



European Centre for Theoretical Studies  
in Nuclear Physics and Related Areas



# Emergent phenomena in strong dynamics

**Daniele Binosi**

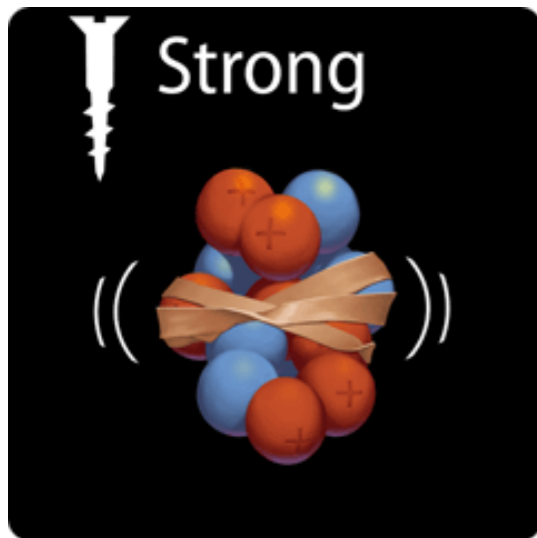
**ECT\* - Fondazione Bruno Kessler, Italy**

*Emergent mass and its consequences in the Standard Model*

Trento, Italy

September 17, 2018

# Standard Model: QCD



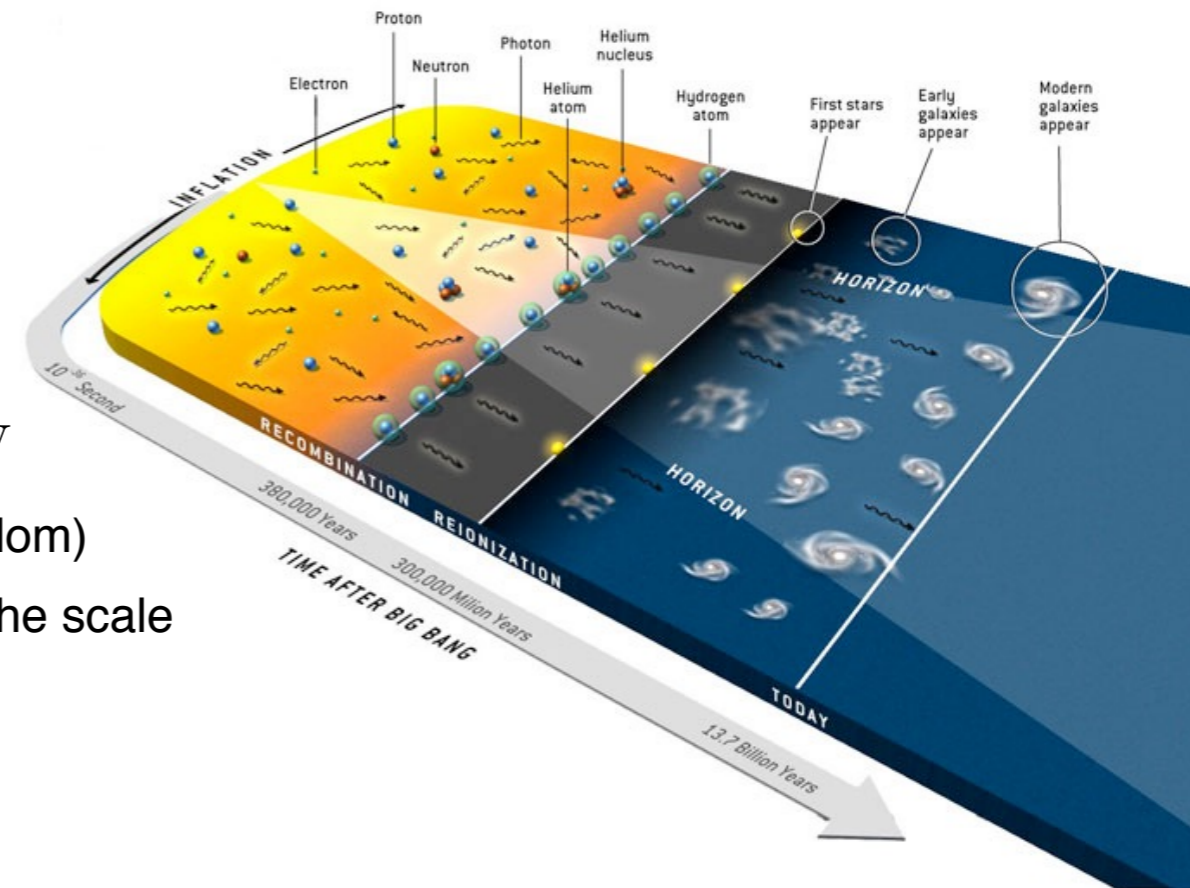
- **QCD started  $10^{-6}$  secs. after the Big-Bang:** it is the glue that binds us all, and understanding its dynamics has profound implications
  - **Explain how massless gluons and light quarks are confined** and bind together to form hadrons...
  - **...thereby explain the origin of ~98% of the mass in the visible Universe**

- **QCD is *likely* a perfect theory** nothing needs to be added or changed

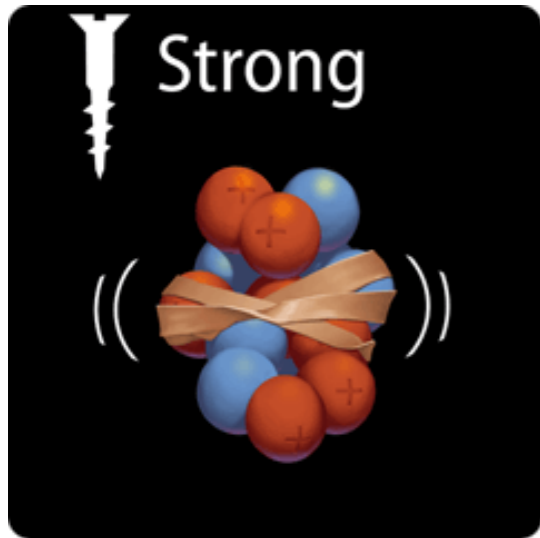
- **Validated** over an incredible energy range:  $0 \lesssim E \leq 8000 \text{ GeV}$
- **Unlikely to break down** at any energy scale (asymptotic freedom)
- **No intrinsic parameters**, just need one observable to define the scale  $\Lambda_{\text{QCD}} \simeq 200 - 300 \text{ MeV}$  QCD's “standard kilogram”

- **QCD is *a theory* not an effective theory**

- **However, it is innately nonperturbative** a priori no idea what such theory can produce



# QCD: degrees of freedom



- **QCD basic degrees of freedom:**  
matter (quarks); gauge (gluons)

$$q_f^i \begin{cases} \text{color} & i = 1, 2, 3 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

$$A_\mu^a \begin{cases} \text{color} & a = 1, \dots, 8 \\ \text{spin} & \epsilon_\mu^{\pm, 0} \end{cases}$$

- **QCD action:**  
encodes all the dynamics

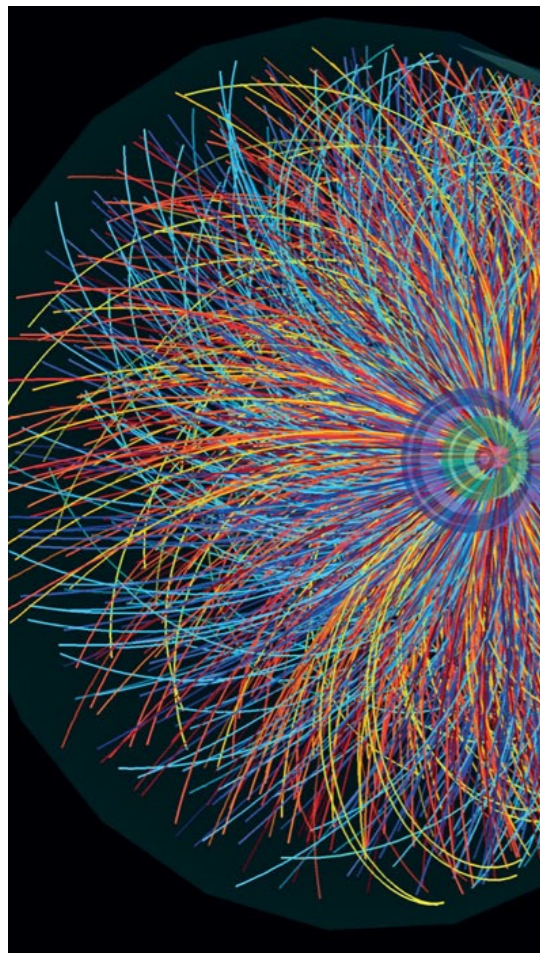
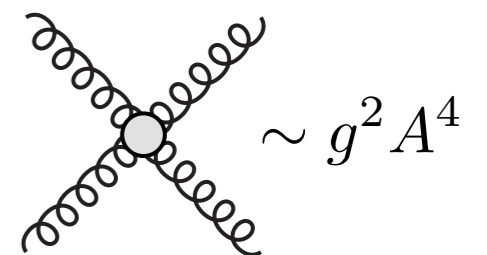
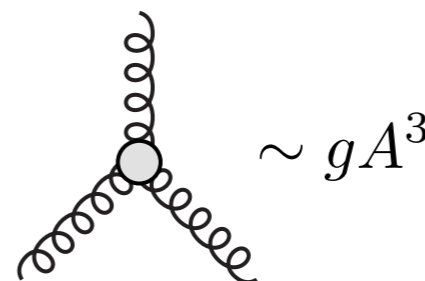
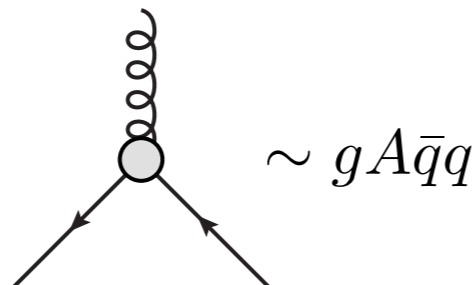
$$S_{\text{QCD}} = \int d^4x (\mathcal{L}_I + \mathcal{L}_{\text{GF+FPG}})$$

$$\mathcal{L}_I = \bar{q}_f^i (i\gamma^\mu D_\mu - m)_{ij} q_f^j - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

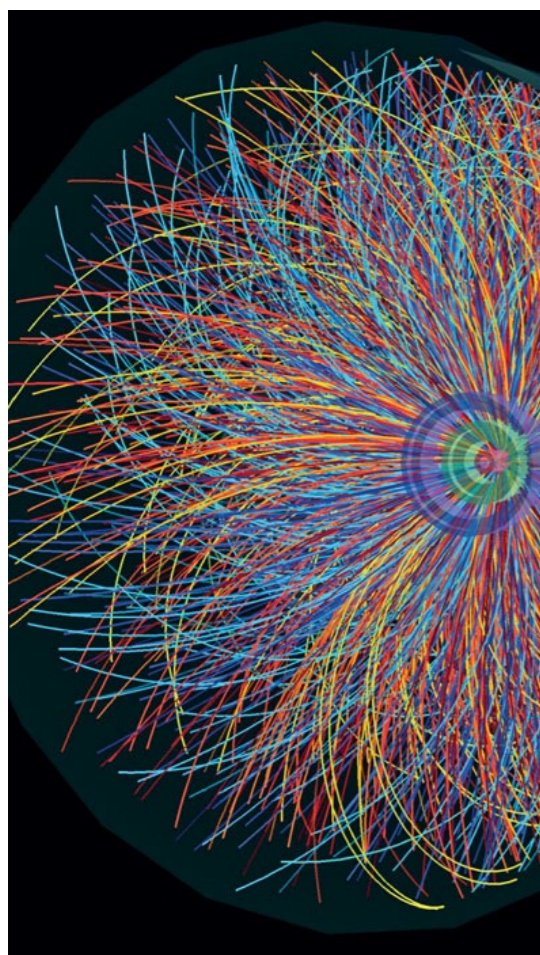
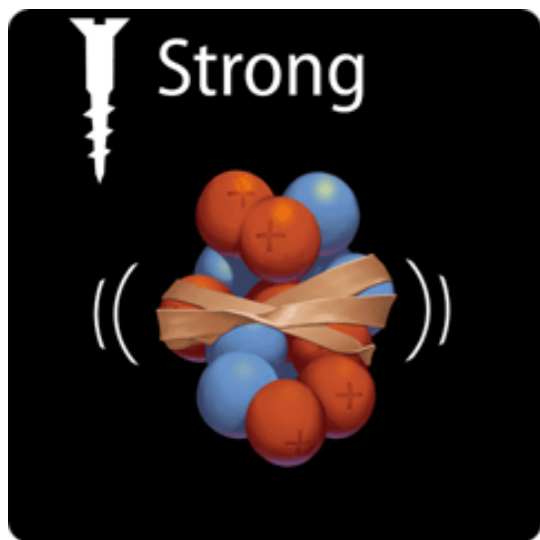
$$D_\mu = \partial_\mu - igA_\mu^a T^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$

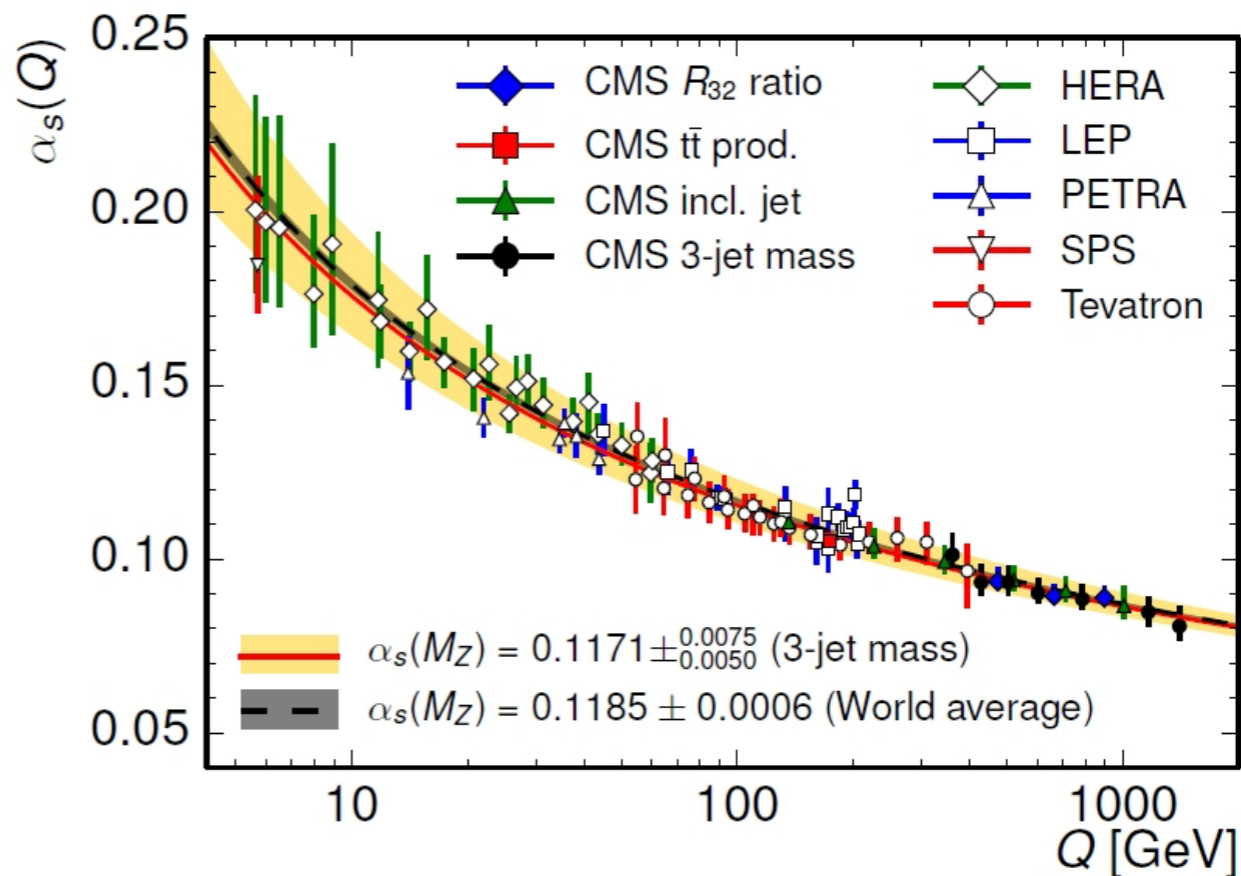
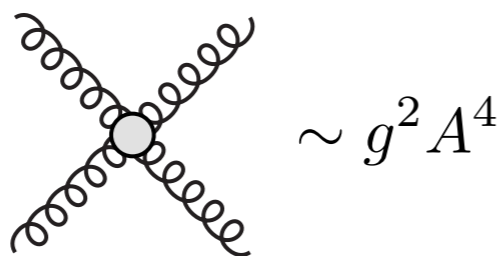
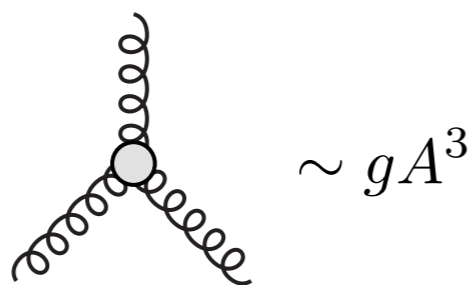
- **QCD (self-)interactions**  
dictate the theory's behavior



