Analytic properties of the gluon propagator in a generic covariant gauge

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Emergent mass and its consequences in the Standard Model
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The Gluon Mass

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The Gluon Mass

but, by Perturbation Theory!

Analytic properties of the gluon propagator in a generic covariant gauge

Last step of a recent attempt to study "Non-Perturbative" QCD by Perturbation Theory

- "Massive-Expansion" for Yang-Mills theory
  F.S. 1509.05891; Nucl.Phys.B907(2016) 572-596.
- Inclusion of Quarks and analytic properties
  F.S. PRD 94 (2016)
- Extension to finite temperature
  F.S. PRD 96 (2017); G. Comitini + F.S. PRD 97 (2018)
- Dynamical mass generation (variational argument)
  F.S. 1701.00286; G. Comitini + F.S. PRD 97 (2018)
- Extension to a generic covariant gauge and optimization
  F.S. + G. Comitini PRD 98 (2018)

The outcome is a self-contained optimized perturbation theory from first principles

Standard Perturbation Theory

Our understanding of QFT relies mainly on PT

Historically based on PT (QED, SM, etc.)
PT has many merits:

- explicit calculations
- analytical results at lowest order and 1-loop
- order by order improved accuracy
- important symmetries embedded in the formalism (gauge inv.)

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Unfortunately, PT breaks down in the IR of QCD

It is a pity since:

- Important phenomenology occurs in the IR (e.g. bound states)
- QCD is believed to be a complete consistent theory at any scale, containing its necessary cut-off

Non-Perturbative Effects

They cannot be addressed by any finite-order truncation

- Typically described by an infinite resummation
- They might be the sign of a wrong expansion point (rather than a failure of PT)

They are not intrinsic if can be cured by a change of the expansion point. (Well known issue of PT in QM where the accuracy depends on the "good" choice of $\hat{H}_0$)

What is "perturbative" and what is not?

It might depend on the Expansion Point

Trivial Example of a Wrong Expansion Point

$$\mathcal{L} = \frac{1}{2}\phi\left(-\partial^2 - m^2\right)\phi \;=\; \frac{1}{2}\phi\left(-\partial^2\right)\phi \;-\; \frac{m^2}{2}\phi^2, \quad \Delta_0(p) = \frac{1}{p^2}$$

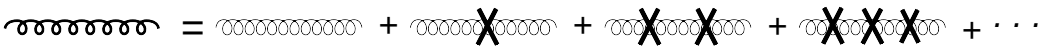
$$\Delta = \frac{1}{p^2}\left[1 - \frac{m^2}{p^2} + \frac{m^4}{p^4} + \cdots\right]$$

The pole is at $p = 0$ at any finite order, but

$$\Delta = \frac{1}{p^2}\frac{1}{1+\frac{m^2}{p^2}} = \frac{1}{m^2+p^2}$$

The shift of the pole emerges as NP effect by an infinite resumm. of the Dyson expansion.

resummation $\Longleftrightarrow$ change of expansion point



Which Expansion point is the best?

$$\text{Gauge inv. (BRST)} \Longrightarrow \begin{cases} \text{No gluon mass at any} \\ \text{finite order of PT} \end{cases}$$

$$\text{Dyn. Mass Generation} \Longrightarrow \begin{cases} \text{exact resumm. (NP approach)} \\ or \\ \text{change the exp. point of PT} \\ \text{BUT give up exact gauge inv.} \\ \text{at any finite order} \end{cases}$$

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We can build a viable PT in the IR

but we must give up exact gauge invariance at any finite order.

Exact vs. Approximate Invariance

Suppose we want a SQUARE to be drawn

1) By a computer using a "silly" algorithm which however preserves exact symmetries like

- Rotat. Inv. by $\theta = \frac{\pi}{4}$
- Inversion of axes

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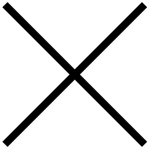

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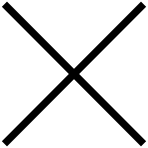

2) Hand-drawn by a human (giving up exact symmetries)

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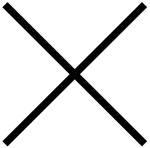

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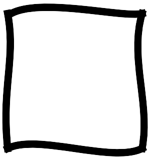


Does not satisfy any of the symmetries!

