FLUCTUATIONS FROM LATTICE QCD AND HRG MODEL

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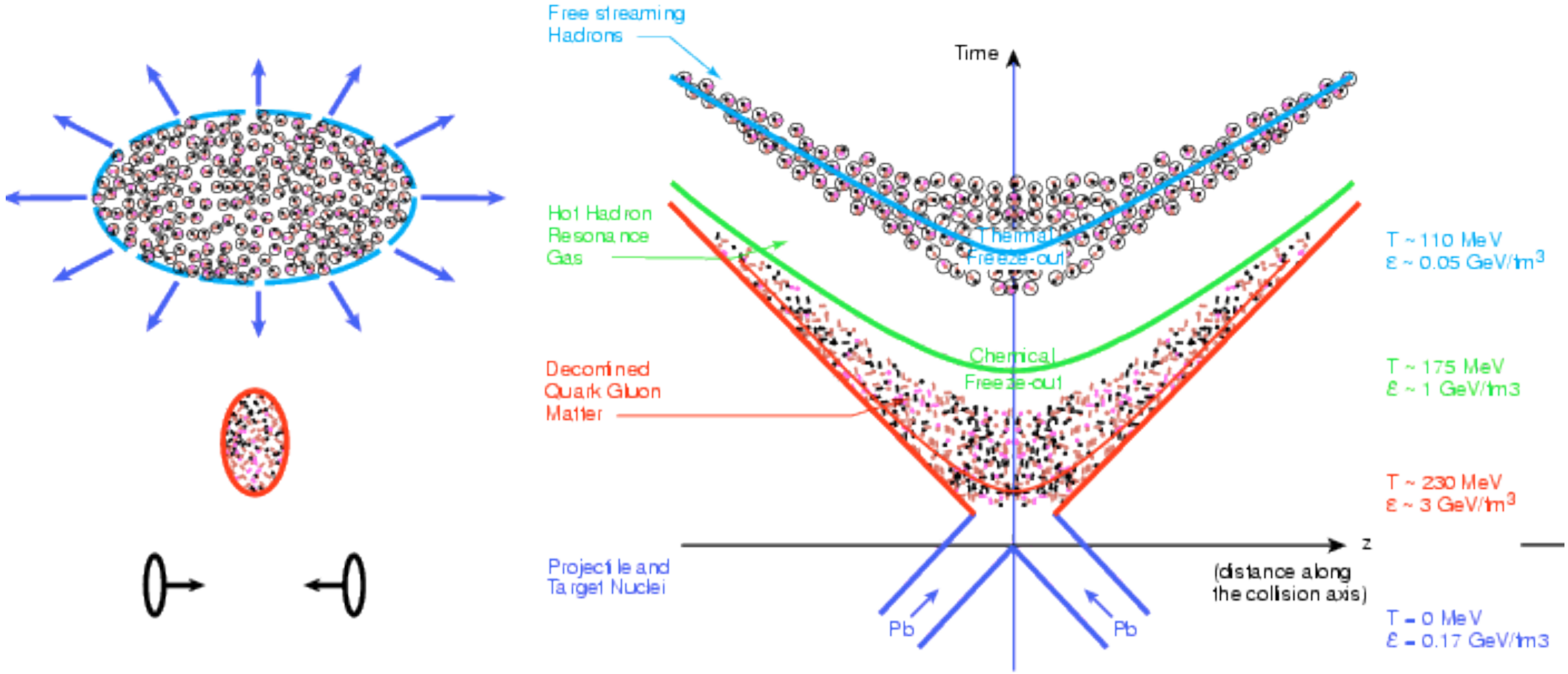
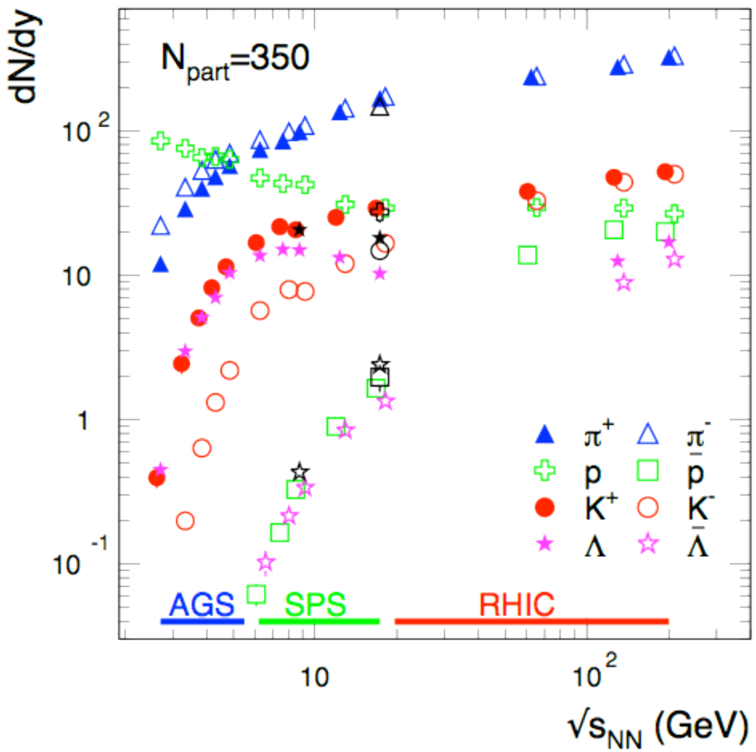
Evolution of a Heavy Ion Collision



- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Hadron yields

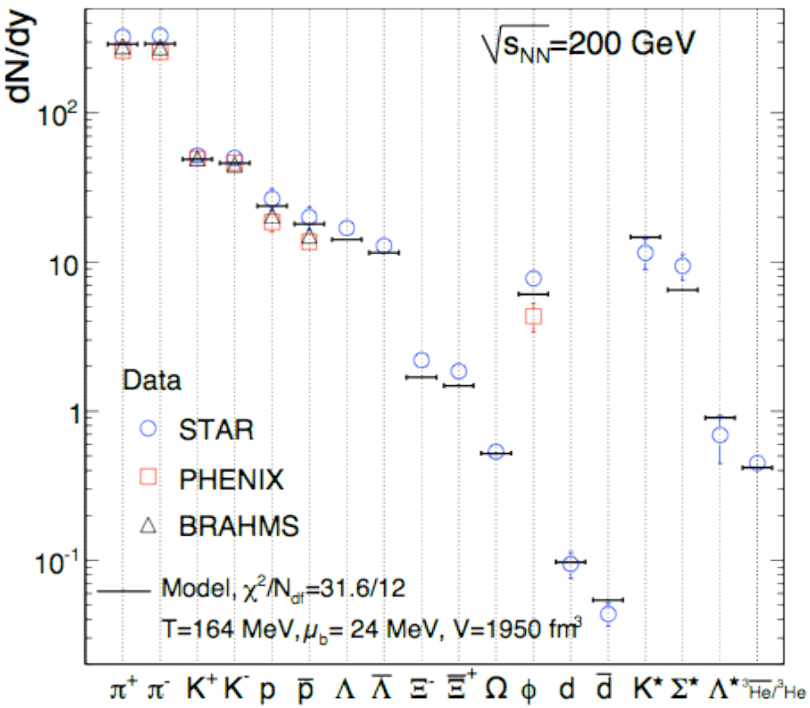


- E=mc²: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays

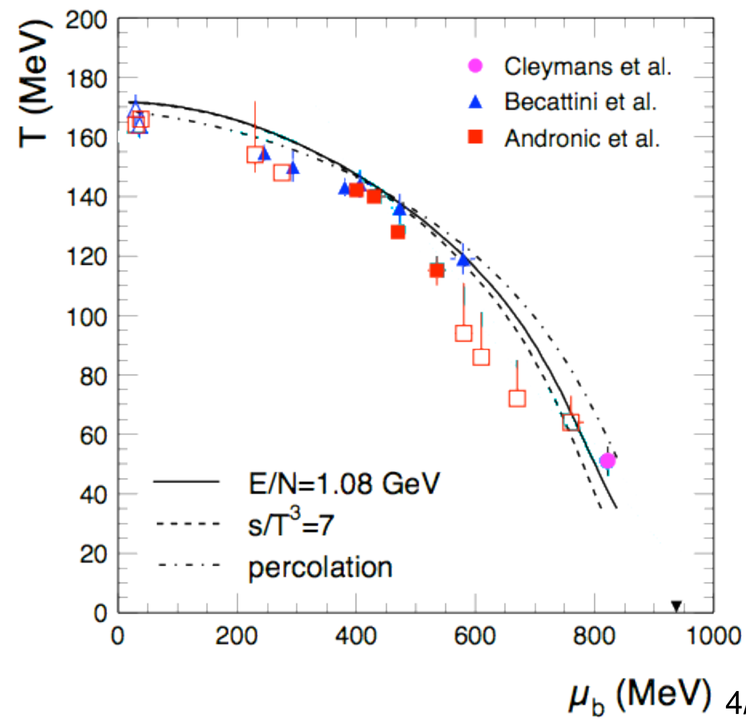
- HRG model: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

The thermal fits



- Fit is performed minimizing the χ^2
- Fit to yields: parameters T, μB, V
- Fit to ratios: the volume V cancels out



- Changing the collision energy, it is possible to draw the freeze-out line in the T, μB plane

Cleymans et al., Becattini et al., Andronic et al.

Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{\,l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned}
\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\
\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\
\mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.
\end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

- The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0 \qquad \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

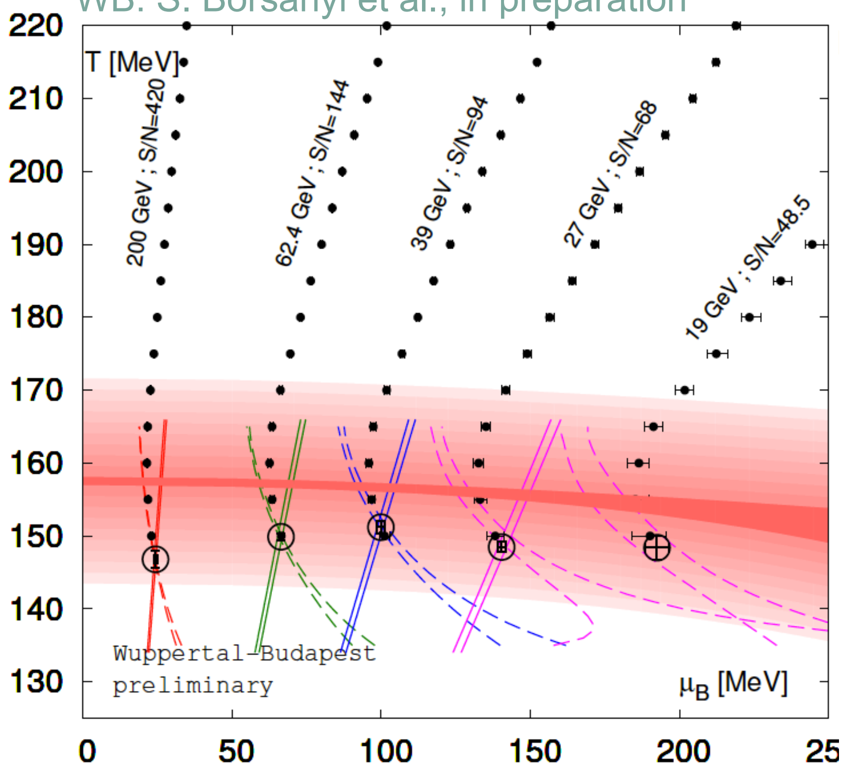
Freeze-out line from first principles

- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation

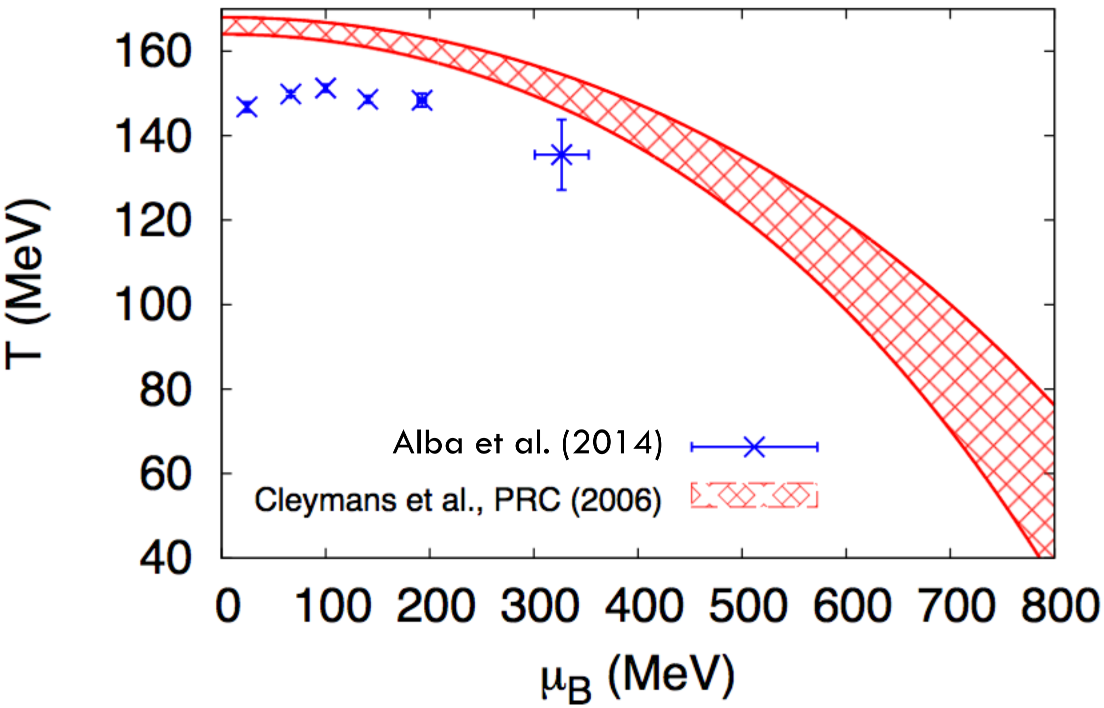


- Net-p and net-Q results from STAR collaboration STAR, PRL112 (2014) STAR, PRL113 (2014)
- The freeze-out points from net-p and net-Q lie at the lower end of the crossover region
- We reconstruct the isentropic trajectories (constant S/N) starting from the freeze-out points

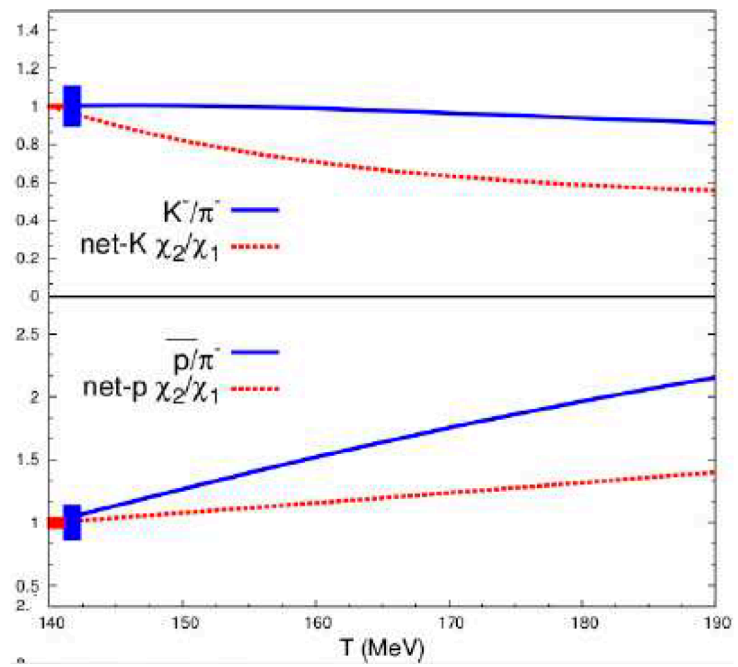
Freeze-out line from the HRG model

- A combined fit of the net-p and net-Q fluctuations yields a lower freeze-out temperature than the one from yields
- Yields include (multi-)strange particles
- For some particles, fluctuations are more sensitive than yields to T_{fo}

