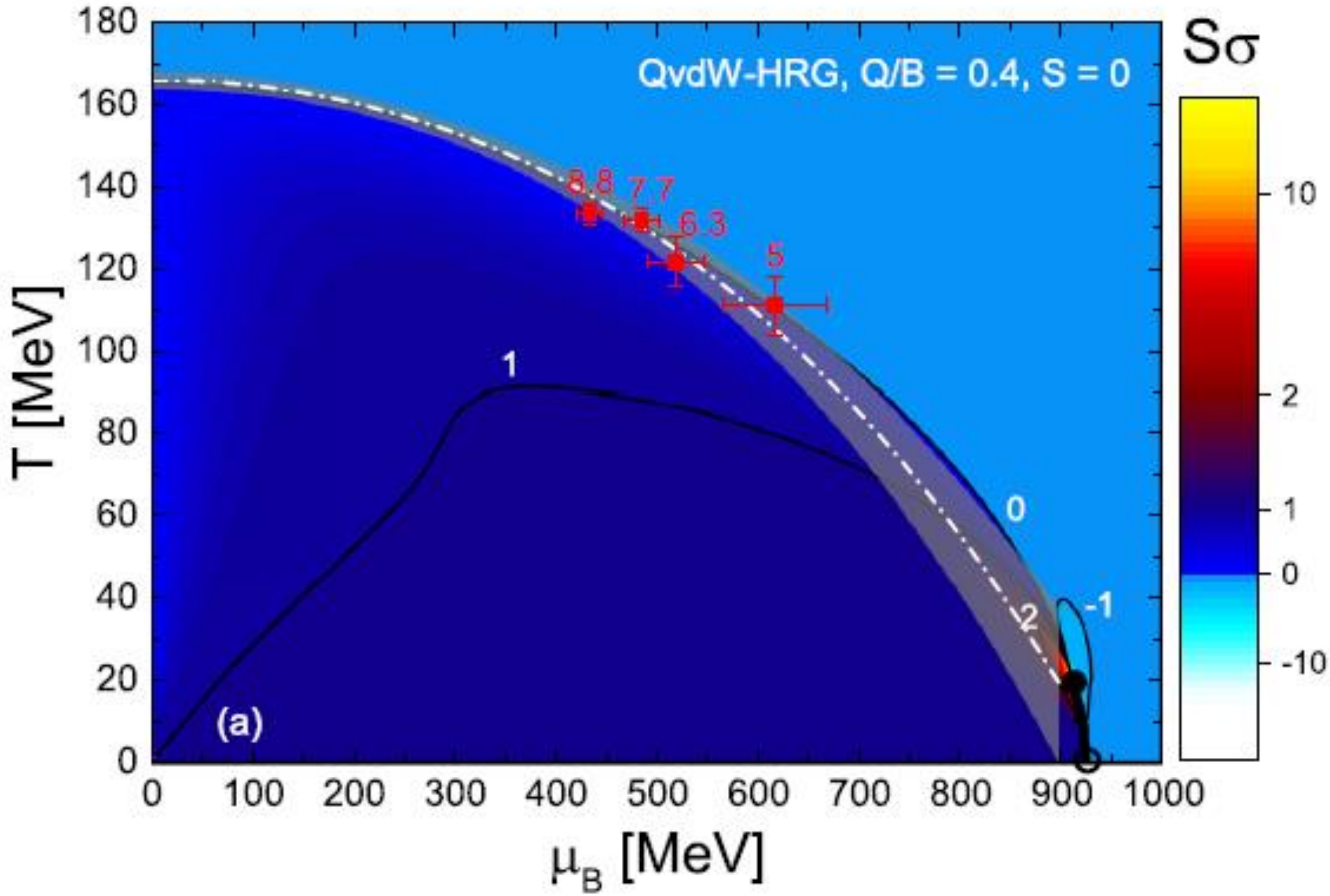
Vovchenko, Lijia Jiang, M.I.G., Stoecker,
Phys. Rev. C (2018)

$$\chi_n = \frac{\partial^n \left(p / T^4 \right)}{\partial \left(\mu_B / T \right)^n} = \chi_n^B + \left(-1 \right)^n \chi_n^{\bar{B}}$$

