Fluctuations of conserved charges along the QCD phase boundary

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HotQCD Collaboration

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Spontaneous chiral symmetry breaking:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \bar{\psi}^f \left(i\gamma^\mu \partial_\mu - m_f \right) \psi^f + g \bar{\psi}^f \left(\gamma^\mu A_\mu \right) \psi^f$$

$m_f \longrightarrow 0$

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{F}}^{\mathrm{L}} + \mathcal{L}_{\mathrm{F}}^{\mathrm{R}}$$

Chiral Symmetrie

$$SU_{\mathrm{L}}(n_f) \times SU_{\mathrm{R}}(n_f) \times U_{\mathrm{L}}(1) \times U_{\mathrm{R}}(1)$$

$$=$$

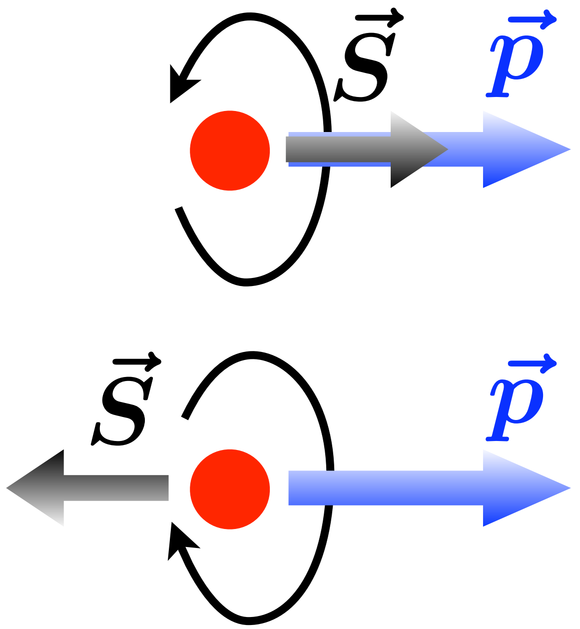
$\left\langle \bar{\psi}_{\mathrm{L}}^f \psi_{\mathrm{R}}^f \right\rangle \neq 0$

Spontaneously broken

$$SU_{\mathrm{V}}(n_f) \times SU_{\mathrm{A}}(n_f) \times U_{\mathrm{V}}(1) \times U_{\mathrm{A}}(1)$$

ψ_{R}^f

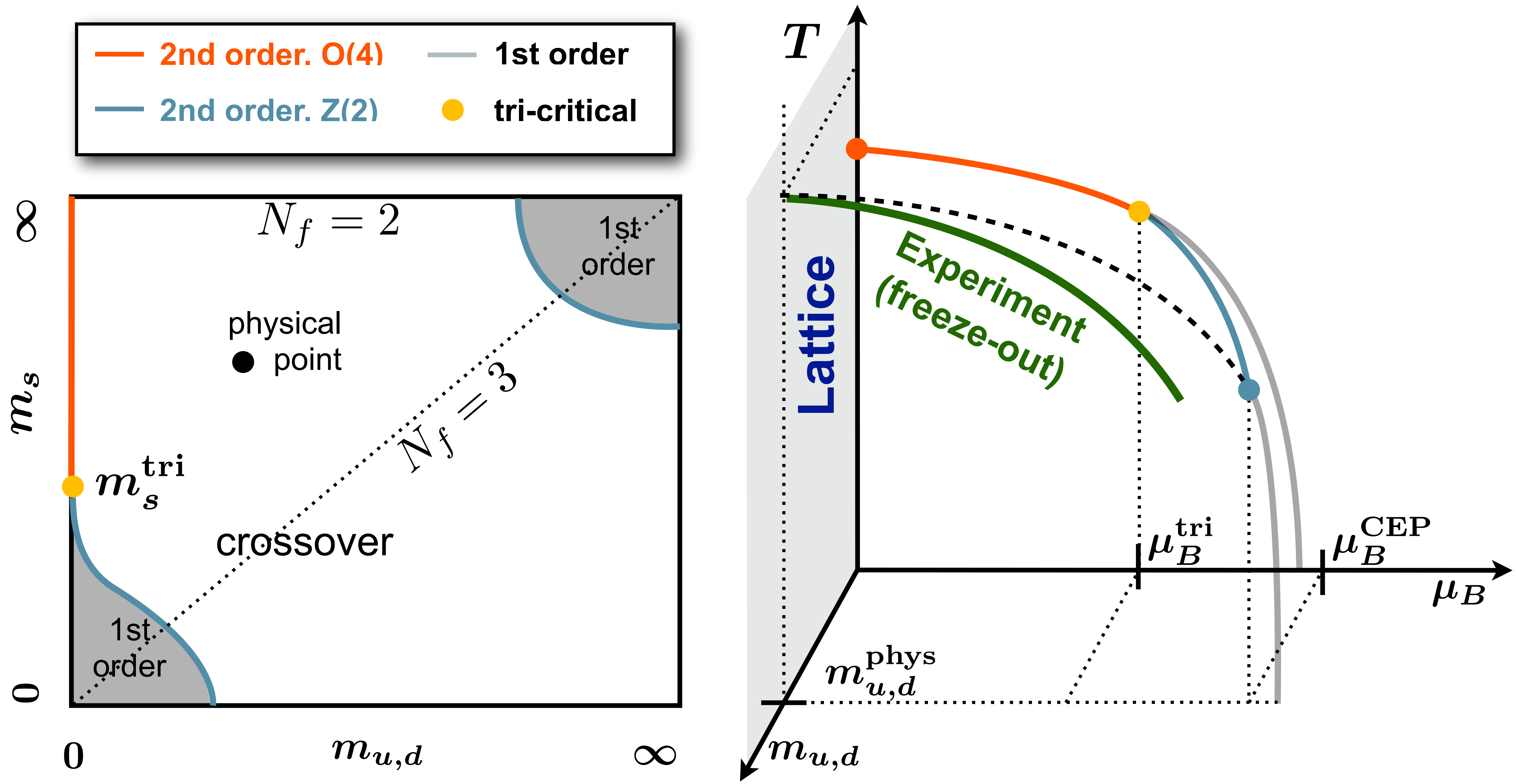
ψ_{L}^f



$\longrightarrow$ 3 massless pions as Nambu-Goldstone modes

Nobel Preis 2008: Y. Nambu

The QCD phase diagram:



- Expectations based on Pisarski, Wilczek, PRD 29 (1984)

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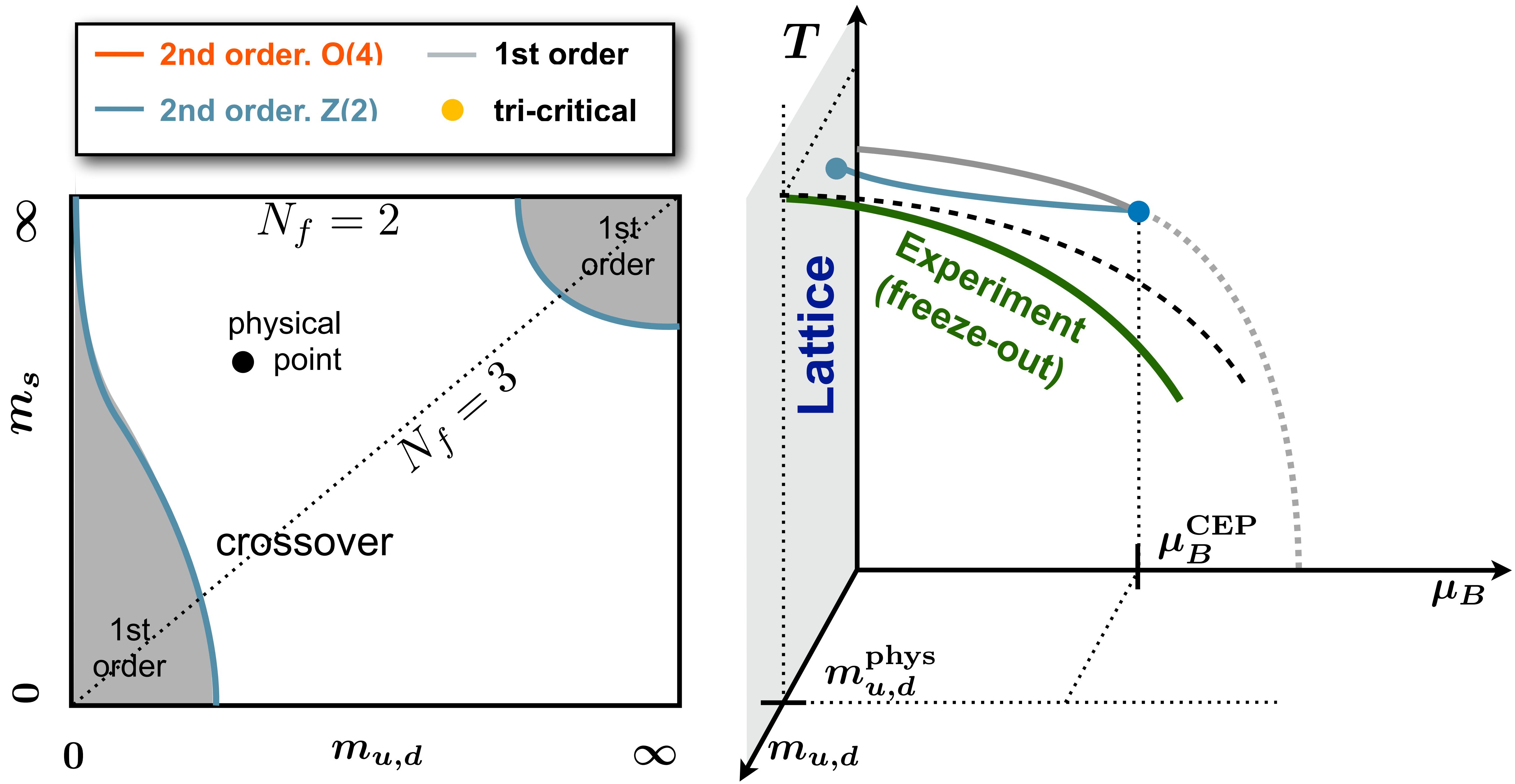
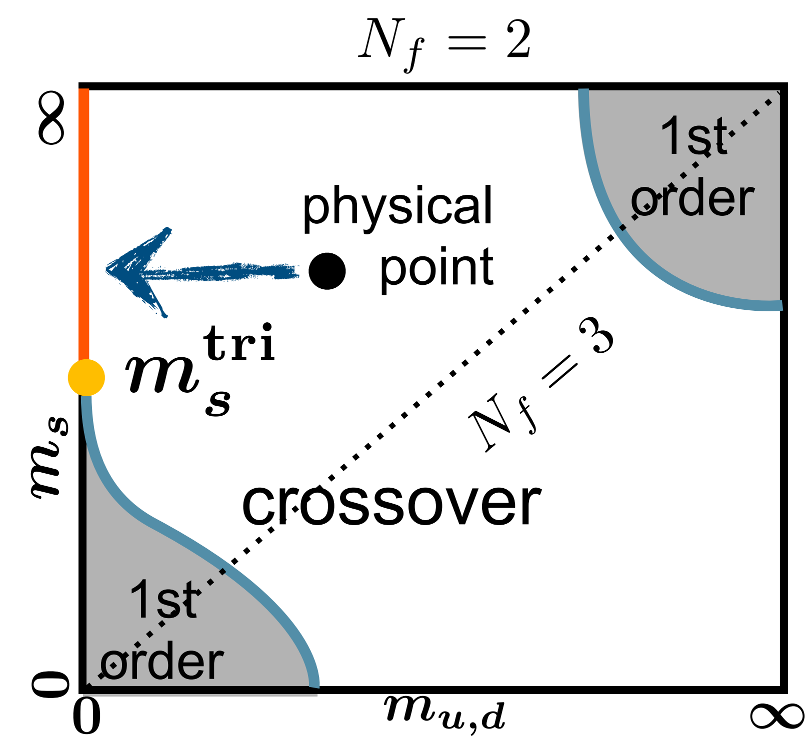


- Expectations based on Pisarski, Wilczek, PRD 29 (1984)
- Other possible scenario: Pilipsen, Pinke, PRD 93 (2016)

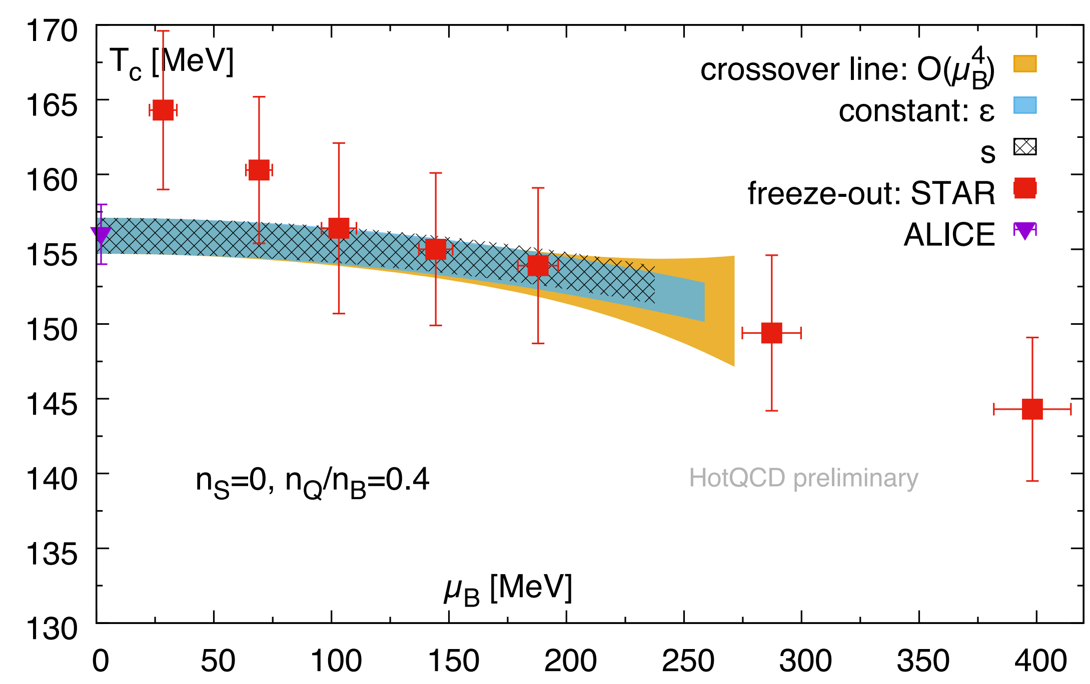
Overview:

- The lattice setup

- Scaling analysis in (2+1)-flavor QCD (sending $m_{ud} \to 0, \quad m_s$ **fixed**)
  - ➡Towards the order of the transition in the chiral limit
  - ➡A determination of the transition temperature in the chiral limit



- The crossover transition at physical quark masses and **nonzero density**
  - ➡Update on the crossover temperature
  - ➡The curvature of the crossover line (introducing a small chemical potential)



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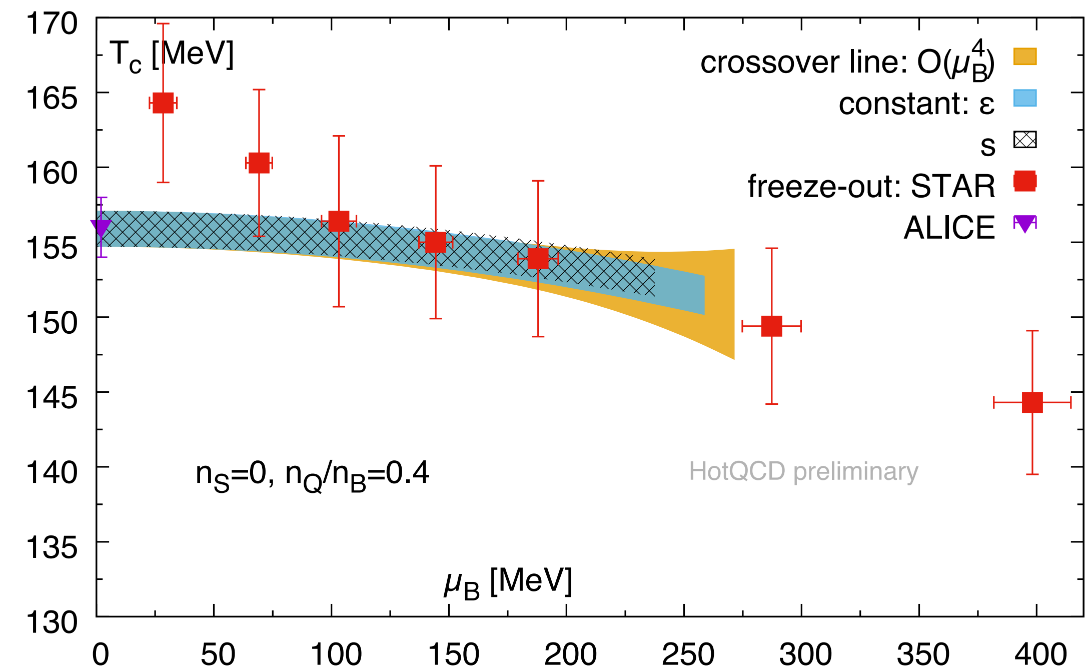
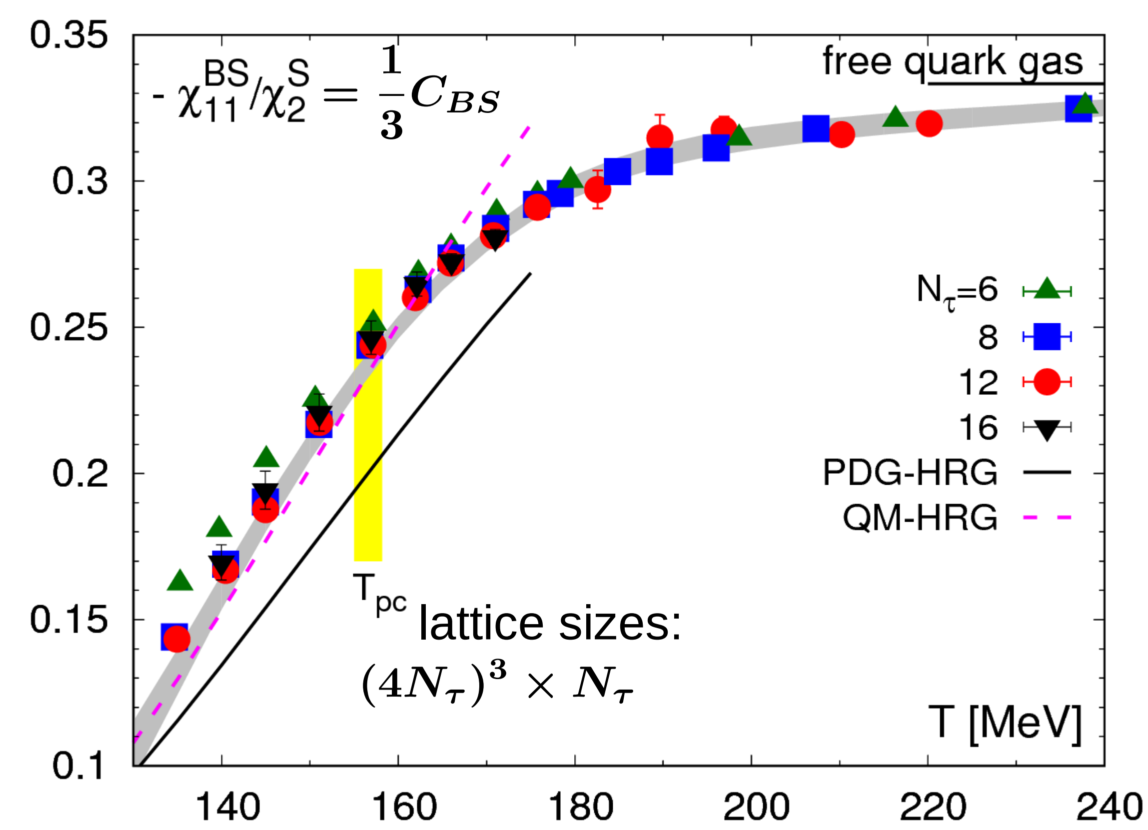


