

On propagators and vertices of Yang-Mills theory from their equations of motion



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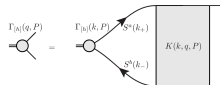
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Graz, Austria

September 20, 2018



Hadronic bound states

Bound state equations: E.g., meson



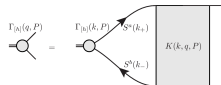
Ingredients:

- Interaction kernel K

- Quark propagator S

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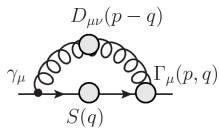
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Approaches:

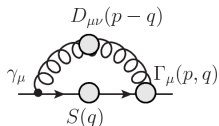
- Phenomenological (bottom-up):
Model interactions

- From first principles (top-down):
Piecing together the **elementary pieces**

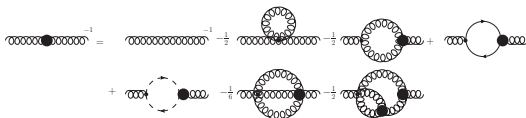
The elementary pieces



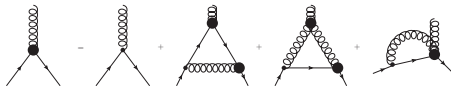
The elementary pieces



Gluon propagator $D_{\mu\nu}(p^2)$:



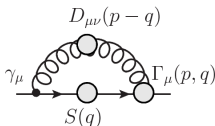
Quark-gluon vertex $\Gamma_{\mu}(p, q)$:



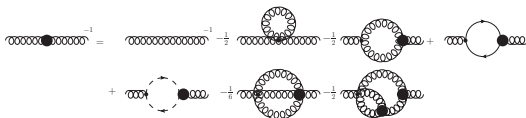
→ Couple to infinity of equations.

→ Gluonic part is crucial.

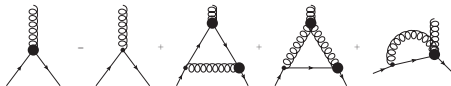
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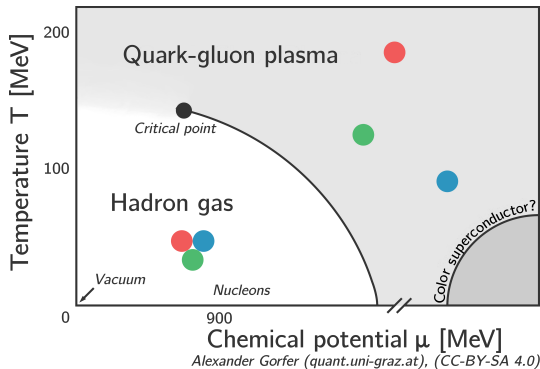
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Note: Effective interaction via $g^2 D_{\mu\nu}(p) \Gamma_{\mu}(p, q) \rightarrow Z_2 \tilde{Z}_3 D_{\mu\nu}^{(0)}(p) \gamma_{\mu} \mathcal{G}((p+q)^2)$

Another example: QCD phase diagram

Questions:

- Phases and transitions between them, critical point
- Experimental signatures



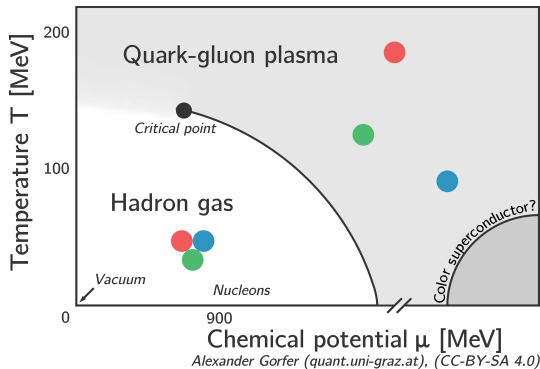
Theoretical challenges:

- Model description
- Mathematical, e.g., complex action for lattice QCD
- Complexity, e.g., truncations of function eqs.
- ...

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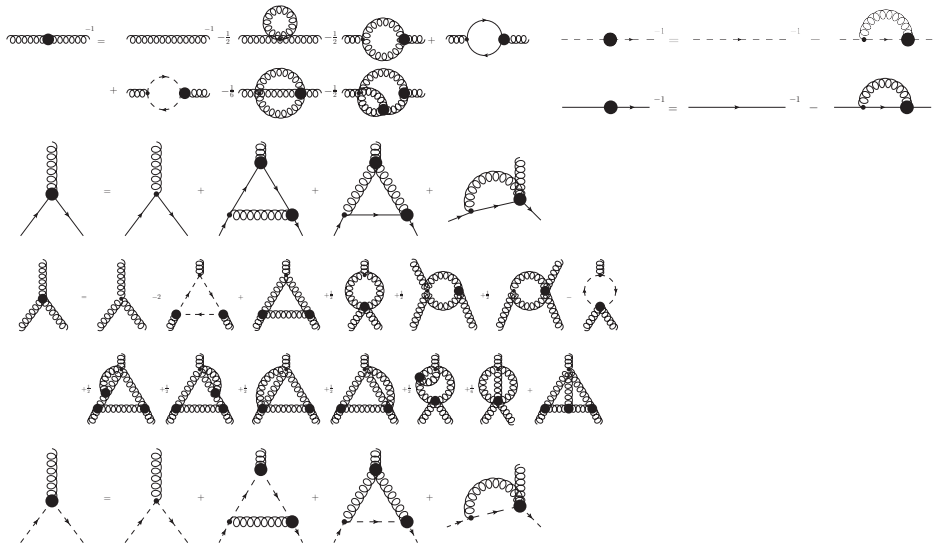
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- Systematics and tests?
comparison to other methods, self-tests?