If it looks like a square $\Rightarrow$ approximate symmetries

Exact symmetries $\not\Rightarrow$ correct result

Screened Expansion in a generic covariant gauge

Standard BRST invariant SU(N) YM Lagrangian:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{fix} + \mathcal{L}_{FP} \leftarrow \text{from Faddeev-Popov Determinant}$$

$$\mathcal{L}_{YM} = -\frac{1}{2}\mathrm{Tr}\left(\hat{F}_{\mu\nu}\hat{F}^{\mu\nu}\right), \quad \mathcal{L}_{fix} = -\frac{1}{\xi}\mathrm{Tr}\left[(\partial_\mu \hat{A}^\mu)(\partial_\nu \hat{A}^\nu)\right]$$

$$S_0 = \frac{1}{2}\int A_\mu(x)\Delta_0^{-1\,\mu\nu}(x,y)A_\nu(y)\mathrm{d}^4x\,\mathrm{d}^4y + \int \omega^\star(x)\mathcal{G}_0^{-1}(x,y)\omega(y)\mathrm{d}^4x\,\mathrm{d}^4y$$

$$\Delta_0{}^{\mu\nu}(p) = \Delta_0(p)\left[t^{\mu\nu}(p) + \xi\ell^{\mu\nu}(p)\right]$$

$$\Delta_0(p) = \frac{1}{p^2}, \qquad \mathcal{G}_0(p) = -\frac{1}{p^2}$$

$S_I = \int \mathrm{d}^dx\,[\mathcal{L}_{gh} + \mathcal{L}_3 + \mathcal{L}_4]$ where:

$$\mathcal{L}_3 = -gf_{abc}(\partial_\mu A_{a\nu})A_b^\mu A_c^\nu, \quad \mathcal{L}_4 = -\frac{1}{4}g^2 f_{abc}f_{ade}A_{b\mu}A_{c\nu}A_d^\mu A_e^\nu$$

$$\mathcal{L}_{gh} = -gf_{abc}(\partial_\mu \omega_a^\star)\omega_b A_c^\mu$$

Screened Expansion in a generic covariant gauge

Same standard, BRST invariant, SU(N) YM Lagrangian:

$$S = \left[S_0 + \frac{1}{2}\int A_\mu\, \delta\Gamma^{\mu\nu}\, A_\nu\right] + \left[S_I - \frac{1}{2}\int A_\mu\, \delta\Gamma^{\mu\nu}\, A_\nu\right]$$

↖ not BRST inv. ↗

P.T. does not satisfy exact relations imposed by BRST at any finite order

$$\begin{cases} \Delta_m^{\mu\nu}(p) = \dfrac{1}{p^2+m^2}\, t^{\mu\nu}(p) + \dfrac{\xi}{p^2}\, \ell^{\mu\nu}(p) & \text{(free propagator)} \\ \qquad\qquad \nwarrow \text{Exact since } \Pi^L = 0 & \\ \delta\Gamma^{\mu\nu} = \left[\Delta_m^{-1\,\mu\nu} - \Delta_0^{-1\,\mu\nu}\right] = m^2\, t^{\mu\nu}(p) & \text{(2-point vertex)} \end{cases}$$

P.T. with the new vertex set

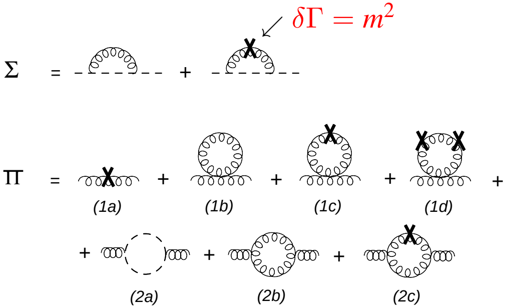
$$\mathcal{L}_3 = -g f_{abc}(\partial_\mu A_{a\nu}) A_b^\mu A_c^\nu, \qquad \mathcal{L}_4 = -\frac{1}{4} g^2 f_{abc} f_{ade} A_{b\mu} A_{c\nu} A_d^\mu A_e^\nu$$

$$\mathcal{L}_{gh} = -g f_{abc}(\partial_\mu \omega_a^\star)\omega_b A_c^\mu, \qquad \mathcal{L}_m = -\frac{1}{2}\delta_{ab}\delta\Gamma_{\mu\nu} A_a^\mu A_b^\nu$$

Screened Expansion in a generic covariant gauge

At variance with Curci-Ferrari model:

$$\Delta_T(p) = \frac{1}{(p^2+m^2) - \Pi^T} = \frac{1}{(p^2+m^2) - (m^2 + \Pi^T_{Loops})} = \frac{1}{p^2 - \Pi^T_{Loops}}$$



- The pole shift cancels at tree level
- All spurious diverging mass terms cancel without counterterms and/or parameters
- Standard UV behavior