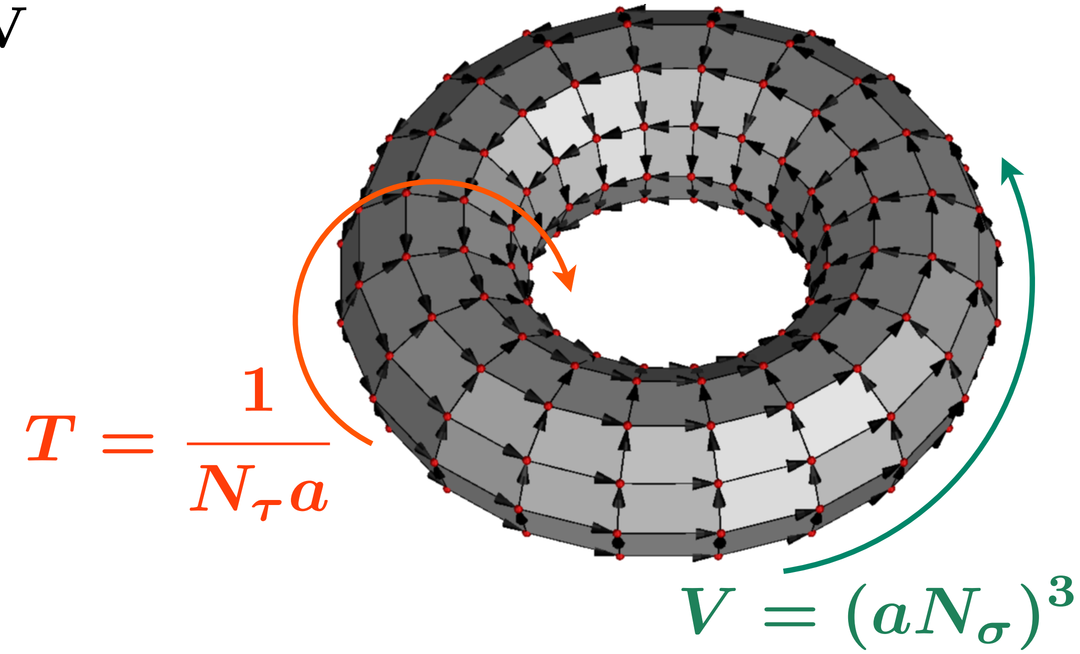
- Fluctuations of conserved charges at zero and **nonzero density**
  - ➡Discuss liberation of quark degrees of freedom
  - ➡Evidence for experimentally not yet observed strange baryons?
  - ➡Where is the critical endpoint?



- Summary

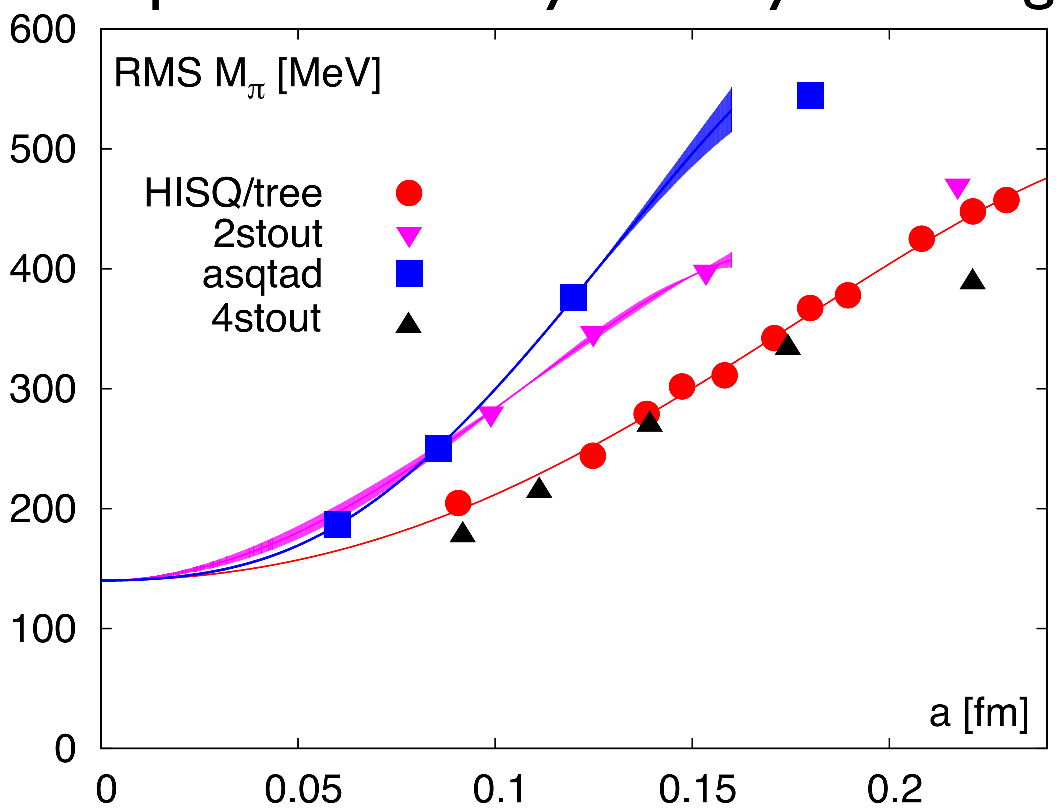
Lattice setup:

- We use (2+1)-flavor of HISQ fermions at physical quark masses in the range $135\ \mathrm{MeV} \leq T \leq 175\ \mathrm{MeV}$
- Our lattice sizes are $N_\sigma^3 \times N_\tau$, with $N_\sigma = 4N_\tau$ and $N_\tau = 6, 8, 12, 16$

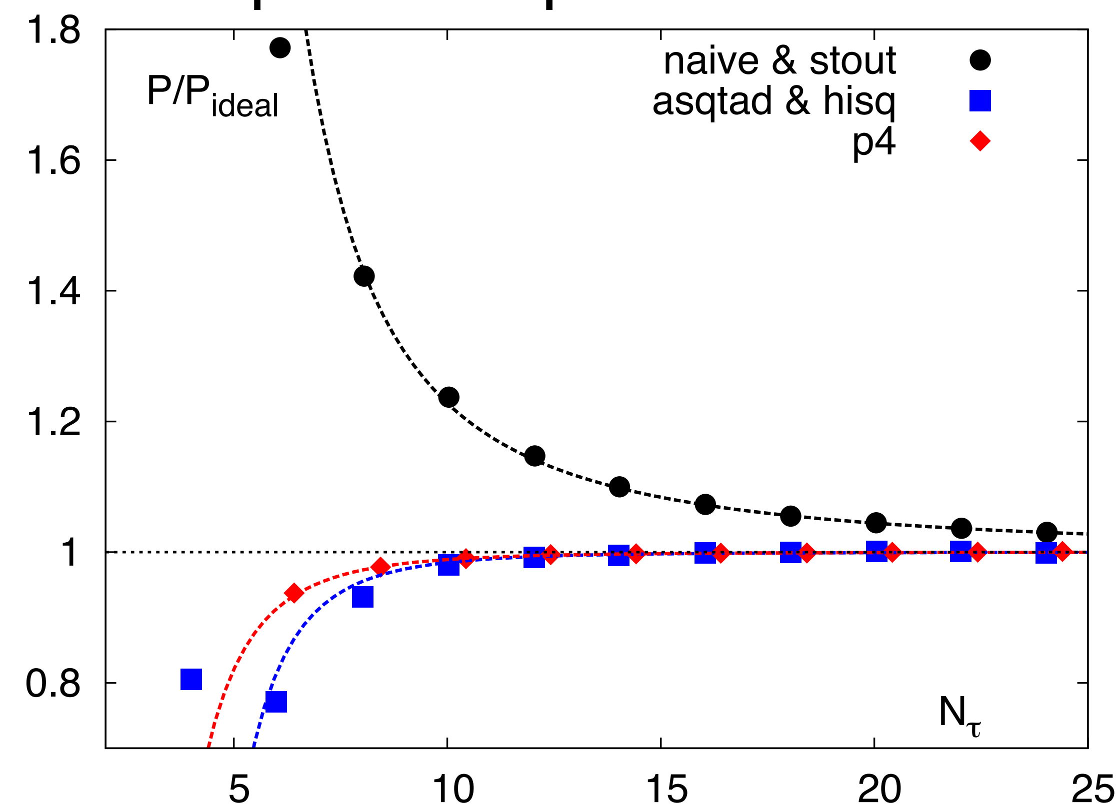


HISQ properties:

improved taste symmetry breaking



improved dispersion relation



Lattice observables

- Equilibrium thermodynamic behavior is determined by the partition function of the system

$$p(T,V,\vec{\mu}) = \frac{T}{V}\ln Z(T,V,\vec{\mu})$$ (thermal EoS)

- Thermodynamic expectation values are calculated numerically by means of Monte Carlo methods (with importance sampling) at $\vec{\mu}\equiv 0$

$$\langle \mathcal{O}\rangle = \frac{1}{Z}\int \Big(\prod_{x,\nu} \mathrm{d}U_{x,\nu}\Big)\, \mathcal{O}\, \underbrace{\det M(U,\vec{m},\vec{\mu})\, e^{-\beta S_G(U)}}$$

positive-definite weight function at $\vec{\mu}\equiv 0$,

at $|\vec{\mu}| > 0$: $\det(M)$ becomes complex

Sign problem!

- Studie derivatives of $\ln Z$ at $\vec{\mu}\equiv 0$

Chiral condensate

$$\frac{\partial \ln Z}{\partial m} = \langle \mathrm{Tr}[M^{-1}]\rangle$$

Order-parameter of spontaneous chiral symmetry breaking

Baryon number density

$$\frac{\partial \ln Z}{\partial \mu_B} = \langle \mathrm{Tr}[M^{-1}M']\rangle$$

$$M' := \frac{\partial M}{\partial \mu}$$

Baryon number fluctuations

$$\frac{\partial^2 \ln Z}{\partial \mu_B^2} = \langle \mathrm{Tr}[M^{-1}M'']\rangle - \langle \mathrm{Tr}[M^{-1}M'M^{-1}M']\rangle$$

$\rightarrow$ Fermion matrix need to be inverted: computationally demanding

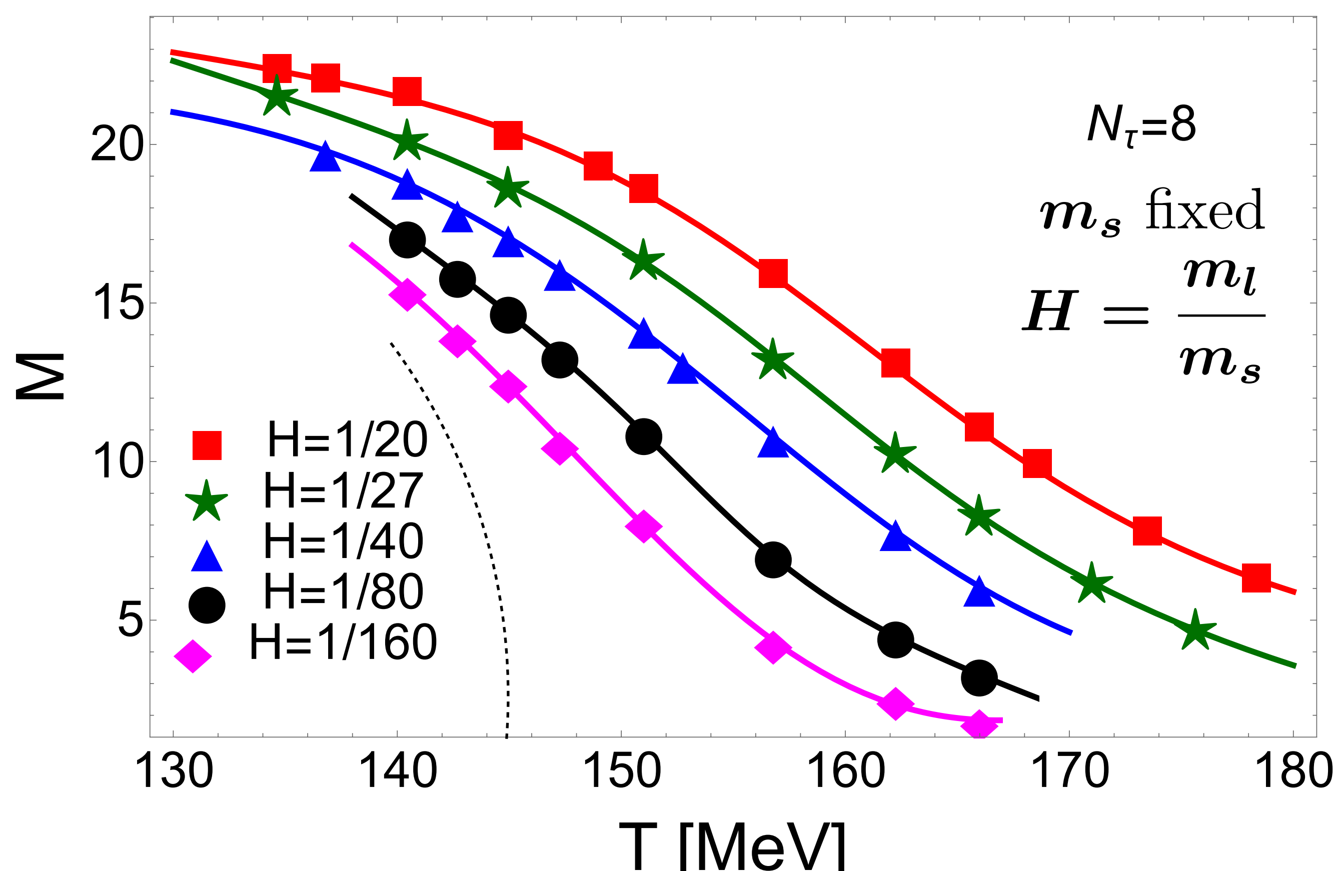
Spontaneous chiral symmetry breaking

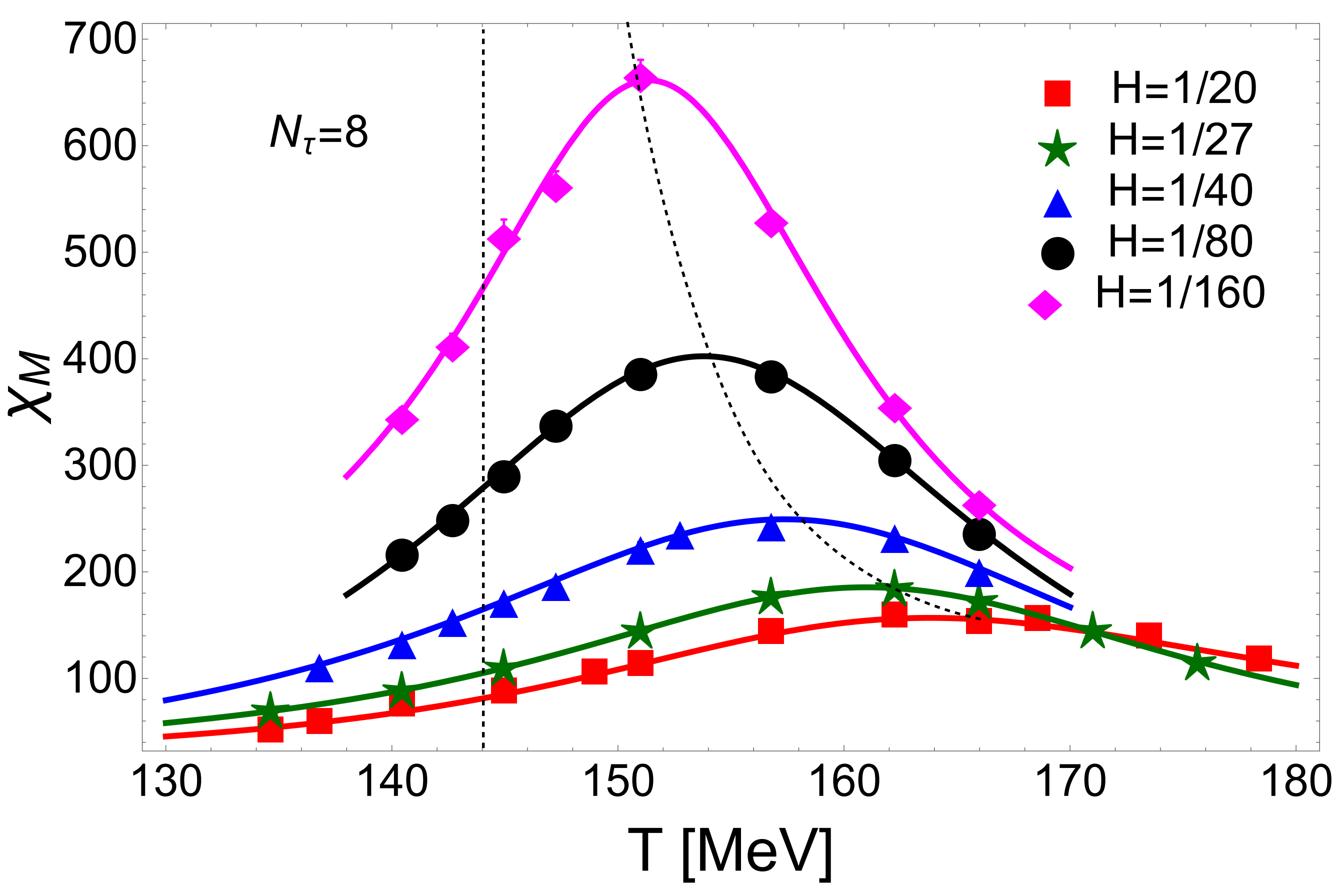
Scaling variables

	Spin-Model	QCD
Symmetry-breaking field:	h	$\frac{1}{h_0}\frac{m_l}{m_s}$
Reduced temperature:	t	$\frac{1}{t_0}\frac{T-T_c^0}{T_c^0}$
Order-parameter:	M	$\frac{m_s}{f_K^4}\langle \mathrm{Tr} M_l^{-1}\rangle - \dots$

→ Need to fix 3 none-universal parameters: h_0, t_0, T_c^0

- T_c^0 is determined from the peak position of the susceptibility χ_M, or alternatively from 60% of the peak hight.