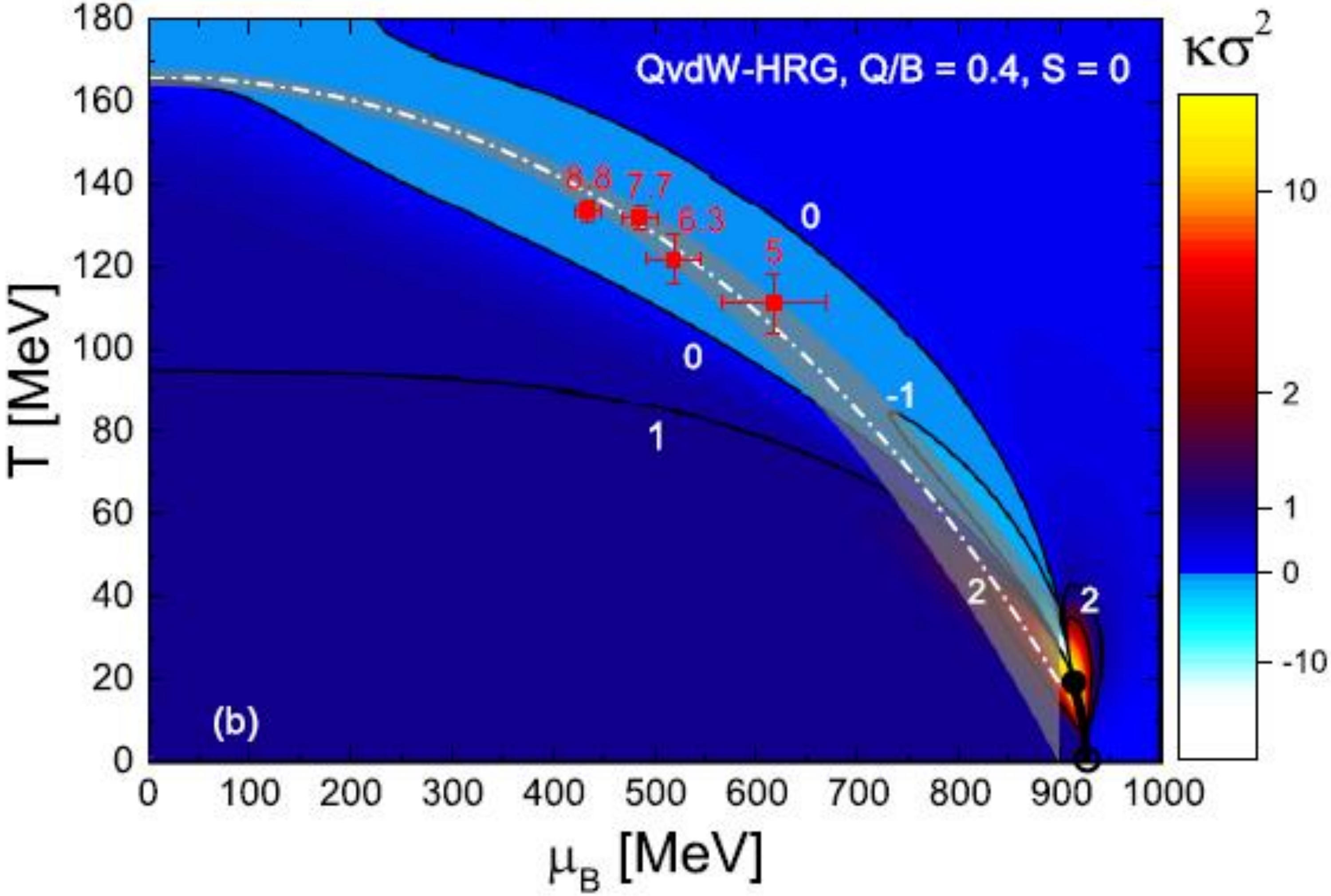
$$S\sigma = \frac{\chi_3}{\chi_2}, \qquad \kappa\sigma^2 = \frac{\chi_4}{\chi_2},$$

$$P\left(n\right) = \sum_{N=n}^{\infty} \Pi\left(N\right) \frac{N!}{n!\left(N-n\right)!} q^n \left(1-q\right)^{N-n}$$

