Mass-dependence of pseudoscalar meson electromagnetic form factors

[arXiv:1808.09461, Muyang Chen, Minghui Ding, Lei Chang, and Craig D. Roberts]

Speaker : Muyang Chen

Nankai University, Tianjin, China

Emergent mass and its consequences in the Standard Model,
@Trento, Italy

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Introduction

- The electromagnetic form factors are fundamental quantities, which describe the internal, dynamical structure of hadrons (non pointlike particles).
- The fundamental role played by form factors was recognized since the electron–proton elastic scattering experiments showed that the proton has a composite structure, when R. Hofstadter was rewarded by the Nobel prize in 1961.
  "For his pioneering studies of electron scattering in atomic nuclei and for his consequent discoveries concerning the structure of nucleons".
- At small momenta form factors probe the size of the hadrons. At high energies they probe the quark and gluon structure. Their behavior should follow scaling laws, predicted by perturbative quantum-chromodynamics (pQCD). The precise knowledge of form factors in a wide kinematical region should probe the transition from non perturbative to pQCD.
- In a P and T invariant theory, the structure of any particle of spin S is parametrized in terms of $(2S+1)$ form factors.

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Introduction

- $\langle \pi(p_2)|j^\mu|\pi(p_1)\rangle = (p_1+p_2)^\mu F_\pi(Q^2),\ Q^2 = (p_1-p_2)^2.$
- Pion is the lightest hadron, much lighter than other hadrons, playing an importance role in strong interaction.
- The pion' s $\bar{q}q$ valence structure is relatively simple. The electric form factor of the pion, F_π , is one of the best observables for the investigation of the transition of QCD effective degrees of freedom in the soft regime to the perturbative regime at higher Q^2.
- Early measurements of the pion elastic electromagnetic form factor via scattering high-energy pions from atomic electrons:
  - *E. B. Dally et al., Phys. Rev. D 24, 1718 (1981).*
    E. B. Dally et al., Phys. Rev. Lett. 48, 375 (1982).
  - *S. R. Amendolia et al., Phys. Lett. B 146, 116 (1984).*
    S. R. Amendolia et al., Nucl. Phys. B 277, 168 (1986).

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Introduction

- Perturbation theory in quantum chromodynamics (QCD) is applicable to hard exclusive processes; and for almost forty years the leading-order factorised result for the electromagnetic form factor of a pseudoscalar meson has excited experimental and theoretical interest.
- $\exists Q_0 > \Lambda_{\rm QCD}$ such that

$$Q^2 F_{0^-}(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_{0^-}^2 \omega_{0^-}^2(Q^2), \tag{1}$$

where f_{0^-} is the meson’s leptonic decay constant; $\alpha_s(Q^2)$ is the leading-order strong running-coupling

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{\rm QCD}^2)}, \tag{2}$$

with $\beta_0 = 11 - (2/3)n_f$, n_f is the number of active quark flavours; and

$$\omega_{0^-}(Q^2) = \frac{1}{3}\int_0^1 dx \frac{1}{x}\varphi_{0^-}(x;Q^2), \tag{3}$$

where $\varphi_{0^-}(x;Q^2)$ is the meson's dressed-valence-quark parton distribution amplitude (PDA).

- The PDA, $\varphi_{0^-}(x; Q^2)$, is determined by the meson's light-front wave function and relates to the probability that, with constituents collinear up to the scale $\sqrt{Q^2}$, a valence-quark within the meson carries light-front fraction x of the bound-state's total momentum.
- The value of Q_0 is not predicted by perturbative QCD.

Experimental measurements

- Early measurements of the pion elastic electromagnetic form factor via scattering high-energy pions from the electrons of a liquid hydrogen target gave us the electromagnetic form factor on $Q^2 \in [0, 0.26]\text{GeV}^2$, and a sound measurement of the pion's charge radius $r_\pi^2 = 0.439 \pm 0.008\text{fm}^2$.