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Scaling relations

- Renormalization group invariant definition of the order parameter

$$M = \frac{m_s}{f_K^4}\left\{\left(\langle\psi\bar{\psi}\rangle_u + \langle\psi\bar{\psi}\rangle_d\right) - \frac{m_u + m_d}{m_s}\langle\psi\bar{\psi}\rangle_s\right\}$$

- Renormalization group invariant definition of the susceptibility

$$\chi_M = \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right) M$$

- Close to the chiral limit the singular part behaves as

$$M = h^{1/\delta} f_G(z)$$

$$\chi_M = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z)$$

The scaling variable is $z = \frac{t}{h^{1/\beta\delta}}$

The scaling functions $f_G(z)$ and $f_\chi(z)$ are universal and known from various spin models

Scaling functions

- Traditional method to determine T_c^0: determine pseudo-critical line from peak location of χ_M

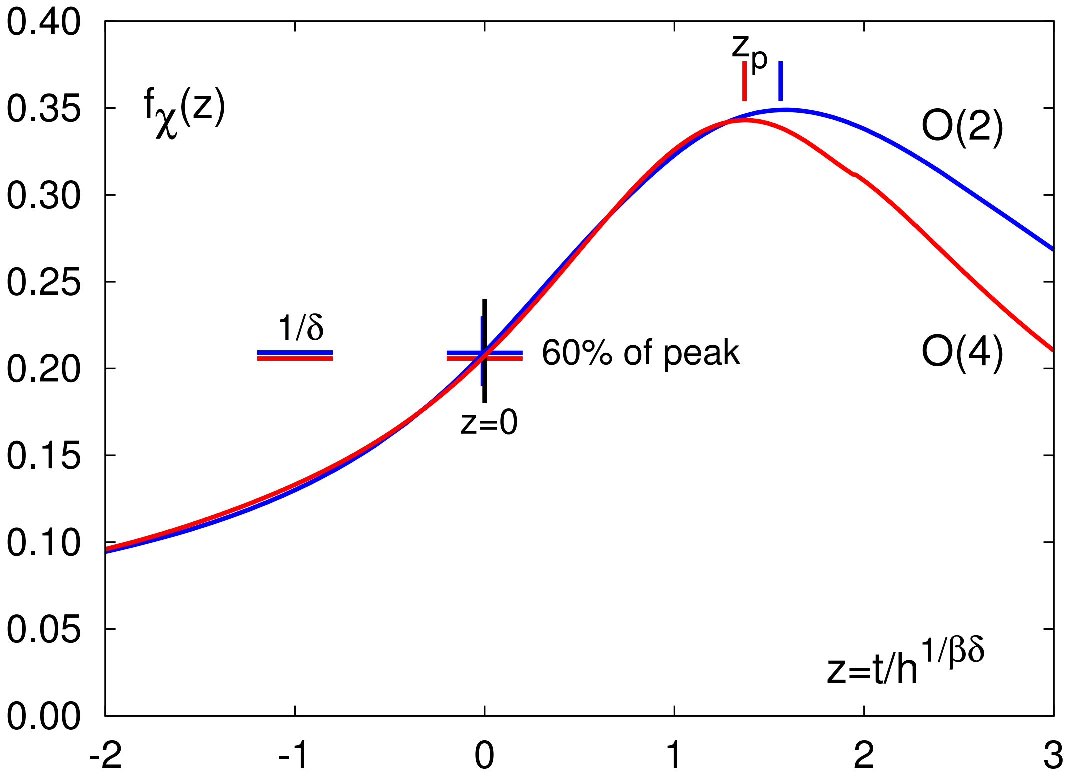
$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{1/\delta}\right)$$

- Our new approach: determine pseudo-critical line from 60% of peak hight χ_M

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_{\overline{60\%}}}{z_0} H^{1/\delta}\right)$$

➡Reduces influence of the regular part

➡Simplifies scaling analysis



	z_p	$z_{\overline{60\%}}$
$O(2)$	1.56	-0.009
$O(4)$	1.37	-0.01

Dependence on quark mass (H) is reduced by two orders of magnitude

Spontaneous chiral symmetry breaking

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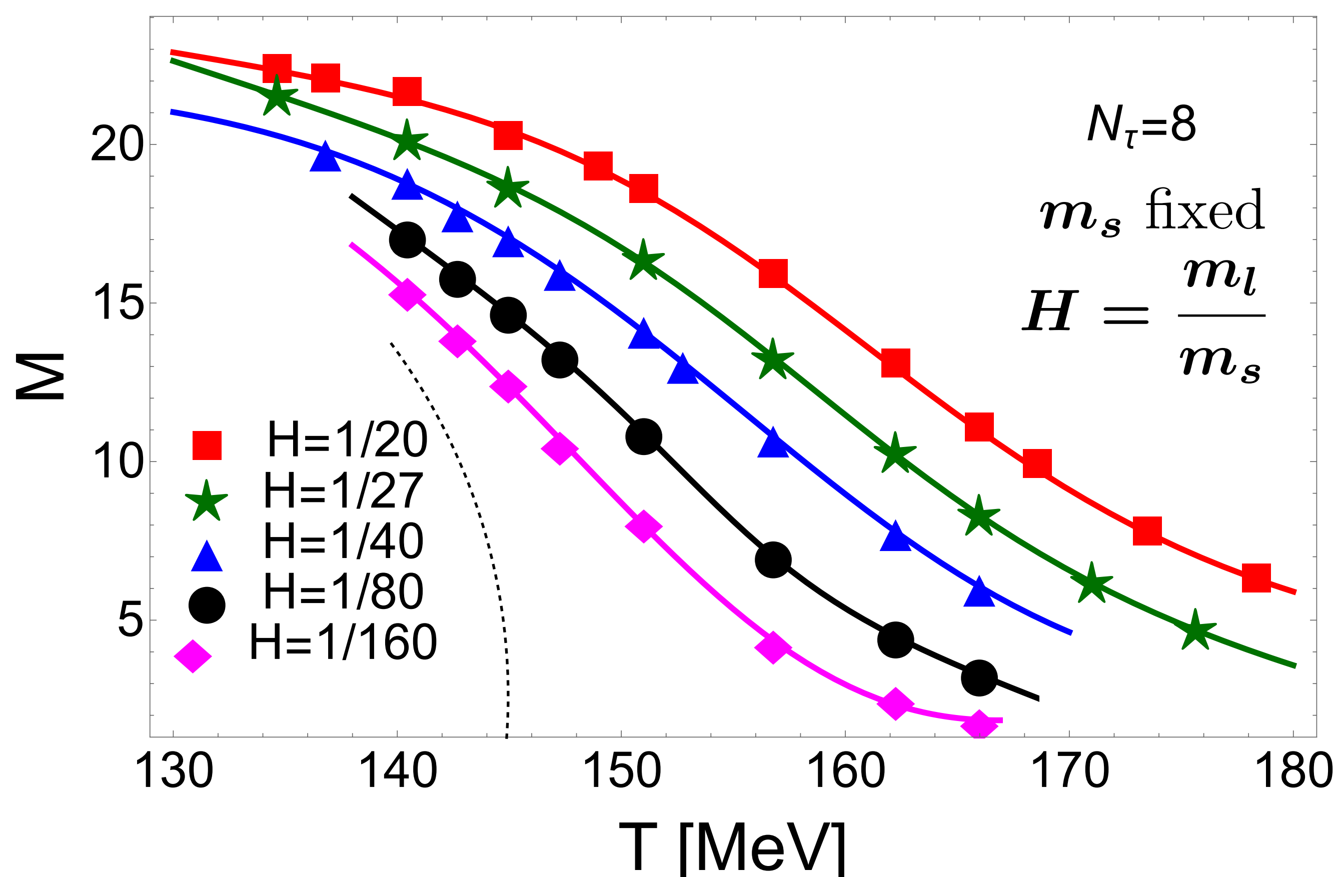

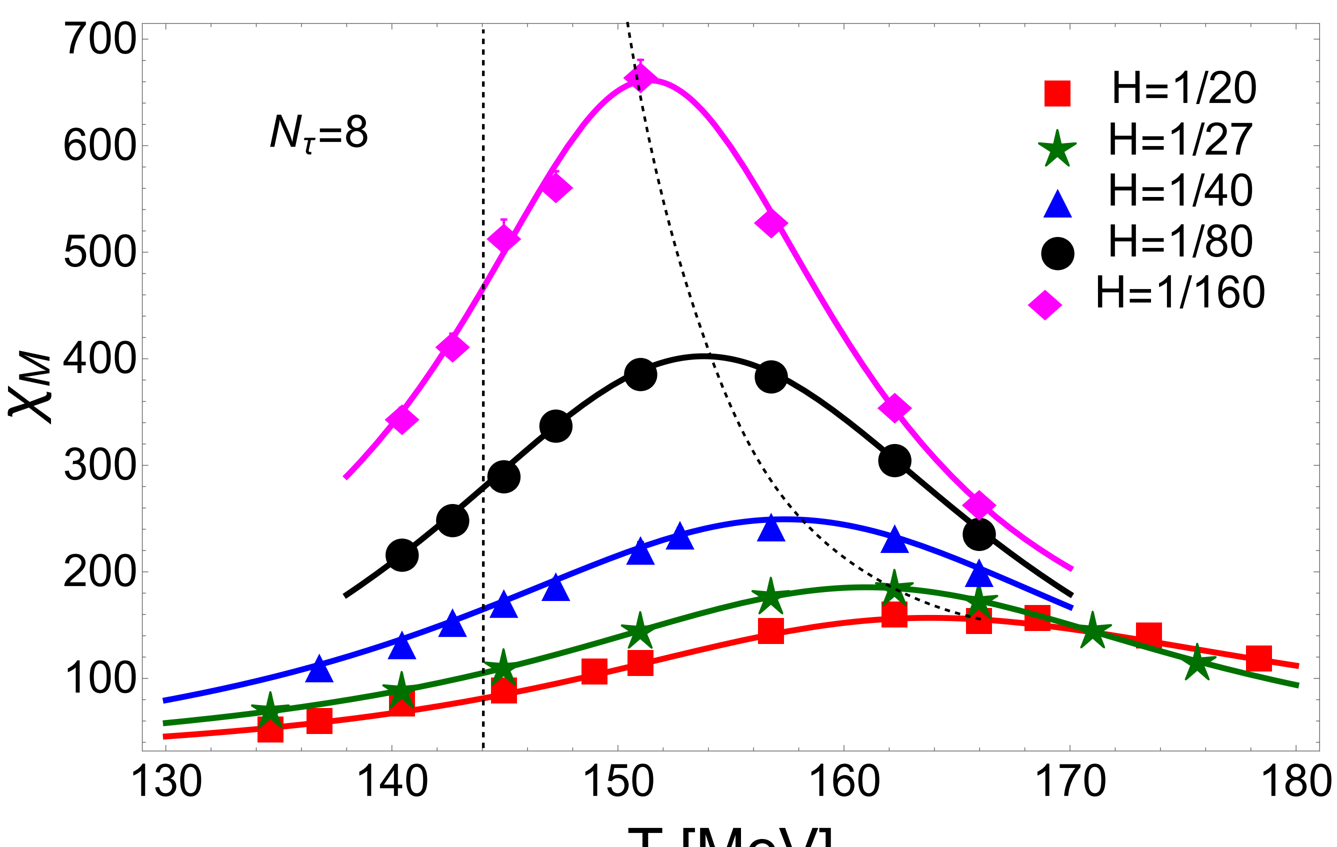

A. Lahiri, QM 2018

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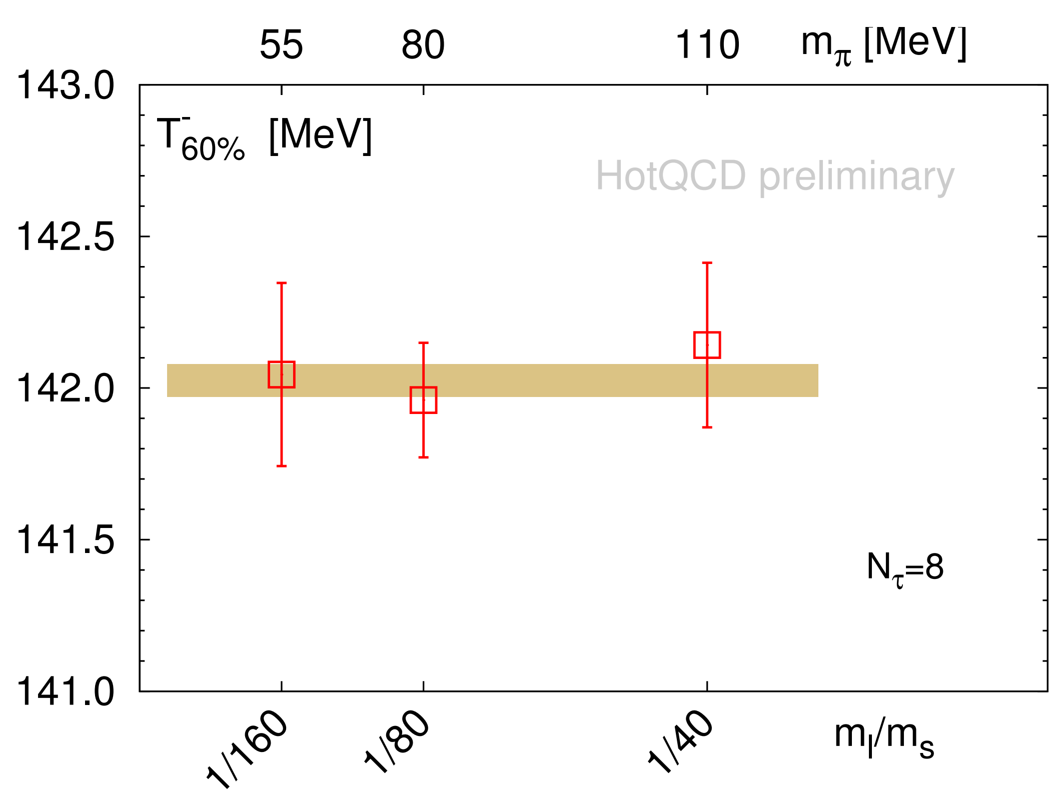
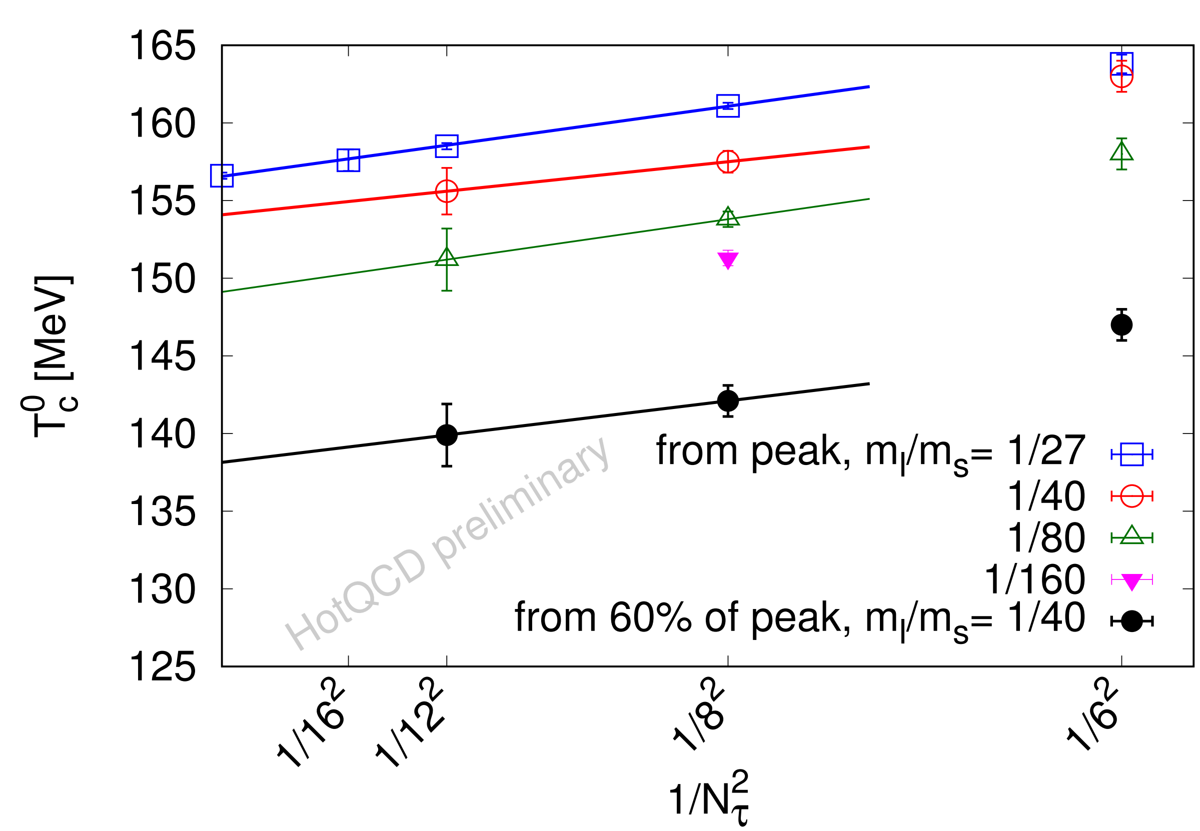
- We find $T_c^0 = 138(5)$ MeV in the continuum.





