Gluon-mass-induced triply heavy baryon masses

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K. Serafin, M. Gómez-Rocha, J. More, S.D. Głazek, arXiv: 1805.03436
S. Głazek, M. Gómez-Rocha, J. More, K. Serafin, Phys.Lett. B773 (2017) 172-178

Emergent mass and its consequences in the Standard Model,
ECT*, 20th of September 2018

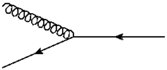
Framework: front form

- We use front form (FF) of Hamiltonian dynamics:
  - positions $(x^\mu) = (x^+, x^-, x^\perp)$,
  - momenta $(p^\mu) = (p^-, p^+, p^\perp)$,
  - longitudinal momentum fraction $x = p^+/P^+$.
- Canonical quantization
  - Lagrangian of QCD, $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{2}\mathrm{Tr}F^{\mu\nu}F_{\mu\nu}$, $A^+ = 0$
  - Energy momentum tensor $\mathcal{T}^{\mu\nu}$
  - FF Hamiltonian density, $\mathcal{T}^{+-}$
  - Canonical Hamiltonian $H_{\mathrm{can}} = \frac{1}{2}\int dx^- d^2x^\perp \; \mathcal{T}^{+-}|_{x^+=0}$
- Advantages
  - Simple vacuum
  - Boost invariant interactions and wave functions
  - 7 kinematical symmetries
  - Wave function formulas for form factors
  - Difficult aspects (confinement, chiral symmetry breaking) contained in zero modes (small-x divergences)
- Challenges
  - Small-x divergences!

Framework: renormalization

- We use renormalization group procedure for effective particles (RGPEP).
- Regulate the canonical Hamiltonian
  - UV divergences come from large transverse momenta
  - Small-x divergences come from small longitudinal momentum fractions

$$r_{21.3} = \exp\left[-\frac{m^2+\kappa^2}{(1-x)\Delta^2} - \frac{\delta^2+\kappa^2}{x\Delta^2}\right]$$



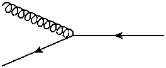
- Define effective particles

$$a_t = \mathcal{U}_t^\dagger a_0 \mathcal{U}_t \ , \quad a_t^\dagger = \mathcal{U}_t^\dagger a_0^\dagger \mathcal{U}_t \ ,$$

$$\frac{d}{dt}\mathcal{H}_t = [[\mathcal{H}_f, \tilde{\mathcal{H}}_t], \mathcal{H}_t]$$

- Effective vertexes have form factors $\leftrightarrow$ effective particles have nonzero size $s = t^{1/4}$

$$f_{21.3} = \exp\left[-t(\mathcal{M}_{12}^2 - m^2)^2\right]$$



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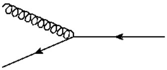

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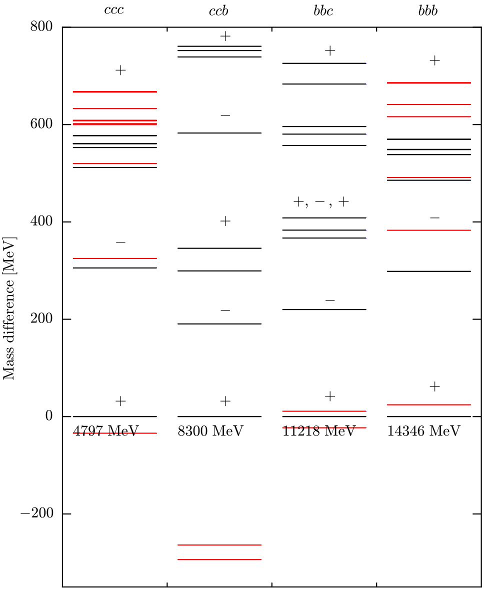
- Effective vertexes have form factors $\leftrightarrow$ effective particles have nonzero size $s = t^{1/4}$
- Effective Hamiltonians must not depend on the regularization parameter Δ when $\Delta \to \infty \Rightarrow$ counterterms at $t = 0$
- However, it does depend on $\epsilon = \delta/\Delta$

Why heavy quarks?

