Quark Number Susceptibilities & Equation of State at finite μ

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Introduction

- Exciting and Challenging results from BES at RHIC. $\Longrightarrow$
  Are we sighting new land(s) ? So far, all we know is,

"These observables show a centrality and energy dependence, which are neither reproduced by non-CP transport model calculations, nor by a hadron resonance gas model. " — STAR Collaboration PRL (2014).

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- Two problems for Lattice approach : 1) Quark Type & 2) Phase/Sign Problem. Quark Type issue resolved : Action for Overlap fermions with $\mu \neq 0$ **and** continuum like chiral symmetry defined (RVG-Sharma, PLB '15, '12 & PRD '12).

- Many innovative methods proposed for studying nonzero chemical potential. QCD with light flavours has been investigated with quite a few. Re-weighting, Imaginary μ, Canonical Approach are some examples (Fodor-Katz '02, de Forcrand-Philipsen '02, Liu et al. '02).

- I will concentrate on the Taylor expansion method in which ΔP is expanded in powers of $z = \mu_B/T$ (Mumbai '03, Bielefeld-Swansea'03):

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_n \frac{\chi_B^n}{T^{4-n}} \frac{z^n}{n!}$$

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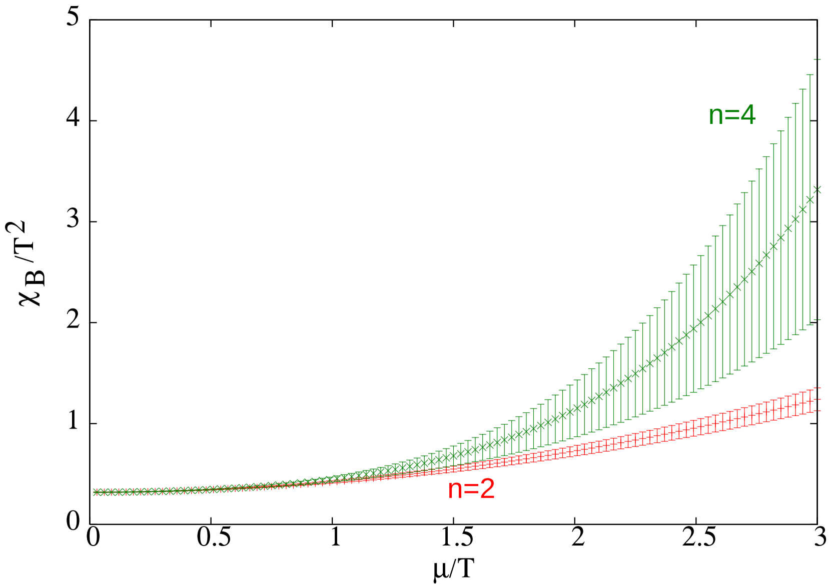
- Using these coefficients, one looks for estimators of radius of convergence of the baryonic susceptibility $\chi_B(\mu_B, T)$ by varying temperature T :

$$\mu_{B;m/n} = \left[\frac{(n-2)!\chi_B^m}{(m-2)!\chi_B^n}\right]^{\frac{1}{(n-m)}}$$

- Mumbai group obtained results for (μ_B^E, T^E) on $N_t = 4$, 6 and 8 lattices. Additionally, Padè approximants were used for the finer lattice to verify that the resultant estimates are in the same ball park.(Mumbai, PRD '05, '08,'15,'17).

Cross Check on μ^E/T^E

♠ Use the series directly to construct χ_B for nonzero $\mu \longrightarrow$ smooth curves with no signs of criticality.



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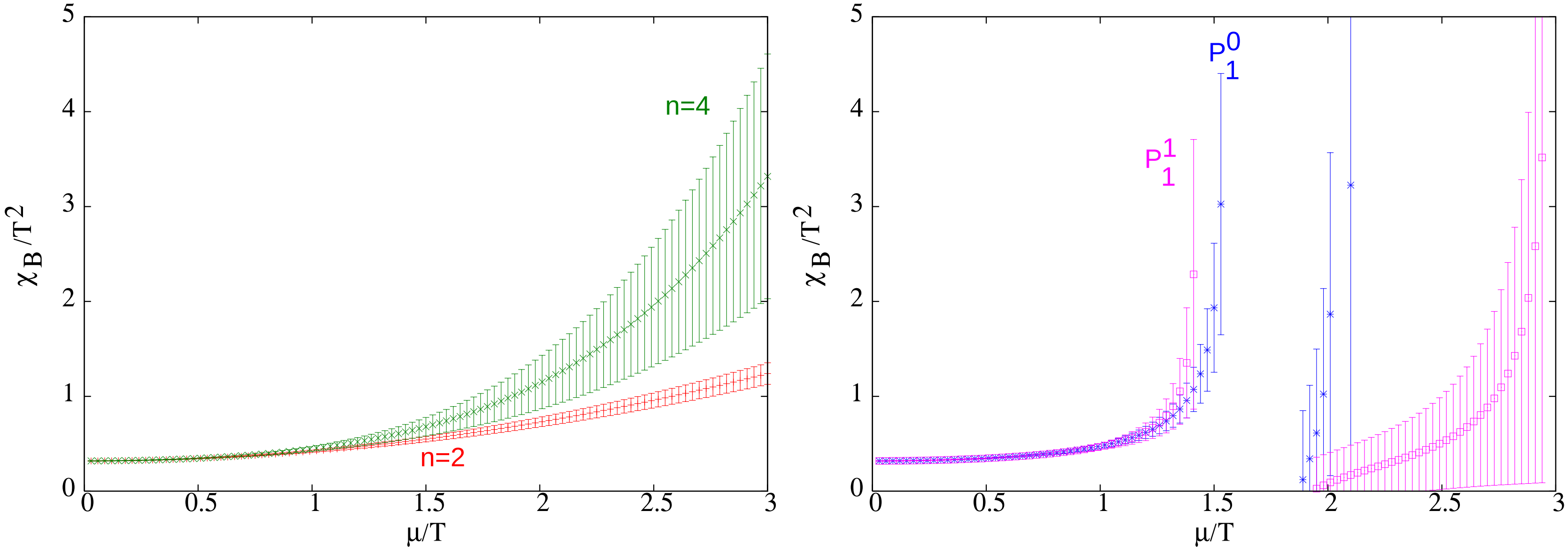

♠ Use Padé approximants for the series to estimate the radius of convergence, as suggested earlier (Lombardo LAT '06).

♡ Consistent Window with our ratio estimate for $N_t = 6$ (Mumbai PRD '08).

- Further, Lattice QCD predictions along the phenomenologically well-established "Freeze-out" curve were made using the Padè for ratios of higher baryon susceptibilities, $m_1 = \frac{\chi_3(\mu_B,T)}{\chi_B(\mu_B,T)}$, $m_2 = \frac{\chi_4(\mu_B,T)}{\chi_B(\mu_B,T)}$, to demonstrate non-monotonicity resulting due to this criticality (Mumbai, PLB '11).