# QCD: asymptotic freedom

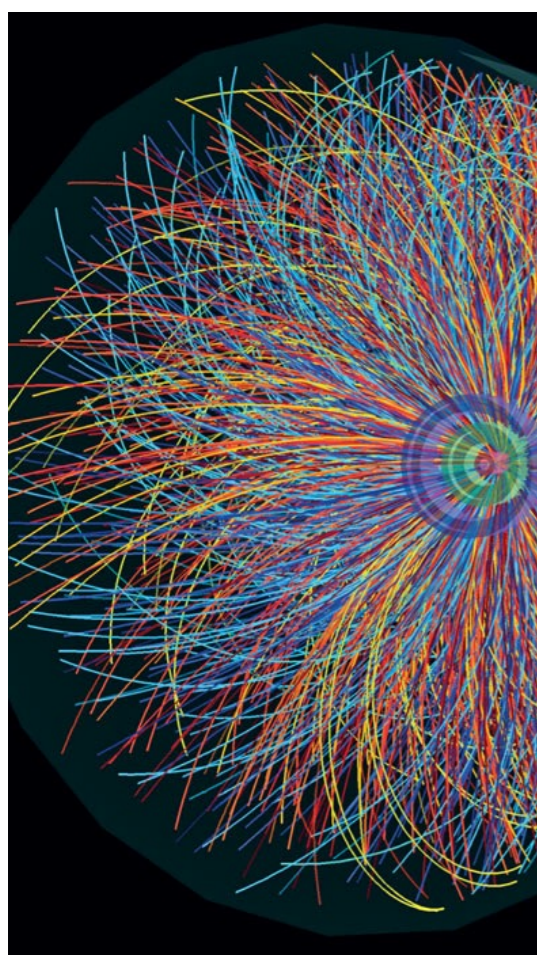
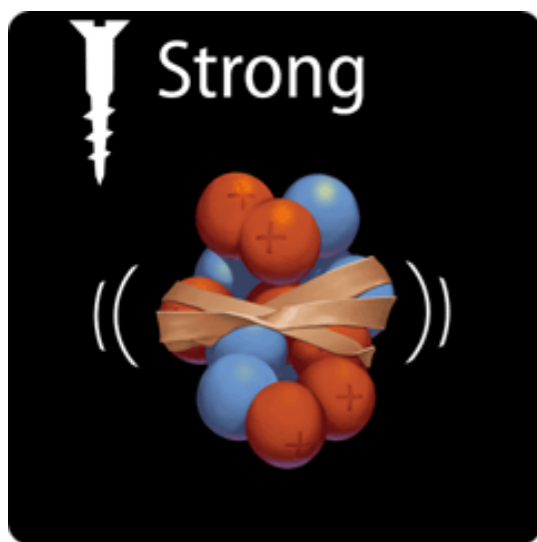



- **Gluon self-interaction runs**  
implies QCD is non perturbative and quark/gluons are confined



- **QCD interaction strength:**  
grows with separation between gluons and quarks
- **Typical scale of hadron physics**  
 $r \sim 2 \text{ fm}$       $\alpha \sim 0.5$
- **Perturbation theory breaks down at this distance**  
QCD is entirely nonperturbative across almost the entire proton's volume

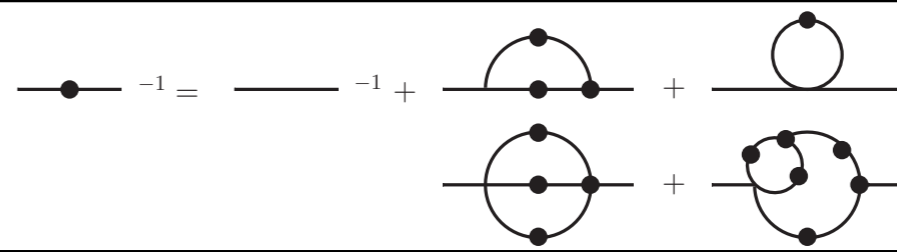
# QCD: Schwinger-Dyson equations



- **Understanding the origin of mass in QCD** is quite likely inseparable from understanding confinement
- **One possible way of addressing this are Schwinger-Dyson eqs** which are quantum eom of Green's functions
- **SDEs: nonperturbative, covariant, IR/UV, light/heavy quarks; but:** infinite system of coupled integral equations
  - **Needs reliable truncation schemes** plus requires a gauge to be chosen (Landau)
- **Concentrate on SDE for 2-point functions**  <sup>-1</sup>
  - **Three equations to be considered** quarks, gluons and ghosts
 
$$\mathcal{L}_{GF+FPG} = s(\bar{c}^a \mathcal{F}^a - \xi/2 \bar{c}^a b^a)$$
    - **Gauge fixing + FP ghost:** BRST exact, does not appear in the spectrum
- **Capture two emergent phenomena**
  - Dynamical mass generation
  - Confinement (?)

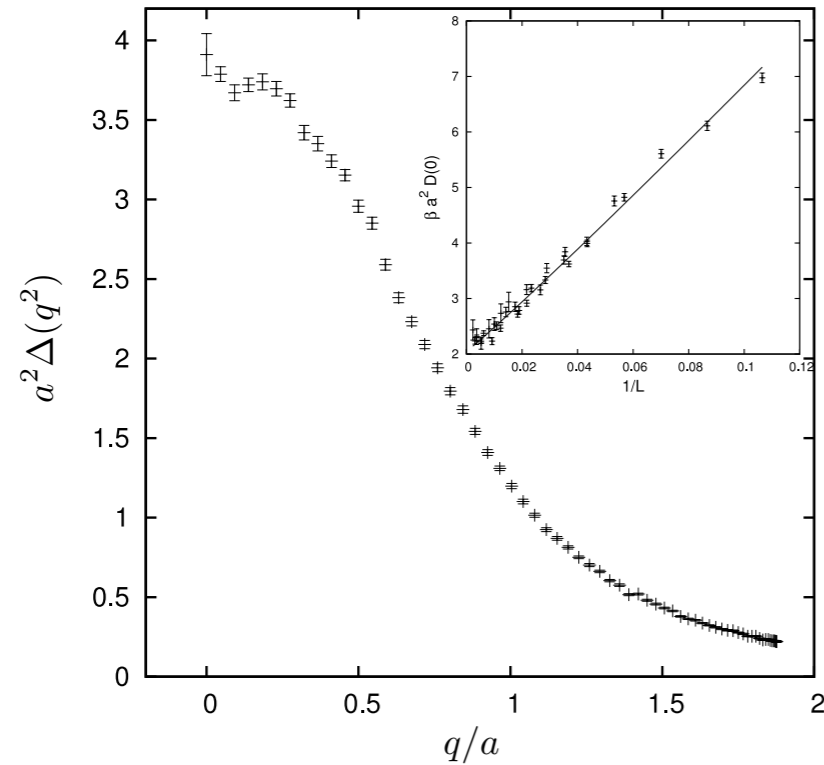


# What about gluons?

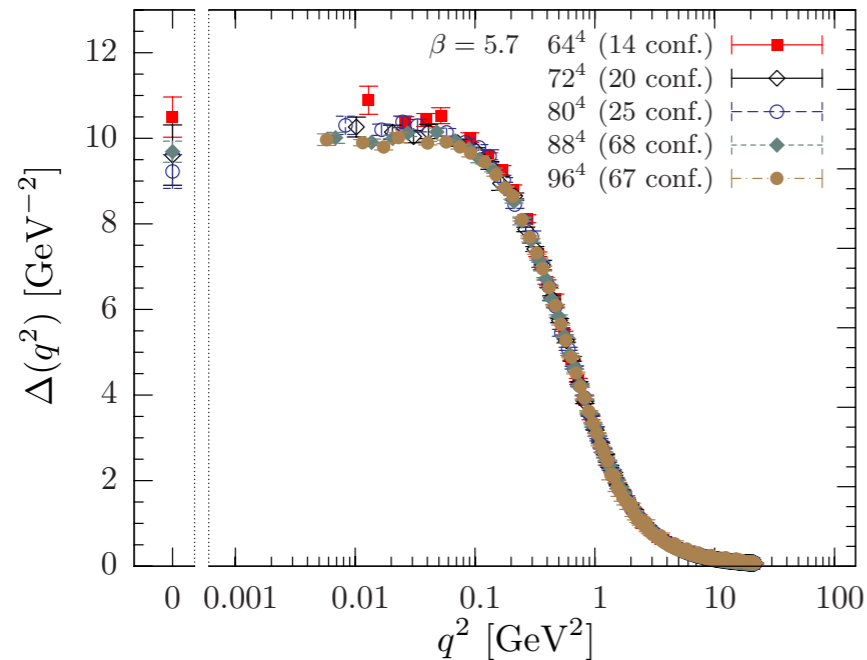


- **Landau gauge (quenched)**

Cucchieri, Mendes POS LATTICE (2007)

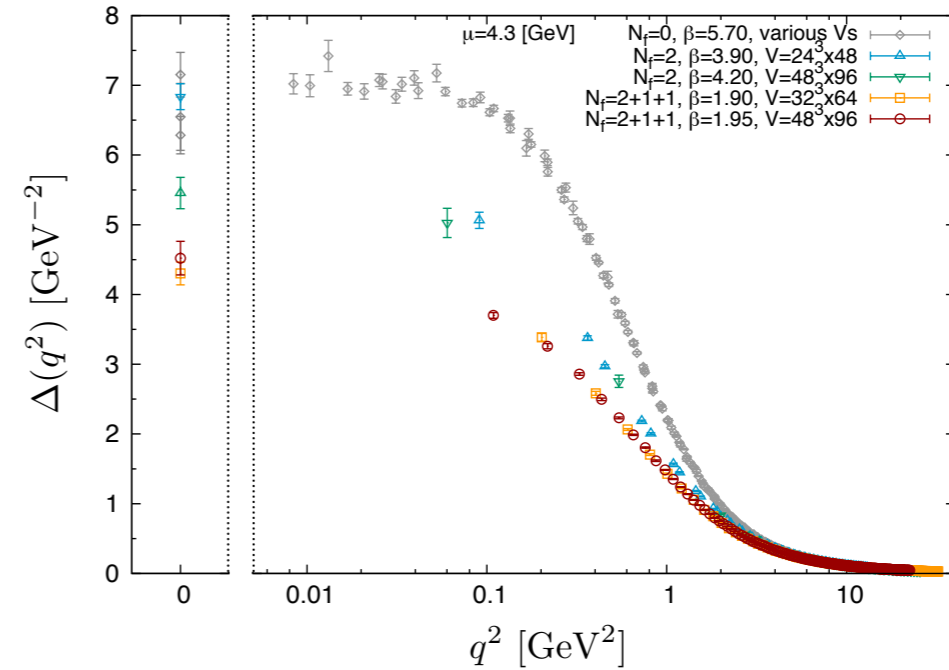


Bogolubsky, Ilgenfritz, Muller-Preussker, Sternbeck, PLB 676 (2009)



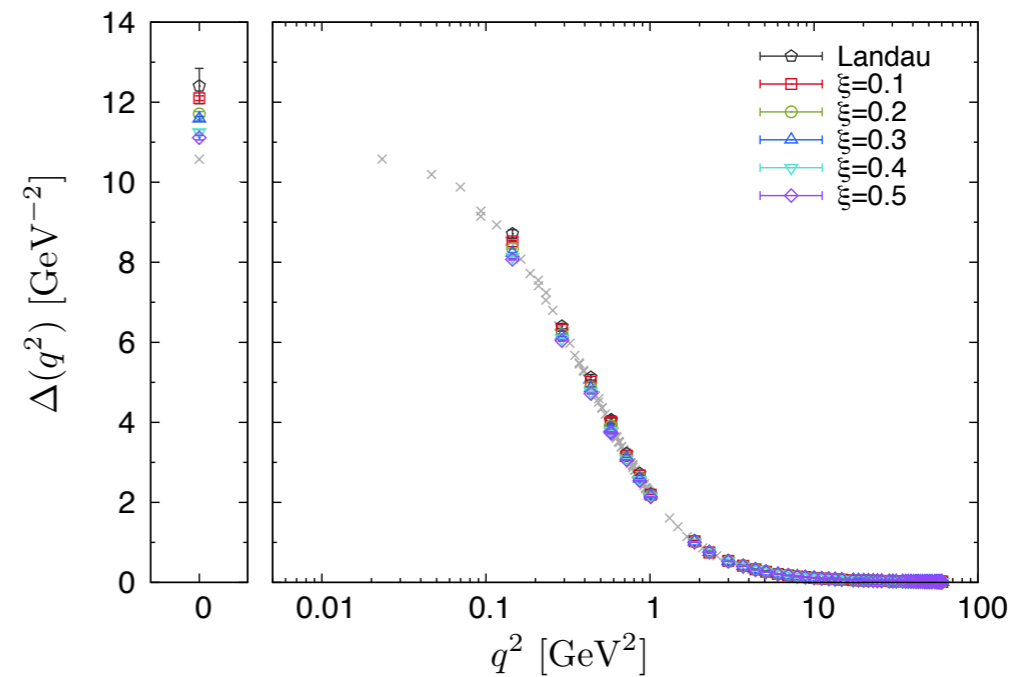
- **Landau gauge (unquenched)**

Ayala, Bashir, DB, Cristoforetti, Rodriguez-Quintero, PRD 86 (2012)

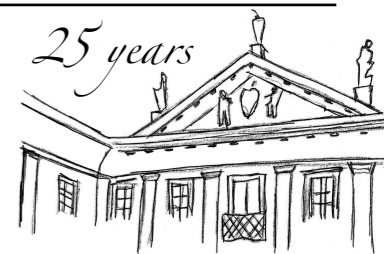
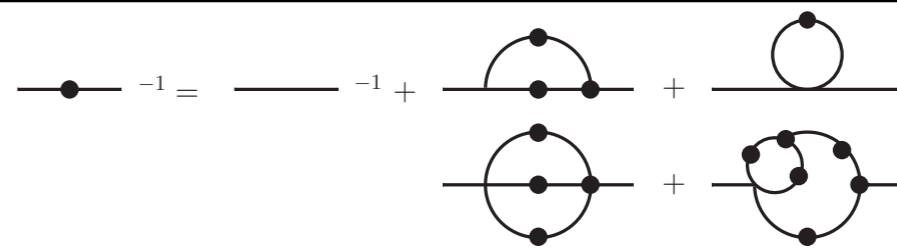


- **Linear gauges (quenched)**

Bicudo, DB, Cardoso, Oliveira, Silva, PRD 92 (2015)



# What about gluons?



- **IR Saturating/massive gluon propagator** challenging from a continuous perspective

- **Best understood within PT-BFM framework**

DB, Papavassiliou, PRD 77 (2008); JHEP 0811 (2008); PR 479 (2009)

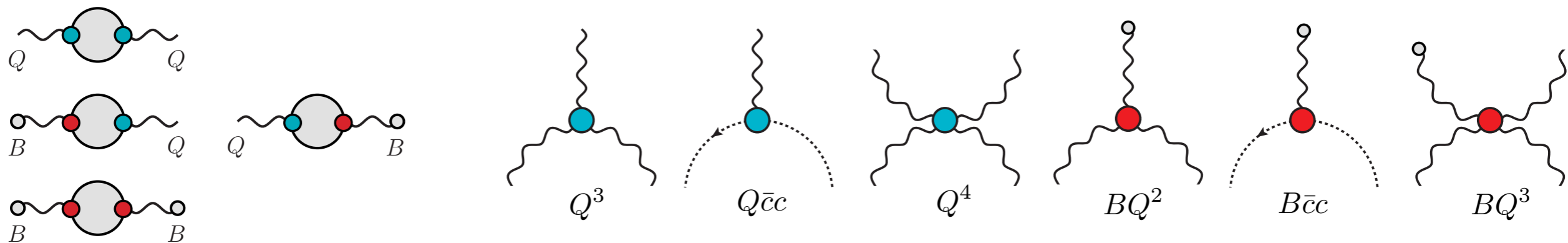
- **Split gauge field**

into background ( $B$ ) and quantum fluctuating ( $Q$ ) parts

Abbott, NPB 185 (1981)