$0 \le q \le 1$ Acceptance parameter

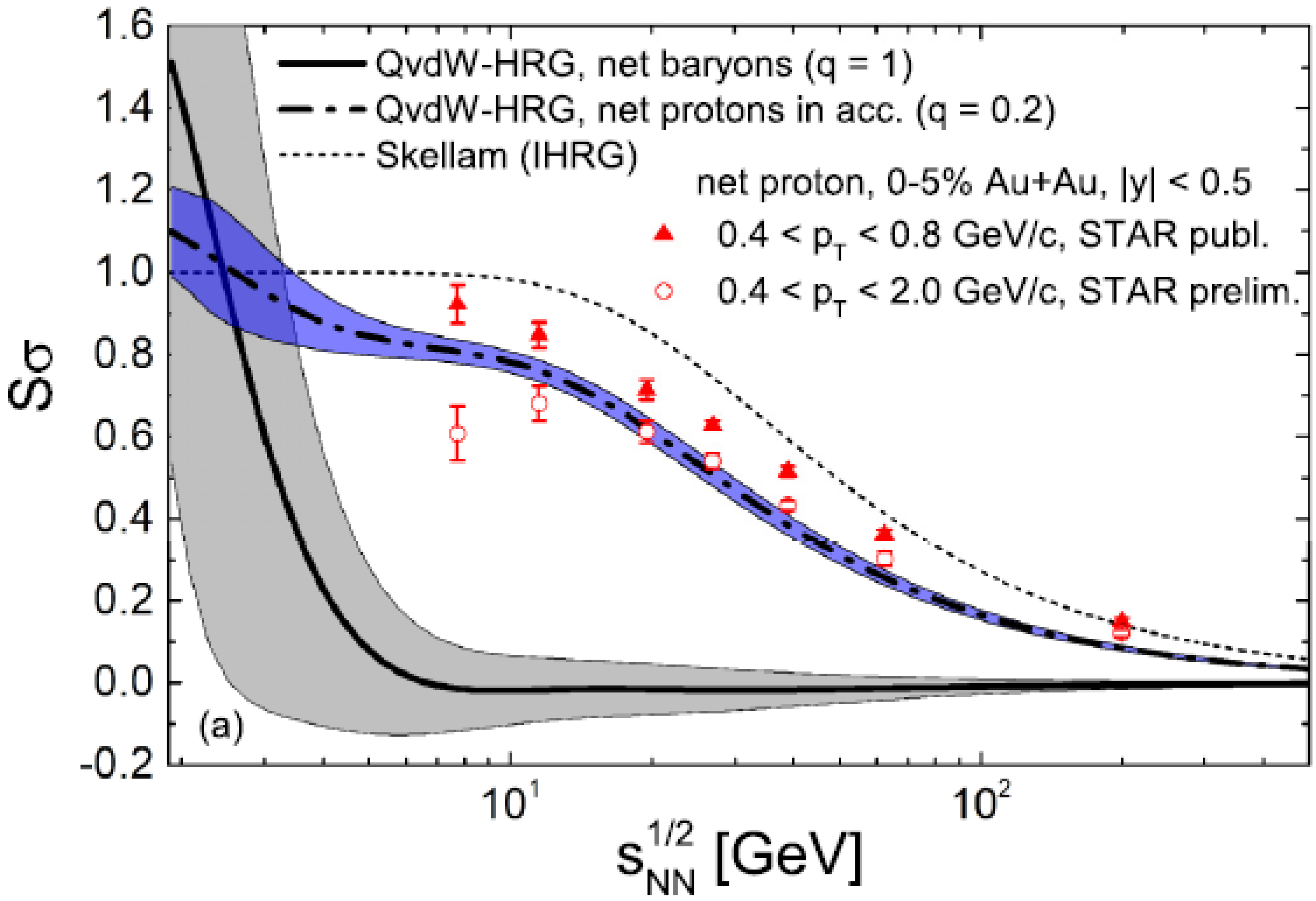


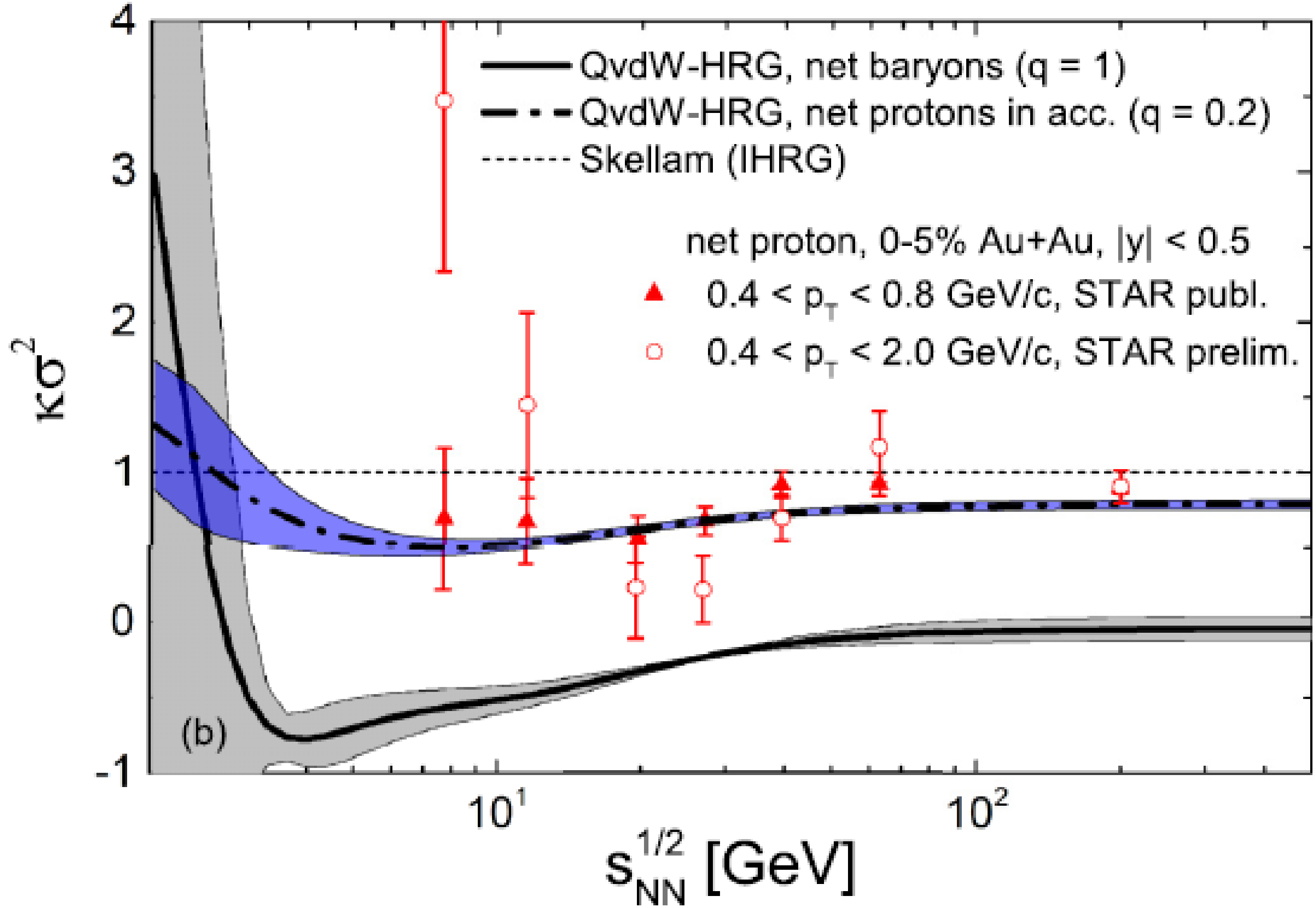
QvdW-HRG, Q/B = 0.4, S = 0
Sσ
T [MeV]
μ_B [MeV]
8.8
7.7
6.3
5
1
0
-1
2
(a)