P. Alba, C.R. et al., PLB (2014)

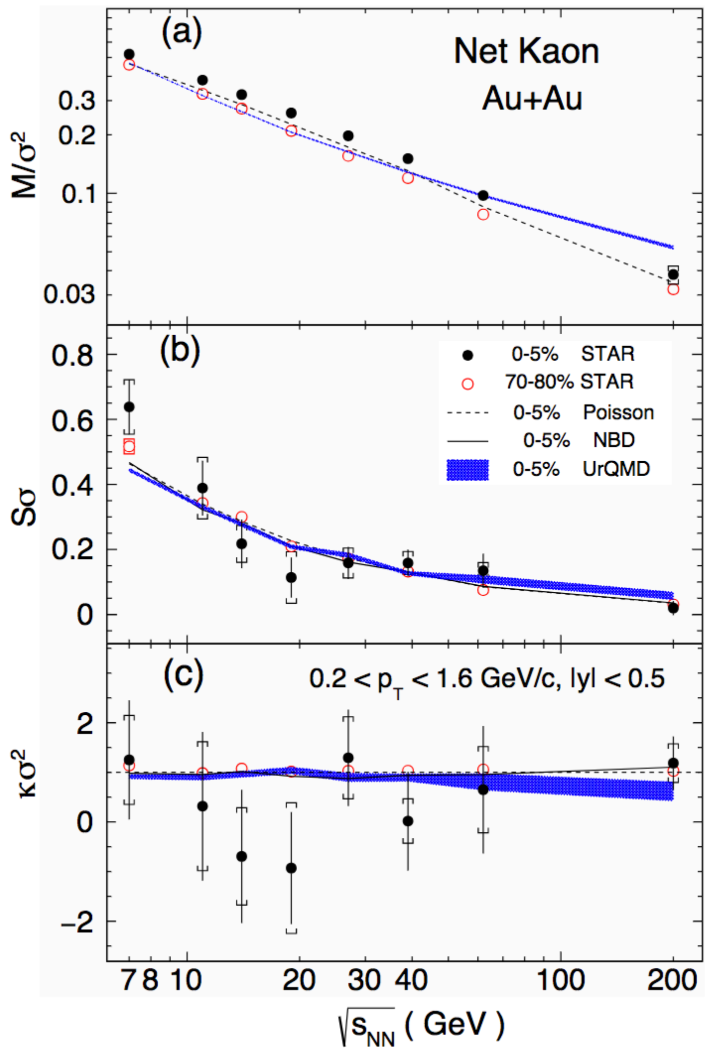


P. Alba, C.R. et al., PRC (2015)



Freeze-out of kaons in the HRG model

STAR Collaboration 1709.00773

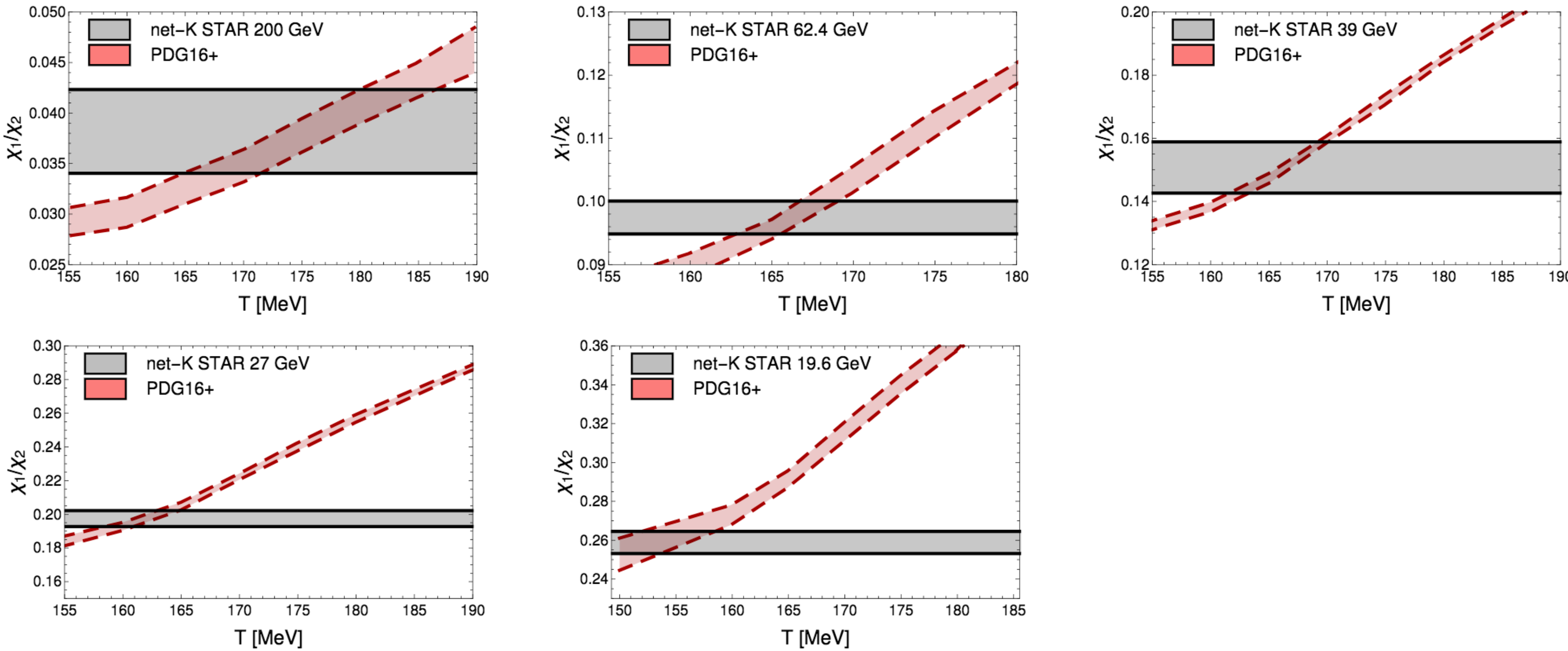


- Calculate χ_1/χ_2 for kaons in the HRG model, including resonance decays and acceptance cuts
- Calculate it along the isentropes
- Fit χ_1/χ_2 and extract T_{fo}
- Obtain μ_{Bfo} from the isentropes

R. Bellwied, C. R. et al., 1805.00088

Freeze-out of kaons in the HRG model

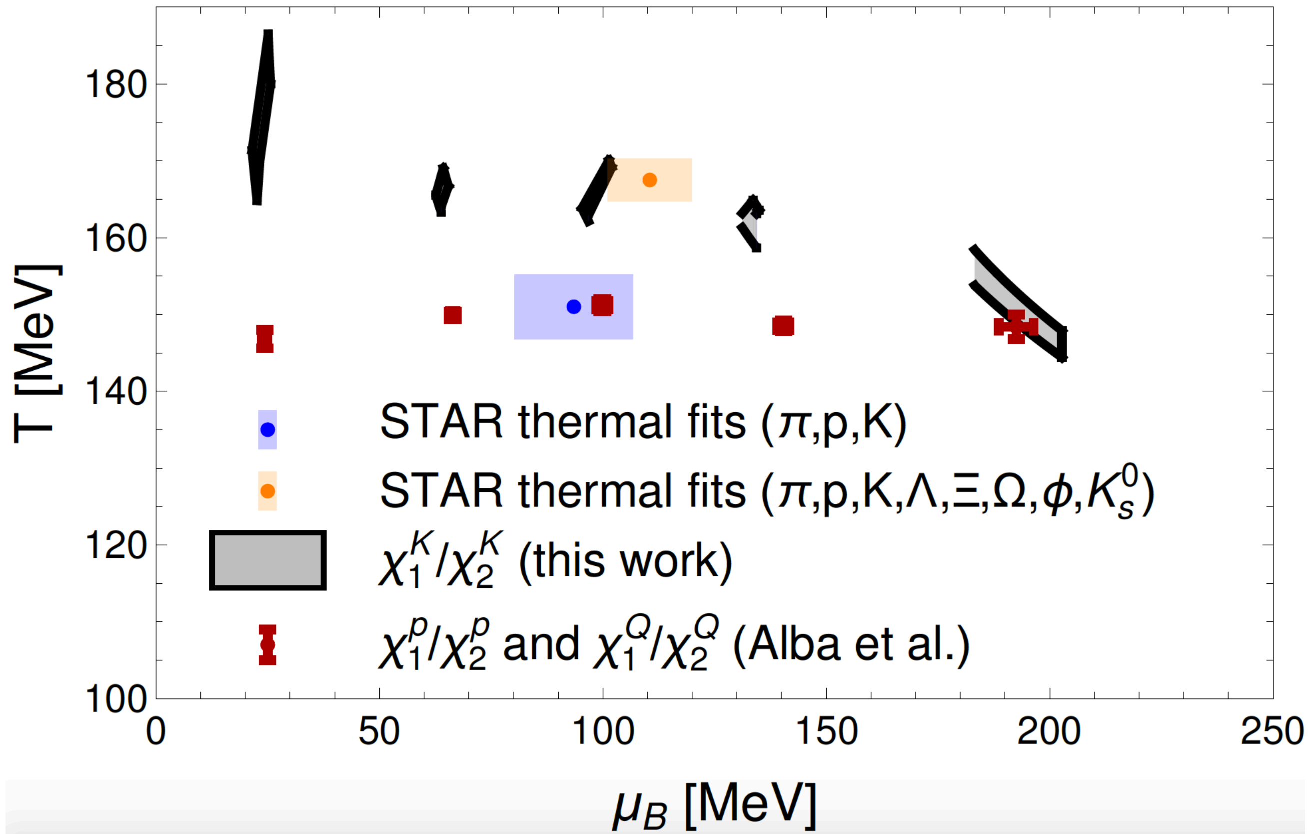
R. Bellwied, C. R. et al., 1805.00088



- χ_1/χ_2 for kaons needs a higher freeze-out temperature than net-p/net-Q

Freeze-out of kaons in the HRG model

R. Bellwied, C. R. et al., 1805.00088

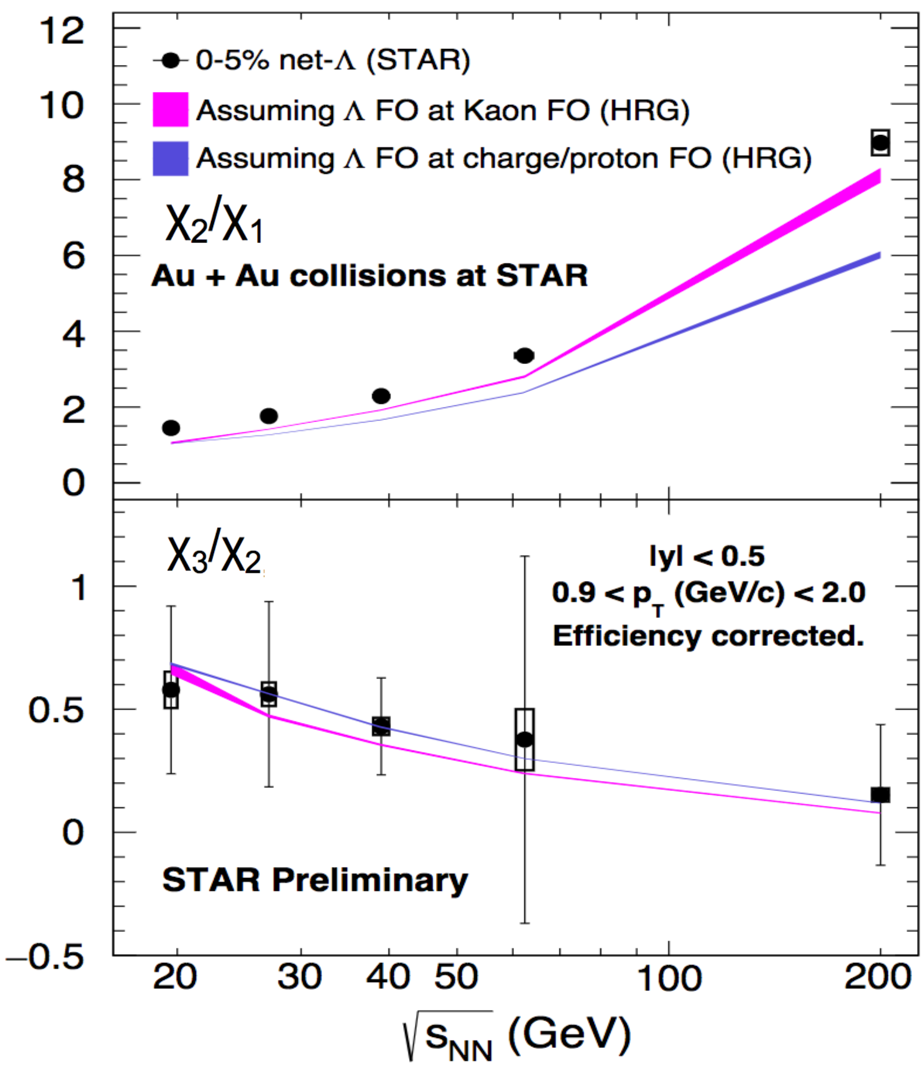


- χ_1/χ_2 for kaons needs a higher freeze-out temperature than net-p/net-Q
- The f.o. parameters agree with the STAR fit of yields (including strange particles)

STAR Collaboration, PRC (2017)