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Universal scaling

Scaling variables

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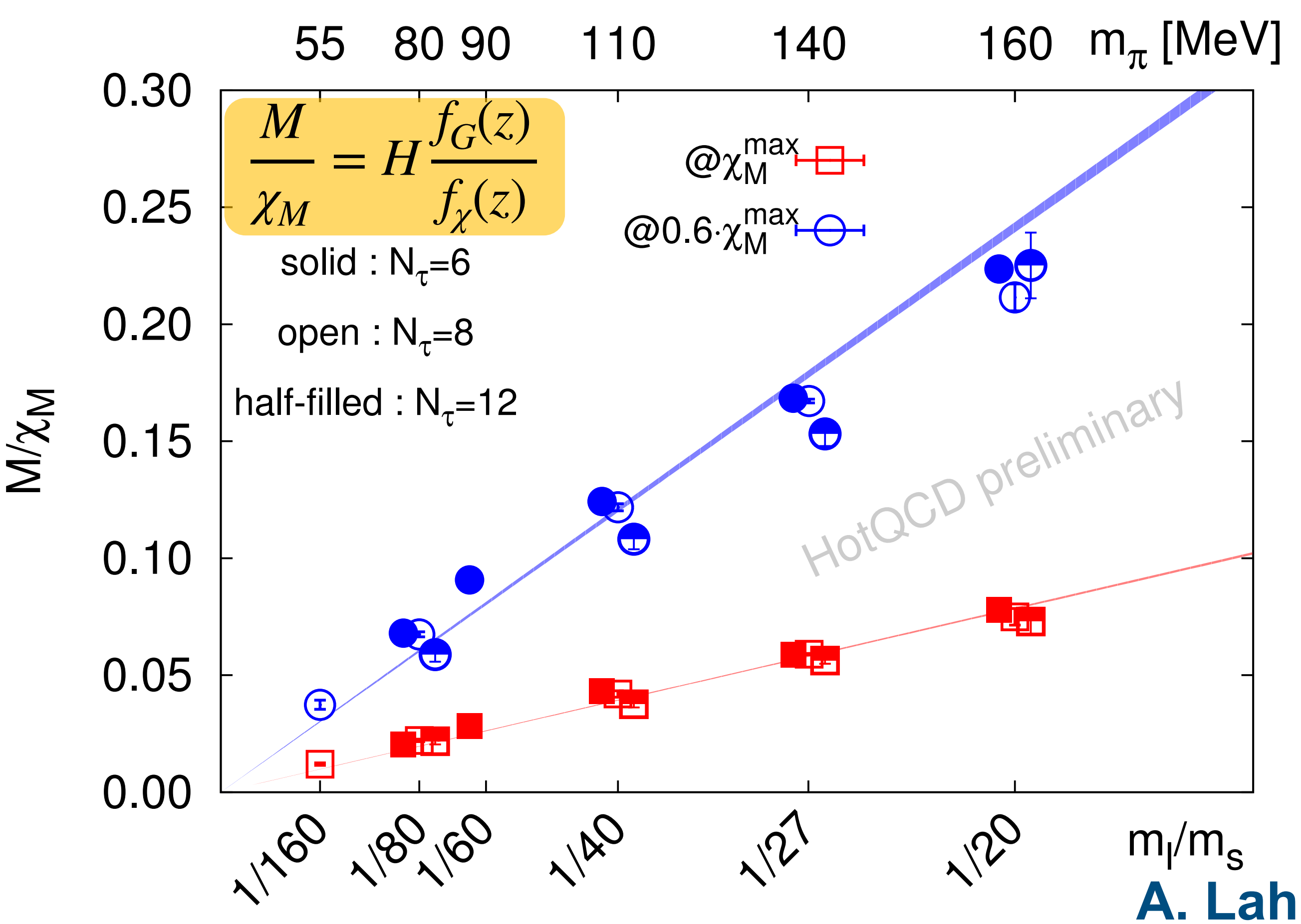

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- Find universal scaling behavior, consistent with an O(4) symmetric 2nd order phase transition in the chiral limit (m_s fixed, $m_l \rightarrow 0$), as predicted by [Pisarksi, Wilczek, PRD 29 (1984)]
- Cannot exclude 1st order transition for $H < 1/160$, or equivalently $m_\pi < 55$ MeV.

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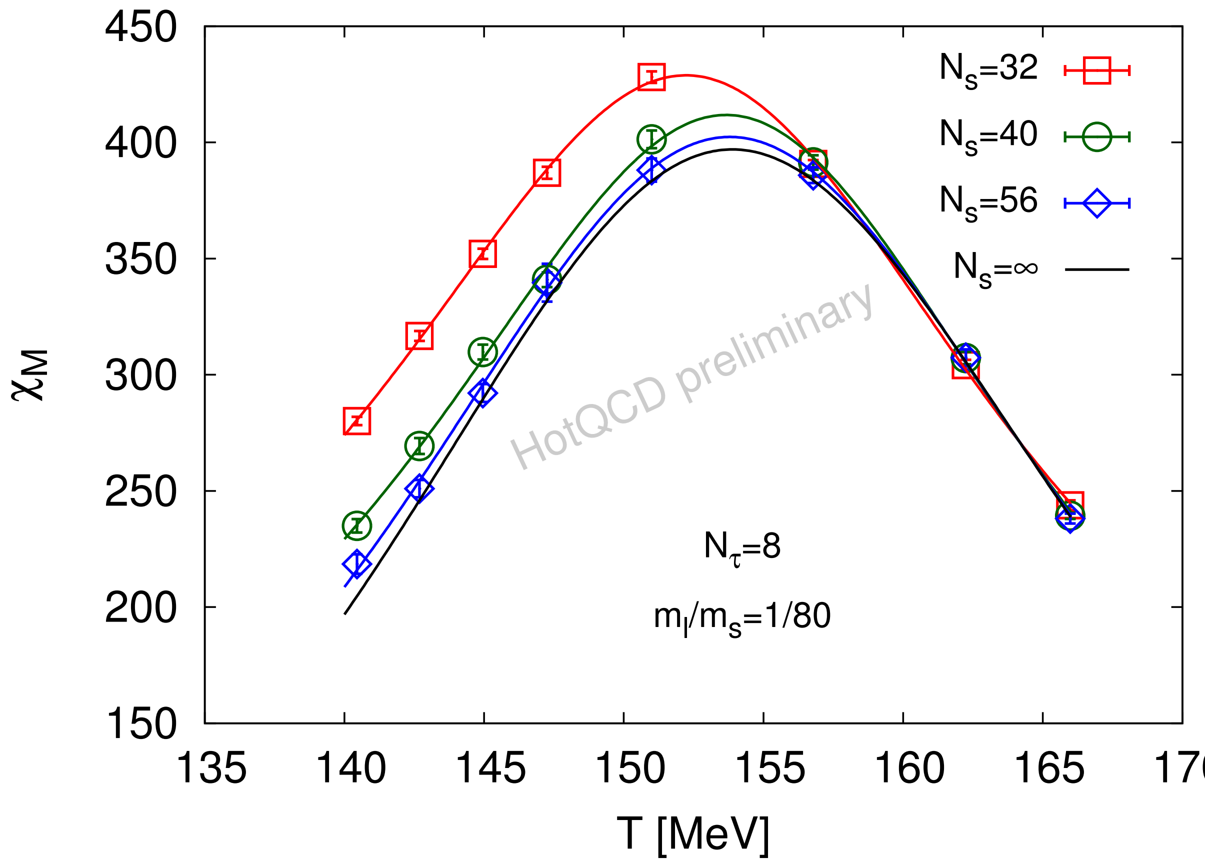

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- Cannot exclude 1st order transition for $H < 1/160$, or equivalently $m_\pi < 55$ MeV.
- Direct check at 1/80: crossover

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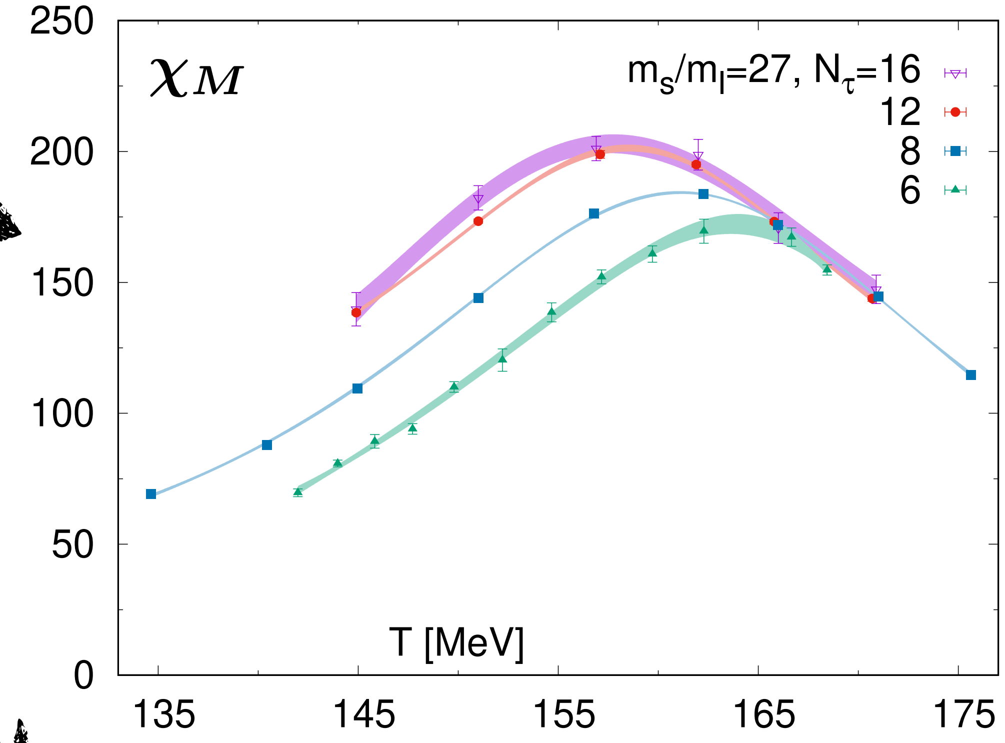
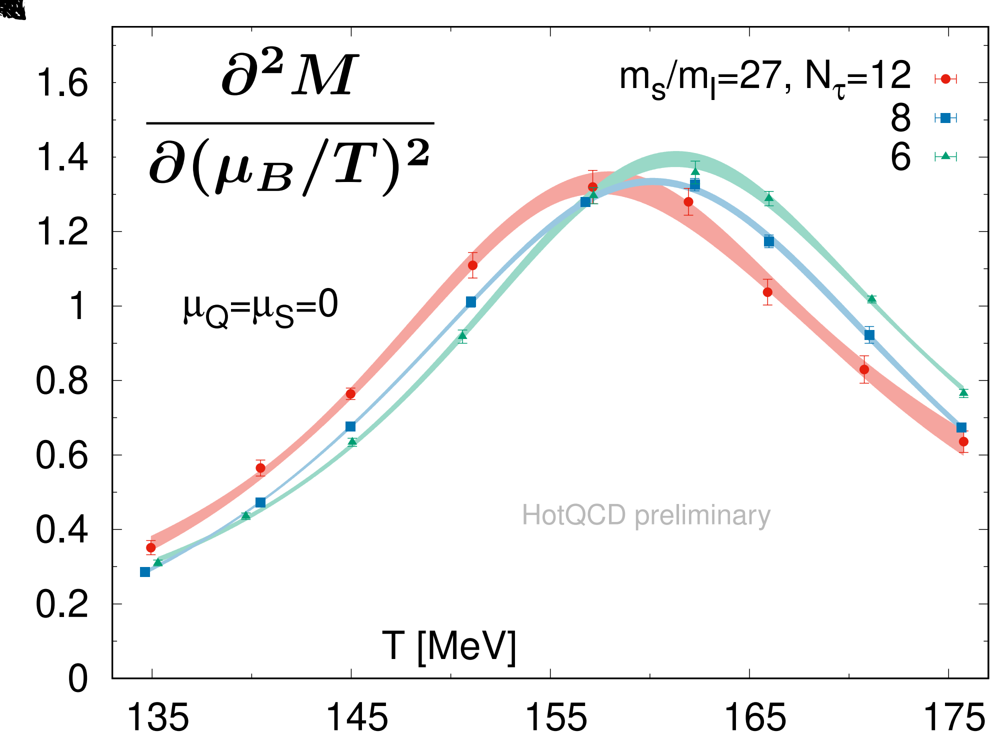
The crossover temperature at physical masses

- At physical quark masses ($H = 1/27$) the transition is a crossover. Different definitions of T_{pc} do not need to agree. We study 5 different definitions:

  - Inflection point of M ($\frac{\partial^2 M}{\partial T^2} \equiv 0$)
  - Peak of χ_M ($\frac{\partial \chi_M}{\partial T} \equiv 0$)
  - Peak of disconnected part $\chi_{\rm disc}$ ($\chi_M = \chi_{\rm disc} + \chi_{\rm con}$)
  - Peak of $\frac{\partial^2 M}{\partial(\mu_B/T)^2}$
  - Peak of $\frac{\partial^2 \chi_M}{\partial(\mu_B/T)^2}$



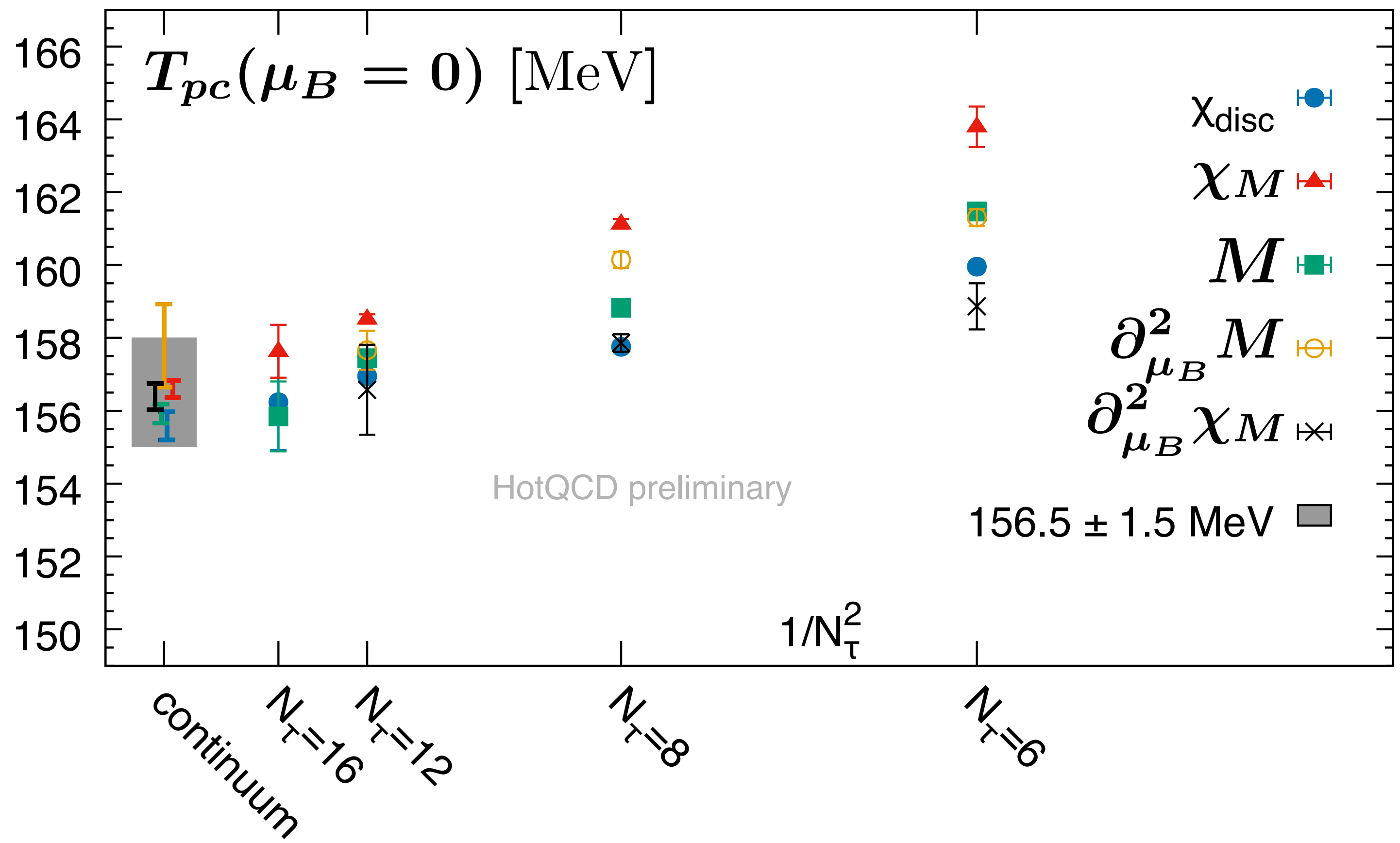


All definitions approach the unique chiral transition temperature T_c^0 in the chiral limit.