Dyson-Schwinger equations



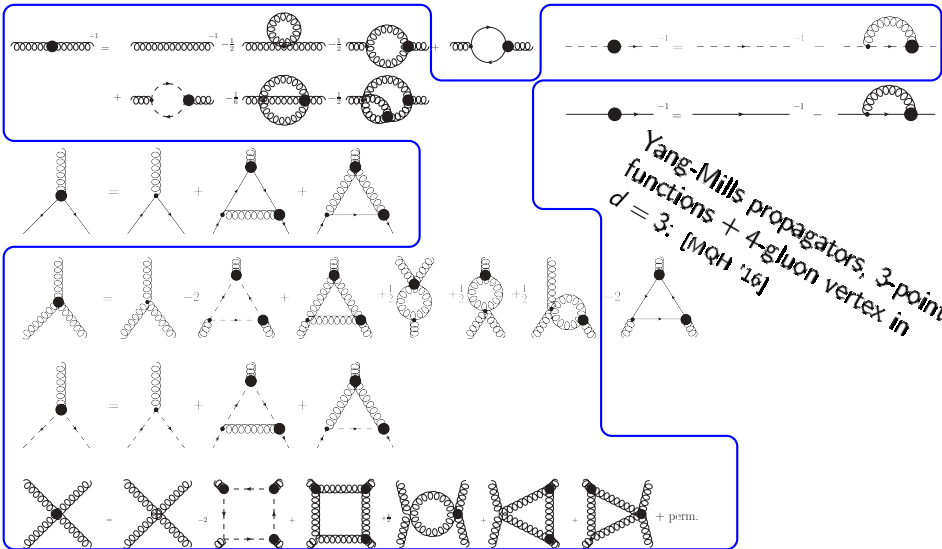
Coupled systems of Dyson-Schwinger equations

Diagrammatic equations for a fermion propagator (solid line) and a ghost propagator (dashed line). The fermion equation shows a self-energy correction (loop with a ghost) and a ghost loop correction. The ghost equation shows a ghost self-energy correction (loop with a fermion).

Diagrammatic equations for a 3-point vertex function. The first equation shows a vertex with a fermion line and a ghost line. The second equation shows a vertex with a ghost line and a fermion line. The third equation shows a vertex with two fermion lines and one ghost line. Each equation is a sum of diagrams representing different truncations of the Dyson-Schwinger equation.

quark propagator + 3-point functions: [Williams, Fischer, Heupel '15] \rightarrow application to bound states

Coupled systems of Dyson-Schwinger equations



Coupled systems of Dyson-Schwinger equations

Diagrammatic equations for the self-energy Σ and ghost self-energy Π . The first equation shows Σ^{-1} as a sum of a tree-level self-energy, a ghost loop, a fermion loop, and a gluon loop. The second equation shows Π^{-1} as a sum of a tree-level ghost self-energy and a ghost loop.

Diagrammatic equation for the ghost-gluon vertex Γ , showing it as a sum of a tree-level vertex, a ghost loop, and a gluon loop.

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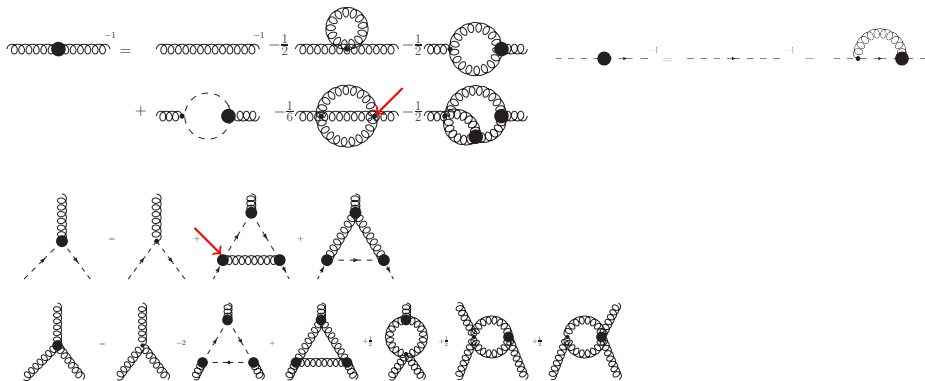
Diagrammatic equation for the ghost-gluon vertex Γ including gluon loops. The equation shows Γ as a sum of a tree-level vertex, a ghost loop, and a gluon loop, with additional terms involving gluon loops on the external lines.

Diagrammatic equation for the four-point function Γ . The equation shows Γ as a sum of a tree-level four-point function, a ghost loop, a gluon loop, and a ghost-gluon loop, plus a permutation term.

Three- and four-point functions
[MQH '17]

3PI system of equations

Three-loop expansion of PI effective action [Berges '04]:



Setting the scale

Only external input for Yang-Mills theory is the coupling α_s . It is related to the scale Λ_{YM} .