Prediction: net-Λ fluctuations

R. Bellwied, C. R. et al., 1805.00088

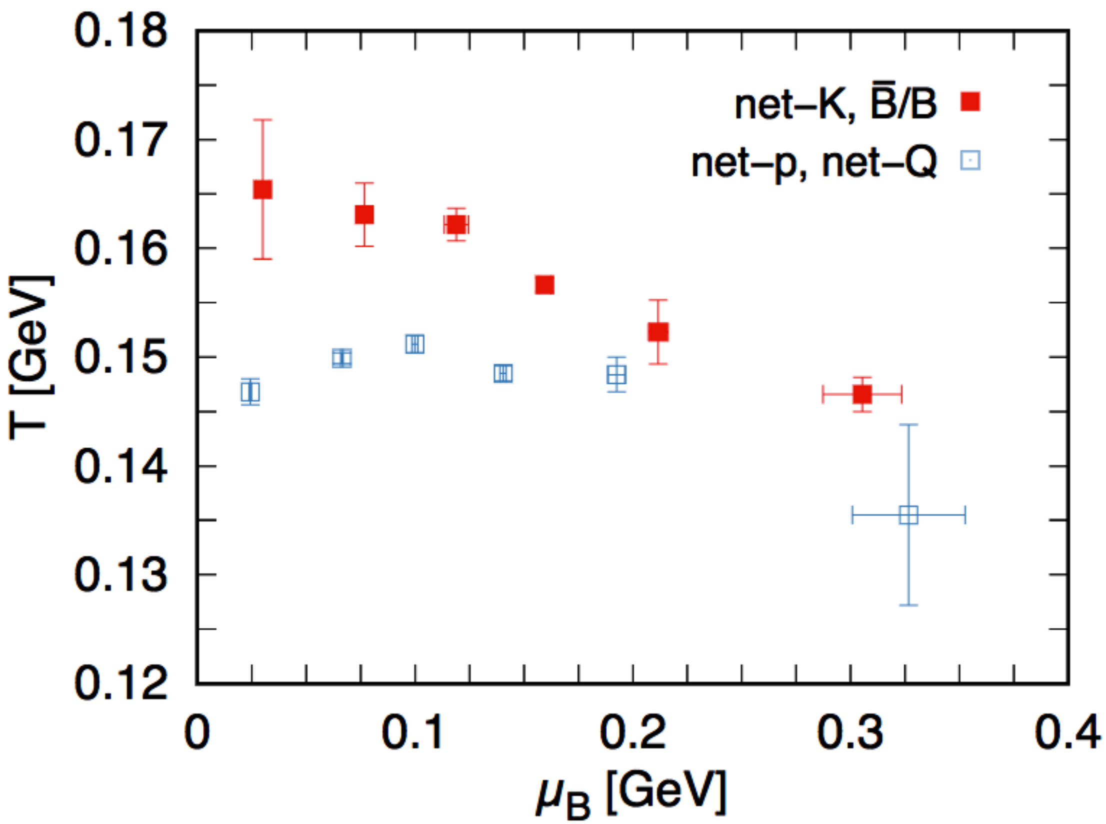


- We predict the net-Λ χ_2/χ_1 and χ_3/χ_2, assuming that they freeze-out with the charge/ protons or the kaons
- The χ_2/χ_1 data prefer the net-kaon freeze-out conditions

STAR preliminary data (see talks by Z. Ye and T. Nonaka and poster by N. Kulatunga at QM2018)

A different approach

M. Bluhm and M. Nahrgang., 1806.04499



- Simultaneous fit of χ_1/χ_2 for kaons and of the strange anti-baryon over baryon yield ratios to determine T and μB
- Qualitative agreement with our results

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527

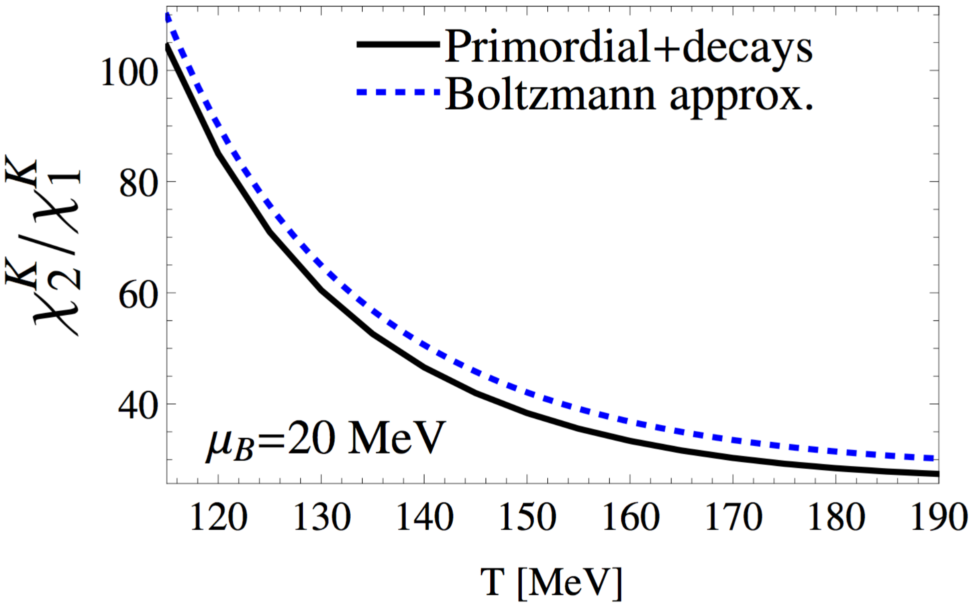
- Lattice QCD works in terms of conserved charges
- Challenge: isolate the fluctuations of a given particle species
- Assuming an HRG model in the Boltzmann approximation, it is possible to write the pressure as:

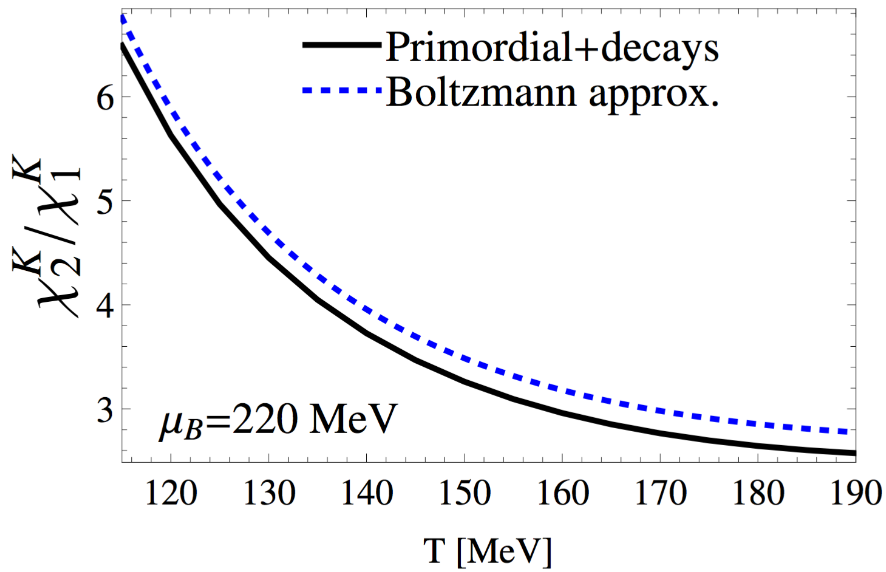
$$P(\hat{\mu}_B, \hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS}\cosh(\hat{\mu}_B) + P_{01}^{BS}\cosh(\hat{\mu}_S)$$
$$+ P_{11}^{BS}\cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS}\cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{13}^{BS}\cosh(\hat{\mu}_B - 3\hat{\mu}_S)$$

- Kaons in heavy ion collisions: primordial + decays
- Idea: calculate χ_2^K/χ_1^K in the HRG model for the two cases: only primordial kaons in the Boltzmann approximation vs primordial + resonance decay kaons

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527





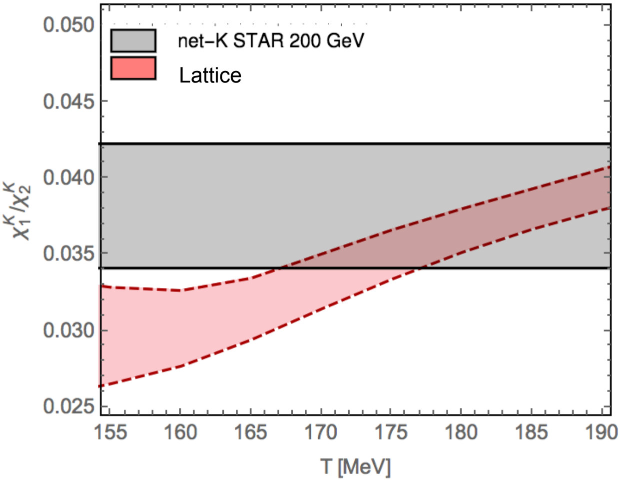
- Boltzmann approximation works well for lower order kaon fluctuations

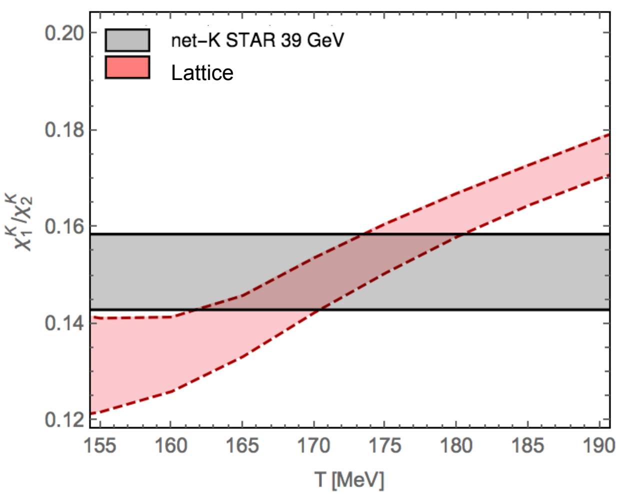
$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