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- Many improvements are clearly desired :
  - More ratios, *i., e.* more orders. : Linear chemical potential in action can help.
  - Reducing cut-off effects, *i., e.* bigger lattices : $N_t = 8$ and more statistics.
  - Better efforts/methods to control errors. : Employ DLOG Padè.
  - Compare with the STAR data on cumulants quantitatively.

More Orders

- Need as many terms of the expansion as possible. However, higher orders entail many more terms. Difficult to handle the statistical problem due to their cancellations.

- Adding chemical potential (RVG-Sharma, '12,'15) linearly in the quark matrix $M(\mu = 0)$ leads to simplifications. Its successive derivatives with μ are
  $M' = \sum_{x,y} N(x,y)$, and $M'' = M''' = M''''... = 0$,
  in contrast to the popular exp($\pm a\mu$)-choice where *all* derivatives are nonzero:
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- Note *only* linear μ-actions have a well defined $\hat{H}$ and $\hat{N}_B$ on lattice, and thus a connection with canonical ensembles. True for the exponential action only in the continuum limit, $a \to 0$.

Our Set-up

- Our calculation: $N_f = 2$ QCD, temperature range 0.9 – 2.1 T_c .
- $32^3 \times 8$ Lattices with a $=$ $1/8$T. Cutoff effect estimated by comparing with earlier calculations at a $=$ $1/4$T and $1/6$T.
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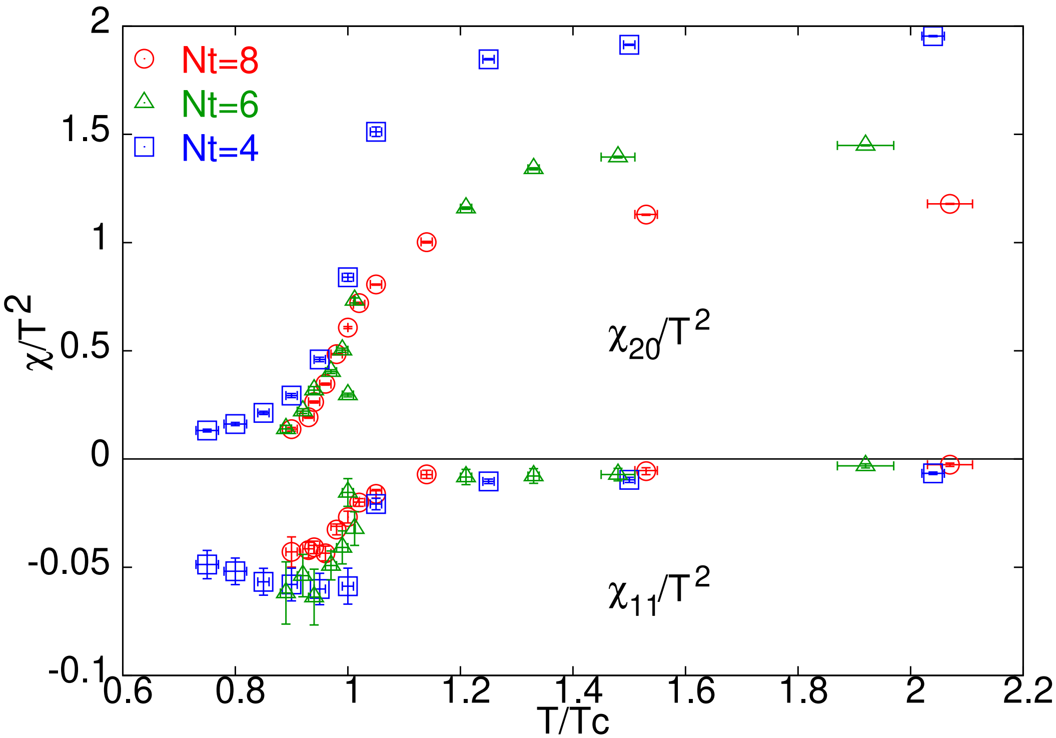
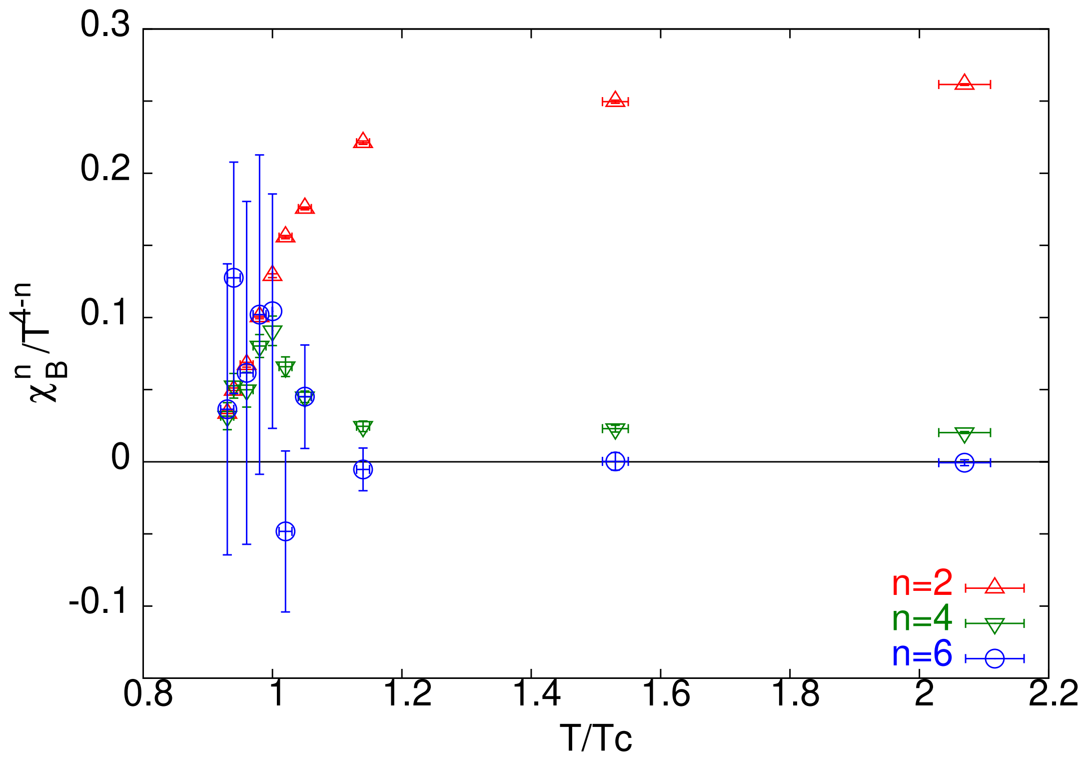
- Degenerate u, d quarks, $m_\pi \simeq 220$ MeV.
- T_c determined from peak of the susceptibility of the Polyakov loop, and its physical value set using w_0 (Wilson flow).
- Other temperatures set with a combination of w_0 and two-loop running.
- Broad features of susceptibility measurements agree with results on susceptibilities with other discretizations.

