The QCD crossover line from lattice simulations

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Observables of Hadronization and the QCD Phase Diagram in the Cross-over Domain

ECT*, Trento 15-19 October 2018

OUTLINE

- **The QCD crossover for small μ_B**

  updated lattice results on the curvature

- **The QCD crossover and deconfinement**

  μ_B dependence of the Polyakov loop signalling the QCD crossover

Crossover nature of the “transition” known since a few years

Y. Aoki *et al.* Nature 443, 675 (2006) [hep-lat/0611014].

Temperature of the transition (from the chiral condensate):

S. Borsanyi *et al.* JHEP 1009, 073 (2010) $T_c = 155(6)$ **MeV** (stout link stag. discretization, $a_{min} \simeq 0.08$ fm)

A. Bazavov *et al.*, PRD 85, 054503 (2012) $T_c = 154(9)$ **MeV** (HISQ/tree stag. discretization, $a_{min} \simeq 0.1$ fm)

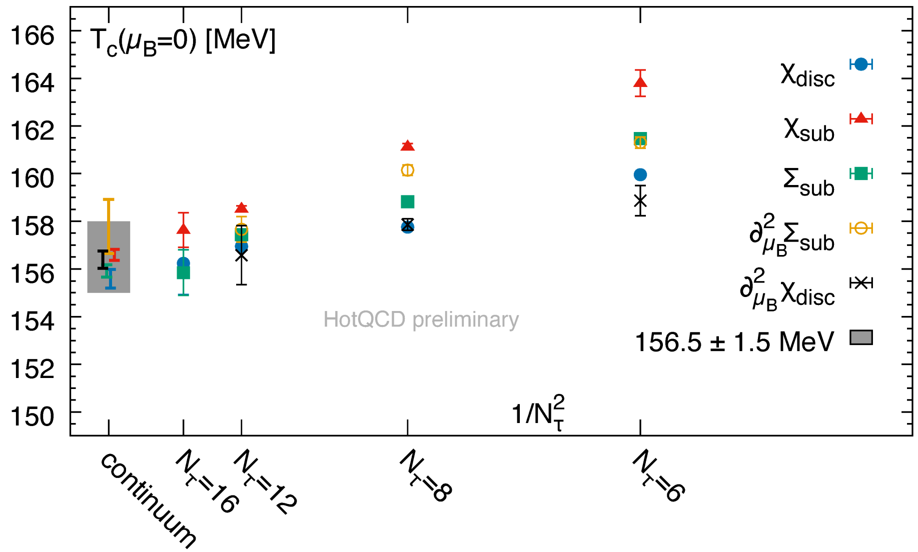
Various refinements in recent years

Update at QM2018

P. Steinbrecher (HotQCD), arXiv:1807.05607

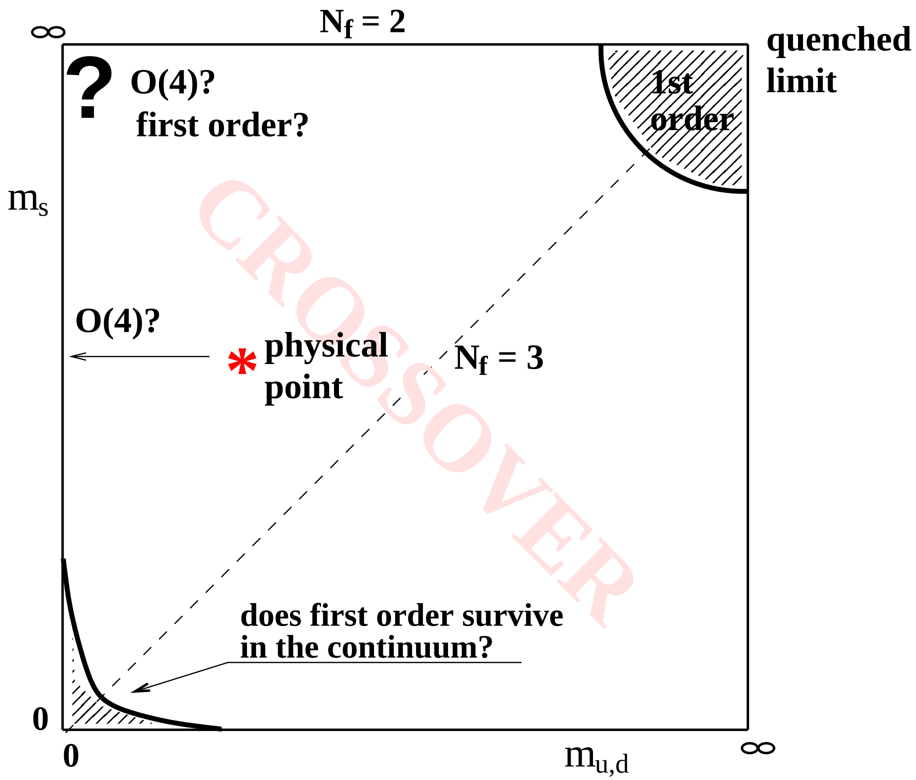
$$T_c \simeq 156.5 \pm 1.5$$

$$T = 1/(N_t a) \quad \rightarrow \quad a \propto 1/N_t$$



T_c usually determined by observables related to chiral symmetry, which is the symmetry most relevant to QCD with physical quark masses.

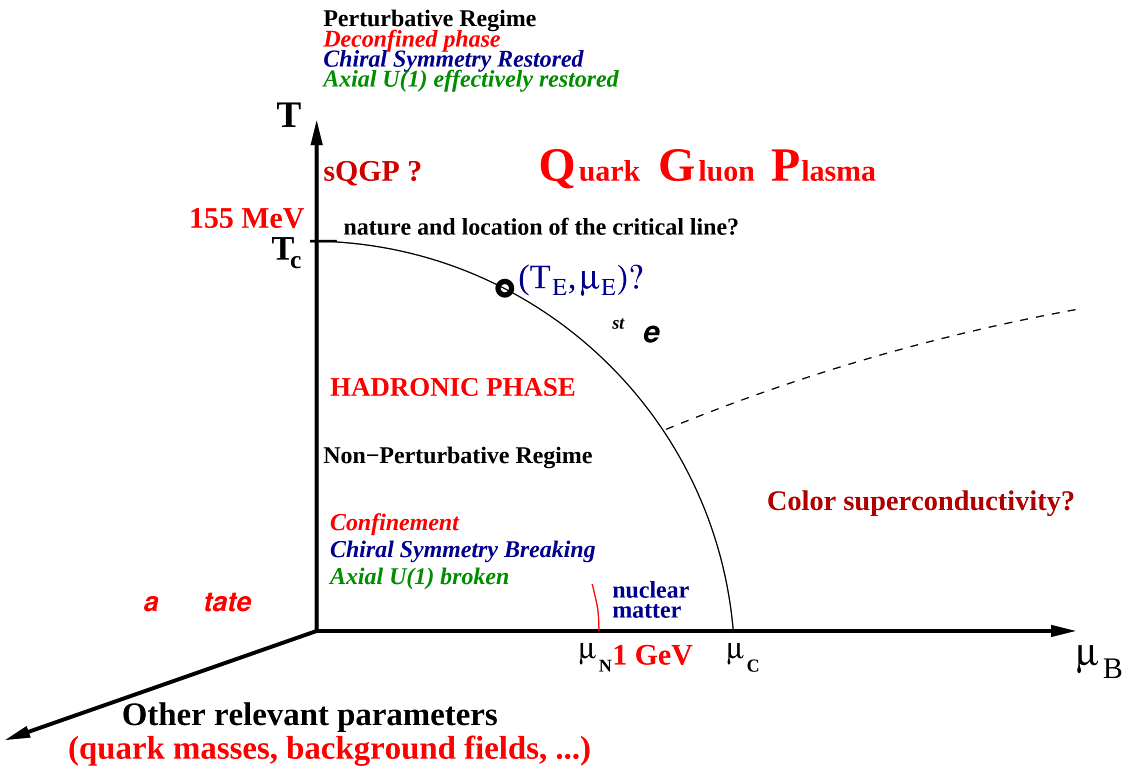
How deep are we into the crossover region?



