Hadron structure in Continuum QCD

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Outline

1. **Background**
2. **Results**
   - Parton distribution amplitudes (PDAs) and form factors of light meson
   - PDAs for heavy system
   - Extracting weight function of wave function with Maximum Entropy Method
3. **Summary**

Quantum ChromoDynamics

Challenges in QCD:

QCD is a non-abelian gauge theory which describes the strong interaction between hadrons and quarks, gluons inside.

- *Asymptotic free behavior at high energy*
- *Dynamical chiral symmetry breaking (DCSB) and confinement at low energy*

  - **DCSB** is a complex phenomenon. It occurs in QCD because the **effective coupling runs**, becoming large at infrared momenta. However, the mechanisms are sophisticated and can be understood via the gap equation.
  - **Confinement** is not understood, it is the special structure of semi simple Lie algebra, and it is likely that **confinement is closely related to the dynamical gluon mass generation**.

Quantum ChromoDynamics

One can approach these problems by:

1. Using temperature and density (in medium)
2. **Studying the properties of strong-interaction bound-states (in vacuum)**
3. Connecting these areas of study

Dyson Schwinger Equations

- Dyson-Schwinger equations framework is a powerful continuum method on studying QCD.
- DSEs are the equations of motion for n-point Green function in quantum field theory derived from principle of least action (S action, ϕ field, J source):

$$\left\langle \frac{\delta S[\phi]}{\delta \phi(x)} \right\rangle_J = J(x)$$

Quark sector:

- **DCSB:** The running mass function describes the almost-massless pseudoscalar meson states and the massive baryon states.
- **Confinement:** The p^2-concave shape of quark propagator at low momentum violates spectral positivity.

Practical challenge in studying hadron properties:

The solution of DSEs is obtained in Euclidean space:

- That is where all the results of perturbation theory, the renormalisation group and lattice-QCD are known
- This enables reliable constraints on input and checks on output

One needs to return to the Minkowski space which is essential to study hadrons, for instance, the mass pole of hadron lies on the second sheet of real axis.

Three feasible ways to go:

- Fitting the DSEs solutions with algebraic formula (Nakanishi-like representation) which can be easily used for analytic continuation
- Brute-force computation
- Extracting the weight function of Nakanishi representation for BS wave function

Light Front

Light Mesons' PDAs and Form Factor

Benefits of the light-front:

- Quantum mechanics-like wave functions can be defined;
- Quantum-mechanics-like expectation values can be defined and evaluated;
- Parton distributions are correlation functions at equal LF-time x_+; namely, within the initial surface $x_+ = 0$ and can thus be expressed directly in terms of ground state LF wavefunctions.

The computation in light front is very natural in the framework of DSEs, since the light front wave function of meson is just the light front projection of meson's Bethe-Salpeter wavefunction.

Light Vector Meson PDAs

There are two independent vector meson's PDAs at leading twist:

- $\phi_\perp(x,\zeta)$ transversely polarized

$$f^T\phi_\perp(x) = \frac{\eta^{\lambda\mu}P_\nu}{P^2} tr_{CD}Z_2\int_{dq}^{\zeta}\delta(\frac{n\cdot q_\eta}{n\cdot P}-x)\sigma_{\mu\nu}\chi_\lambda(q;P)$$

- $\phi_\parallel(x,\zeta)$ longitudinally polarized

$$fM\frac{-(n\cdot P)^2}{P^2}\phi_\parallel(x) = tr_{CD}Z_2\int_{dq}^{\zeta}\delta(\frac{n\cdot q_\eta}{n\cdot P}-x)n\cdot\gamma n_\lambda\chi_\lambda(q;P)$$

where $\chi_\lambda(q;P) = S(q+\eta P)\Gamma_\lambda(q;P)S(q-\bar{\eta}P)$.

