Perturbative Curci-Ferrari and its consequences in infrared QCD

M. Peláez[1], J. Gracey[2], U. Reinosa[3], J. Serreau[4], M. Tissier[5] and N. Wschebor[1]

[1]Instituto de Física, Facultad de Ingeniería, Udelar, Montevideo, Uruguay,
[2]University of Liverpool, United Kingdom,
[3]Centre de Physique Théorique, Ecole Polytechnique, France,
[4]APC, Université Paris Diderot, France,
[5]LPTMC, Université de Paris VI, France.

Emergent mass and its consequences in the Standard Model
21st September 2018, Trento, Italy

1. Curci-Ferrari model in Landau gauge
2. Preliminary two loop results
3. Quark propagator
4. Conclusions and perspectives

1. Curci-Ferrari model in Landau gauge
2. Preliminary two loop results
3. Quark propagator
4. Conclusions and perspectives

Landau gauge Euclidean QCD Lagrangian

- Computation of correlation functions analytically requires gauge fixing.
- Euclidean gauge fixed Lagrangian via Faddeev-Popov in Landau gauge

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i (\gamma_\mu D_\mu + M_i) \psi_i + \underbrace{ih^a \partial_\mu A^a_\mu}_{\text{Landau gauge}} + \underbrace{\partial_\mu \bar{c}^a (D_\mu c)^a}_{\text{Ghosts}}.$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,$$

$$D_\mu \psi = \partial_\mu \psi - i g A^a_\mu t^a \psi$$

$$(D_\mu c)^a = \partial_\mu c^a + g f^{abc} A^b_\mu c^c.$$

- **Standard** perturbation theory:
  - Asymptotic freedom [1973, Politzer, Gross, Wilczec; Nobel prize 2004]
  - Landau pole in the infrared

Faddeev-Popov ≠ Perturbation theory

➡ **Nonperturbative approaches:**

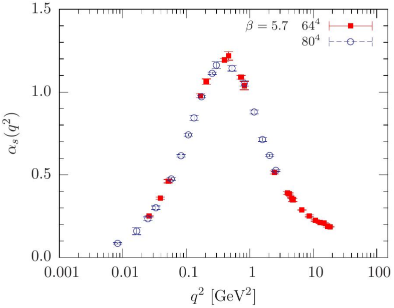
fRG [Cyrol, Fister, Mitter, Pawlowsky, Strodthoff ...]

Dyson-Schwinger equations [Aguilar, Alkofer, Binosi, Eichmann, Fischer, Huber, Papavassiliou, Roberts ...]

nPI, ...

Lattice simulations

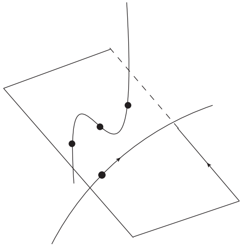
- Lattice simulations in the infrared:
  - moderate coupling constant
  - no evidence for a Landau pole
  - some kind of perturbation theory should be possible



[*Bogolubsky et al. Phys. Lett. B676 (2009)*]

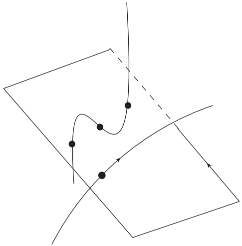
Problems fixing the gauge

- Faddeev-Popov procedure is not completely justified in the infrared.
- Presence of Gribov copies



Problems fixing the gauge

- Faddeev-Popov procedure is not completely justified in the infrared.
- Presence of Gribov copies



- Faddeev-Popov action needs to be extended or modified

  ➡ How to find the appropriate gauge-fixed Lagrangian?

  - Studies trying to restrict the integrals to a region without Gribov copies: Gribov-Zwanziger action and refined-Gribov-Zwanziger action. [*Zwanziger (1989)*, *Dudal et al (2008)*]
  - Phenomenological approach: include new operators to complete the gauge-fixing model and try to constraint their coupling

Lattice simulations:

- Several Gribov copies are found by lattice simulations.
- Fortunately, Lattice simulations are able to choose one Gribov copy for each orbit.

However

The equivalent action in the continuum is not known.