- Comparing the second order quark number susceptibility as a function of cut-off decreasing a, or equivalently increasing N_t,



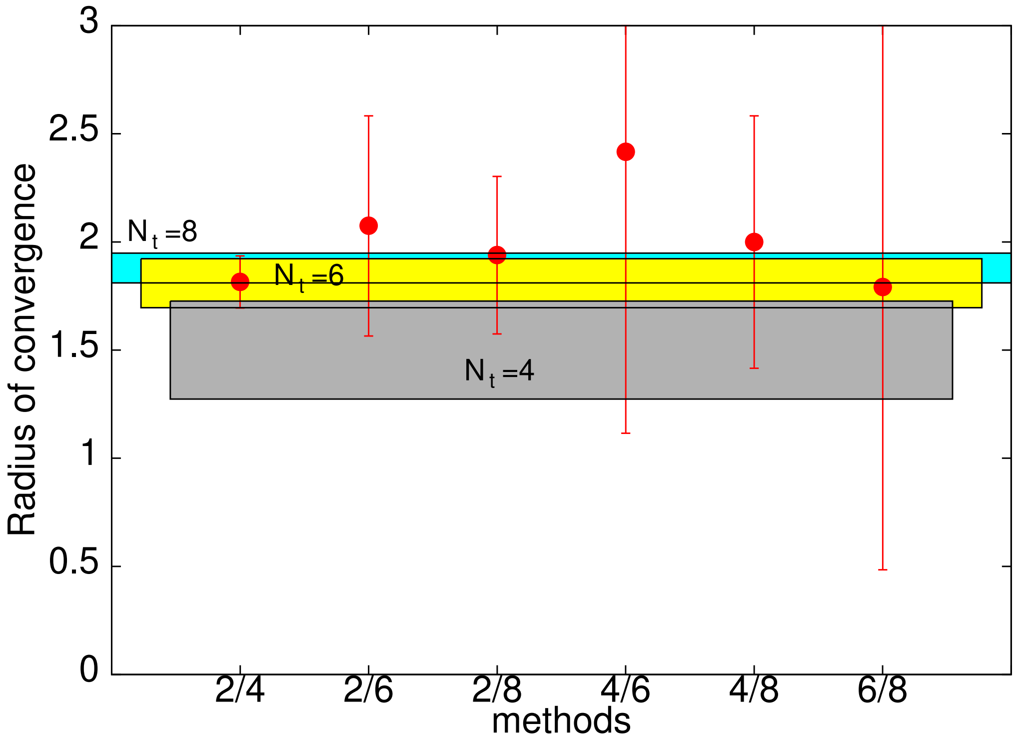
- Cut-off effects seem to be relatively smaller for the two biggest lattices for $T < T_c$.

- The Taylor coefficients or higher order baryon number susceptibility as a function of T are :



- Critical Point estimate for $N_t = 8$ is $\frac{T^E}{T_c} = 0.94 \pm 1$ and $\frac{\mu_B^E}{T^E} = 1.85 \pm 0.04$ (Mumbai PRD '17).

- Comparing the results as a function of N_t :



A good agreement with our earlier $N_t = 6$ result (Mumbai PRD '08) is seen.

DLOG Padè

- Baryonic susceptibility expected to diverge at the critical point :

  $$\frac{\chi_B(z,T)}{T^2} \propto |z^2 - z_E^2|^{-\psi}.$$

- DLog Padè approximants are common in CP-analysis in SM. For us,

  $$m_1 = \frac{\partial \log \ \chi_B(z,T)/T^2}{\partial z} = \frac{\chi_3(z,T)/T}{\chi_2(x,T)/T^2} \sim \frac{\psi z}{z^2 - z_E^2}$$

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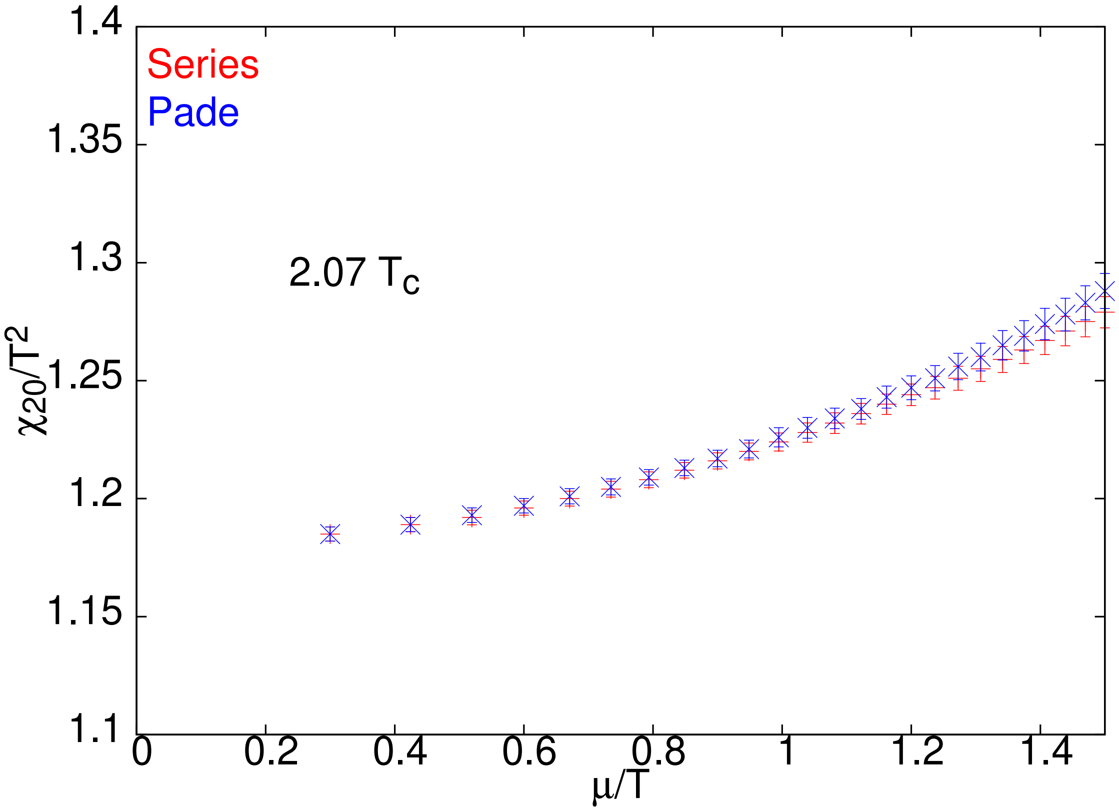
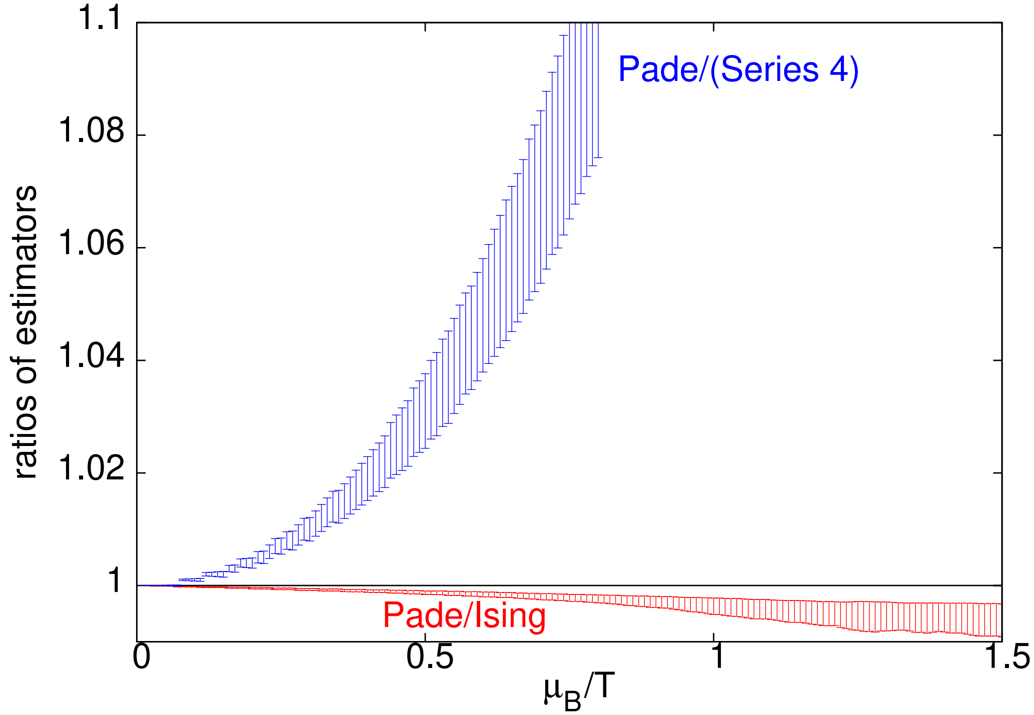
- Convert series for baryonic susceptibility to that for m_1, and use Padè for it to locate critical point. Errors on coefficients determine the error-size on its location.

