QCD correlation functions from the FRG

Results and Applications

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Universität Heidelberg & ExtreMe Matter Institute

Trento, September 20th 2018

GEFÖRDERT VOM

Bundesministerium
für Bildung
und Forschung

ISOQUANT
SFB1225

EMMI

Motivation

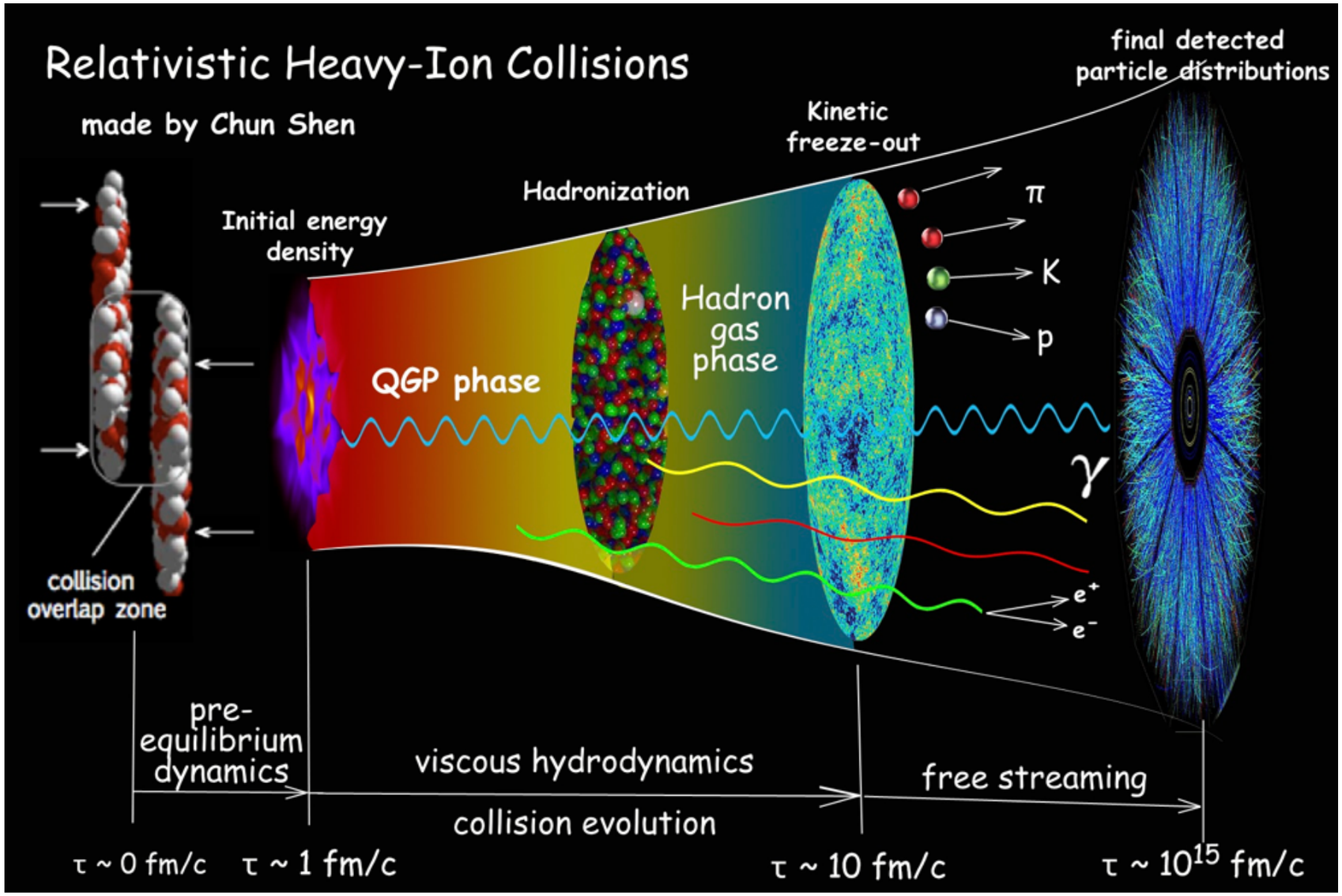
Motivation

'Never underestimate the joy people derive from hearing something they already know'

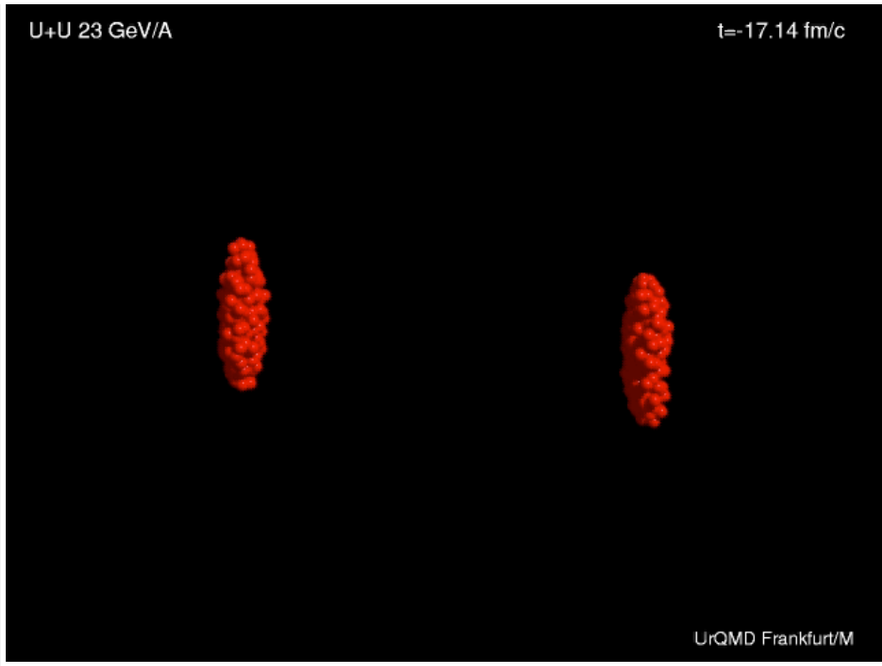
attributed to Fermi

Heavy ion collisions

'Phases/Epochs' of a heavy ion collision & time scales



Simulation of a heavy ion collision



UrQMD Frankfurt/M

$$1\mathrm{fm/c} \sim 3 \times 10^{-24}\mathrm{seconds}$$

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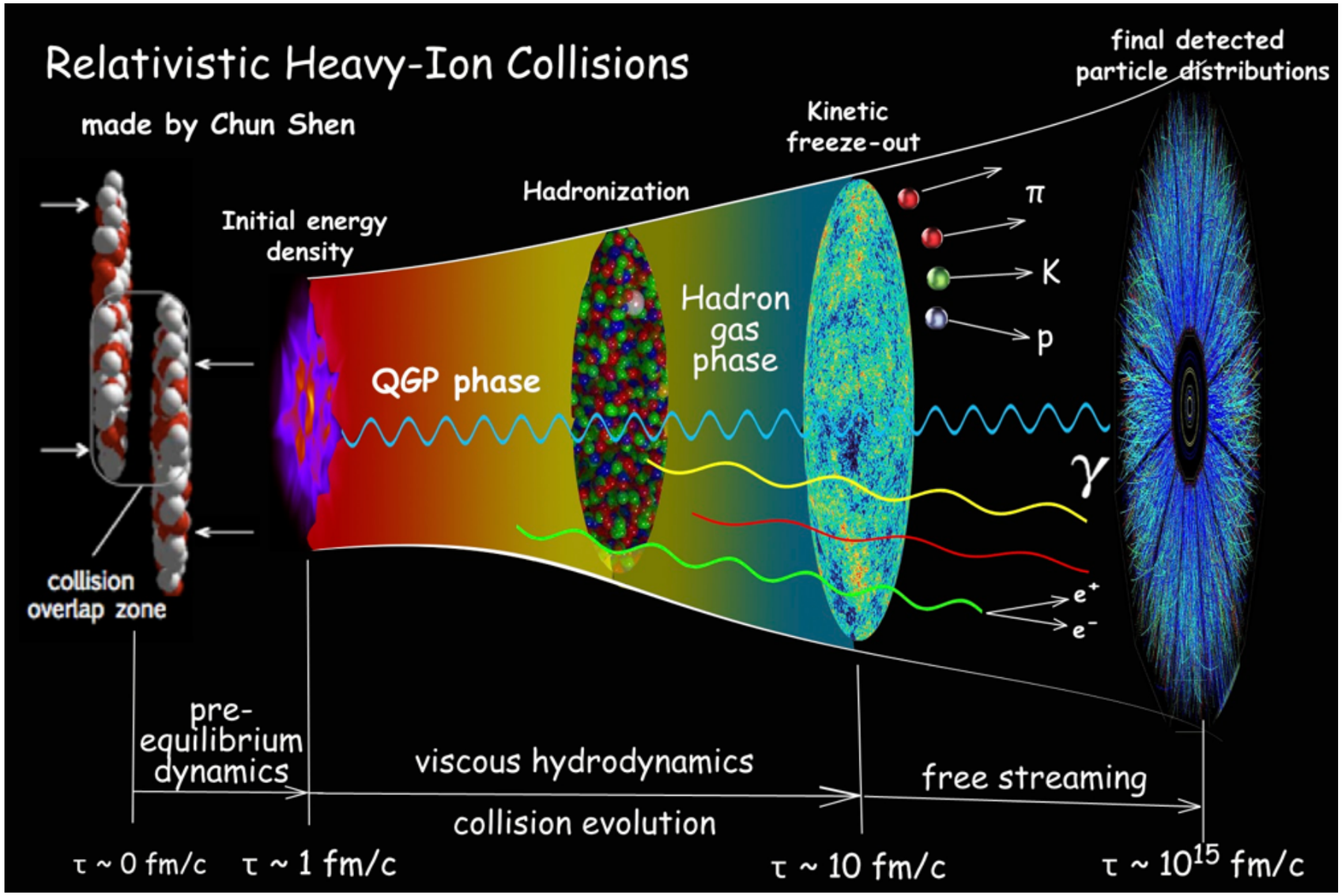

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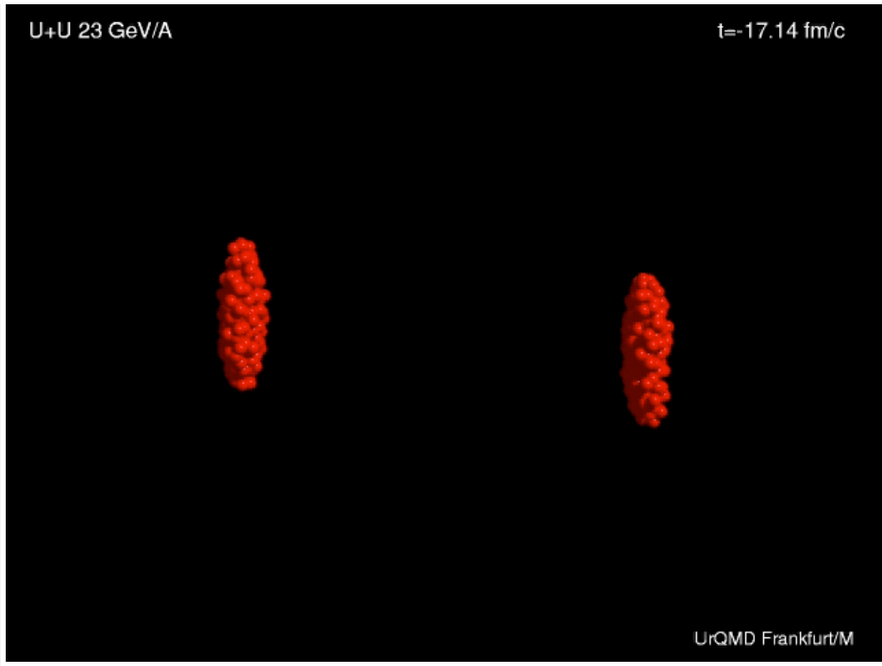

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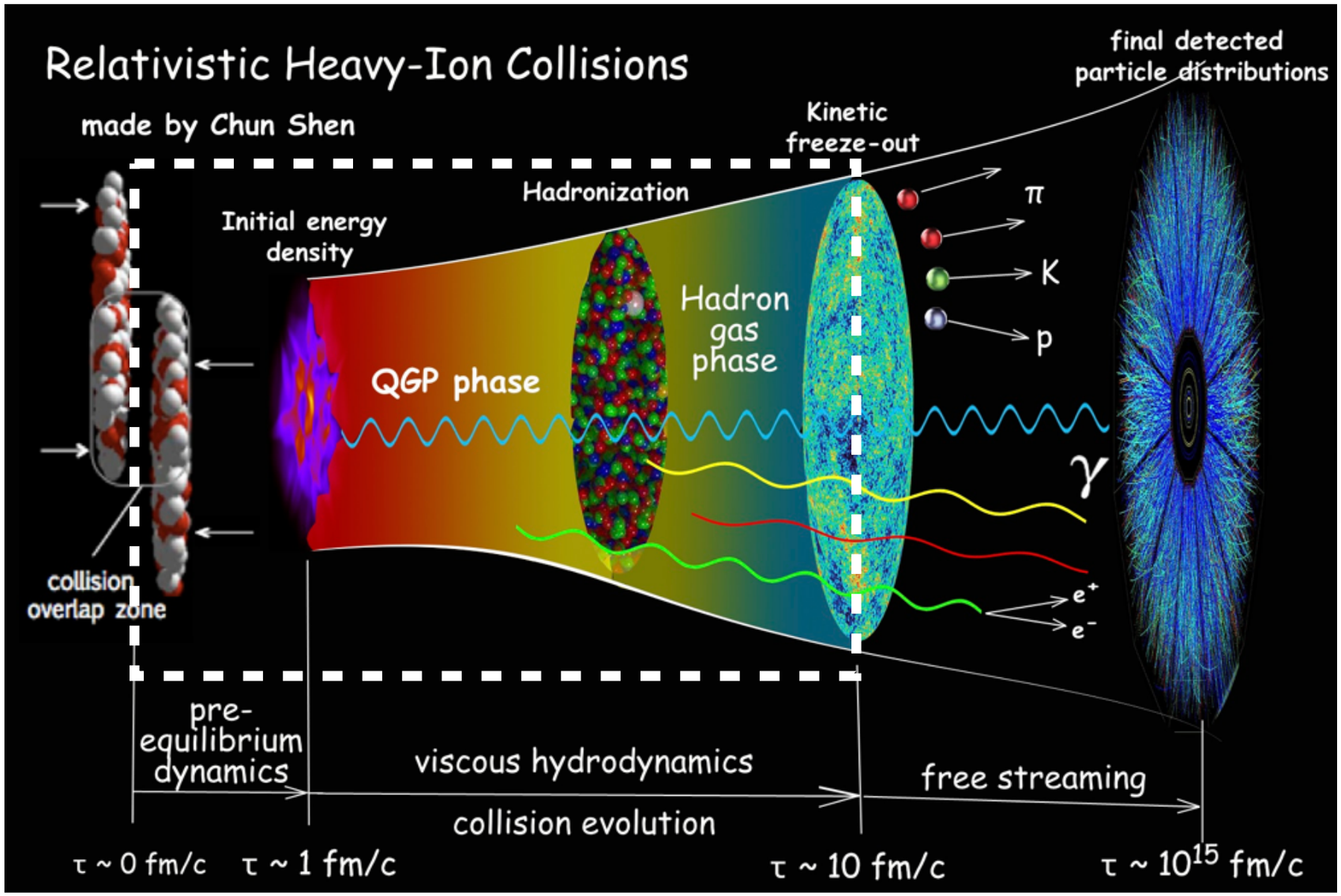

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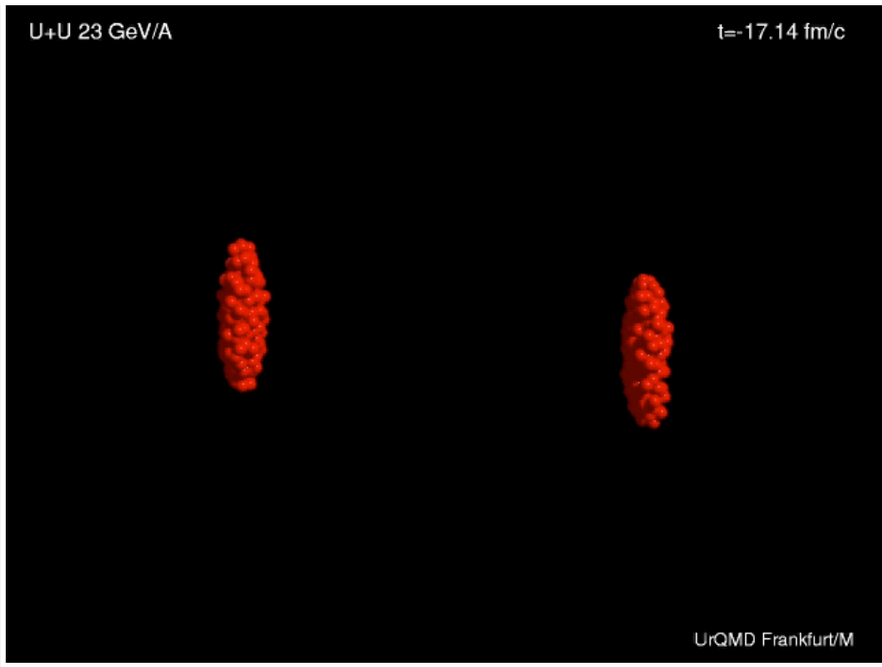
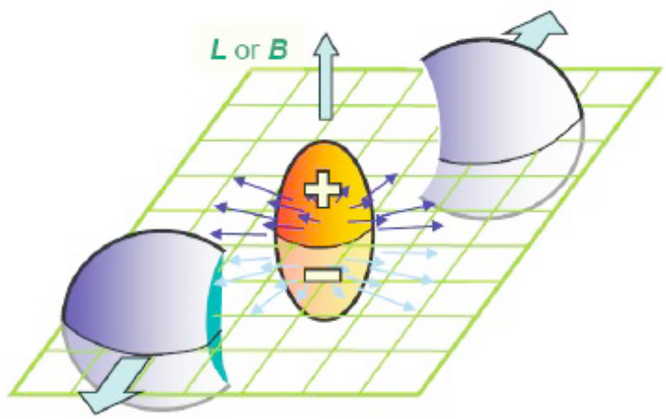


UrQMD Frankfurt/M

Flow

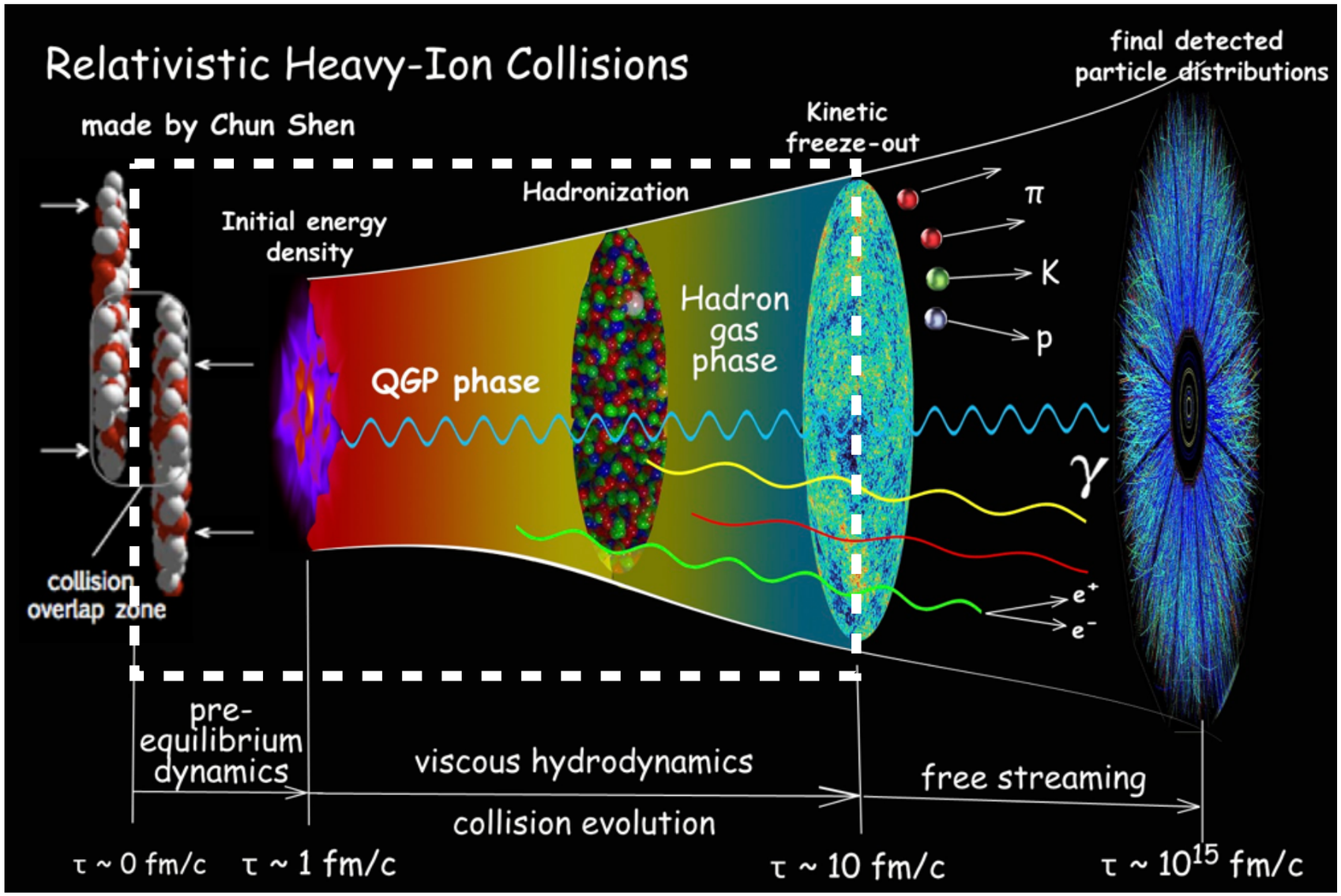


$$1\mathrm{fm/c} \sim 3 \times 10^{-24}\mathrm{seconds}$$

$$\frac{dN}{d(\varphi - \Psi_R)} = \frac{N_0}{2\pi}\left(1 + 2\sum_n v_n \cos[\,n\,(\varphi - \Psi_R)]\right)$$

Heavy ion collisions

'Phases/Epochs' of a heavy ion collision & time scales



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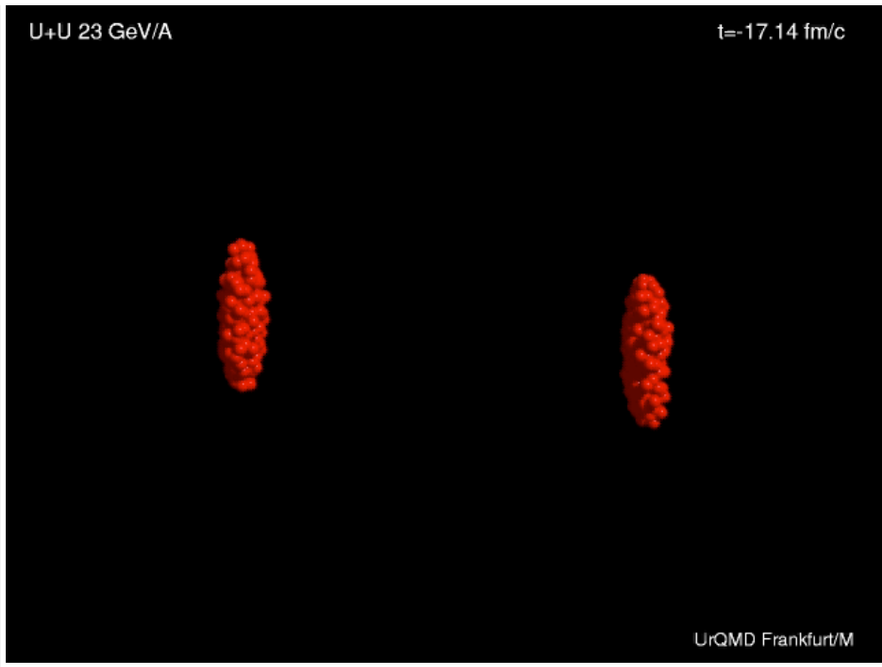

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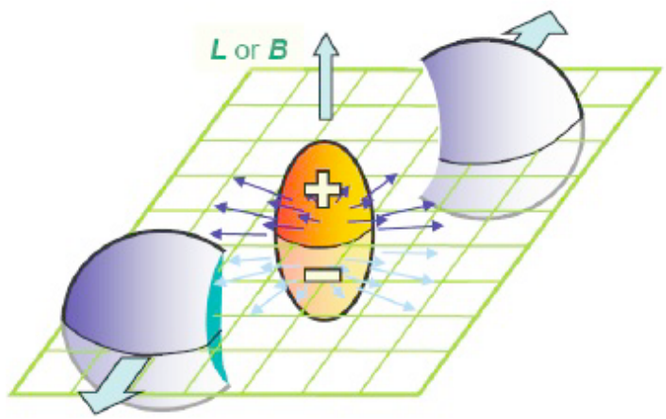