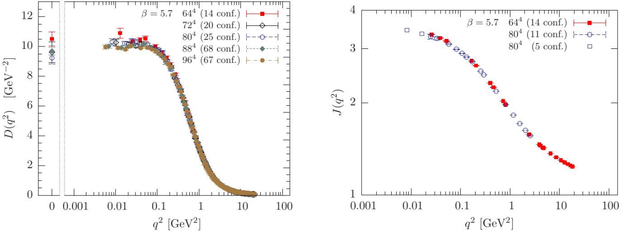
Lattice simulations:

- Several Gribov copies are found by lattice simulations.
- Fortunately, Lattice simulations are able to choose one Gribov copy for each orbit.

However

The equivalent action in the continuum is not known.

- Finite coupling constant.
- Massive gluons and massless ghosts.



[*I. L. Bogolubsky, E. M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, Phys. Lett. B676 (2009)*]

The model: Massive gluons (Curci-Ferrari)

What is the simplest Lagrangian that allows us to do perturbation theory reproducing lattice results?

- Let's try just adding a gluon mass term:

$$\mathcal{L} = \mathcal{L}_{\text{inv}} + ih^a\partial_\mu A^a_\mu + \partial_\mu \bar{c}^a (D_\mu c)^a + \mathbf{\frac{m^2}{2} A^a_\mu A^a_\mu}$$

[*Curci-Ferrari (1975)*]

- **This term breaks BRST symmetry**. [*Becchi, Rouet, Stora (1975) and Tyutin (1975)*] But it still has a modified-BRST symmetry which allows to prove renormalizability.
- It violates positivity ... but also lattice simulations do [Cucchieri, Mendes,Taurines Phys.Rev.D71 (2005)].

We would like to check

... if the perturbative analysis reproduces the lattice data

Renormalization Scheme

Infrared safe scheme:

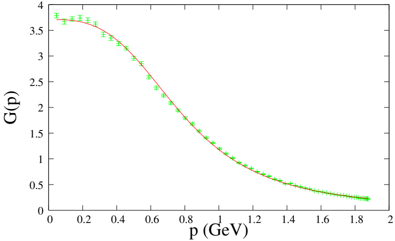
$$\Gamma^{(2)}_{AA}(p=\mu,\mu) = \mu^2 + m^(\mu),$$
$$\Gamma^{(2)}_{C\bar{C}}(p=\mu,\mu) = \mu^2,$$
$$Z_g\sqrt{Z_A}Z_c = 1,$$
$$Z_{m^2}Z_AZ_c = 1$$

As a consequence, the gluon and the ghost propagators are given explicitely in terms of the running parameters.

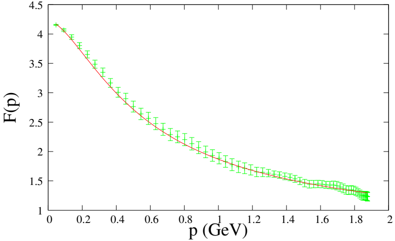
$$D(p) = \frac{g^2(\mu_0)}{m^4(\mu_0)}\frac{m^4(p)}{g^2(p)}\frac{1}{p^2+m^2(p)}, \qquad J(p) = \frac{m^2(\mu_0)}{g^2(\mu_0)}\frac{g^2(p)}{m^2(p)}$$

- $d = 4$

Gluon propagator



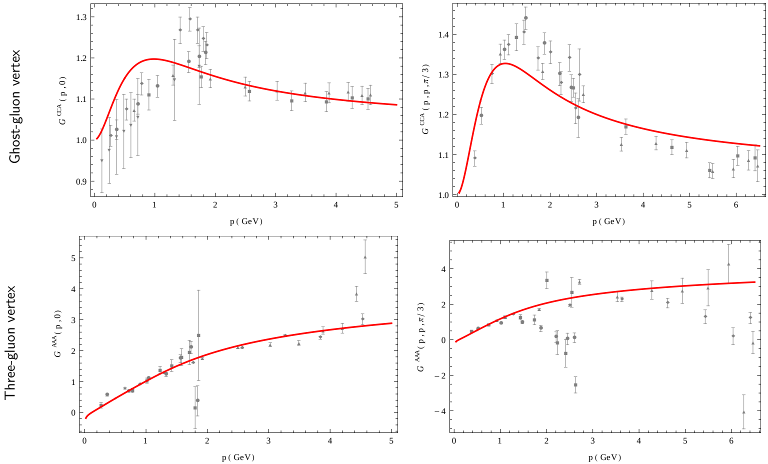
Ghost dressing function



$g(1\ \text{GeV}) = 7{,}5$ and $m(1\ \text{GeV}) = 0{,}77$ GeV

Lattice data from [*A. Cucchieri, A. Maas and T. Mendes, Phys.Lett. D77, 2008*]. Results from [*M. Tissier and N. Wschebor, Phys.Rev. D84, 2011*]