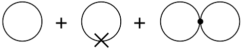
In the $\overline{MS}$ scheme: $\Pi^{diverg.} = \frac{Ng^2}{(4\pi)^2}\left(\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2}\right) p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right)$

Standard UV behavior $\Longrightarrow \Pi^{finite} \sim -\frac{Ng^2}{(4\pi)^2} p^2 \left(\frac{13}{6} - \frac{\xi}{2}\right) \log\frac{p^2}{\mu^2}$

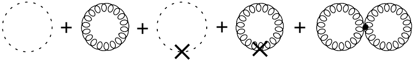
P.M. Stevenson PRD 32 (1985); P.M. Stevenson Z.Phys.C35 (1987)

$$m_B = 0 \quad \Longrightarrow \quad \mathcal{L} = \left[\frac{1}{2}\phi\left(-\partial^2 - m^2\right)\phi\right] - \left[\frac{\lambda}{4!}\phi^4 - m^2\phi^2\right]$$

$$V_{GEP}(\ \langle\phi\rangle,\ m^2) = \begin{cases} \\ \end{cases}$$



SCALAR



SU(N)

"Precarious" renormalization in $d = 4 + \epsilon$, P.M. Stevenson, (1987):

$$\frac{\partial V_{GEP}(\ \langle\phi\rangle = 0,\ m^2)}{\partial m^2} = 0 \Longrightarrow \begin{cases} m = m_0 \neq 0 \\ V_{GEP}\left(\ \langle\phi\rangle = 0,\ m_0^2\right) = -\frac{m_0^4}{128\pi^2} \ < \ 0 \end{cases}$$

Gluon Mass

Same identical result for SU(N) YM in any covariant ξ-gauge (gauge parameter independent! G. Comitini + F.S. PRD 97 (2018))

Setting $s = p^2/m^2 \leftarrow$ the scale m cannot be fixed by theory!

$$\Pi^T_{Loops} = -\frac{3Ng^2}{(4\pi)^2}\, p^2 \; [F(s) + \xi\, F_\xi(s)] + \Pi^{diverg.}$$

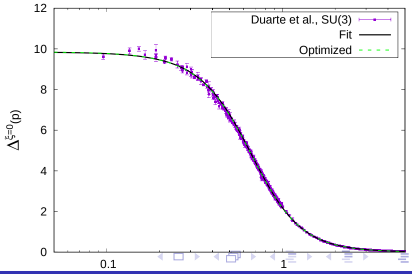
After subtraction (wave function renormalization):

$$\Delta(p) = \frac{Z_\mu}{p^2 + \frac{3Ng^2}{(4\pi)^2}\, p^2 \left[F(s) + \xi\, F_\xi(s) - F\left(\frac{\mu^2}{m^2}\right) - \xi\, F_\xi\left(\frac{\mu^2}{m^2}\right)\right]}$$

$$\boxed{\Delta(p) = \frac{Z}{p^2\, [F(s) + \xi\, F_\xi(s) + F_0]}} \qquad \mu \Leftrightarrow F_0$$

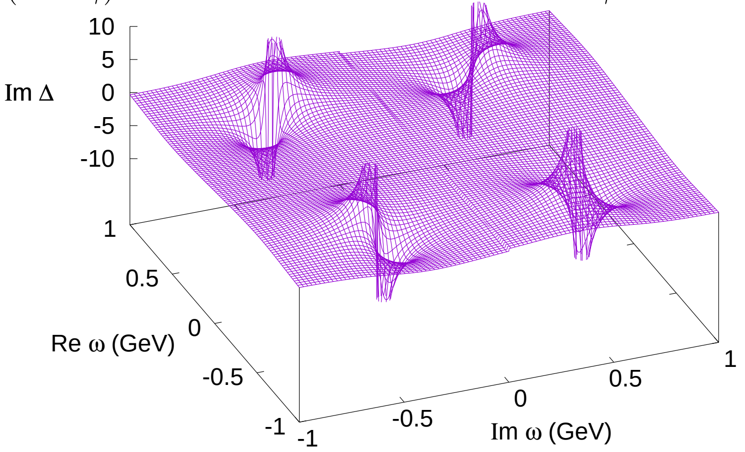
- Results depend on $\mu/m \to F_0$
- Nielsen Identities (BRST) are NOT exactly satisfied

Best fit at $\xi = 0$: $\begin{cases} m = 0.654 \text{ GeV} \\ F_0 = -0.887 \end{cases}$