P. Steinbrecher, QM 2018

The crossover temperature at physical masses

- At physical quark masses ($H = 1/27$) the transition is a crossover. Different definitions of T_{pc} do not need to agree. We study 5 different definitions.
- We find in the chiral limit $T_{pc}(\mu_B = 0) = 156.5 \pm 1.5$ MeV .



- Compares well with previously obtained results [Borsanyi 2010, Bazavov 2012, Bonati 2015]

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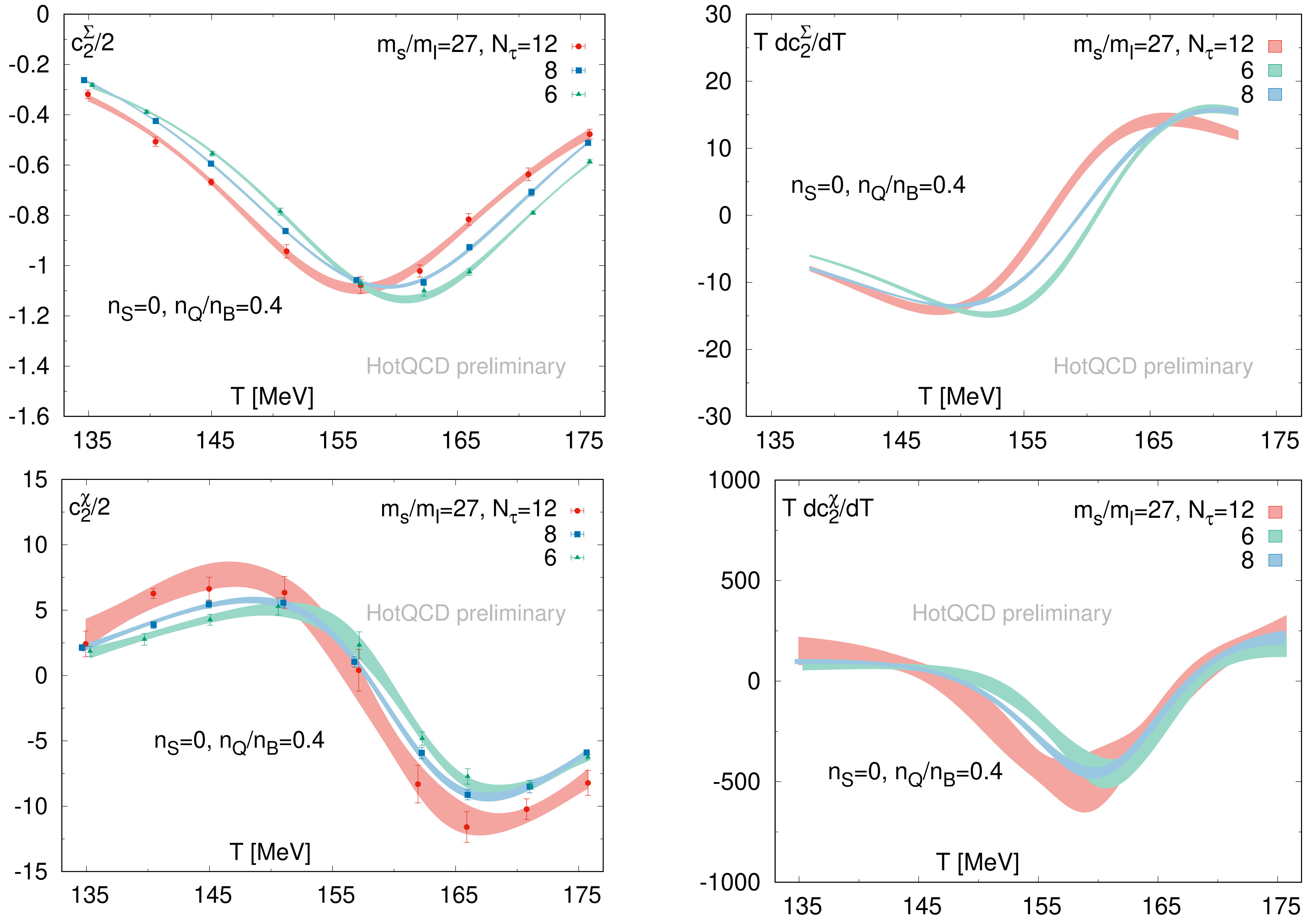
The curvature of the transition line

$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

- Coefficients are obtained from strangeness neutral Taylor expansion of the disconnected chiral condensate M and chiral susceptibility ($\chi_{\rm disc}$)



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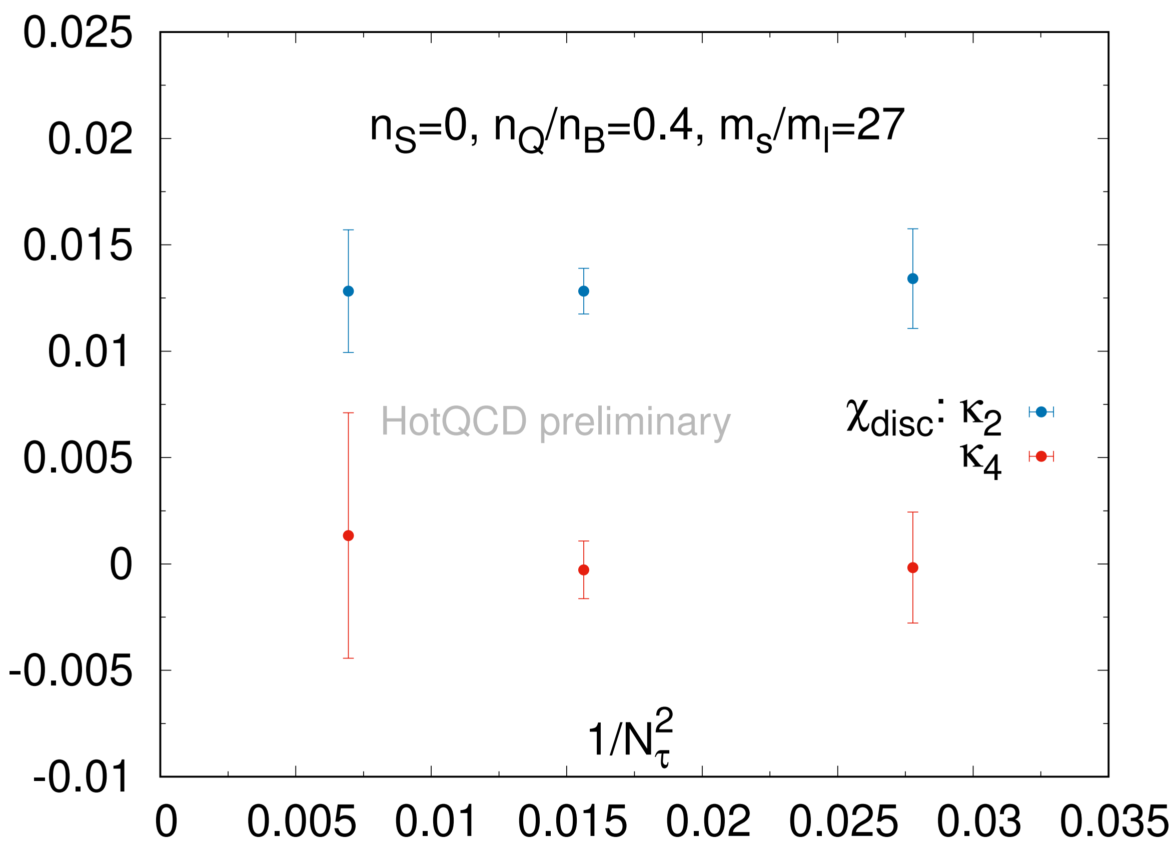

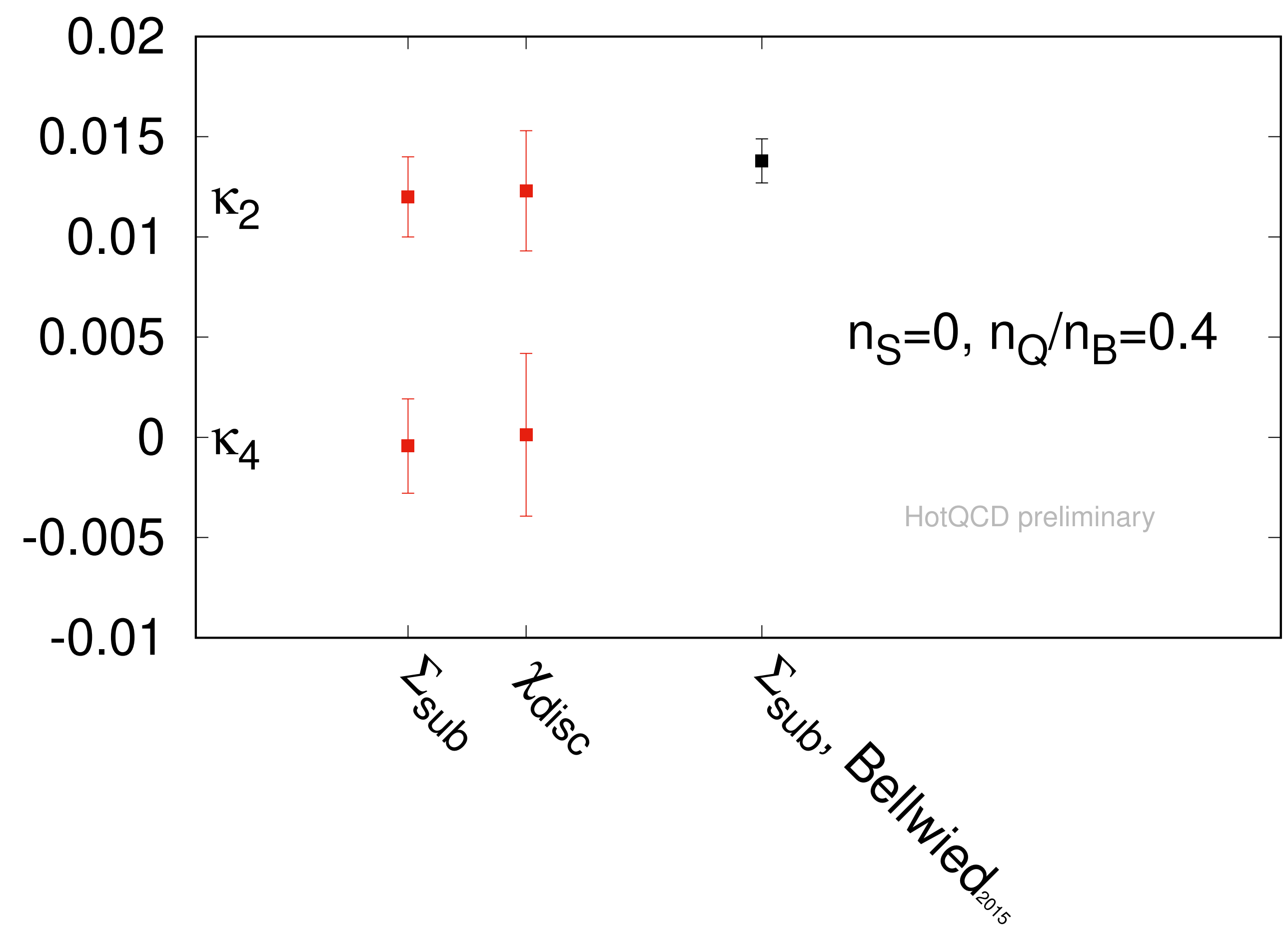

- Continuum extrapolation is flat
- Crossover line shows only quadratic dependence on μ_B
- Consistent results from different observables and collaborations

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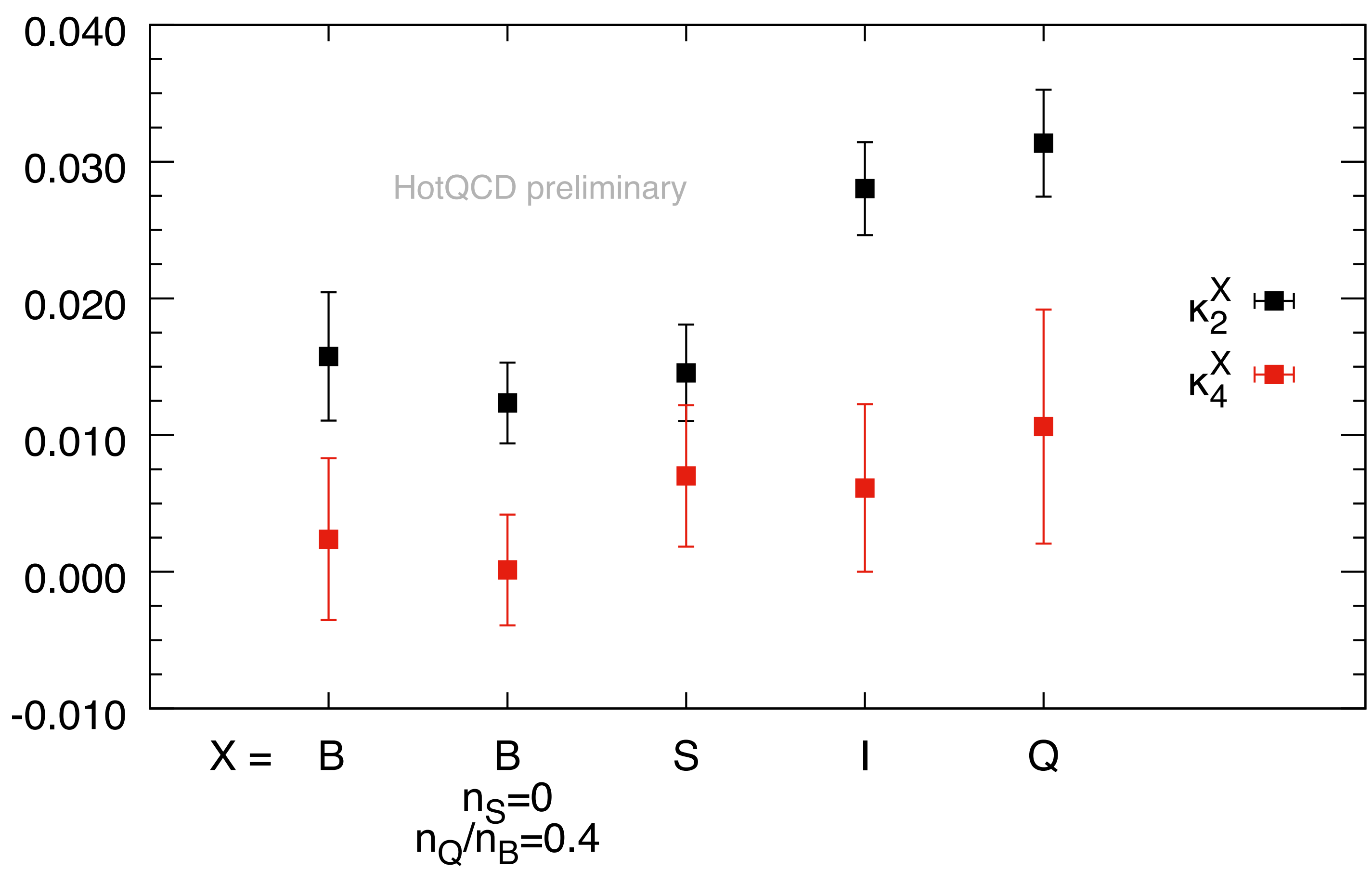

- Continuum extrapolation is flat
- Crossover line shows only quadratic dependence on μ_B
- Consistent results from different observables and collaborations
- Dependence on the type of chemical potential

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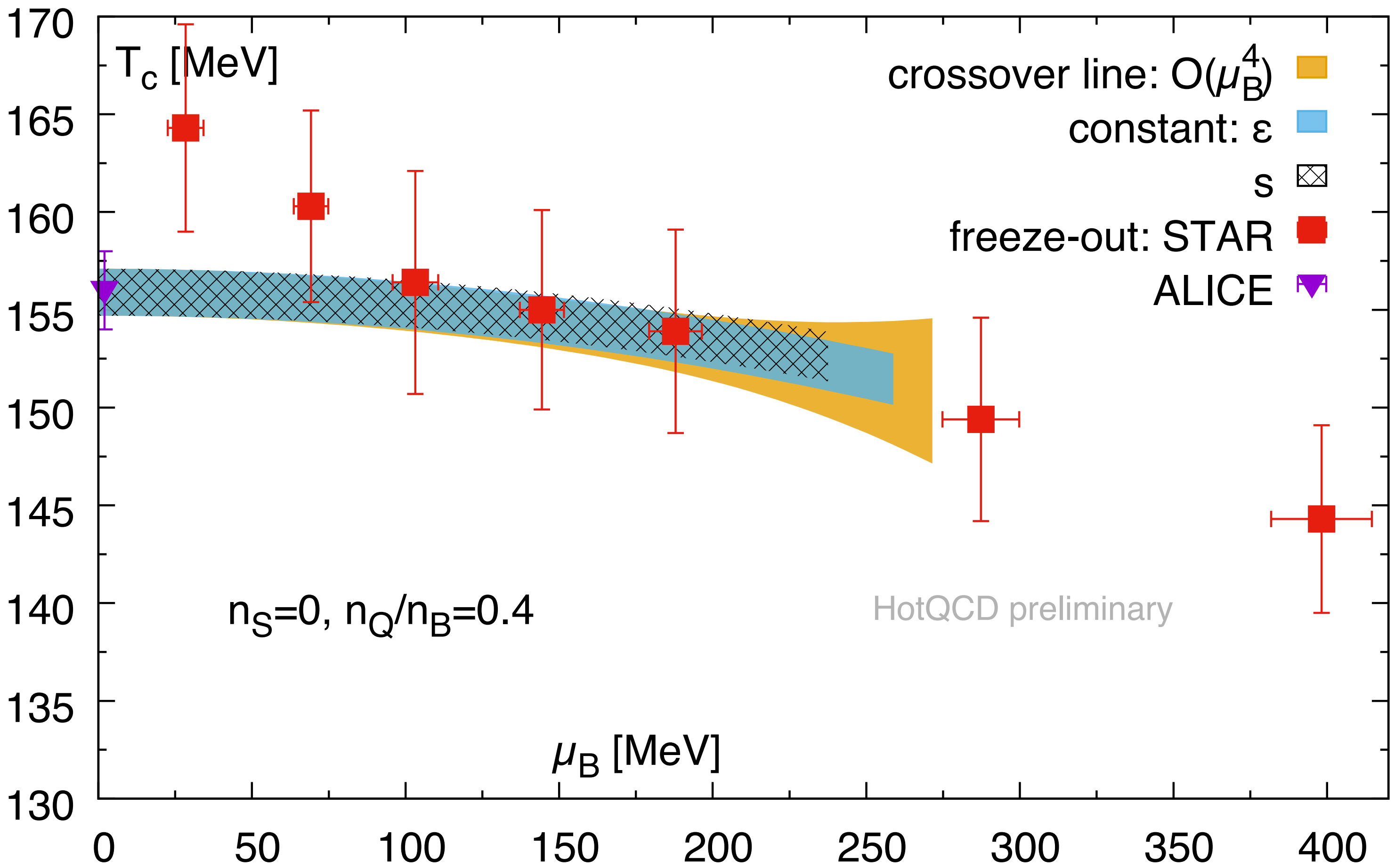
The crossover line