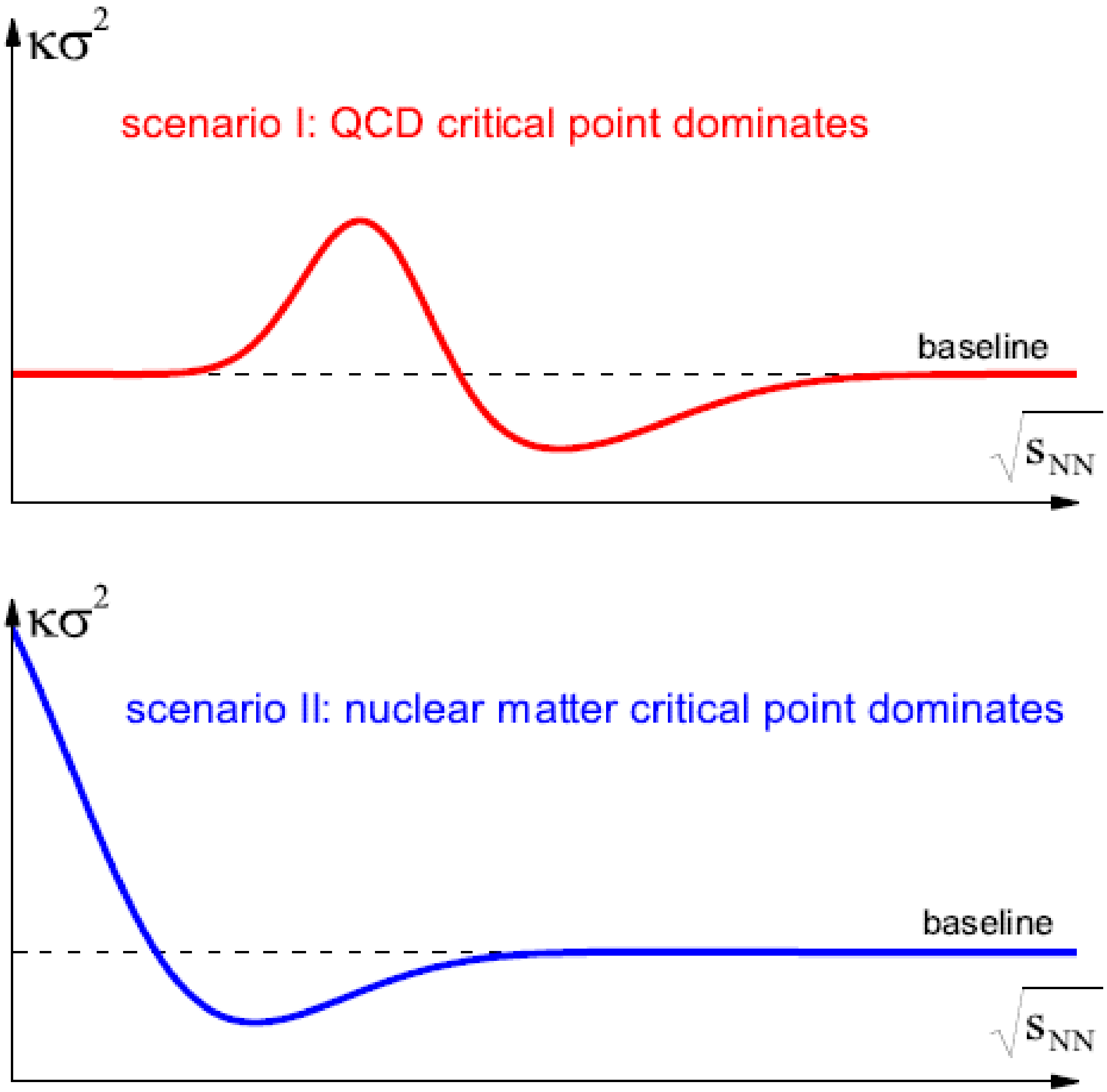
QvdW-HRG, Q/B = 0.4, S = 0
κσ²
(b)
T [MeV]
μB [MeV]
8.8
7.7
6.3
5
0
0
-1
1
2
2
180
160
140
120
100
80
60
40
20
0
100
200
300
400
500
600
700
800
900
1000
10
2
1
0
-10