- **Without any extra fitting.**



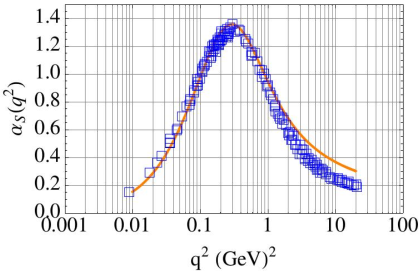
Configurations: gluon vanishing momentum and all equal momenta.

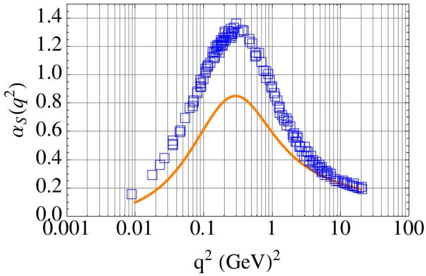
Lattice data from [*A. Cucchieri, A. Maas and T. Mendes, Phys.Lett. D77, 2008*]. Results from [*M. Peláez, M. Tissier and N. Wschebor, Phys.Rev. D88, 2013*]

- The three-gluon vertex diverges as $\log p$ in the infrared.

One loop pCF: alpha strong for $SU(3)$ and $d = 4$.

Fitting the IR **Fitting the UV**





Lattice data from [*A. G. Duarte, O. Olivera, P.J. Silva, Phys. Rev D94, 2016*]

Results from [*M. Tissier and N. Wschebor, Phys.Rev. D84, 2011*]

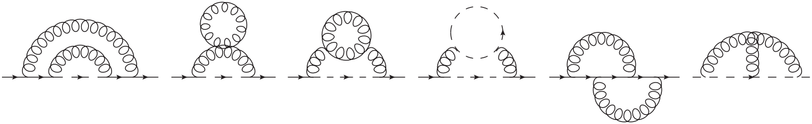
1 Curci-Ferrari model in Landau gauge

2 Preliminary two loop results

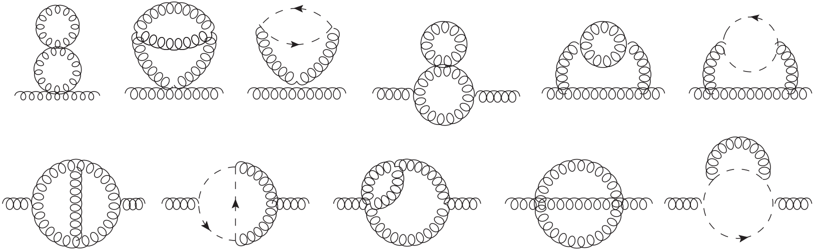
3 Quark propagator

4 Conclusions and perspectives

- Two loop diagrams for the ghost propagator



- Two loop diagrams for the gluon propagator



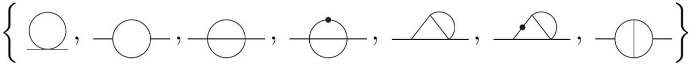
[JaxoDraw: A Graphical user interface for drawing Feynman diagrams, D. Binosi, L. Theussi]

- We use Laporta algorithm to decompose the two-loop two-point functions into master integrals

$$\Gamma^{(2)}_{AA}(p) = p^2 + m^2 + \sum_{D\in\mathcal{M}} \mathcal{R}_{AA}(D)\mathcal{I}(D)$$