- Stable sources of gluons
- Gluons much lighter than quarks
- Coupling constant relatively small
- Simplifications due to nonrelativistic limit
- No complications from light quarks

The result



- Assuming a gluon mass, we obtain a sketch of the meson and baryon spectra in heavy-quark QCD that does not depend on the assumed gluon mass value.
- Black = ours; red = lattice (Meinel 2012, Brown et.al. 2014, Padmanath et.al. 2014); ± denotes parity of the states below.
- This stage of our theory is analogous to quenched QCD on "$3 \times 3 \times 3 \times 3$ lattice."
- It can be systematically improved.

How did we do it?

1. FF quantization and renormalization $\to H_t$
2. Heavy quarks allow for simplifications
3. Eigenvalue equation for H_t and the gluon mass ansatz
4. Reduction in the Fock space
5. Nonrelativistic limit

Many-sectors problem

$$H_s|\psi_s\rangle \quad = \quad P^-|\psi_s\rangle$$

$$|\psi_s\rangle \quad = \quad \begin{bmatrix} \ldots \\ |4Q_s\,\bar{Q}_s\rangle \\ \ldots \\ |3Q_s\,3G_s\rangle \\ |3Q_s\,2G_s\rangle \\ |3Q_s\,G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

Heavy quarks allow for several simplifications

- We choose s such that

$$\frac{1}{\Lambda_{\rm QCD}} \gg s \gtrsim \frac{1}{m}$$

- For $s \ll 1/\Lambda_{\rm QCD}$ effective coupling constant is small in accordance with asymptotic freedom, we can expand in powers of g,

$$H_s = H_{s\,0} + gH_{s\,1} + g^2 H_{s\,2} + \ldots$$

- Choosing $s \gtrsim 1/m$ we can neglect Fock sectors with extra $Q_s \bar{Q}_s$ pairs.
- Gluons, however, still pose a problem, bacause they are massless.

Gluon mass ansatz

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & H_{s\,0} + g^2 H_{s\,2} & g H_{s\,1} \\ \cdot & \cdot & g H_{s\,1} & H_{s\,0} + g^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = P^- \begin{bmatrix} \cdot \\ \cdot \\ |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} H_{s\,0} + \frac{\mu_s^2}{P^+} & g H_{s\,1} \\ g H_{s\,1} & H_{s\,0} + g^2 H_{s\,2} \end{bmatrix} \begin{bmatrix} |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = P^- \begin{bmatrix} |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix}$$

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$$\downarrow$$

$$H_{\mathrm{eff}\ s}|3Q_s\rangle = \frac{M^2 + P^{\perp 2}}{P^+}|3Q_s\rangle \ ,$$

$$\langle l|H_{\mathrm{eff\ s}}|r\rangle = \langle l| \Big[H_{s\,0} + g^2 H_{s\,2} + \frac{1}{2} g H_{s\,1} \left(\frac{1}{E_l - H_{s\,0} - \mu_s^2/P^+} + \frac{1}{E_r - H_{s\,0} - \mu_s^2/P^+} \right) g H_{s\,1} \Big] |r\rangle \ .$$

How to improve?

$$\left\{ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & H_{s\,0} & gH_{s\,1} & g^2 H_{s\,2} \\ \cdot & gH_{s\,1} & H_{s\,0} + g^2 H_{s\,2} & gH_{s\,1} + g^3 H_{s\,3} \\ \cdot & g^2 H_{s\,2} & gH_{s\,1} + g^3 H_{s\,3} & H_{s\,0} + g^2 H_{s\,2} + g^4 H_{s\,4} \end{bmatrix} - P^- \right\} \begin{bmatrix} \cdot \\ |3Q_s\, 2G_s\rangle \\ |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = 0\ .$$