STAR data for net-proton fluctuations





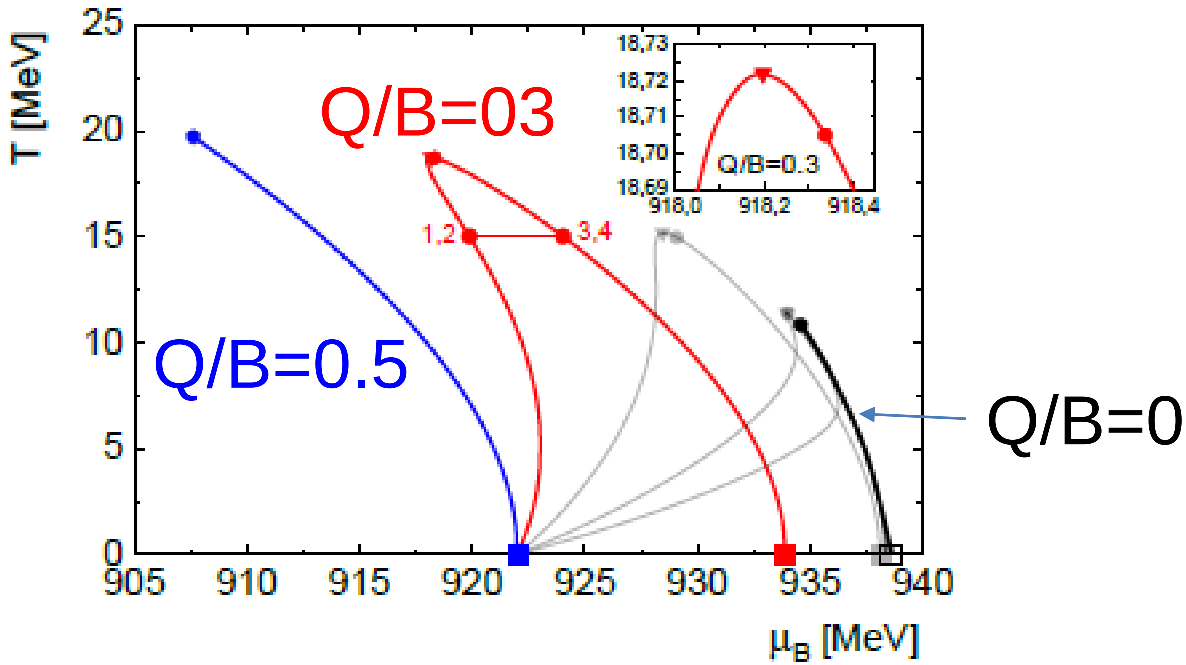
QvdW-HRG, net baryons (q = 1)
QvdW-HRG, net protons in acc. (q = 0.2)
Skellam (IHRG)
net proton, 0-5% Au+Au, |y| < 0.5
0.4 < p_T < 0.8 GeV/c, STAR publ.
0.4 < p_T < 2.0 GeV/c, STAR prelim.
(b)
κσ^2
s_NN^1/2 [GeV]
4
3
2
1
0
-1
10^1
10^2


κσ²
scenario I: QCD critical point dominates
baseline
√s_NN
κσ²
scenario II: nuclear matter critical point dominates
baseline
√s_NN