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QCD: $\boxed{v_n}$

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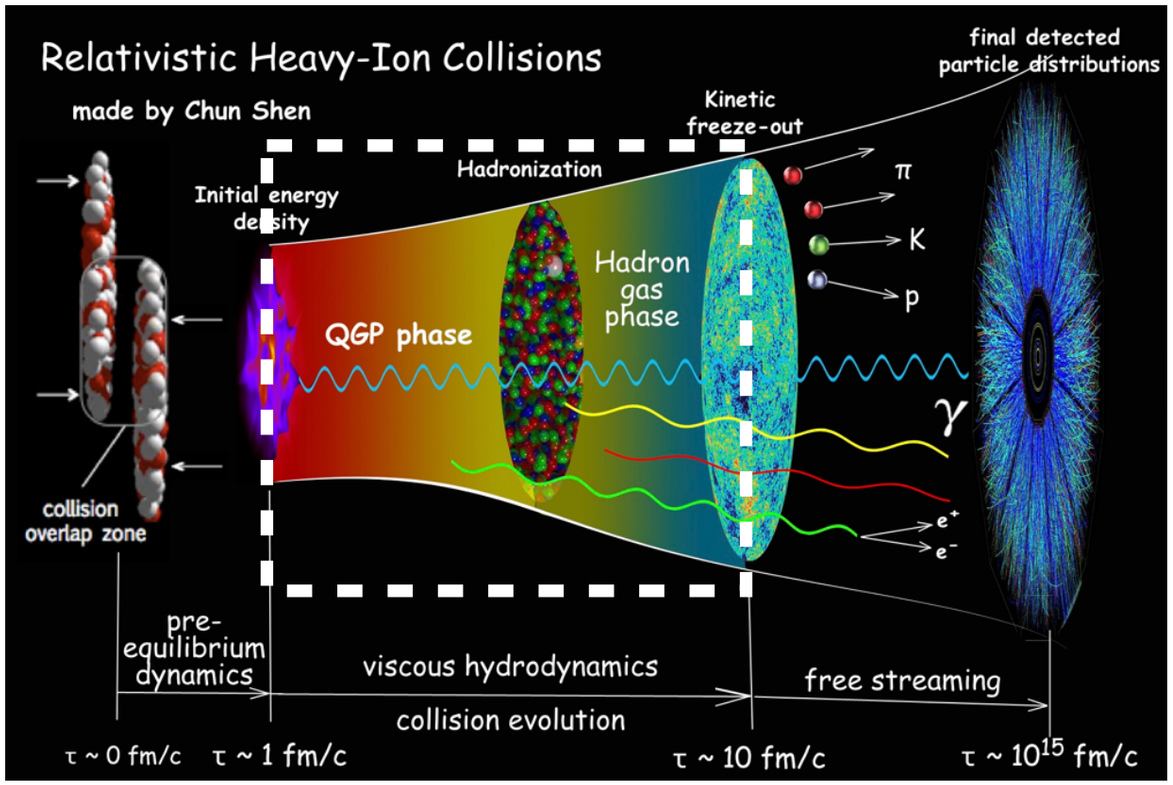
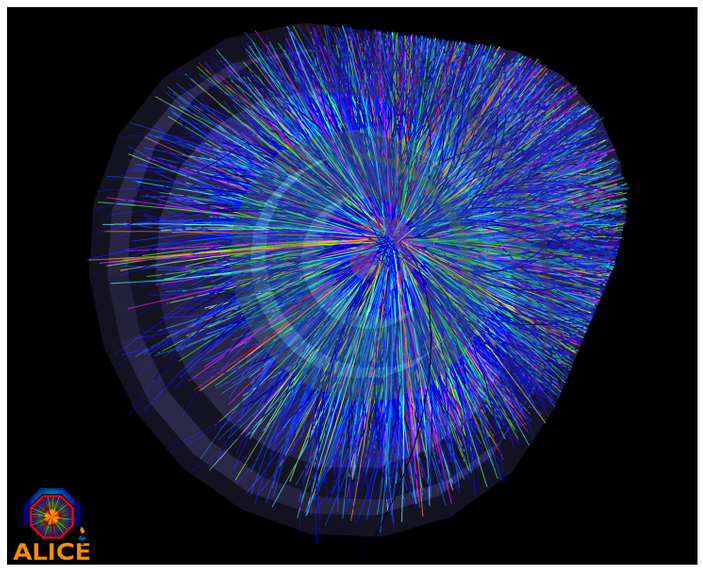
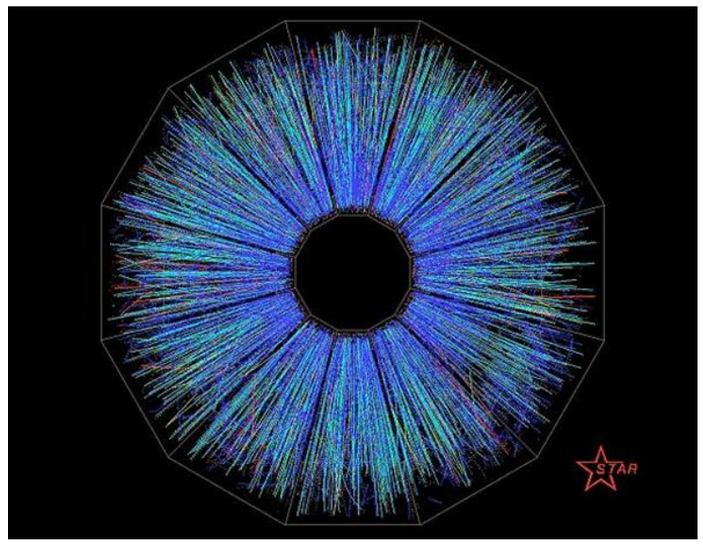
Heavy ion collisions



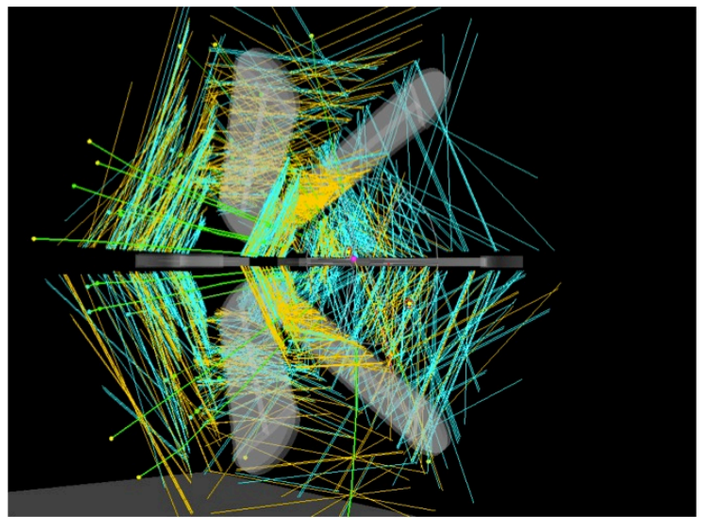
LHC

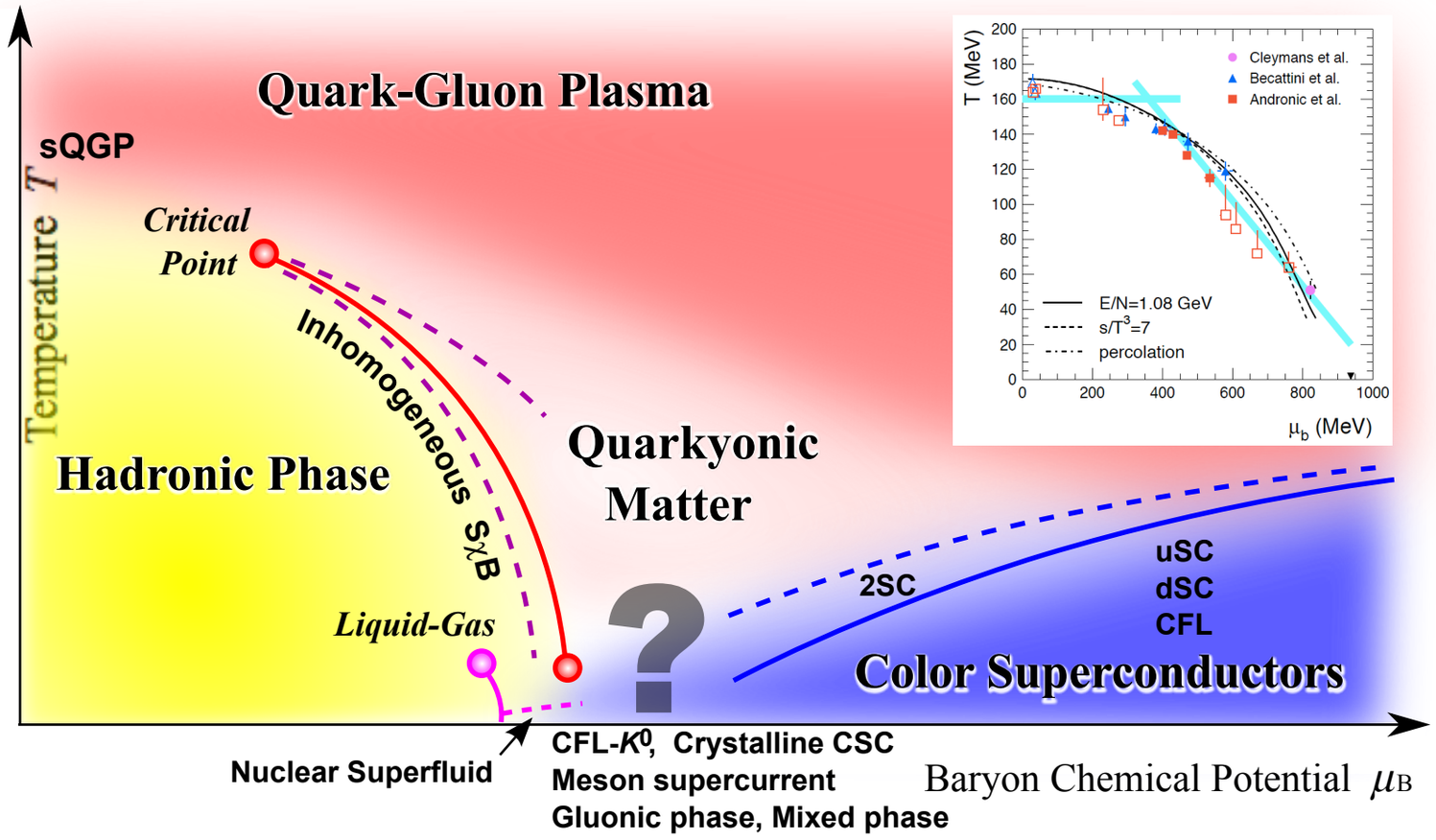


RHIC



HADES





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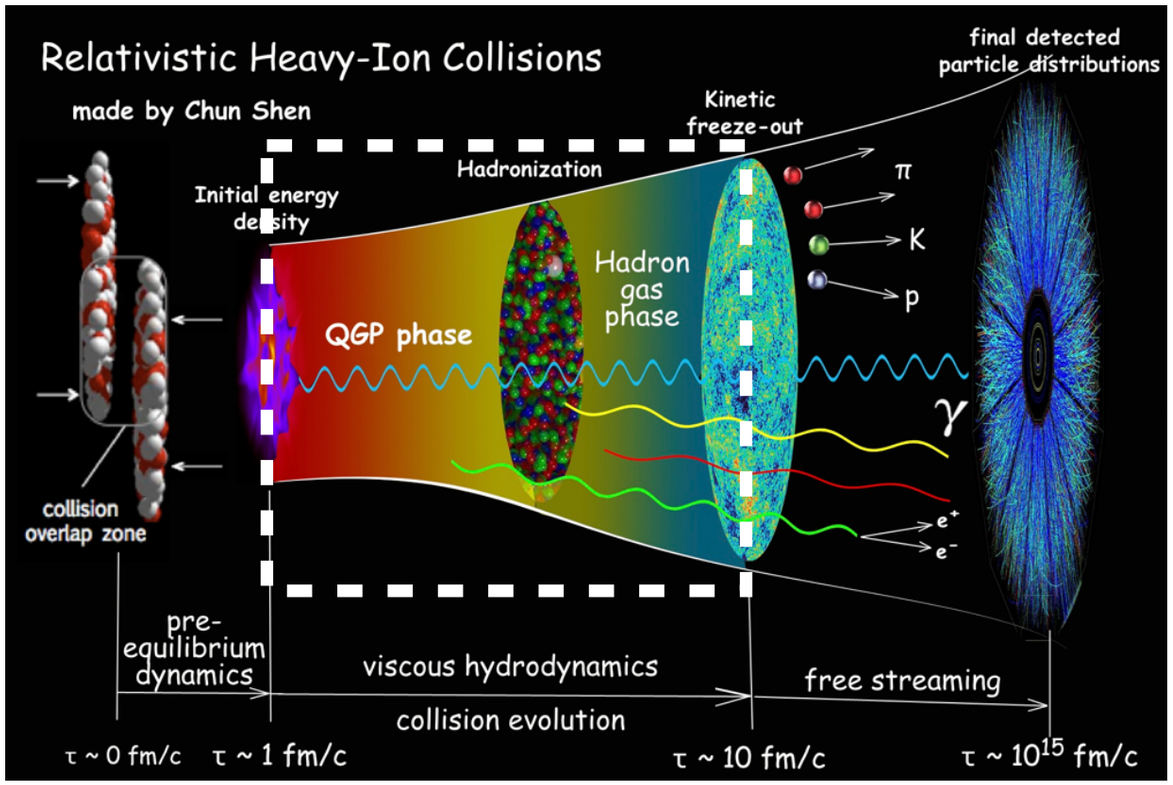

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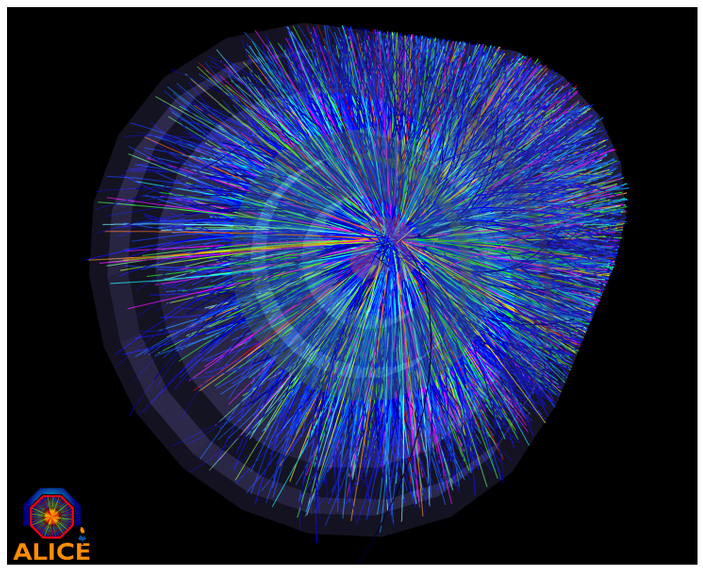

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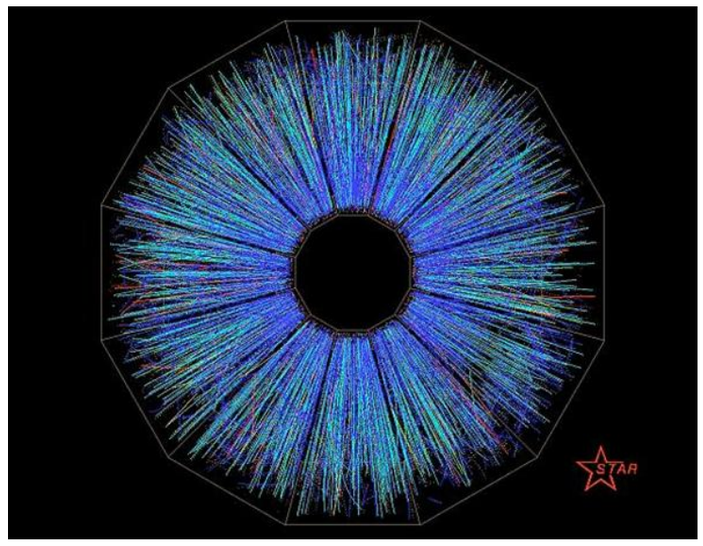
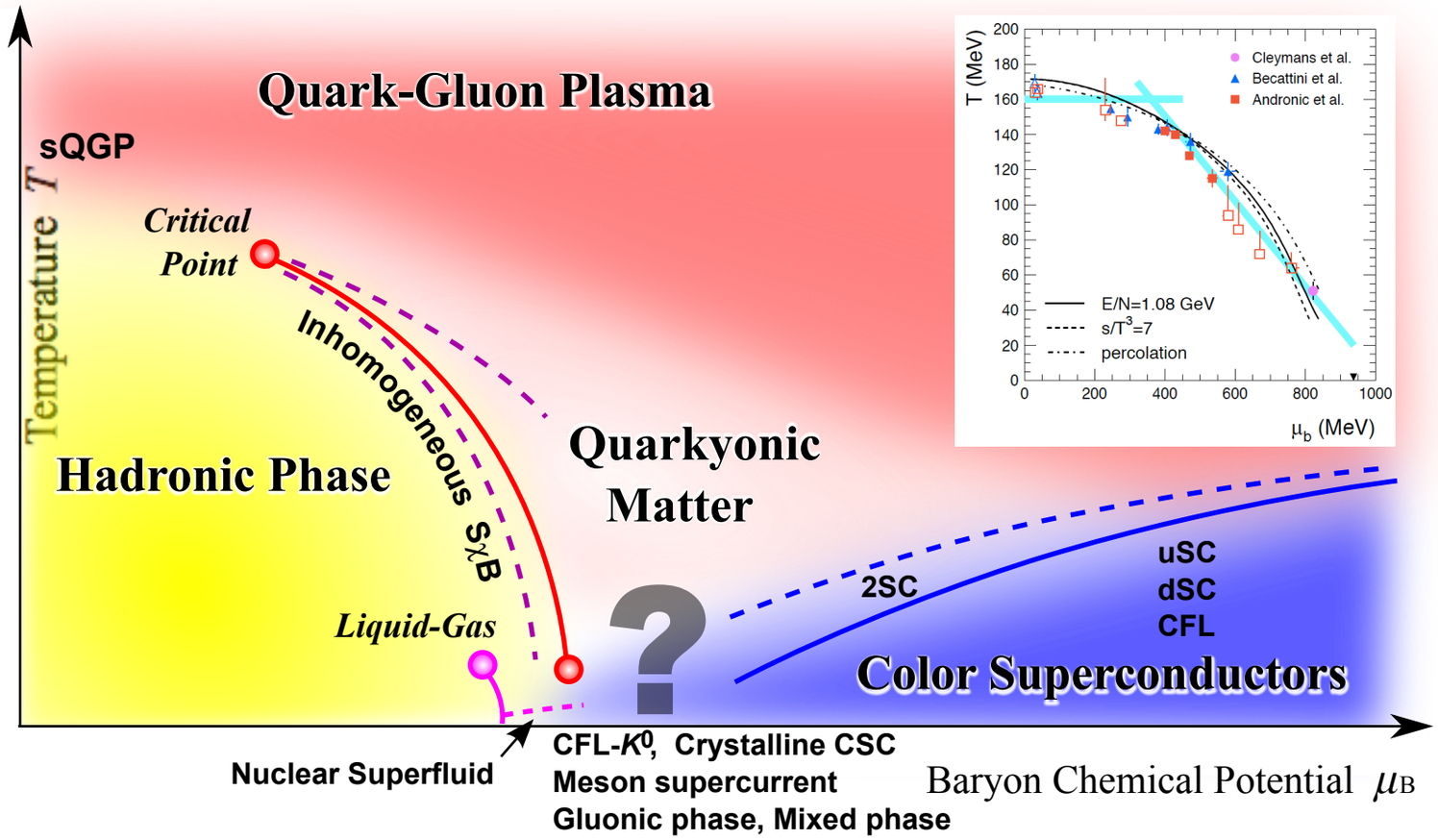
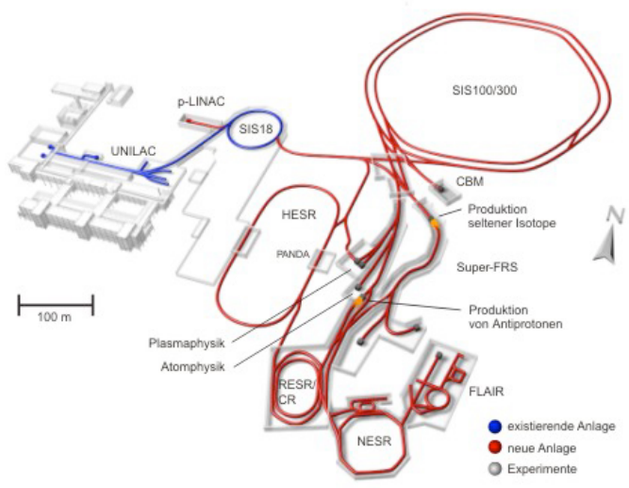


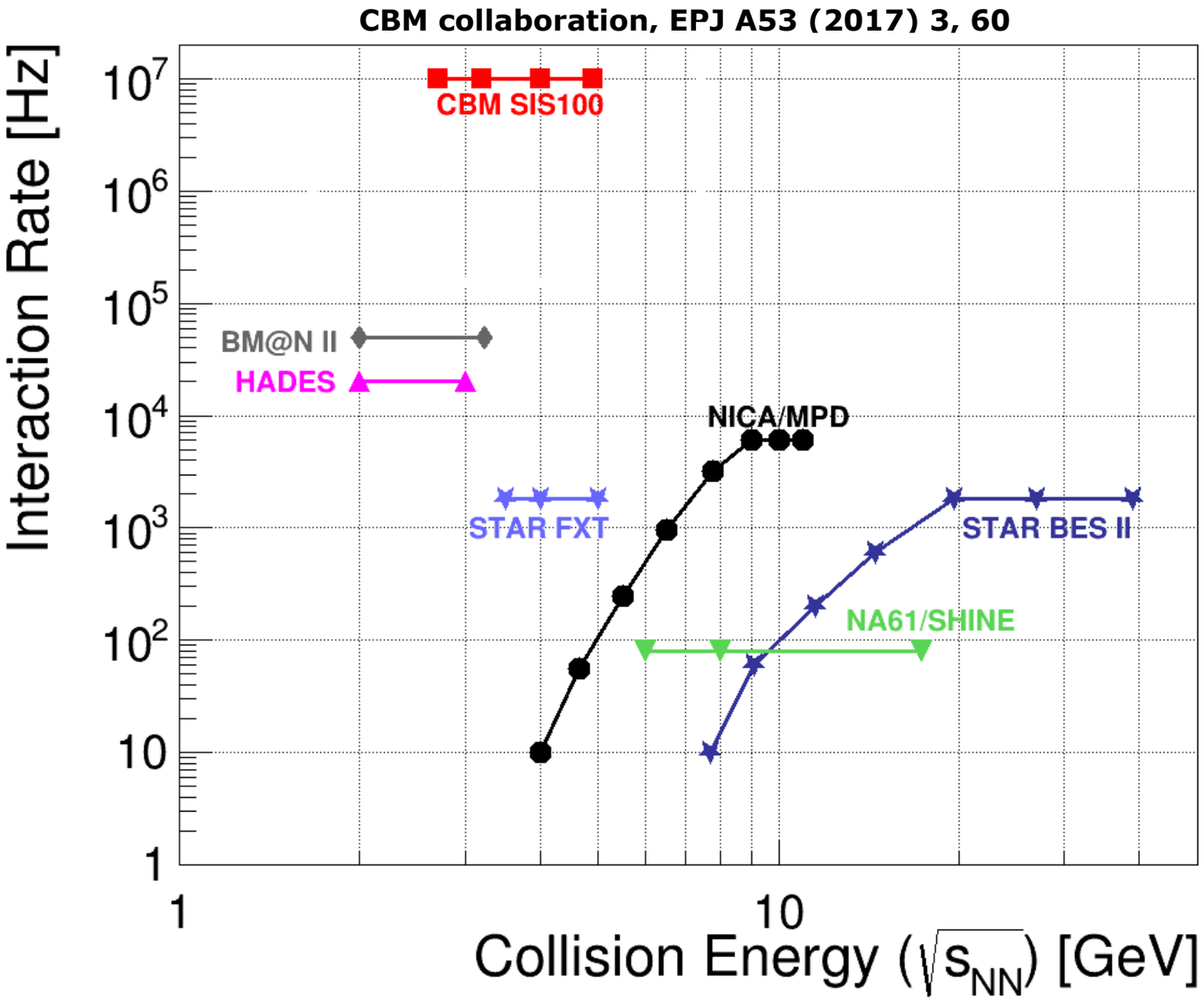


FAIR



NICA

Heavy ion collisions



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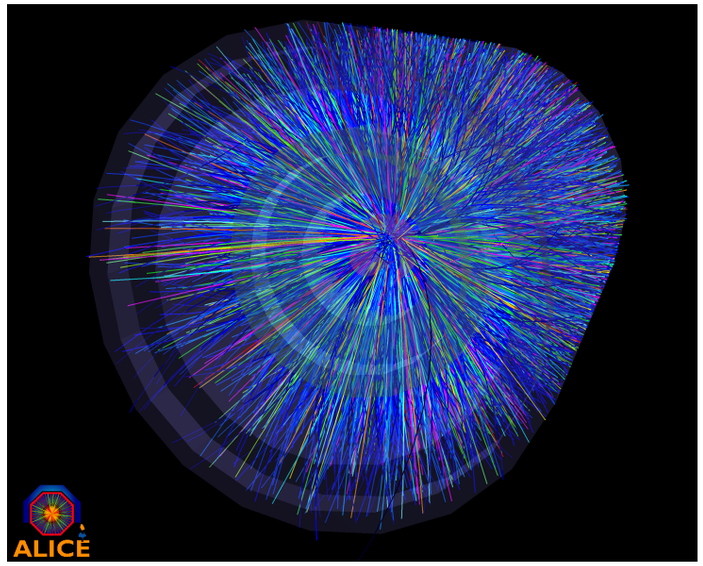