$$\downarrow$$

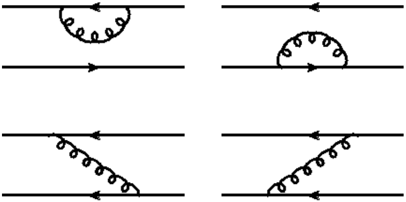
$$\left\{ \begin{bmatrix} H_{s\,0} + \frac{\mu_s^2}{P^+} & gH_{s\,1} & g^2 H_{s\,2} \\ gH_{s\,1} & H_{s\,0} + g^2 H_{s\,2} & gH_{s\,1} + g^3 H_{s\,3} \\ g^2 H_{s\,2} & gH_{s\,1} + g^3 H_{s\,3} & H_{s\,0} + g^2 H_{s\,2} + g^4 H_{s\,4} \end{bmatrix} - P^- \right\} \begin{bmatrix} |3Q_s\, 2G_s\rangle \\ |3Q_s\, G_s\rangle \\ |3Q_s\rangle \end{bmatrix} = 0\ .$$

$$\downarrow$$

$$H_{\mathrm{eff}\ s}|3Q_s\rangle = \frac{M^2 + P^{\perp 2}}{P^+}|3Q_s\rangle\ ,$$

Self energy versus gluon exchange

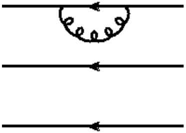
Mesons



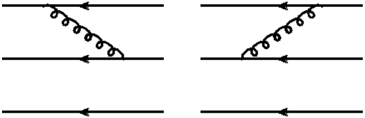
Color factors allow for small-x divergences cancellation:
$2 \times \frac{4}{3} - 2 \times \frac{4}{3}$

Baryons

$3\times$



$3\times$



Color factors allow for small-x divergences cancellation:
$3 \times \frac{4}{3} - 6 \times \frac{2}{3}$

The result (mesons)

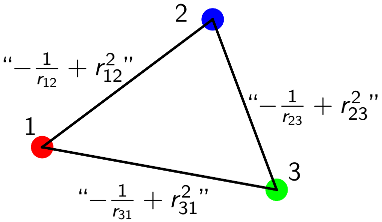
- Schrödinger equation:

$$\left[\frac{K_{12}^2}{2\mu_{12}} - \frac{\mu_{12}\omega_{12}^2\Delta_K^2}{2}\right] \psi_t(\vec{K}_{12})$$
$$- \int \frac{d^3q}{(2\pi)^3} \frac{4}{3}\frac{g^2}{q^2} e^{-16t[K_{12}^2-(\vec{K}_{12}-\vec{q})^2]^2} \psi_t(\vec{K}_{12}-\vec{q}) = E\ \psi_t(\vec{K}_{12}) ,$$

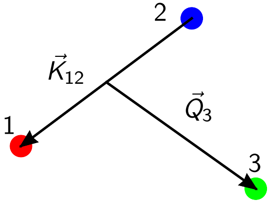
$$\mu_{12} = \frac{m_1 m_2}{m_1 + m_2} ,$$
$$\omega_{12}^2 = \frac{\alpha(\lambda)}{18\sqrt{\pi}\,\mu_{12}}\left(\frac{\lambda^2}{\sqrt{m_1^2+m_2^2}}\right)^3 ,$$
$$M = (m_1+m_2)\sqrt{1+\frac{2E}{m_1+m_2}} .$$

The result (baryons)

Three Coulomb potentials and three harmonic oscillator potentials.



In relative variables

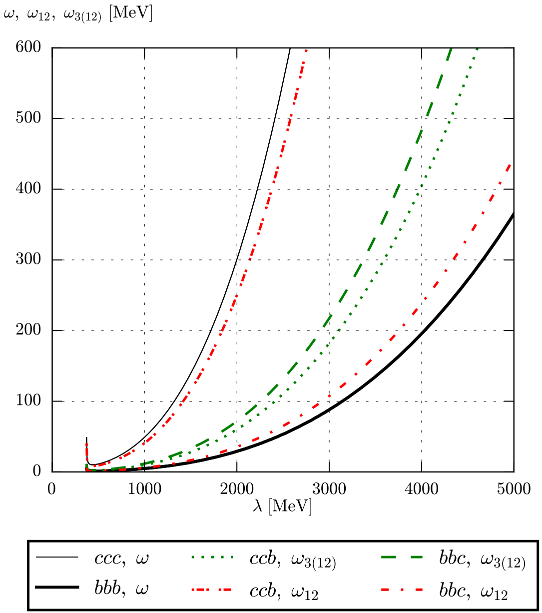


three Coulomb potentials and two collective harmonic oscillators with frequencies ω_{12} and $\omega_{3(12)}$.