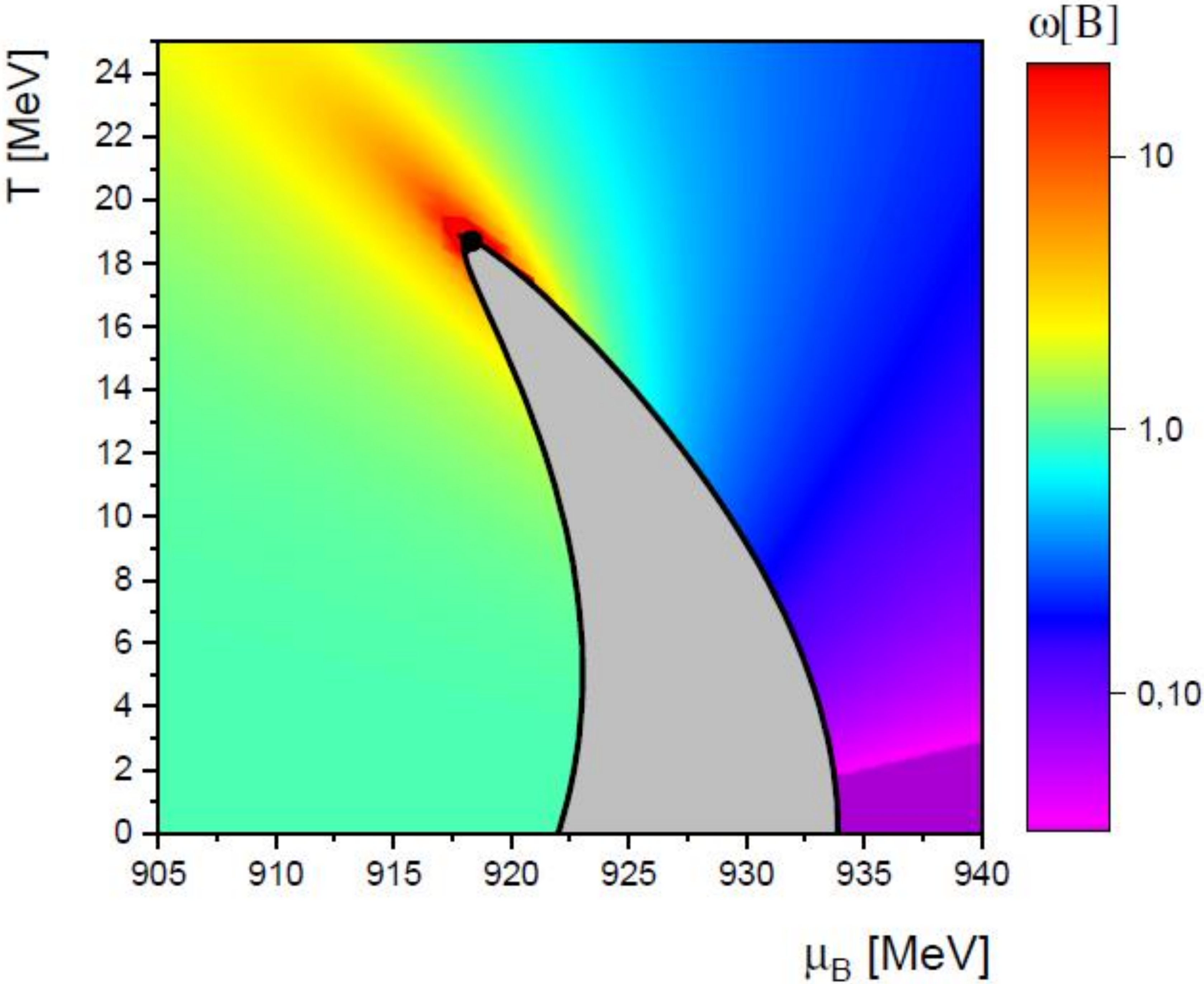
V. Electric Charge Fluctuations

$$p\left(T,\mu_B,\mu_Q\right)=p_n^{\mathrm{id}}\left(T,\mu_B^*\right)+p_p^{\mathrm{id}}\left(T,\mu_B^*+\mu_Q\right)-an_B^2,$$