- **Proliferation of Green's functions**

three possibilities in two-point gluon sector



- **Symmetry induced identities**

relate  $B$  and  $Q$  functions; in 2-point sector:

DB, Papavassiliou, PRD 66 (2002)

$$[1 + G(q^2)]^{-1} \times \text{Diagram } B \xrightarrow{q} = \text{Diagram } Q \xrightarrow{q}$$

- **$G$  function known**

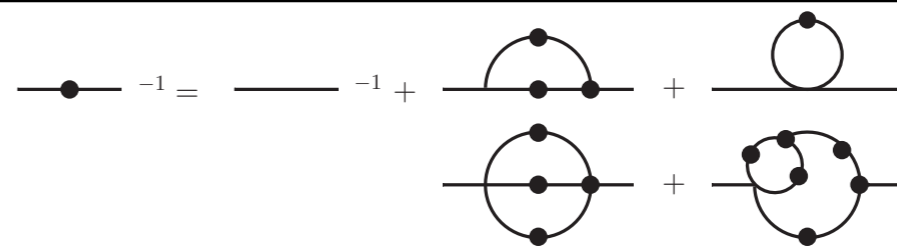
constrained by antiBRST symmetry

DB, Quadri, PRD 88 (2013)

$$1 + G(0) = F^{-1}(0)$$



# What about gluons?



- **Schwinger mechanism**  
propagator Dyson resums to

$$\Delta(q^2) = \frac{1}{q^2 [1 + \Pi(q^2)]}$$

Schwinger, PR 125 (1962)  
Schwinger, PR 128 (1962)

- If  $\Pi(q^2)$  has a pole at  $q^2 = 0$  the gauge boson becomes massive even if it was massless in the absence of interactions

- **Yes, but in QCD?**

- **PT-BFM theorem:** in any covariant gauge  $\Delta(0) = 0$  in the absence of vertex non-analyticities

Aguilar, DB, Figueiredo, Papavassiliou, PRD 94 (2016)

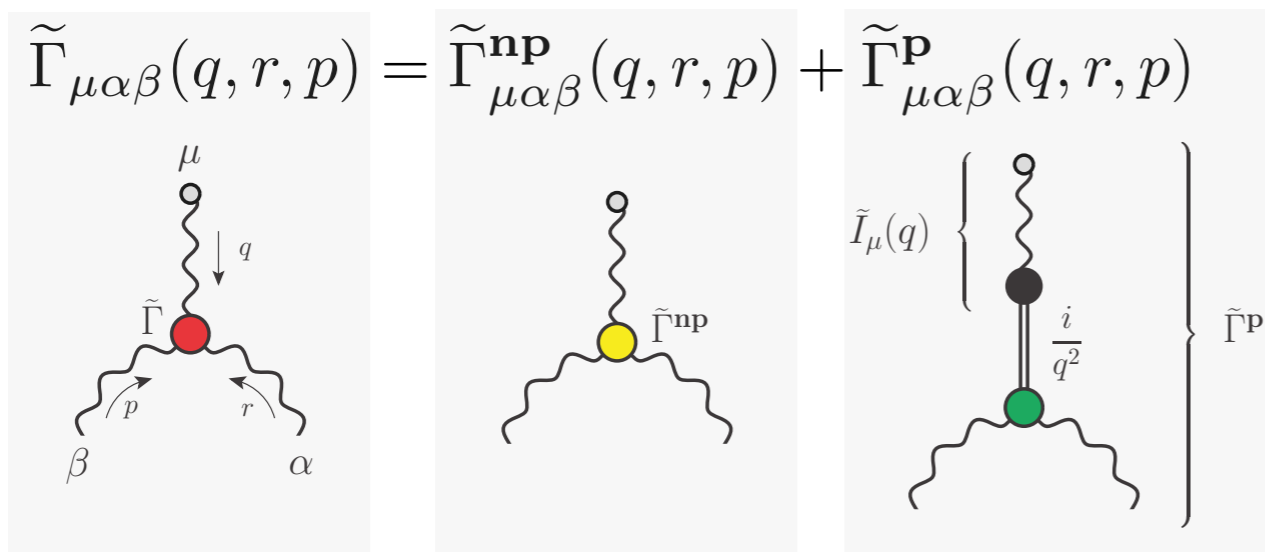
- **Way out:** require massless, longitudinally coupled Goldstone like poles  $1/q^2$

- **Occur dynamically** (even in the absence of canonical scalar fields) as composite (colored) excitations in a strongly coupled gauge theory

Jackiw, Johnson, PRD 8 (1973)

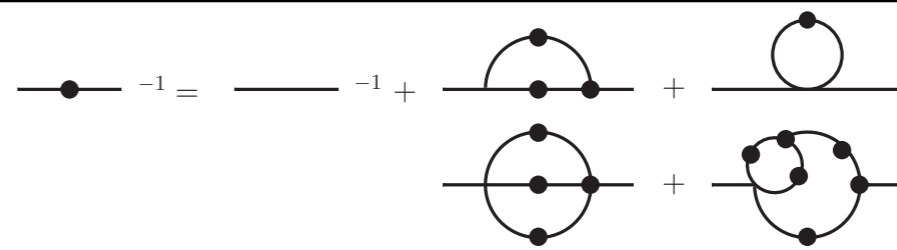
Cornwall, Norton, PRD 8 (1973)

Eichten, Feinberg, PRD10 (1974)

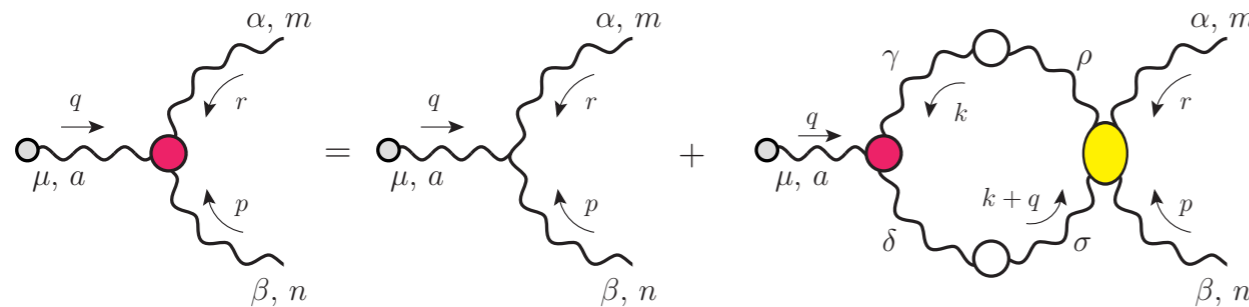


- **3-gluon vertex** contains non-analyticities
- **Not kinematic singularities** composite excitations produced by strong dynamics
- **Do not appear in the S-matrix** (longitudinally coupled)

# What about gluons?

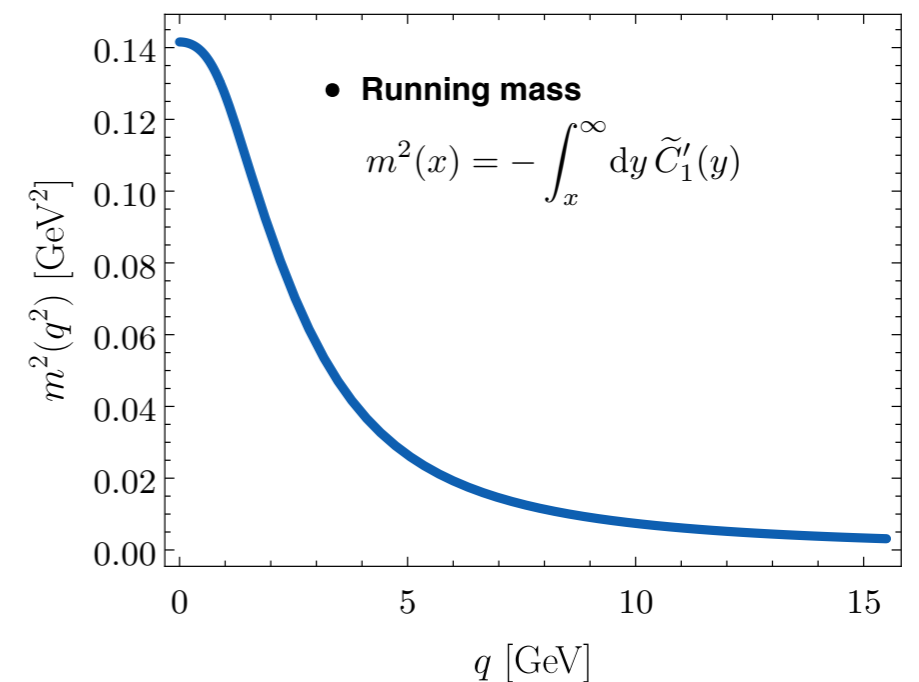
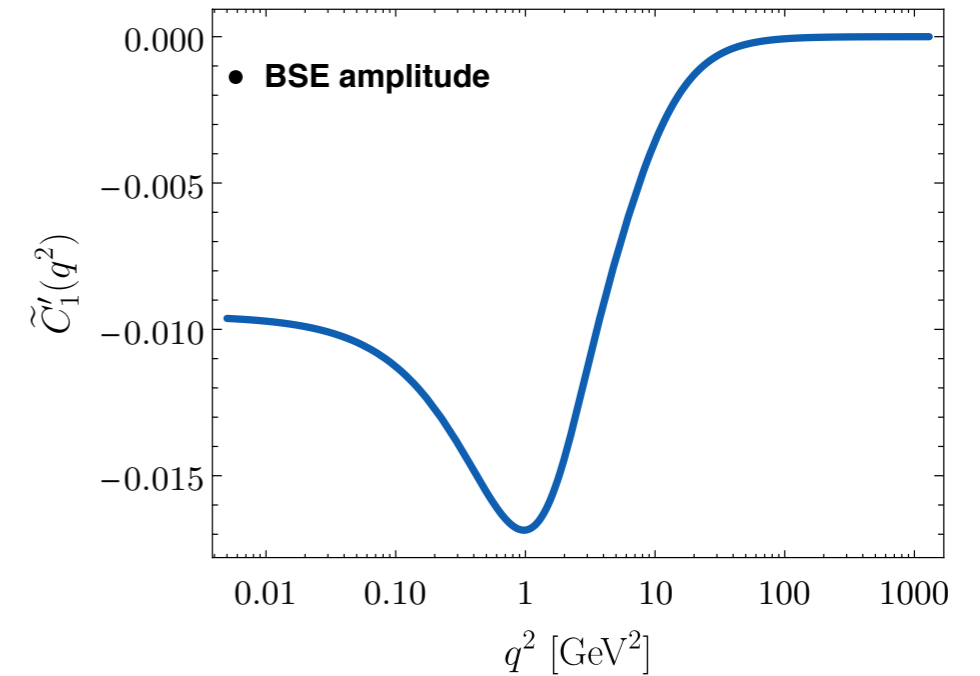
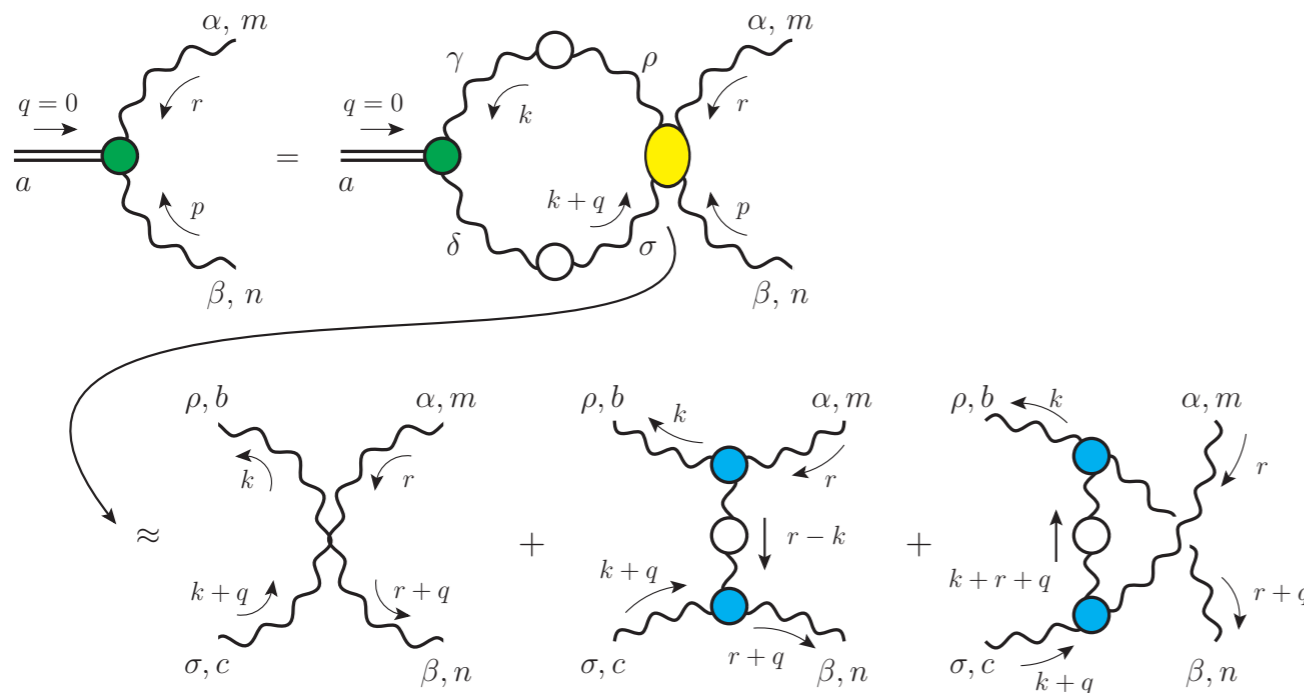


- Consider BSE for the full vertex



- Replace vertex:  $\Gamma \rightarrow \Gamma^{\text{np}} + \Gamma^{\text{p}}$   
expand and equate terms linear in  $q$

DB, Papavassiliou, PRD 97 (2018)



# And ghosts?



- **Ghost remains massless**  
even at non-perturbative level

- **Lattice results**  
confirmed by continuous studies

- **Ghost dressing function saturates**

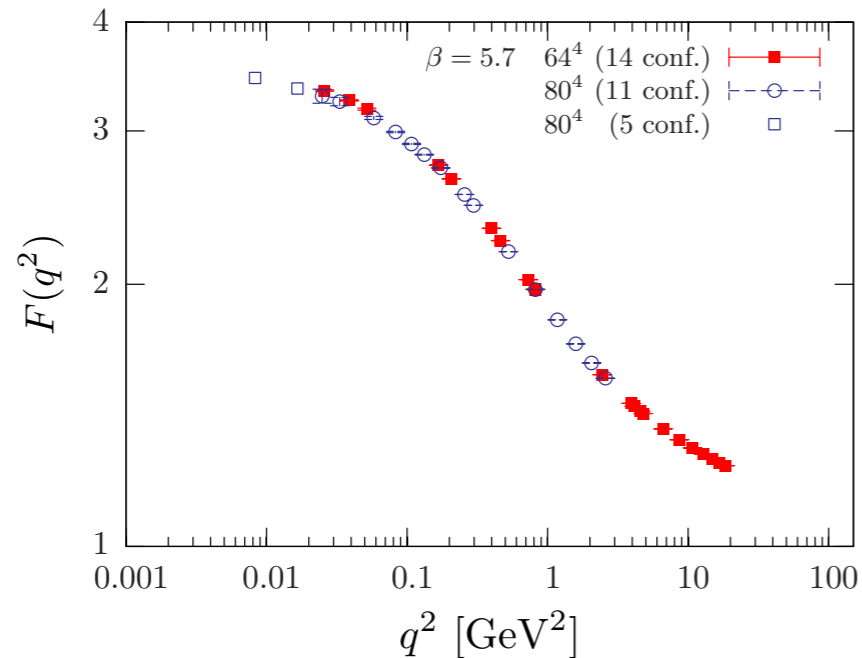
$$F(q^2) = q^2 D(q^2)$$

- **IR propagator diverges**

$$D(q^2) \sim c/q^2$$

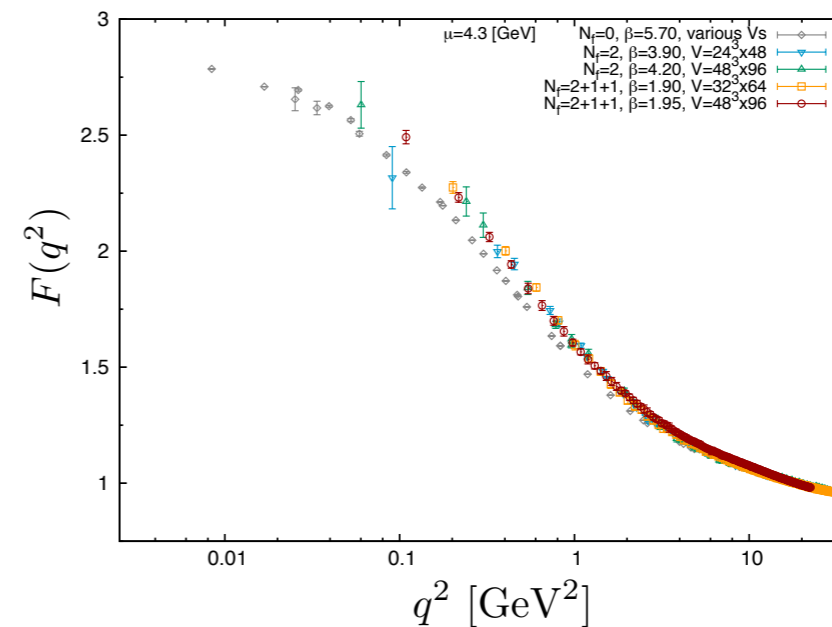
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Bogolubsky, Ilgenfritz, Muller-Preussker,  
Sternbeck, PLB 676 (2009)