- Successive integrations of m_1 with z lead to $\chi_2(z,T)$, $n(z,T)$ and $\Delta P(z,T)$. Series for m_1 can be resummed in any region including where singularity dominates.

- For temperatures away from T^E, Padè approximant and series summation give mutually consistent results.



- Near T^E on the other hand, the two differ:



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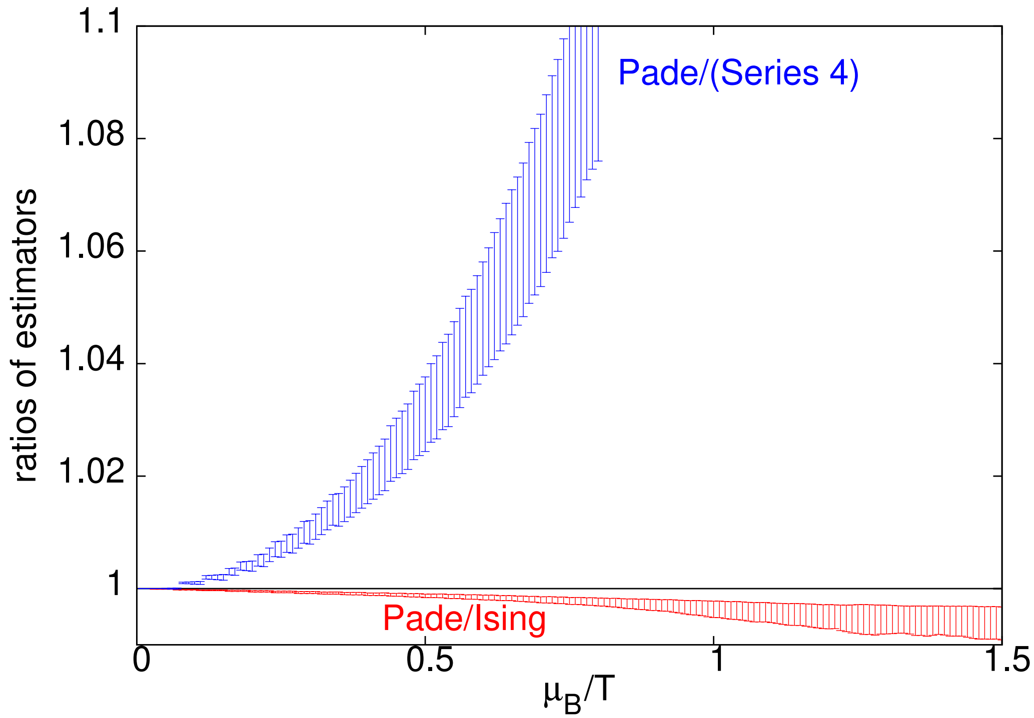
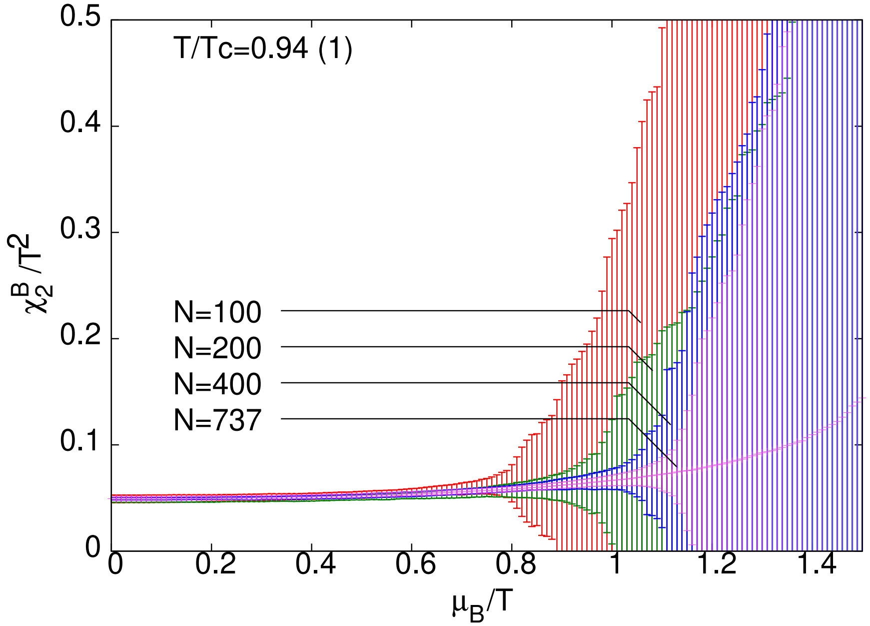
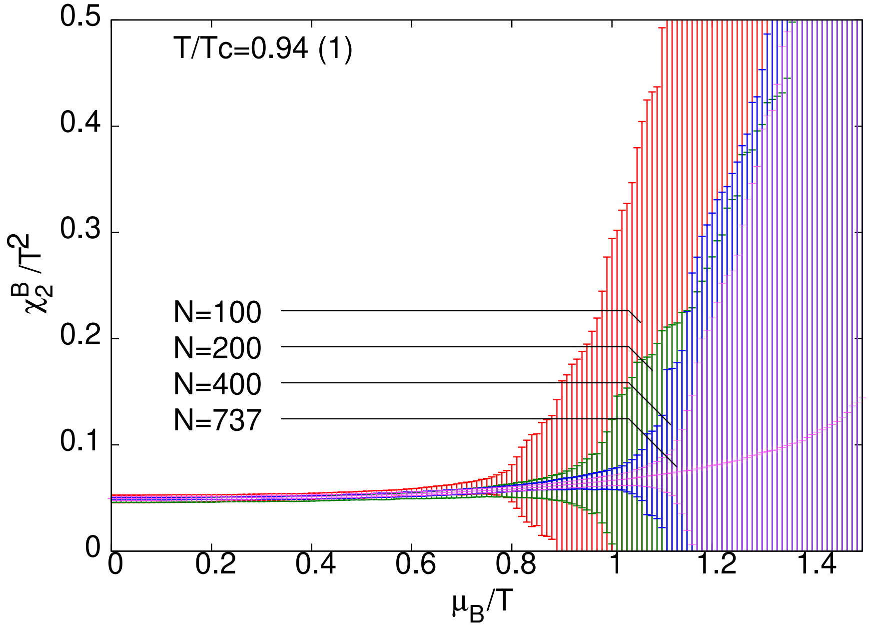