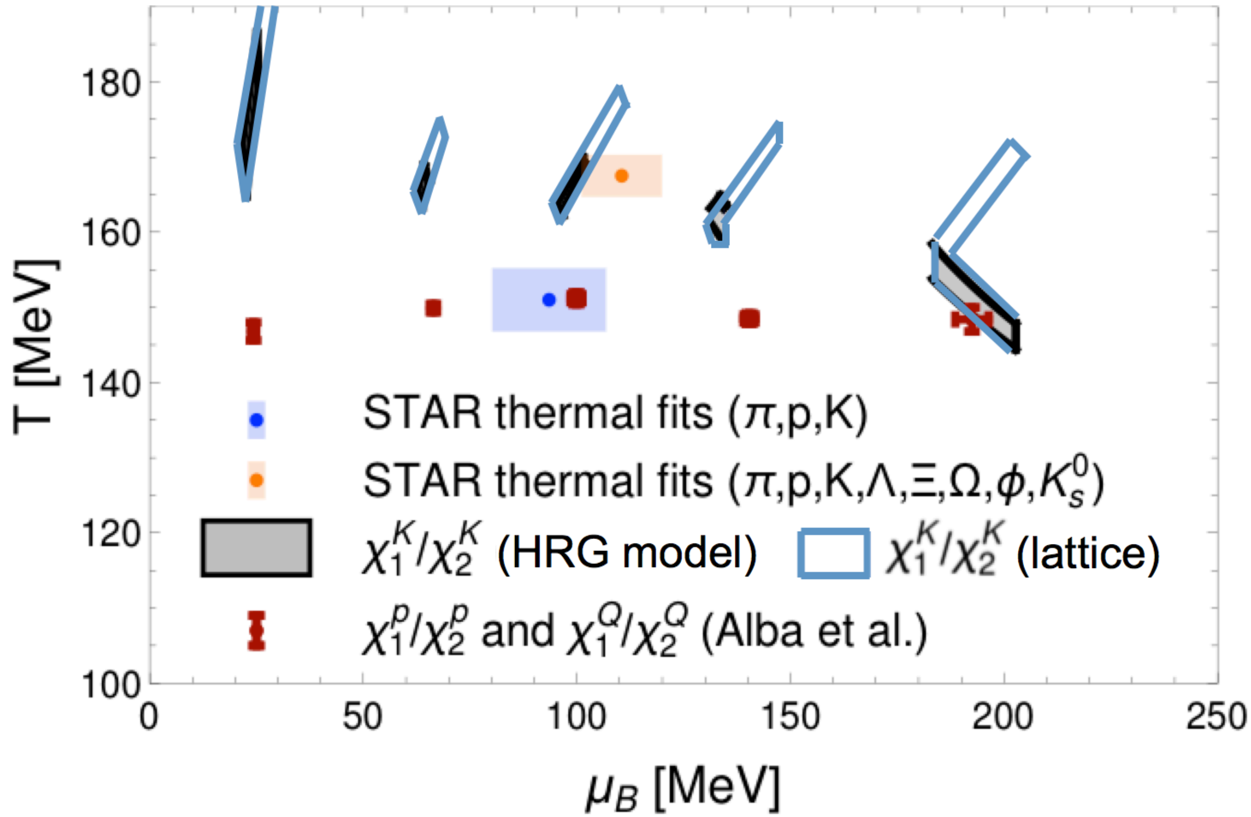
- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al. forthcoming



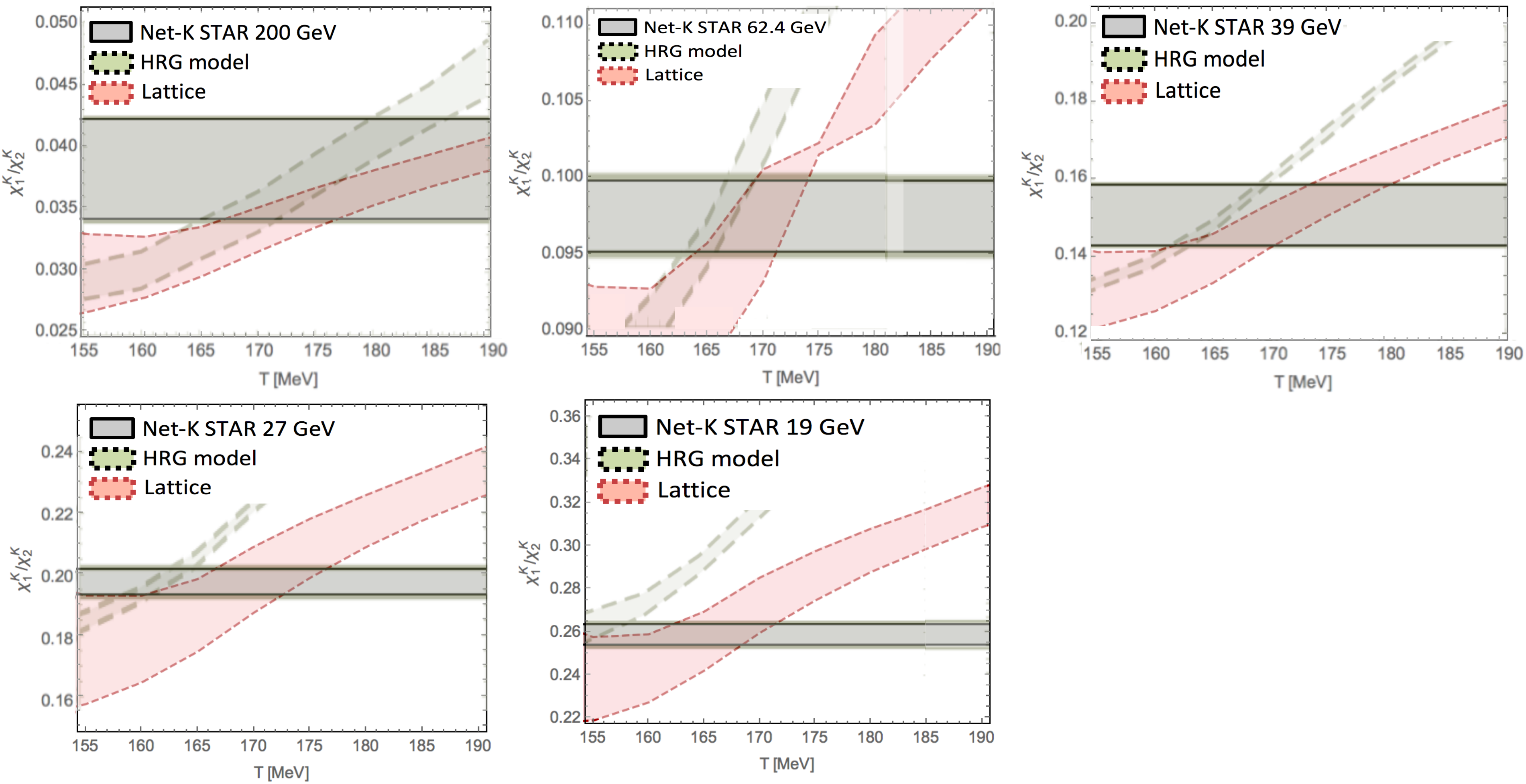




- Lattice QCD results have a larger uncertainty but agree with HRG model ones within error-bar

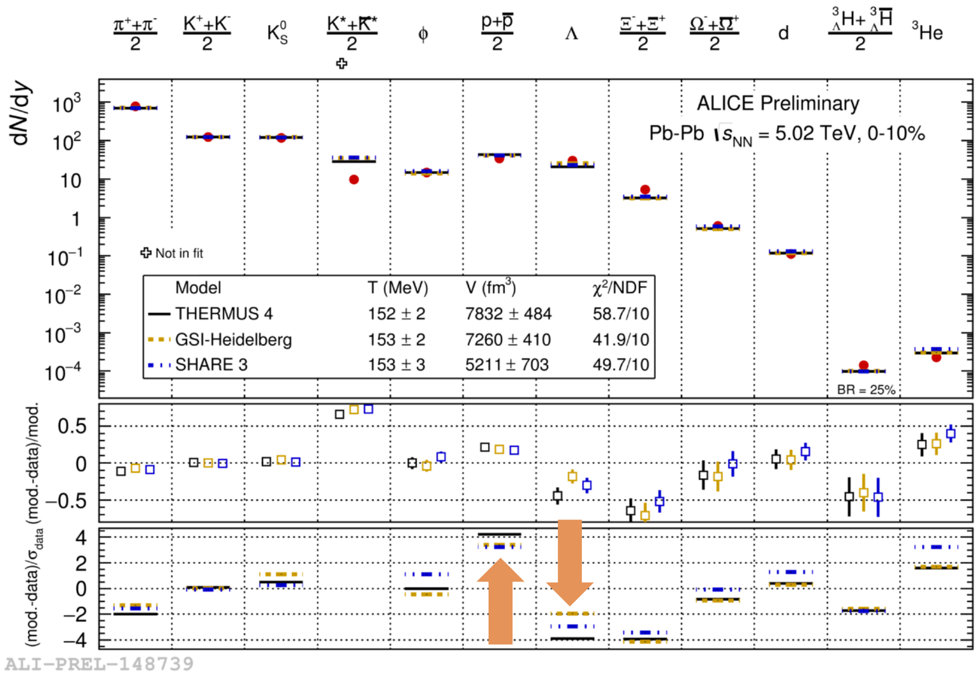
Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al. forthcoming



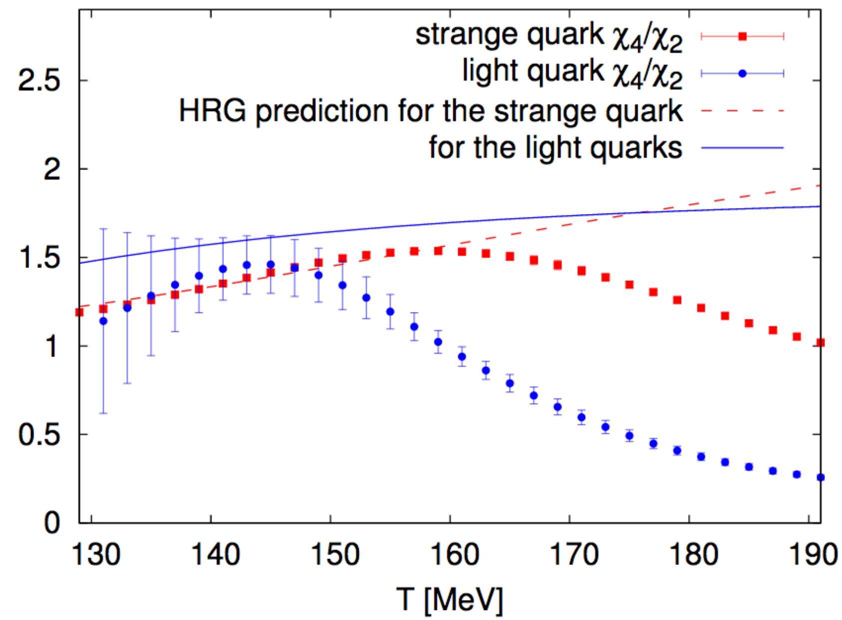
Possible explanations

- Flavor-dependent freeze-out temperature?