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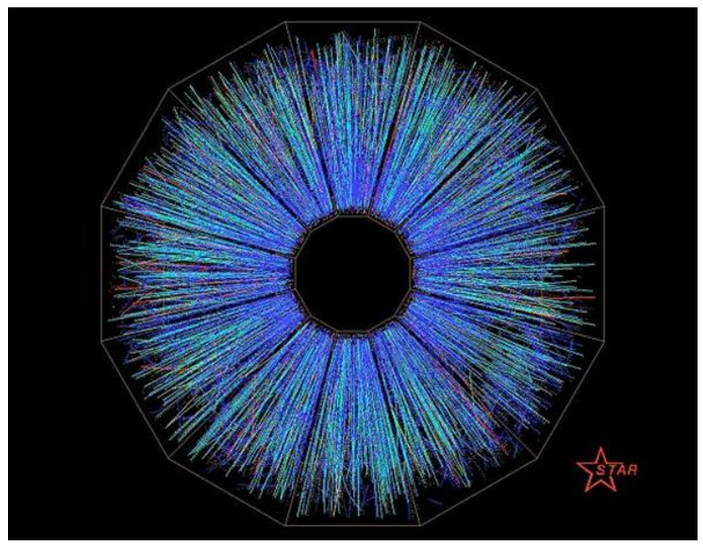

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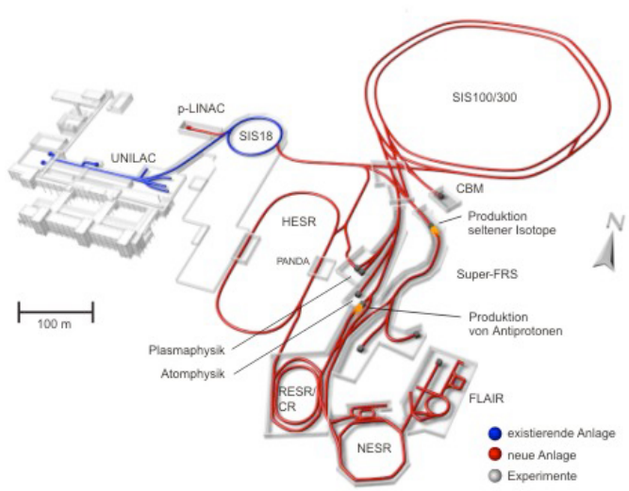

NICA

Outline

- QCD from functional methods

Applications

- QCD-assisted hydrodynamics

- QCD-assisted transport

- Summary & outlook

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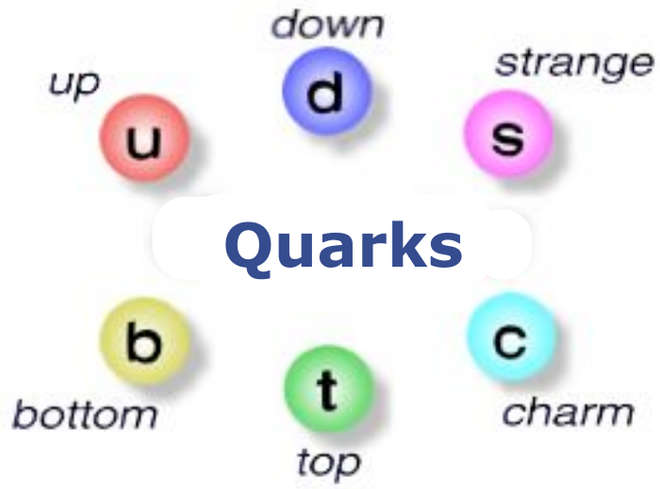
- QCD from functional methods



Gluons

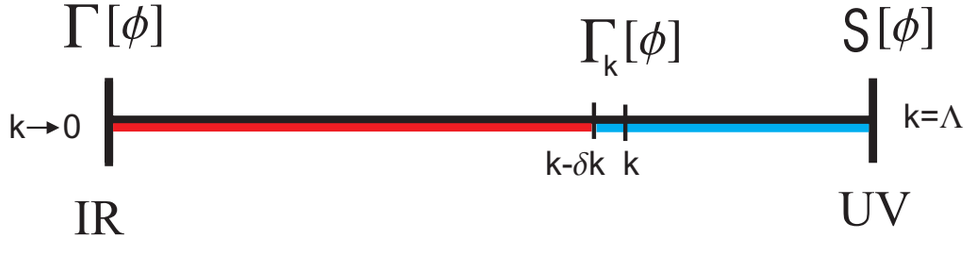
Applications

- QCD-assisted hydrodynamics
- QCD-assisted transport
- Summary & outlook

Functional RG for QCD

eg. JMP, AIP Conf.Proc. 1343 (2011)
NPA 931 (2014) 113

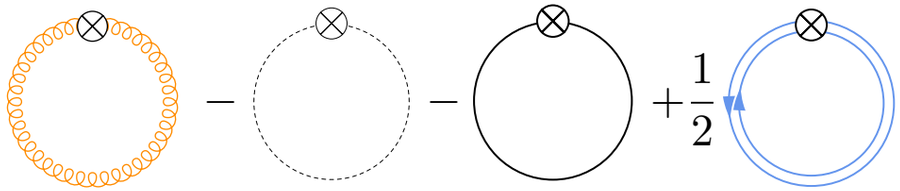
free energy at momentum scale k



ab initio

glue
quantum fluctuations

hadronic
quantum fluctuations

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$+\frac{1}{2}$$

free energy/
grand potential

quark
quantum fluctuations

RG-scale k: $t = \ln k$

properties

closed form

- **access to physics**
- **numerically tractable, also at real time**
  no sign problem
  systematic error control via closed form
- **low energy models naturally encorporated**

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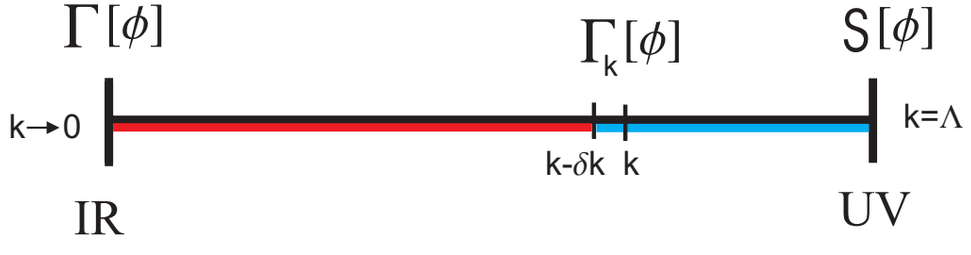

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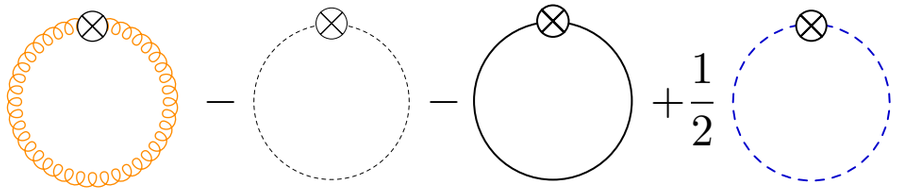
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$+\frac{1}{2}$$

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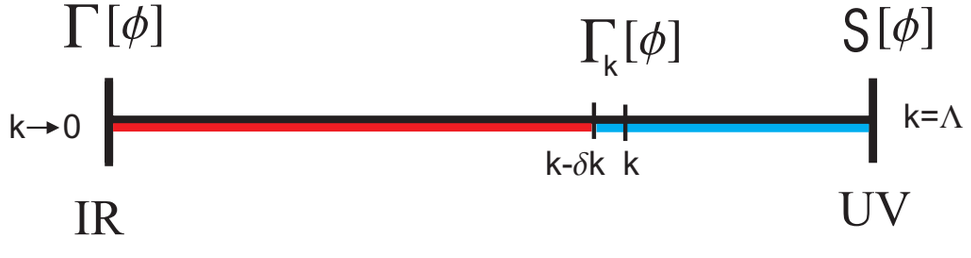

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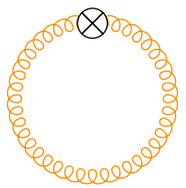
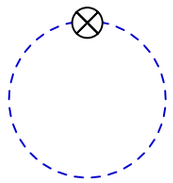
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$- \quad - \quad + \frac{1}{2}$$ 

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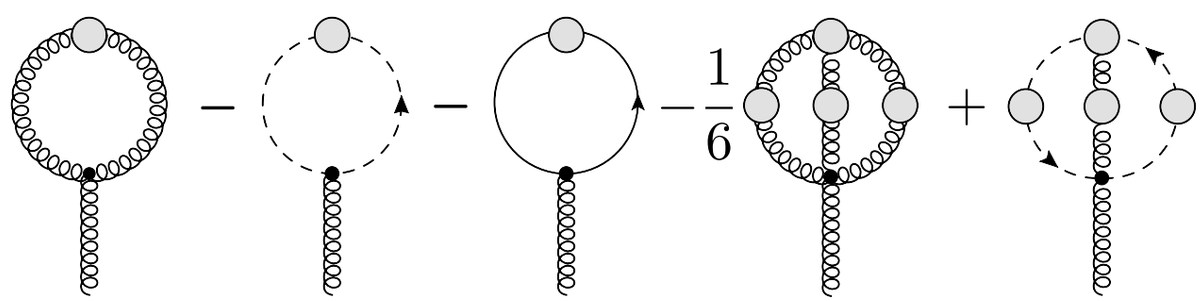
functional DSE : $\dfrac{\delta(\Gamma - S)}{\delta A_0} = \dfrac{1}{2}$ 

A_0 **: background field**

see talks of …sorry, too many people

Functional RG for QCD

Vision: solve coherently for the timeline of a heavy ion collision

fQCD collaboration: **Braun, Corell, Cyrol, Fu, Huang, Leonhardt, Mitter, JMP, Pospiech, Rennecke, Wen, Wink, Yin**

Heidelberg, Dalian, Darmstadt

Agenda

QCD at finite T & mu

- **Phase structure**
- **Fluctuations**
- **Phenomenology**

Real time correlation functions

- **Hadron spectrum & decays**
- **Transport coefficients**
- **Dynamics**

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'Wer Visionen hat, sollte zum Arzt gehen'

Helmut Schmidt '80

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Selection of papers

quenched QCD: Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

unquenched QCD: Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

vector mesons: Rennecke, PRD 92 (2015) 076012

pure glue: Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

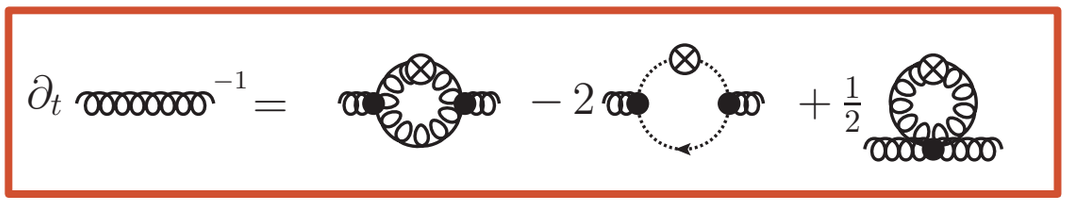
finite T: Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

finite density: *fluctuations:* Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

phase structure: Braun, Leonhardt, Pospiech, PRD 96 (2017) 076003

YM-theory: gluonic correlation functions

$\langle A\, A\rangle(p^2)$



Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

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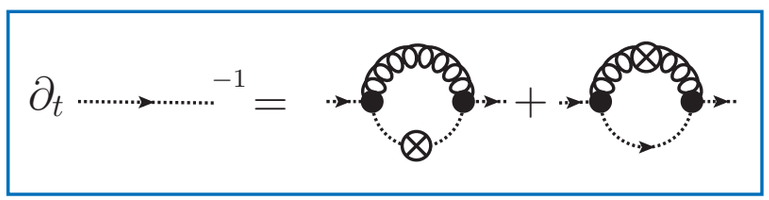
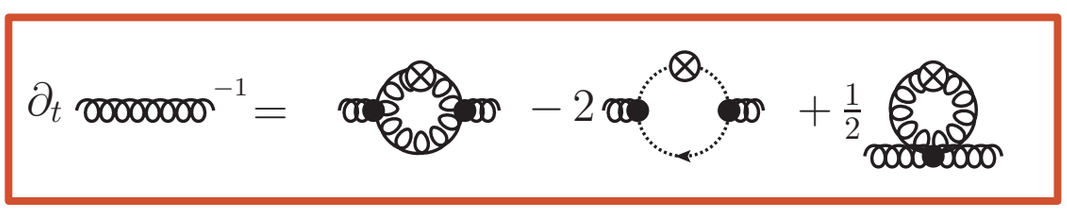
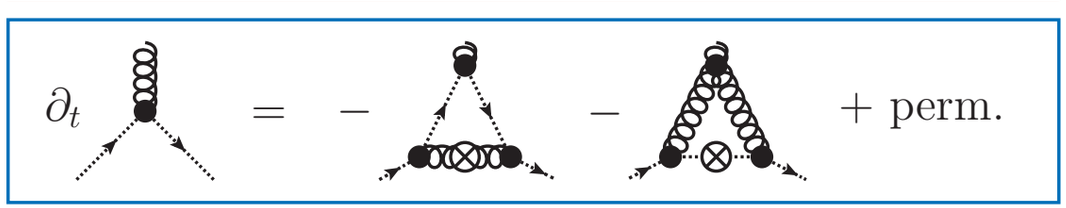
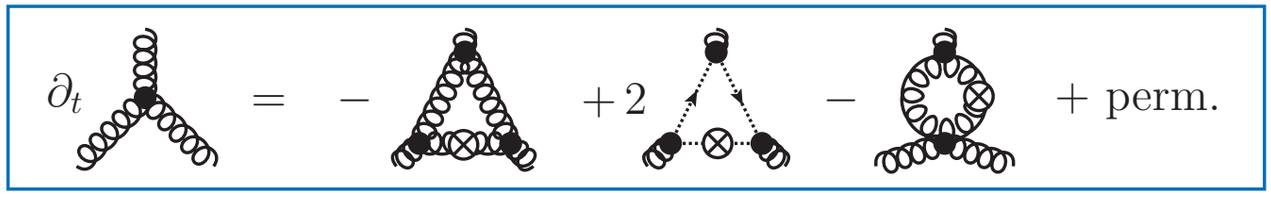
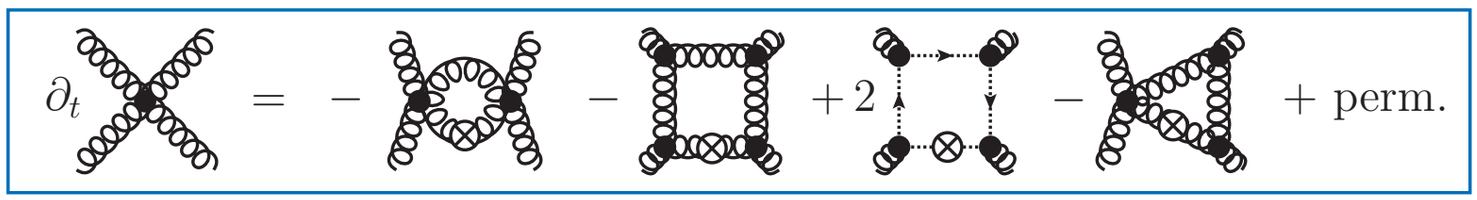




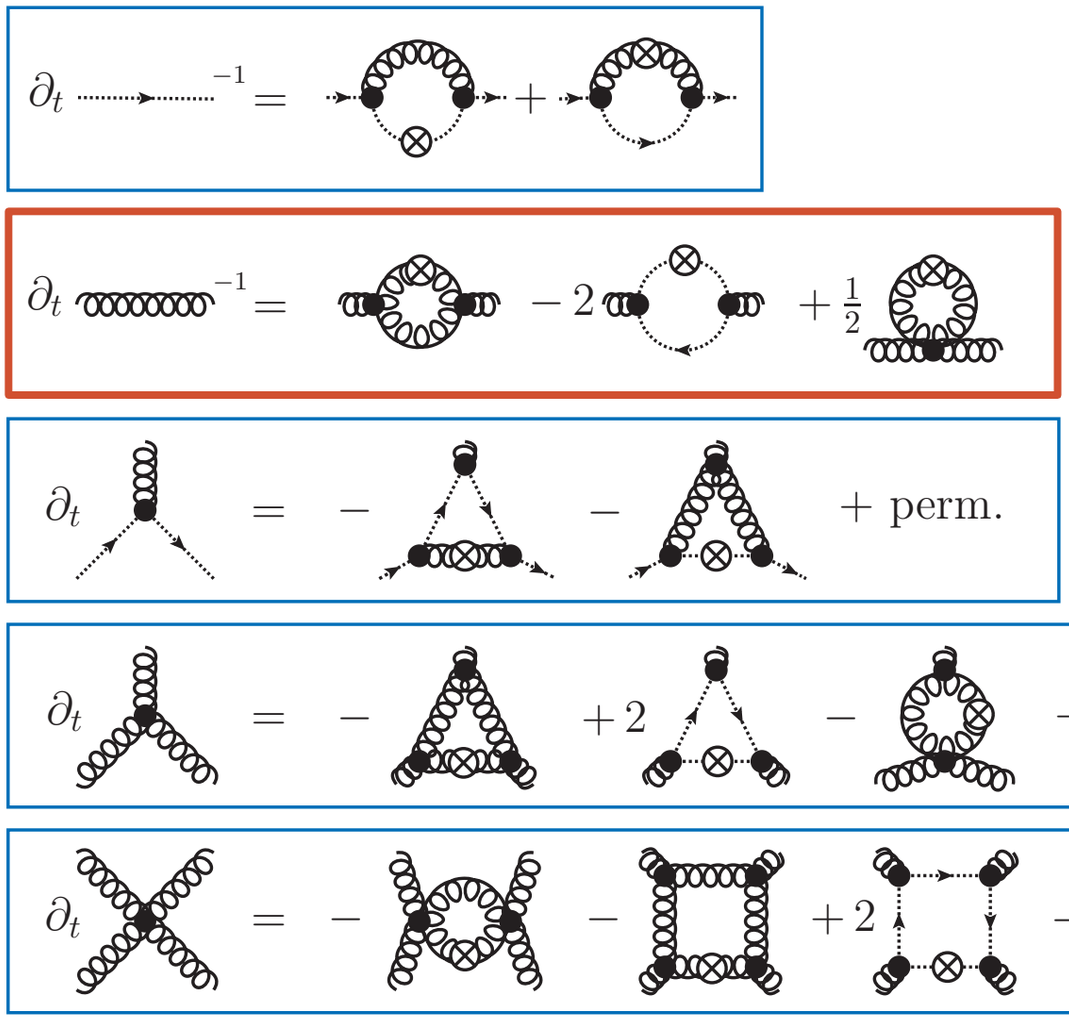


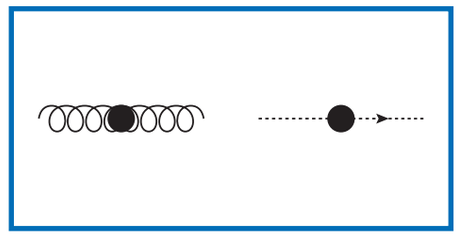




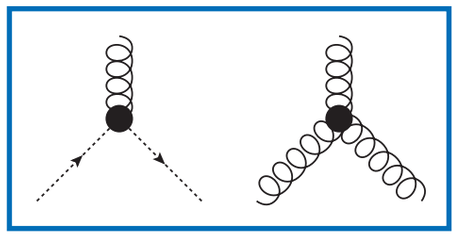
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions

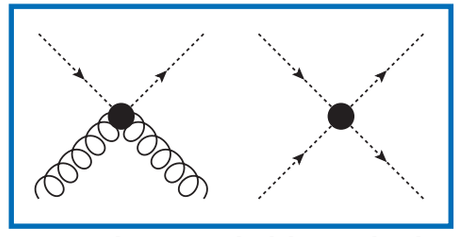




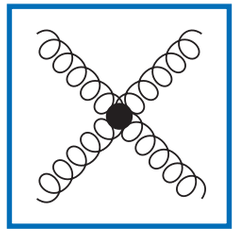
full. mom. dep.



full. mom. dep.
classical tensor structures



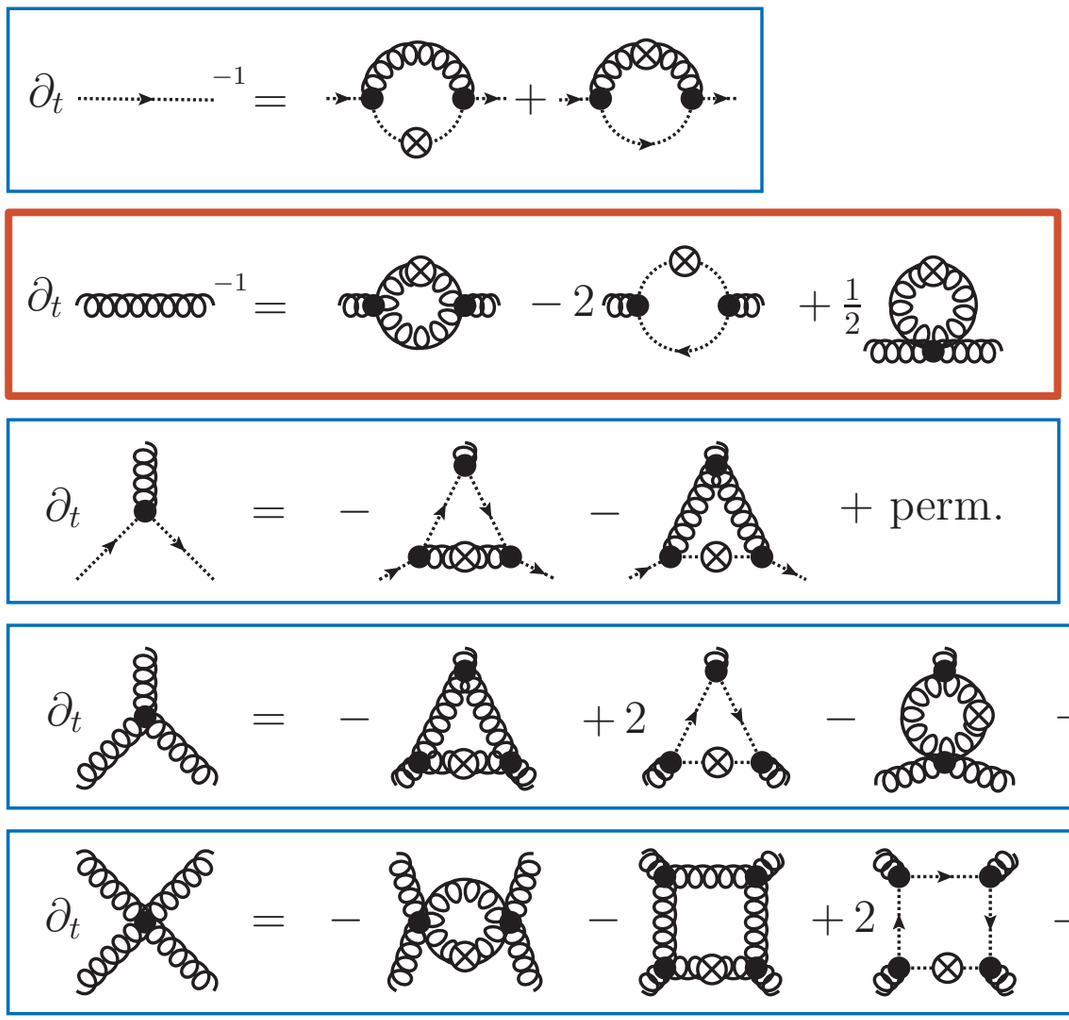
mom. dep. needed by tadpoles
full tensor basis

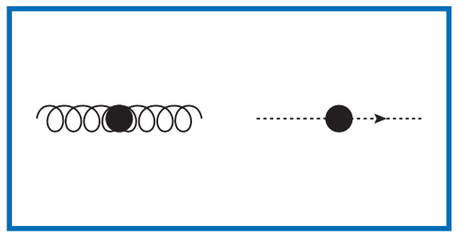


sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: gluonic correlation functions