Setting the scale

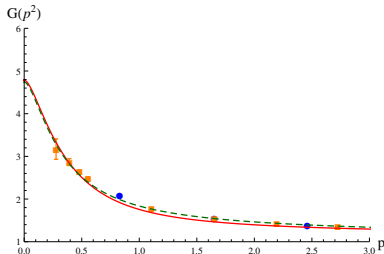
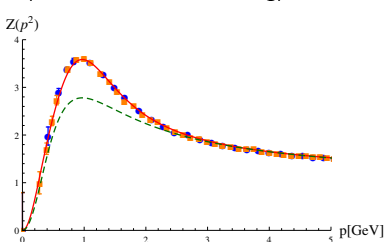
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Observables of Yang-Mills theory, e.g., glueballs to fix the scale. \rightarrow Impractical.

More convenient: Take scale from lattice calculations of the gluon propagator.
Scale via string tension of $\sigma = (440 \text{ MeV})^2$.

Example of a bottom-up calculation

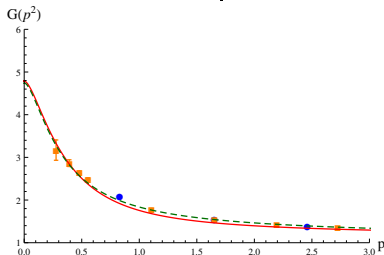
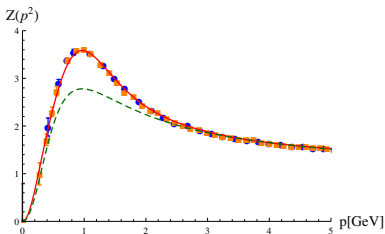
Propagators and ghost-gluon vertex with three-gluon vertex model:
 One-loop truncation of gluon propagator with an **optimized effective model**
 (contains zero crossing) [MQH, von Smekal '13; lattice: Sternbeck '06]:



Good quantitative agreement for ghost and gluon dressings.

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QCD is only this:

$$\mathcal{L} = -\frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \sum_j \bar{\psi}_j [i \not{\partial} - m_j] \psi_j$$

$$\text{WOBEL} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$\text{UND} \quad D_\nu = \partial_\nu + igA_\nu$$

Can we do with only that?

A lesson from three dimensions?

Three-dimensional Yang-Mills theory is finite.

→ No renormalization

→ Leading perturbative contributions $\propto g^2/p$

⇒ Testbed for functional calculations.

Various methods employed:

- Lattice: [Bornyakov, Cucchieri, Maas, Mendes, Mitrjushkin, Rogalyov, ...]
- Coupled propagator DSEs: [Maas, Wambach, Grüter, Alkofer '04]
- (R)GZ: [Dudal, Gracey, Sorella, Vandersickel, Verschelde '08]
- DSEs of PT-BFM: [Aguilar, Binosi, Papavassiliou '10]
- YM + mass term: [Tissier, Wschebor '10, '11]
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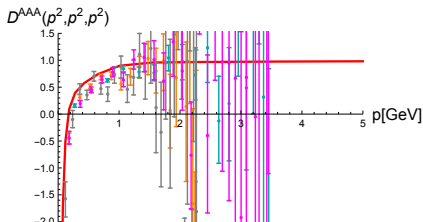
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Study effect of individual diagrams. . .

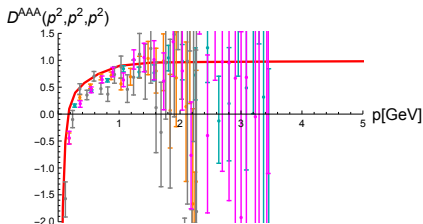
Cancellations in three-gluon vertex



[MQH '16; lattice: Cucchieri, Maas, Mendes '08]

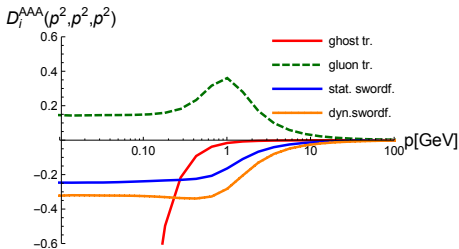
- Close to tree-level above 1 GeV
- Good agreement with lattice data.
- Linear IR divergence [Pelaez, Tissier, Wschebor '13; Aguilar et al. '13]
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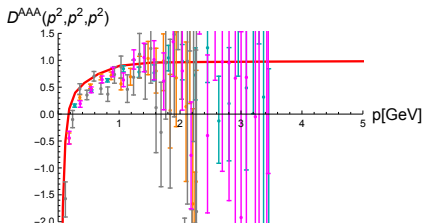
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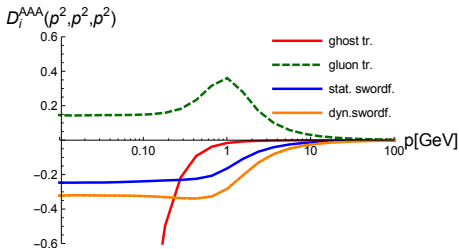
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→ In four dimensions similar qualitative effects, but renormalization complicates things.

UV behavior of the gluon propagator

Resummed **one-loop** order: anomalous dimension $\gamma = -13/22$

$$\left(1 + \frac{\alpha(s)11N_c}{12\pi} \ln \frac{p^2}{s}\right)^\gamma$$

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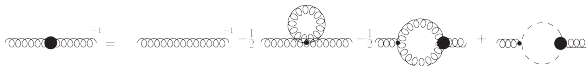
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Origin in resummation of higher order diagrams.