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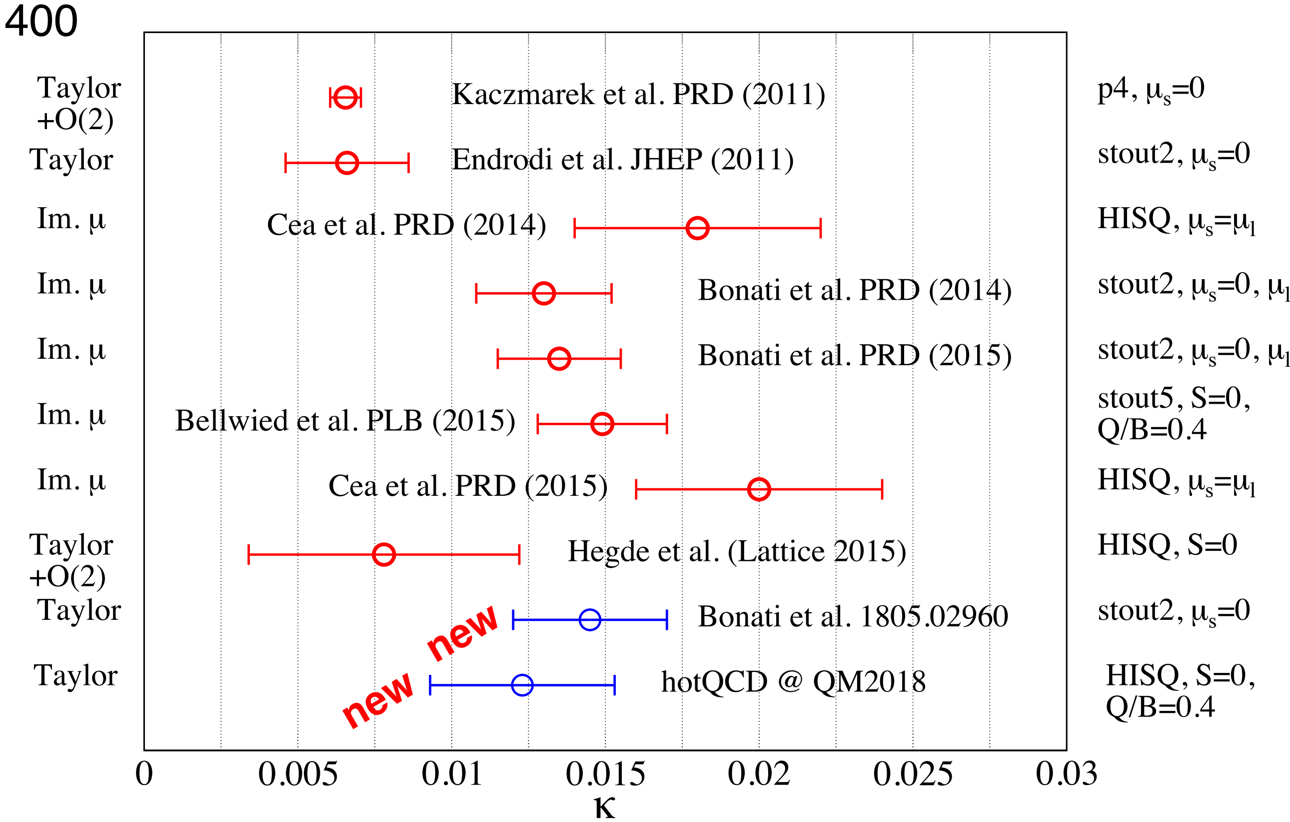
STAR, PRC 96 (2017)

ALICE, Nucl.Phys.A (2014)

$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(0)}\right)^2 - \cdots$$

- The crossover line is quadratic in μ_B, with small curvature
- Agrees with line of constant energy density and entropy
- Agrees with current estimates of the freeze-out line from ALICE and STAR data



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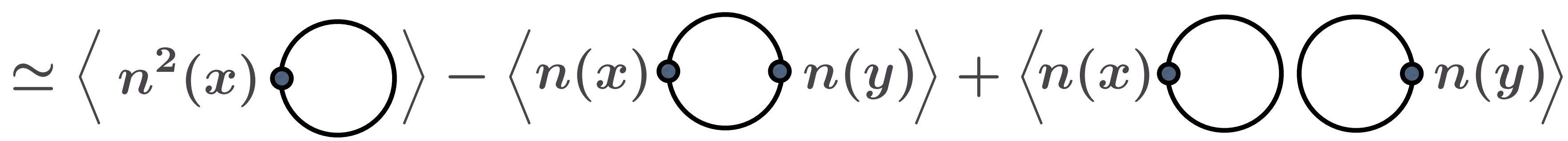
Fluctuations of conserved charges

$$\frac{p(\vec{\mu},T)}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

with $\chi_{i,j,k}^{BQS}(T) = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln Z(\vec{\mu},T)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\vec{\mu}=0}$ and $\hat{\mu} = \mu/T$

Example:

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \left\langle \mathrm{Tr}\left[M^{-1}M''\right]\right\rangle - \left\langle \mathrm{Tr}\left[M^{-1}M'M^{-1}M'\right]\right\rangle + \left\langle \mathrm{Tr}\left[M^{-1}M'\right]^2\right\rangle$$

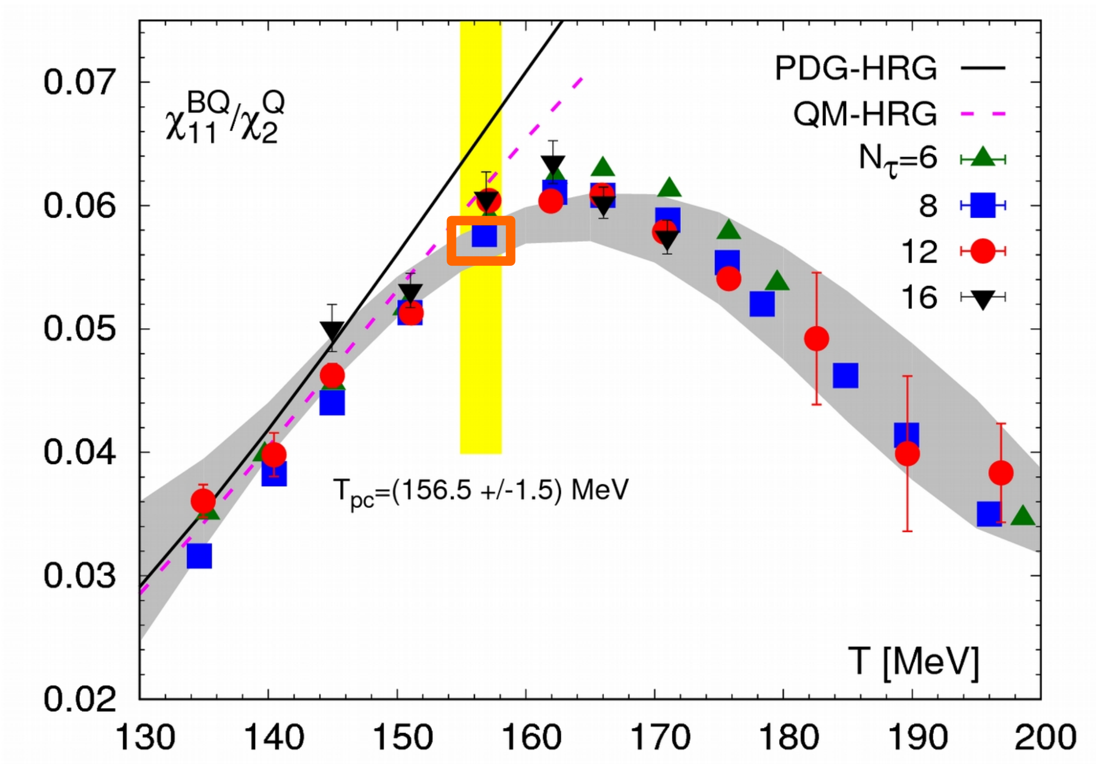


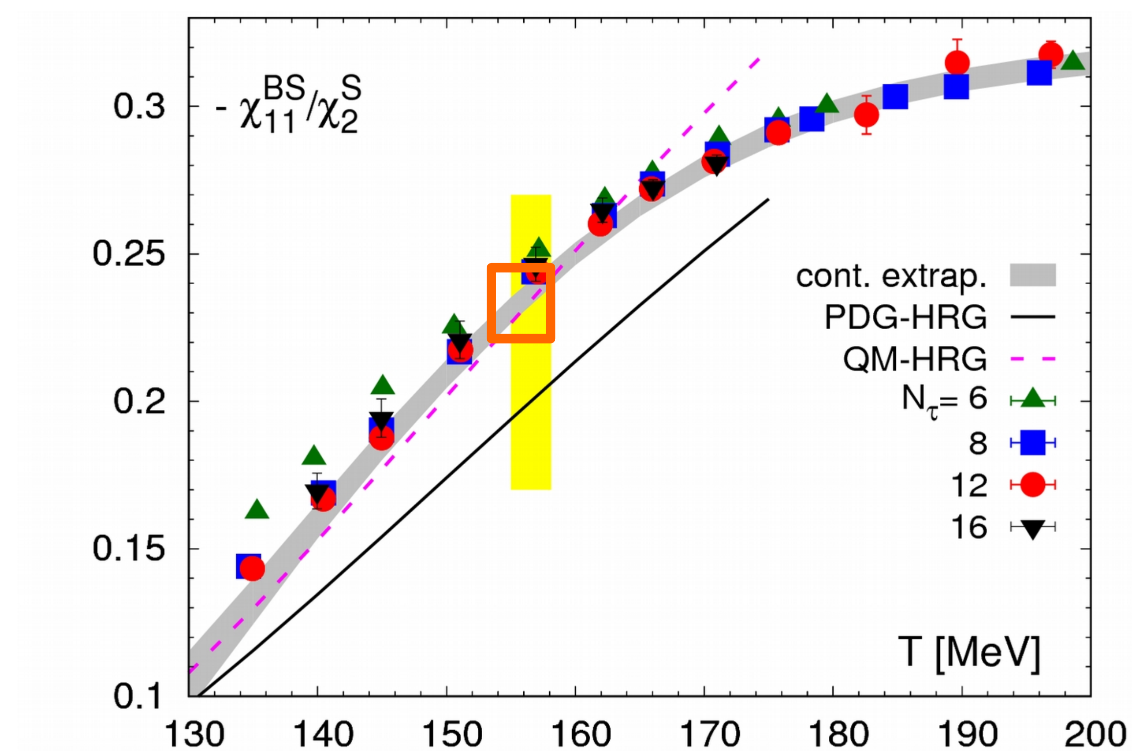
Lattice		**Experiment**
$R_{12}^X(T,\mu_B) \equiv \frac{\chi_1^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$\frac{M_X}{\sigma_X^2}$
$R_{32}^X(T,\mu_B) \equiv \frac{\chi_3^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$S_X \sigma_X$
$R_{42}^X(T,\mu_B) \equiv \frac{\chi_4^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$\kappa_X \sigma_X^2$

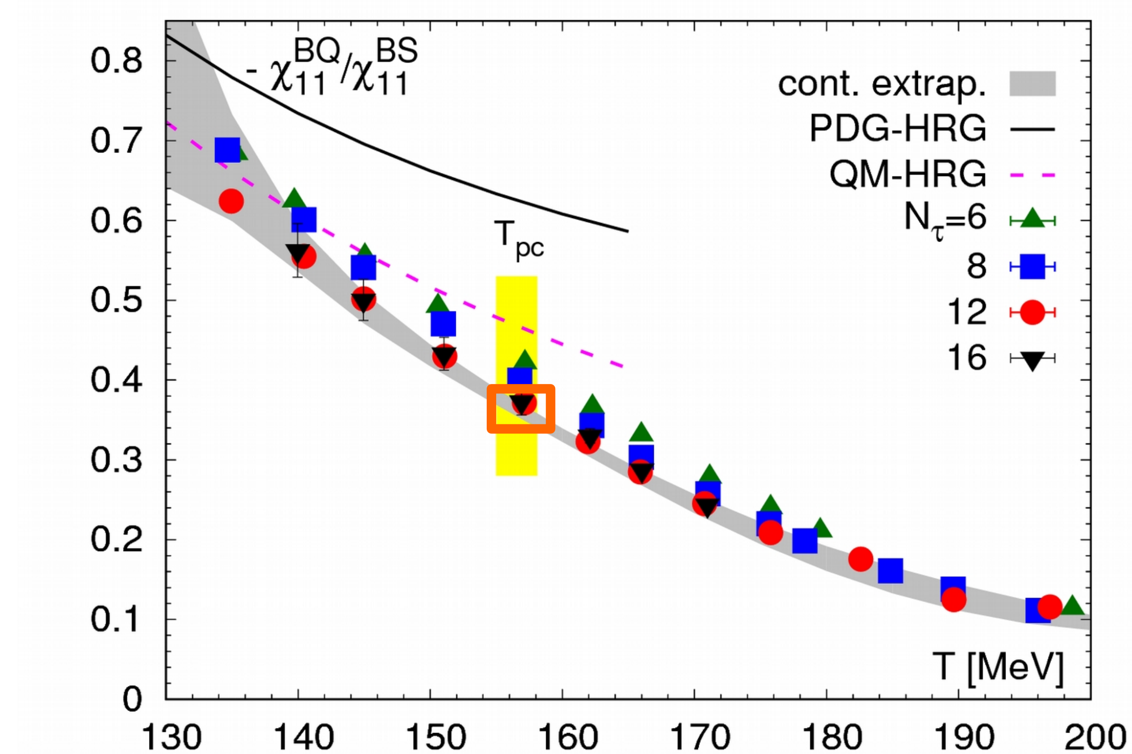
M := mean
σ^2 := variance
S := skewness
κ := kurtosis

Fluctuations of conserved charges at LHC

- 2nd order fluctuations: BQ and BS correlations







at ALICE freeze-out temperature
== QCD crossover temperature

$$T_{fo} = 156.5(1.5)\mathrm{MeV}$$

$$\chi_{11}^{BQ}/\chi_2^Q = 0.058(2)$$

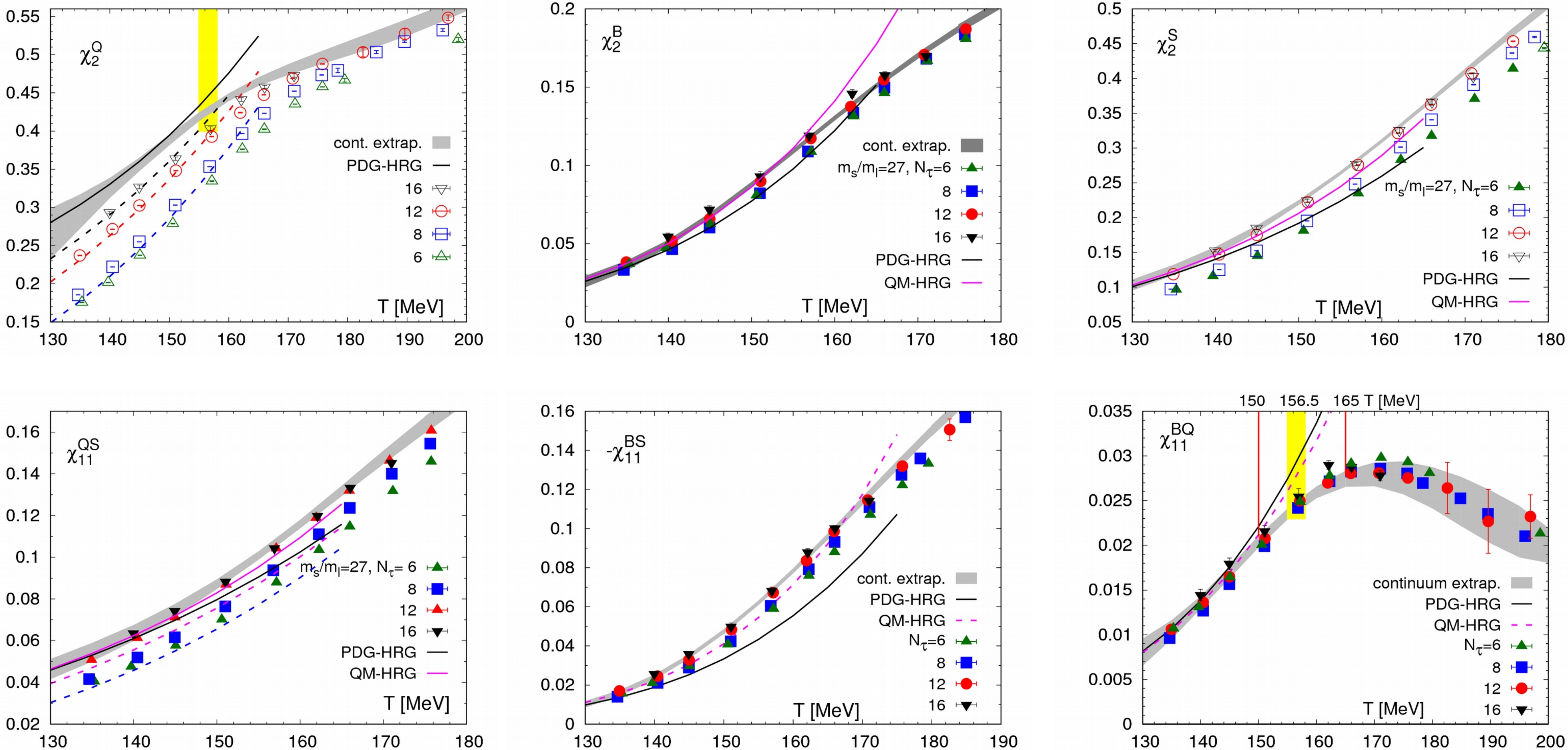
$$\chi_{11}^{BS}/\chi_2^S = -0.235(15)$$

$$\chi_{11}^{BQ}/\chi_{11}^{BS} = -0.37(3)$$

Fluctuations of conserved charges at LHC

- Continuum results for all 2nd order observables

$T_c = (156.5 \pm 1.5)\mathrm{MeV}$ $\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$ $\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$