$$\omega_{12}^2 = \frac{1}{m_1} \frac{\alpha\lambda^3}{18\sqrt{\pi}} \left[\left(\frac{\lambda^2}{2m_1^2} \right)^{3/2} + \frac{1}{2} \left(\frac{\lambda^2}{m_1^2 + m_3^2} \right)^{3/2} \right], \quad \lambda = 1/s$$

$$\omega_{3(12)}^2 = \frac{2m_1 + m_3}{2m_1 m_3} \frac{\alpha\lambda^3}{18\sqrt{\pi}} \left(\frac{\lambda^2}{m_1^2 + m_3^2} \right)^{3/2} .$$

λ dependence



We choose

$$\lambda_{Q\bar{Q}} = \sqrt{\alpha}\,(a\ \bar{m}_{Q\bar{Q}} + b)$$

$$\lambda_{3Q} = \sqrt{\alpha}\,(a\ \bar{m}_{3Q} + b)$$

Sketch of hadron spectra

Treating Coulomb as a perturbation to harmonic oscillator we obtain analytical formulas for masses of mesons and baryons.

$$\alpha(\lambda) = \left[\beta_0 \log(\lambda^2/\Lambda^2_{\rm QCD})\right]^{-1} , \quad \beta_0 = (33 - 2n_f)/(12\pi) , \quad n_f = 2$$
$$\alpha(M_Z) = 0.1181 .$$

Fit to $\Upsilon(1S)$, $\Upsilon(2S)$ and $\chi_{b1}(1P)$ $\rightarrow$ $m_b = 4698$ MeV, $\lambda_{b\bar{b}} = 4258$ MeV

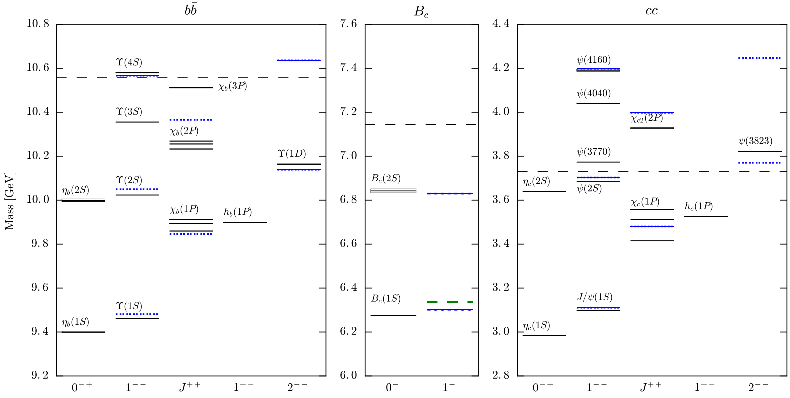
Fit to J/ψ, $\psi(2S)$ and $\chi_{c1}(1P)$ $\rightarrow$ $m_c = 1460$ MeV, $\lambda_{c\bar{c}} = 1944$ MeV

Values of $\lambda_{b\bar{b}}$, $\lambda_{c\bar{c}}$, and m_b, m_c fix

$$a = 1.589 ,$$
$$b = 783 \text{ MeV} .$$

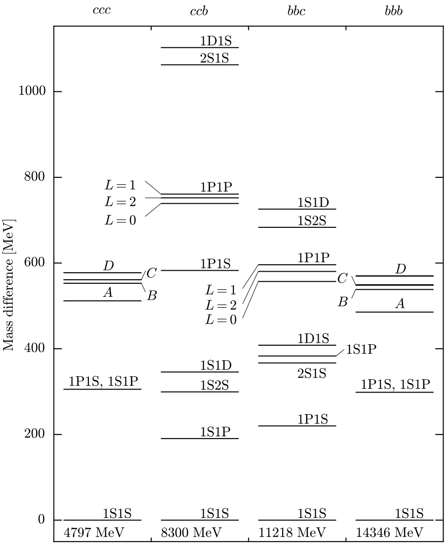
These coefficients imply λs and, hence, ωs for all other systems.