- **Landau gauge (unquenched)**

Ayala, Bashir, DB, Cristoforetti,  
Rodriguez-Quintero, PRD 86 (2012)



# And ghosts?



- **Ghost masslessness**

phenomenologically very important

[Aguilar, DB, Ibañez, Papavassiliou, PRD 89 \(2014\)](#)

$$\Delta^{-1}(q^2) \sim q^2 \left[ 1 + \log \frac{q^2 + m^2(q^2)}{\mu^2} + \log \frac{q^2}{\mu^2} \right] + m^2(q^2)$$

gluon loops
ghost loops

- **IR behavior of derivative of the gluon propagator**

entirely determined by ghost loops

$$\partial_{q^2} \Delta^{-1}(q^2) \sim \log \frac{q^2}{\mu^2} \xrightarrow{q^2 \rightarrow 0} -\infty$$

- **Related to 3-gluon vertex form factor**  
proportional to tree-level tensor structure

[Aguilar, DB, Ibañez, Papavassiliou, PRD 89 \(2014\)](#)

- **Suppression wrt tree-level value**

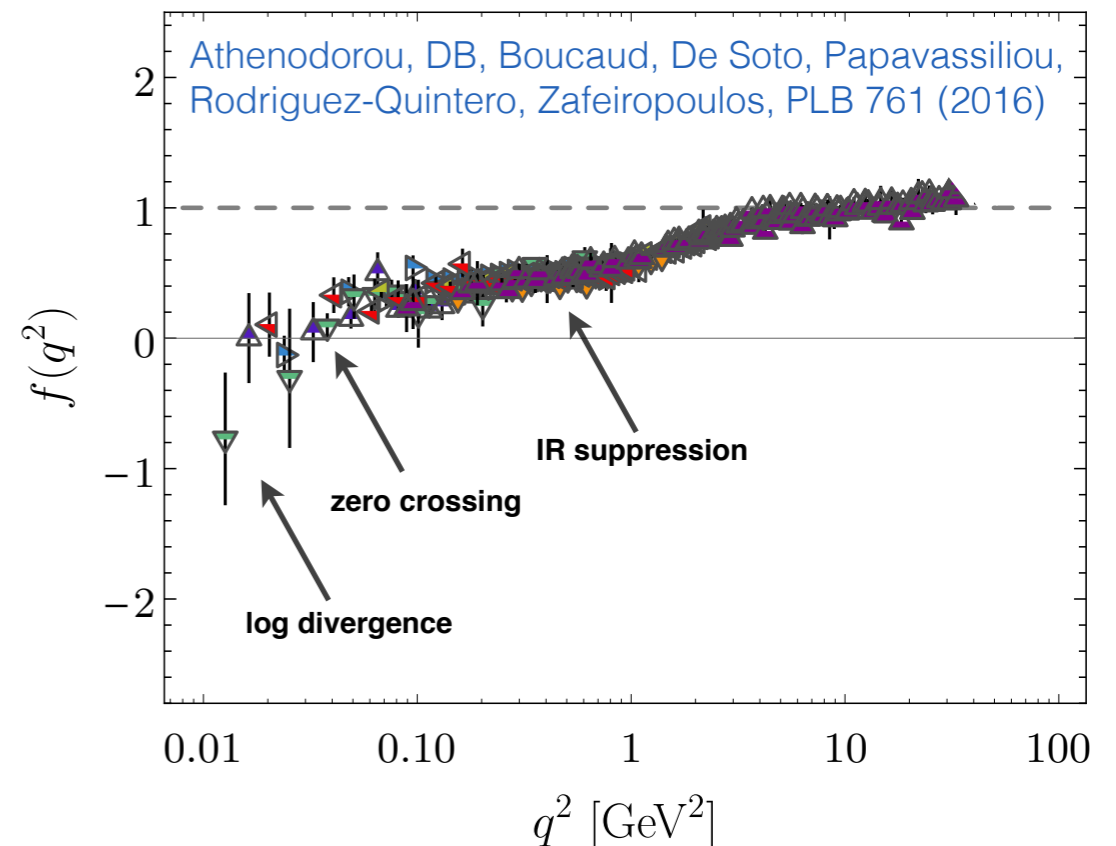
vertex must drop below 1

- **Zero crossing**

in the (deep?) IR followed by

- **Log divergence**

as  $q^2 \rightarrow 0$



# PDA's within a DS framework



- **Meson leading twist PDA**

projection of BS wave-function onto the light-front

Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, PRL 110 (2013)

$$f_{\text{PS}} \varphi(x, \zeta) = Z_2(\zeta, \Lambda) \text{Tr}_{\text{CD}} \int_{\text{d}q}^{\Lambda} \delta(n \cdot q_+ - x n \cdot P) \gamma_5 (\gamma \cdot n) \chi_{\text{PS}}(q, P)$$

$f_{\text{PS}}$

- **Meson's decay constant** (normalization of PDAs)  $\int_0^1 dx \phi(x, \zeta) = 1$

$Z_2(\zeta, \Lambda)$

- **Quark wave-function renormalization** evaluated at  $\zeta$  (2 GeV)

$\int_{\text{d}q}^{\Lambda}$

- **Poincaré invariant regularization** of 4-dimensional integral

$\delta(n \cdot q_+ - x n \cdot P)$

- **$n$  light-cone vector;  $P$  meson momentum**  
 $n \cdot P = -m_{\text{PS}} \quad P^2 = -m_{\text{PS}}^2$

$\chi_{\text{PS}}(q, P)$

- **BS wave-function:**  $\chi_{\text{PS}}(q, P) = S_{f_1}(q_+) \Gamma_{\text{PS}}(q, P) S_{f_2}(q_-)$

$$q_+ = q + \eta P$$

$$q_- = q - (1 - \eta)P$$

$\Gamma_{\text{PS}}(q, P)$

- **BS amplitude**  
solution of the homogeneous BS equation

$$\Gamma_{\text{PS}}^{\alpha\beta}(k, P) = \int_q \mathcal{K}^{\alpha\alpha'; \beta\beta'}(q_{\pm}, k_{\pm}) [S_{f_1}(q_+) \Gamma_{\text{PS}}^{\alpha\beta}(q, P) S_{f_2}(q_-)]_{\alpha'\beta'}$$

$S_{f_1}(q_+)$

- **Quark propagator**  
solution of the gap equation

$$S_f^{-1}(k) = Z_2(-i\gamma \cdot k + m_f) + Z_1 \int_{\text{d}q}^{\Lambda} g^2 D^{\mu\nu}(k - q) \frac{\lambda^a}{2} \gamma_{\mu} S(q) \frac{\lambda^a}{2} \Gamma_{\nu}(q, k)$$



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- **Not accessible** in Euclidean space

- **Mellin moments**

used for reconstructing the PDA

$$\langle x^m \rangle = \frac{1}{f_{\text{PS}}} Z_2(\zeta; \Lambda) \text{Tr}_{\text{CD}} \int_{\text{d}q}^{\Lambda} \frac{(n \cdot q_+)^m}{(n \cdot P)^{m+1}} \gamma_5 (\gamma \cdot n) \chi_{\text{PS}}(q; P)$$

- **Highly oscillatory integral** in Euclidean space

- **Two possible representations**

for determining the full amplitude

- **Gegenbauer**

$$\varphi(x) = \mathcal{N} x^\alpha (1-x)^\beta$$

- **Gaussian**

$$\varphi(x) = \mathcal{N} [1 - (2x - 1)^2] e^{a^2(1 - (2x - 1)^2) + b^2(2x - 1)}$$

- **Requires damping factor** and extrapolation

$$1/(1 + r^2 k^2)^m; \quad r^2 \rightarrow 0$$

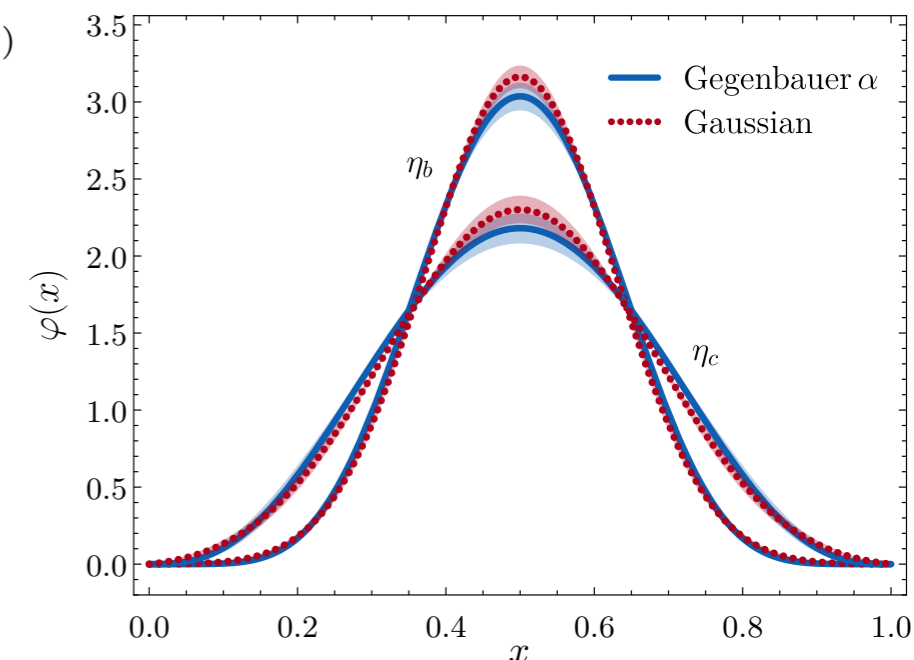
- **Main difference**

behavior as  $x \rightarrow 1$

- **Affects the inverse moment**  $\int_0^1 dx \phi(x)/(1-x)$   
large  $Q^2$  behavior of form factors

- **Heavy-heavy limit:**  $\varphi(x) \rightarrow \delta(1 - x/2)$

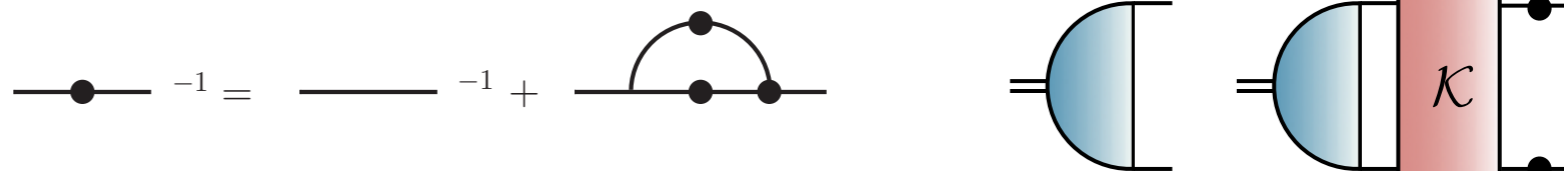
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# Propagators and BS amplitudes



- **Use RL truncation** for gap and BS equation



$$\Gamma_\nu = \gamma_\nu$$

$$\mathcal{K}[S\Gamma_{\text{PS}}S] = -P^{\mu\nu} \mathcal{G} \gamma_\mu [S\Gamma_{\text{PS}}S] \gamma_\nu$$

- **For the interaction** we set

Qin, Chang, Liu, Roberts and Wilson, PRC 84 (2011)

$$\mathcal{I}(k^2) = k^2 \frac{\mathcal{G}_{\text{IR}}(k^2) + \mathcal{G}_{\text{UV}}(k^2)}{4\pi}$$

$$\mathcal{G}_{\text{IR}}(k^2) = \frac{8\pi^2}{\omega^5} \varsigma^3 e^{-k^2/\omega^2}$$

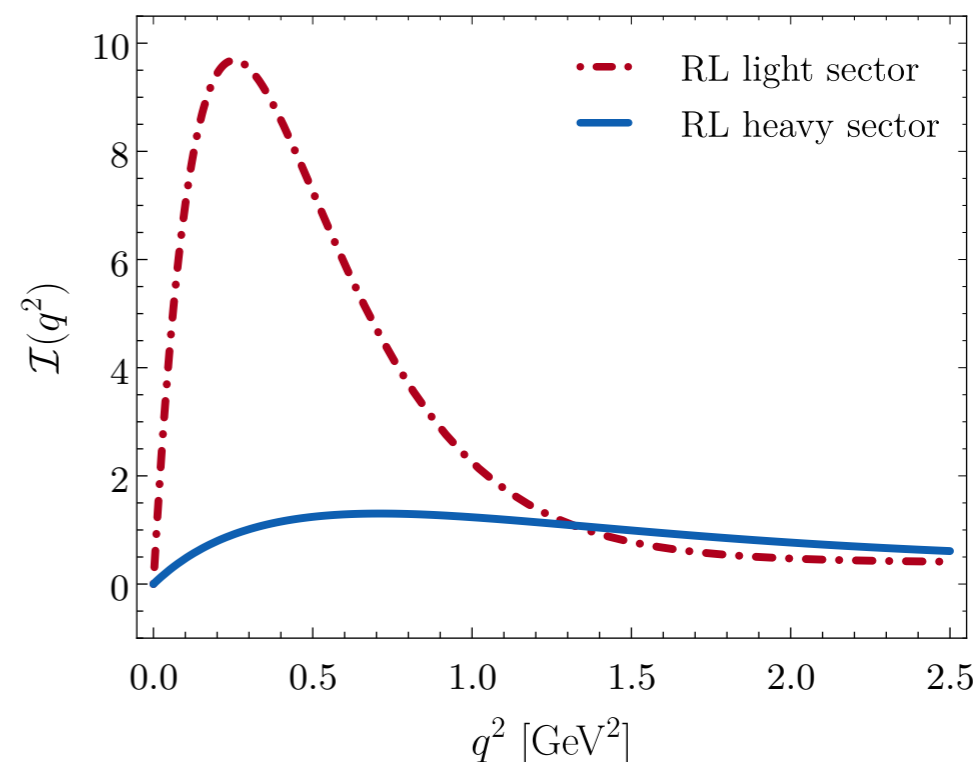
$$\mathcal{G}_{\text{UV}}(k^2) = \frac{96\pi^2}{25} \frac{1 - e^{-k^2/1[\text{GeV}^2]}}{k^2 \log[e^2 - 1 + (1 + k^2/\Lambda^2)^2]}$$