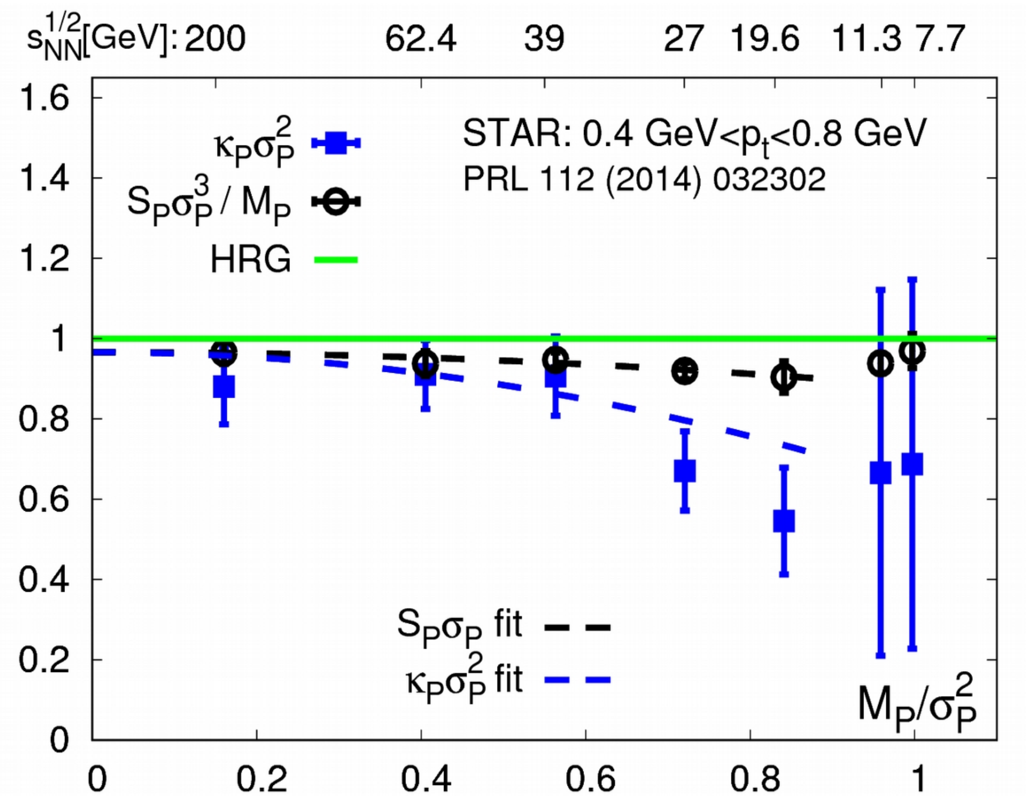
➡Only 4 out of 6 observables are independent

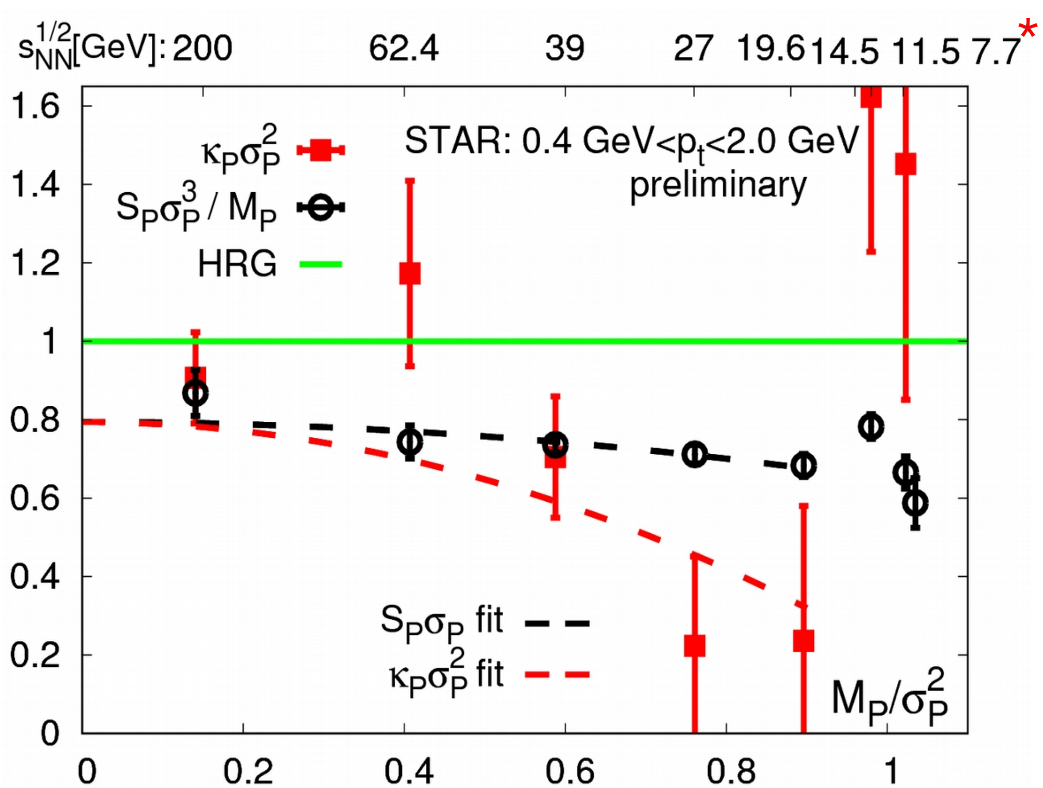
Fluctuations of conserved charges at LHC

- Kurtosis and skewness

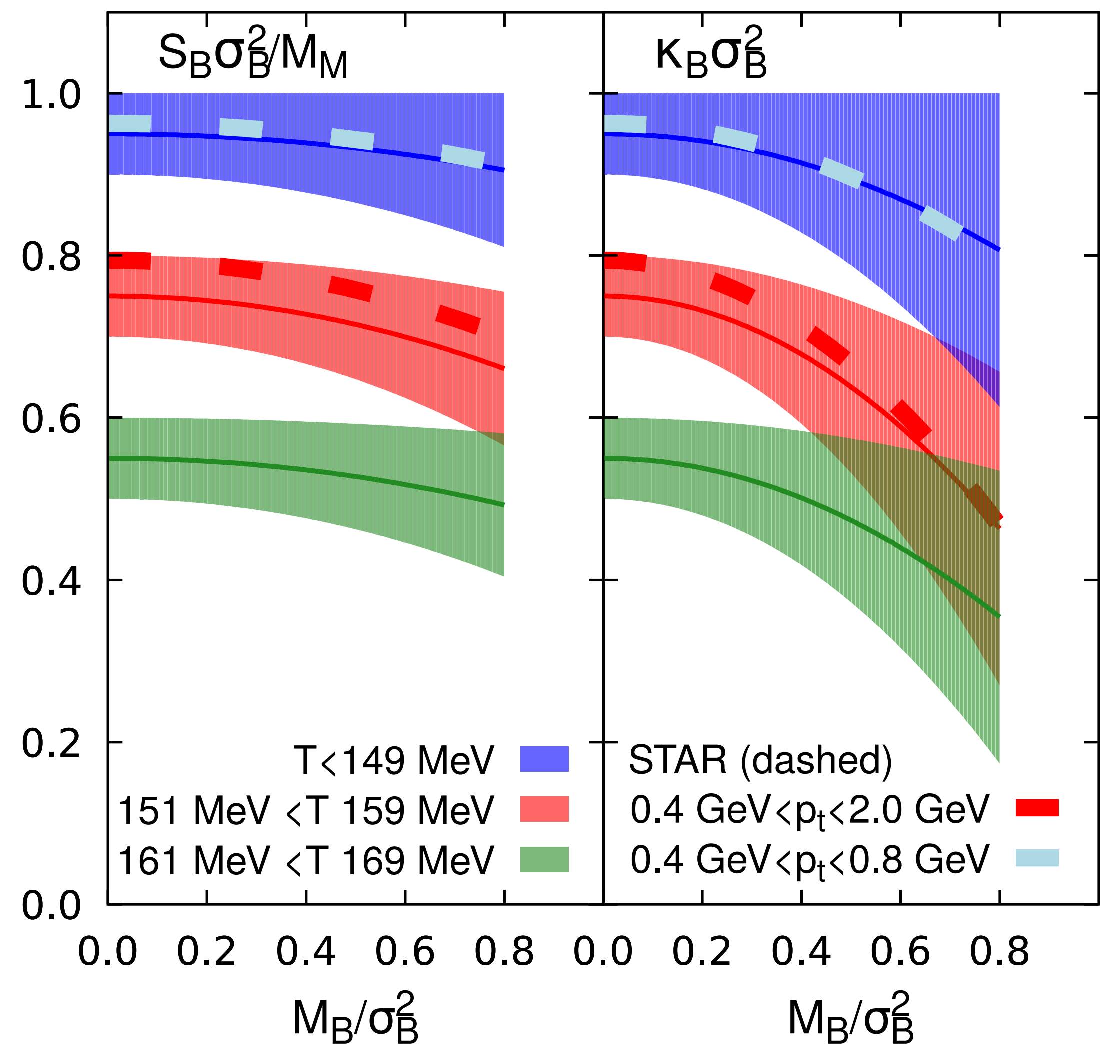
$$R_{42}^{B} = \frac{\chi_4^B}{\chi_2^B} = \kappa_B \sigma_B^2$$

$$R_{31}^{B} = \frac{\chi_3^B}{\chi_1^B} = S_B \sigma_B^3 / M_B$$



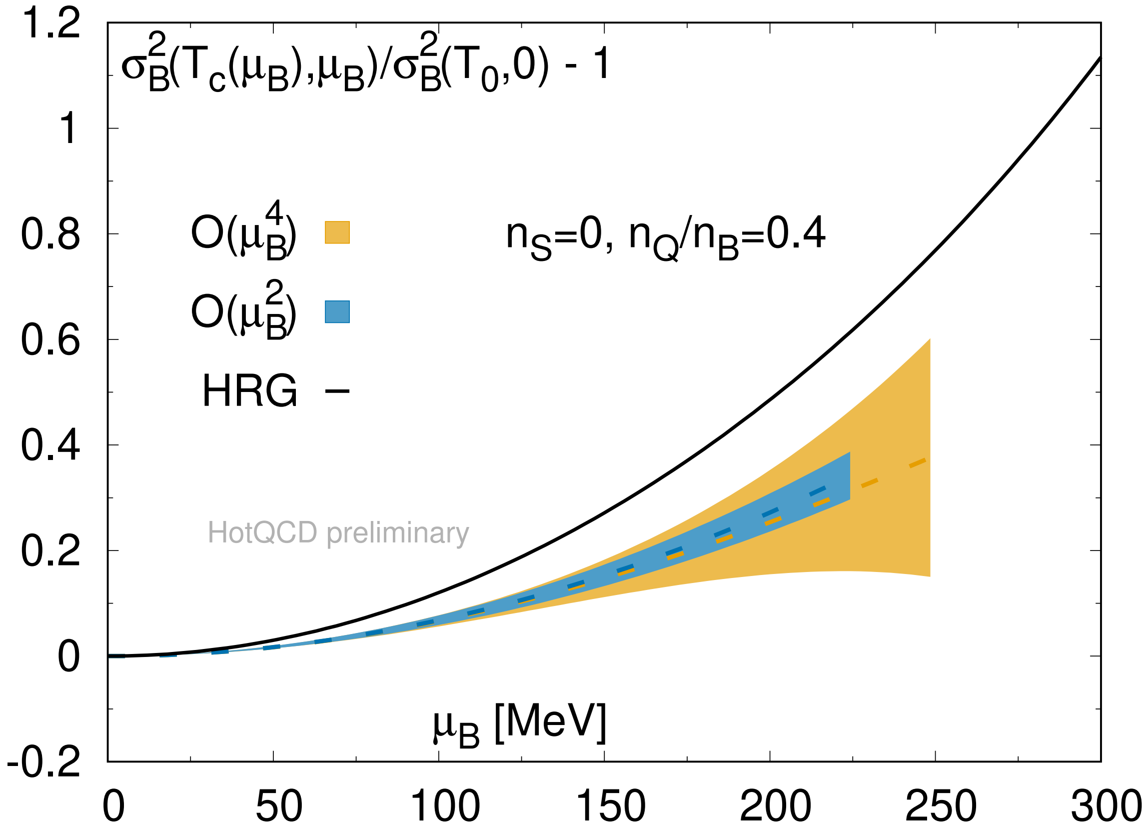


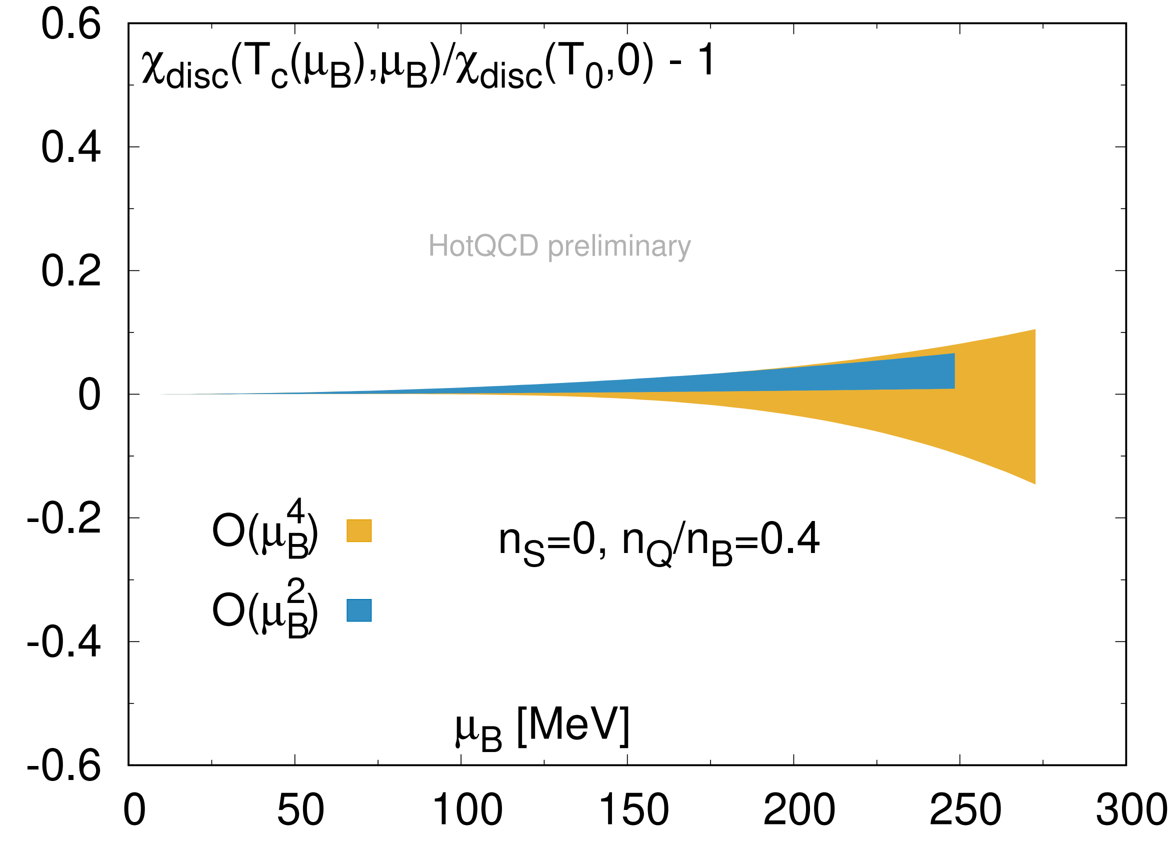
(not yet continuum extrapolated!)