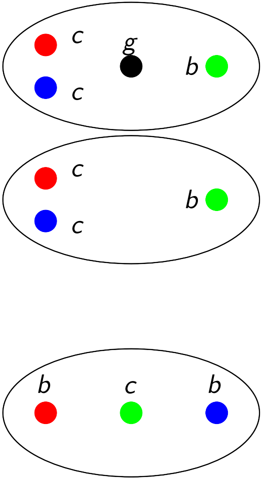
Meson spectra



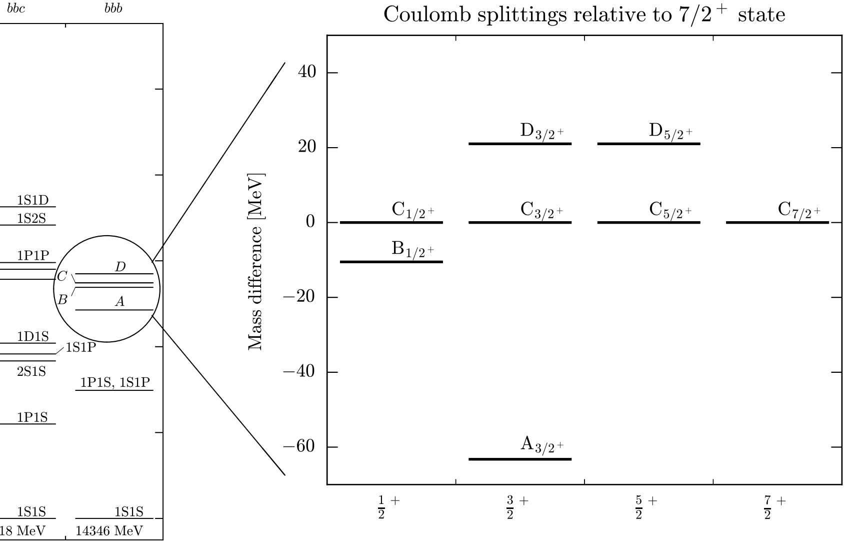
Dotted blue: our masses.
Solid black: PDG2017.
Dashed green: Gómez-Rocha, Hilger, Krassnigg, PRD93 074010 (2016)

Baryon spectra



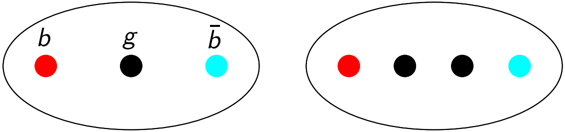


Splittings of the second band of harmonic oscillator



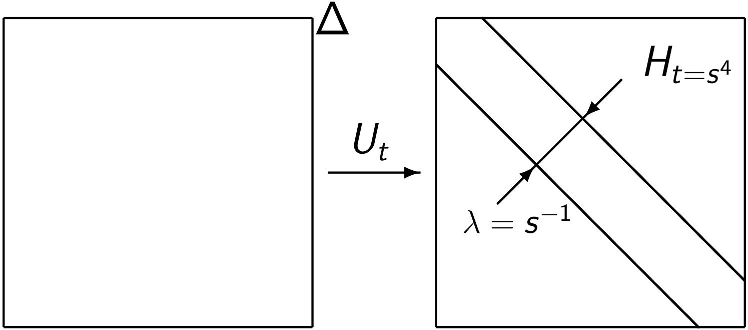
Outlook

- Systematic method of improving the calculation.
- 4^{th}-order RGPEP studies of H_s in QCD.
- Nonperturbative solutions of RGPEP equation.
- Gluon strings in a Hamiltonian approach?



- Hybrid mesons, glueballs, tetraquarks ...
- Light quarks after gluons (gluon mass).

Renormalization Group Procedure for Effective Particles



- The narrowing of the Hamiltonian is realized by the following differential equation,

$$\frac{d}{dt}\mathcal{H}_t = \left[[\mathcal{H}_{\text{free}}, \tilde{\mathcal{H}}_t], \mathcal{H}_t\right] \ , \qquad \mathcal{H}_t = H_t(q_0) \ .$$

- It allows for nonperturbative calculations.