Most recent ALICE fit shown at QM2018

R. Bellwied et al. (WB Collaboration): PRL2013



- Non-equilibrium effects (hadronic re-interactions)? J. Steinheimer et al., PRL (2013)
- Interacting HRG model? K. Redlich et al.
- Non-thermal effects? A. Rustamov et al.

Conclusions

- Lattice QCD and HRG model give consistent results on freeze-out conditions from fluctuations
- Fits of net-p and net-charge χ_2/χ_1 yield freeze-out parameters at the lower end of the crossover region
- Fits of net-K χ_2/χ_1 yield freeze-out temperatures that are ~15-20 MeV higher than the ones from net-p/net-charge fits
- New data for net-Λ fluctuations are closer to the net-K freeze-out parameters

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013), P. Braun-Munzinger et al., NPA (2017),
V. Begun and M. Mackowiak-Pawlowska (2017)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

- Final-state interactions in the hadronic phase
  - Consistency between different charges = fundamental test

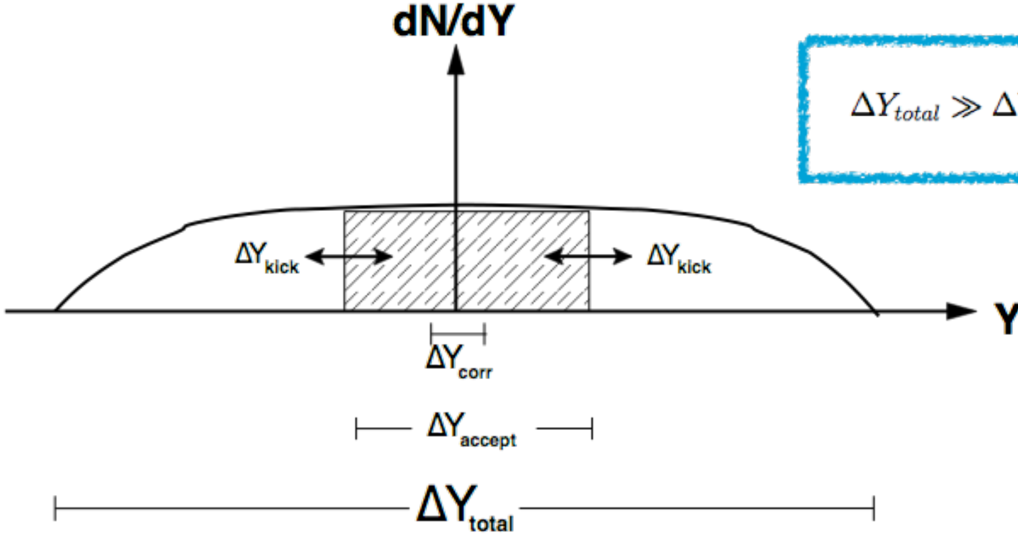
J.Steinheimer et al., PRL (2013)

Fluctuations of conserved charges

V. Koch (2008)

* If we look at the entire system, none of the conserved charges will fluctuate

* By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful

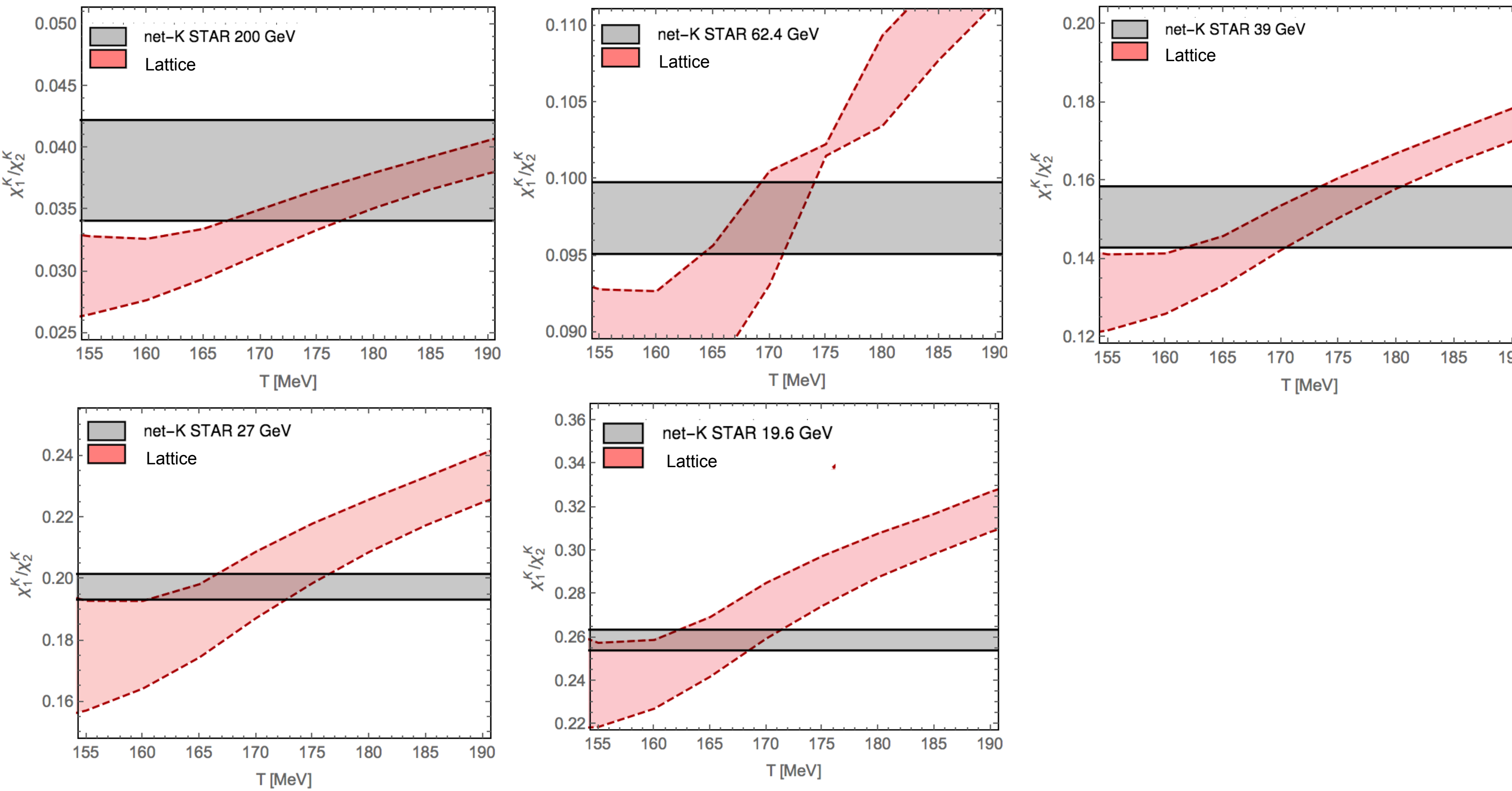


$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

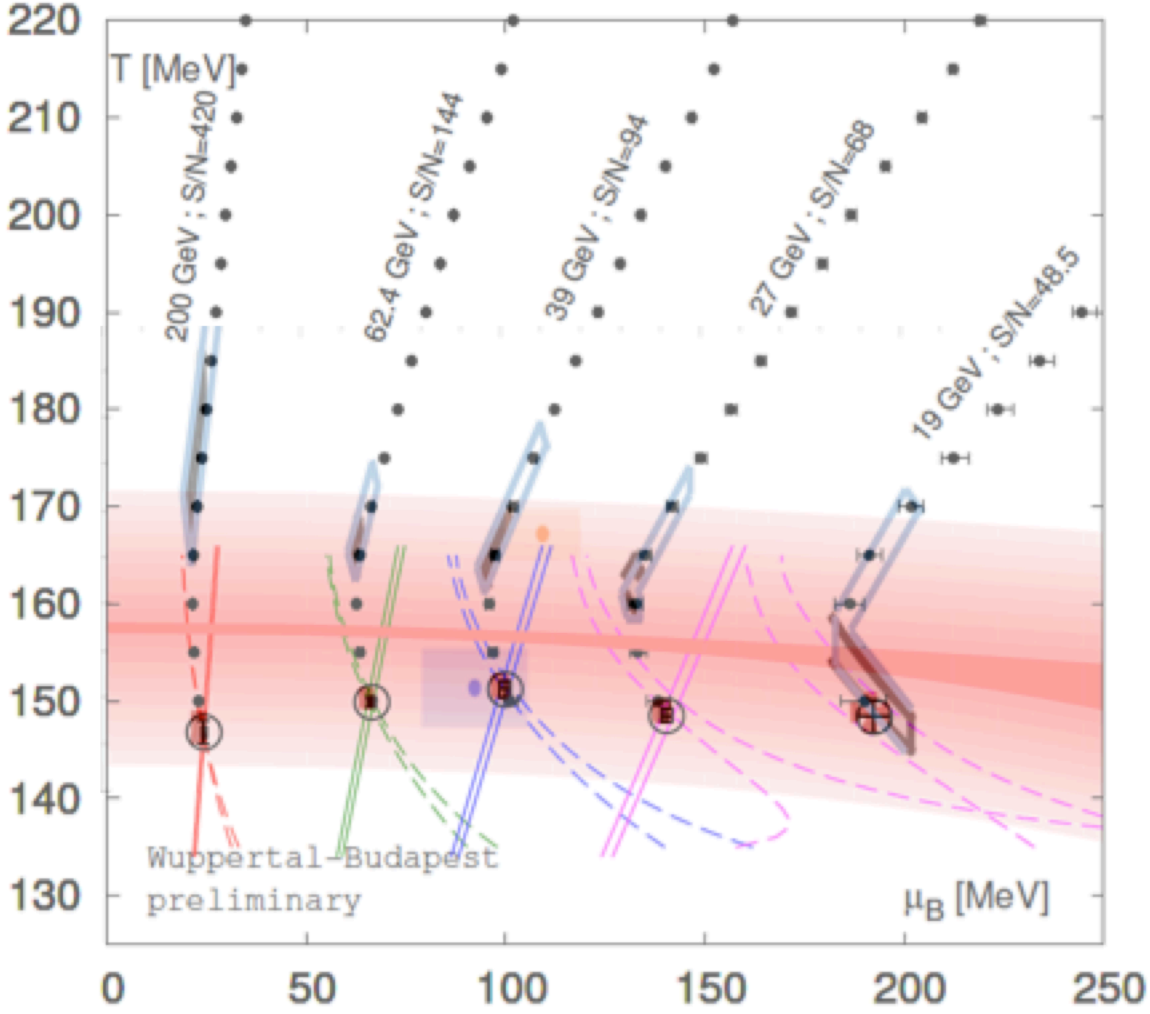
- ❑ ∆Ytotal: range for total charge multiplicity distribution
- ❑ ∆Yaccept: interval for the accepted charged particles
- ❑ ∆Ykick: rapidity shift that charges receive during and after hadronization