To perform the analytic continuation, it's more convenient to compute Mellin moments $< x^n >$:

$$< x^n >= \int dx\, x^n \phi(x)$$

- *The moments contain the factor $\left(\frac{n \cdot q_\eta}{n \cdot P}\right)^m$, the complex $n \cdot q_\eta$ results in oscillation.*
- *For light mesons, $n \cdot P$ is small, the oscillation is hard to control in the numerical calculations.*

Fit the quark propagator and BS amplitudes into the algebraic formula:

- It can be used for analytic continuation
- It will convert $\frac{n \cdot q_\eta}{n \cdot P}$ to $\frac{x(1+z)}{2} + y$, where $z \in (-1, 1)$ and $x + y \in (0, 1)$

A Simple Example

Fit the quark propagator and BS amplitude with the following form ($\Delta_M^\nu(z) = 1/(z+M^2)^\nu$):

$$
\begin{aligned}
S(p) &= [-i\gamma\cdot p + M]\Delta_M(p^2)\,,\\
\rho_\nu(z) &= \frac{1}{\sqrt{\pi}}\frac{\Gamma(\nu+3/2)}{\Gamma(\nu+1)}(1-z^2)^\nu\,,\\
\Gamma_\lambda(q;P) &= i\gamma_\lambda\frac{M^3}{f_\rho}\int_{-1}^{1}dz\,\rho_\nu(z)\Delta_M^\nu(q_{+z}^2)\,,
\end{aligned}
$$

After Feynmann parametrization, we could get the analytic form of distribution amplitude.

- *If $\nu = 1$, the ultraviolet behaviors of BS amplitudes act as $1/q^2$ which is the asymptotic behavior of QCD and we found the PDA go back to the asymptotic form 6x(1-x)*
- *If $\nu = 0$, it's a point-like structure and would obtain the constant amplitude*

Realistic Computation

Fit all Lorentz structures in the vector meson's BS amplitudes, even though some of them seem small in magnitude, they are crucial to get the correct decay constant and ultraviolet behavior.

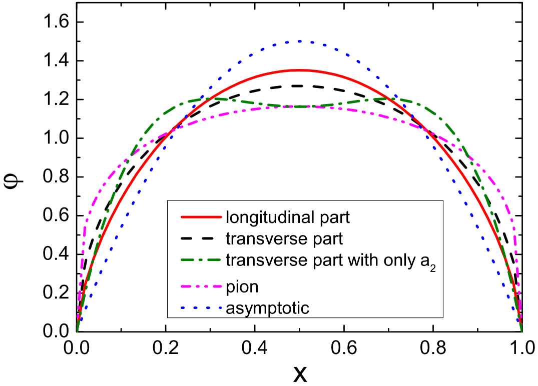
Fit the scalar and vector part of quark propagator respectively with pairs of complex poles.
For ρ meson, we need two pairs, while for ϕ meson one pair is enough.

Computing the moments and then reconstruct PDAs with Gegenbauer polynomials of order α:

$$\phi(x) \approx \phi_m(x) = N_\alpha [x(1-x)]^{\alpha - 1/2} [1 + \sum_{j=2,4,\ldots}^{j_{max}} a_j^\alpha C_j^\alpha (2x-1)]$$

ρ Distribution Amplitudes

We shown the comparison of ρ, π and asymptotic form[1].

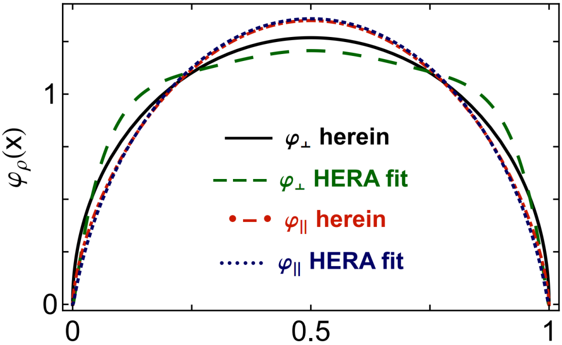


- Light mesons' PDAs are all broader than asymptotic form
- Double-humped shape caused by the incomplete expansion with $\alpha = 3/2$ basis

[1] F. Gao, L. Chang, Y. X. Liu, C. D. Roberts, et al. Phys. Rev. D 90 014011(2014).

ρ Distribution Amplitudes

Compared our predictions for ρ meson with those fitted to data



- agreement of the longitudinal amplitude
- data-fit oscillates mildly around our calculated result of transverse PDA, which indicates the higher order information loss

Form Factor

Within the same framework, we also obtained the Kaon electromagnetic form factor.