A. Lahiri (HotQCD) at QM2018 m_s **fixed,** $m_l \to 0$**: 1st order excluded down to** $m_\pi \simeq 80$ **MeV**

$N_f = 3$ **chiral region:** despite universality arguments, it is not clear if first order region survives in the continuum limit G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, arXiv:0710.0998; X. Y. Jin, Y. Kuramashi, Y. Nakamura, S. Takeda and A. Ukawa, arXiv:1612.05371; P. de Forcrand, MD,, arXiv:1702.00330

How does T_c move in the $T - \mu_B$ plane?



Problems in lattice QCD at $\mu_B \neq 0$

$$Z(\mu_B, T) = \mathrm{Tr}\left(e^{-\frac{H_{\mathrm{QCD}} - \mu_B N_B}{T}}\right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

$\det M$ **complex** $\Longrightarrow$ **MC simulations not feasibile (sign problem)**

The two approximate solutions for small μ_B/T which are presently mostly used are:

- **Taylor expansion of physical quantities around $\mu = 0$**

  Bielefeld-Swansea collaboration 2002; R. Gavai, S. Gupta 2003

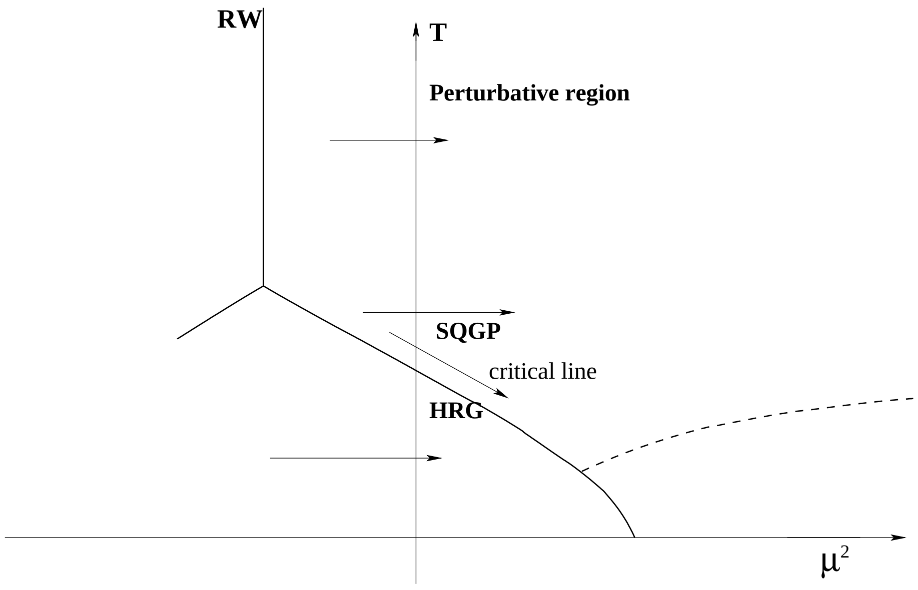
- **Simulations at imaginary chemical potentials (plus analytic continuation)**

  Alford, Kapustin, Wilczek, '99; Lombardo '00; de Forcrand, Philipsen, '02; MD, Lombardo '03.

Reliable results achievable for small μ_B/T

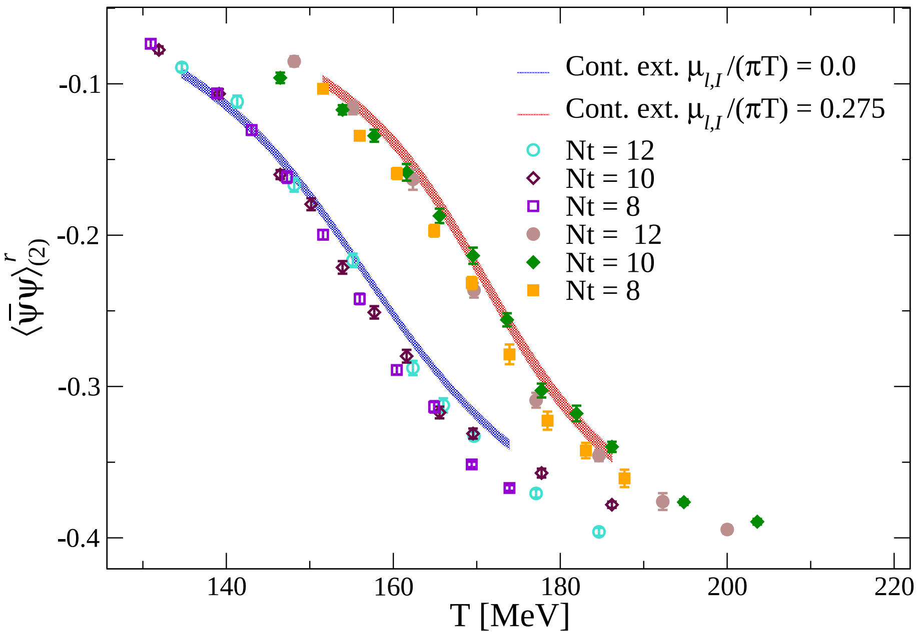
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + O(\mu_B^4)$$

The curvature κ can be determined by following explicitly how T_c moves at imaginary μ_B and then continuing to real μ_B (analytic continuation)