[Bazavov et al. (hotQCD), PRD 96 (2017), 074510]

- Consider baryon number fluctuations and fluctuations of the chiral condensate along the crossover line





➡ No sign of critical fluctuations / critical point
➡ No sign that crossover gets stronger

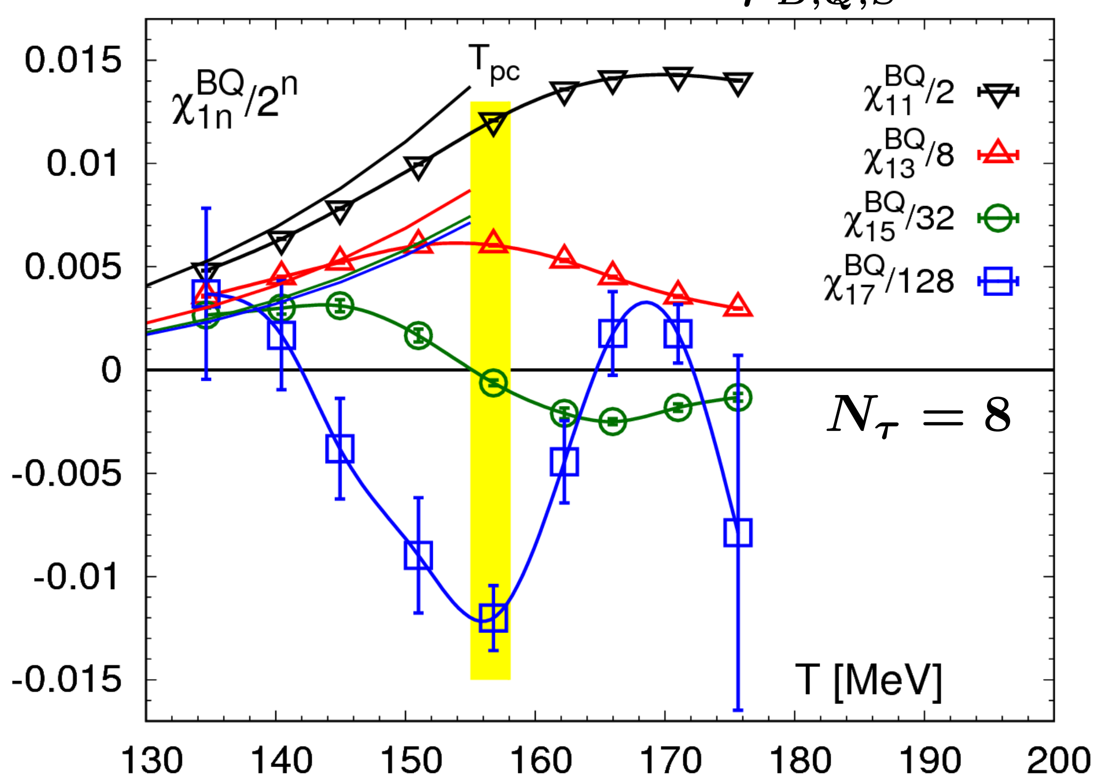
Will a critical point ever be seen by a Taylor expansion?

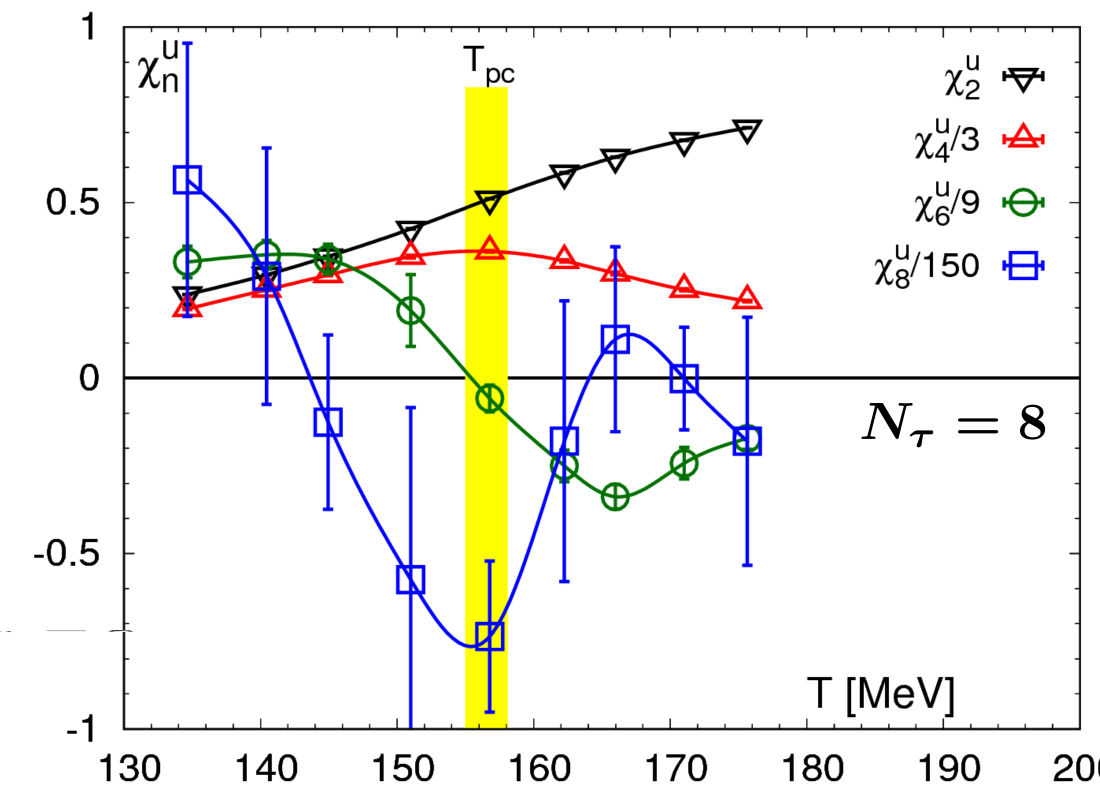
Theoretical Method: estimate the radius of convergence from successive expansion coefficient. So far, not conclusive.

Where is the critical point?

- All expansion coefficients need to be positive

$$\chi_{1n}^{BQ} = \left.\frac{\partial^{n+1} P/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q^n}\right|_{\mu_{B,Q,S}=0} \qquad \chi_n^u = \left.\frac{\partial^n P/T^4}{\partial \hat{\mu}_u^n}\right|_{\mu_{u,d,s}=0}$$





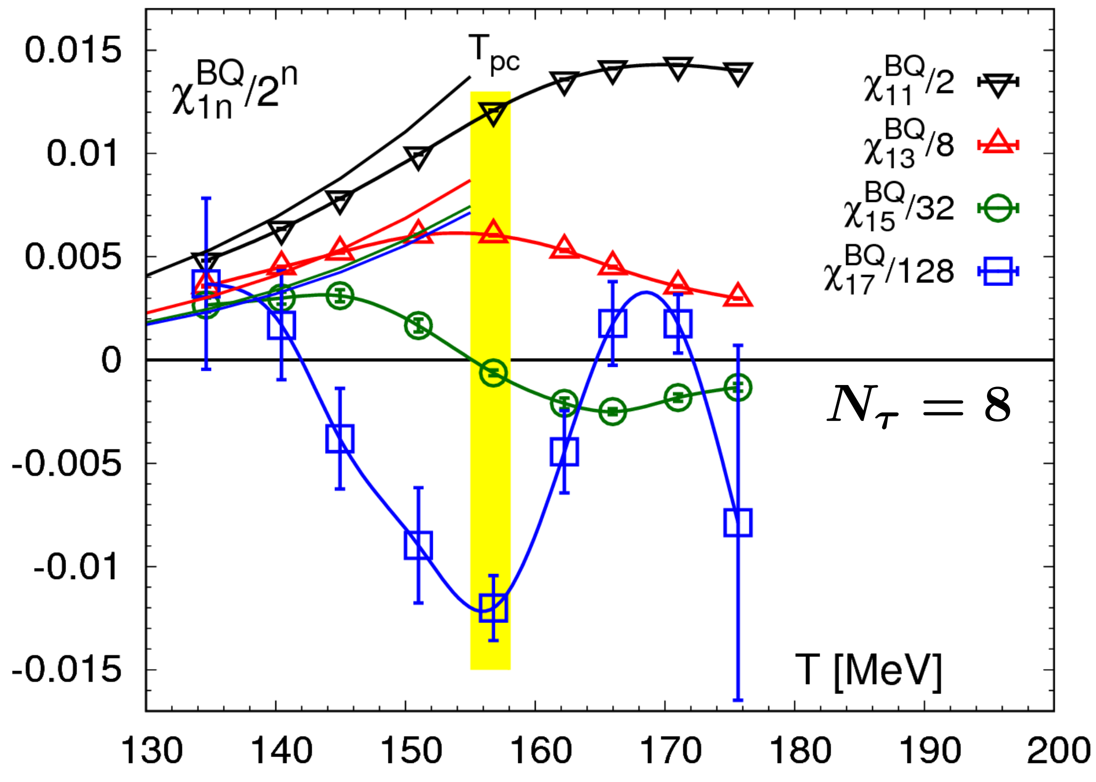
- Structure is consistent with critical scaling

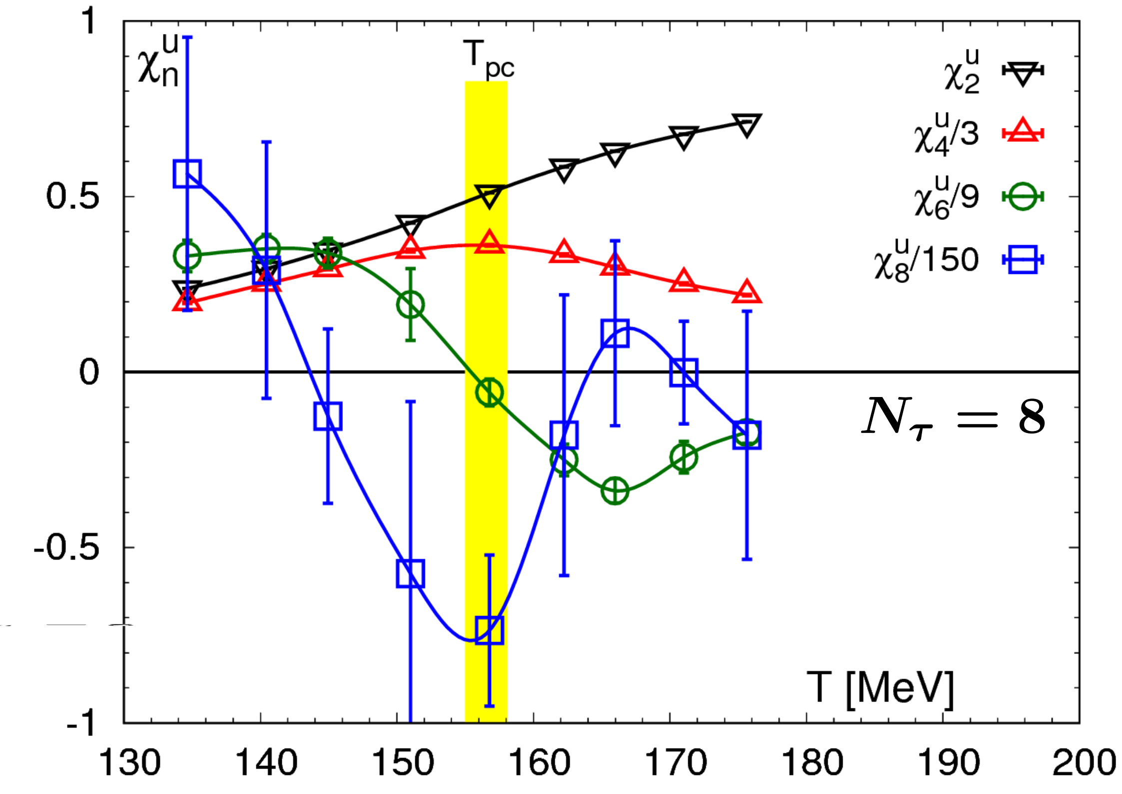
$$\frac{\partial^2}{\partial(\mu_B/T)^2} \simeq \frac{\partial}{\partial T}$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2$$

- All expansion coefficients need to be positive

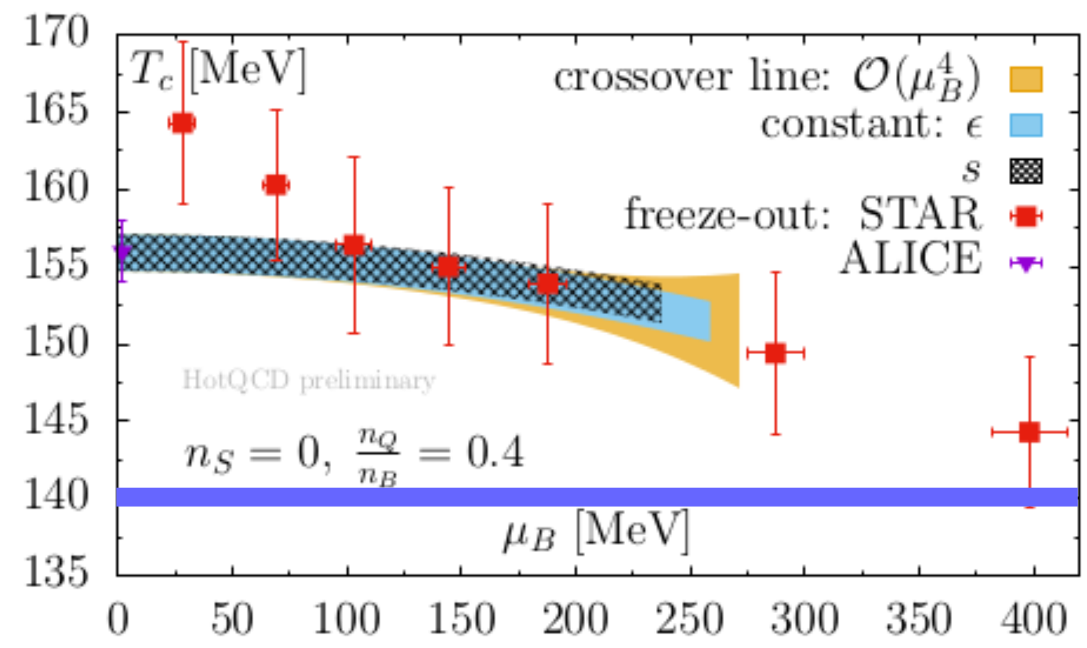
$$\chi_{1n}^{BQ} = \left.\frac{\partial^{n+1} P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_Q^n}\right|_{\mu_{B,Q,S}=0} \qquad \chi_n^u = \left.\frac{\partial^n P/T^4}{\partial\hat{\mu}_u^n}\right|_{\mu_{u,d,s}=0}$$





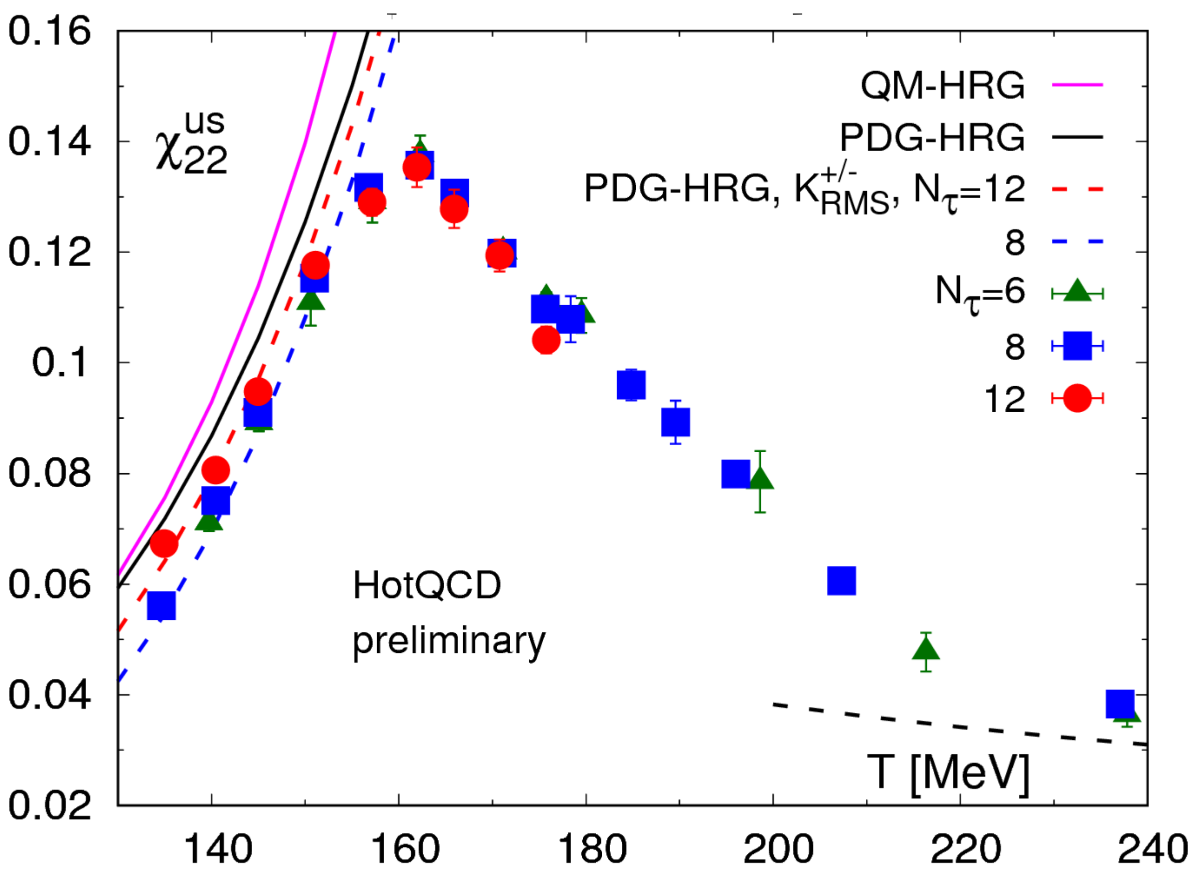
➡ Temperature of CEP is likely below 140 MeV

➡ μ_B^{CEP} likely above 400 MeV



Liberation of quark degrees of freedom

- Liberation of quark degrees of freedom starts quickly above the crossover



Perturbative correction of order $\mathcal{O}(g^3)$

- Very difficult to describe any fluctuation result by HRG above 160 MeV

Summary

- ➡ Scanning behavior that consistent with O(4)-critical point in the chiral limit (cannot exclude 1st-order transition at very small masses)
- ➡ New precise transition temperature: $\boldsymbol{T_{pc} = 156.5 \pm 1.5}$ MeV
- ➡ Equation of state (phase diagram) accessible to $\boldsymbol{\mu_B/T < 2}$
- ➡ Curvature of the crossover line is small ….
  - $\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$
  - $\kappa_2 = 0.0123 \pm 0.003$
  - $\kappa_4 = 0.000131 \pm 0.0041$
- ➡ No indication for critical point, limit: $\boldsymbol{\mu_B^{\mathrm{CEP}} > 400\mathrm{MeV}}$
- ➡ Structure of cumulants show a universal pattern