Form factors are of primary importance in hadron physics.

They provide Poincare-invariant information about the nonpointlike nature of QCD's observable bound-states, and the distribution of gluons and quarks within them.

Electromagnetic Form Factor

- The perturbative QCD (pQCD) has given the predictions for meson form factors.
- At leading-order and leading twist, perturbative QCD (pQCD) yields:

$$\exists Q_0 > \Lambda_{QCD} Q^2 F_P(Q^2) \overset{Q^2 > Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_P^2 w_{\phi_P}^2,$$

with Q^2 spacelike and

$$\begin{aligned} w_{\phi_P}^2 &= e_{q_1} w_{\phi_{q_1}}^2 + e_{\bar{q}_2} w_{\phi_{q_2}}^2, \\ w_{\phi_{q_1}} &= \frac{1}{3}\int_0^1 du \frac{1}{1-u}\phi_P(u), w_{\phi_{q_2}} = \frac{1}{3}\int_0^1 du \frac{1}{u}\phi_P(u), \end{aligned}$$

Electromagnetic Form Factor

Disagreement between experiment and supposed pQCD theory:

- At $Q^2 = 4$ GeV^2, the pQCD prediction yields this value to be $Q^2 F_\pi(Q^2 = 4\text{GeV}^2) = 0.15$
- the empirical value at $Q^2 = 2.45$ GeV^2: $0.41^{+0.04}_{-0.03}$.

This mismatch for pion form factor has been clarified via DSEs:

The PDA at large energy is still far away from the asymptotic form.

Electromagnetic Form Factor

The electromagnetic form factor at leading order in the systematic and symmetry-preserving DSE truncation scheme is given by:

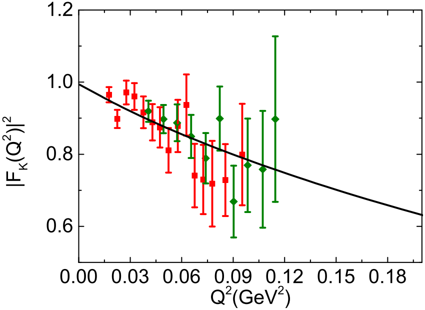
$$K_\mu F(Q^2) = N_c tr_D \int \frac{d^4k}{(2\pi)^4} \chi_\mu(k+p_f, k+p_i) \times \Gamma(k_i; p_i) S(k) \Gamma(k_f; -p_f),$$

where Q is the incoming photon momentum, $p_{f,i} = K \pm Q/2$, $k_{f,i} = k + p_{f,i}/2$.

- *$S(k)$ and $\Gamma_K(k;P)$ are computed in Euclidean space in the framework of DSEs*
- *Employ the algebraic formula for these quantities.*

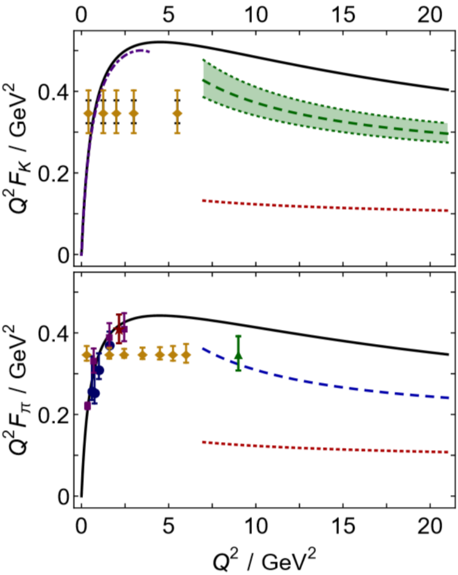
Kaon Electromagnetic Form Factor

We firstly compared the predicted form factor with the experimental data at low energies.



- Obtained result:
  $r_K^2 = 0.36$ fm^2
- Empirical value:
  $r_K^2 = 0.34 \pm 0.05$ fm^2

Kaon Electromagnetic Form Factor



- Consistent with the previous DSE computation up to 4 GeV^2.
- New result has a maximum at $Q^2 \sim 5$ GeV^2.
- Consistent with the hard scattering formula.