Effective-particle picture of nucleons

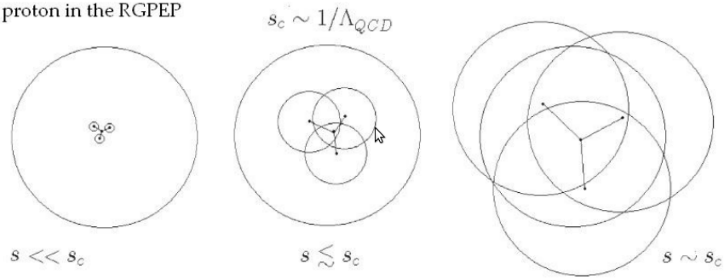


FIG. 6: The RGPEP scale-dependent proton picture

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Path to Schrödinger equation

- QCD Lagrangian → canonical Front Form Hamiltonian H_{can}.
- H_{can} is ill-defined → needs regularization and counterterms.
- The counterterms are determined using RGPEP.
- RGPEP gives us also a family of renormalized Hamiltonians parametrized by the size of effective particles.
- We reduce the Fock space to two sectors only at the prize of introducing gluon mass function.
- Taking advantage of asymptotic freedom we reduce Fock space perturbatively to the lowest sector obtaining the effective Hamiltonian.
- Taking nonrelativistic limit we obtain Schrödinger equation for three quarks.

Baryon and meson Schrödinger equations I

The effective eigenvalue equation for heavy baryons in QCD with two heavy flavors, implied by our gluon mass hypothesis, is

$$\begin{aligned}
&\left[\frac{K_{12}^2}{2\mu_{12}} + \frac{Q_3^2}{2\mu_{3(12)}} + \frac{\mu_{12}\omega_{12}^2\Delta_K^2}{2} + \frac{\mu_{3(12)}\omega_{3(12)}^2\Delta_Q^2}{2}\right]\psi_t(\vec{K}_{12},\vec{Q}_3) \\
+ &\int \frac{d^3q}{(2\pi)^3} V_C(q)\ \psi_t\left(\vec{K}_{12} - \vec{q}, \vec{Q}_3\right) \\
+ &\int \frac{d^3q}{(2\pi)^3} V_C(q)\ \psi_t\left(\vec{K}_{12} + \frac{1}{2}\vec{q}, \vec{Q}_3 + \vec{q}\right) \\
+ &\int \frac{d^3q}{(2\pi)^3} V_C(q)\ \psi_t\left(\vec{K}_{12} + \frac{1}{2}\vec{q}, \vec{Q}_3 - \vec{q}\right) \\
= &\ E\ \psi_t(\vec{K}_{12},\vec{Q}_3)\ ,
\end{aligned}$$

where Δ denotes Laplacian, reduced masses are $\mu_{12} = m_1/2$, $\mu_{3(12)} = 2m_1 m_3/(2m_1 + m_3)$ and

$$V_C(q) = -\frac{2}{3}\frac{g^2}{q^2}\ .$$

Baryon and meson Schrödinger equations II

The baryon mass eigenvalue is obtained from the eigenvalue E,

$$M = (2m_1 + m_3)\sqrt{1 + \frac{2E}{2m_1 + m_3}} \, .$$

We omitted the BF spin-dependent terms and the RGPEP form factors whose numerical inclusion requires the fourth-order RGPEP calculation. The associated quarkonium eigenvalue equation is

$$\left[\frac{K_{12}^2}{2\mu_{12}} + \frac{\mu_{12}\omega_{12}^2\Delta_K^2}{2}\right] \psi_t(\vec{K}_{12}) + 2\int \frac{d^3q}{(2\pi)^3} V_C(q)\, \psi_t(\vec{K}_{12} - \vec{q}) = E\, \psi_t(\vec{K}_{12}) \, ,$$

$$\omega_{12}^2 = \frac{\alpha(\lambda_{bc})}{18\sqrt{\pi}\,\mu_{bc}} \left(\frac{\lambda_{bc}^2}{\sqrt{m_b^2 + m_c^2}}\right)^3 ,$$

$$M = (m_1 + m_2)\sqrt{1 + \frac{2E}{m_1 + m_2}} \, ,$$

λs for baryons