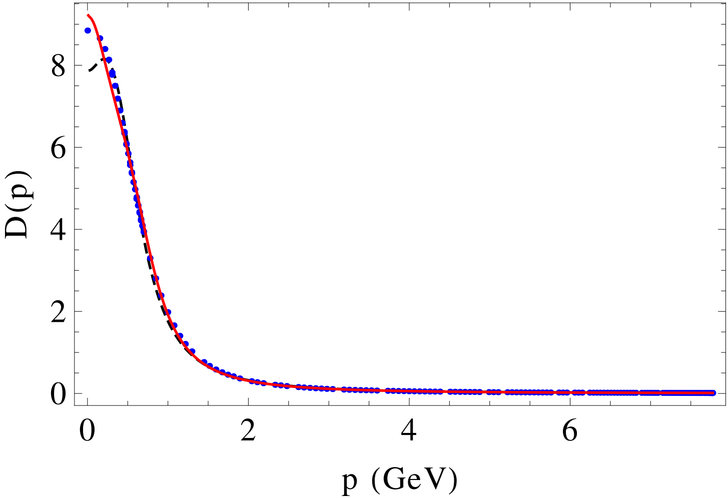
$$\Gamma^{(2)}_{C\bar{C}}(p) = p^2 + \sum_{D\in\mathcal{M}} \mathcal{R}_{C\bar{C}}(D)\mathcal{I}(D)$$

where $\mathcal{R}_{AA}(D)$ and $\mathcal{R}_{C\bar{C}}(D)$ are rational functions of p^2 and m^2, and $\mathcal{I}(D)$ is a master Feynman integral, with D among

$D \in \mathcal{M} =$ 

- We then evaluate each of the master integrals using the TsiL package. [https://www.niu.edu/spmartin/TSIL/]

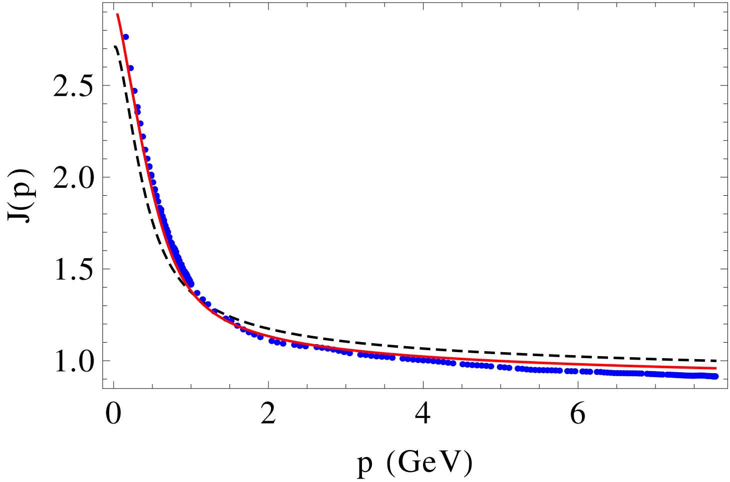
dashed: one-loop correction full line: two-loop correction



[J.A. Gracey, M. Peláez, U. Reinosa, M. Tissier, in preparation]

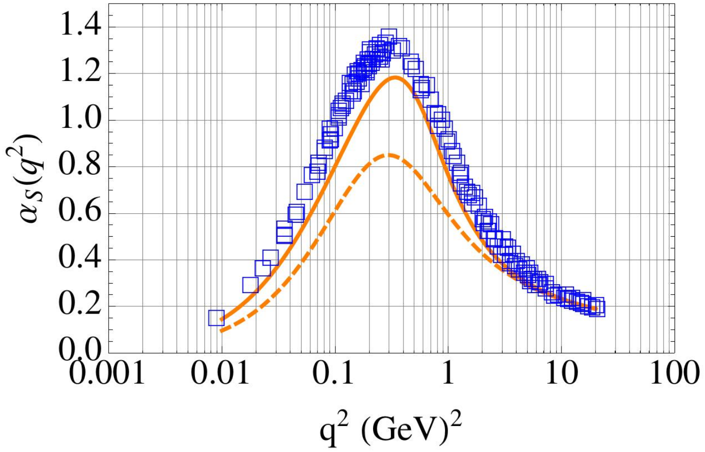
Two loop results: Ghost dressing function

dashed: one-loop correction full line: two-loop correction



[J.A. Gracey, M. Peláez, U. Reinosa, M. Tissier, in preparation]

Two loop results: Coupling constant



[J.A. Gracey, M. Peláez, U. Reinosa, M. Tissier, in preparation]