ANALYTIC CONTINUATION AND CONFINEMENT

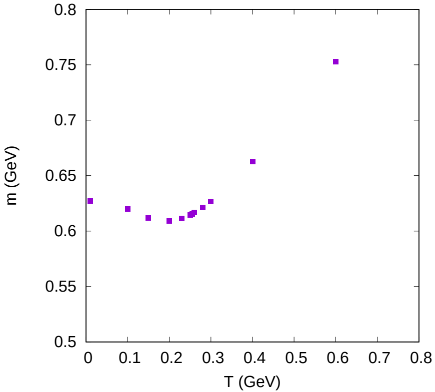
In the long wave-length limit $p^2 = \omega^2 - \mathbf{k}^2 \to \omega^2$ the poles are at $\omega = \pm(M \pm i\gamma)$ where $\mathbf{M = 0.581}$ **GeV** and $\gamma = \mathbf{0.375}$ **GeV**.

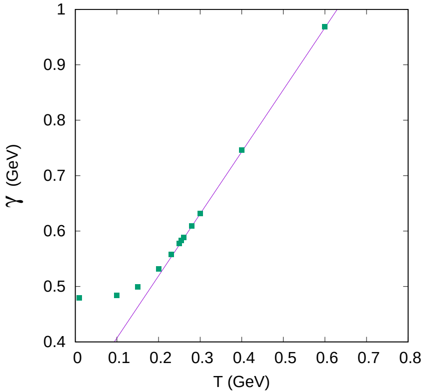


No violation of unitarity and casuality (Stingl, 1996):

short-lived quasigluons with lifetime $\tau = 1/\gamma$ are canceled from the asymptotic states

In the limit $\mathbf{k} \to 0$ the pole $\omega = \pm(m \pm i\gamma)$ is the same for Δ_L, Δ_T.
Using $m_0 = 0.73$ GeV and $F_0 = -1.05$ (fixed at $T = 0$):





The line is the fit $\gamma = \gamma_0 + bT$ with $\gamma_0 = 0.295$ GeV and $b = 1.12$.
(Hard thermal loops: $\gamma/T = 3.3\alpha_s$)

N.I. $\rightarrow$ $\boxed{\frac{\partial}{\partial\xi}\frac{1}{\Delta(p)} = G^T(p)\left[\frac{1}{\Delta(p)}\right]^2}$ $\quad G \sim \langle T\left[D^\mu\omega_a A_a^\nu \omega_b^\star B_b\right]\rangle$

The pole $p_0(\xi)$ must be gauge-parameter-independent:

$$\frac{1}{\Delta\left(p_0(\xi)\right)} = 0; \quad \frac{\mathrm{d}}{\mathrm{d}\xi}\frac{1}{\Delta\left(p_0(\xi)\right)} = 0 \quad \Longrightarrow \quad \boxed{\frac{\mathrm{d}}{\mathrm{d}\xi}p_0(\xi) = 0}$$

The residues are also ξ-independent (first suggested by D.Dudal):

$$\frac{\partial}{\partial\xi}\left[\frac{\mathrm{d}}{\mathrm{d}p^2}\frac{1}{\Delta}\right] = \left[\frac{\mathrm{d}}{\mathrm{d}p^2}G^T\right]\left[\frac{1}{\Delta}\right]^2 + 2G^T\frac{1}{\Delta}\left[\frac{\mathrm{d}}{\mathrm{d}p^2}\frac{1}{\Delta}\right]$$

$$R = \lim_{p\to p_0}\Delta(p)(p^2 - p_0^2) = \lim_{p\to p_0}\left[\frac{\mathrm{d}}{\mathrm{d}p^2}\frac{1}{\Delta(p)}\right]^{-1} \quad \Longrightarrow \quad \boxed{\frac{\partial}{\partial\xi}R = 0}$$

ξ-independent *Principal Part* $\quad \Delta^P(p) = \frac{R}{p^2 - p_0^2} + \frac{R^\star}{p^2 - p_0^{\star 2}}$ (RGZ)

Assume that **if** $\begin{cases} F_0 = F_0(\xi) \\ m = m(\xi) \end{cases} \implies$ N.I. are satisfied

and define the complex variable: $z^2 = -p_E^2 = p_M^2, \quad z = x + iy$

$$\Delta = \frac{Z}{p^2\,\Psi} \quad \text{where} \quad \boxed{\Psi(z,\xi,F_0,m) = F(-z^2/m^2) + \xi\, F_\xi(-z^2/m^2) + F_0}$$