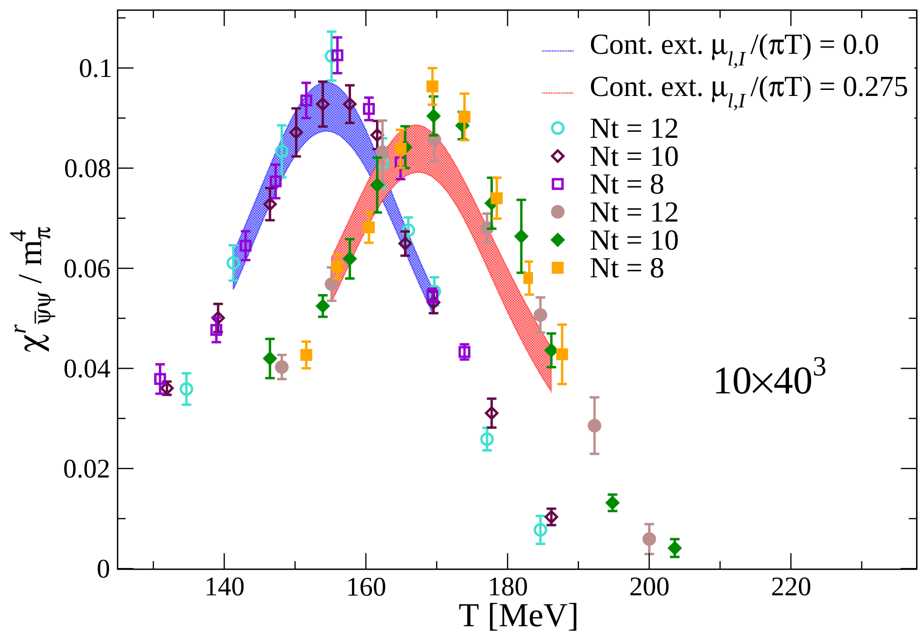


or by finding $dT_c/d\mu_B^2$ implicitly in terms of derivatives at $\mu_B = 0$ (Taylor expansion)

In the imaginary chemical potential approach, T_c is computed as a function of μ_I from various quantities (e.g., inflection point of the renormalized chiral condensate or peak of the renormalized susceptibility)



chiral condensate

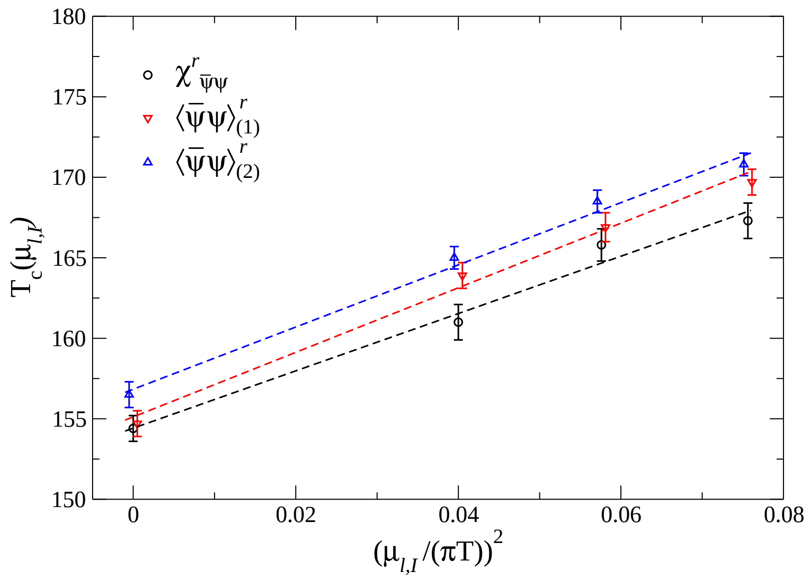


chiral susceptibility

Localizing the pseudocritical temperature for various imaginary chemical potentials from various observables (continuum extrapolation)

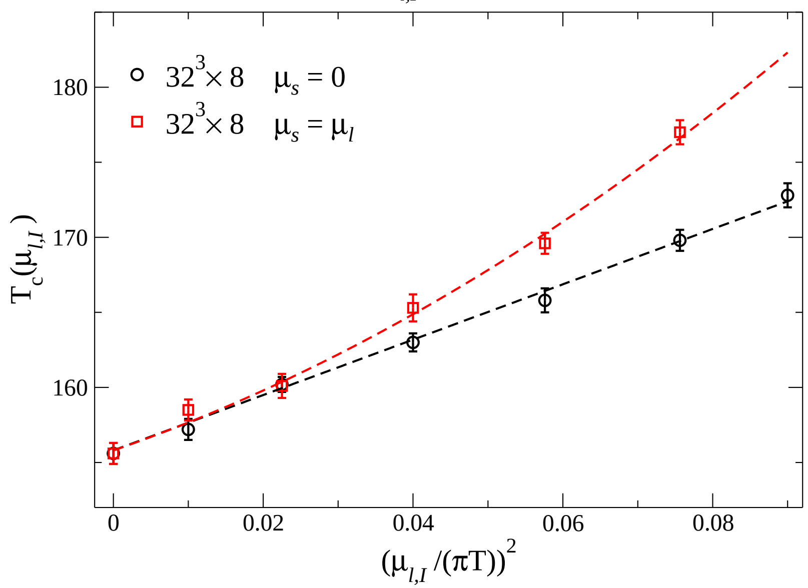
results from Bonati et al., arXiv:1507.03571, $N_f = 2+1$ QCD, stout staggered discretization with physical quark masses

then, assuming analyticity, κ is extracted by fitting a linear dependence in μ_I^2 for small μ_I.
T_c location depends on the observable, slope in μ_I^2 is much less sensitive
$\kappa = 0.0135(20)$ from Bonati *et al*,arXiv:1507.03571



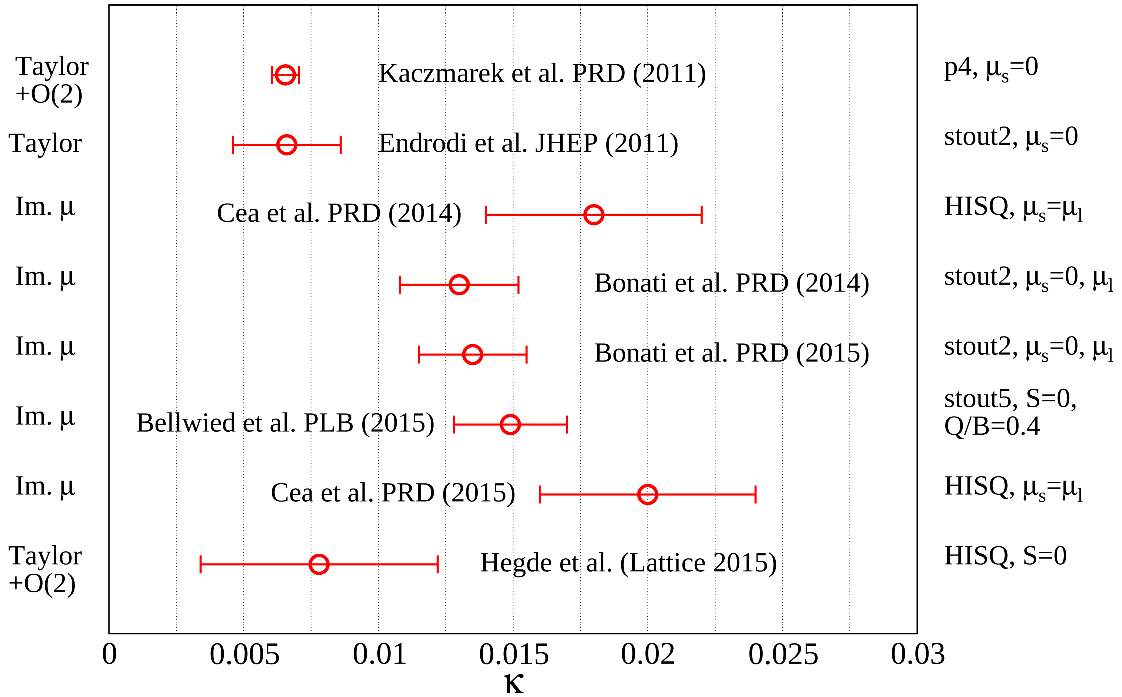
Results obtained for $\mu_u = \mu_d = \mu_l;\ \ \mu_s = 0$