  [S. R. Amendolia et al., Nucl. Phys. B 277, 168 (1986)]
- For the determination of the pion form factor at higher values of Q^2 people has to use high-energy electroproduction of pions on a nucleon, i.e. employ the ${}^1H(e, e'\pi^+)n$. In 2001 a long-planned continuous electron beam accelerator facility (CEBAF) experiment collected data on $0.6 < Q^2/\text{GeV}^2 < 1.6$, [Phys. Rev. Lett. 86, 1713-1716(2001)] and reached $Q^2 = 2.45$ GeV^2 in 2006, [Phys. Rev. Lett. 97, 192001(2006)]
- No signal for the behavior of pQCD has yet been claimed.
- Experiments at the updated Jefferson Lab aim for precision measurements of $F_\pi(Q^2)$ upto $Q^2 = 6$ GeV^2.

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- Based on the conformal limit PDA

$$\varphi_\pi(x;Q^2) \overset{\Lambda^2_{\rm QCD}/Q^2\simeq 0}{\approx} \varphi^{cl}(x) = 6x(1-x), \tag{4}$$

the leading-order factorised formula gives

$$Q^2 F_\pi(Q^2) \overset{Q^2=4\text{GeV}^2}{\approx} 0.15, \tag{5}$$

which is a factor of 2.7 smaller than the empirical value at $Q^2 = 2.45\text{GeV}^2$: $0.41^{+0.04}_{-0.03}$.

- However, continuum and lQCD studies of the pseudoscalar meson bound-state problem have revealed that $\varphi_\pi(x;Q^2 \sim (2m_p)^2)$ is a concave function, much broader than $\varphi^{cl}(x)$ owing to emergent mass generation.
- A continuum calculation of $F_\pi(Q^2)$ on a large domain of spacelike momenta predicted that the approved JLab 12 experiments are capable of validating the leading-order factorised formula Eq.(1), as the estimate in Eq.(5) is too small by a factor of approximately 2.

LQCD simulations at small Q^2

- LQCD results with pion masses near the physical value are currently restricted to small Q^2.
- HPQCD Collaboration gives electromagnetic form factor on $Q^2 \in [0, 0.13]$ GeV^2, $r_\pi^2 = 0.403$ fm^2. [J. Koponen *et al.* Phys. Rev. D93, 054503(2016).]
- ETM Collaboration gives electromagnetic form factor on $Q^2 \in [0, 0.25]$ GeV^2, $r_\pi^2 = 0.443$ fm^2. [C. Alexandrou *et al.* Phys. Rev. D97, 014508(2018).]
- No lQCD predictions at m_π are available on the full domain accessible to JLab 12.

Difficulties in LQCD simulations:

- Large lattice volume to represent light pions.
- Small lattice spacing to reach large Q^2.
- High statistics to compensate for decaying signal-to-noise ratio as form factors drop rapidly with increasing Q^2.

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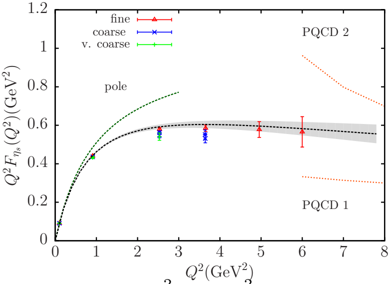
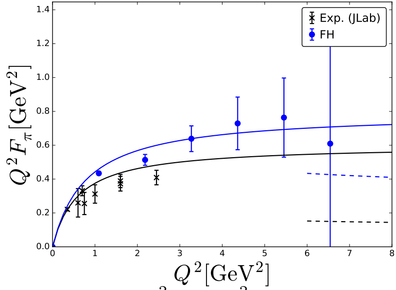
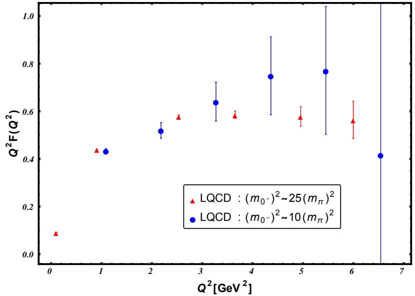
Preliminary LQCD simulations at larger Q^2



$m^2_{0^-} \approx 25 m^2_\pi$



$m^2_{0^-} \approx 10 m^2_\pi$



- $m^2_{0^-} \approx 10 m^2_\pi$, QCDSF/UKQCD/CSSM Collaborations, Phys. Rev. D 96, 114509 (2017).
- $m^2_{0^-} \approx 25 m^2_\pi$, HPQCD Collaboration, Phys. Rev. D 96, 054501 (2017).