- **Light quark sector (GeV)**

$$\varsigma = 0.8; \quad \omega = 0.5$$

- **Heavy quark sector (GeV)**

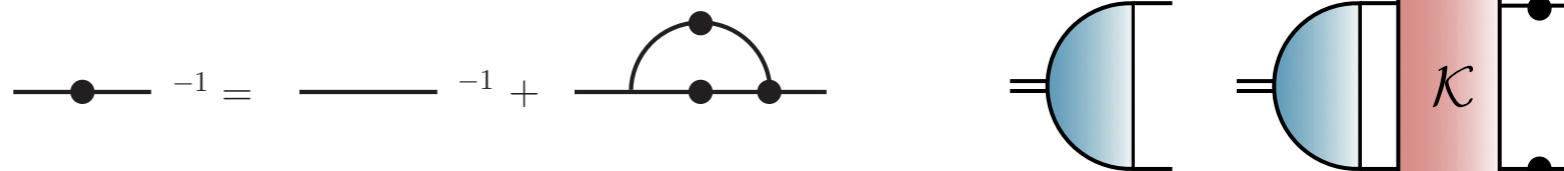
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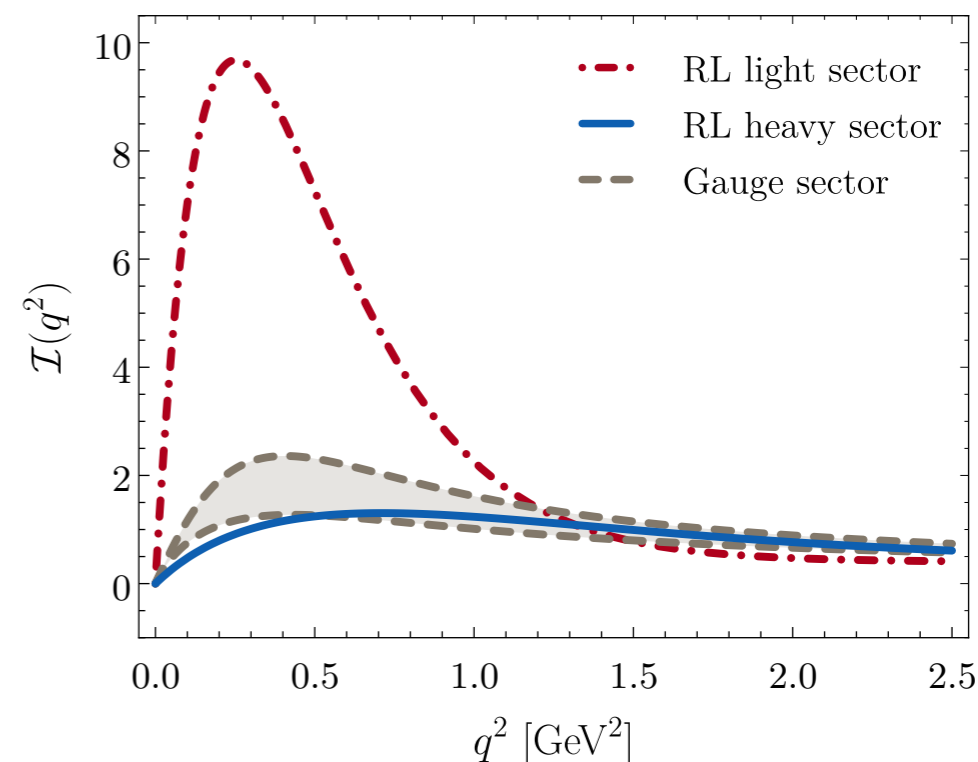
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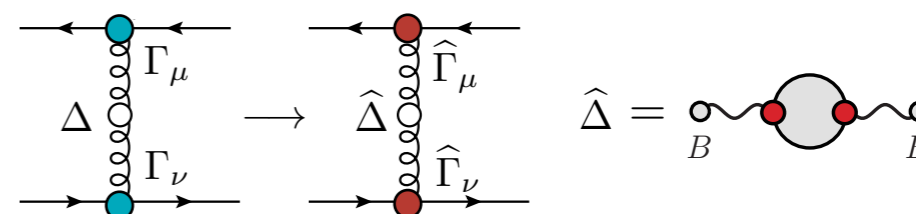
- **Heavy quark sector (GeV)**

$$\varsigma = 0.6; \quad \omega = 0.8$$



- **Heavy sector interaction** overlaps with gauge sector kernel determination

[DB, Chang, Papavassiliou, Roberts, PLB 742 \(2015\)](#)

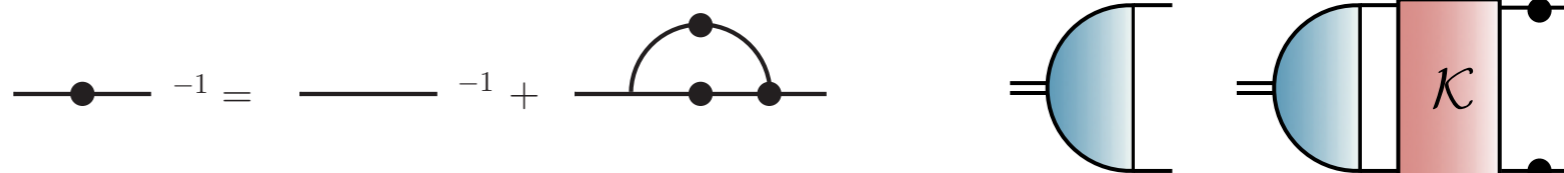




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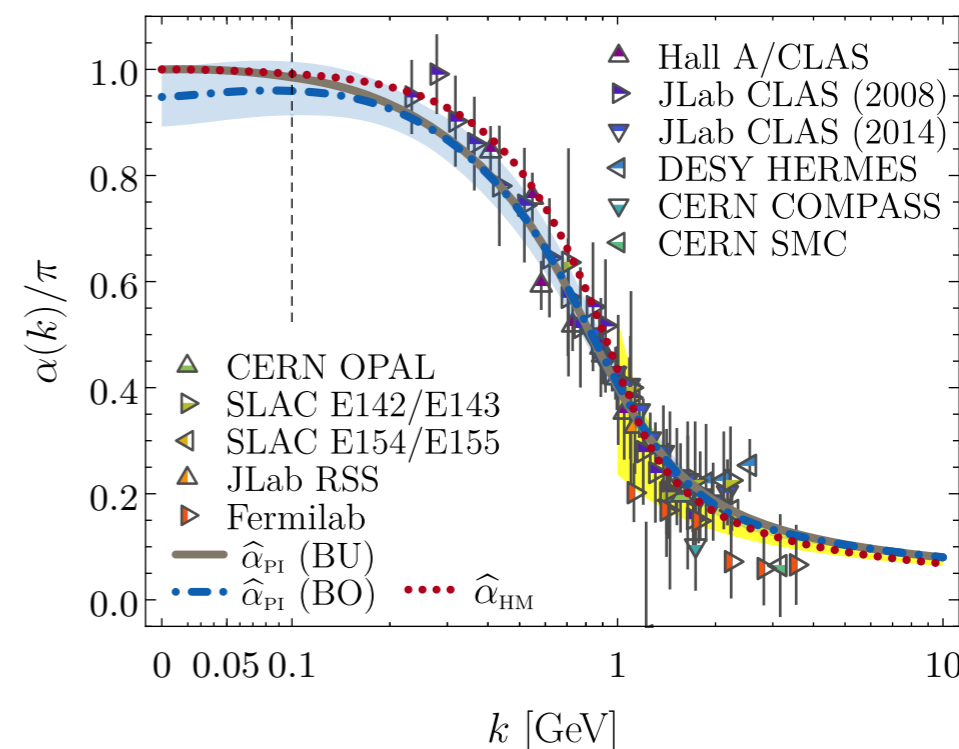
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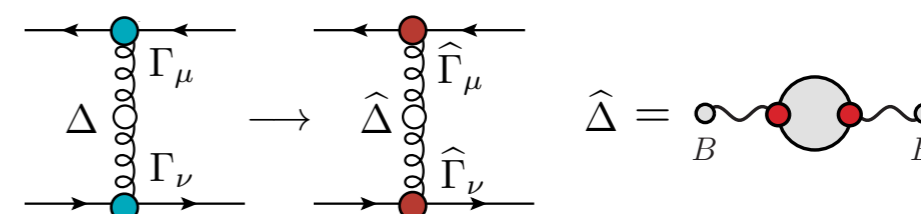
$$\varsigma = 0.6; \quad \omega = 0.8$$



- **Heavy sector interaction**

overlaps with gauge sector kernel determination

DB, Chang, Papavassiliou, Roberts, PLB 742 (2015)



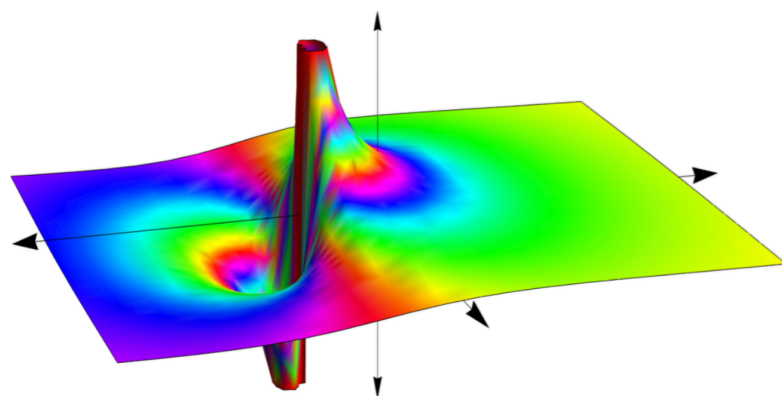
# Propagators and BS amplitudes

- **SDE solutions**

dramatic change in the analytic structure

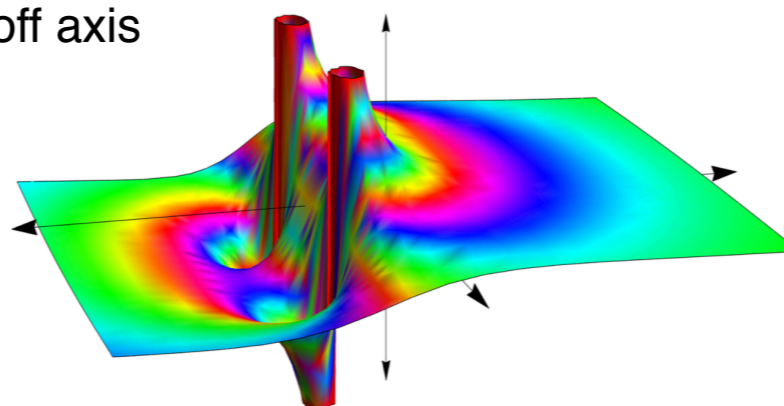
- **Normal particle**

mass pole on the real axis



- **SDE solutions**

interactions move the mass pole off axis



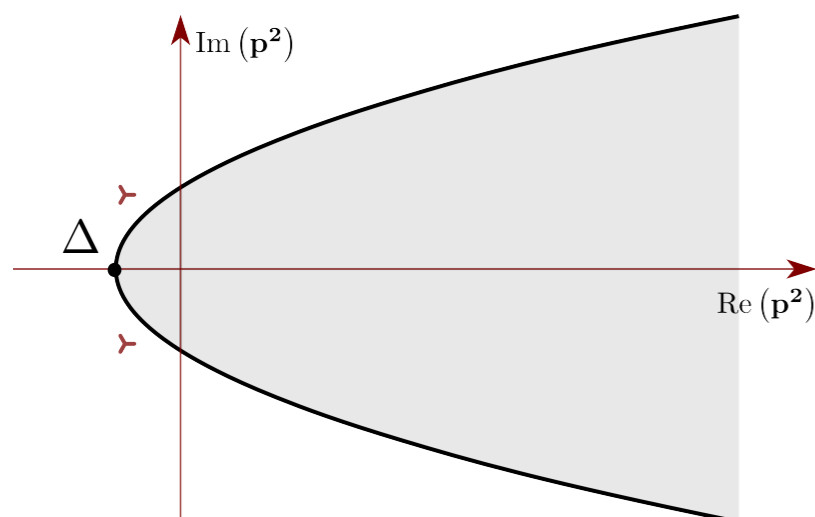
- **Complex conj. singularities** characterized by a dynamically generated mass scale

- **SDE singularities**

impacts our ability to find BS solutions

- **Quark propagators**

need to be known on a parabolic region



- **Cauchy th. not valid**

if singularities are inside the contour

- **Maximize parabola apices**

probe bound states up to  $M = \Delta_h + \Delta_l$  ( $\Delta_h \gg \Delta_l$ )

- **Use freedom to unbalance legs**

in BS equation

$$q_+ = q + \eta P$$

$$q_- = q - (1 - \eta)P$$

$$\eta \in [\Delta_h/M, 1 - \Delta_l/M]$$

- **Still, if gap is too big**

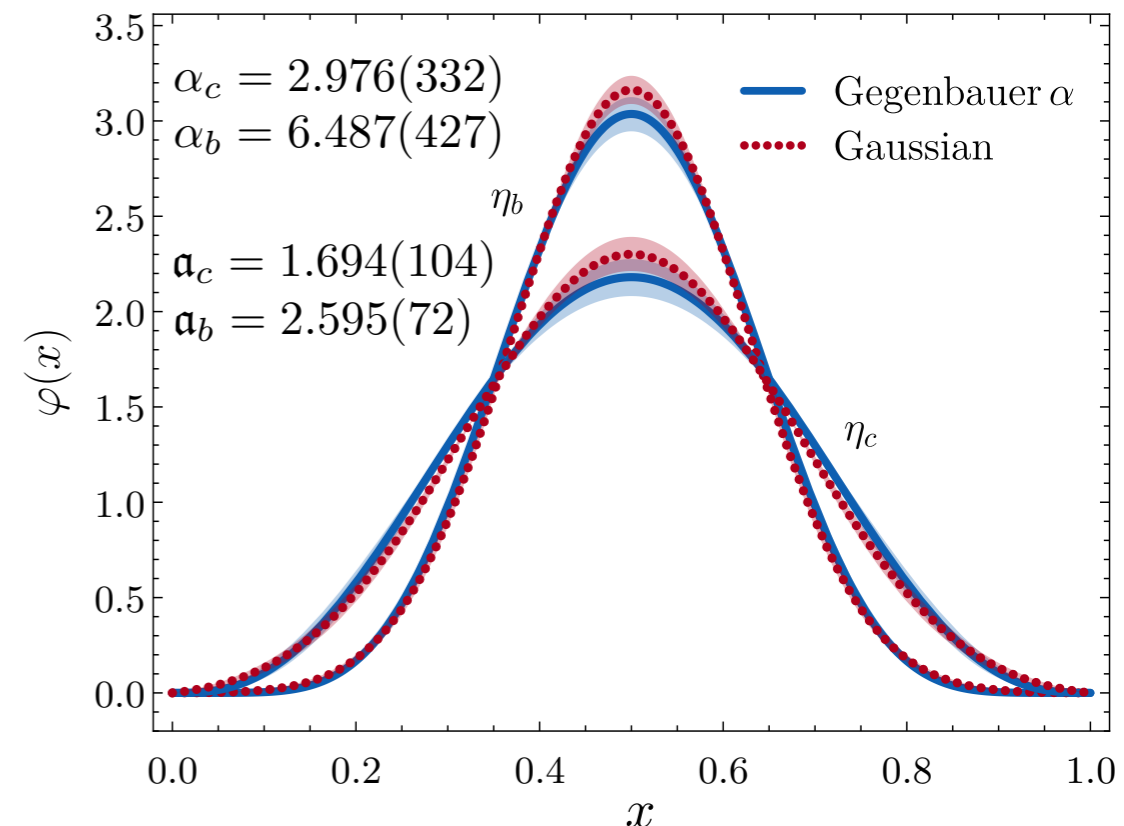
no BS solution exists anyway

# Heavy-light systems in RL



- **Choose current masses**  
for heavy quarks (in GeV):  $m_c = 1.25$ ,  $m_b = 4.35$
- **Fix interaction parameters**  
to heavy sector values:  $\varsigma = 0.6$ ;  $\omega = 0.8$
- **Evaluate properties of  $\eta$  mesons**

	Exp.		lQCD	
	$m_i$	$f_i$	$m_i$	$f_i$
$\eta_c$	2.98	0.272	2.98	0.238
$\eta_b$	9.38	0.501	9.39	/