Comparison with $\mu_s = \mu_u = \mu_d = \mu_l$ shows deviations, but limited to higher order terms in μ^2, curvature is unaffected



Experimental conditions: $\mu_s \sim 0.25\,\mu_l$ (to ensure strangeness neutrality)

This was the situation up to last year, pointing to a possible tension between results obtained from Taylor expansion or analytic continuation



For this reason we decided to repeat our study by Taylor expansion, adopting exactly the same discretization adopted in our previous determination by analytic continuation.

C. Bonati, MD, F. Negro, F. Sanfilippo and K. Zambello, arXiv:1805.02960

We have considered a Taylor expansion of the renormalized condensate, with T-dependent coefficients

$$\langle\bar{\psi}\psi\rangle^r(T,\mu_B) = A(T) + B(T)\mu_B^2 + O(\mu_B^4)$$

The curvature can then be defined implicitly by either finding how the point with zero T-second-derivative (inflection point) of the condensate moves with μ_B^2

$$\kappa_2 = \frac{B''(T_c)}{A'''(T_c)} T_c = \frac{\frac{\partial^2}{\partial T^2}\left(\frac{\partial\langle\bar{\psi}\psi\rangle^r(T,\mu_B)}{\partial(\mu_B^2)}\big|_{\mu_B=0}\right)\big|_{T=T_c}}{\frac{\partial^3}{\partial T^3}\langle\bar{\psi}\psi\rangle^r(T,0)\big|_{T=T_c}} T_c\,.$$

or by finding how the point at fixed chiral condensate moves with μ_B^2

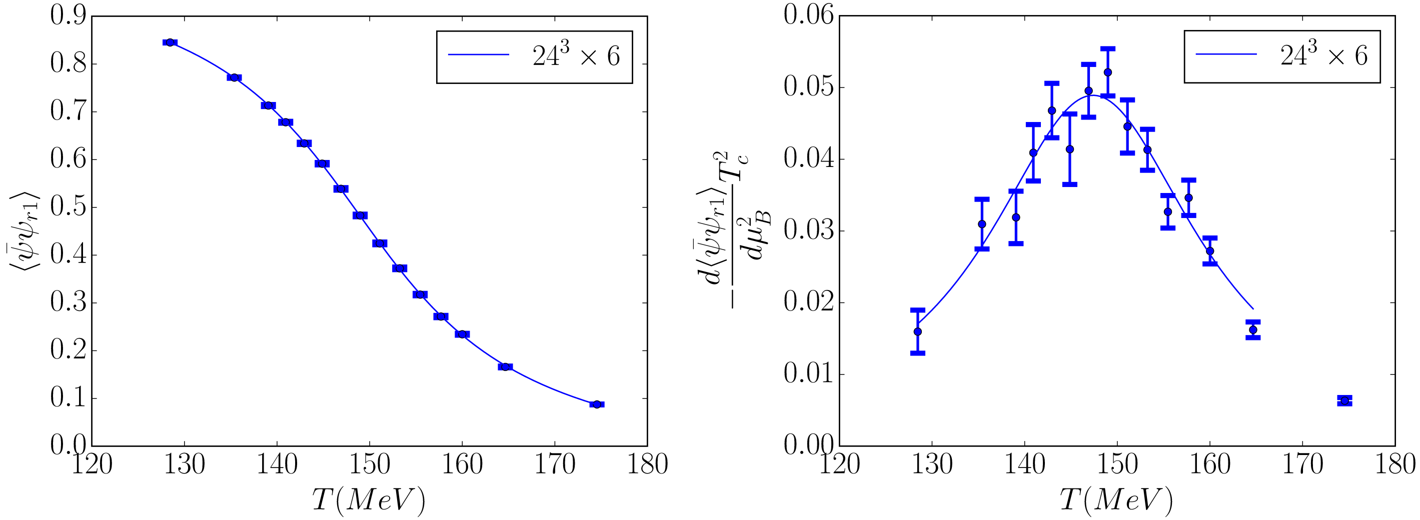
$$\kappa_1 = -T_c\frac{dT_c}{d(\mu_B^2)} = T_c\frac{\frac{\partial\langle\bar{\psi}\psi\rangle^r}{\partial(\mu_B^2)}\big|_{\mu_B=0,T=T_c}}{\frac{\partial\langle\bar{\psi}\psi\rangle^r}{\partial T}\big|_{\mu_B=0,T=T_c}}\,.$$

The κ_2 definition is more rigorous but much noisier

the relevant quantity in both cases is the mixed (mass-μ_B^2) susceptibility, which has a peak at T_c:

$$\frac{\partial\langle\bar{\psi}\psi\rangle_f}{\partial(\mu_B^2)}\Big|_{\mu_B=0} = \frac{1}{T^2}\frac{\partial\langle\bar{\psi}\psi\rangle_f}{\partial((\frac{\mu_B}{T})^2)}\Big|_{\mu_B=0} = \frac{1}{T^2}\frac{1}{2}\frac{\partial^2\langle\bar{\psi}\psi\rangle_f}{\partial(\frac{\mu_B}{T})^2}\Big|_{\mu_B=0}$$

$$= \frac{1}{T^2}\frac{1}{2}\left(\langle(n^2+n')\bar{\psi}\psi_f\rangle - \langle n^2+n'\rangle\langle\bar{\psi}\psi_f\rangle + \langle 2n\bar{\psi}\psi'_f + \bar{\psi}\psi''_f\rangle\right)$$