Perturbation theory within Curci-Ferrari

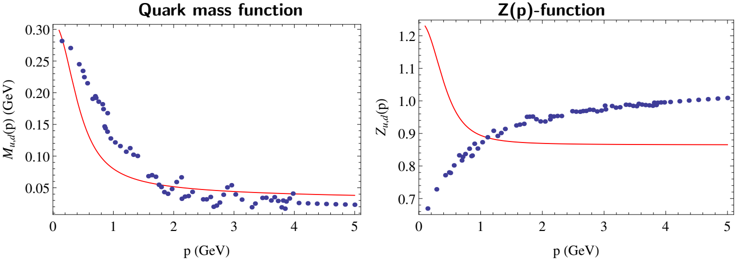
partial- Conclusions: Yang-Mills sector

- Yang-Mills quantities can be reproduced perturbately using Curci-Ferrari model.
- Curci-Ferrari model seems to encode the main effects of the complete Landau gauge-fixed Yang-Mills Lagrangian.

partial- Conclusions: Yang-Mills sector

- Yang-Mills quantities can be reproduced perturbately using Curci-Ferrari model.
- Curci-Ferrari model seems to encode the main effects of the complete Landau gauge-fixed Yang-Mills Lagrangian.

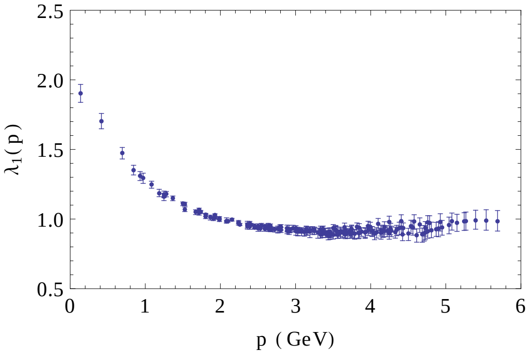
➡ However, in the **quark sector** results are not as good as Yang-Mills ones.



The points are lattice data of [*Bowman et al, Phys.Rev. D70 (2004)*] [*M. Peláez, M. Tissier, N. Wschebor, Phys. Rev D90 (2014)*].

Quark-Gluon coupling VS Ghost-Gluon coupling

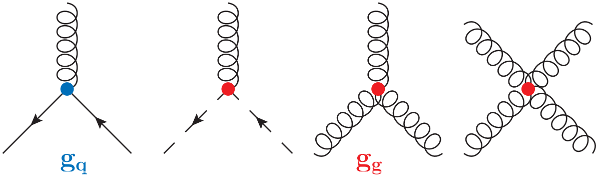
- Quark-gluon coupling constant not too small. $g_q(\mu) = g_g(\mu)\lambda_1(\mu)$



Data from [*Skullerud et al. JHEP* **0304**, *047 (2003)*]

Quark-Gluon coupling VS Ghost-Gluon coupling

- As the quark-gluon g_q and YM g_g running coupling constants are different in the infrared, we treat them separetly,



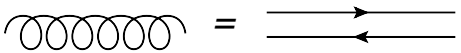
- g_g is considered as small parameter. Yang-Mills sector can be studied perturbately in the infrared.
- g_q is not a small parameter.

Large N_c limit

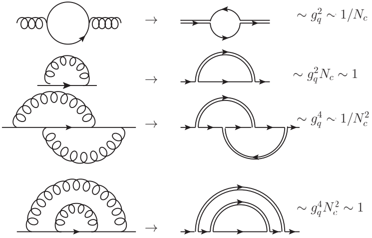
- Large N_c limit shows the same general features of QCD.

  [*G. 't Hooft, Nucl. Phys. B* **75**, *461 (1974). Witten, Nucl. Phys. B* **160**, *57 (1979)]*

In the large N_c limit, gluon propagators can be replaced by double color lines and



$g_q \sim 1/\sqrt{N_c}$



Organizing the systematic expansion:

- How to implement the systematic expansion, ℓ-order improved expansion:
  - We write all diagrams until ℓ-loops
  - We count the powers of g_g and $1/N_c$
  - We also add higher loop order diagrams with the same powers of g_g and $1/N_c$.