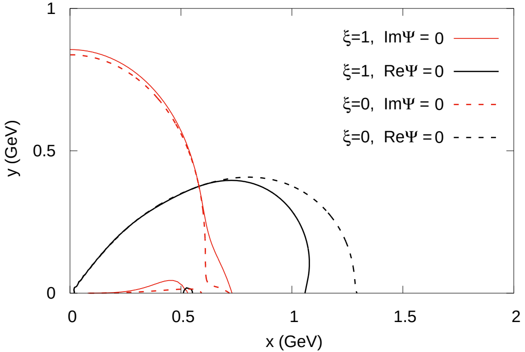
Conformal map $\rightarrow \Psi\Big(z_1,\xi_1,F_0(\xi_1),m(\xi_1)\Big) = \Psi\Big(z_2,\xi_2,F_0(\xi_2),m(\xi_2)\Big)$

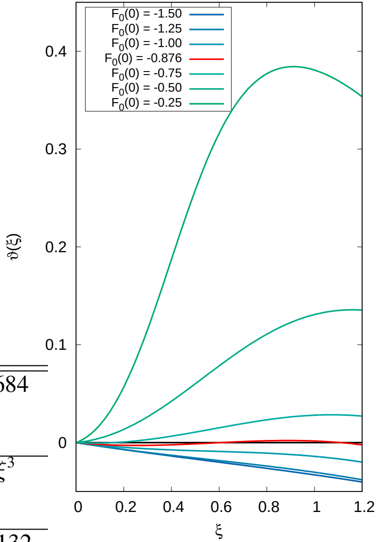
Fixed Point: $\Psi\Big(z_0,\xi_1,F_0(\xi_1),m(\xi_1)\Big) = \Psi\Big(z_0,\xi_2,F_0(\xi_2),m(\xi_2)\Big) = 0$

$$\left.\begin{aligned} \xi_1 &= 0 \\ m(\xi_1) &= m(0) \\ F(\xi_1) &= F_0(0) \end{aligned}\right\} \implies F_0(\xi_2),\ m(\xi_2) \quad \text{(two real equations)}$$

BUT $\nwarrow$ free param.

$$R(\xi) = R(0)\, e^{i\theta(\xi)}, \quad \theta(\xi) = \text{Arg}\left\{\frac{\frac{\mathrm{d}}{\mathrm{d}z}\Psi\left(z,0,F_0(0),m(0)\right)}{\frac{\mathrm{d}}{\mathrm{d}z}\Psi\left(z,\xi,F_0(\xi),m(\xi)\right)}\right\}_{z=z_0} \neq 0$$





$$F_0(0) = -0.876, \quad m_0 = m(0) = 0.656 \text{ GeV}, \quad Z(0) = 2.684$$

$$|\theta(\xi)| < 2.76 \cdot 10^{-3}, \qquad 0 < \xi < 1.2$$

$$F_0(\xi) \approx -0.8759 - 0.01260\xi + 0.009536\xi^2 + 0.009012\xi^3$$

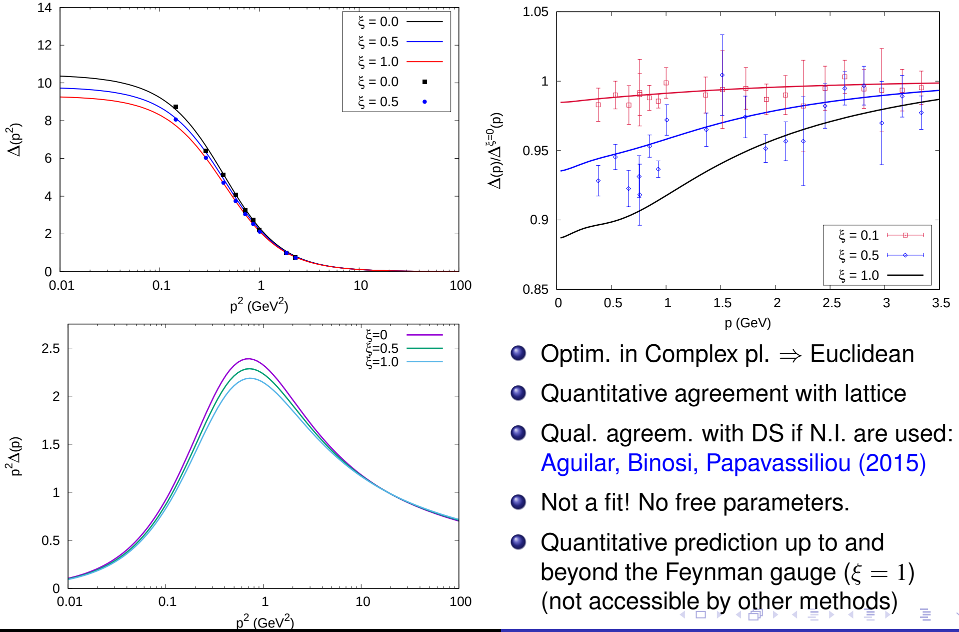
$$m^2(\xi)/m_0^2 \approx 1 - 0.39997\xi + 0.064141\xi^2$$

$$z_0/m_0 = 0.8857 + 0.5718\, i, \qquad t_R = \operatorname{Im} R(0)/\operatorname{Re} R(0) = 3.132$$

$$M = 0.581 \text{ GeV}, \gamma = 0.375 \text{ GeV} \quad \text{(invariant pole)}$$