However, one-loop truncation discards some terms.



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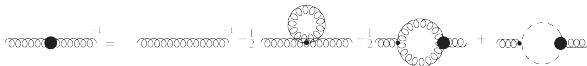
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→ Puts constraints on UV behavior of vertices if one wants a self-consistent solution [von Smekal, Hauck, Alkofer '97].

Fixing the UV behavior of the gluon propagator I

First possibility:

Modify the **UV behavior** of the integrand, e.g., replace the renormalization constant by a momentum dependent function [von Smekal, Hauck, Alkofer '97] ('RG improvement'), adapt employed models.

$$\tilde{Z}_1 \rightarrow f(p^2)$$

Part of the modeling.

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Examples:

- Yang-Mills propagators, e.g., [von Smekal, Hauck, Alkofer '97; Fischer, Alkofer '02; MQH, von Smekal '12, '14]
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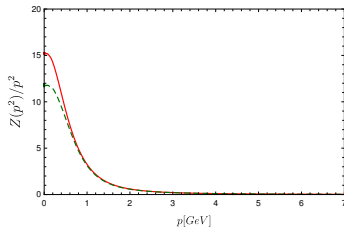
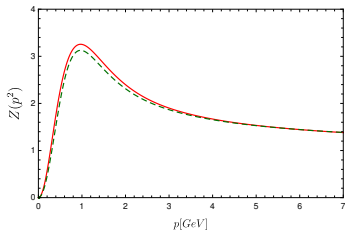
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IR completion has an effect on the gluon propagator [MQH, von Smekal '14]:



Fixing the UV behavior of the gluon propagator II

Second possibility:

Include higher perturbative terms.

Worked out analytically for ϕ^3 -theory [MQH '18].

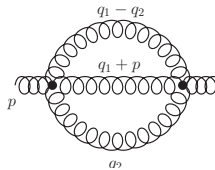
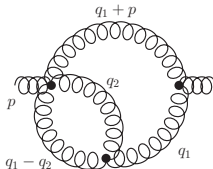
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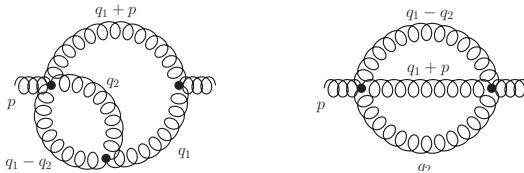
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→ Contributions also from renormalization constants in front of one-loop diagrams.

$$\text{Gluon Propagator}^{-1} = \text{Gluon Propagator}^{-1} - \frac{1}{2} \text{Gluon Loop}^{-1} - \frac{1}{2} \text{Gluon Loop}^{-1} + \text{Gluon Loop}^{-1} + \text{Gluon Loop}^{-1}$$

Z_1 \tilde{Z}_1

⇒ All two-loop contributions in the gluon propagator are included.
And higher contributions...

Resummed behavior

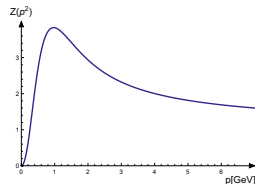
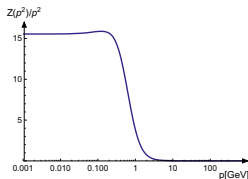
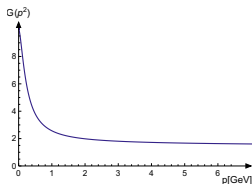
Minimal requirements to obtain one-loop resummed behavior:

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Resummed behavior

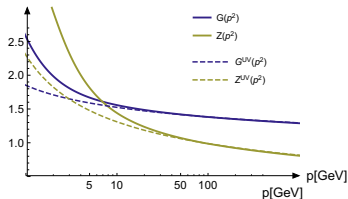
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[propagator+ghost-gluon eqs. full, 3-gluon vertex model, bare 4-gluon vertex]

- Resummed behavior is recovered [MQH '17].



Extending truncations

Various ways to extend truncations:

- Vertex tensors beyond tree-level tensor
- Include neglected diagrams
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Extensions also test the previous truncations!

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In the following:

- Three-gluon vertex
- Four-point functions
- Coupling the equations

Three-gluon vertex DSE

Talk by Papavassiliou: “Three-gluon vertex: The new frontier”

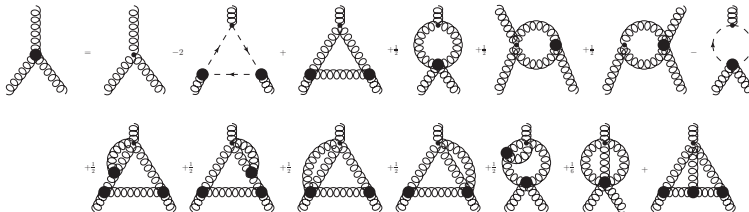
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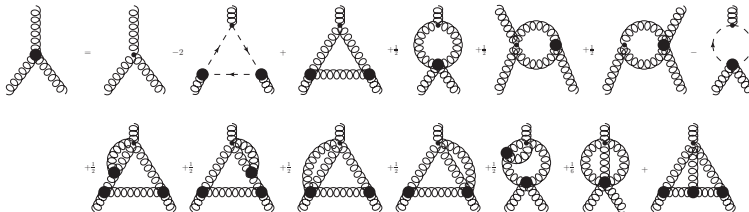