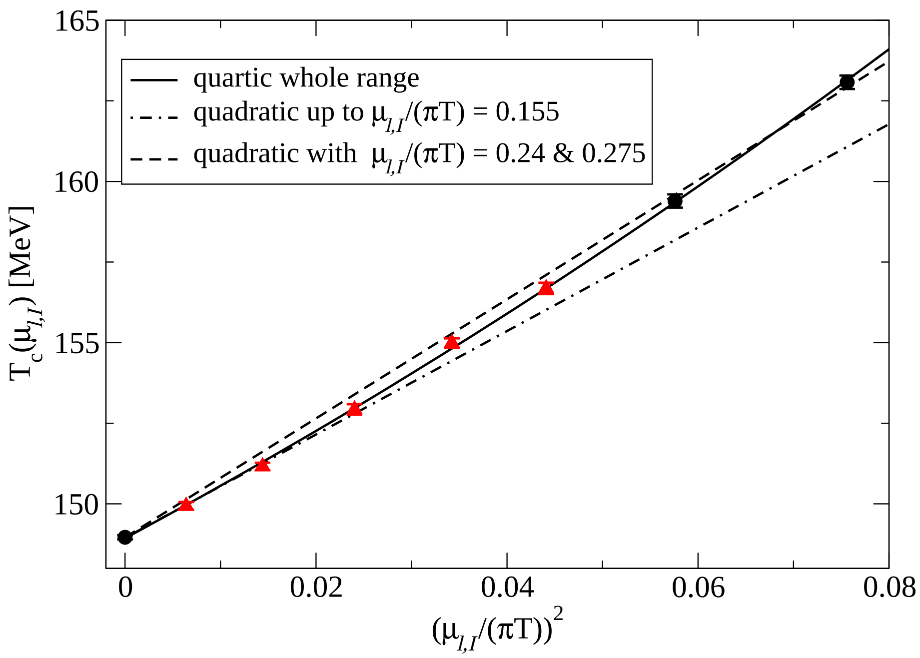
Lattice	$\kappa_1(\bar\psi\psi_{r1})$	$\kappa_2(\bar\psi\psi_{r1})$	$\kappa_1(\bar\psi\psi_{r2})$	$\kappa_2(\bar\psi\psi_{r2})$
$16^3 \times 6$	$0.0119(4)$	$0.016(5)$	$0.0119(4)$	$0.016(5)$
$24^3 \times 6$	$0.0122(4)$	$0.015(4)$	$0.0122(4)$	$0.015(4)$
$32^3 \times 8$	$0.0126(14)$	$0.014(9)$	$0.0121(13)$	$0.012(8)$
$40^3 \times 10$	$0.0146(19)$	-	$0.0154(21)$	-

Curvature coefficients κ obtained for different definitions and lattice sizes

A careful check of consistency between Taylor expansion and analytic continuation on the $24^3 \times 6$ lattice:

a badly taken analytic continuation (i.e. not properly estimating systematic effects) yields $\kappa = 0.0140(3)$

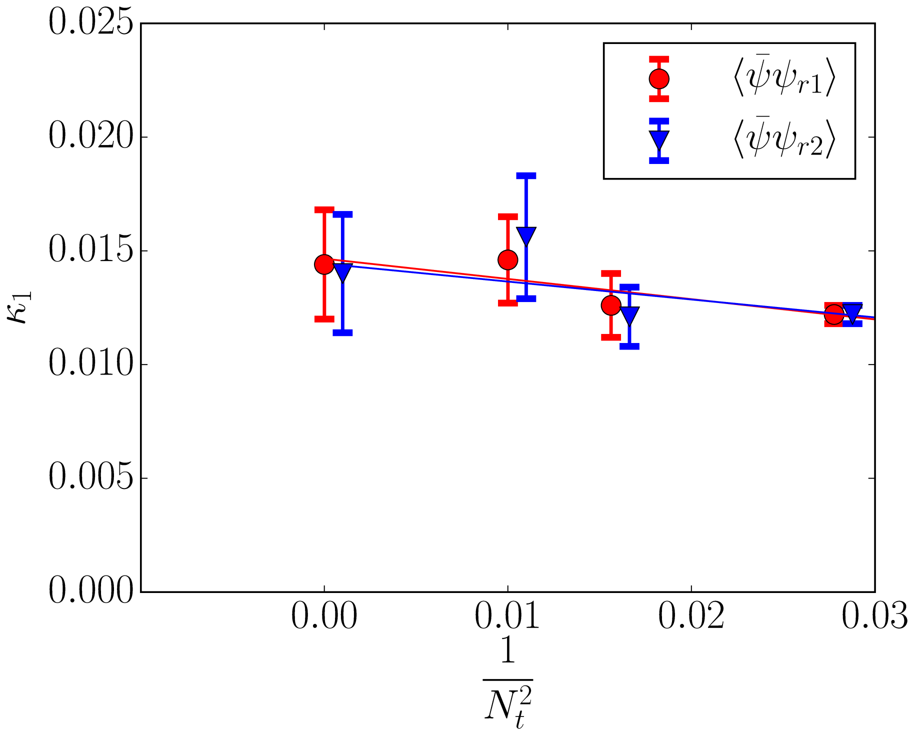
a properly taken analytic continuation yields $\kappa = 0.0121(5)$, in perfect agreement with Taylor expansion



Our continuum extrapolation

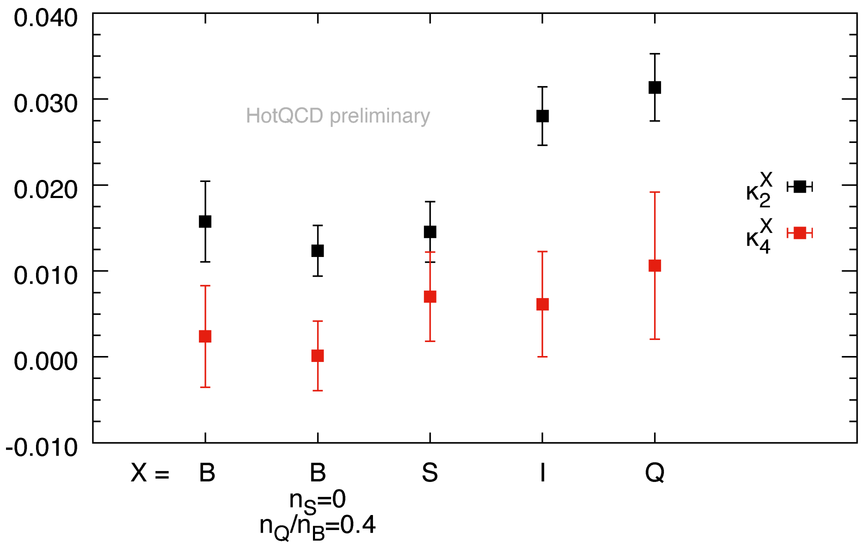
C. Bonati, MD, F. Negro, F. Sanfilippo and K. Zambello, arXiv:1805.02960

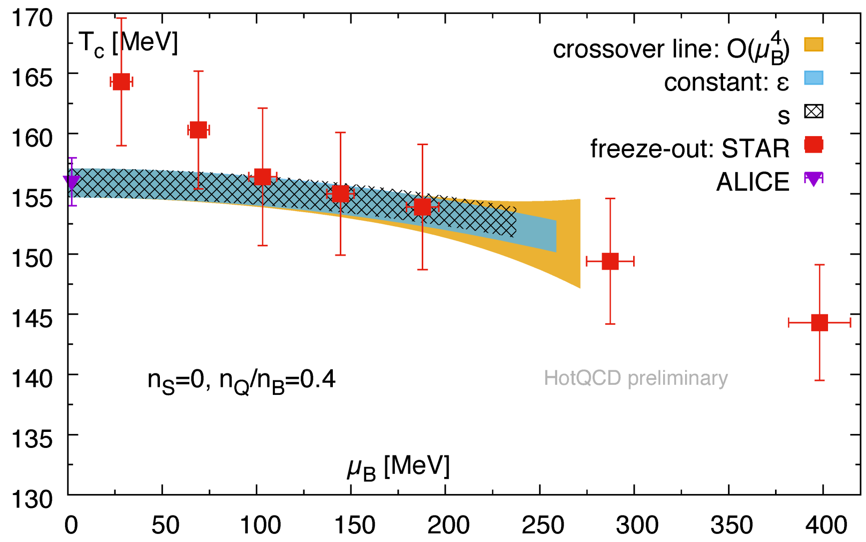
$N_f = 2+1$ **QCD by stout staggered quarks,** $\mu_u = \mu_d = \mu_B/3$, $\mu_s = 0$.