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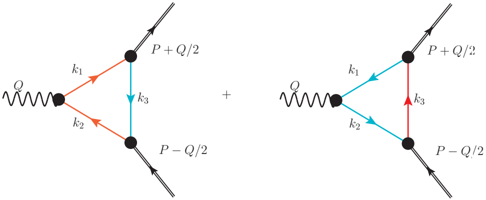
$$\begin{aligned} 2P_\nu F_{0^-}(Q^2) &= \Lambda_\nu^{a\bar{b}}(P,Q), \\ \Lambda_\nu^{a\bar{b}}(P,Q) &= \hat{Q}^a \Lambda_\nu^{a\bar{b}a}(P,Q) + \hat{Q}^{\bar{b}} \Lambda_\nu^{a\bar{b}\bar{b}}(P,Q). \end{aligned}$$

The pion scatter amplitude in the impulse approximation is:

$$\begin{aligned} \Lambda_\nu^{a\bar{b}a}(P,Q) &= N_c \int_{\mathrm{d}k}^{\Lambda} \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{tr}\Big[S^b(k_3)\, \bar{\Gamma}^{a\bar{b}}(k_1,k_3;-P_+) \\ &\quad \times S^a(k_1)\, i\Gamma_\nu^a(k_2,k_1;Q) \times S^a(k_2)\, \Gamma^{a\bar{b}}(k_3,k_2;P_-) \Big], \end{aligned} \tag{6}$$

where $\Gamma^{q\bar{q}}$-Meson BSA, Γ_α^q-quark-photon vertex, $S^q(k_i)$-dressed quark propagator.



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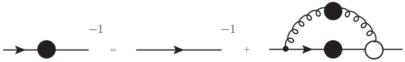
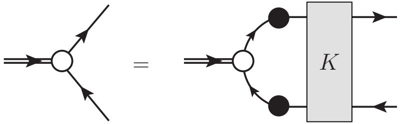
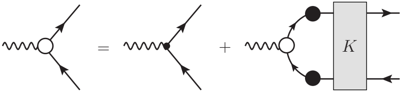
Computing the form factor in rainbow-ladder truncation



[DSE for quark propagator.]



[BSE for meson BSA.]



[BSE for quark-photon vertex.]

Rainbow Ladder approximation:

- Quark-gluon vertex $\Gamma^\mu = \gamma^\mu$.
- Scattering kernel $K = \gamma^\mu G^{\mu\nu}(q^2)\gamma^\nu$.
- Gluon propagator $G^{\mu\nu}(q^2) = G_0^{\mu\nu}(q^2)\mathcal{F}(q^2)$.
- $G_0^{\mu\nu}(q^2) = \delta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$, $\mathcal{F}(q^2) = \frac{8\pi^2}{\omega^5}\varsigma^3 e^{-q^2/\omega^2} + \frac{8\pi^2\gamma_m\mathcal{P}(q^2)}{\ln[\tau+(1+q^2/\Lambda_{QCD}^2)^2]}$, $\tau = e^2 - 1$, $\mathcal{P}(q^2) = (1 - exp(-\frac{q^2}{4m_t^2}))/q^2$.
- Satisfies the axial vector Wald-Takahashi-Idendity(WTI) and quark-photon vertex WTI.

Determining the meson PDA

- Compute the moments,

$$\langle x^m \rangle = \int_0^1 dx x^m \varphi(x;\varsigma) \tag{7}$$

$$= \frac{Z_2 N_c}{f_{0^-}} \mathrm{tr} \int_{dk}^{\Lambda} \frac{(n \cdot k_+)^m}{(n \cdot P)^{m+1}} \gamma_5 \gamma \cdot n \chi(k;P). \tag{8}$$

  where $\varphi(x;\varsigma)$ is the PDA, normalized as $\int_0^1 dx \varphi(x;\varsigma) = 1$, $n^2 = 0$, $\chi(k;P) = S(k_+)\Gamma(k;P)S(k_-)$ is the wave function of the meson.
- A convergence factor $1/(1 + k^2 r^2)$ is included in the integrand to stabilise the computation; the moment is computed as a function of r^2; and the final value is obtained by extrapolation to $r^2 = 0$.
- Use the moment $\langle \varepsilon^2 \rangle$, where $\varepsilon = 2x - 1$, to reconstruct a realistic approximation to the PDA,

$$\tilde{\varphi}(x) = \mathcal{N} x^\alpha (1 - x)^\alpha, \tag{9}$$

  where $\mathcal{N} = \frac{\Gamma(2\alpha+2)}{\Gamma(\alpha+1)^2}$ is the normalization constant.