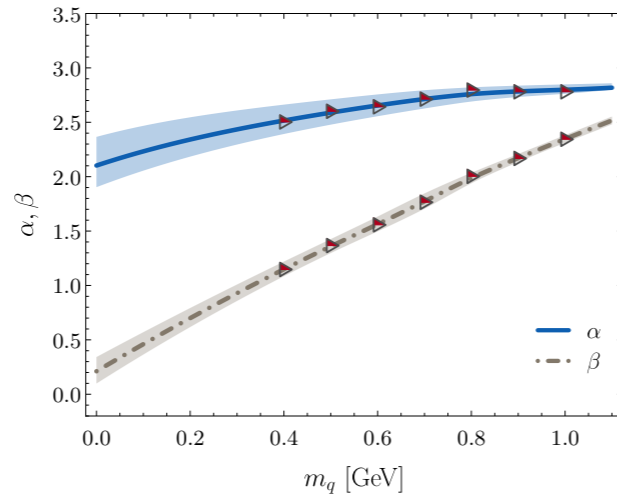
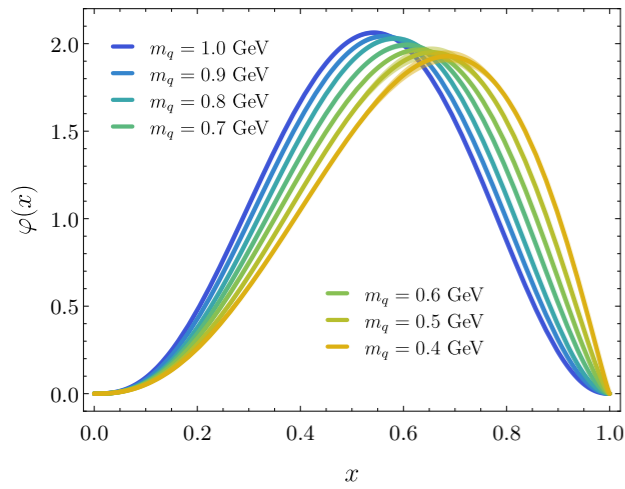
- **Construct an array of fictitious  $h\bar{q}$  pseudoscalar mesons**  
down to the lowest  $m_q$  RL can accommodate
  - **Charm sector:**  $m_q \sim 0.4$
  - **Bottom sector:**  $m_q \sim 1.3$
- **Estimate PDAs parameters**  
use them to (SPM) extrapolate to lower  $m_q$  masses

# Charm sector

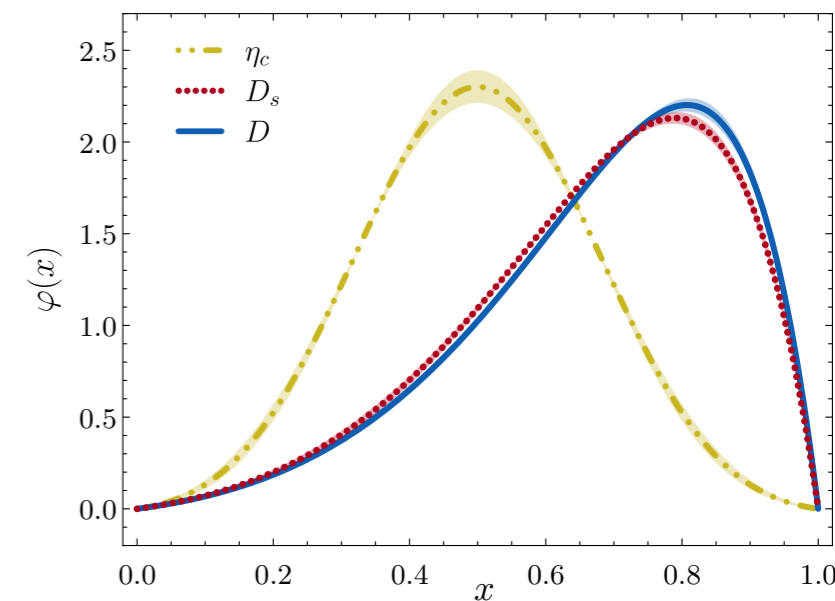
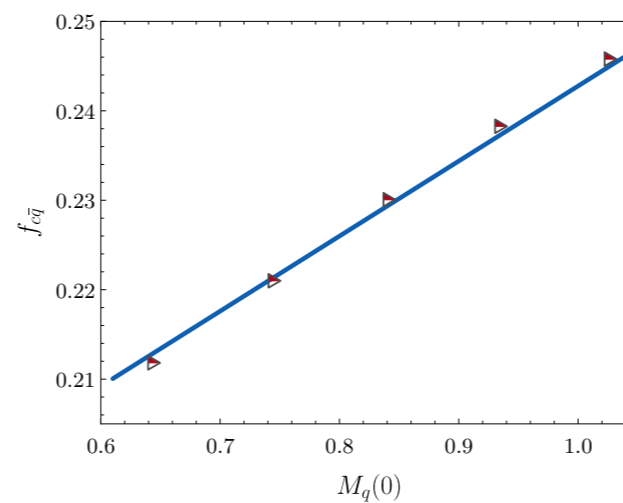
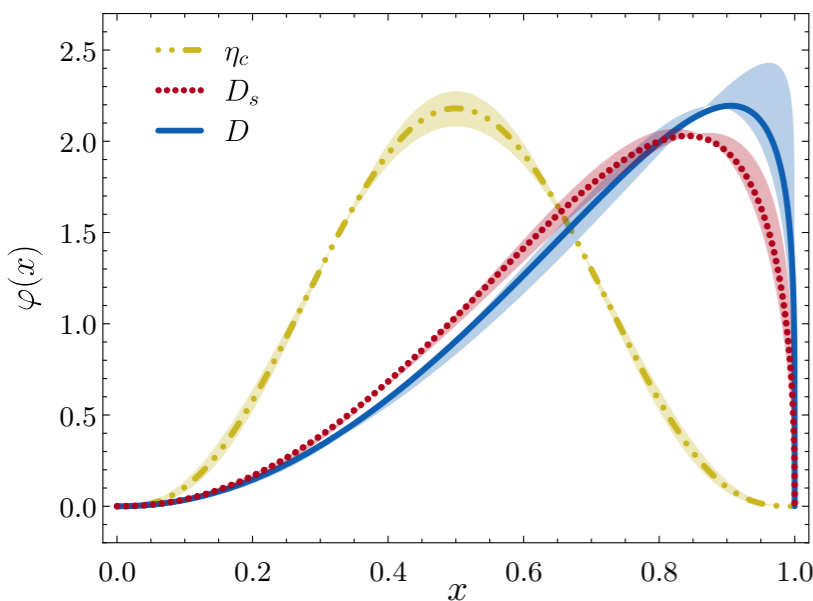
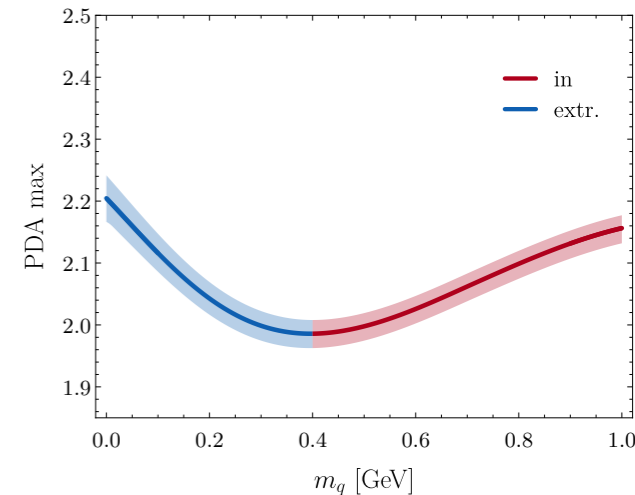
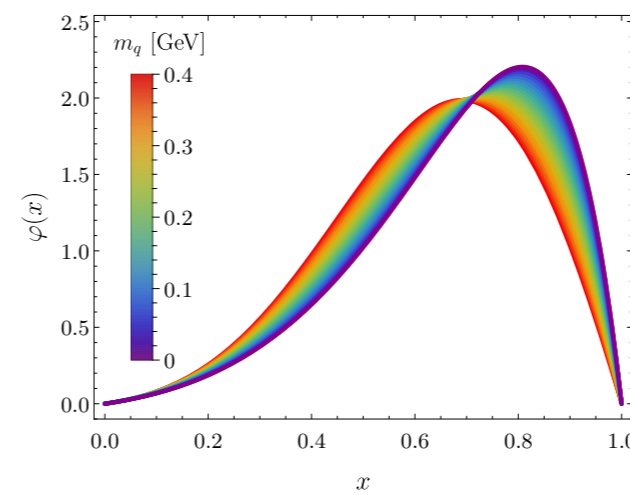
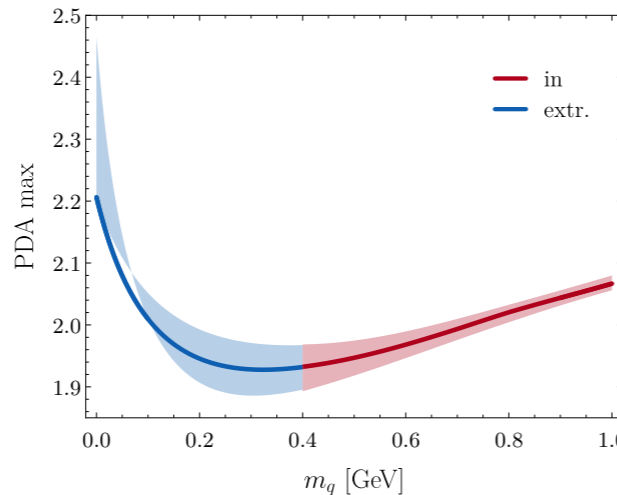
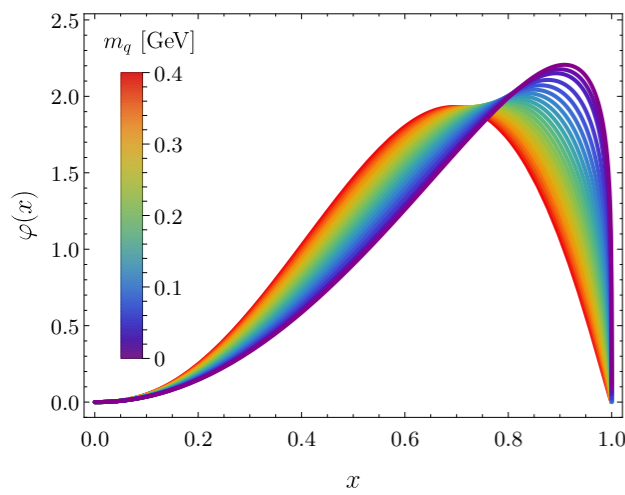
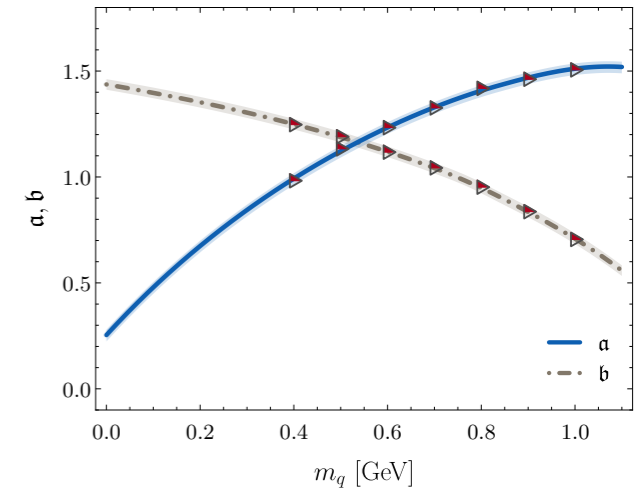
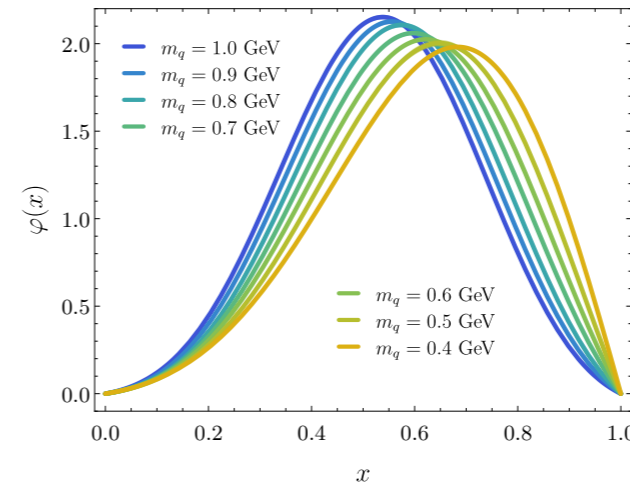
$m_q$	$\alpha$	$\beta$	$a$	$b$
$m_s$	$2.212^{0.228}_{0.184}$	$0.418^{0.102}_{0.123}$	$0.442^{0.025}_{0.030}$	$1.404^{0.025}_{0.025}$
$m_u$	$2.107^{0.254}_{0.168}$	$0.220^{0.107}_{0.143}$	$0.262^{0.025}_{0.030}$	$1.436^{0.025}_{0.025}$



## • Gegenbauer



## • Gaussian

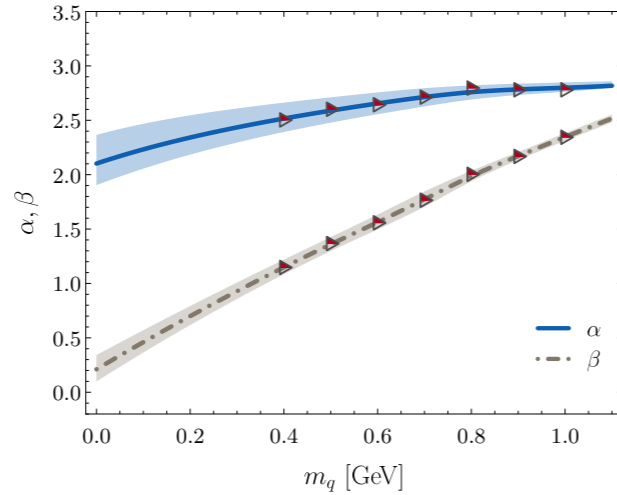
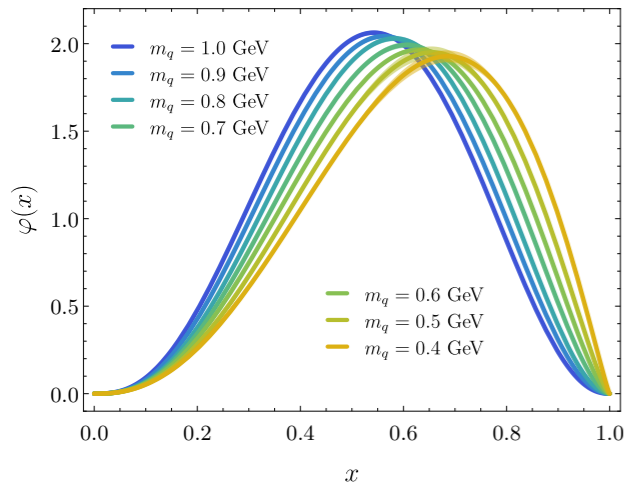