$$\kappa = 0.0142(25)$$

Similar results from HotQCD (P. Steinbrecher at QM2018, arXiv:1807.05607)

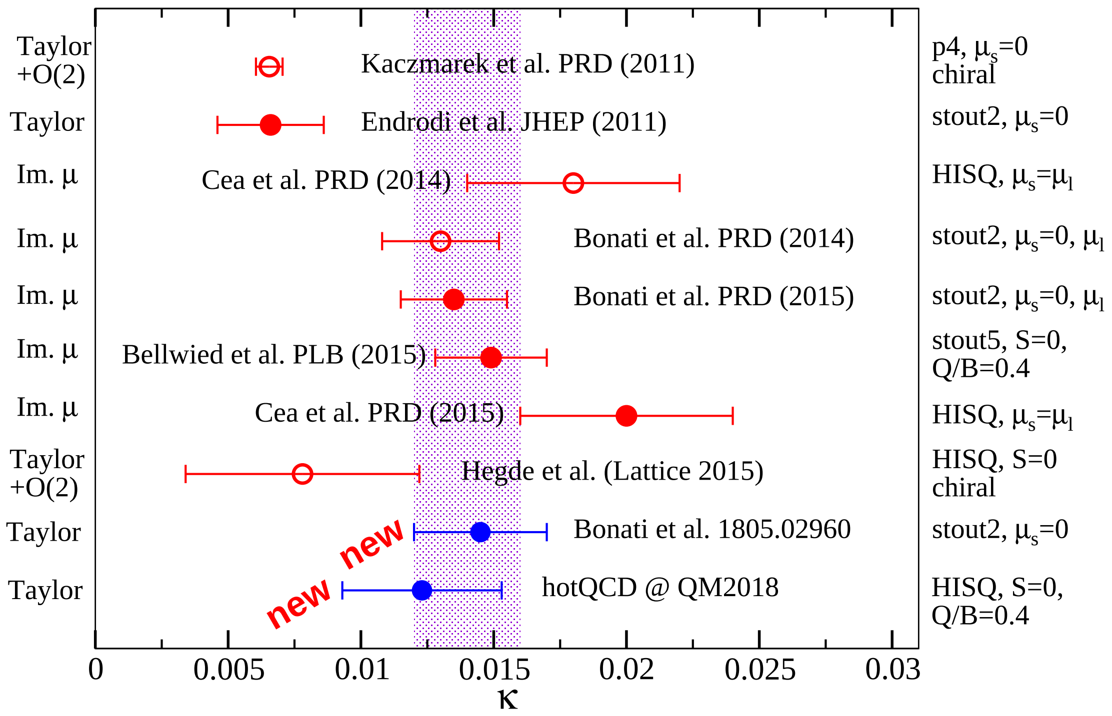




$$\frac{T_c(\mu_X)}{T_c} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_c}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_c}\right)^2 + O(\mu_X^6)$$

- $N_f = 2+1$ **QCD by HISQ staggered quarks, various setup of chemical potentials**
- **curvature determined from the maximum of the disconnected chiral susceptibility**
- **imposing strangeness neutrality** $\kappa = 0.0123(30)$

Quantitative agreement of the most recent determinations at or close to the physical point confirms the reliability of analytic continuation and Taylor expansion



(conservative) average of most recent continuum extrapolated results (filled dots):

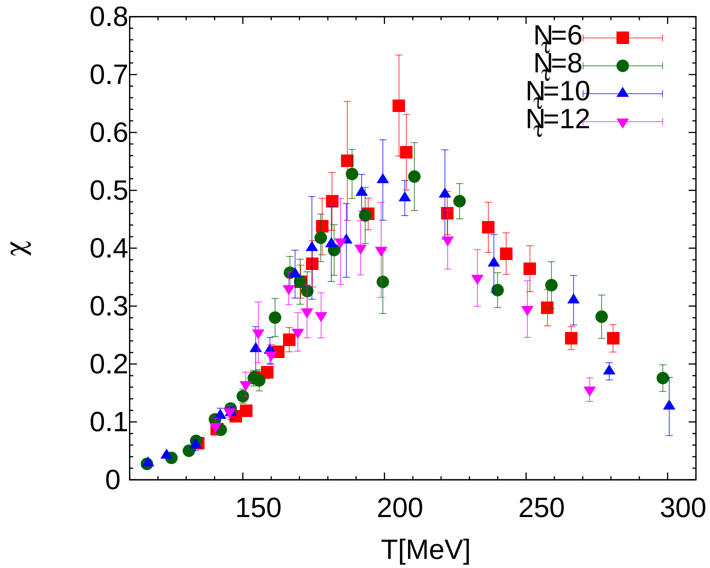
$$\kappa = 0.014(2)$$

The QCD crossover, deconfinement, and the Polyakov loop

observables related to confinement, like the Polyakov loop, usually point to higher crossover temperatures

an example: the Polyakok loop susceptibility

A. Bazavov, N. Brambilla, H.-T. Ding, P. Petreczky, H.-P. Schadler, A. Vairo and J. H. Weber, arXiv:1603.06637



there are exceptions, like the Polyakov loop entropy

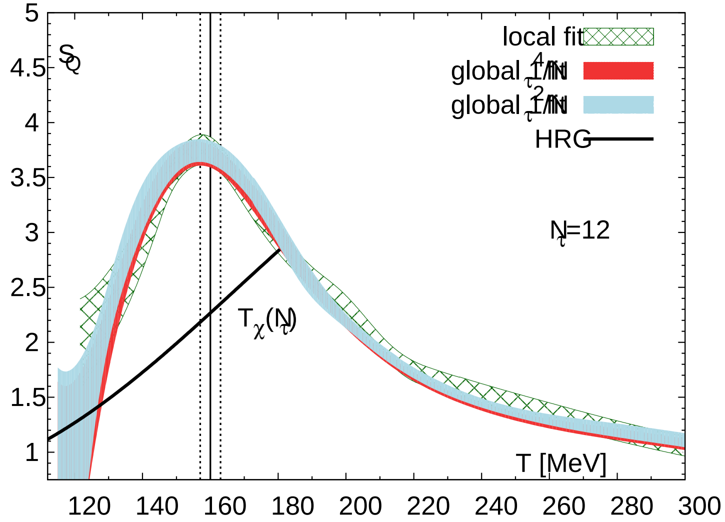
$$S_Q = -\frac{\partial F_Q}{\partial T}$$