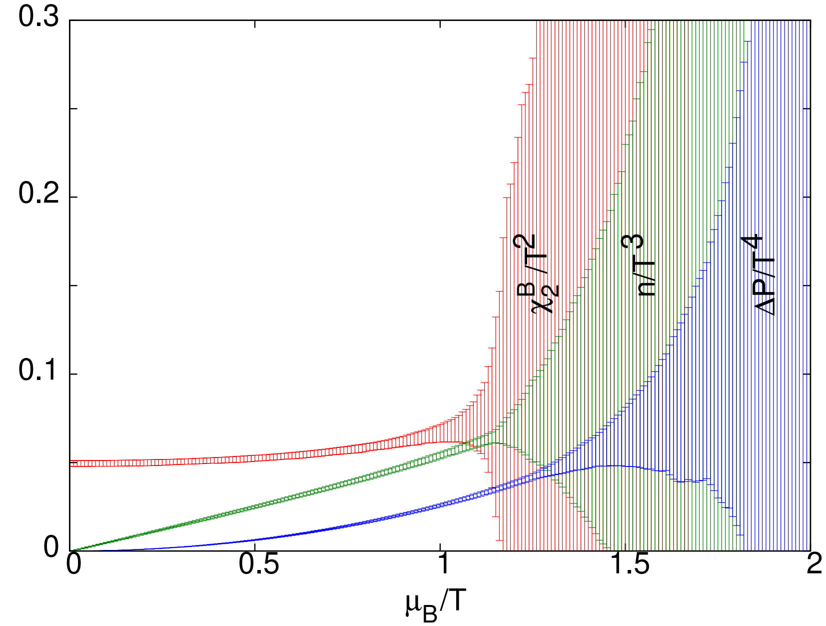
- While the ratio of the Padè approximant to the series approximation which keeps all BNS up to order 4(Series 4), is clearly different from unity, the ratio of the Padè approximant to the one with fixed Ising critical index is much closer to unity.



♠ Errors blow in a region near z_E, which decreases in size with increasing # of gauge configurations N.

♡ Slow rate of decrease ⇒ Critical slowing ?





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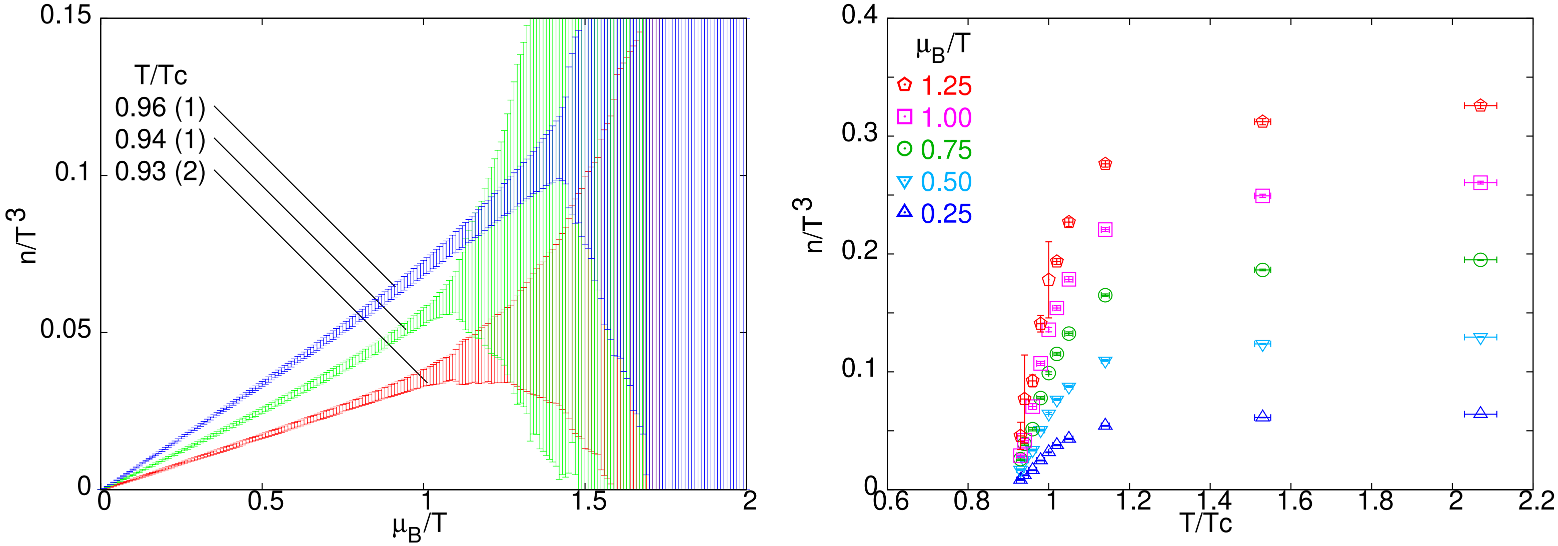
♣ Note χ is constant in a range of z, leading to n and P are respectively linear and quadratic.

♢ Successive integrations reduce singular behaviour, or extend range of better error control.