# Charm sector

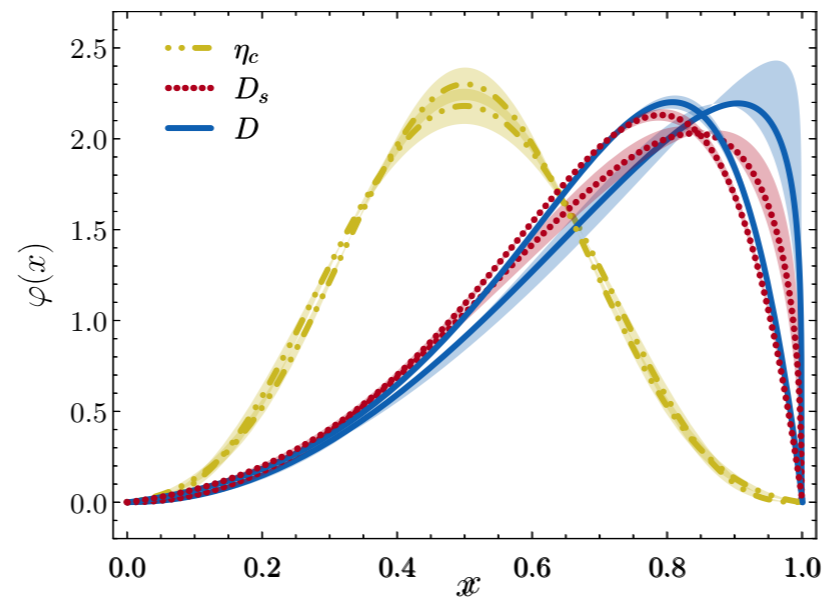
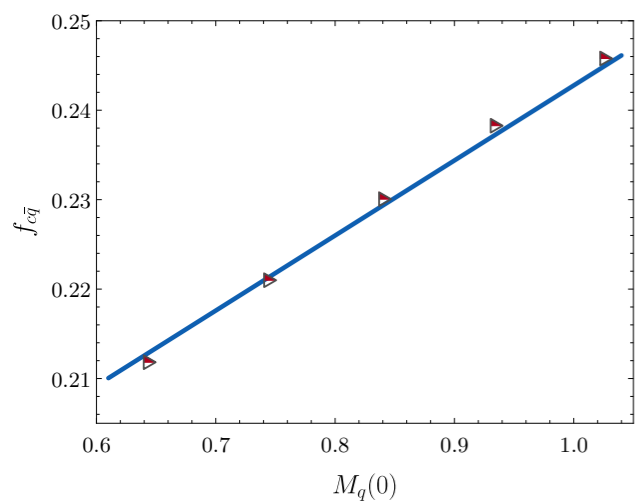
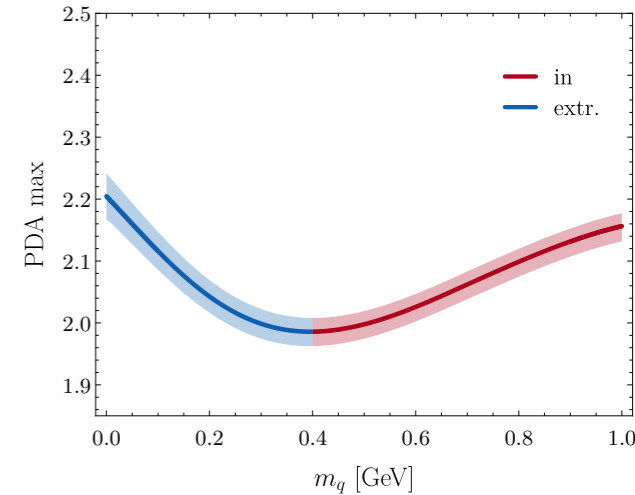
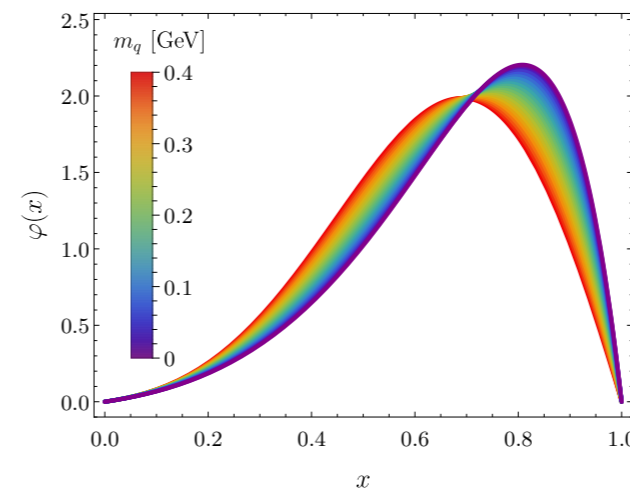
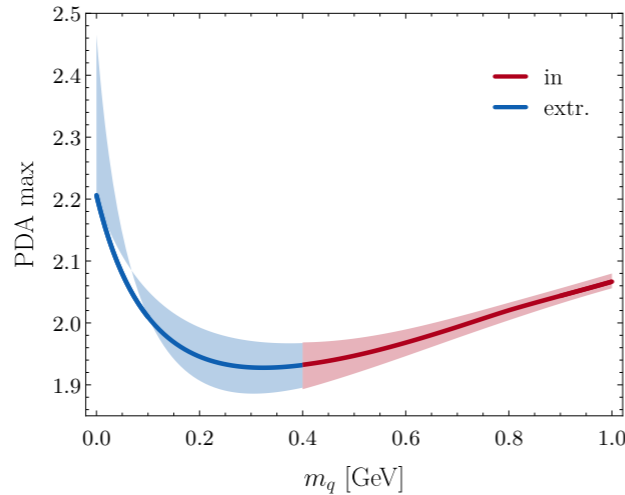
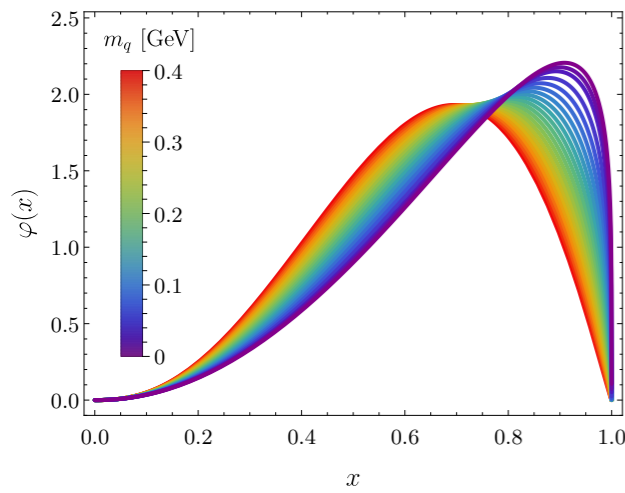
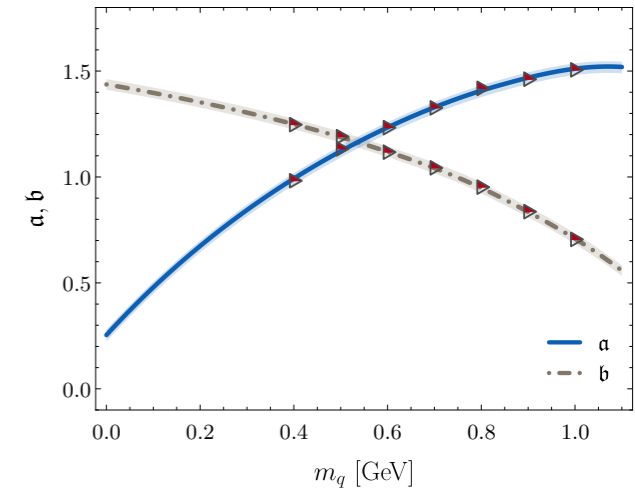
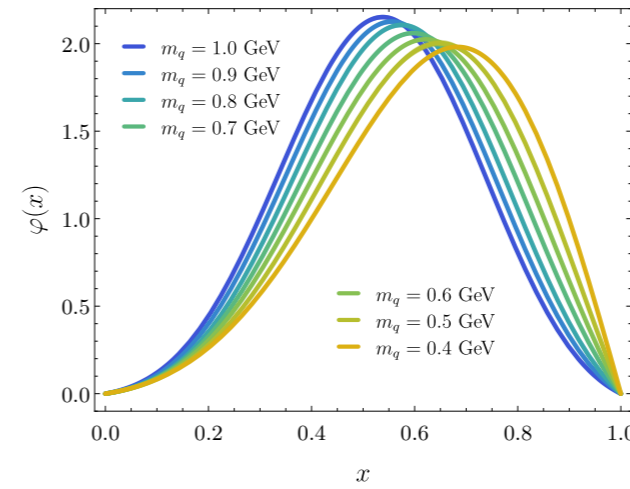


$m_q$	$\alpha$	$\beta$	$a$	$b$
$m_s$	$2.212_{0.184}^{0.228}$	$0.418_{0.123}^{0.102}$	$0.442_{0.030}^{0.025}$	$1.404_{0.025}^{0.025}$
$m_u$	$2.107_{0.168}^{0.254}$	$0.220_{0.143}^{0.107}$	$0.262_{0.030}^{0.025}$	$1.436_{0.025}^{0.025}$

## • Gegenbauer

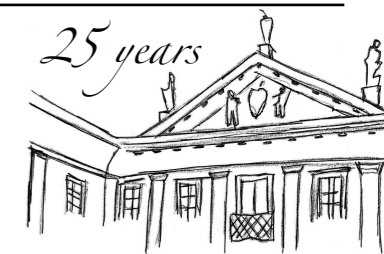


## • Gaussian

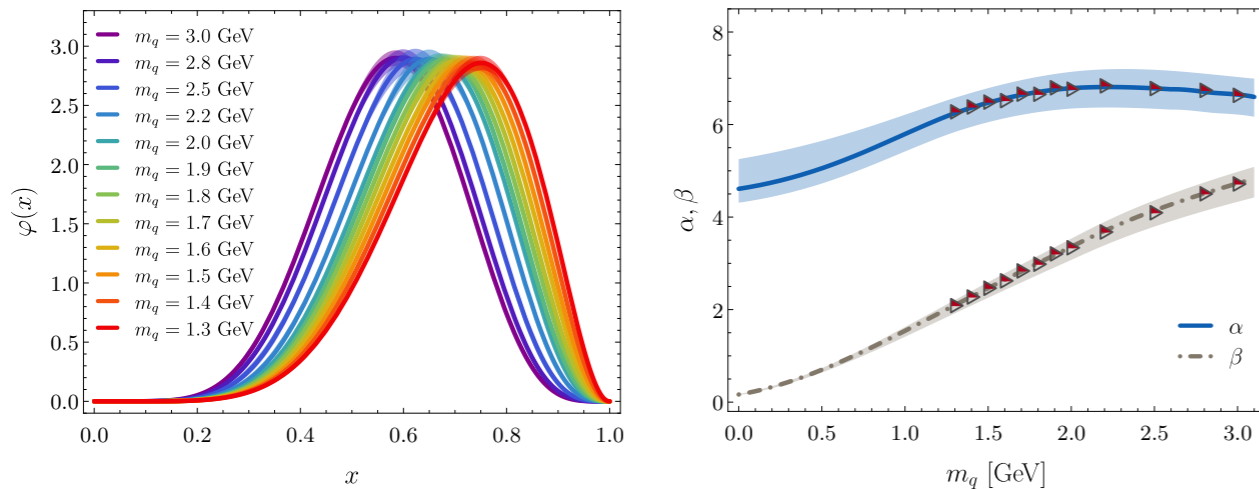


# Bottom sector

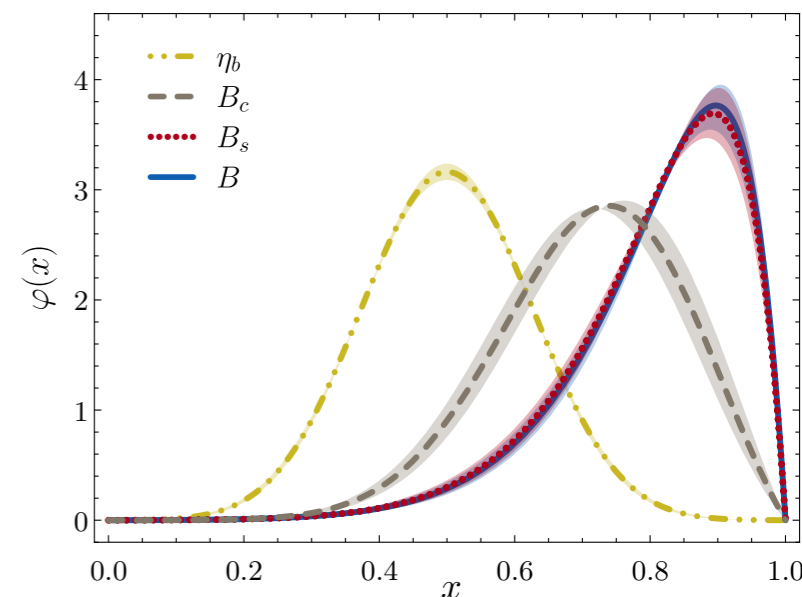
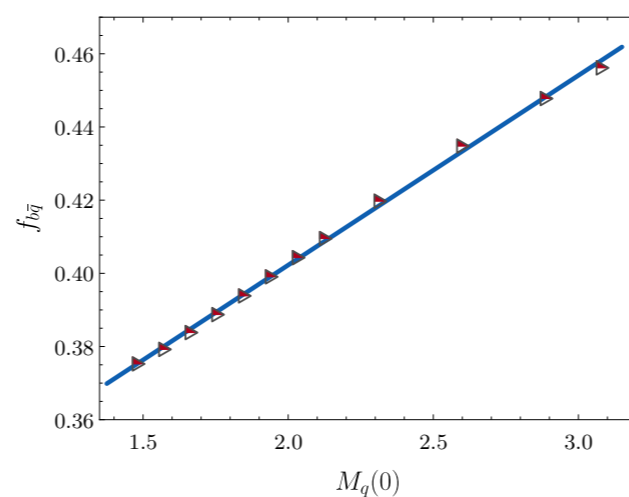
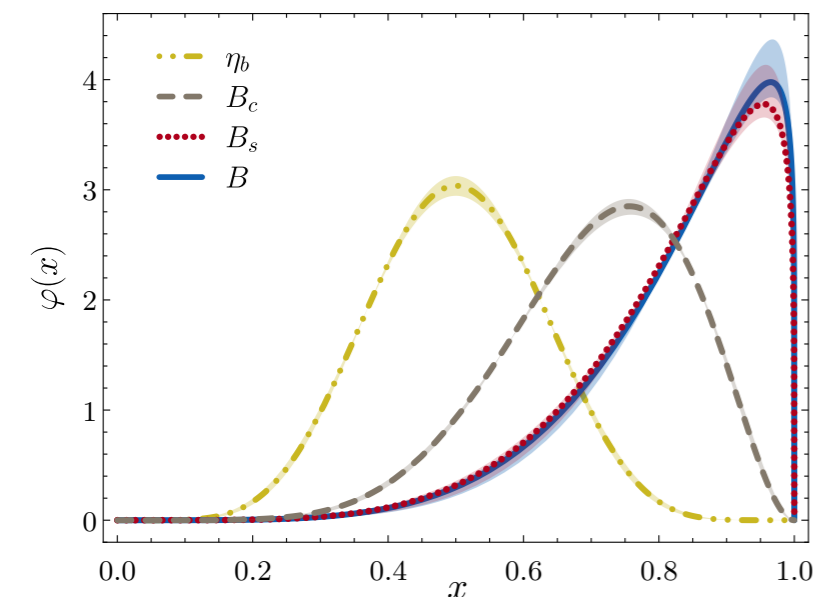
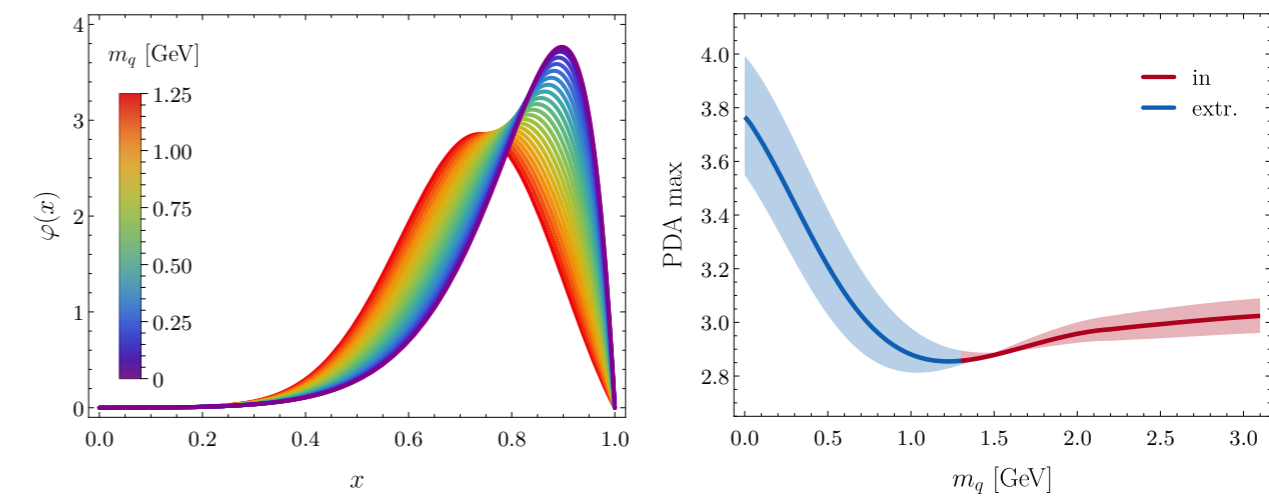
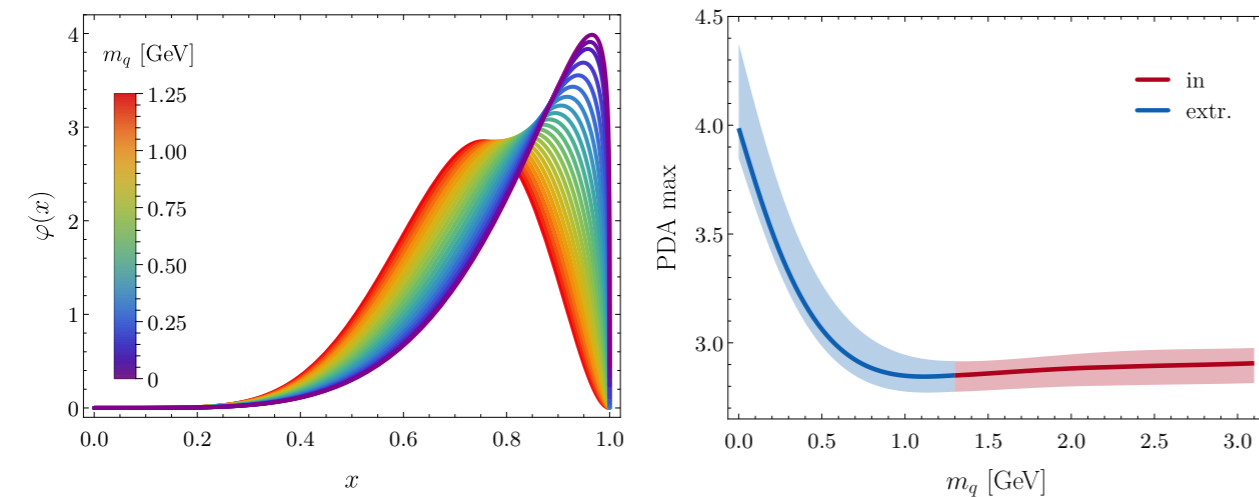
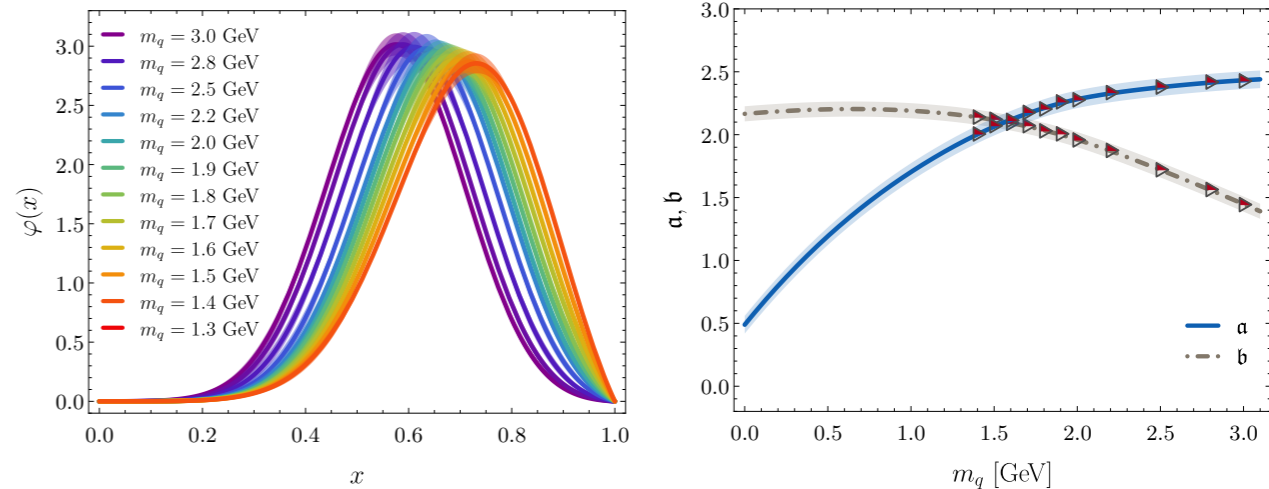
$m_q$	$\alpha$	$\beta$	$a$	$b$
$m_c$	$6.179^{0.398}_{0.379}$	$1.994^{0.342}_{0.141}$	$1.901^{0.070}_{0.070}$	$2.163^{0.060}_{0.060}$
$m_s$	$4.660^{0.760}_{1.240}$	$0.230^{0.125}_{0.108}$	$0.621^{0.070}_{0.070}$	$2.174^{0.060}_{0.060}$
$m_u$	$4.612^{0.714}_{1.287}$	$0.144^{0.054}_{0.118}$	$0.495^{0.070}_{0.070}$	$2.166^{0.060}_{0.060}$