A. Bazavov, N. Brambilla, H.-T. Ding, P. Petreczky, H.-P. Schadler, A. Vairo and J. H. Weber, arXiv:1603.06637

or the transverse susceptibility

$$\chi_T \equiv V(\langle(\mathrm{Im}L)^2\rangle - \langle\mathrm{ImL}\rangle^2)$$

P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, arXiv:1306.5094, arXiv:1307.5958



Here we consider the dependence of the static quark free energy on μ_B

The free energy F_Q of a static quark is given by

$$2F_Q(T) = -T \log |\langle \mathsf{L} \rangle|^2$$

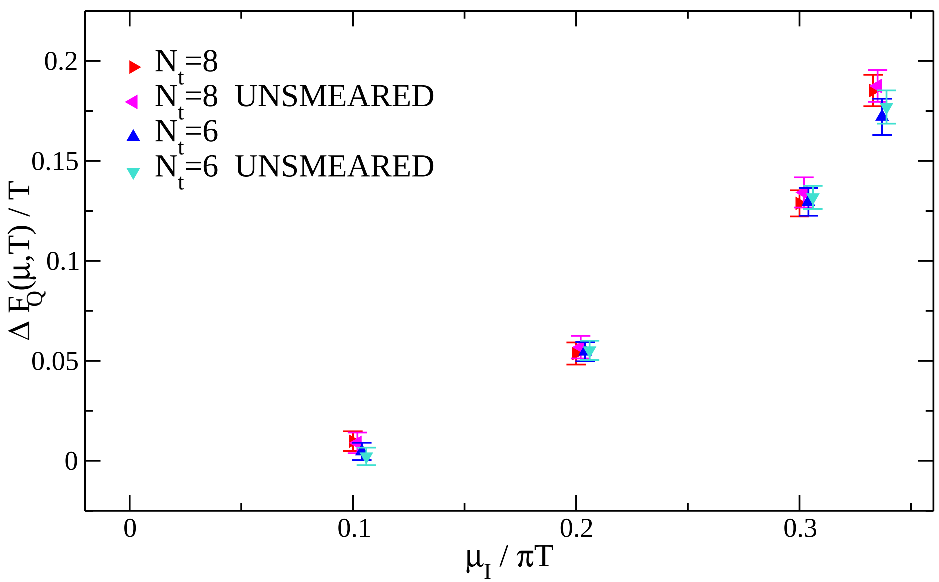
i.e. the asymptotic value of the Polyakov loop correlator.

The quantity we measure is

$$\frac{\Delta F_Q(T, \mu_B, \beta)}{T} = \frac{F_Q(T, \mu_B, \beta) - F_Q(T, 0, \beta)}{T}$$
$$= -\log \left(\frac{|\langle \mathrm{Tr} L \rangle (T, \mu_B, \beta)|}{|\langle \mathrm{Tr} L \rangle (T, 0, \beta)|} \right)$$

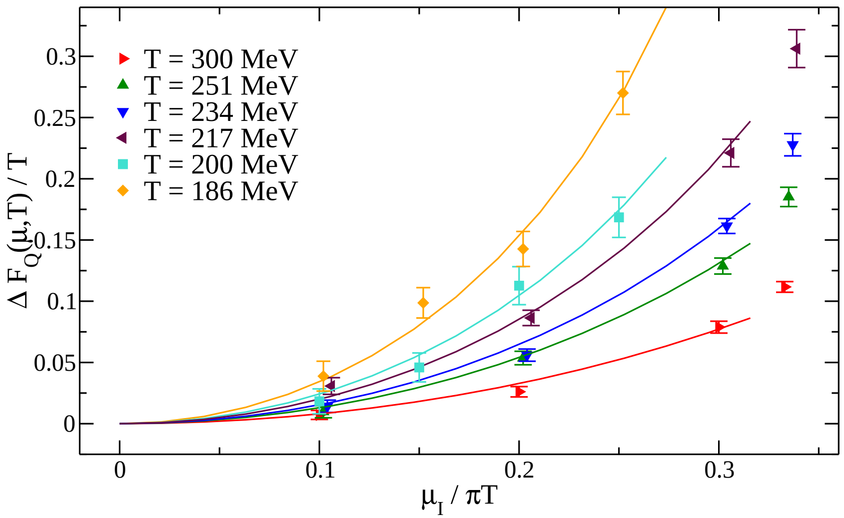
M. Andreoli, C. Bonati, MD, M. Mesiti, F. Negro, A. Rucci, F. Sanfilippo, arXiv:1712.09996

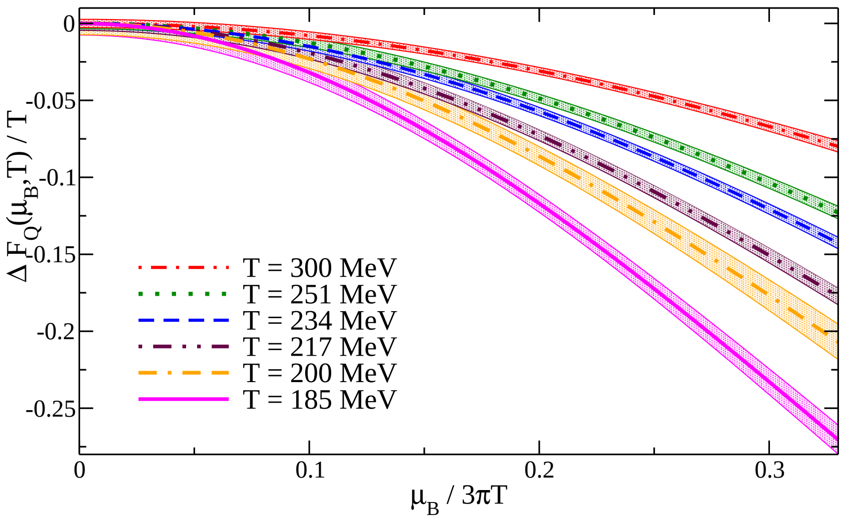
it is an already subtracted quantity which does not need any further renormalization



from M. Andreoli *et al* arXiv:1712.09996

In arXiv:1712.09996 we computed $\Delta F_Q(T, \mu_B, \beta)$ by analytic continuation