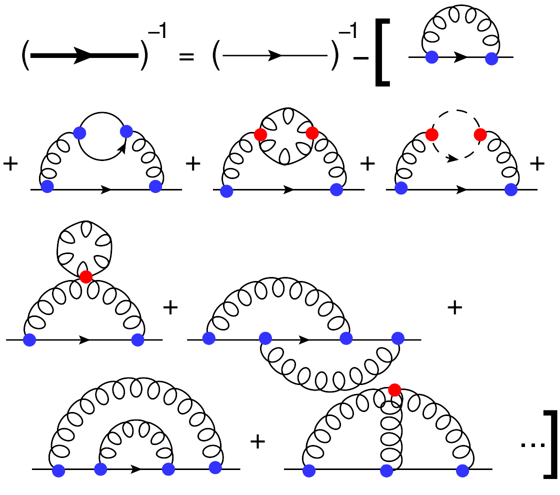
1 Curci-Ferrari model in Landau gauge

2 Preliminary two loop results

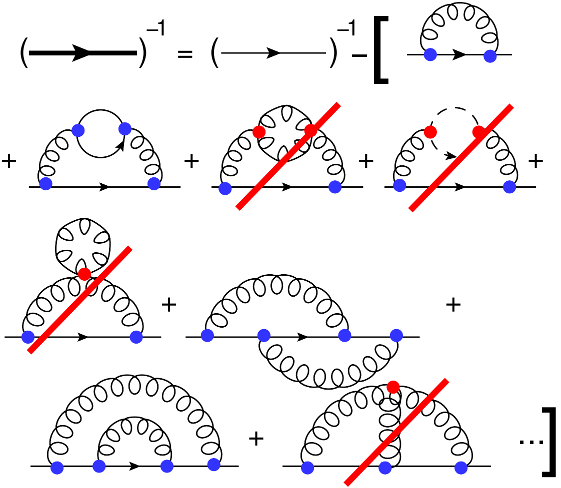
3 Quark propagator

4 Conclusions and perspectives

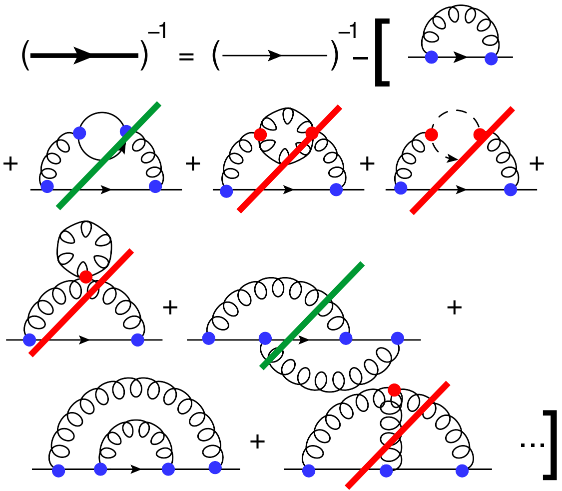
Quark propagator



Quark propagator

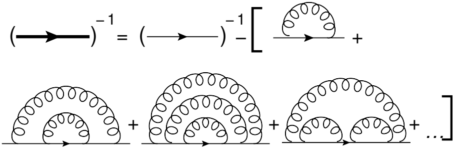


Quark propagator

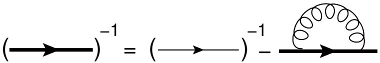


Rainbow equation

- Only Rainbow diagrams survive



- They can be resummed in:

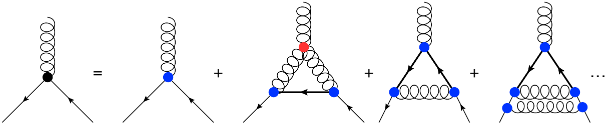


which is the well-known Rainbow approximation for the quark propagator.

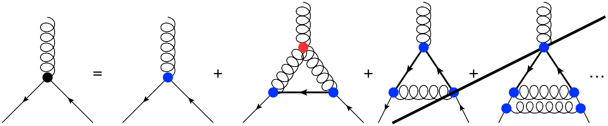
see e.g. [*Johnson et al, PRB (1964). Maskawa, PTP (1975). Atkinson et al, PRD (1988). Miransky et al, PRC (2004).*]

[Maris et al, IJMP (2003). Roberts et al,EPJST (2007). Eichman et al, PRC (2008).]