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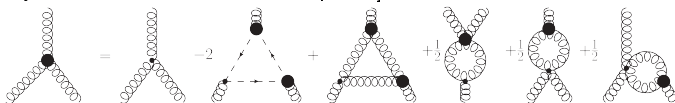
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Perturbative one-loop truncation [Blum, MQH, Mitter von Smekal '14; Eichmann, Alkofer, Vujanovic '14; Williams, Fischer, Heupel '16]:

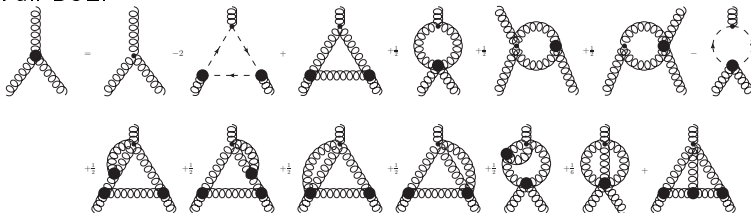


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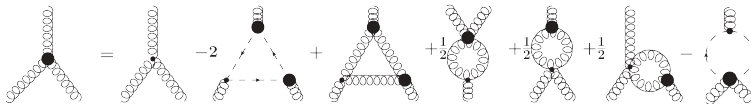
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Full DSE:



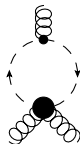
Non-perturbative one-loop truncation [MQH '17]:



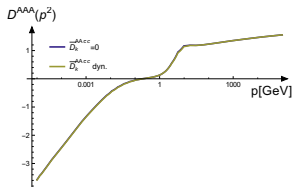
Influence of two-ghost-two-gluon vertex



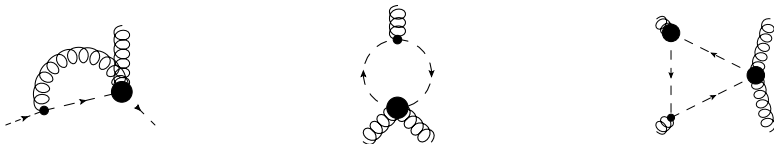
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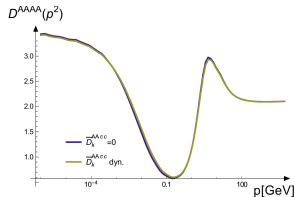
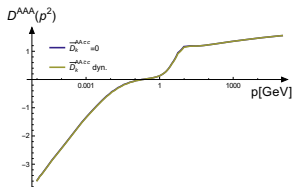
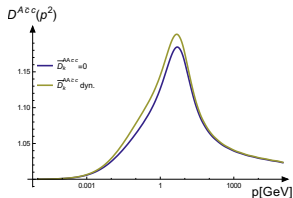
Coupled system of ghost-gluon, three-gluon and four-gluon vertices **with and without** two-ghost-two-gluon vertex [MQH '17]:



Influence of two-ghost-two-gluon vertex

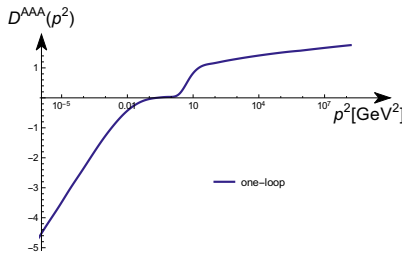


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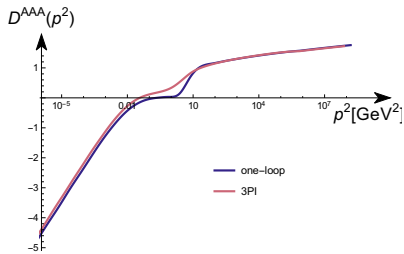


- **Small** influence on ghost-gluon vertex ($< 1.7\%$)
- **Negligible** influence on three- and four-gluon vertices.

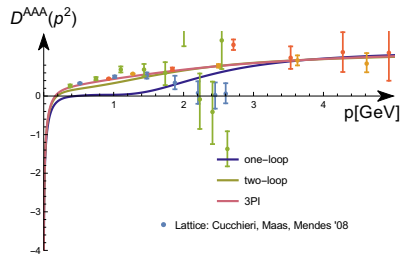
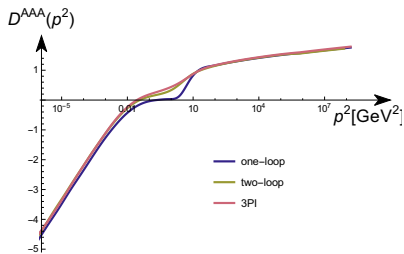
Three-gluon vertex results



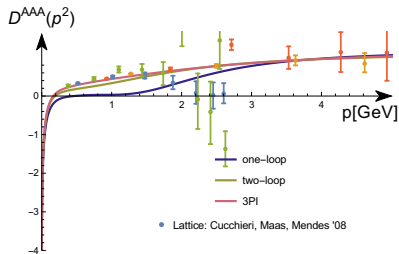
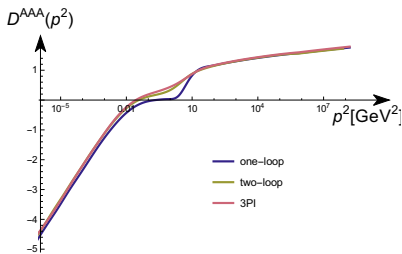
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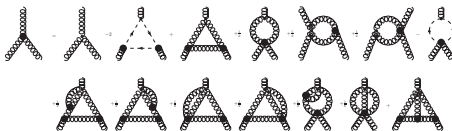
Three-gluon vertex results