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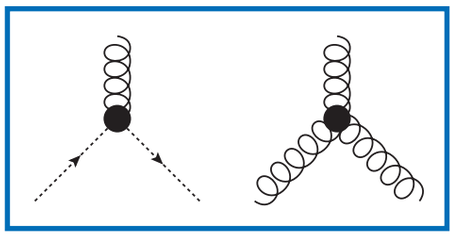
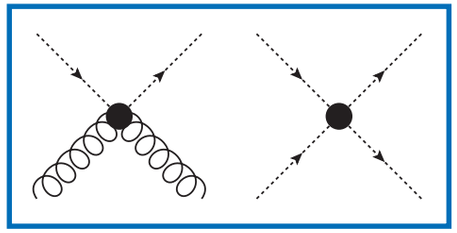
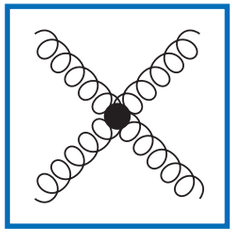


full. mom. dep.
classical tensor structures



mom. dep. needed by tadpoles
full tensor basis



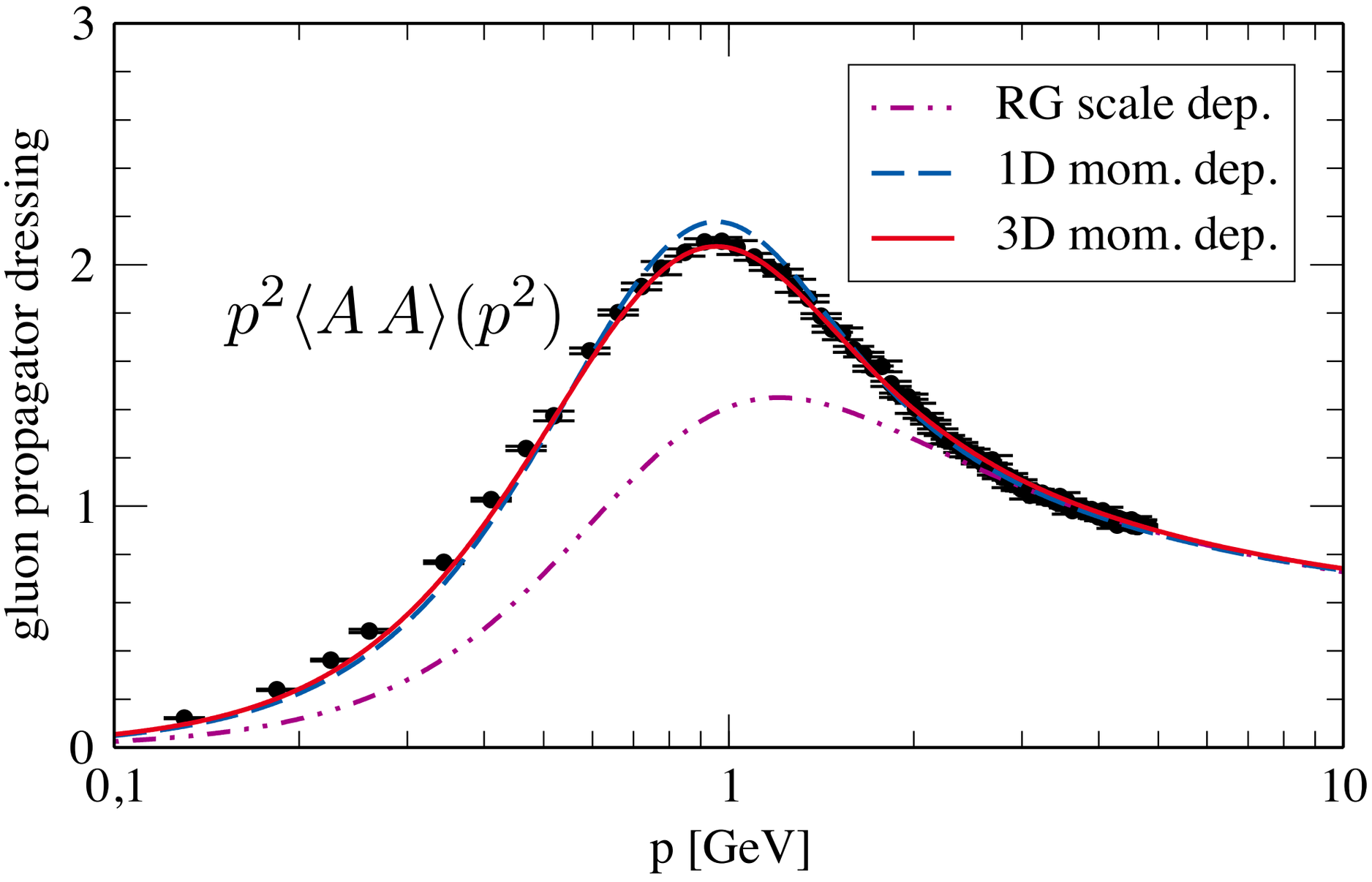
sym. point mom. dep. and
mom. dep. needed by tadpole
classical tensor structure

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

YM-theory: Euclidean gluon propagator

Functional Renormalisation



Lattice: Sternbeck, Ilgenfritz, Müller-Preussker, Schiller, Bogolubsky, PoS LAT2006, 076

Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

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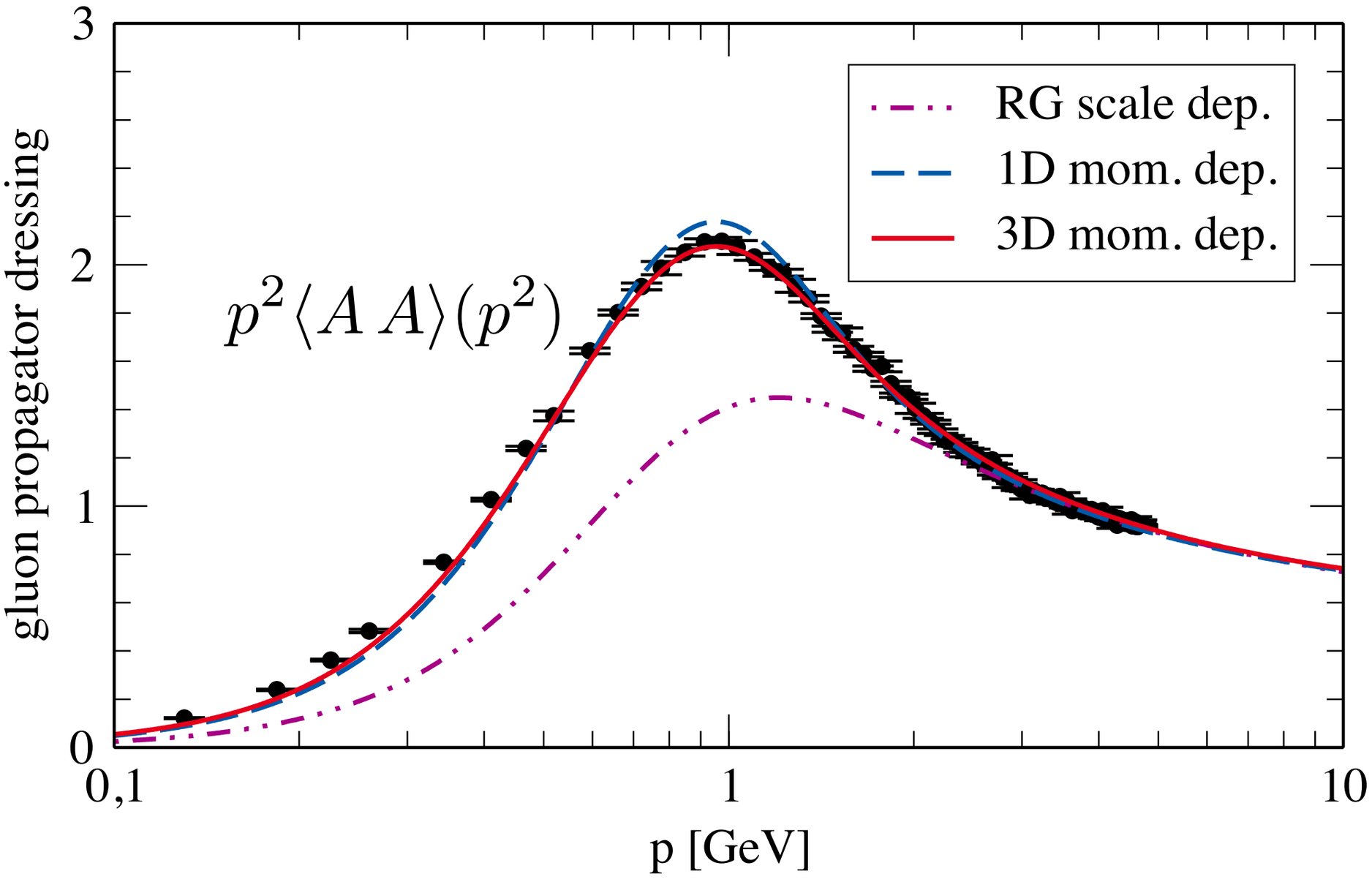


Lattice: Sternbeck, Ilgenfritz, Müller-Preussker, Schiller, Bogolubsky, PoS LAT2006, 076

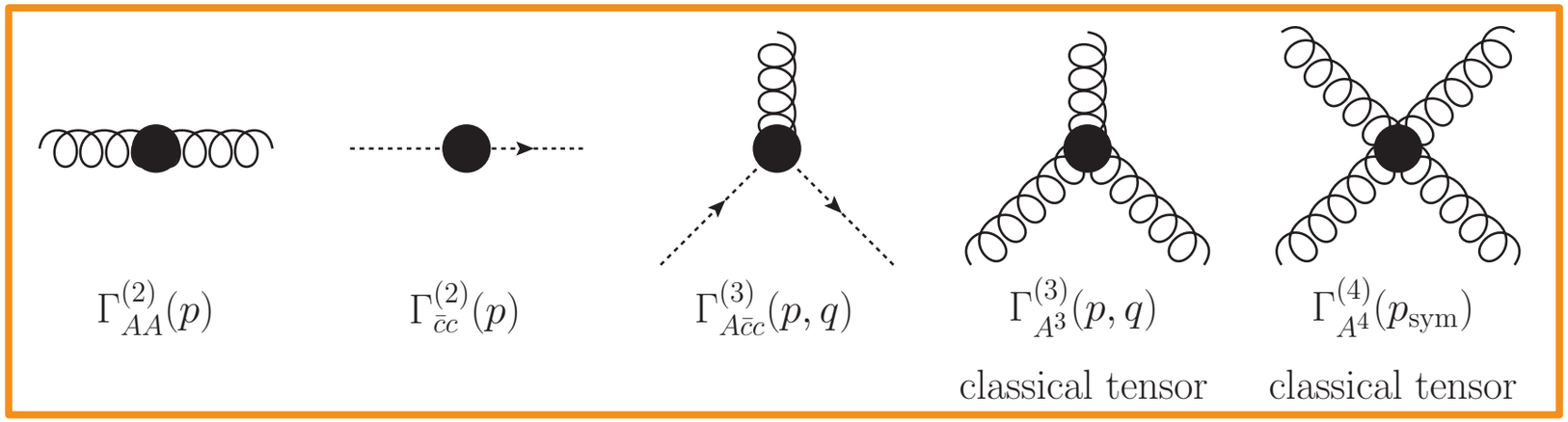
Aiming at apparent convergence

up to date pinch technique:
Aguilar, Binosi, Papavassiliou, PRD 89 (2014) 085032

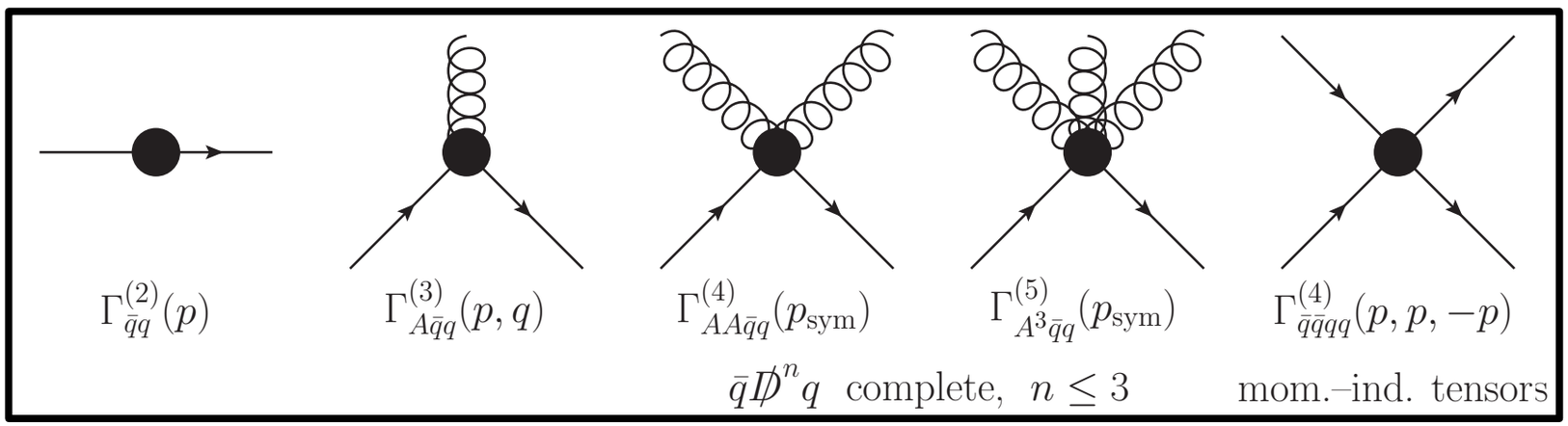
up to date DSE:
Cyrol, Huber, Smekal, EPJ C75 (2015) 102

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

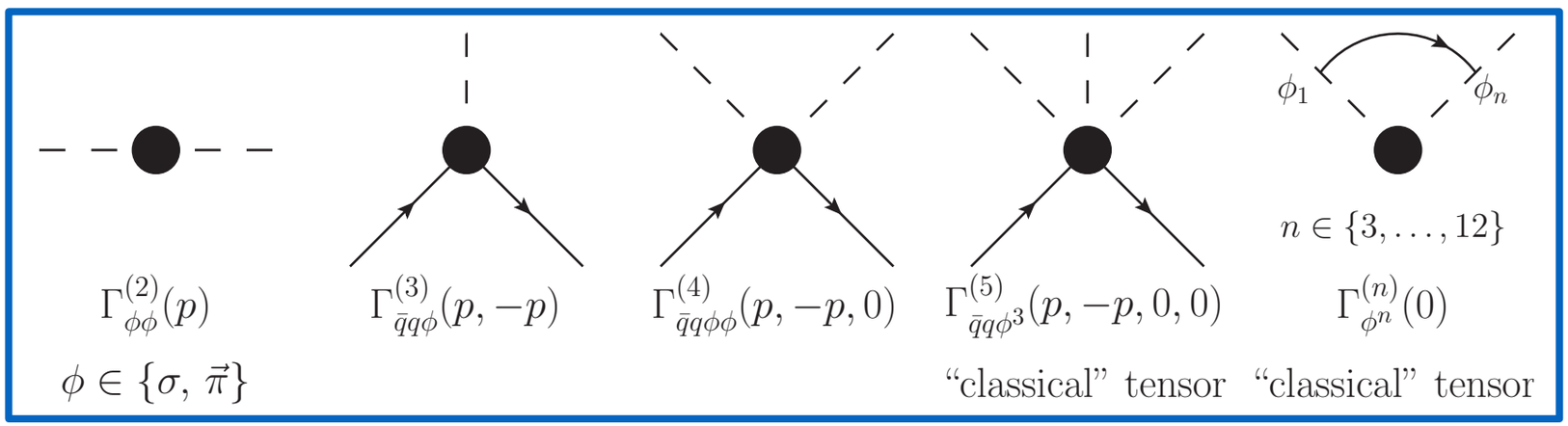
QCD: current set of correlation functions



glue sector



quark-glue sector



quark-meson sector

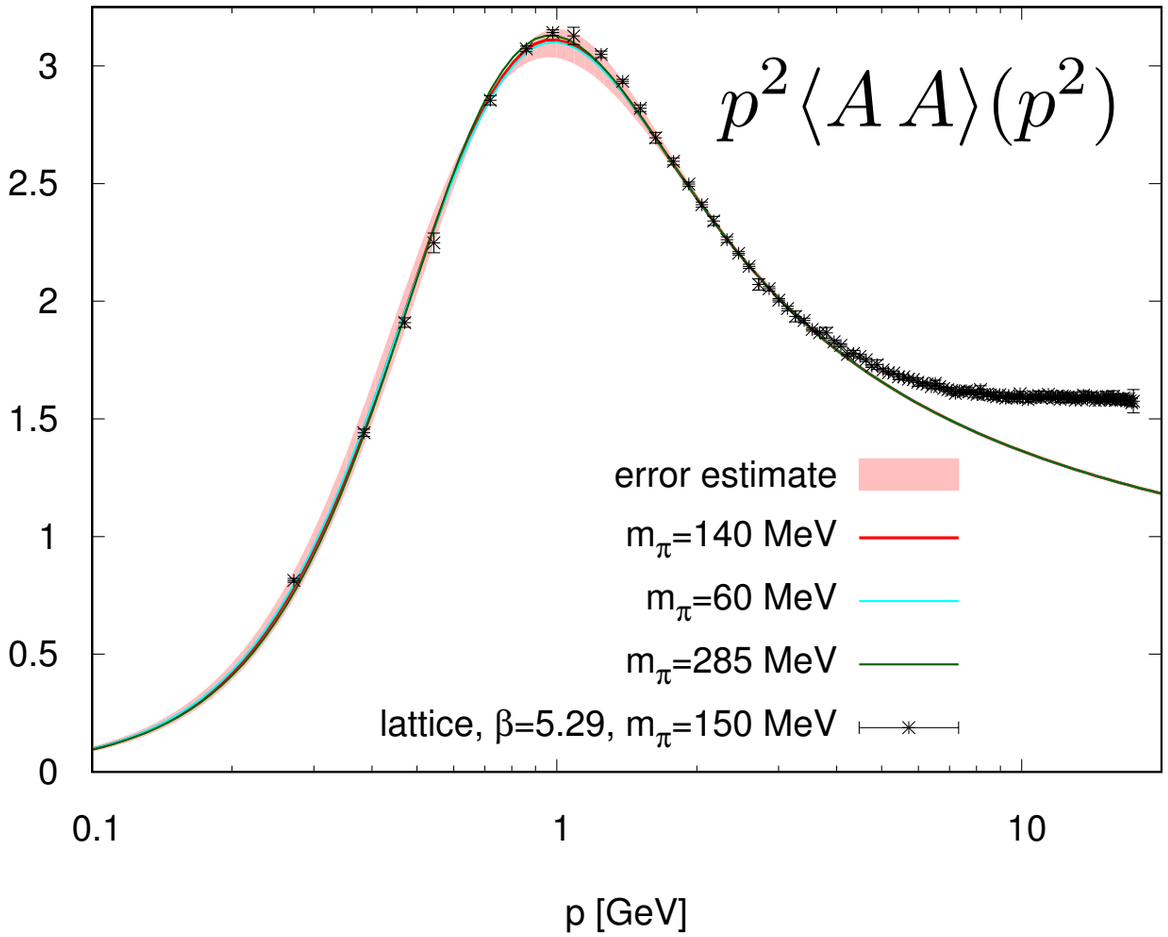
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

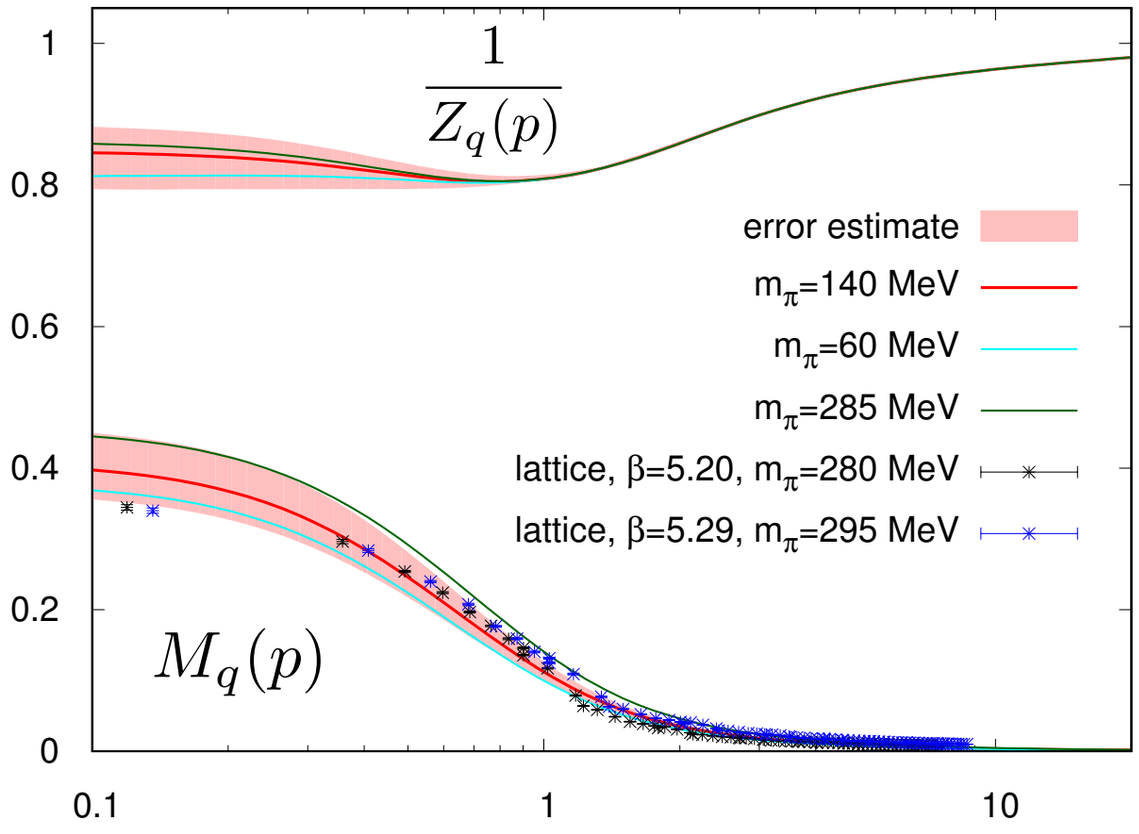
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

QCD: Euclidean propagators



lattice, e.g.: Oliviera et al, Acta Phys.Polon.Supp. 9 (2016) 363
Sternbeck et al, PoS LATTICE2016 (2017)
A. Athenodorou et al, PLB 761 (2016) 444

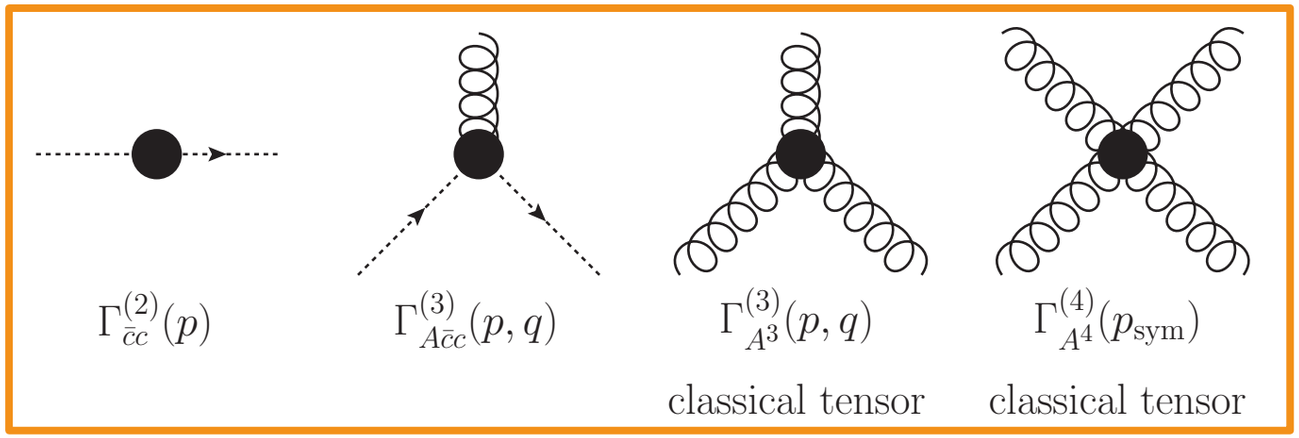
$$\frac{1}{Z_q(p)}\;\frac{1}{i\,\not{p}+M_q(p)}$$



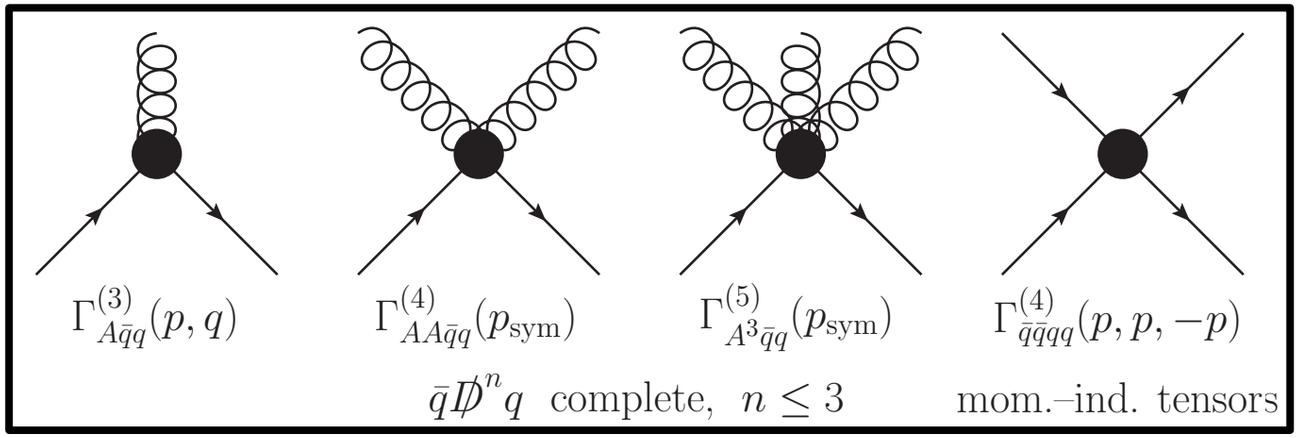
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