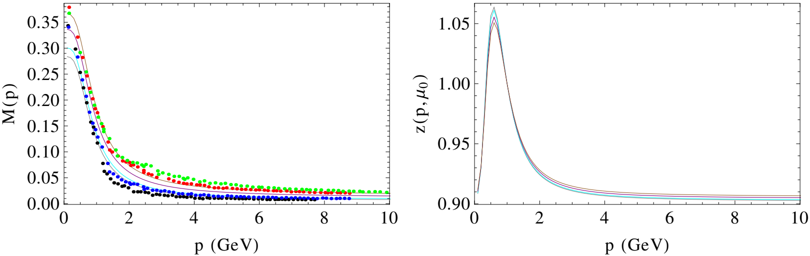
- Corrections for the quark-gluon vertex:



- Corrections for the quark-gluon vertex:



- One-loop-diagram with three-gluon vertex must be included.
- The quark propagator appears always in its full form.
- We define the quark-gluon coupling through $\lambda_1' = -\frac{1}{4g_B(d-2)} Im \sum \mathrm{Tr}(\gamma_\epsilon \Gamma_\mu P^\perp_{\mu\nu}(k) P^\perp_{\nu\rho}(r) P^\perp_{\rho\epsilon}(p))$ in the kinematic configuration corresponding to two equal and ortogonal quark-antiquark momenta (OTE).



M(p), with initial condition M(10GeV)=0.008, 0.01, 0.015, 0.02 GeV, y $g_g(10GeV) = 1{,}85$
[M. Peláez, U. Reinosa, J. Serrau, N. Wschebor, in preparation]

- We reproduce spontaneus chiral symmetry breaking.
- Same UV behaviour as in [Aguilar et al, PRD **83** (2011).]

1 Curci-Ferrari model in Landau gauge

2 Preliminary two loop results

3 Quark propagator

4 Conclusions and perspectives

Conclusions

To sumarize:

- **pCF** gives very accurate results for two and three-point correlation function in **Yang-Mills sectors**.
- However pCF does not reproduce the hole picture for quarks.
- We propose a **systematic expansion** scheme for QCD at low energy based on a **double expansion** in powers of the coupling strength g_g in the Yang-Mills sector of the theory and in powers of $1/N_c$.

Conclusions and perspectives

Conclusions

- At leading order, this scheme reproduces the well-known **rainbow approximation**.
- It allows for a systematic study of higher order corrections.
- We are able to implement a **consistent renormalization group** improvement of the rainbow equations that yields a better control of large logarithms.

Conclusions

- At leading order, this scheme reproduces the well-known **rainbow approximation**.
- It allows for a systematic study of higher order corrections.
- We are able to implement a **consistent renormalization group** improvement of the rainbow equations that yields a better control of large logarithms.

Thanks

Rainbow equation

- Rainbow equation represents a system of two coupled integral equations for the two scalar functions of the quark propagator.

$$S(p) = \left[-iA(p)\not{p} + B(p)\right]^{-1} = i\tilde{A}(p)\not{p} + \tilde{B}(p)\,,$$

where

$$\tilde{A}(p) = \frac{A(p)}{A^2(p)p^2 + B^2(p)}\,,$$
$$\tilde{B}(p) = \frac{B(p)}{A^2(p)p^2 + B^2(p)}\,,$$

- It is well-known that Rainbow resummation reproduces correctly the phenomenology of Spontaneous Chiral Symmetry Breaking.
- Let us stress that the main point here is that the Rainbow approximation is justified when considering g_g and $1/N_c$ as small parameters.

- We introduce the renormalization factors: $A^a_{\mu,\Lambda} = \sqrt{Z_A} A^a_\mu$, $\psi_\Lambda = \sqrt{Z_\psi}\psi$, $m^2_\Lambda = Z_{m^2} m^2$, $M_\Lambda = Z_M M$ and $g_{q,\Lambda} = Z_{g_q} g_q$.
- Renormalization condition: $S^{-1}(p = \mu_0, \mu_0) = -i\not{\mu}_0 + M(\mu_0)$
- The renormalized equations take the form:

$$A(p,\mu_0) = Z_\psi - Z^2_{g_q} Z^2_\psi Z_A g^2_q(\mu_0) C_F \int_{|q|<\Lambda} \frac{f(q,p)\tilde{A}(q,\mu_0)}{Z_A[(p+q)^2 + Z_{m^2} m^2(\mu_0)]},$$