0.15
0.1
0.05
0
T/Tc
0.96 (1)
0.94 (1)
0.93 (2)
n/T 3
0
0.5
1
1.5
2
μB/T
0.4
0.3
0.2
0.1
0
n/T 3
μB/T
1.25
1.00
0.75
0.50
0.25
0.6
0.8
1
1.2
1.4
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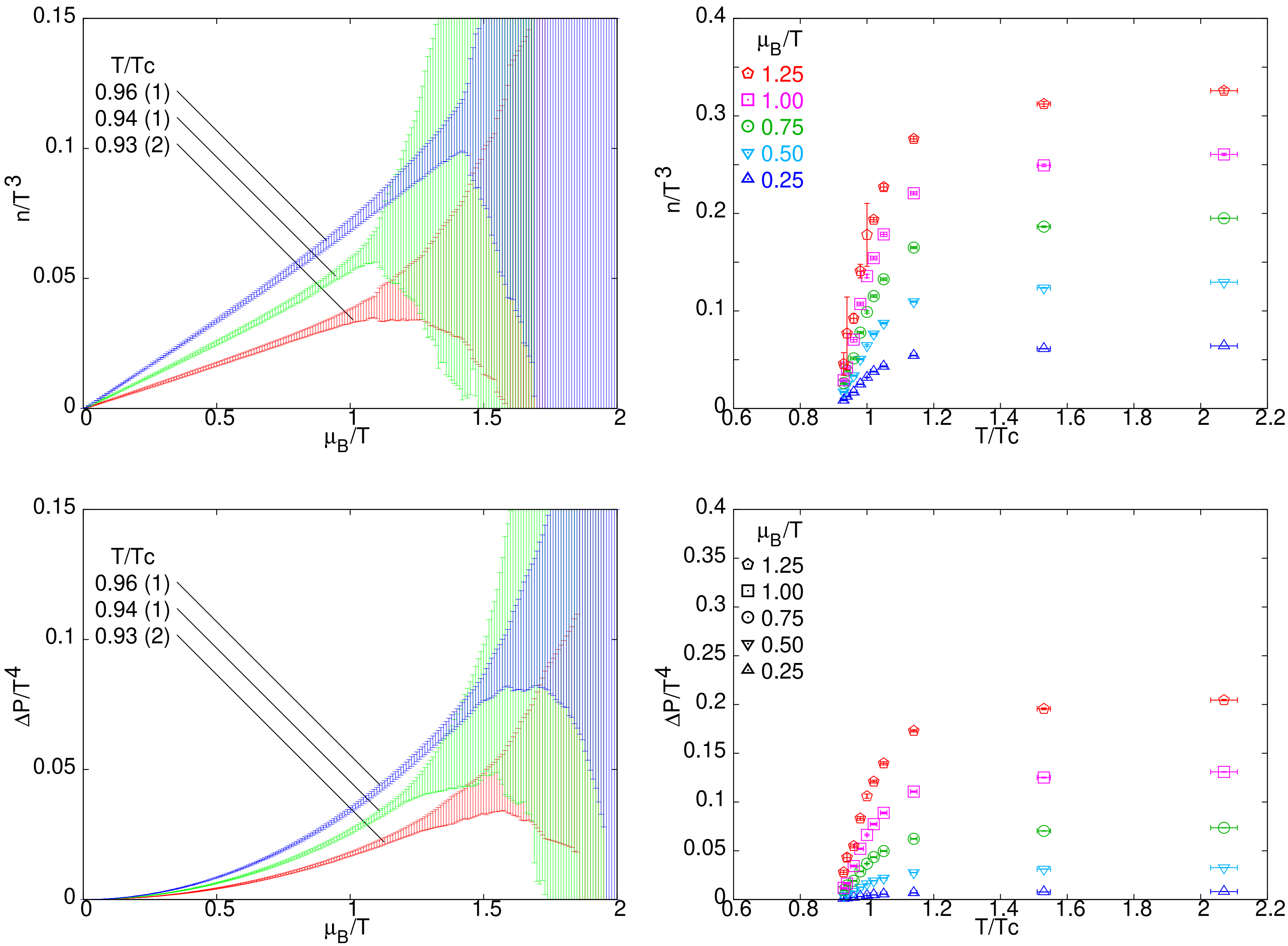
1.8
2
2.2
T/Tc



T/Tc
0.96 (1)
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μB/T
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1.25
1.00
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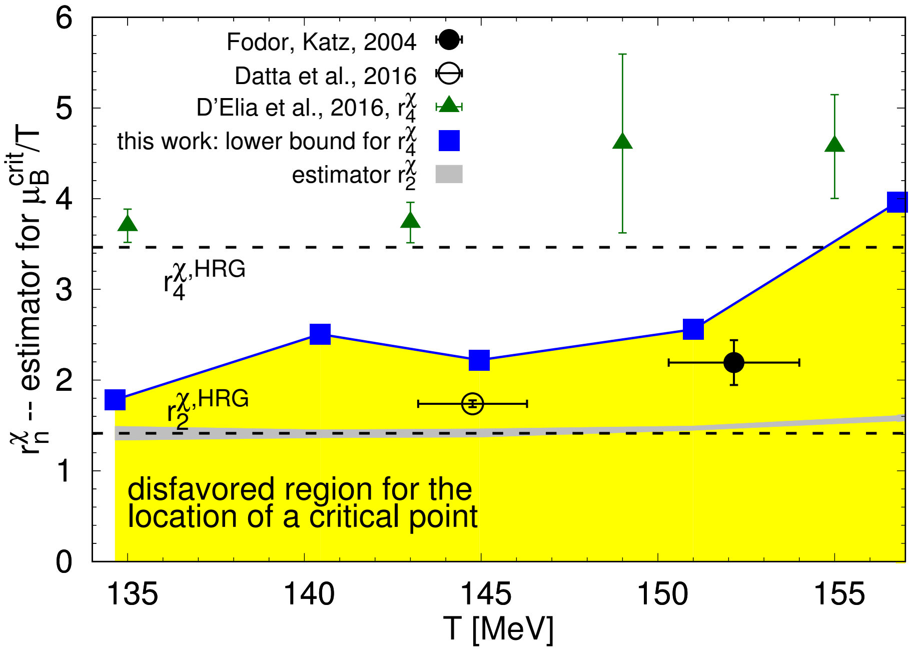
T/Tc
ΔP/T^4

♠ Comparison with other group (Bazavov et al. PRD '17): 1-σ bound disfavours Mumbai & Budapest results.



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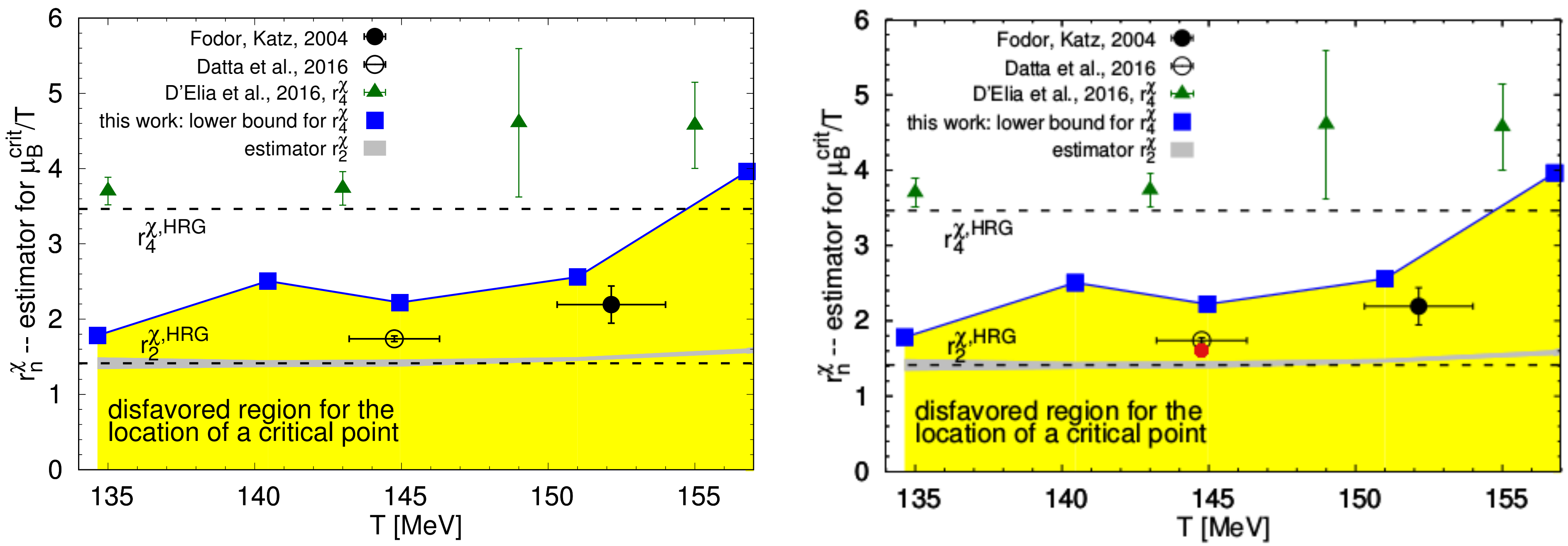