Form factor on the whole range

- Owing to the analytic structure of the quark propagator, the calculation algorithm fails on $Q^2 \gtrsim Q_f^2$, $Q_f^2 = 4$ GeV^2 for the pion.
- *[Phys. Rev. Lett. 111, 141802 (2013)]* solved this problem by using perturbation theory integral representations [PTIRs] for each matrix-valued function in Eq.(6), enabling a reliable computation of the electromagnetic form factor to arbitrarily large-Q^2.
- PTIRs is straightforward but time consuming. Analysis shows that the computed elastic pion form factor can accurately be interpolated by a monopole multiplied by a simple factor that restores the correct QCD anomalous dimension. Assuming on the domain $Q^2 \lesssim 10$ GeV^2,

$$F_{0^-}(Q^2) = \frac{1}{1+Q^2/m_v^2}\mathcal{A}_{0^-}(Q^2), \tag{10}$$

$$\mathcal{A}_{0^-}(Q^2) = \frac{1+a_1 Q^2 + a_2^2 Q^4}{1+Q^4(a_2^2/b_u^2)\ln[1+Q^2/\Lambda_{\rm QCD}^2]}, \tag{11}$$

where m_v is the computed vector meson mass and a_1, a_2, b_u are determined via a least-squares fit to the computed results on $Q^2 \gtrsim Q_f^2$.

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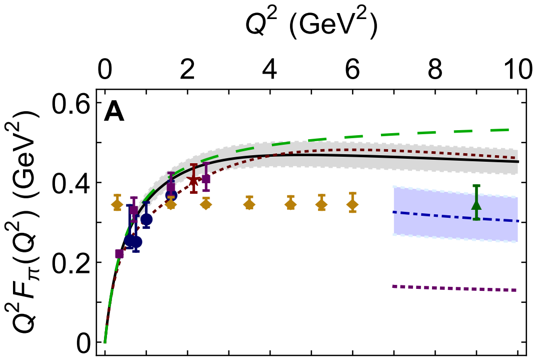


Solid black – our prediction,

m_{0^-}	m_v	a_1	a_2	b_u
0.14	0.77	-0.14	0.50	2.12

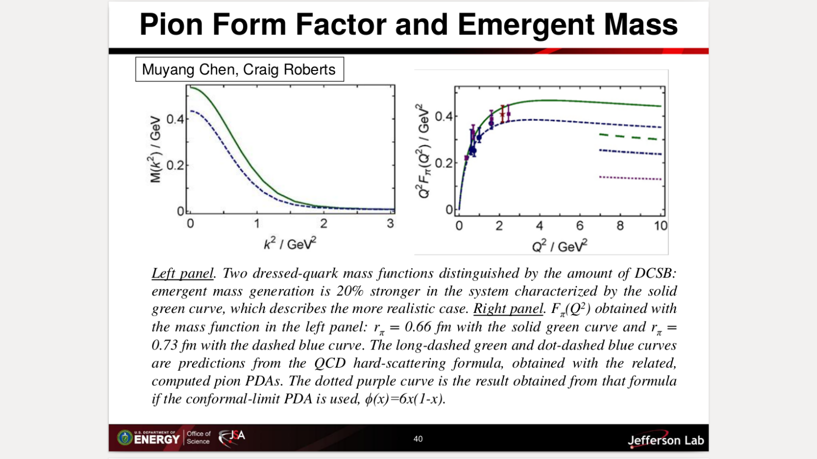
;

Dotted brown – PTIRs result;

Long-dashed green – single-pole vector meson dominance;

Dot-dashed blue – pQCD with the PDA at $\varsigma = 2$ GeV;

Dotted purple – pQCD with the conformal-limit PDA.