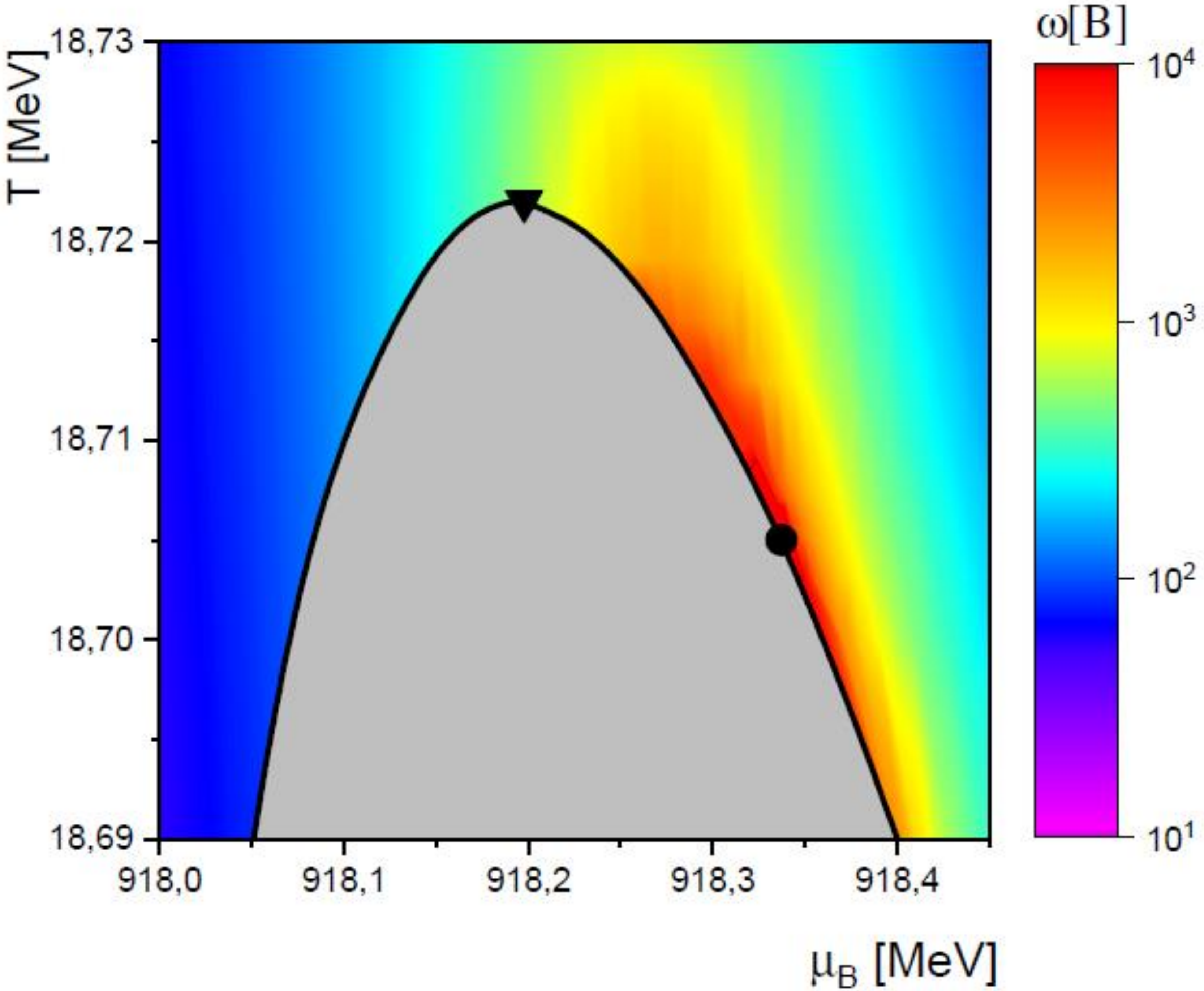
$$\mu_B^*=\mu_B-bp-abn_B^2+2an_B$$



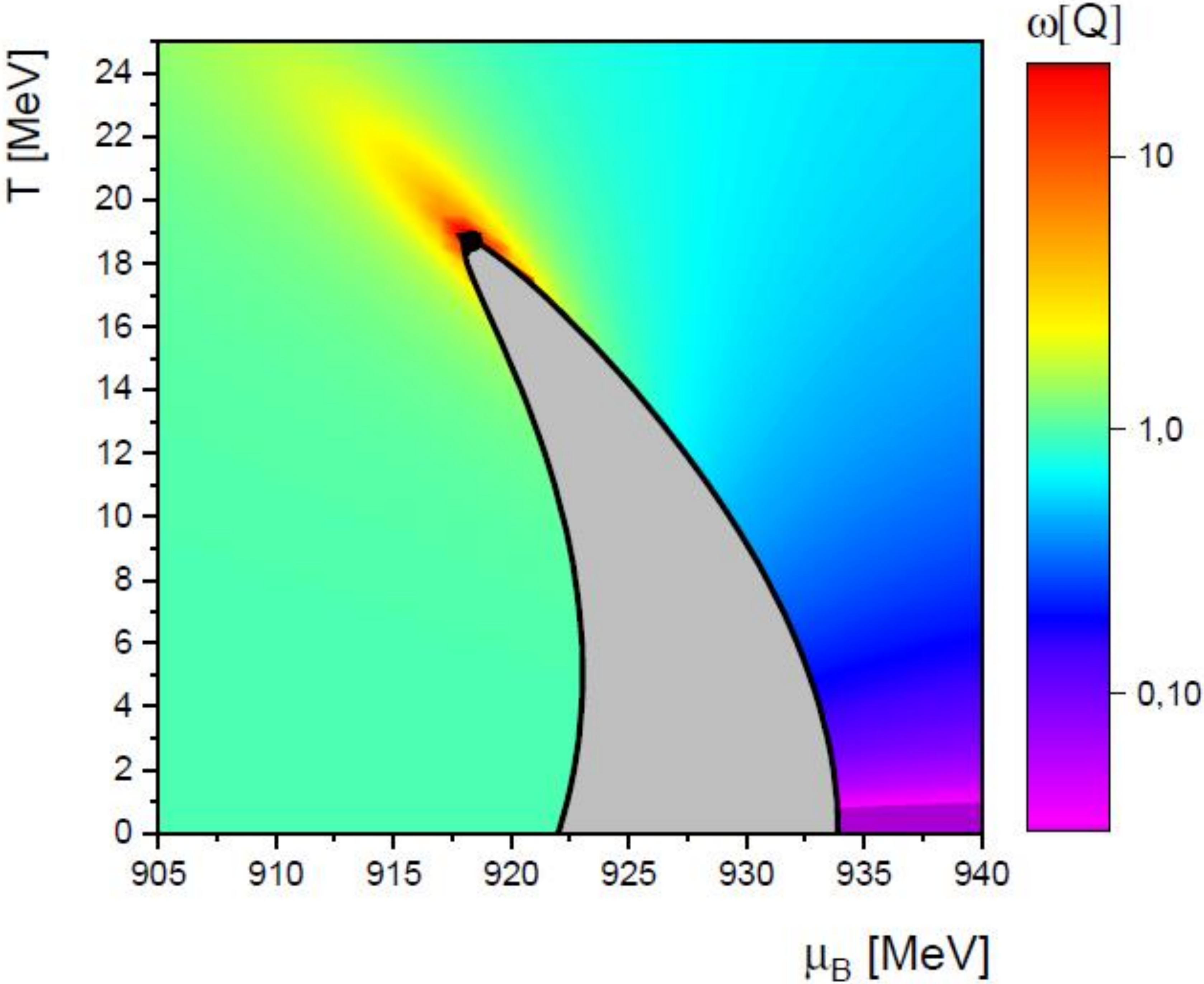
Poberezhnyuk, Vovchenko, M.I.G., Stoecker (2018)