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al. forthcoming



QCD phase diagram and freeze-out



Lattice details

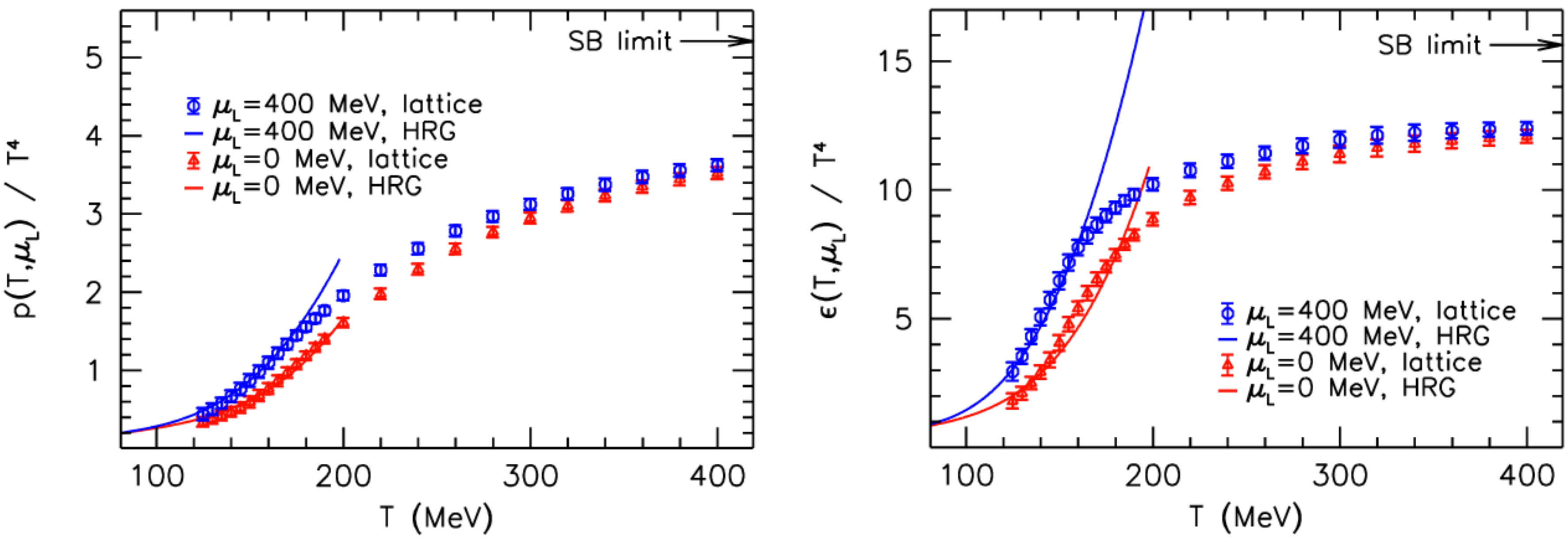
- **The 4stout staggered action**
  - 2+1+1 dynamical flavors
  - 4 levels of stout smearing in the fermionic action
  - The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping m_c/m_s=11.85
  - The scale is set in two ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the source of the systematic error in all of the plots

- **Ensembles**
  - Continuum limit from N_t=10, 12, 16
  - For imaginary μ we have $\mu_B=iT\pi j/8$, with j=3, 4, 5, 6, 6.5, 7

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

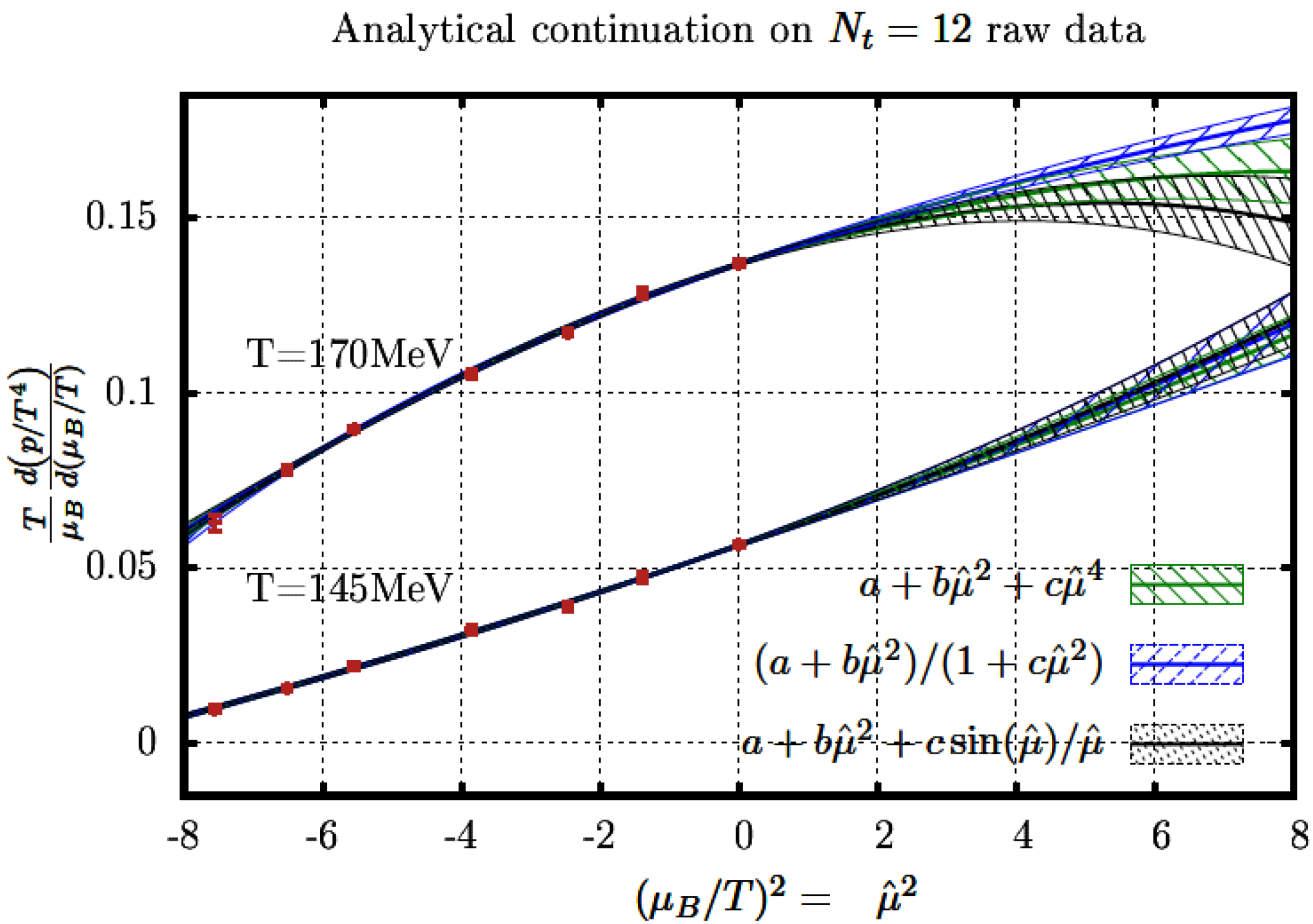
$$\frac{p(T,\{\mu_i\})}{T^4} = \frac{p(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V}\frac{1}{T^2}\left.\frac{\partial^2 \log \mathcal{Z}}{\partial\mu_i\partial\mu_j}\right|_{\mu_i=\mu_j=0}$$



S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Analytical continuation – illustration of systematics



Testing the Taylor expansion

R. Critelli, C. R. et al., PRD (2017)

Taylor expansion of observables in terms of susceptibilities
$\chi_n = \chi_n^B(T, \mu_B = 0)$