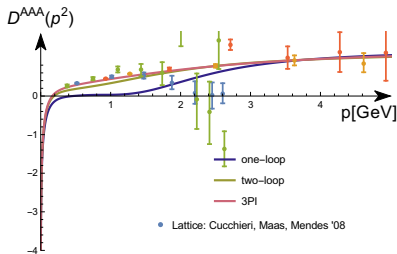
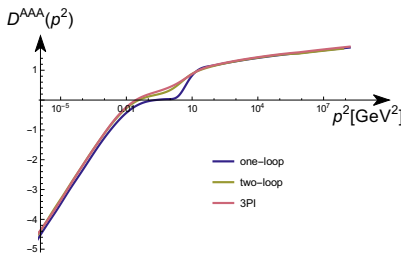
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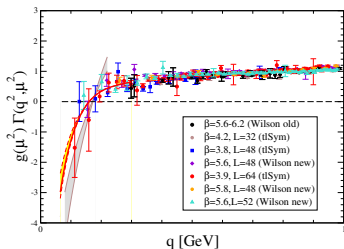
- **Two-loop truncation:** All diagrams except the one with a five-point function.
- One-momentum configuration approximation.



Three-gluon vertex results



- Difference between two-loop DSE and 3PI smaller than lattice error.
- Zero crossing in agreement with other approaches, e.g., [Pelaez et al. '13; Aguilar et al. '13; Cyrol et al. '15; Athenodorou et al. '16; Duarte et al. '16; Sternbeck et al. '17]



[Athenodorou et al. '16, '18]

The two-ghost-two-gluon vertex

Non-primitively divergent correlation function \rightarrow No guide from tree-level tensor. \rightarrow Use full basis.

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Two-ghost-two-gluon vertex



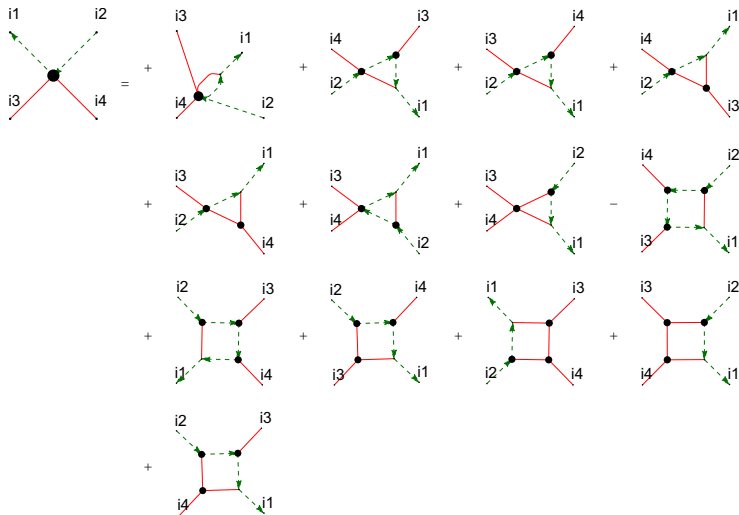
$$\Gamma_{\mu\nu}^{AA\bar{c}c,abcd}(p, q; r, s) = g^4 \sum_{k=1}^{40} \rho_{\mu\nu}^{k,abcd} D_{k(i,j)}^{AA\bar{c}c}(p, q; r, s)$$

with

$$\rho_{\mu\nu}^{k,abcd} = \sigma_i^{abcd} \tau_{\mu\nu}^j, \quad k = k(i, j) = 5(i - 1) + j$$