♡ Noting 2+1 flavours, smaller m_π, different discretization & μ-actions employed, $\Longrightarrow$ Good to see that a 2σ-level consistency is plausible.

Freeze-out Parameters

- Ratios of cumulants like m_1, m_2 and/or similar charge or strangeness cumulants can be, and have been, used to determine freeze-out parameters by comparing $S\sigma$ or $\kappa\sigma^2$ from RHIC data (Gavai and Gupta 2011;Bazavov et al. 2012; Borsanyi et al. 2013).

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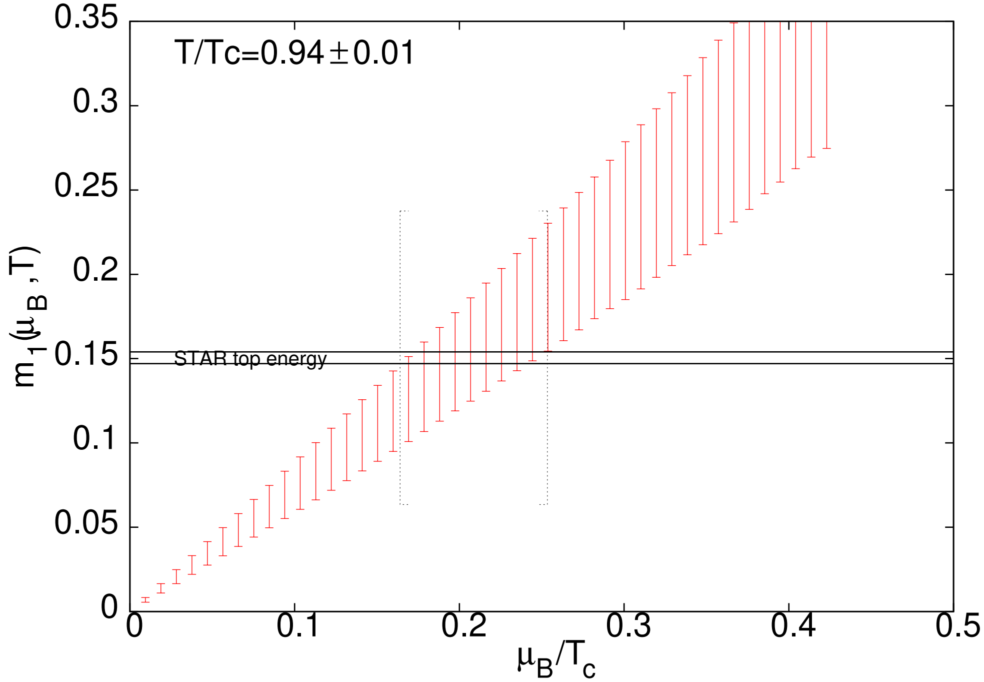
- What can we learn about freezeout curve from m1? Consider the data from STAR collaboration for Au-Au 200 GeV/c: $m_1 = 0.150 \pm 0.004 \pm 0.006$.

- For a given temperature, m_1 will yield a band of μ_B.

◇ STAR collaboration for Au-Au 200 GeV/c: $m_1 = 0.150 \pm 0.004 \pm 0.006$.



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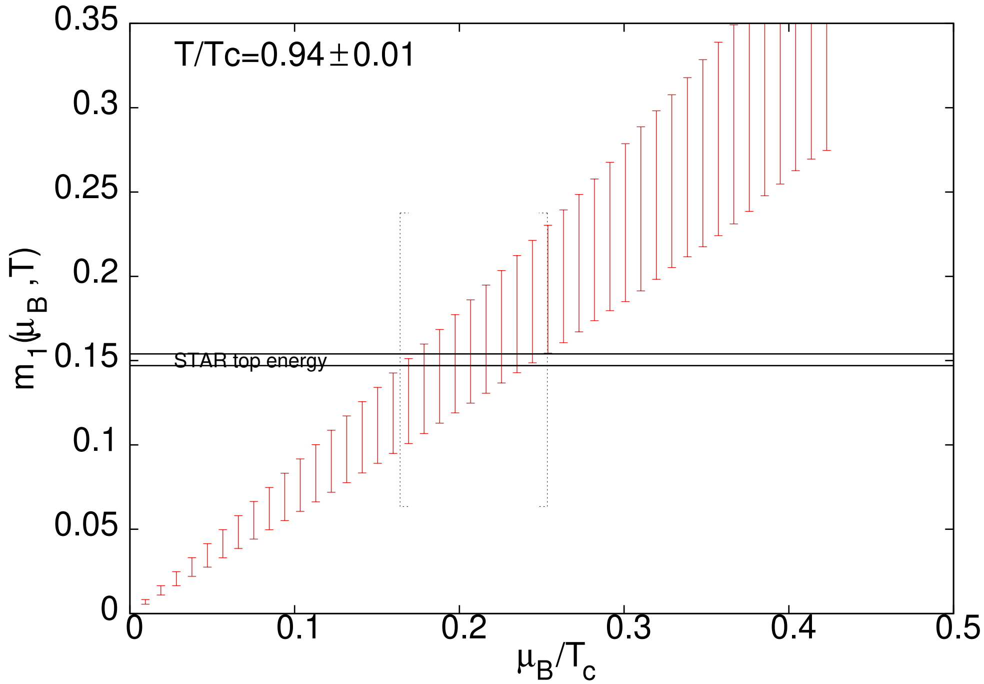