$$B(p,\mu_0) = Z_\psi Z_M M(\mu_0) + Z^2_{g_q} Z^2_\psi Z_A g^2_q(\mu_0) C_F \int_{|q|<\Lambda} \frac{(d-1)\tilde{B}(q,\mu_0)}{Z_A[(p+q)^2 + Z_{m^2} m^2(\mu_0)]},$$

$$f(q,p) \equiv \frac{2p^2q^2 + 3(p^2+q^2)(p\cdot q) + 4(p\cdot q)^2}{p^2(q+p)^2}$$

Renormalization

- **At the order g_g^0 and $1/N_c^0$:**
  - No corrections for the gluon propagator: $Z_A \sim 1$ and $Z_{m^2} \sim 1$.
  - No corrections for the quark-gluon vertex: $Z_{g_q} Z_\psi \sqrt{Z_A} \sim 1$
- The equations can be simplified as

$$A(p,\mu_0) = Z_\psi(\mu_0) - g_q^2(\mu_0) C_F \int_q \tilde{A}(q,\mu_0) \frac{f(q,p)}{(p+q)^2 + m^2(\mu_0)},$$

$$B(p,\mu_0) = M(\mu_0) + g_q^2(\mu_0) C_F (d-1) \int_q \tilde{B}(q,\mu_0) \left(\frac{1}{(p+q)^2 + m^2(\mu_0)} - \frac{1}{(\mu_0+q)^2 + m^2(\mu_0)} \right)$$

Renormalization Group

- **Renormalization Group** equation for the quark propagator: $(\mu\partial_\mu - \gamma_\psi + \beta_{X_i}\partial_{X_i})S^{-1} = 0$
- The **solution** at different renormalization scales:

$$S^{-1}(p,\mu,X_i(\mu)) = z_\psi(\mu,\mu_0)S^{-1}(p,\mu_0,X_i(\mu_0)),$$

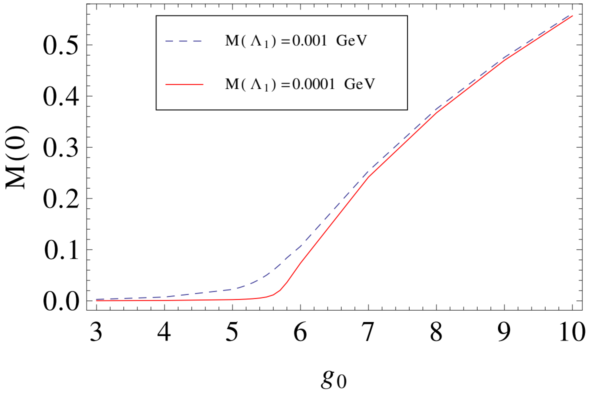
with $\log z_\psi(\mu,\mu_0) = \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\gamma_\psi(\mu')$

$$\begin{aligned} \mathrm{A}(\mathrm{p},\mu_0) &= z_\psi^{-1}(p,\mu_0) \\ \mathrm{B}(\mathrm{p},\mu_0) &= z_\psi^{-1}(p,\mu_0)M(p) \\ \mathrm{z}_\psi(p,\mu_0) &= Z_\psi(p)/Z_\psi(\mu_0) \end{aligned}$$

- The β-function for g_q takes the form:

$$\beta_{g_q} = \mu \frac{dg_q}{d\mu}|_{g_\Lambda} = g_q(\gamma_\psi + \frac{1}{2}\gamma_A) + g_q \mu \frac{d\lambda_1^\Lambda(ren)}{d\mu}$$
$$\beta_{g_g} = g_g(\gamma_C + \frac{1}{2}\gamma_A)$$

- γ_C is computed in its one loop form.
- In order to compute γ_A we include one loop diagrams in gluon propagator, considering full quark propagators in the diagram with a quark loop.
- λ_1, γ_A and γ_ψ are also coupled with $M(p)$ and $z_\psi(p)$.



Constituent quark mass $M(p=0)$ as a function of the coupling parameter g_0 for two values of the ultraviolet mass $M(\Lambda_1)$. The variation of g_0 is done by keeping Λ_{QCD} fixed.