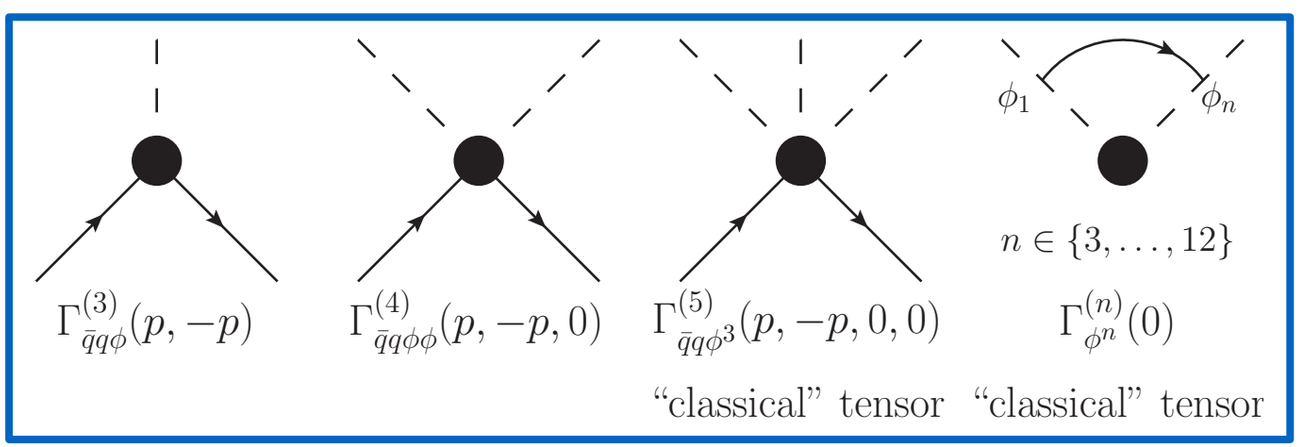
QCD: current set of correlation functions



glue sector



quark-glue sector



quark-meson sector

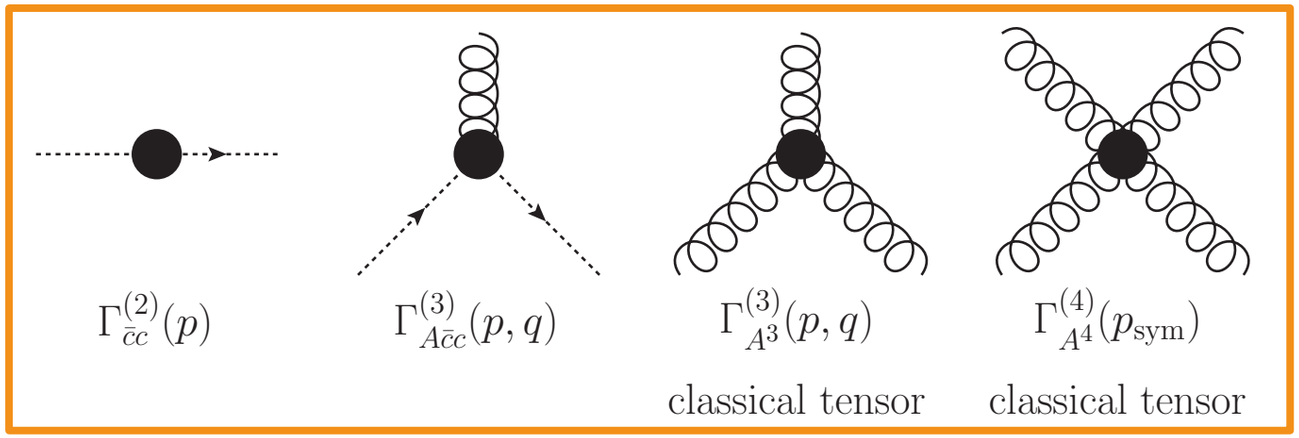
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

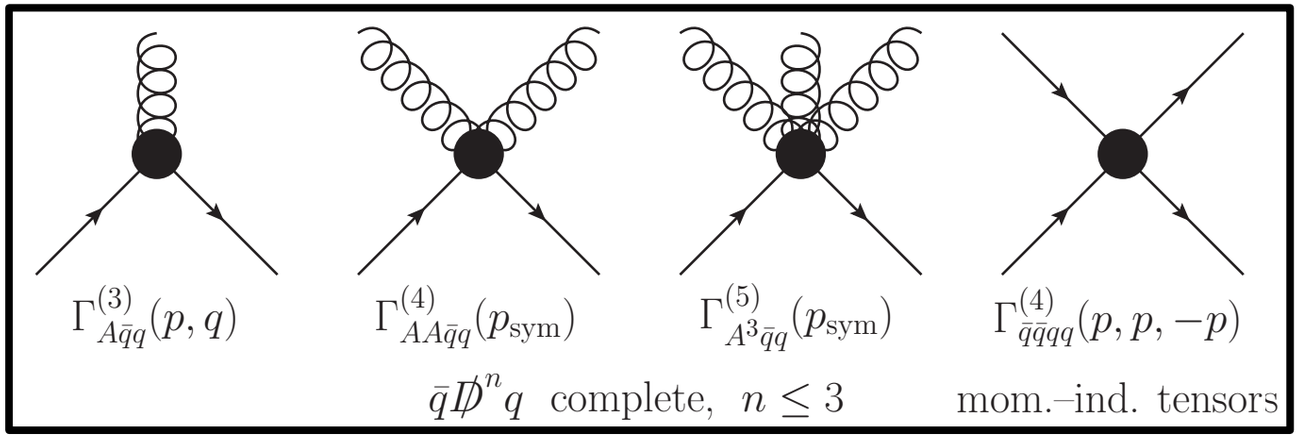
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

QCD: current set of correlation functions

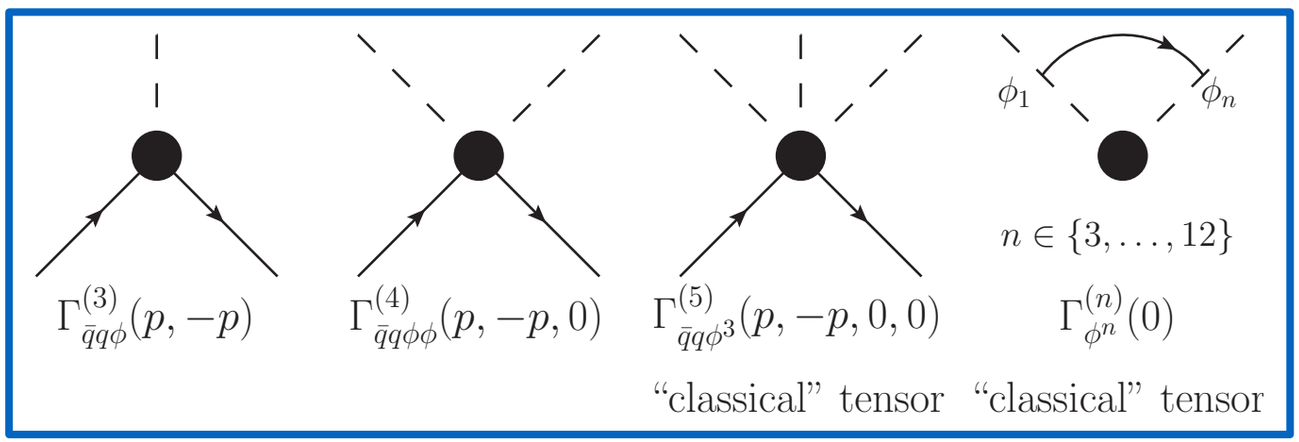


glue sector



quark-glue sector

Welches Schweinderl hätten's denn gerne?



quark-meson sector

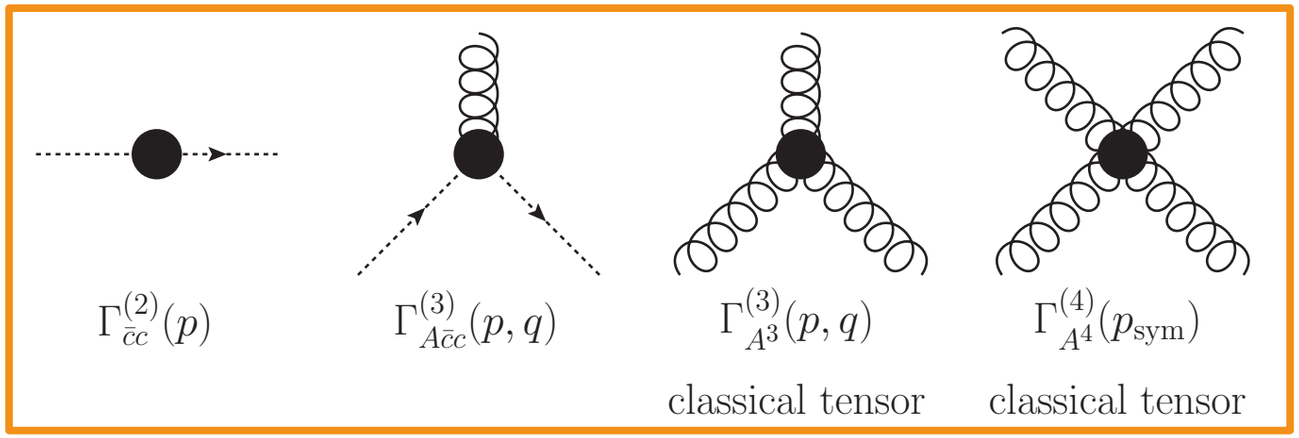
Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

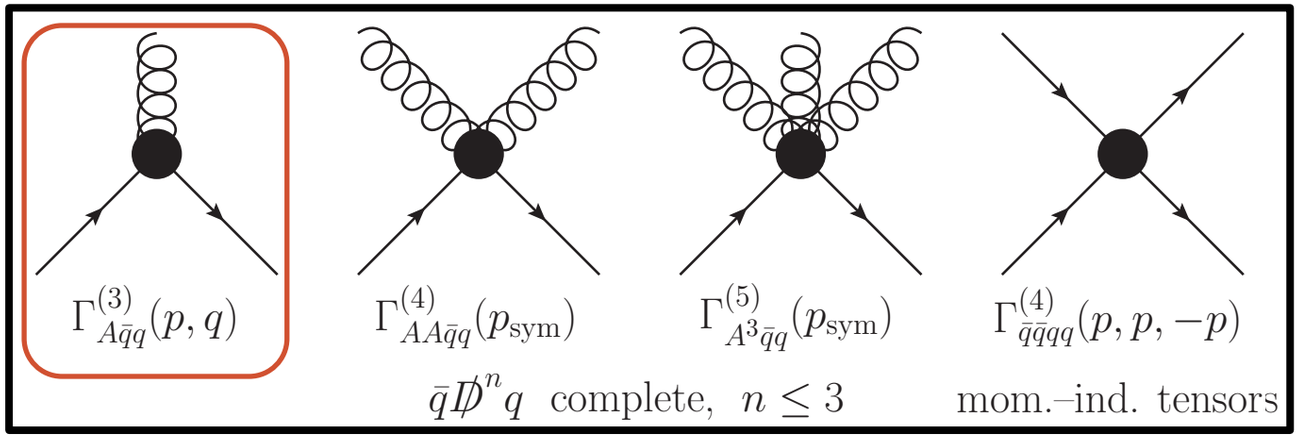
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

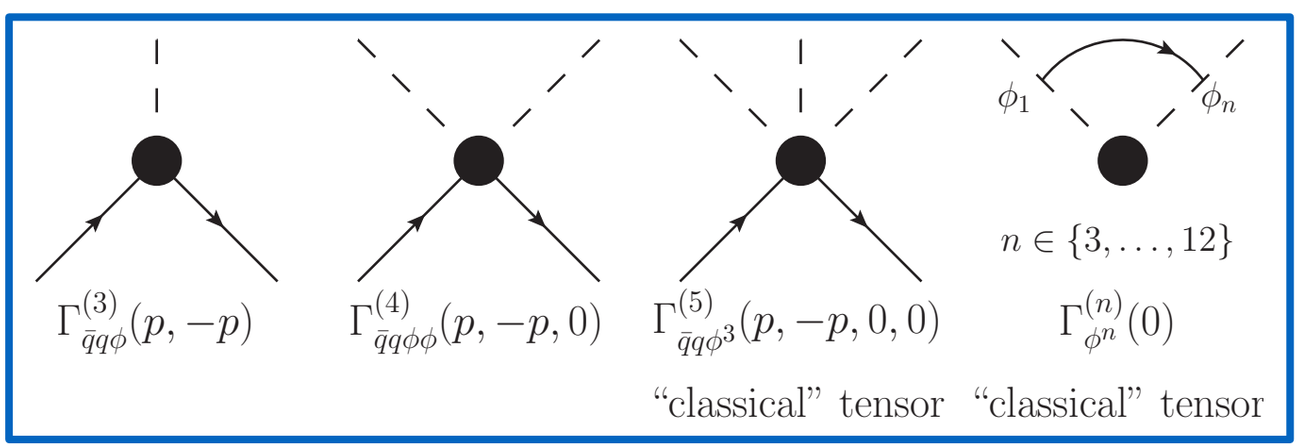
QCD: current set of correlation functions



glue sector



quark-glue sector



quark-meson sector

Welches Schweinderl hätten's denn gerne?

Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006, PRD 97 (2018) 054015

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 94 (2016) 054005

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Quark-gluon vertex

$$\left[\Gamma^{(3)}_{\bar{q}qA}\right]^{a}_{\mu}(p,q) = 1^{\text{flav}}_{2\times 2}\, T^a \sum_{i=1}^{8} \lambda_i(p,q) \left[\mathcal{T}^{(i)}_{\bar{q}qA}\right]_{\mu}(p,q)$$

covariant expansion scheme

$\bar{q}\not{D}q$: $\left[\mathcal{T}^{(1)}_{\bar{q}qA}\right]_{\mu}(p,q) = -i\,\gamma_\mu$

$\bar{q}\not{D}^2 q$: $\left[\mathcal{T}^{(2)}_{\bar{q}qA}\right]_{\mu}(p,q) = (p-q)_\mu\, 1_{4\times 4}$

$\left[\mathcal{T}^{(3)}_{\bar{q}qA}\right]_{\mu}(p,q) = (\not{p}-\not{q})\gamma_\mu$

$\left[\mathcal{T}^{(4)}_{\bar{q}qA}\right]_{\mu}(p,q) = (\not{p}+\not{q})\gamma_\mu$

$\bar{q}\not{D}^3 q$: $\left[\mathcal{T}^{(5)}_{\bar{q}qA}\right]_{\mu}(p,q) = i\,(\not{p}+\not{q})(p-q)_\mu$

$\left[\mathcal{T}^{(6)}_{\bar{q}qA}\right]_{\mu}(p,q) = i\,(\not{p}-\not{q})(p-q)_\mu$

$\left[\mathcal{T}^{(7)}_{\bar{q}qA}\right]_{\mu}(p,q) = \frac{i}{2}[\not{p},\not{q}]\gamma_\mu$

Aiming at apparent convergence

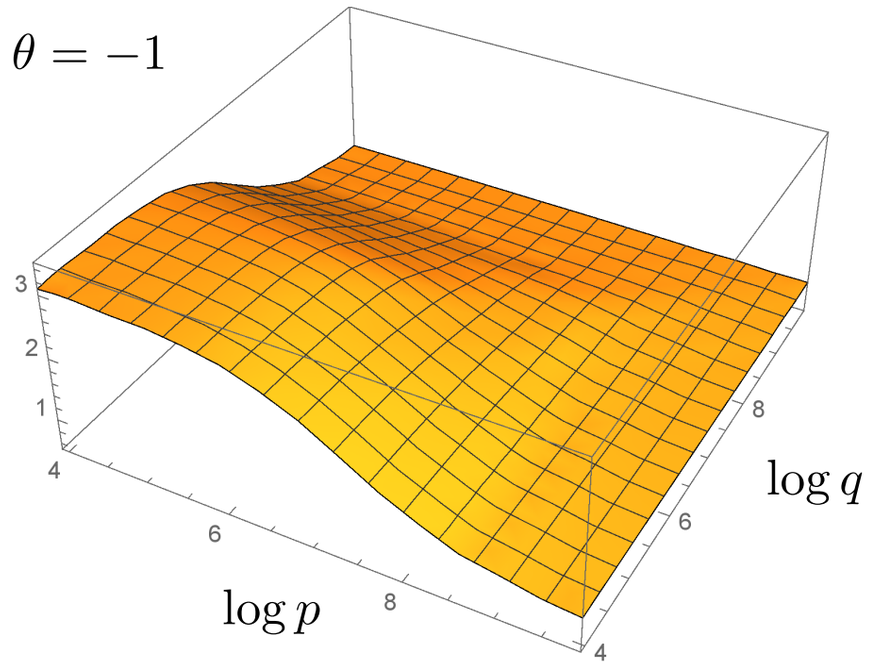
quenched: **Mitter, JMP, Strodthoff, PRD 91 (2015) 054035**

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

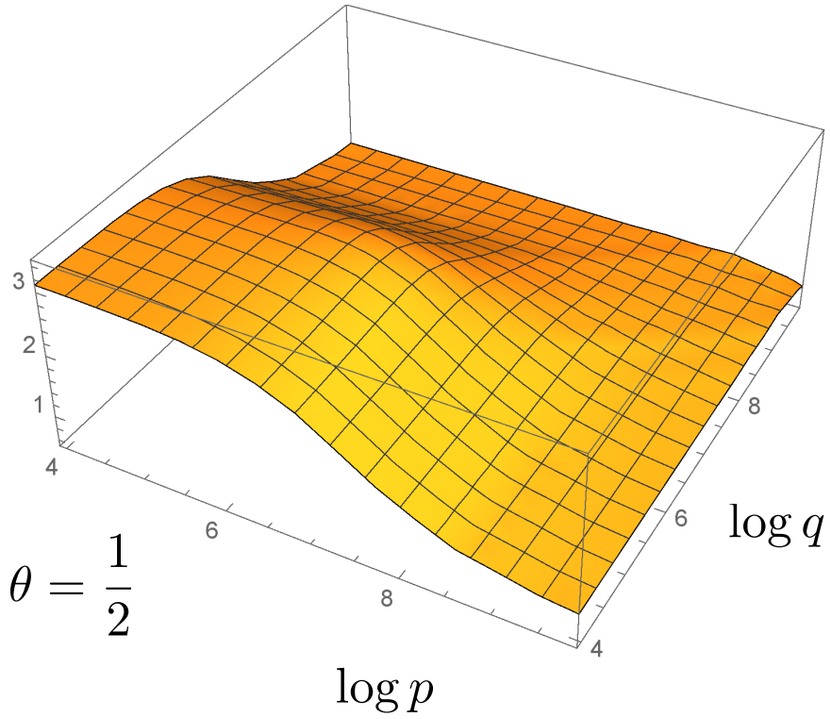
Quark-gluon vertex

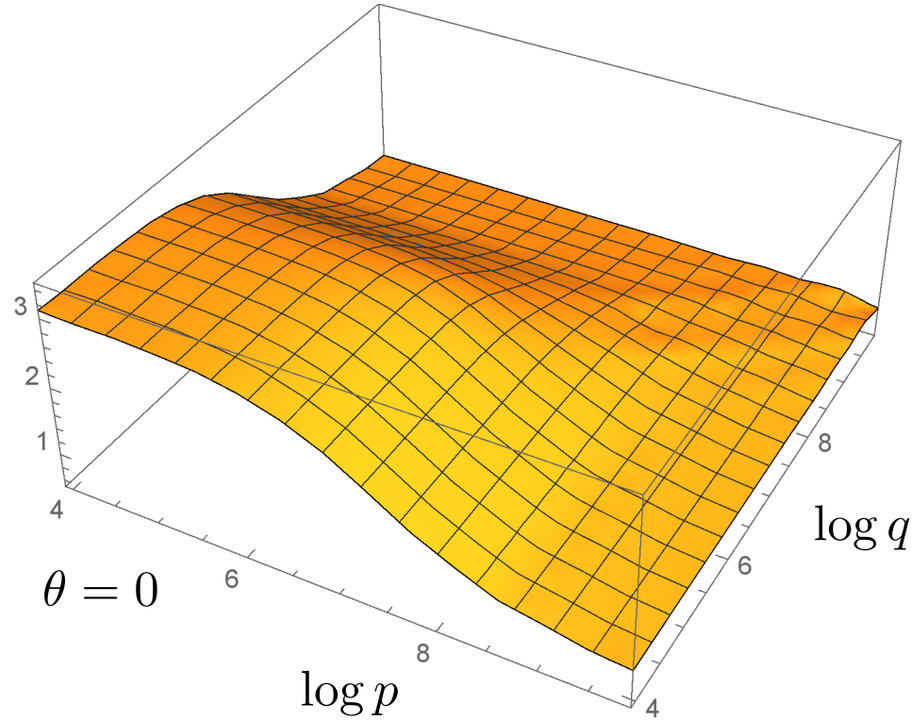
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV



$\lambda_1(p,q)$





Aiming at apparent convergence

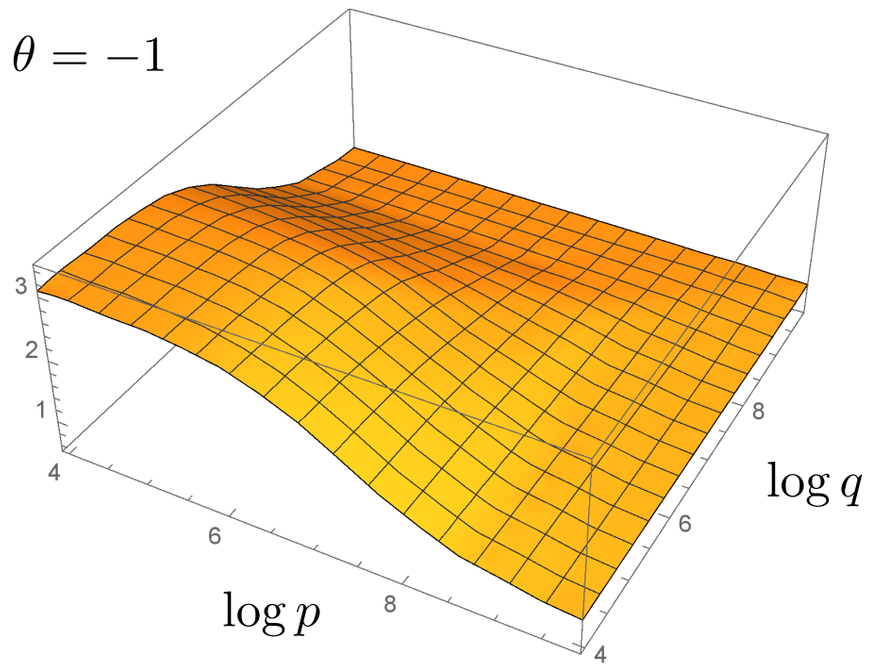
Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

Quark-gluon vertex

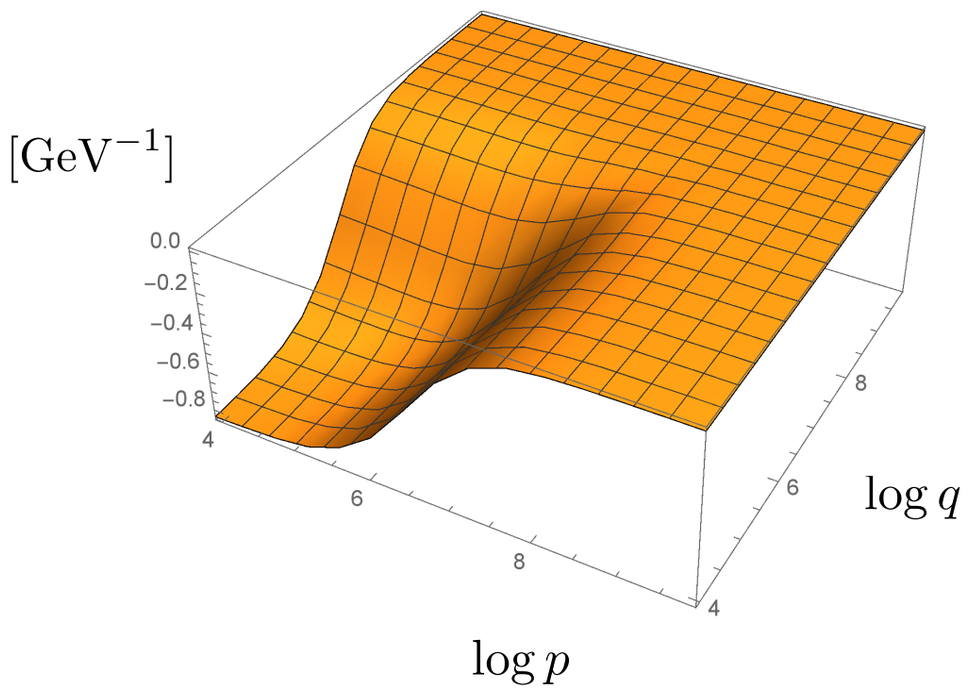
$$\theta = \frac{p \cdot q}{\sqrt{p^2\, q^2}}$$

p,q in MeV

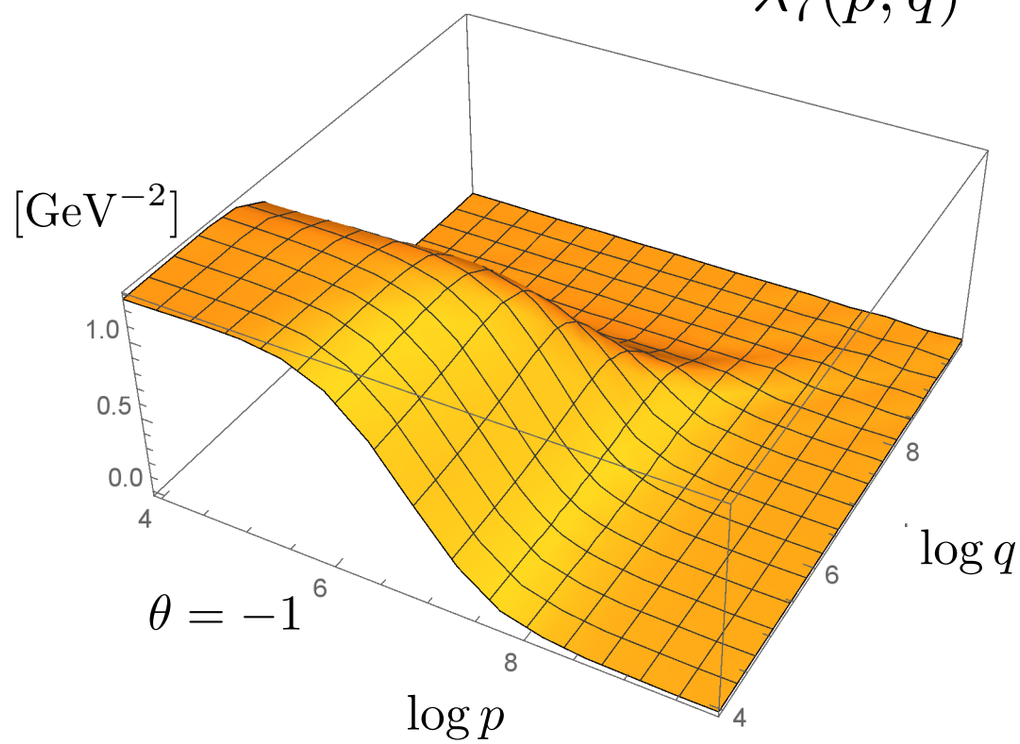
$\lambda_1(p,q)$



$\lambda_4(p,q)$



$\lambda_7(p,q)$



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

QCD: Quark-gluon vertex

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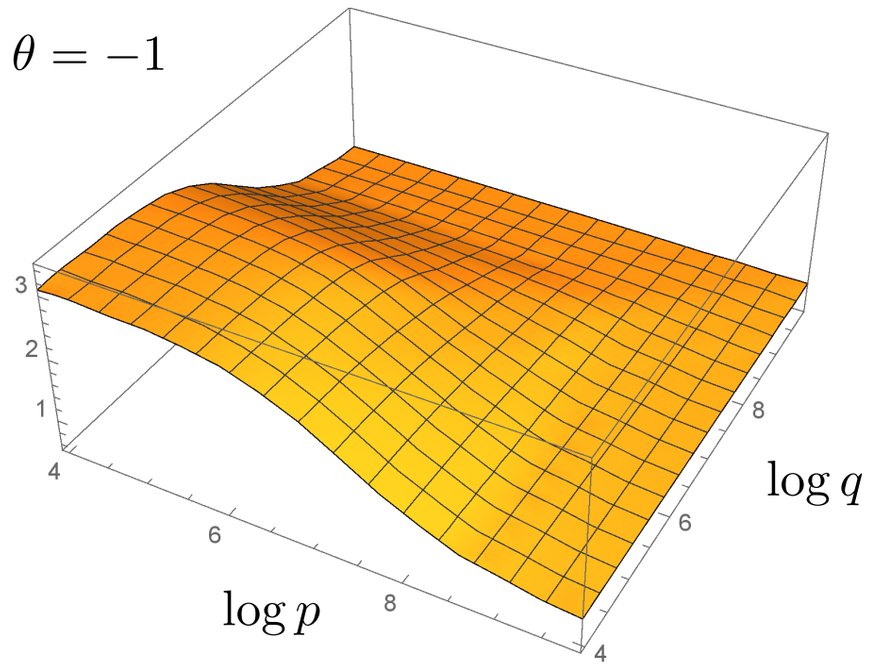
p,q in MeV