Slide from Rolf Ent's talk "Pion and Kaon Structure at an EIC"

Mass dependence

Table 1: Input current-quark masses m^{ζ_2} for four pion-like mesons and related results computed with $\omega = 0.5 \pm 0.1$ GeV. All dimensioned quantities listed in GeV, except r_{0^-} in fm.

m^{ζ_2}	m_{0^-}	$\omega = 0.4$				$\omega = 0.5$				$\omega = 0.6$			
		f_{0^-}	r_{0^-}	$\langle\xi^2\rangle$	α	f_{0^-}	r_{0^-}	$\langle\xi^2\rangle$	α	f_{0^-}	r_{0^-}	$\langle\xi^2\rangle$	α
0.0046	0.14	0.092	0.63	0.255	0.46	0.094	0.66	0.265	0.39	0.097	0.68	0.273	0.33
0.053	0.47	0.115	0.53	0.217	0.80	0.115	0.55	0.226	0.71	0.115	0.56	0.229	0.68
0.107	0.69	0.135	0.47	0.196	1.05	0.133	0.49	0.207	0.92	0.133	0.49	0.211	0.87
0.152	0.83	0.147	0.43	0.180	1.28	0.145	0.45	0.193	1.09	0.145	0.45	0.200	1.00

- lQCD results, QCDSF/UKQCD/CSSM Collaborations, $m_{0^-} = 0.47$ GeV, $f_{0^-} = 0.111(2)$ GeV, $r_{0^-} = 0.56(1)$ fm.
  Phys. Rev. D 96, 114509 (2017).
- lQCD results, HPQCD Collaboration, $m_{0^-} = 0.69$ GeV, $f_{0^-} = 0.128$ GeV, $r_{0^-} = 0.498(4)$ fm.
  Phys. Rev. D 96, 054501 (2017).

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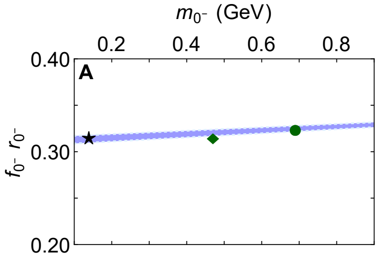
Mass dependence



- The charge radius decreases with increasing mass, i.e. the bound-states become more pointlike; and $r_{0^-} \propto 1/f_{0^-}$, up to $\ln(m_{0^-})$ -corrections.
- r_{0^-} is an intrinsic length-scale in these systems. The meson becomes a more highly correlated state as it diminishes. Hence, steadily increasing values of Q^2 are required to reach the domain upon which the factorised formula, Eq.(1), provides a useful guide to $F_{0^-}(Q^2)$.

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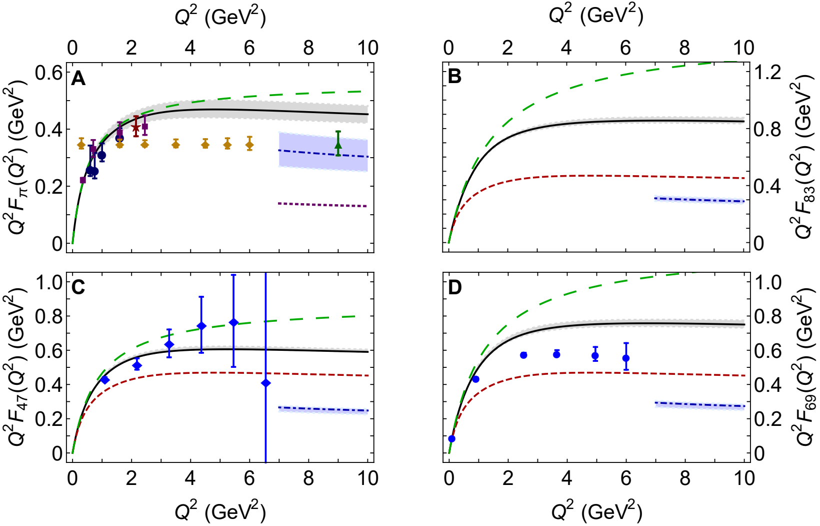

Proceeding anticlockwise from A → C → D → B, the mismatch increases between the direct calculation [solid black curve] and the result obtained using Eq.(1) with the consistent f_{0^-}, $\varphi(x;\varsigma)$ [dot-dashed blue curve].