- Pressure

$$\frac{p(T,\mu_B) - p(T,\mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

- Baryonic density

$$\frac{\rho_B(T,\mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n-1}$$

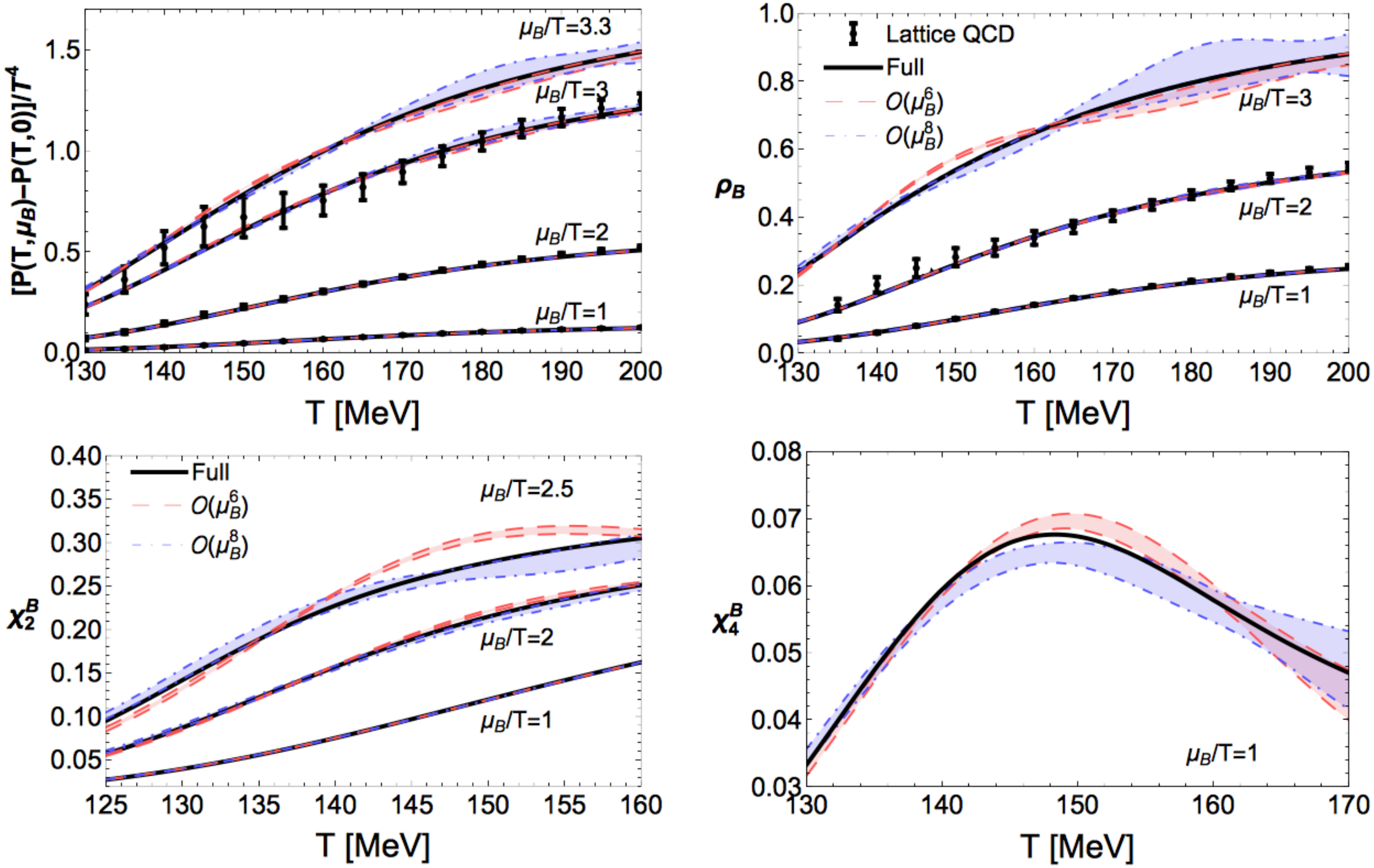
- Susceptibilities χ_2 and χ_4

$$\chi_2(T,\mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_4(T,\mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Reconstruction of thermodynamic quantities at different values of μ_B/T via Taylor series from calculations at $\mu_B = 0$.

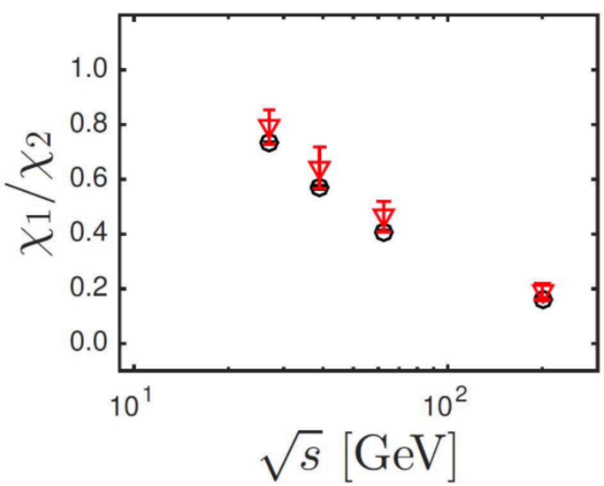


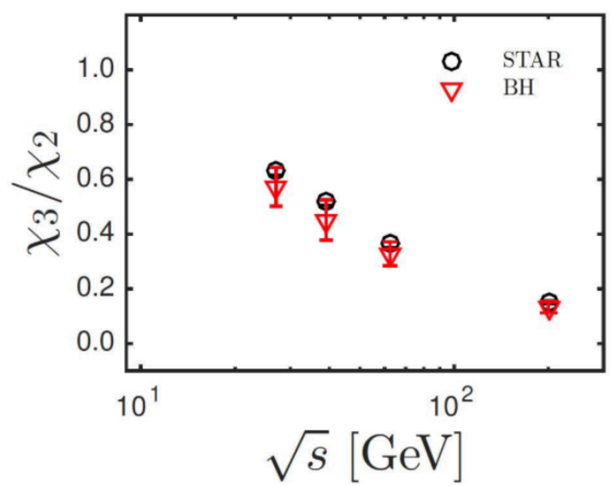
Lattice results: [HotQCD] Phys. Rev. D, **95**, 054504 (2017).

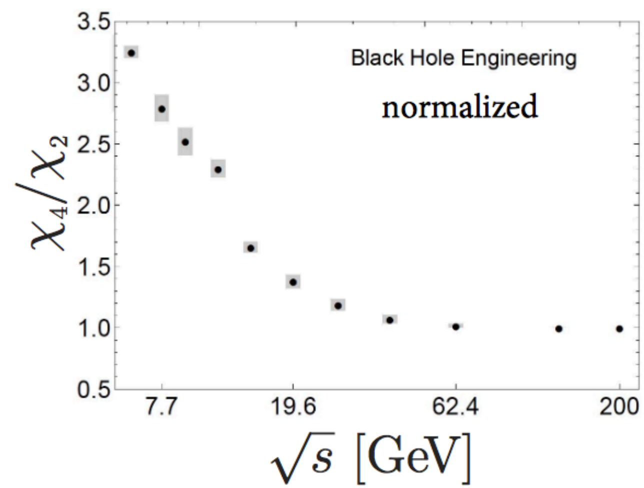
Connection to experiment

R. Critelli, C. R. et al., (2017)

- We want to estimate the collision energy we need to find the critical point in experiments
- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR
- Freeze-out parameters are extracted by fitting the experimental values for $\chi 1/\chi 2$ and $\chi 3/\chi 2$



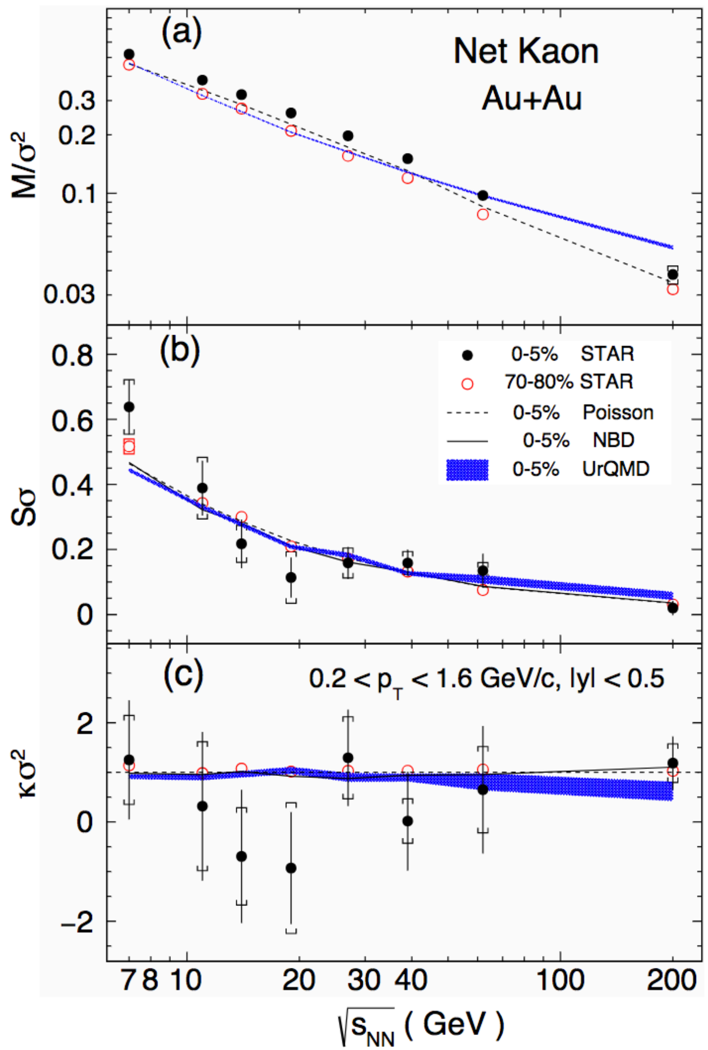




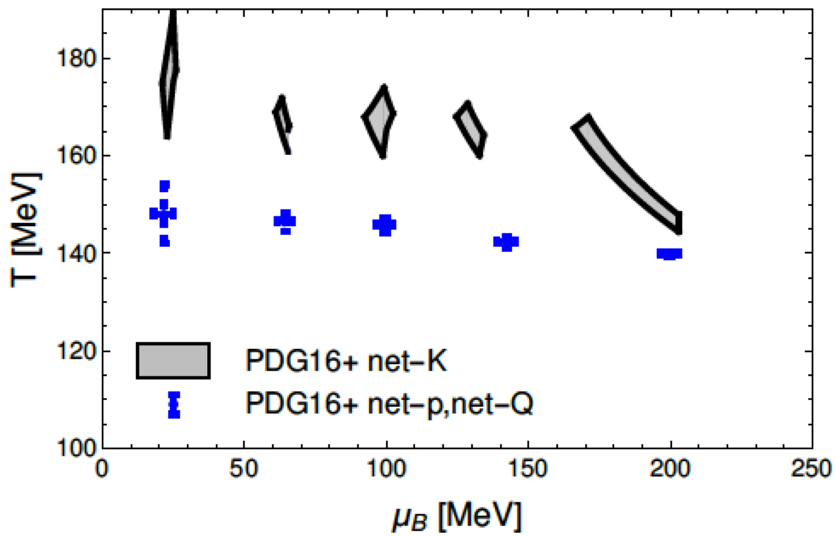
[STAR] Phys. Rev. Lett. **112** (2014)

Freeze-out of kaons in the HRG model

STAR Collaboration (2017)



- Calculate χ_1/χ_2 for kaons in the HRG model, including resonance decays and acceptance cuts
- Calculate it along the isentropes



C. R. et al., in preparation