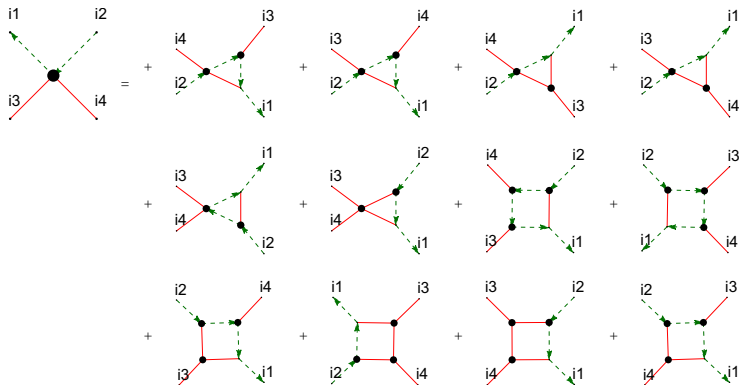
The two-ghost-two-gluon vertex DSE

2 DSEs, choose the one with the ghost leg attached to the bare vertex
 → Truncation discards only one diagram.



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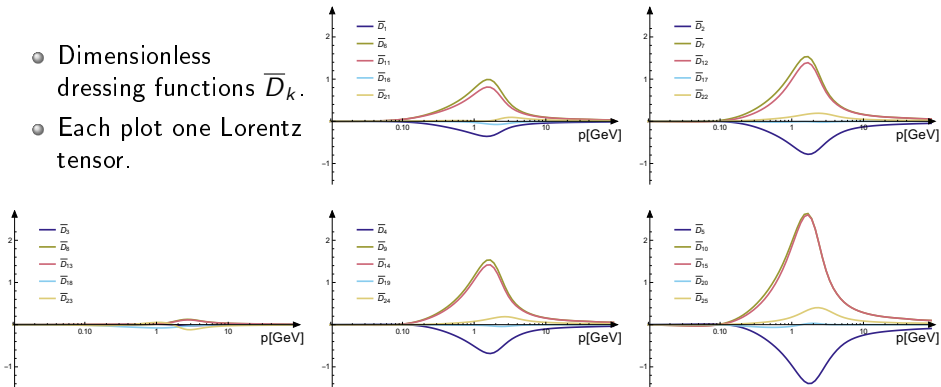
2 DSEs, choose the one with the ghost leg attached to the bare vertex
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Results for the two-ghost-two-gluon vertex

Kinematic approximation: one-momentum configuration

- Dimensionless dressing functions \bar{D}_k .
- Each plot one Lorentz tensor.



→ Two classes of dressings: 13 very small, 12 not small

→ No nonzero solution for $\{\sigma_6, \sigma_7, \sigma_8\}$ found.

[MQH '17]

Results for the four-ghost vertex

Kinematic approximation: one-momentum configuration

Four-ghost vertex



$$\Gamma^{\bar{c}\bar{c}cc,abcd}(p, q, r, s) = g^4 \sum_{k=1}^8 \sigma^{k,abcd} E_k^{\bar{c}\bar{c}cc}(p, q, r, s).$$

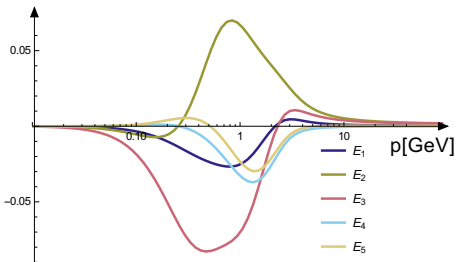
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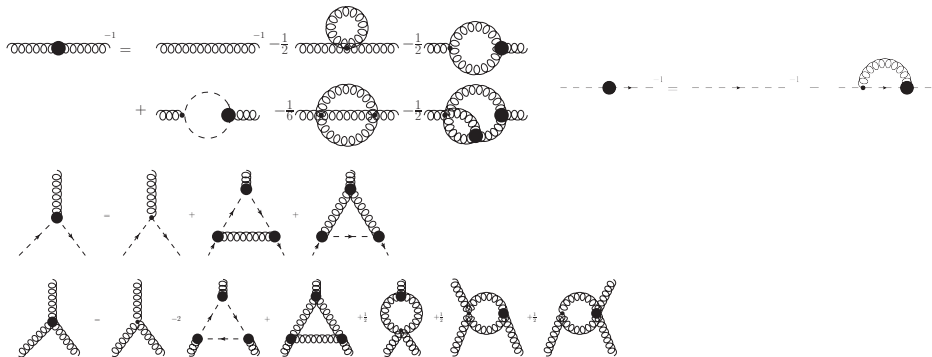
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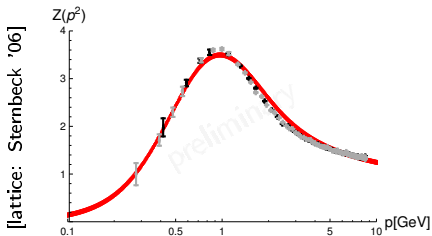
→ All dressings very small.

[MQH '17]

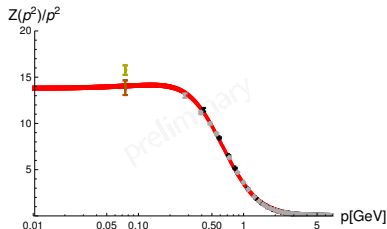
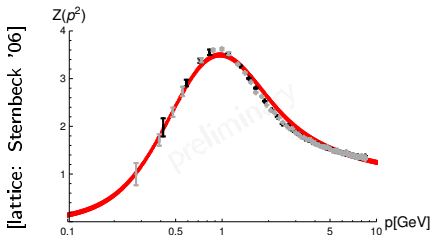
3PI system of primitively divergent correlation functions



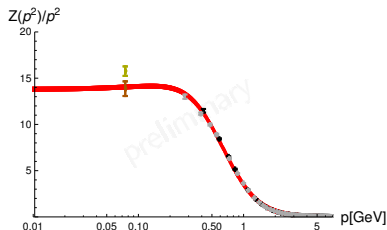
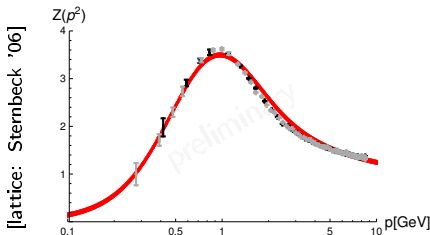
Results for fully coupled 3PI system



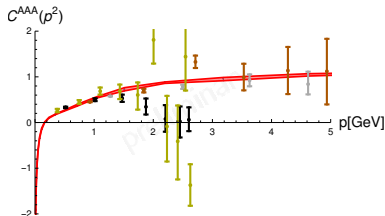
Results for fully coupled 3PI system



Results for fully coupled 3PI system



- Details of renormalization crucial!
- Other details also important.
- Very small angle dependence of three-gluon vertex.
- Slight bending down of gluon propagator in IR.