Figure: *(red dashed curve)*—previous

Kaon Electromagnetic Form Factor

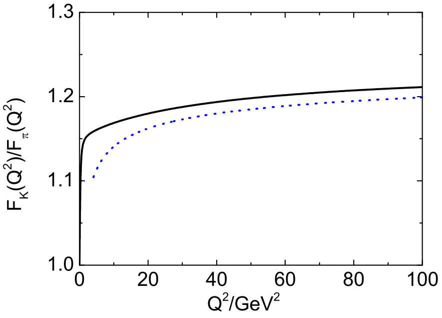


Figure: (*dotted curve*)— obtained using the PDAs at 4 GeV^2 and its evolution equation.

- Increase as the transferred energy increases
- Conformal limit is $f_{K^+}^2/f_\pi^2 = 1.4$

Flavour Separation

The form factor for K^+ could be separated into
$F_{K^+} = \frac{2}{3}F_{u\bar{s}u}(Q^2) + \frac{1}{3}F_{u\bar{s}\bar{s}}(Q^2)$

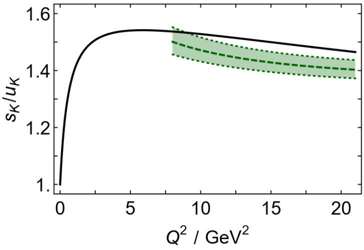
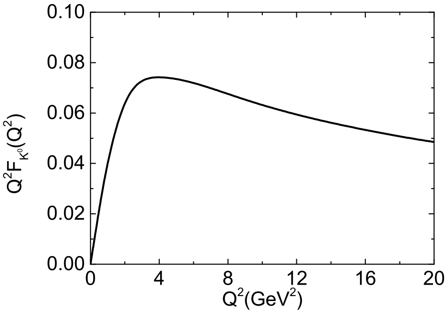


Figure: (*dotted curve*)—obtained using the PDAs at 4 GeV^2 and its evolution equation.

- At large energy, the quark mass scale is negligible, and thus the ratio tends to be 1.
- Crucial observation, both DSE and pQCD agree that the s-quark contribution never exceeds 1.5-times the u-quark contribution.

Flavour Separation

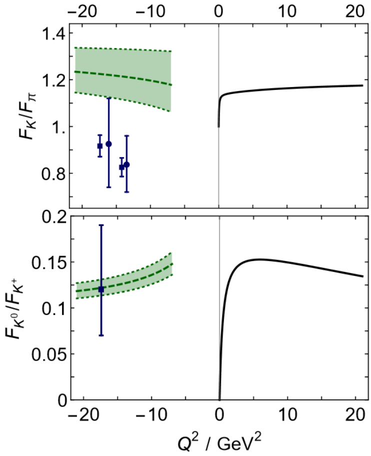
The form factor of K^0, $F_{K^0} = -\frac{1}{3}F_{d\bar{s}d}(Q^2) + \frac{1}{3}F_{d\bar{s}\bar{s}}(Q^2)$, if neglecting the mass difference of u and d quark, the form factor of K^0 could be directly computed:



- $F_{u\bar{s}\bar{s}} > F_{u\bar{s}u}$, which leads to a negative squared charge radius for K^0. $r^2_{K^0} = -0.042\ \text{fm}^2$
- Empirical value: $-0.054 \pm 0.026\ \text{fm}^2$

Flavour Separation

Timelike prediction with hard scattering formula compared with experiments:



- Consistent for the ratio of F_{K^0}/F_{K^+}
- Disagreement for F_K/F_π with experiment data lower than one.

Our results and their implications:

Vector mesons PDAs:

- Confirmed dilated, concave character of ground-state PDAs first observed in connection with pion
- Established an ordering for the light-front size of hadrons

Kaon Form Factor:

- First QCD-connected prediction of kaon electromagnetic form factors on the entire domain of spacelike Q^2
- Direct and favorable comparison with pQCD's hard scattering formulae on $Q^2 > 10\text{GeV}^2$.
  - Including results on the flavor-separation
  - Using the consistently calculated PDAs
- Kaon results have prompted experimentalists at JLab to explore capacity of JLab12 to test these predictions

PDAs for heavy system

In the theory of strong interactions, the cross-sections for many hard exclusive hadronic reactions can be expressed in terms of the leading twist PDAs of the hadrons involved.