Back to Euclidean Space

Optim. S.E. vs. Lattice data of Bicudo,Binosi,Cardoso,Oliveira,Silva PRD 92 (2015)



- Optim. in Complex pl. $\Rightarrow$ Euclidean
- Quantitative agreement with lattice
- Qual. agreem. with DS if N.I. are used: Aguilar, Binosi, Papavassiliou (2015)
- Not a fit! No free parameters.
- Quantitative prediction up to and beyond the Feynman gauge ($\xi = 1$) (not accessible by other methods)

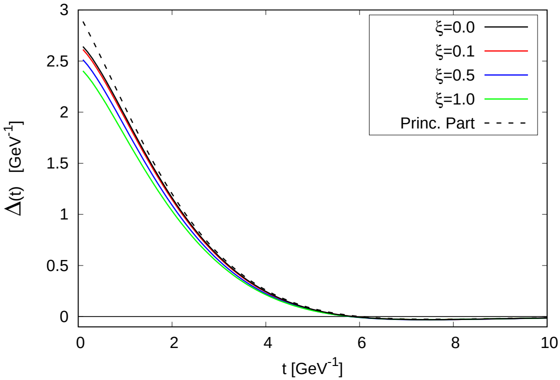
Schwinger function

$$\Delta(t) = \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi}\, e^{ip_4 t}\, \Delta(\vec{p}=0, p_4) \quad (t = \text{Euclidean time})$$

$$\begin{cases} \Delta^P(p) = \frac{R}{p^2 - p_0^2} + \frac{R^\star}{p^2 - p_0^{\star 2}} = Z_{GZ} \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4} \\ \Delta^P(t) = \left[\frac{|R|}{\sqrt{M^2+\gamma^2}}\right] e^{-Mt} \cos(\gamma t - \phi) \end{cases}$$

(RGZ language)

where $\phi = \text{Arg}[R] - \arctan\frac{\gamma}{M}$



- $t_0 \approx 5.8\ \text{GeV}^{-1} \approx$ hadron size: physical gauge-invariant scale ? Conject. by Alkofer, Detmold, Fischer, Maris PRD 70 (2004)
- Large t behavior dominated by singularities (i.e. ξ-independent principal part)
- $t_0 \approx \frac{1}{\gamma}\left(\text{Arg}[R] - \arctan\frac{\gamma}{M} + \frac{\pi}{2}\right)$

Final remarks

- Screened Expansion (S.E.) $\rightarrow$ analyt. and from first principles
  ξ-gauge + N.I. $\Longrightarrow$ self-contained optimization
  (no external inputs and/or parameters are required).

- Optimization by N.I. $\Longrightarrow$ furthrer proof that poles are genuine.
  Complex poles $\rightarrow$ Gluon-Confinement ($\gamma > 0$ at $T = 0$).

- $m \approx 0.6$ GeV $\Longrightarrow$ Gribov copies irrelevant!
  The mass m is as effective as the Gribov parameter for screening the theory (Gao,Qin,Roberts,Rodriguez-Quintero,2018);
  Faddeev-Popov $\rightarrow$ very good approx. if P.T. works well.

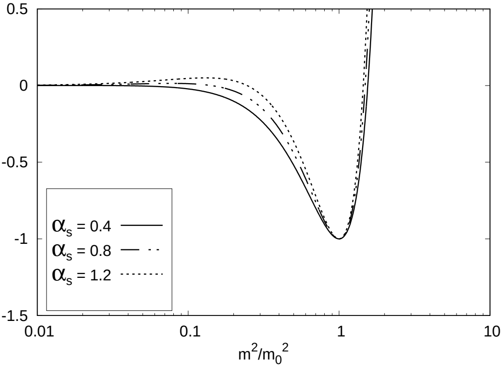
- $\Delta(p)$ almost gauge invariant (slightly depressed for $\xi > 0$).
  Well described by ξ-independent principal part (RGZ).