[lattice: Cucchieri,
Maas, Mendes '08]

Open checks

- Effects of larger tensor bases, in particular of the three-gluon vertex
- Renormalization

What tests can be done?

Couplings

Couplings can be defined from every vertex, e.g., [Allés et al. '96; Alkofer et al., '05; Eichmann et al. '14]:

$$\alpha_{\text{ghg}}(p^2) = \alpha(\mu^2) (D^{A\bar{c}c}(p^2))^2 G^2(p^2) Z(p^2),$$

$$\alpha_{3g}(p^2) = \alpha(\mu^2) (C^{AAA}(p^2))^2 Z^3(p^2),$$

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They must agree perturbatively (STIs).

This agreement is important also in coupled systems of functional equations and constitutes a highly non-trivial

check of a truncation [Mitter, Pawłowski, Strodthoff '14].

Couplings

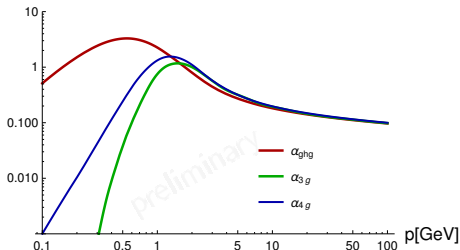
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Ghost-gluon vs. other couplings: Further checks required.

Renormalization with a hard UV cutoff

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Due to the anomalous running, this behaves as (at one-loop resummed level)

$$\frac{\Lambda_{\text{QCD}}^2}{p^2} (-1)^{2\delta} \Gamma(1 + 2\delta, -\ln(\Lambda^2/\Lambda_{\text{QCD}}^2))$$

Note: Appears already **perturbatively!**

Many ways to deal with them, e.g., Brown-Pennington projector, modifications of integrands/vertices, fitting, seagull identities,

Examples for renormalization with a hard UV cutoff

Here: Second renormalization condition

Breaking of gauge covariance \rightarrow mass counter term [Collins '84]

Renormalization condition: $D(0) = c$ [Meyers, Swanson '14]

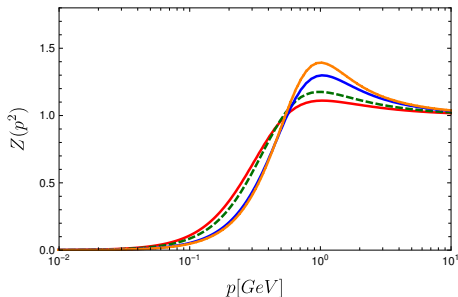
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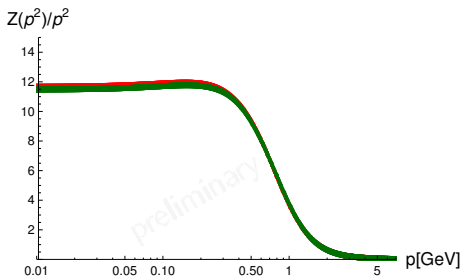
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Extreme example: One-loop truncation with bare vertices in three dimensions [MQH '16].

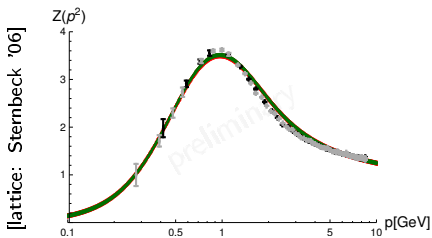


Better example: Full system with one-momentum configuration approximation.



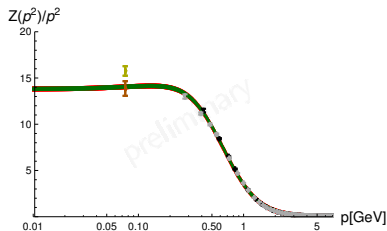
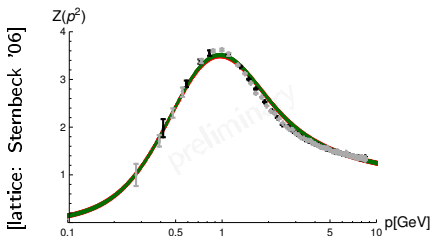
Results for fully coupled 3PI system revisited

Vary the renormalization condition $D(0)$:



Results for fully coupled 3PI system revisited

Vary the renormalization condition $D(0)$:



→ Two solutions on top of each other. $D(0)$ is not a parameter of the system.

Four-gluon vertex

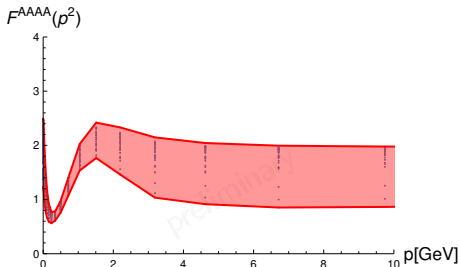
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Deviations from leading perturbative behavior small, but larger angle dependence than three-gluon vertex.

Summary and conclusions

Towards a systematic understanding of truncations of functional equations to establish them as a first principles method.

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- Add quarks
- Finite temperature
- Bound states
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Thank you for your attention!