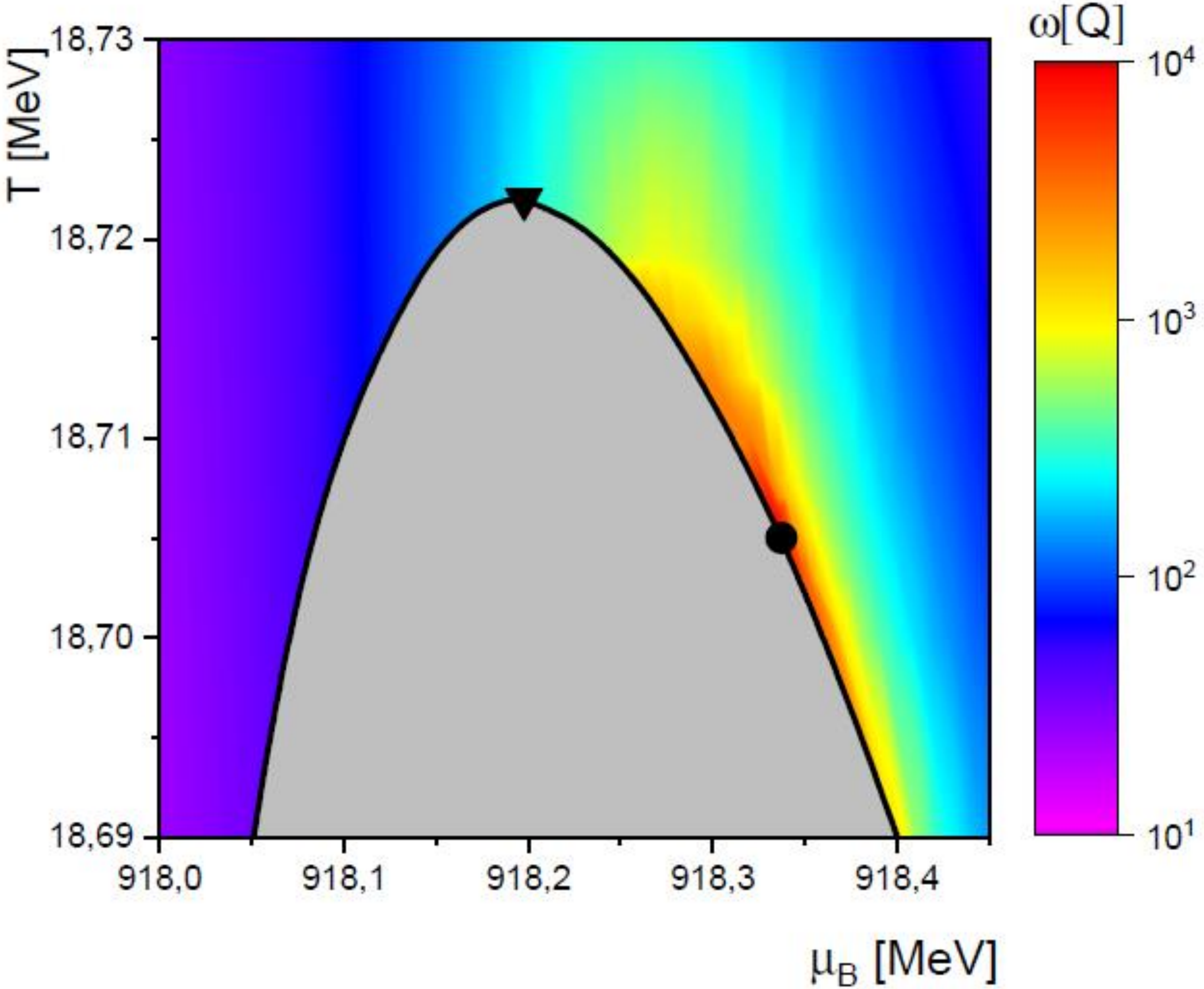
ω[B]
10
1,0
0,10
T [MeV]
24
22
20
18
16
14
12
10
8
6
4
2
0
905
910
915
920
925
930
935
940
μ_B [MeV]



T [MeV]
18,73
18,72
18,71
18,70
18,69
918,0
918,1
918,2
918,3
918,4
μ_B [MeV]
ω[B]
10^4
10^3
10^2
10^1



ω[Q]
T [MeV]
24
22
20
18
16
14
12
10
8
6
4
2
0
905
910
915
920
925
930
935
940
μB [MeV]
10
1,0
0,10



T [MeV]
18,73
18,72
18,71
18,70
18,69
918,0
918,1
918,2
918,3
918,4
μ_B [MeV]
ω[Q]
10^4
10^3
10^2
10^1

Summary

Hadron resonance gas with vdW interaction between baryons and between antibaryons

1. Provides an example of the systems with 1st order liquid-gas phase transition and critical point in nuclear matter.

2. Gives a description of thermodynamical functions and fluctuations of conserved charges in Lattice QCD.

3. Explains the STAR data for skewness and kurtosis of the net proton fluctuations.

Conclusions

Effects of the nuclear matter critical point are seen in the lattice data and in nucleus-nucleus collisions.
No signatures of the QCD critical point have been found.

NA61 CERN, STAR BNL