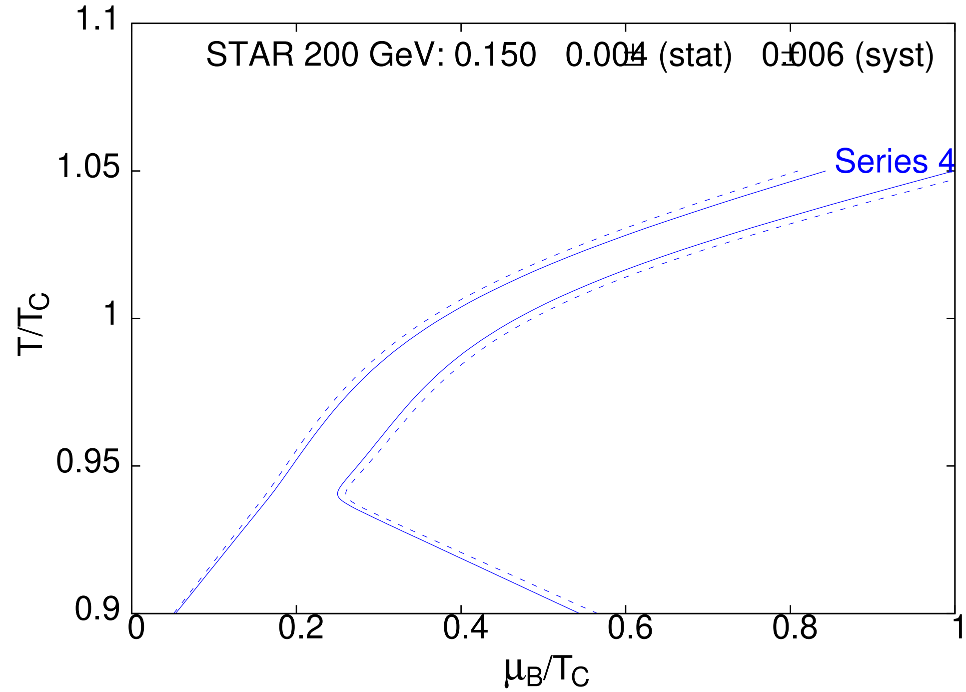

- Error estimate is indicated by the brackets on the left panel. Making an *ansatz* for freeze-out temperature, say T_c, one can read off the μ_B at freeze-out with its associated error.

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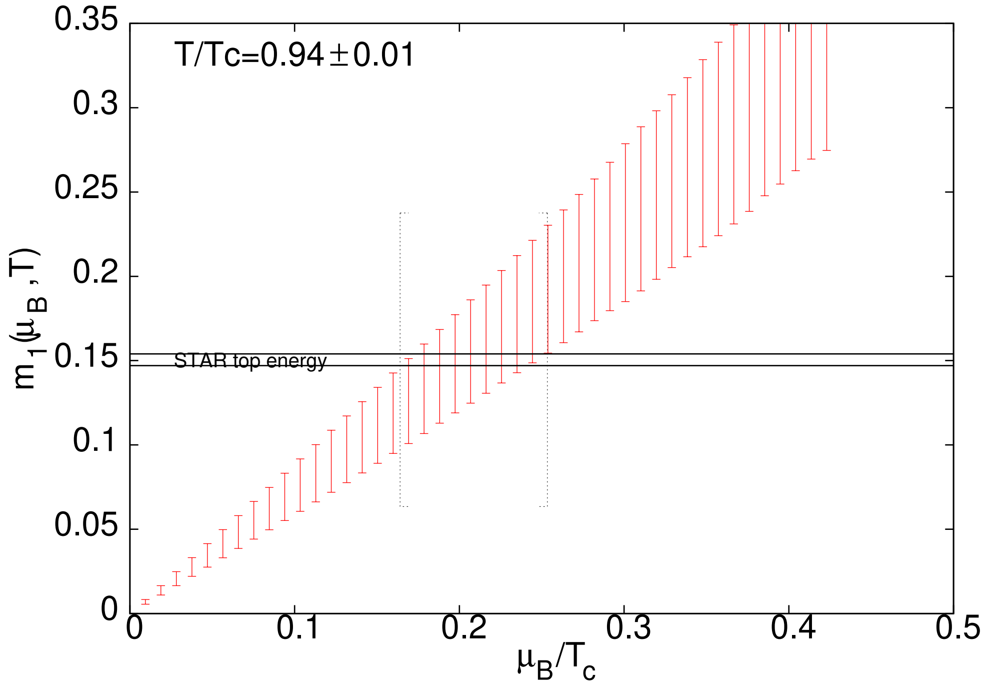

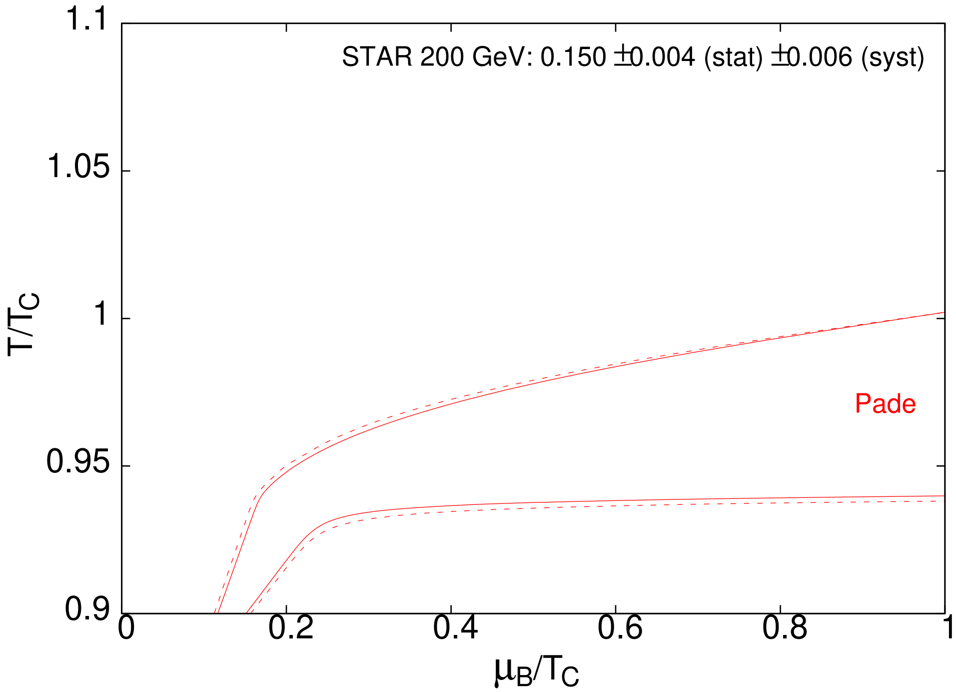

- Determination of freeze-out parameters seems error-prone. Padè suggests freeze-out temperature is likely to be below T_c. Moreover the DLog Padè m_1 is unlikely to be a baryometer.

Summary

- Slow but perhaps steady progress in pinning down QCD Critical Point. DLog Padè technique maybe used to cross-check the ratio method.

- Linear μ-action may make it easier to go to higher orders. Its divergences are easy to remove, as has been explicitly shown.

Summary

- Slow but perhaps steady progress in pinning down QCD Critical Point. DLog Padè technique maybe used to cross-check the ratio method.

- Linear μ-action may make it easier to go to higher orders. Its divergences are easy to remove, as has been explicitly shown.

- We stress the need of using resummation rather than simple series summation for obtaining EoS for temperatures close to T_c.

- Critical slowing down visible in numerical evaluation of DLog Padè, m_1. This leads to rapidly increasing error in it, dramatically decreasing its efficacy in determining the freezeout curve.

- The DLog Padè m_1 is unlikely to be a baryometer. Resummed series shows that freezeout temperature is likely to be below T_c . Extraction of freezeout parameters appears sensitive to higher orders & is likely still error-prone.