The cross section for $B \to \pi\pi$ is:

$$< \pi(p')\pi(q)|Q_i|\bar{B}(p) >= f^{B\to\pi}(q^2)\int_0^1 dx T_i^I(x)\Phi_\pi(x)$$

$$+\int_0^1 d\xi dx dy T_i^{II}(\xi, x, y)\Phi_B(\xi)\Phi_\pi(x)\Phi_\pi(y)$$

PDAs for Heavy System

In the calculation of PDAs, we've found that the reason that the solution of DSEs in Euclidean space cannot be directly used is the oscillator $(\frac{n\cdot q_\eta}{n\cdot P})^m$.

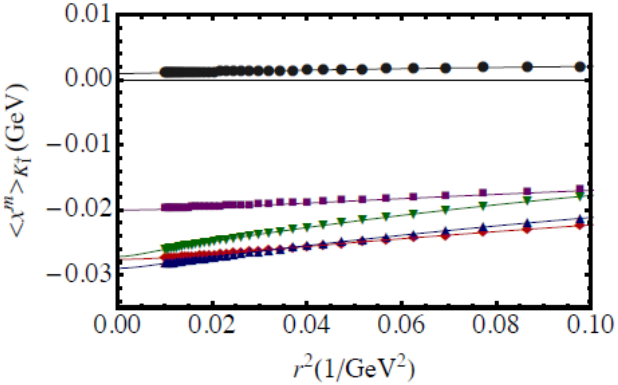
- If meson's mass M is large enough, $n \cdot P$ serves as a damping factor. So the oscillation can be well controlled.
- For heavy mesons, the brute-force computation directly from numerical solution in Euclidean space is practical.[1,2]

[1] M. H. Ding, **F. Gao**, L. Chang, etc, Phys. Lett. B **753**, 330 (2016).
[2] B.L. Li, L. Chang,**F. Gao**, etc, Phys.Rev. D **93** 114033 (2016).

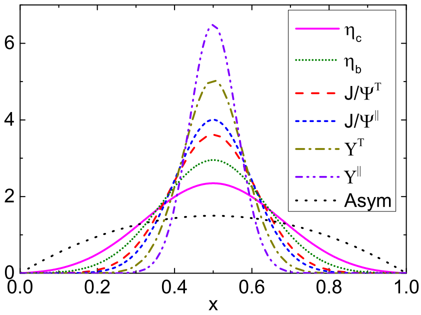
Brute-force Computation

We added a factor $1/(1+k^2r^2)^m$ *in the integration of* d^4k *for each moment* $< 2x-1 >^{2m}$ *to diminish the oscillation*

- Compute moments as a function of r
- Extrapolate to r=0



We then obtain the PDAs after fitting the moments.

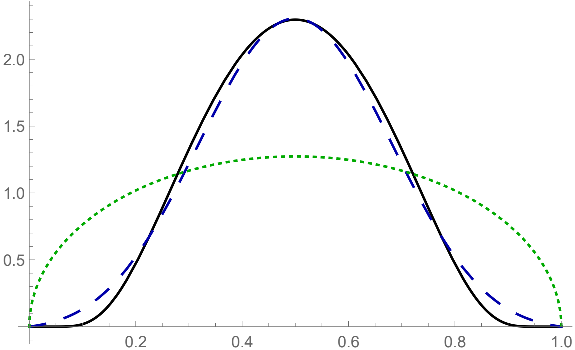


- For the same quark mass, the different polarization affect meson's amplitudes
- As the quark mass goes larger, the PDAs tend to be δ function

$$\phi_{\Upsilon\|} <_N \phi_{\Upsilon\perp} <_N \phi_{J/\Psi\|} <_N \phi_{J/\Psi\perp} <_N \phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi_{asymptotic}$$

Comparison with AdS/QCD Results

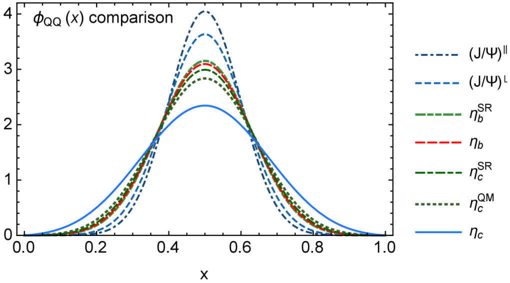
PDAs for η_c meson between DSE and AdS/QCD:



- Setting $Mc(AdS/QCD) = 1.1 GeV$
- In DSE, the current quark mass is $m_c = 1.21$ GeV.
- The $< 1/x >$ moments differ only by 5%

Comparison with Other Results

We compared our results with those using sum rules and light front QM:



The approximation in other calculations bring in wrong results:

- *For ϕ_{η_b}, other models and ours are consistent*
- *For charmonium, the difference is big. For example, $\phi_{\eta_c} \approx \phi_{J/\Psi\|} \approx \phi_{J/\Psi\perp}$ is not true in our calculation.*

Critical Mass Scale

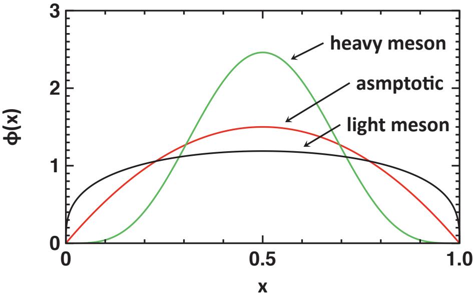
PDAs for light meson:

- Broader than the asymptotic form $\phi^{asy}(x) = 6x(1-x)$;
- The broadest shape of PDA, $\phi(x) = constant$, means the meson is point-like.

PDAs for heavy meson:

- Narrower than the asymptotic form $\phi^{asy}(x) = 6x(1-x)$;
- The narrowest shape of PDA, $\phi(x) = \delta(x - 1/2)$, means the meson is like a two-static-particle system.

Critical Mass Scale



There must exist a critical mass at which $\phi(x) = \phi^{asy}(x)$

	φ_{PS}	$\varphi_{V,\perp}$	$\varphi_{V,\parallel}$
m_{cri}	0.15 GeV	0.13 GeV	0.12 GeV

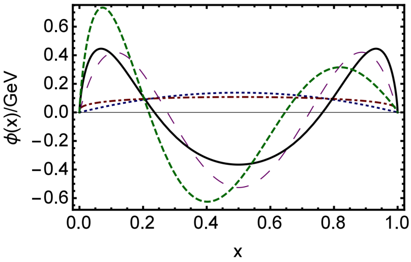
The critical mass typically lies just above the s-quark mass.

PDAs of Excited States

- In the chiral limit, the decay constant of excited states becomes 0 owing to the relation:

$$f_{PS} M_{PS}^2 = 2m_q \rho_{PS}$$

  The zeroth moment of its PDA becomes 0, PDAs become negative.
- The PDAs for excited states π_1 (dark solid) and K_1 (green dashed) compared with those for ground states:



Indicates that PDAs of n-th radial excited states contains 2n zeros, which is similar to the radial wave function in the quantum mechanics.

Our results and their implications:

- First QCD-connected computation of the pointwise behavior of quarkonium PDAs.
  - Confirms expectations regarding the static limit
  - Demonstrates the c-quark systems are far from that limit. This means NRQCD cannot be expected to be reliable for c-quark systems.
- First to put this in print and compute the value of critical mass.
  - Lies in the neighborhood of the s-quark current-mass.
  - Indication that no expansions in the s-quark mass can be reliable (e.g., ChPT) because it defines a transition boundary for internal hadron dynamics.
- The first symmetry-preserving computation, which should serve as a benchmark against which any model should be measured.

Extracting weight function with MEM

The Bethe-Salpeter wave function' integral Nakanishi representation is:

$$\Phi(k, P) = \int_{-1}^{1} dz \int_{0}^{\infty} d\gamma \frac{g(\gamma, z)}{(k^2 + zk \cdot P + \frac{1}{4}P^2 + M^2 + \gamma)^3}$$

- If we know the information of weight function $g(\gamma, z)$, we can obtain the light front information.

Maximum Entropy Method

The Maximum Entropy Method can be used to solve this ill-posed problem, which has been employed to extract the quark spectral density from quark propagator in Euclidean space.