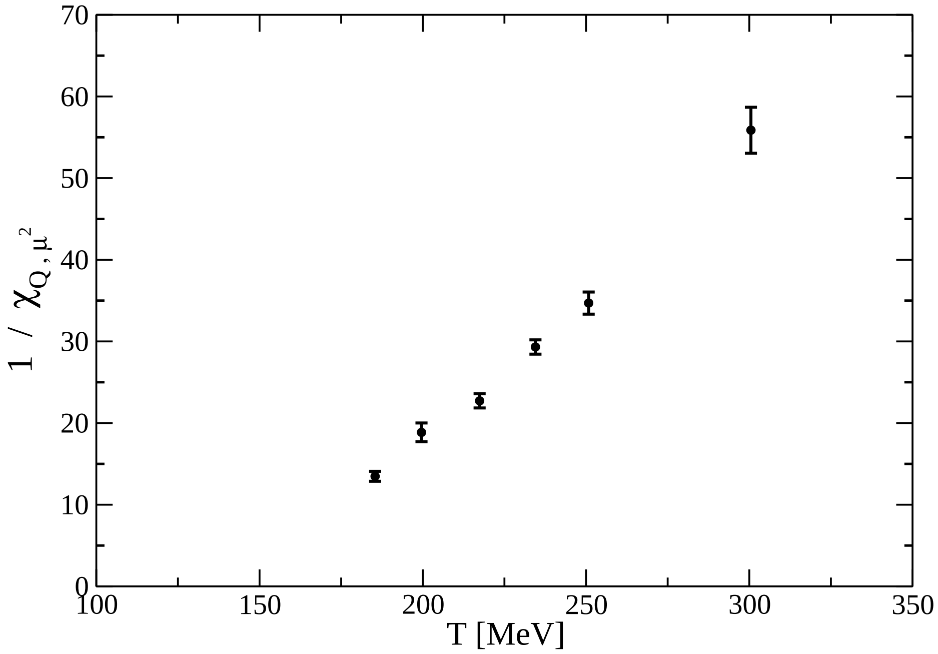
- **chemical potential enhances deconfinement**
- F_Q **decreases with** μ_B**, good description with**

$$\frac{|\langle \mathrm{Tr} L\rangle(T,\mu_B)|}{|\langle \mathrm{Tr} L\rangle(T,0)|} = \exp\left(-\frac{\Delta F_Q(T,\mu_B)}{T}\right) = 1 - \chi_{Q,\mu_B^2}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}\left((\mu_B/T)^4\right)$$

$$\chi_{Q,\mu_B^2} \equiv \frac{\partial^2 F_Q/T}{\partial(\mu_B/T)^2}$$

defines a sort of mixed susceptibility, analogous of $\partial\langle\bar{\psi}\psi\rangle/\partial\mu_B^2$**.**

In arXiv:1712.09996 χ_{Q,μ_B^2} was computed only in the high-T region. Data pointed out to a possibile divergence around T_c (as it would happen if L were a true order parameter).



therefore we decided to extend the analysis to a wider range of temperatures

PRELIMINARY STUDY OF χ_{Q,μ_B^2}

work in progress

mixed strategy (two methods):

- **analytic continuation**: compute $|L|^2$ for several imaginary μs, then fit and extract the (minus) quadratic term

  PROS: simple
  CONS: limited range between T_C and T_{RW}, systematic on the fits, simulations with many μs

- **Taylor exp.**: compute directly the second derivative of F_Q

  PROS: straightforward
  CONS: very noisy and costly observable on the lattice

Taylor expansion details

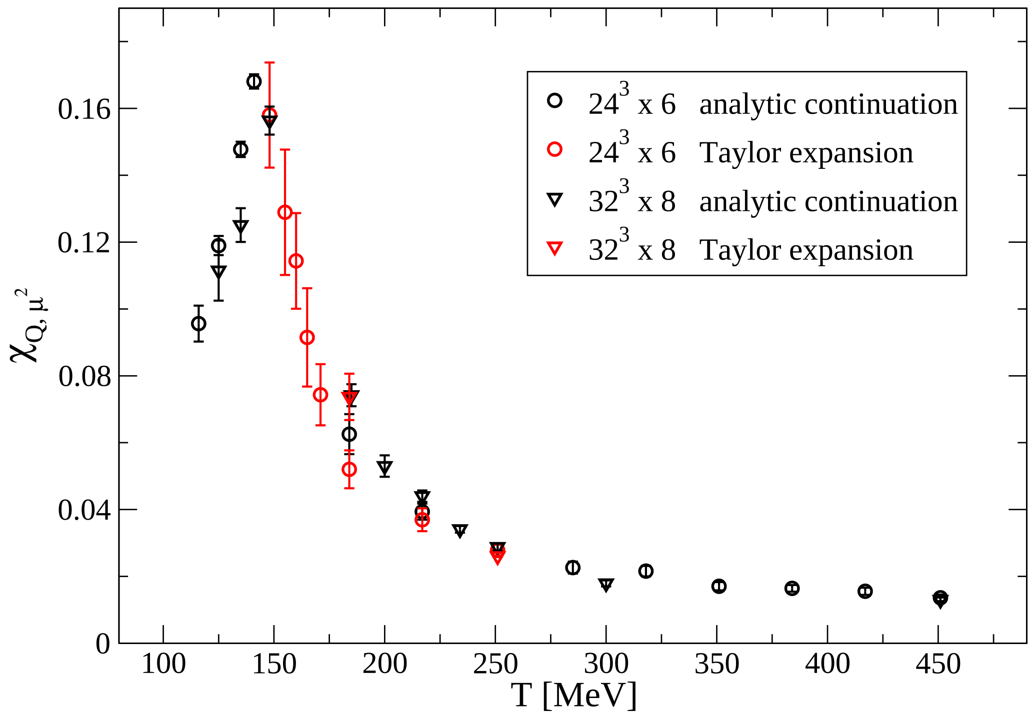
$$\chi_{Q,\mu_B} = \frac{1}{2}\frac{\partial^2}{\partial\mu^2}\frac{|\langle \mathrm{Tr}L(\mu,T)\rangle|^2}{|\langle \mathrm{Tr}L(0,T)\rangle|^2}$$

leads to

$$\begin{aligned}\frac{1}{2}\frac{\partial^2|\langle \mathrm{Tr}L\rangle|^2}{\partial\mu^2} &= \langle \mathrm{ReTrL}\,n\rangle^2 + 3\langle \mathrm{ReTrL}\rangle^2\langle n\rangle^2 - 4\langle \mathrm{ReTrL}\rangle\langle \mathrm{ReTrL}\,n\rangle\langle n\rangle^2 \\ &+ \langle \mathrm{ReTrL}\rangle\langle \mathrm{ReTrL}\,n^2\rangle - \langle \mathrm{ReTrL}\,n\rangle^2\langle n^2\rangle \\ &+ \langle \mathrm{ReTrL}\rangle\langle \mathrm{ReTrL}\,n'\rangle - \langle \mathrm{ReTrL}\rangle^2\langle n'^2\rangle + (\mathrm{ReTrL} \leftrightarrow \mathrm{ImTrL})\end{aligned}$$

$$n = \sum_f n_f = \sum_f \frac{1}{4}\mathrm{Tr}\left(M_f^{-1}\frac{\partial M_f}{\partial\mu}\right) \qquad n' = \frac{\partial n}{\partial\mu}$$

Preliminary Results



similarly to what happens for the μ_B^2-derivative of the chiral condensate, also χ_{Q,μ_B} presents a well defined peak in correspondence of the chiral crossover temperature

SUMMARY

- **Full convergence from different methods for the curvature of the pseudo-critical line around $\mu_B = 0$**
  $\kappa = 0.014(2)$

- **We have introduced a susceptibility related to the μ_B dependence of the Polyakov loop, χ_{Q,μ_B}**

- **χ_{Q,μ_B} presents a well defined peak at $T \sim 150$ MeV, i.e. in correspondence of the QCD chiral crossover.**