BACKUP SLIDES

Gaussian Effective Potential (GEP)

Renormalized Effective Potential in units of the best mass m_0

$$V(m) = \frac{m^4}{128\pi^2}\left[\alpha\left(\log\frac{m^2}{m_0^2}\right)^2 + 2\log\frac{m^2}{m_0^2} - 1\right]$$



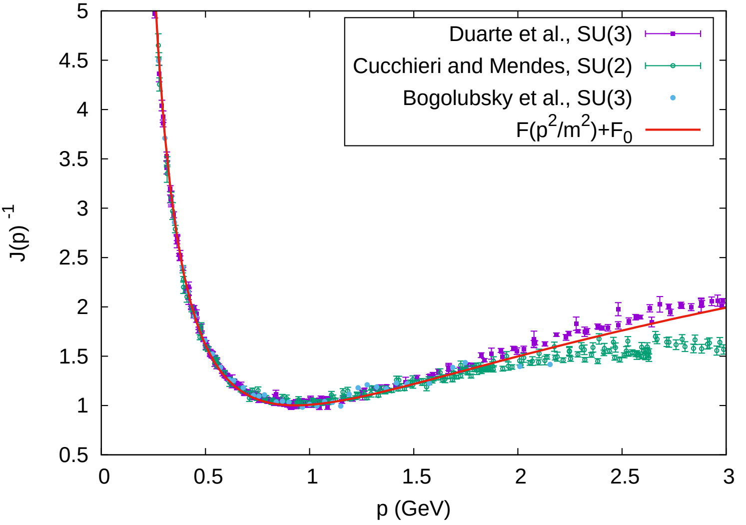
From the gap eq.:

$$\delta_\epsilon = m_0 \exp(-1/\alpha)$$

The vacuum energy does not depend on δ_ϵ and α:

$$V(m_0) = -\frac{m_0^4}{128\pi^2} < 0$$

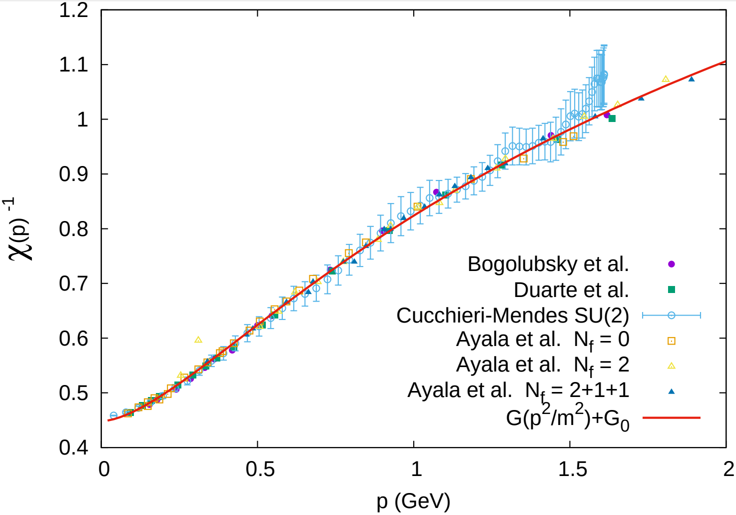
Gluon mass generation: the same identical result for SU(N) Yang-Mills Theory in any covariant ξ-gauge if $\alpha = 9N\alpha_s/(8\pi)$



Duarte et al., SU(3)
Cucchieri and Mendes, SU(2)
Bogolubsky et al., SU(3)
$F(p^2/m^2)+F_0$
$J(p)^{-1}$
p (GeV)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

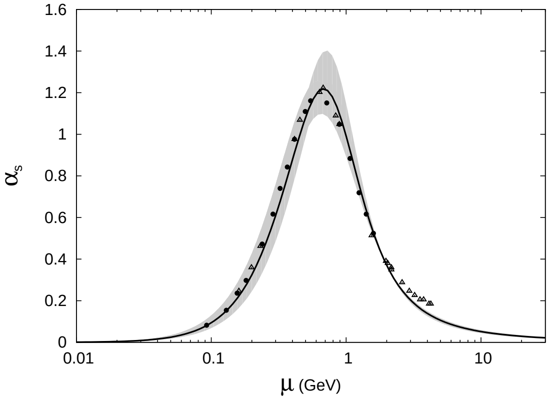
The ghost universal function is just

$$G(s) = \frac{1}{12}\left[2 + \frac{1}{s} - 2s\log s + \frac{1}{s^2}(1+s)^2(2s-1)\log(1+s)\right]$$



RG invariant product (Landau Gauge – MOM-Taylor scheme):

$\alpha_s(\mu) = \alpha_s(\mu_0)\frac{J(\mu)\chi(\mu)^2}{J(\mu_0)\chi(\mu_0)^2}$ What if $\delta F_0 = \delta G_0 = \pm 25\%$?