$\lambda_1(p,q)$

up-to-date 1st principles works:

Beware of BRST

FunMethods: **Williams, EPJ A51 (2015) 57**
Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, Fischer, Heupel, PRD 93 (2016) 034026
Contant, Huber, Fischer, Welzbacher, Williams, arXiv:1805.05885

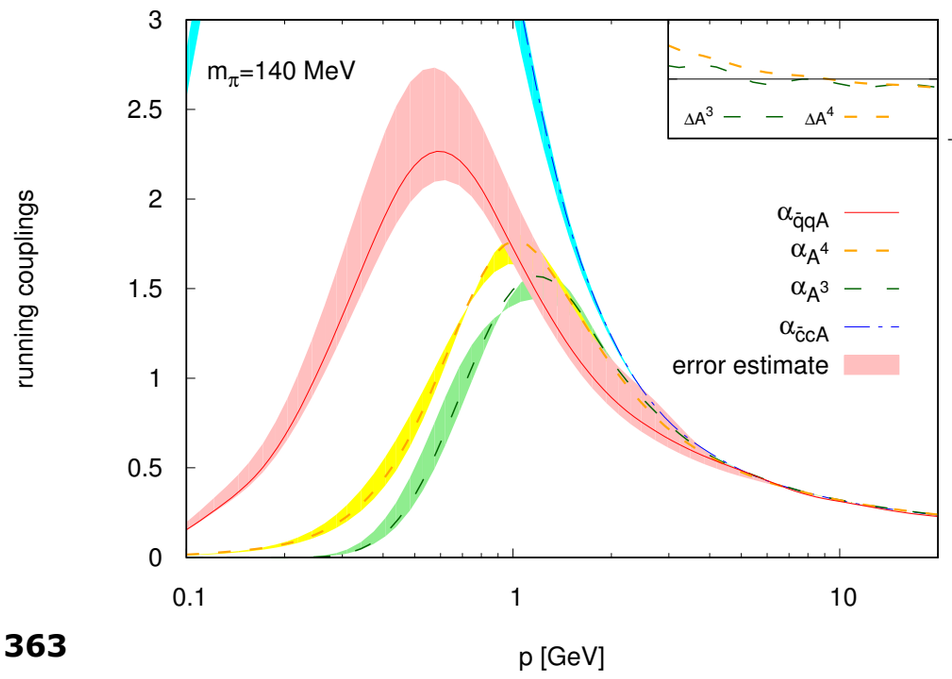
Aguilar, Binosi, Ibanez, Papavassiliou, PRD 89 (2014) 065027
Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 95 (2017) 031501
Aguilar, Cardona, Ferreira, Papavassiliou, PRD 96 (2017) 014029
PRD 98 (2018) 014002

Mitter, JMP, Strodthoff, PRD 91 (2015) 054035

Pelaez, Tissier, Wschebor, PRD 92 (2015) 045012

Eichmann, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016) 1

lattice, e.g.: **Oliveira, Kizilersü, Silva, Skullerud, Sternbeck, Williams, APP Suppl. 9 (2016) 363**



Aiming at apparent convergence

Cyrol, Mitter, JMP, Strodthoff, PRD 97 (2018) 054006

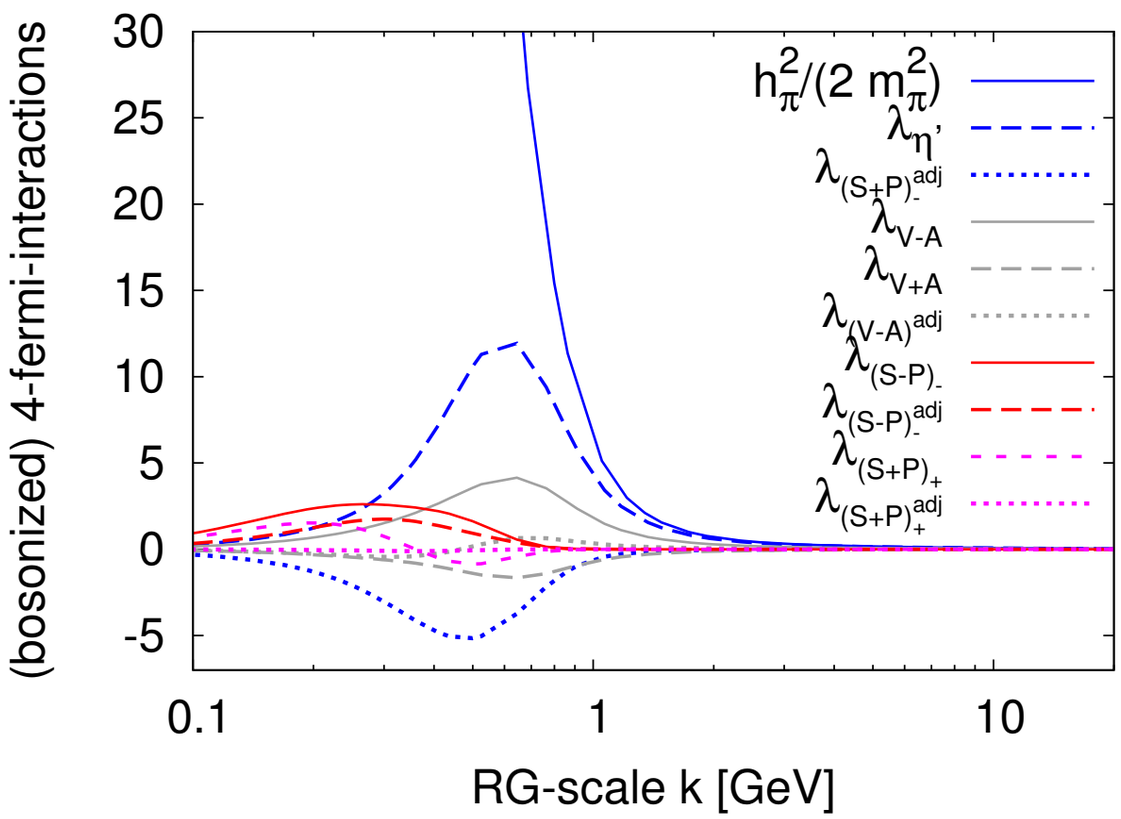
A glimpse at the hadron spectrum

unpublished

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \quad \rightarrow \quad \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2 - m_\pi^2} \quad + \quad \text{finite terms}$$

$\Gamma^{(4)}(p_1, p_2, p_3, p_4)$



Mitter, JMP, Strodthoff, unpublished

A glimpse at the hadron spectrum

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$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

Mitter, JMP, Strodthoff, unpublished

A glimpse at the hadron spectrum

unpublished

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$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2 - m_\pi^2}$$

pion decay constant $\mathbf{f_\pi}$ **via normalisation of** $\boldsymbol{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f_\pi \simeq 99\,MeV}$$

quenched QCD

Mitter, JMP, Strodthoff, unpublished

A glimpse at the hadron spectrum

unpublished

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pion decay constant $\mathbf{f_\pi}$ **via normalisation of** $\mathbf{\Gamma^{(3)}_{\bar{q}\pi q}}$

$$\mathbf{f_\pi \simeq 99\,MeV}$$
quenched QCD

$$\mathbf{f_\pi \simeq 89\,MeV}$$
unquenched QCD

lattice: **Davies et al., PRL 92 (2004) 022001** $\dfrac{\mathbf{f_\pi^{quenched}}}{\mathbf{f_\pi^{unquenched}}} \simeq 1.1$

Mitter, JMP, Strodthoff, unpublished

A glimpse at the hadron spectrum

unpublished

four-fermi scattering amplitude at pion pole

$$\langle \bar{q}\vec{\sigma}\gamma_5 q(p) \quad \bar{q}\vec{\sigma}\gamma_5 q(-p)\rangle \;\to\; \frac{\chi_{\bar{q}\pi q}\bar{\chi}_{\bar{q}\pi q}}{p^2-m_\pi^2} \;+\; \text{finite terms}$$

$$\Gamma^{(4)}_{(\bar{q}\gamma_5\vec{\sigma}q)^2}(p,p,-p,-p) \qquad \frac{\Gamma^{(3)}_{\bar{q}\pi q}\Gamma^{(3)}_{\bar{q}\pi q}}{p^2-m_\pi^2}$$

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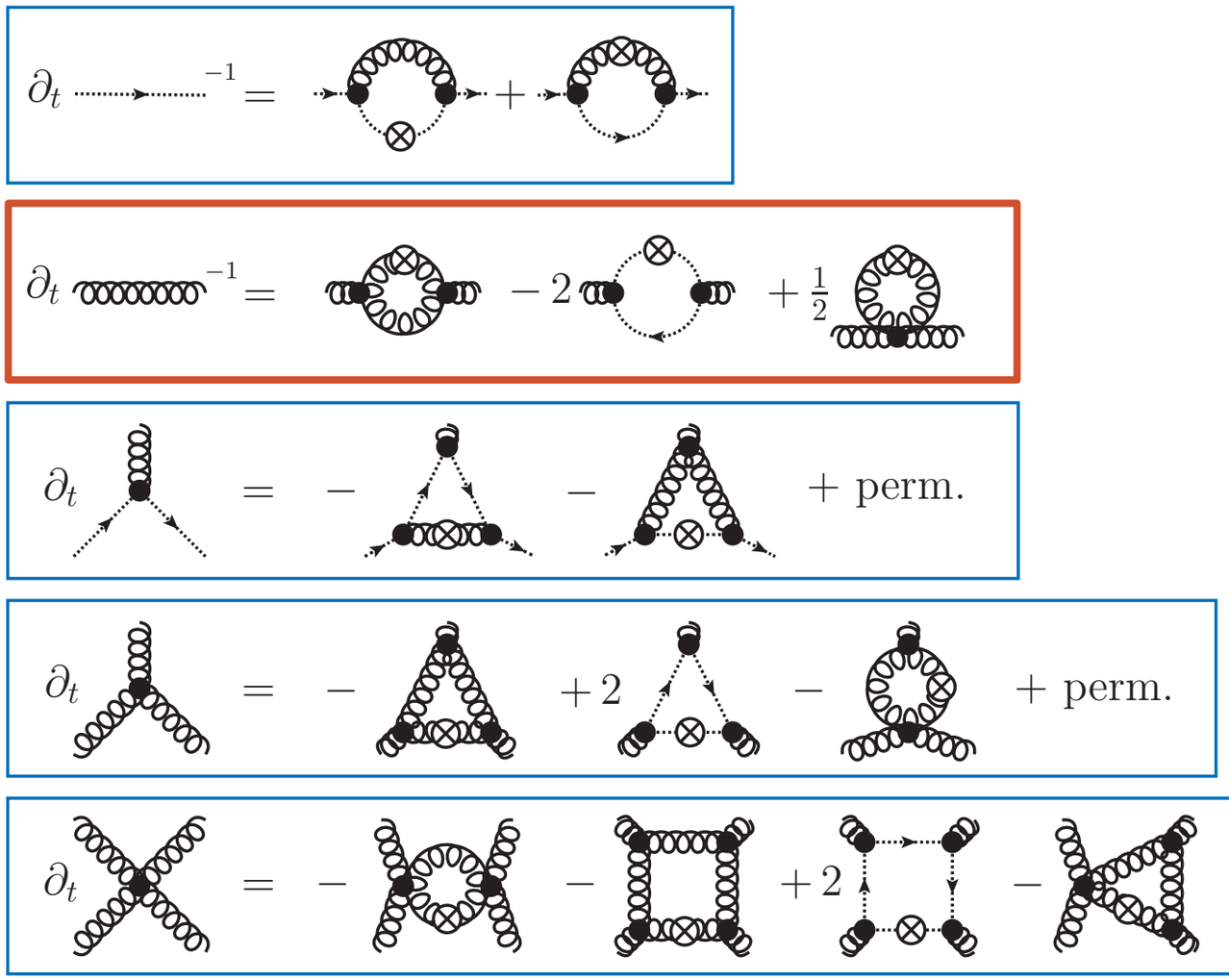
lattice: Davies et al., PRL 92 (2004) 022001

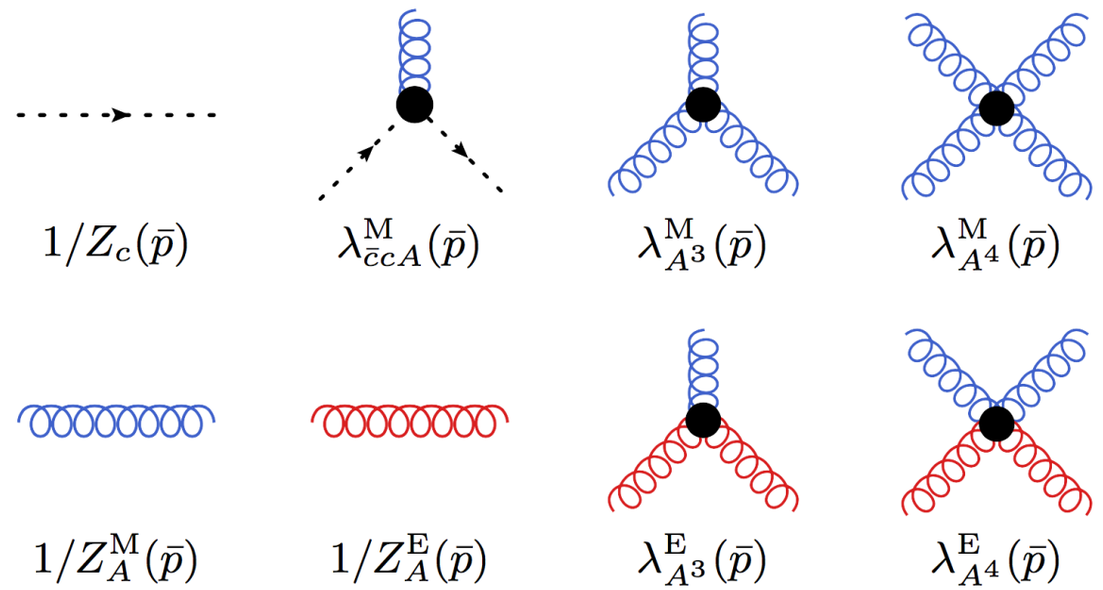
unquenched: e.g. Horsley et al., PLB 732, 41 (2014)

$$f_\pi^{\text{lattice}} \simeq 89\,\text{MeV}$$

Mitter, JMP, Strodthoff, unpublished

YM-theory at finite T





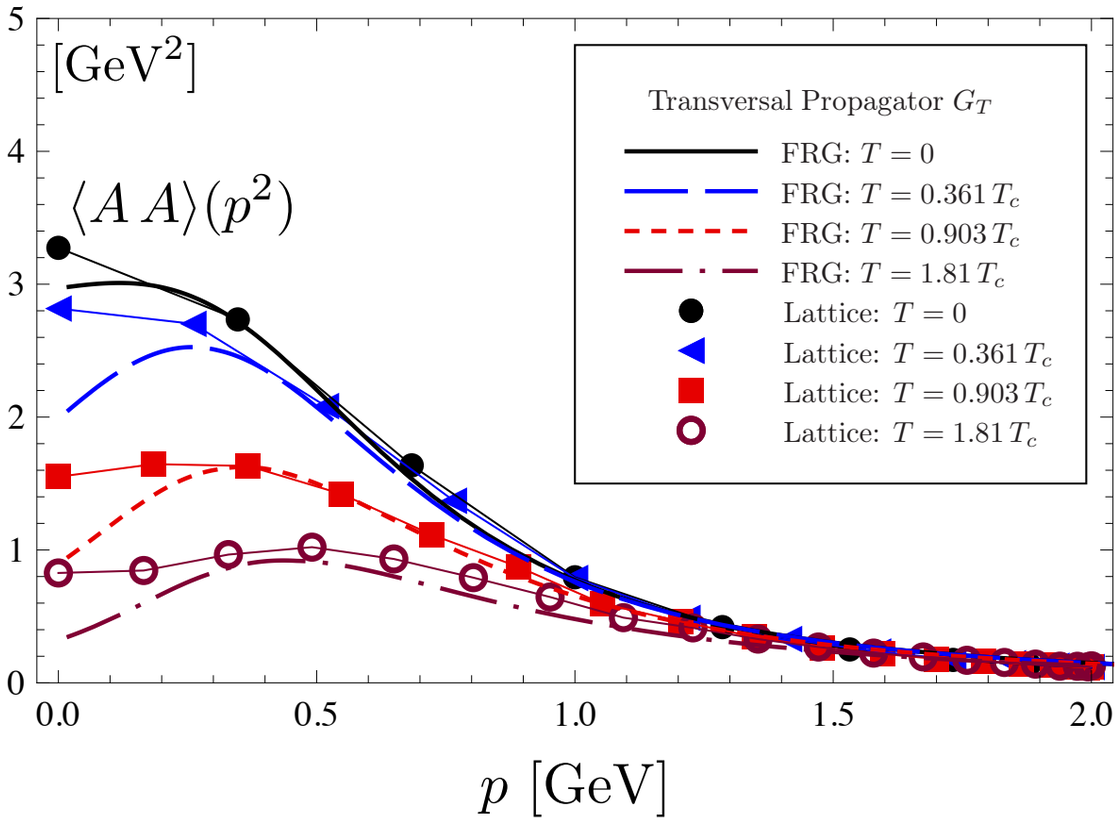
Aiming at apparent convergence

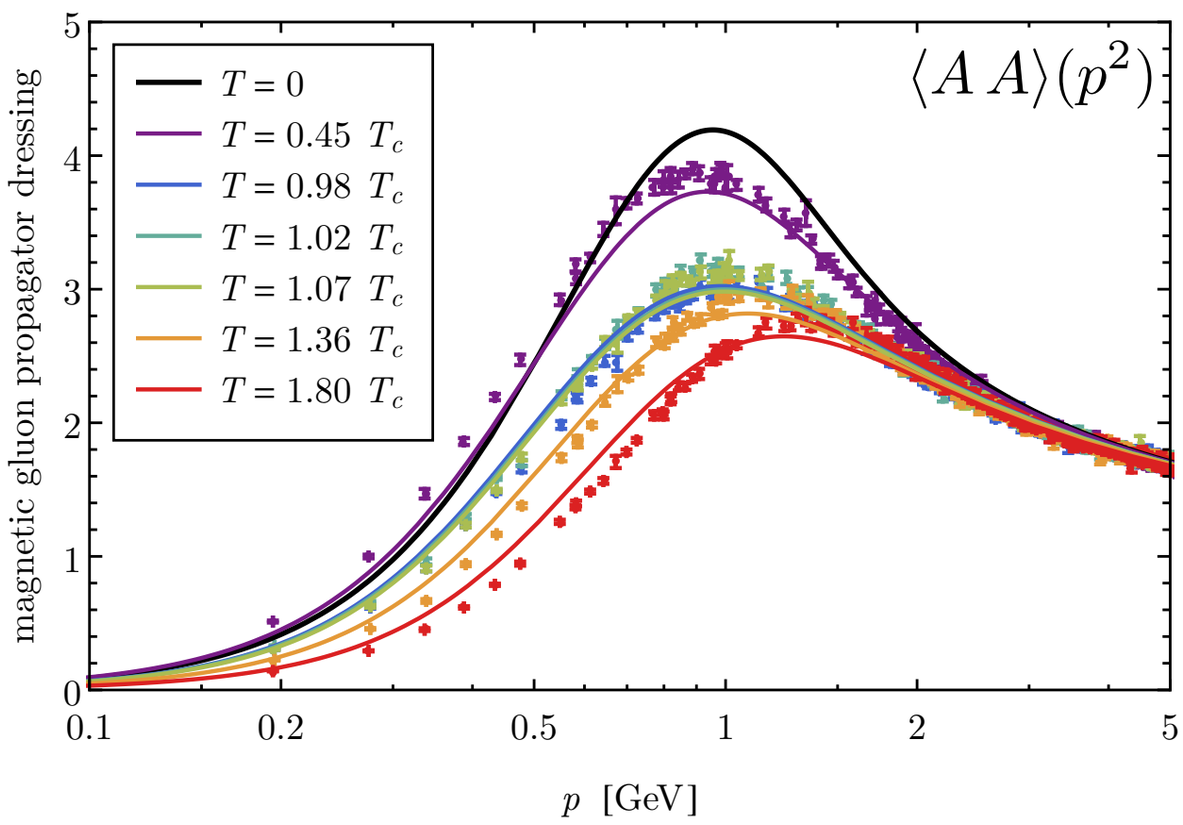
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Euclidean gluon propagator at finite T

Yang-Mills propagators, finite T

chromo-magnetic propagator





Fister, JMP, arXiv:1112.5440

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014

Lattice: Silva, Oliveira, Bicudo, Cardoso, PRD89 (2014) 074503

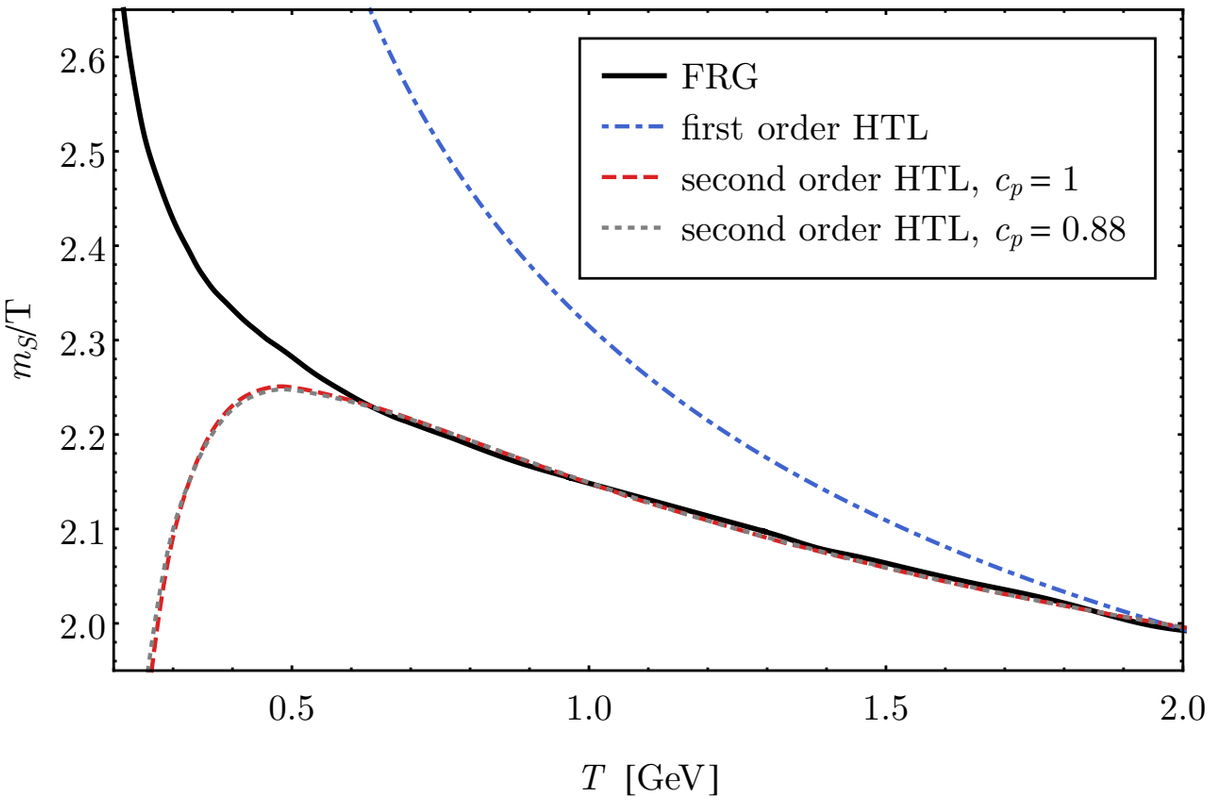
Aiming at apparent convergence

Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Euclidean gluon propagator at finite T