The weight function can also be obtained through Maximum Entropy Method (MEM) when we have known the information of Bethe-Salpeter wave function and input some prior information.

Maximum Entropy Method

MEM is based on the Bayes' theorem

$$P[g|\Phi M] = \frac{P[\Phi|gM]P[g|M]}{P[\Phi|M]},$$

where $P[g|\Phi M]$ is the likelihood probability and $P[g|M]$ is the prior probability, $P[\Phi|M]$ is just a normalization constant.

normal distribution:

$$P[\Phi|gM] = \frac{1}{Z_L} e^{-L[g]},$$

$$L[g] = \sum_{i}^{N_{data}} \frac{(\Phi_{data}(k_{i,E}) - \Phi_g(k_{i,E}))^2}{2\sigma_i^2},$$

entropy:

$$P[g|M(\alpha,\omega)] = \frac{1}{Z_S} e^{\alpha S[g,\omega]},$$

$$S[g,\omega] = \int \left[g(\gamma,z) - \omega(\gamma,z) - g(\gamma,z)\log\frac{g(\gamma,z)}{\omega(\gamma,z)}\right].$$

"default model"$\omega(\gamma,z)$ chosen to be uniform distribution.

A Simple Example

We first analyzed a weight function model:

$$g(\gamma, z) = e^{-(\gamma+1)/(1-z^2)}.$$

We use this model to create the mock data of BS wavefunction, input this data and extract the weight function.

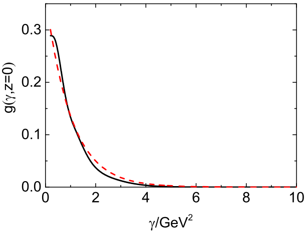


Figure: Extracted weight function (solid) via MEM compared with the original function (dashed)

A Simple Example

The corresponding leading twist two-particle light-front parton distribution can then be defined with the weight function:

$$\varphi(x) = \mathcal{N}\int_0^{\infty} d\gamma \Psi(\gamma, z) = \mathcal{N}\int_0^{\infty} d\gamma \frac{g(\gamma, 1-2x)}{\gamma + M^2 - x(1-x)M^2},$$

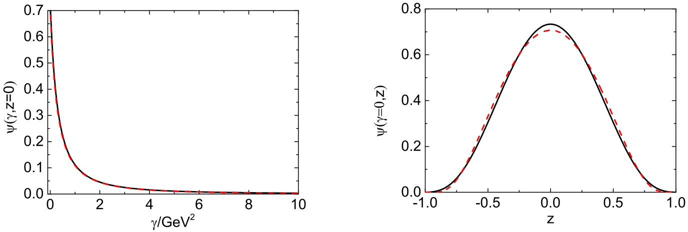
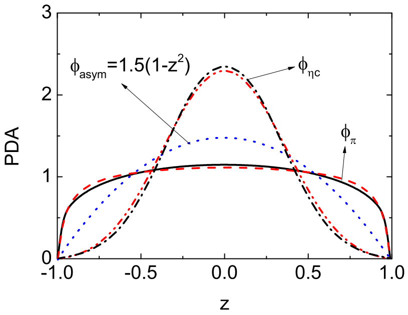


Figure: Obtained light front wave function (solid) compared with the analytical one (dashed)

Realistic Computation

We compared the PDA of pion and η_c with previous results



- Consistent with the results via other methods
- No assumption and robust for all mesons
- Useful for extracting of light front wave function.

Highlights:

1. Developed innovative algebraic and numerical methods for completing the analytic continuation of nonperturbative amplitudes in QCD from Euclidean space to Minkowski space.
   - The methods are applicable no matter what is the origin of the nonperturbative amplitudes, so long as the pointwise behavior is known.
2. Delivered the first QCD-connected unification of the parton distribution amplitudes of light-light and heavy-heavy mesons
3. The first QCD-connected prediction of kaon electromagnetic form factors on the entire domain of spacelike Q^2
   - Unifying the results with those from pQCD using the consistently-calculated PDAs

Immediate future

Adapt these methods for computation of GPDs and TMDs

- Beginning with the pion's light-front wave function
- Proceeding to the nucleon.

Thank You