$\mu_0 = 2$ GeV, $\alpha_s = 0.37$,   data of Bogolubsky et al.(2009).

Quark propagator:

$$S(p) = S_p(p^2)\not{p} + S_M(p^2)$$

NO COMPLEX POLES $\Longrightarrow$ Standard Dispersion Relations

$$\rho_M(p^2) = -\frac{1}{\pi}\,\mathrm{Im}\, S_M(p^2)$$
$$\rho_p(p^2) = -\frac{1}{\pi}\,\mathrm{Im}\, S_p(p^2)$$

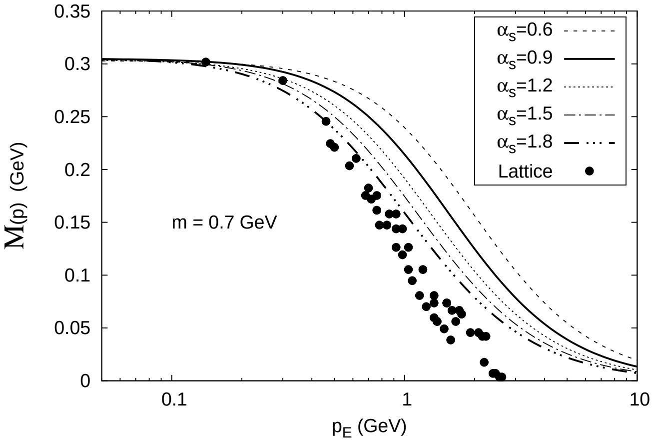
$$S(p) = \int_0^\infty \mathrm{d}q^2 \frac{\rho_p(q^2)\not{p} + \rho_M(q^2)}{p^2 - q^2 + i\varepsilon}.$$

Positivity Conditions:

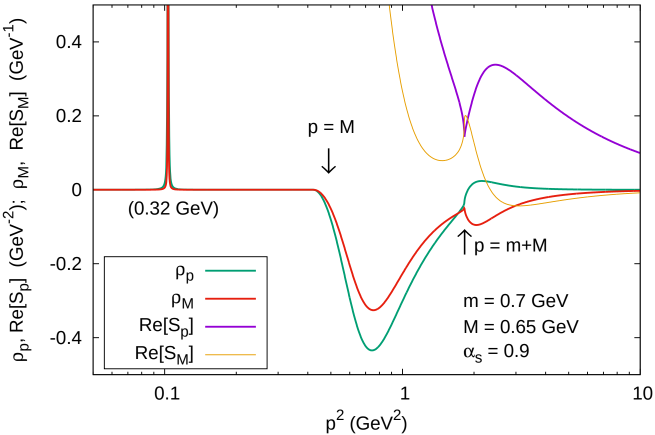
$\rho_p(p^2) \geq 0,\qquad p\,\rho_p(p^2) - \rho_M(p^2) \geq 0$

CHIRAL QCD

Quark sector: $m = 0.7$ GeV, M is fixed by requiring that $M(0) \approx 0.32$ GeV



Quark sector: $N_f = 2$, $M = 0.65$ GeV, $m = 0.7$ GeV

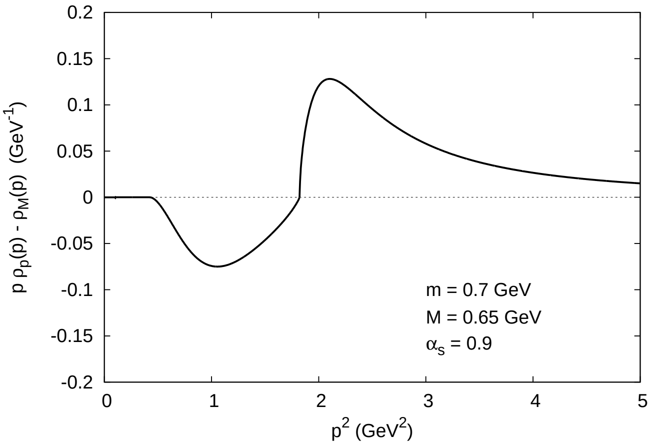


Positivity Conditions:

$$\rho_p(p^2) \geq 0, \qquad p\, \rho_p(p^2) - \rho_M(p^2) \geq 0$$

CHIRAL QCD

Quark sector: $N_f = 2$, $M = 0.65$ GeV, $m = 0.7$ GeV



Positivity Conditions:

$$\rho_p(p^2) \geq 0, \qquad p\ \rho_p(p^2) - \rho_M(p^2) \geq 0$$