## • Gegenbauer



## • Gaussian

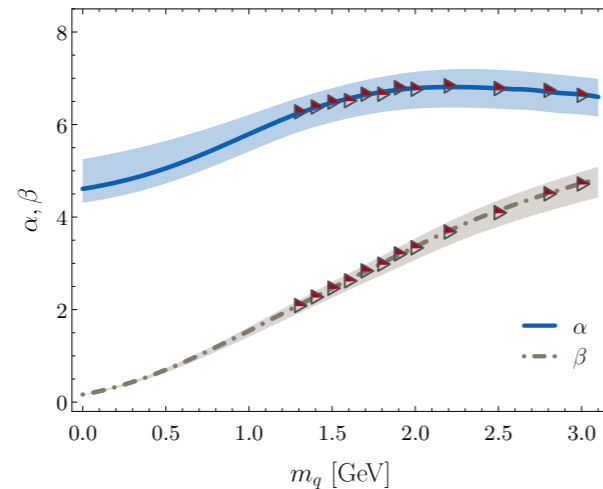
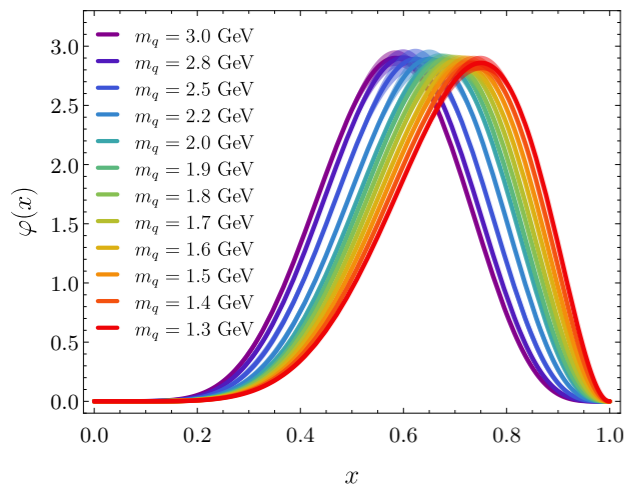


# Bottom sector

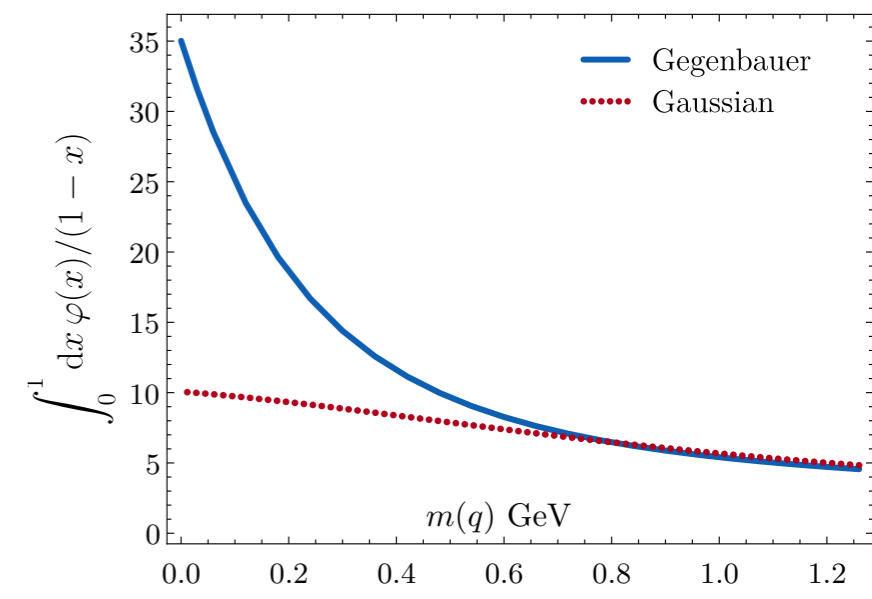
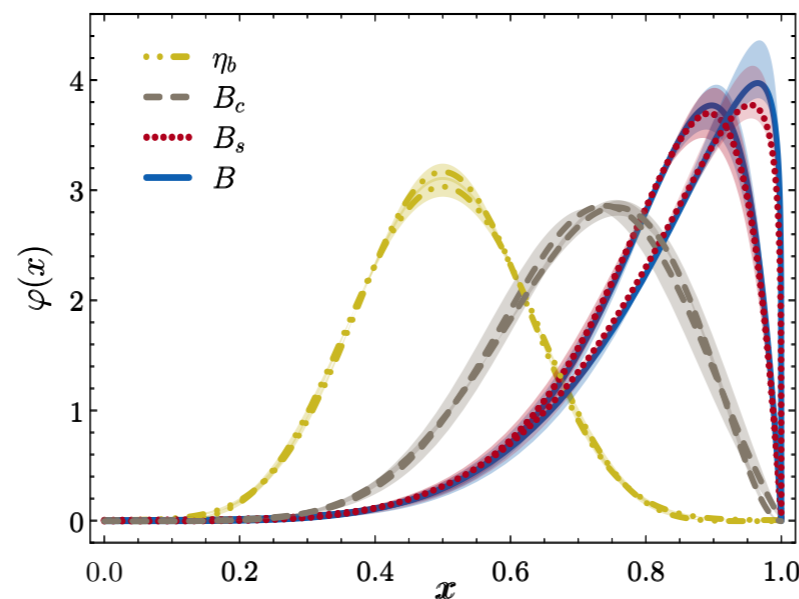
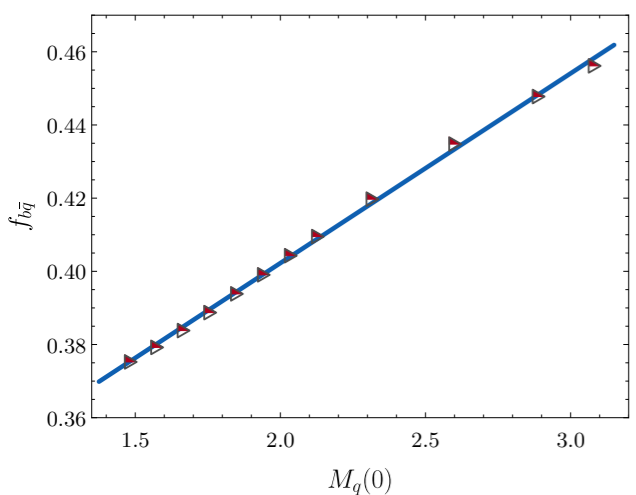
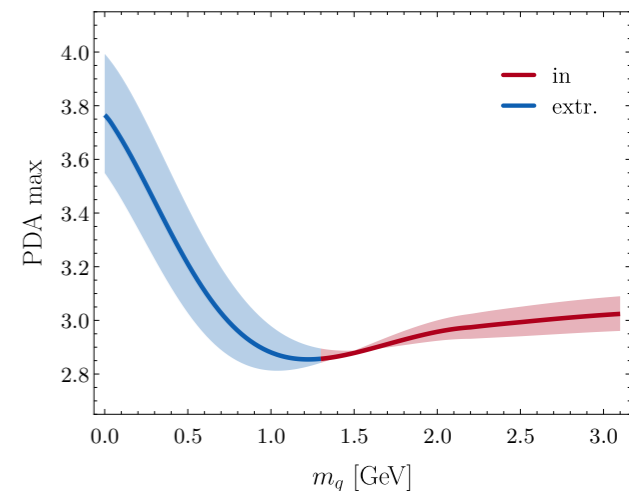
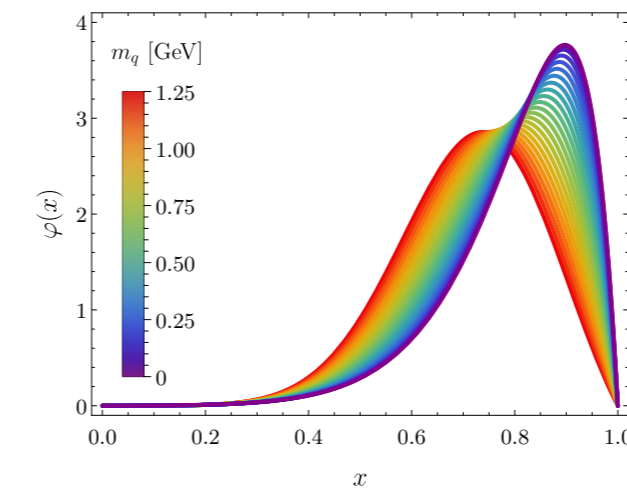
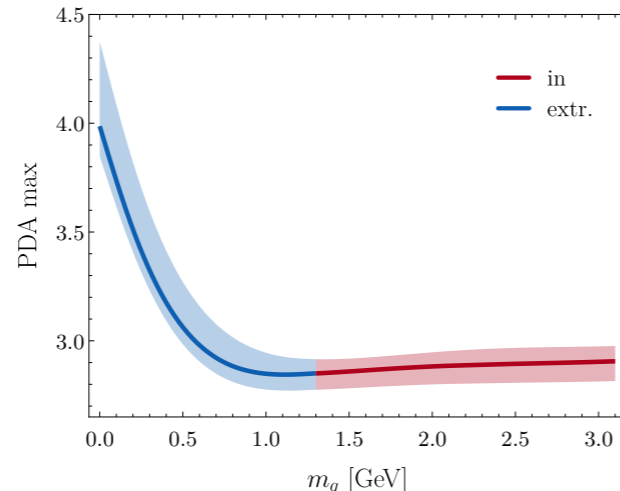
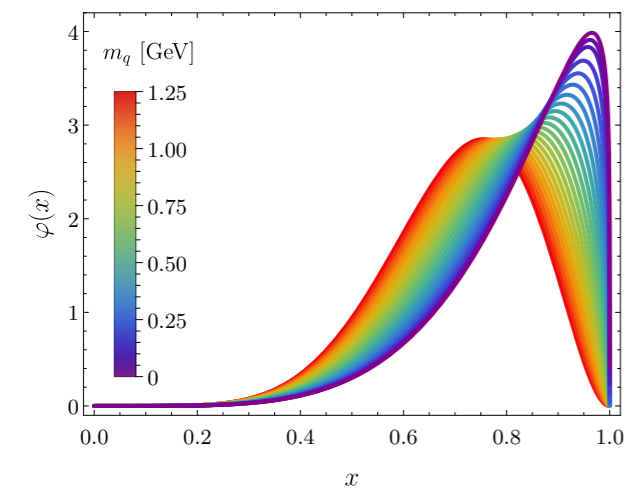
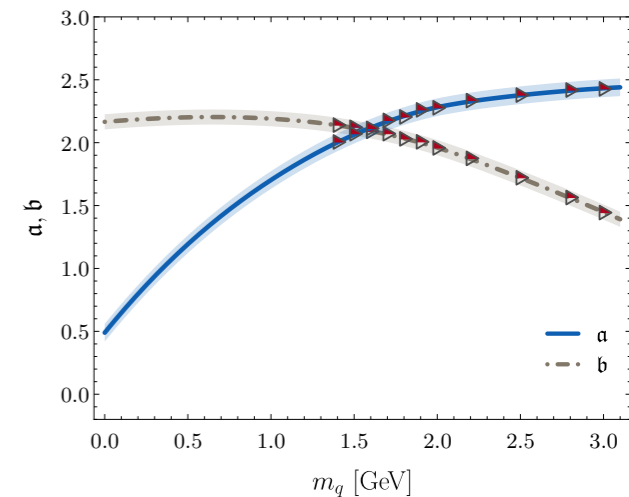
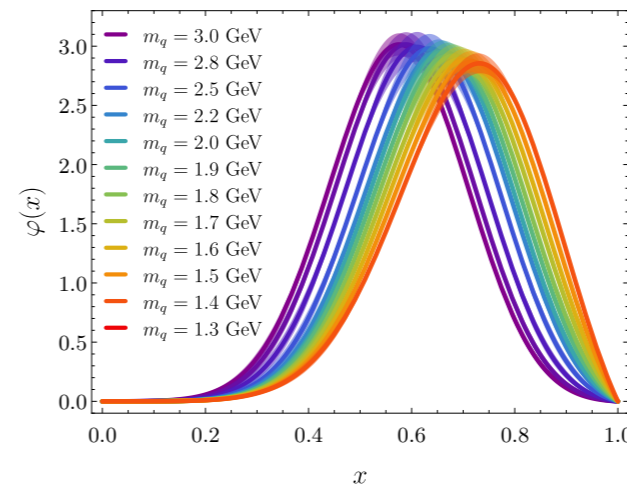
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## • Gegenbauer



## • Gaussian





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