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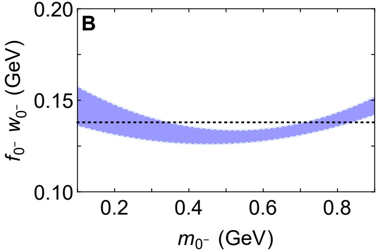


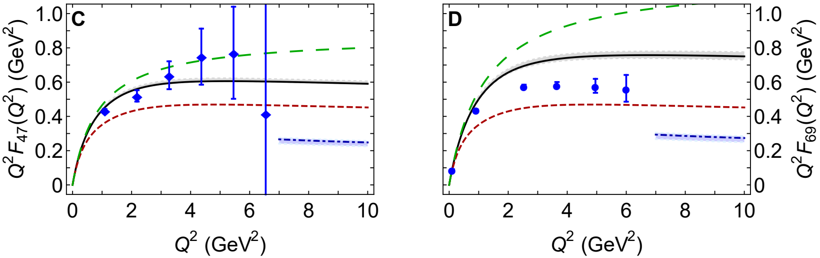
- The failure of the Eq.(1) prediction to increase in magnitude as quickly as the direct calculation is explained by a feature of the meson PDA's $1/x$-moment.
- $f_{0-}\omega_{0-}$ is roughly constant on the domain of meson masses considered: with $\omega = 0.5$ GeV, the integrated relative difference between the computed m_{0-} dependence and the mean value is just 3%.
- The prediction of the hard-scattering formula is weakly varying on $m_{0-} \in [0.1, 0.9]$ GeV, whereas the form factor itself rises steadily with m_{0-}, owing primarily to the decreasing radius [increasing f_{0-}] of the system.
- Therefore, the growing Higgs-generated current-quark mass drives away the domain whereupon the exclusive hard-scattering formula is applicable.
- The minimum of the $f_{0-}\omega_{0-}$ curve occurs in the neighbourhood of the s-quark current-mass, which is a consequence of the fact that $\varphi(x; Q^2) \approx \varphi^{cl}(x)$ in this area.

- The deviation from the single-pole vector-meson-dominance [VMD] prediction is a crucial prerequisite to entering the validity domain of Eq.(1).
- The direct calculation’s deviation from the trajectory defined by the VMD prediction also increases with m_{0^-}, and in each case the departure begins at a steadily decreasing value of Q^2.
- These effects owe to a shift to deeper timelike values of the ground-state vector-meson mass, so that this resonance contribution to the dressed-quark-photon vertex diminishes in importance for the meson-photon coupling, and parallel alterations in the pseudoscalar meson’s internal structure.

Comparing with lQCD results

lQCD in panel C: Phys. Rev. D 96, 114509 (2017).

lQCD in panel D: Phys. Rev. D 96, 054501 (2017).



- It seems that the lQCD results in panel C and D are mutually inconsistent: the lighter meson mass in panel C is associated with an elastic form factor which is larger in magnitude than that describing the internal structure of the heavier 0^--meson in panel D.
- Whilst the low-scale results from both studies match our predictions, only lQCD result in panel C is consistent with our calculations on the domain of larger Q^2.

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Conclusion

- We studied the electromagnetic form factors of pion-like mesons with masses $m_{0^-}/\text{GeV} = 0.14, 0.47, 0.69, 0.83$ on a spacelike domain that extends to $Q^2 \lesssim 10\text{GeV}^2$; and simultaneously computed the parton distribution amplitudes of each system.
- The form factor of the physical pion provides the best opportunity for verification of the leading-order, leading-twist factorised hard-scattering formula for such exclusive processes.
- The lower bound, Q_0, of the domain upon which the factorised hard-scattering formula is valid increases quickly with growing m_{0^-}.
- The lQCD results from HPQCD Collaboration and QCDSF/UKQCD/CSSM Collaborations are mutually inconsistent.
- The low-scale results from both lQCD studies match our predictions, only the QCDSF/UKQCD/CSSM Collaborations' result is consistent with our calculations on the domain of larger-Q^2.

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Thank you!

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