Yang-Mills propagators, finite T

Debye mass (chromo-electric)



Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

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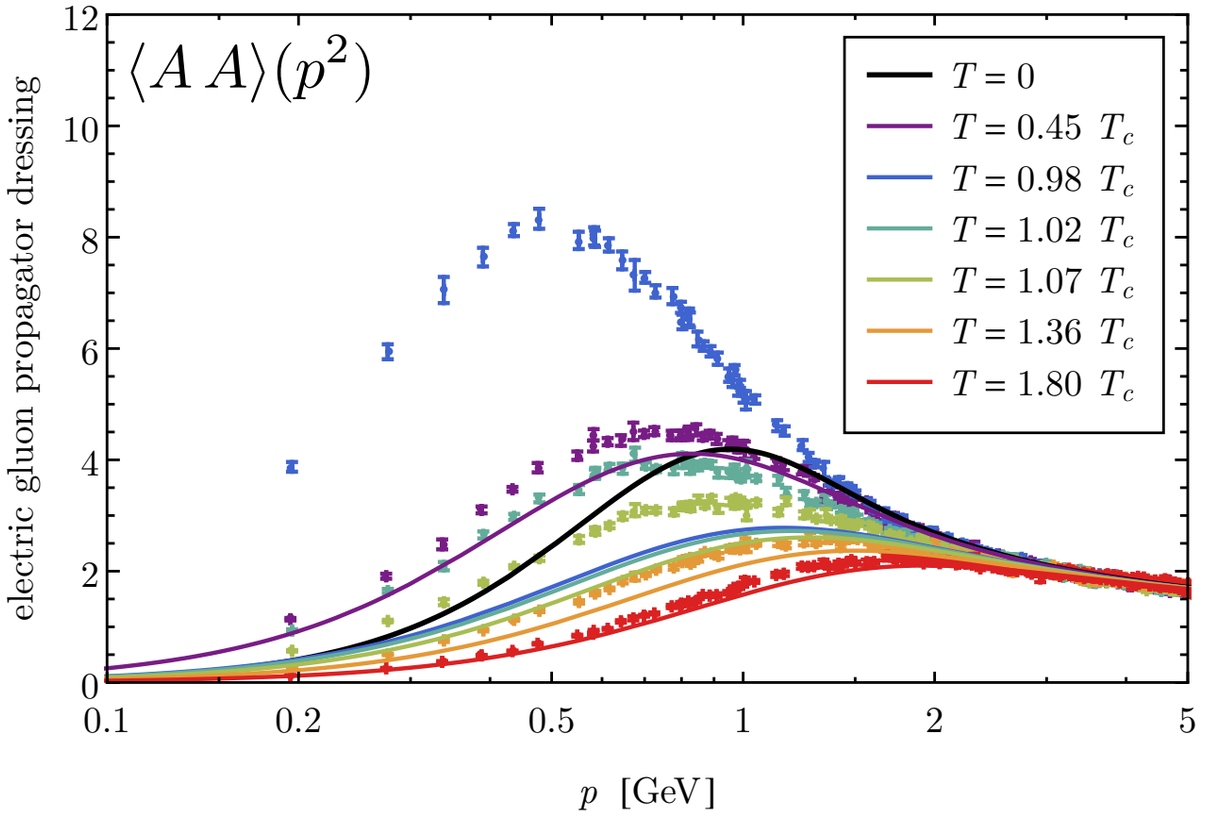
chromo-electric propagator



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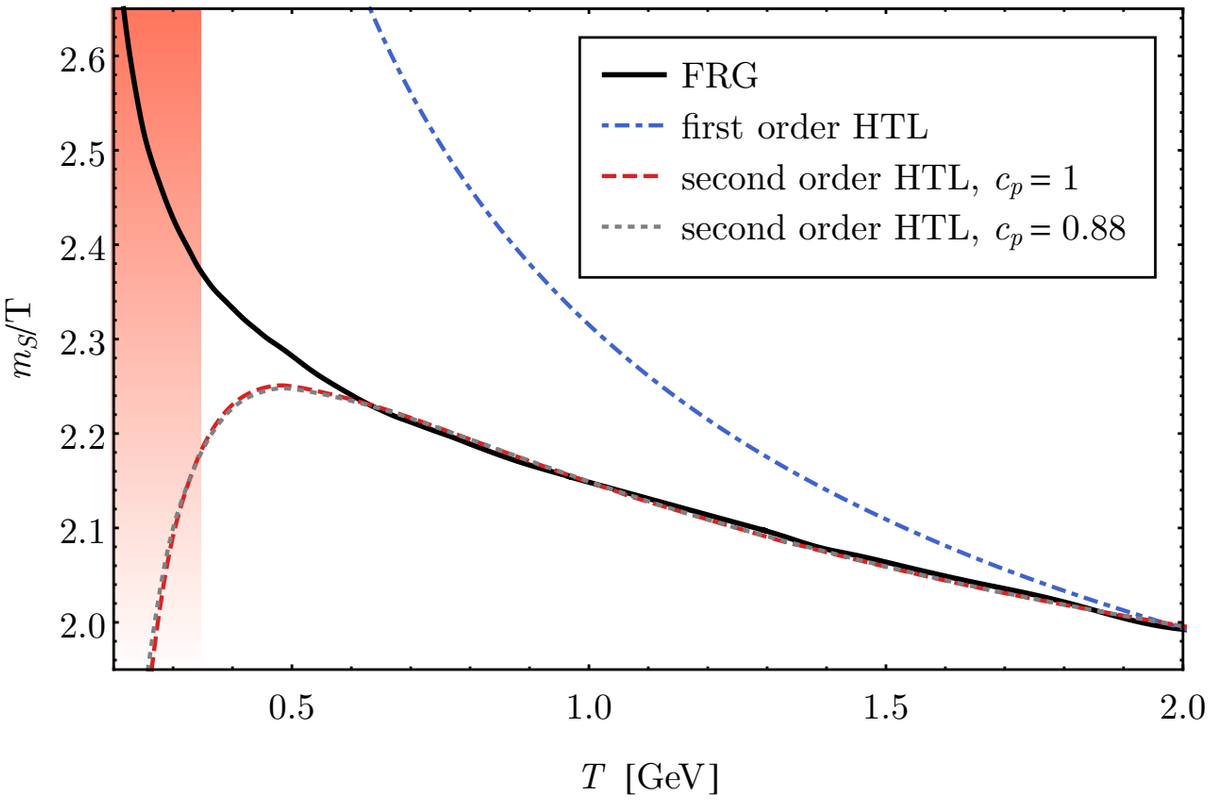
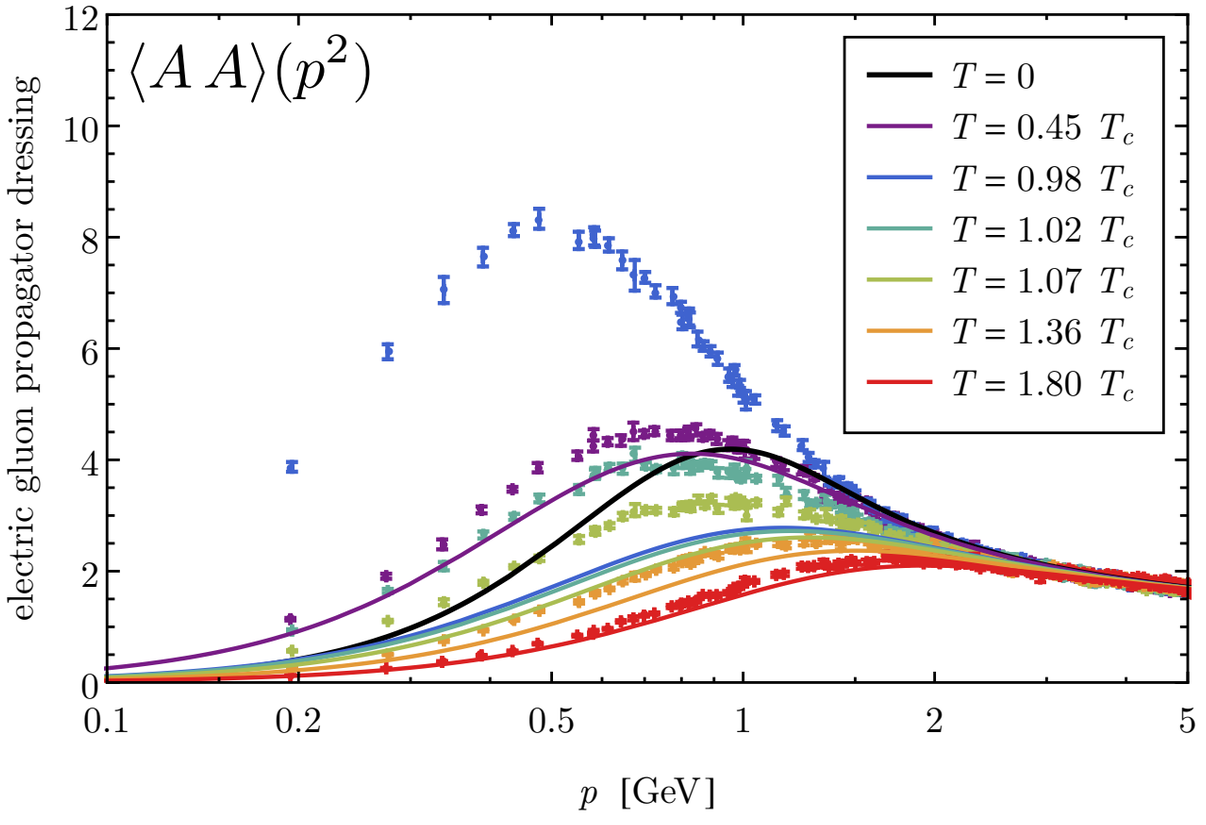


$\langle A_0 \rangle \neq 0$

Lattice: Maas, JMP, Smekal, Spielmann, PRD 85 (2012) 034037

CF model: Reinosa, Serreau, Tissier, Tresmontant, PRD 95 (2017) 045014

chromo-electric propagator



Lattice: Silva, Oliveira, Bicudo, Cardoso, PRD89 (2014) 074503

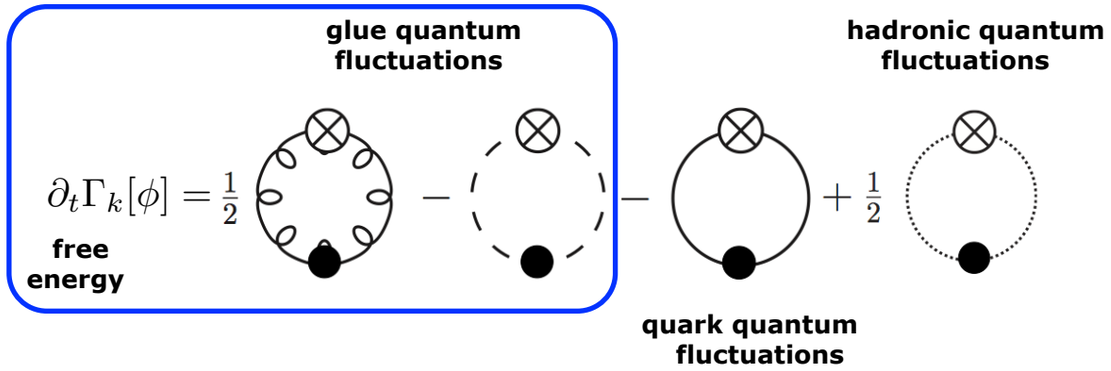
Cyrol, Fister, Mitter, JMP, Strodthoff, PRD 97 (2018) 054015

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{i g \int_0^\beta A_0(x)}$$



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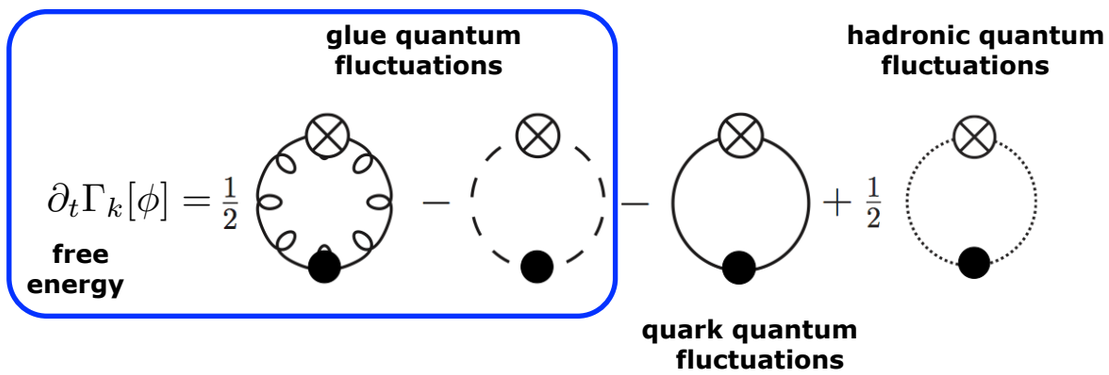
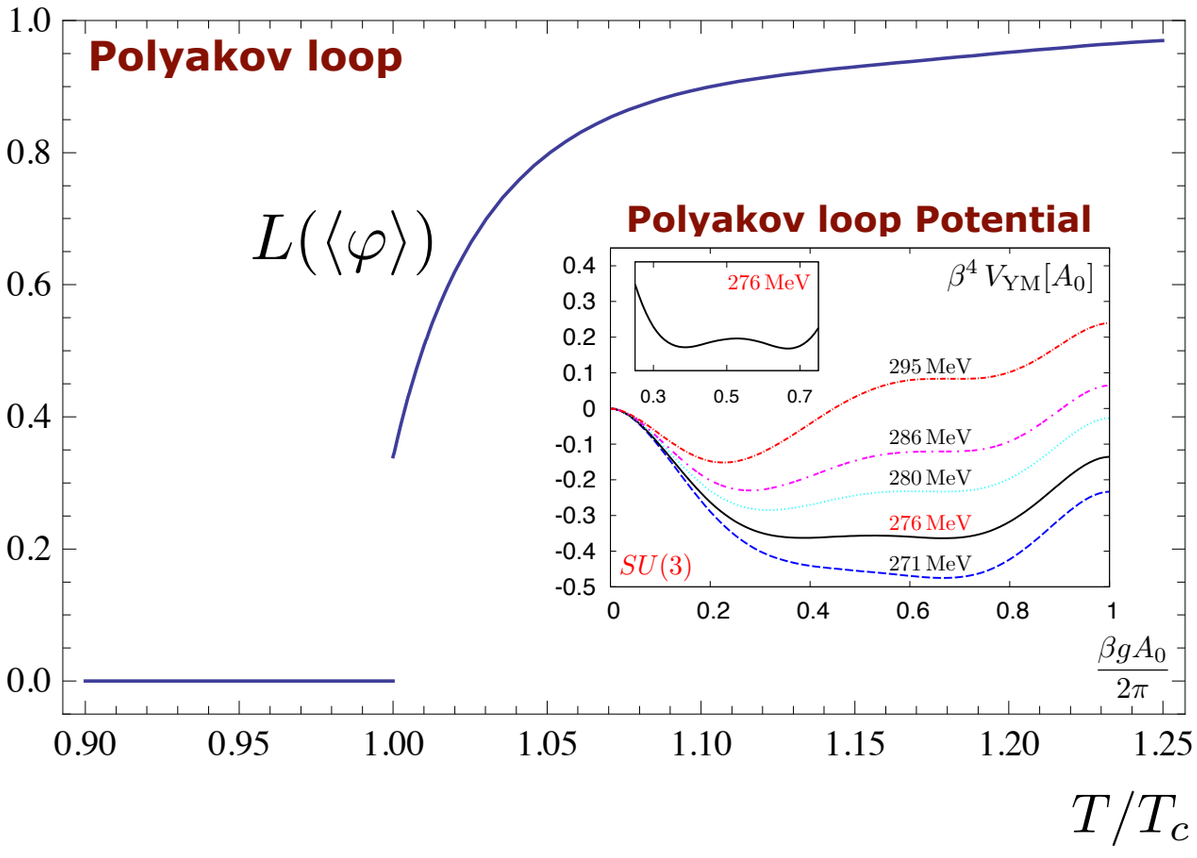


$$\mathcal{P}e^{i\,g\int_0^\beta A_0(x)} = e^{i\varphi}$$



$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

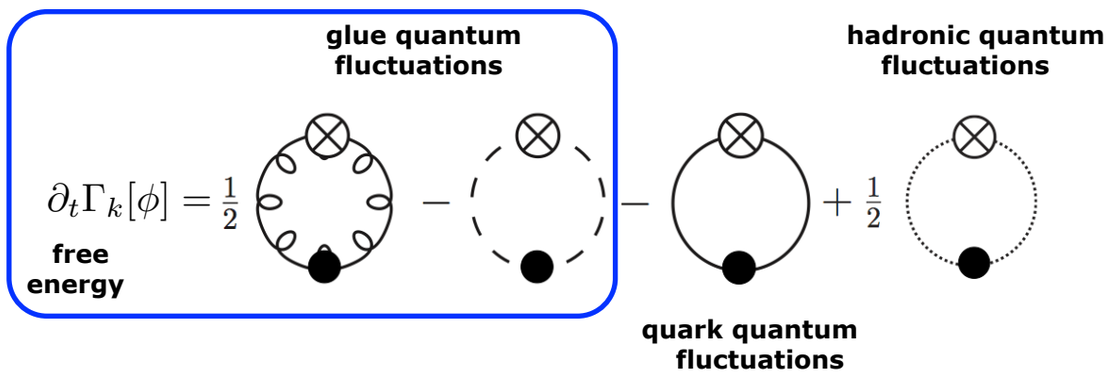
lattice : $T_c/\sqrt{\sigma} = 0.646$

Confinement

FRG: Braun, Gies, JMP, PLB 684 (2010) 262

FRG, DSE, 2PI: Fister, JMP, PRD 88 (2013) 045010

$$L[A_0] = \frac{1}{N_c}\,\mathrm{tr}\,\mathcal{P}e^{ig\int_0^\beta A_0(x)}$$



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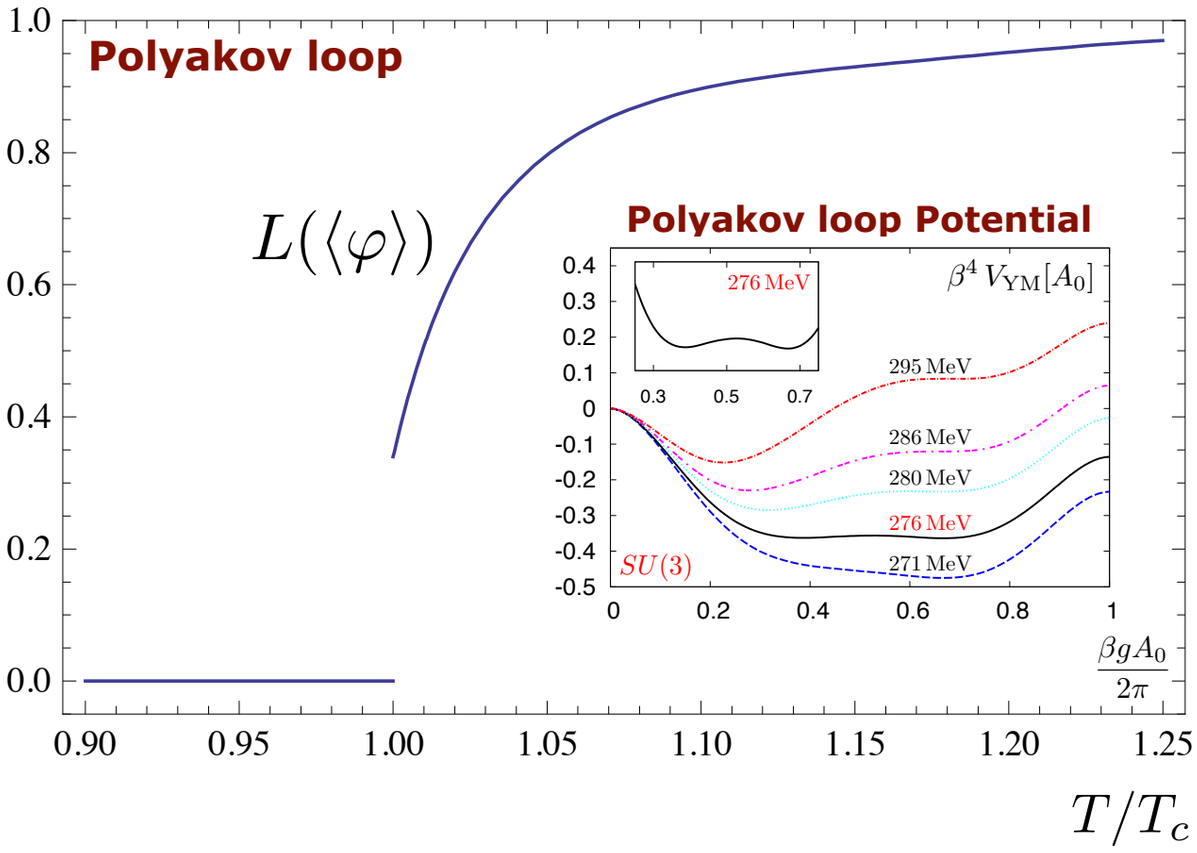
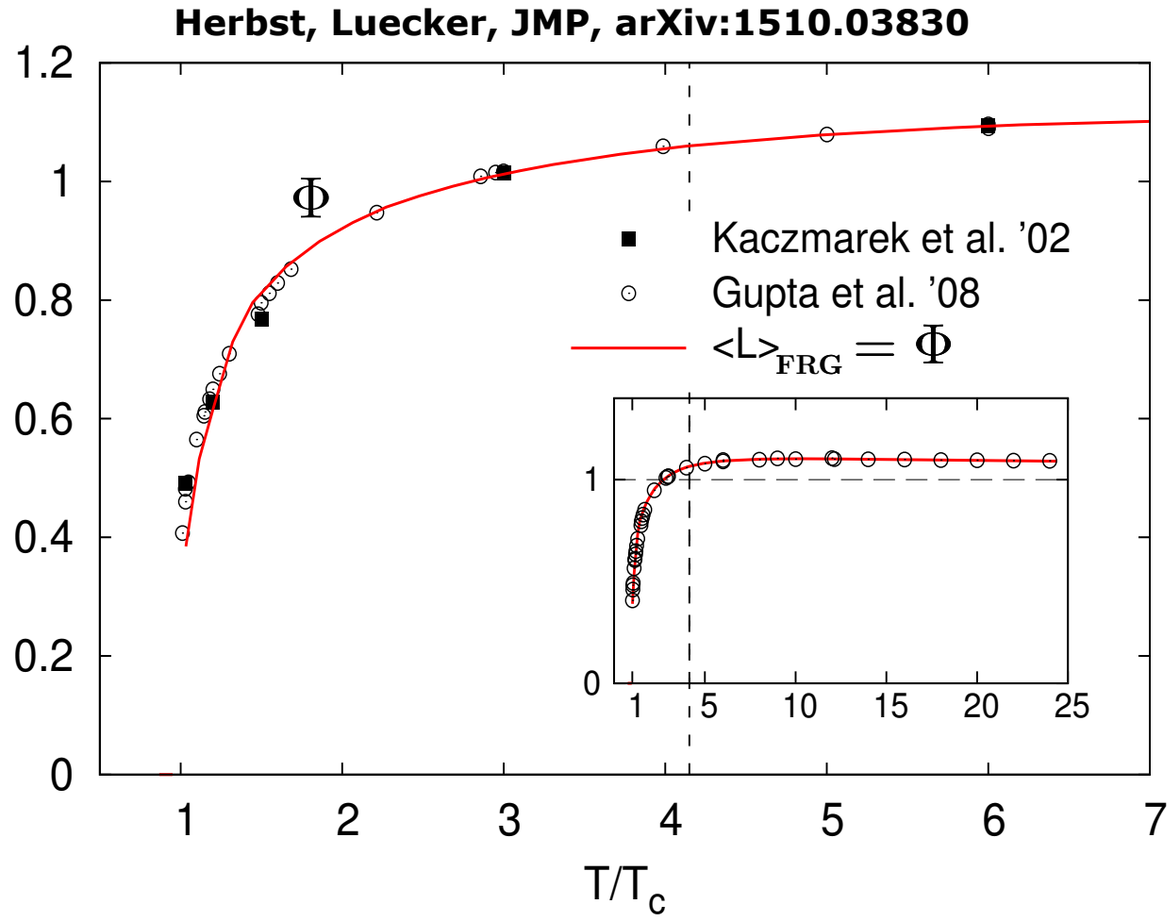


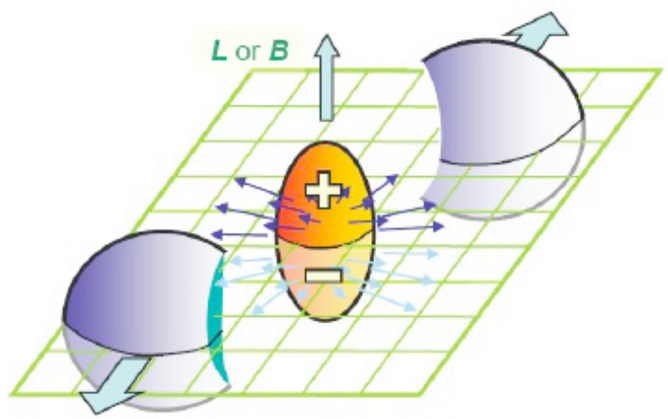


Outline

- QCD from functional methods

Applications

- QCD-assisted hydrodynamics



- QCD-assisted transport

- Summary & outlook

QCD-assisted hydrodynamics

Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985

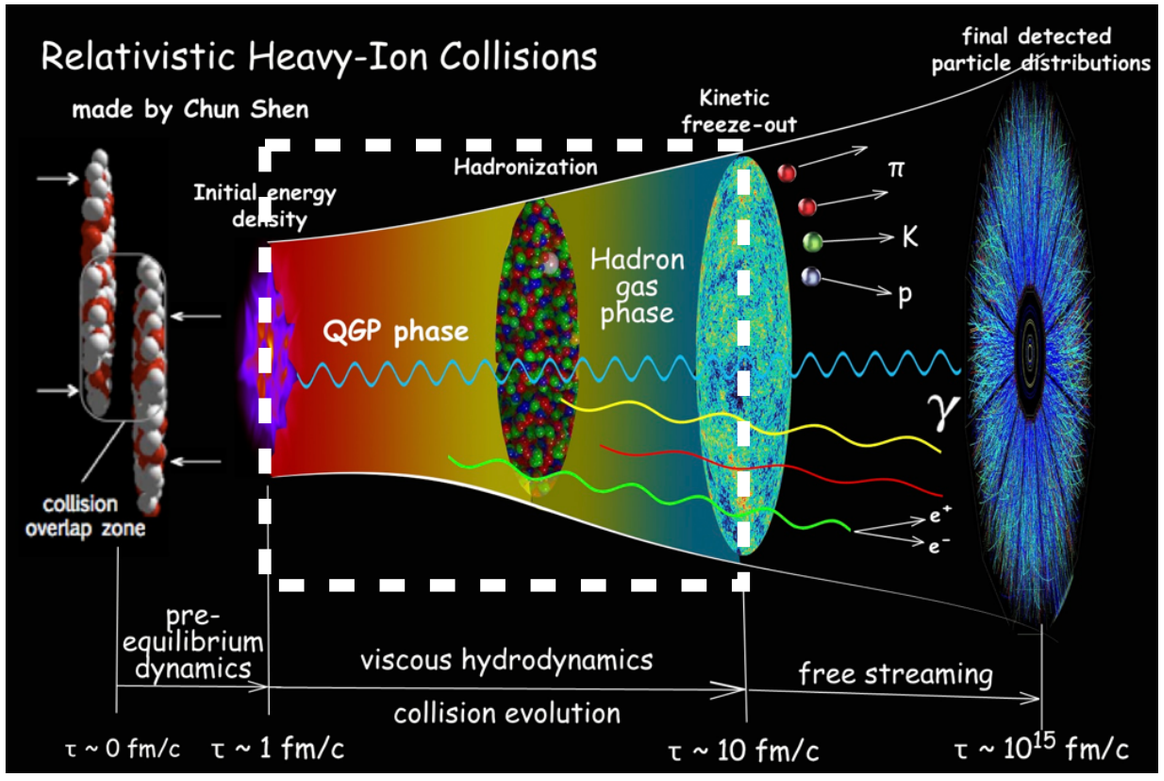
HIC 'phases'

Far from equilibrium initial phase

Kinetic phase

Hydrodynamical phase

Hadronisation & freeze out



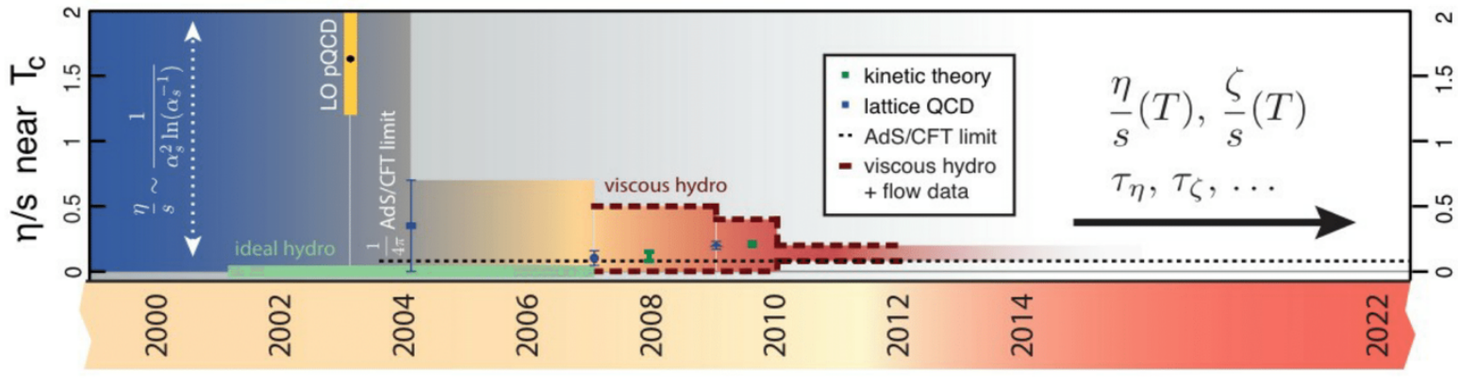
QCD-assisted transport

Hydro with QCD transport coefficients

Equilibrium transport coefficients

'Steady-state' hydro

Constraints for the other phases

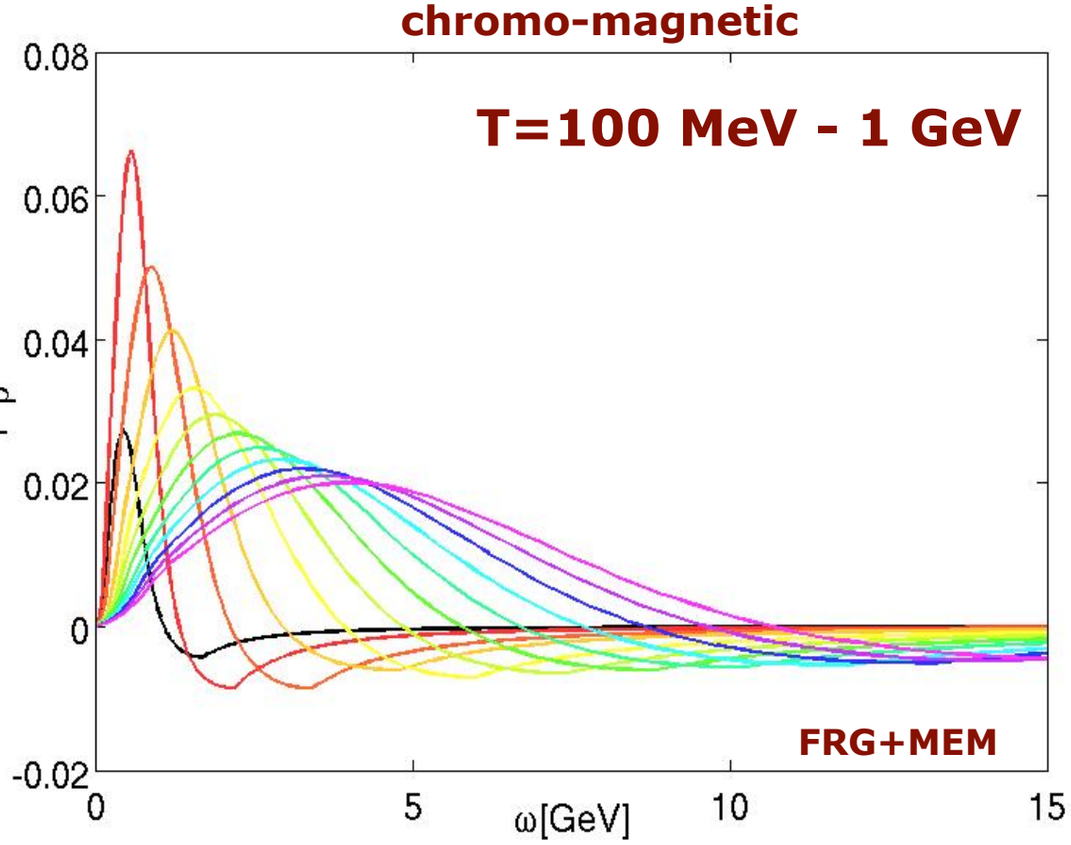


$$\pi^{\mu\nu} = \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\partial_\alpha u^\alpha - \tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta u^\sigma \partial_\sigma \pi^{\alpha\beta}$$

Single particle spectral functions

$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

Single particle spectral functions



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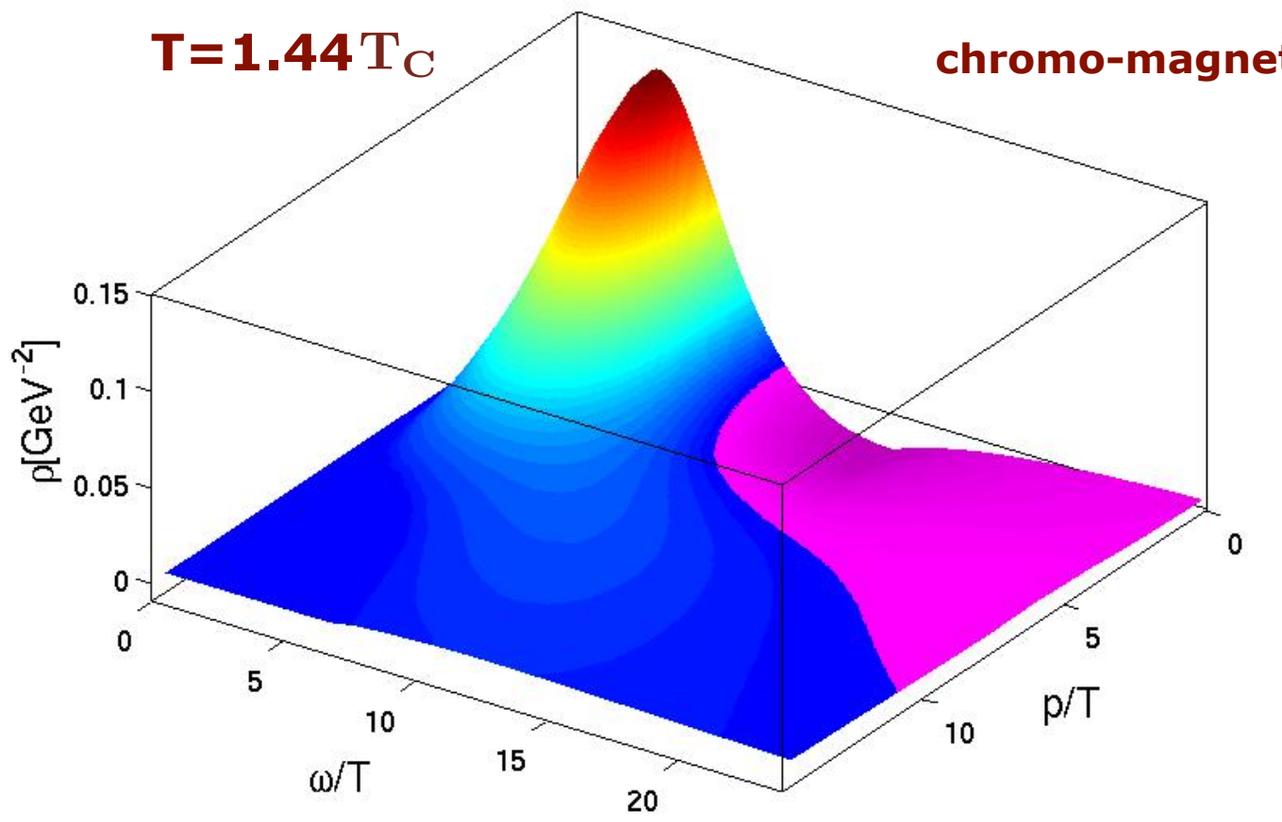
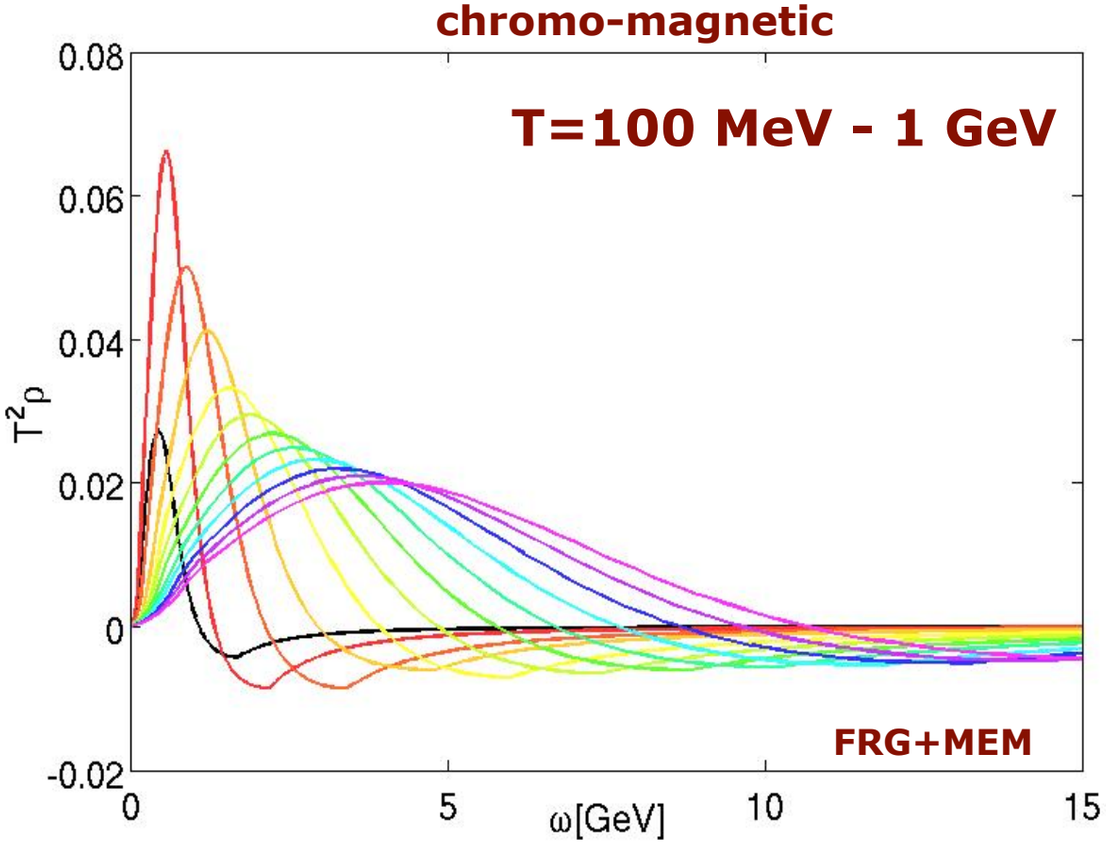


Haas, Fister, JMP, PRD 90 (2014) 091501

Single particle spectral functions



gluon spectral function at T=0

YM: Strauss, Fischer, Kellermann, PRL 109 (2012) 252001
complex frequency DSE

YM: Dudal, Oliveira, Silva, PoS ConfinementX (2012) 033
Tikhonov-continuation

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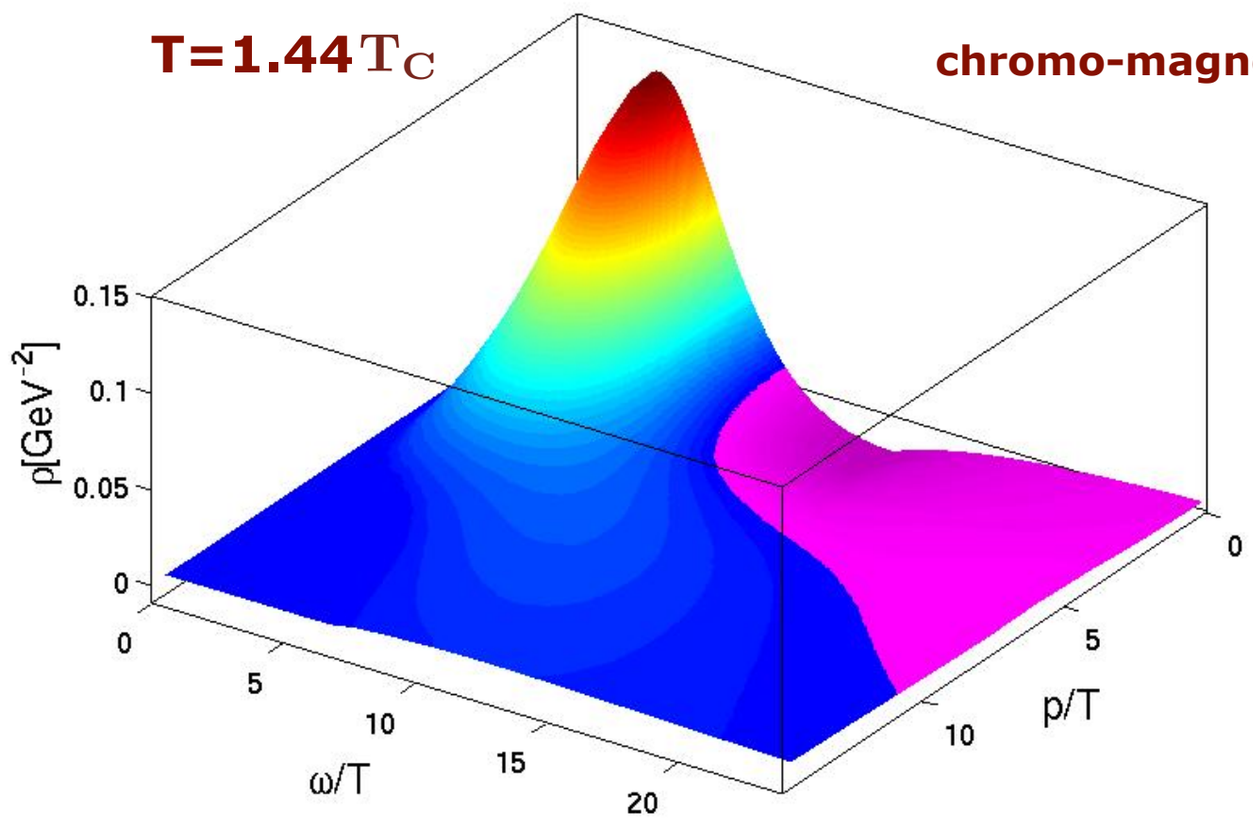

Haas, Fister, JMP, PRD 90 (2014) 091501

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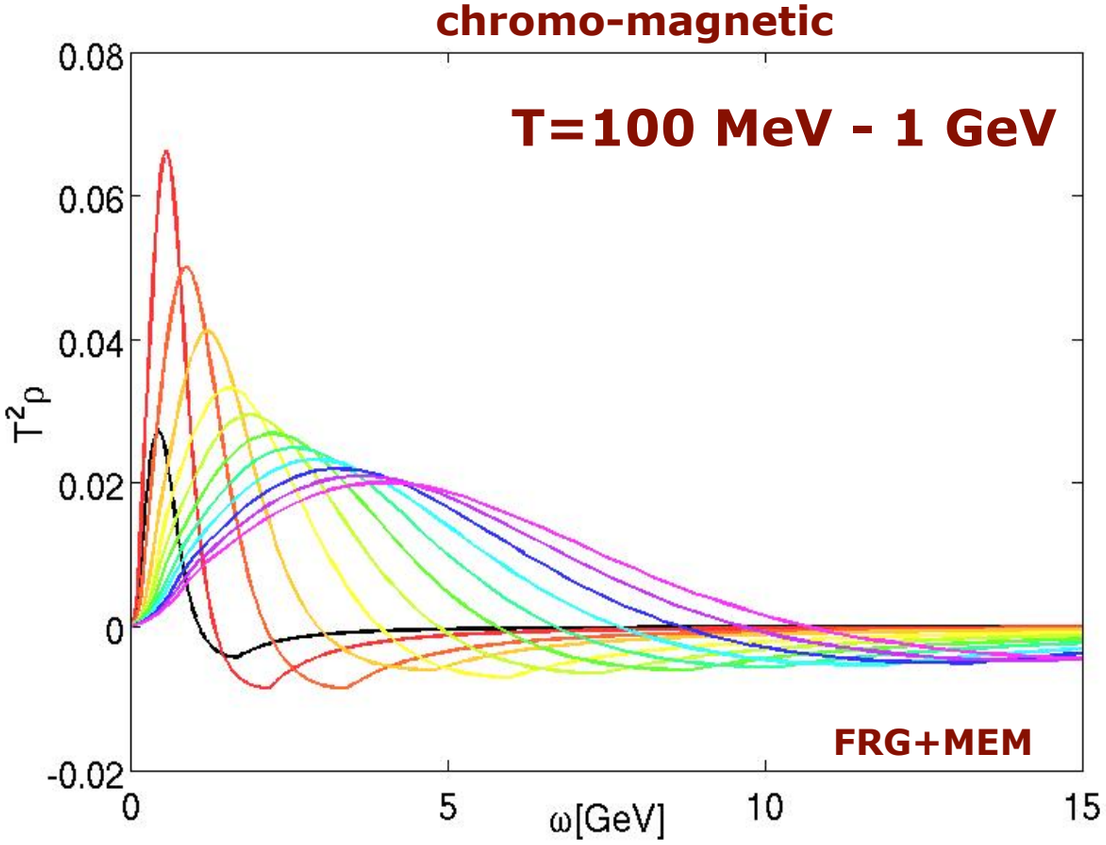

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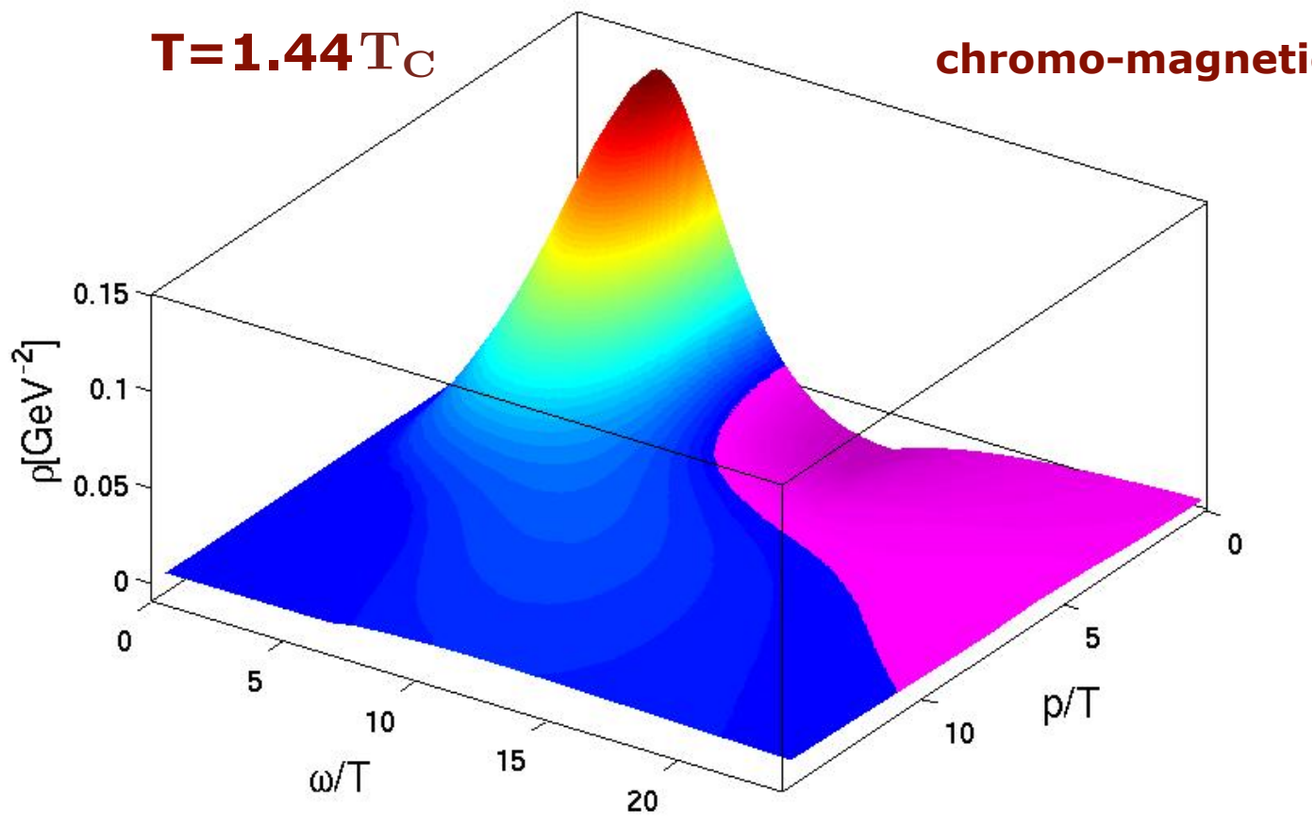
Tikhonov-continuation

gluon spectral function at T≠0

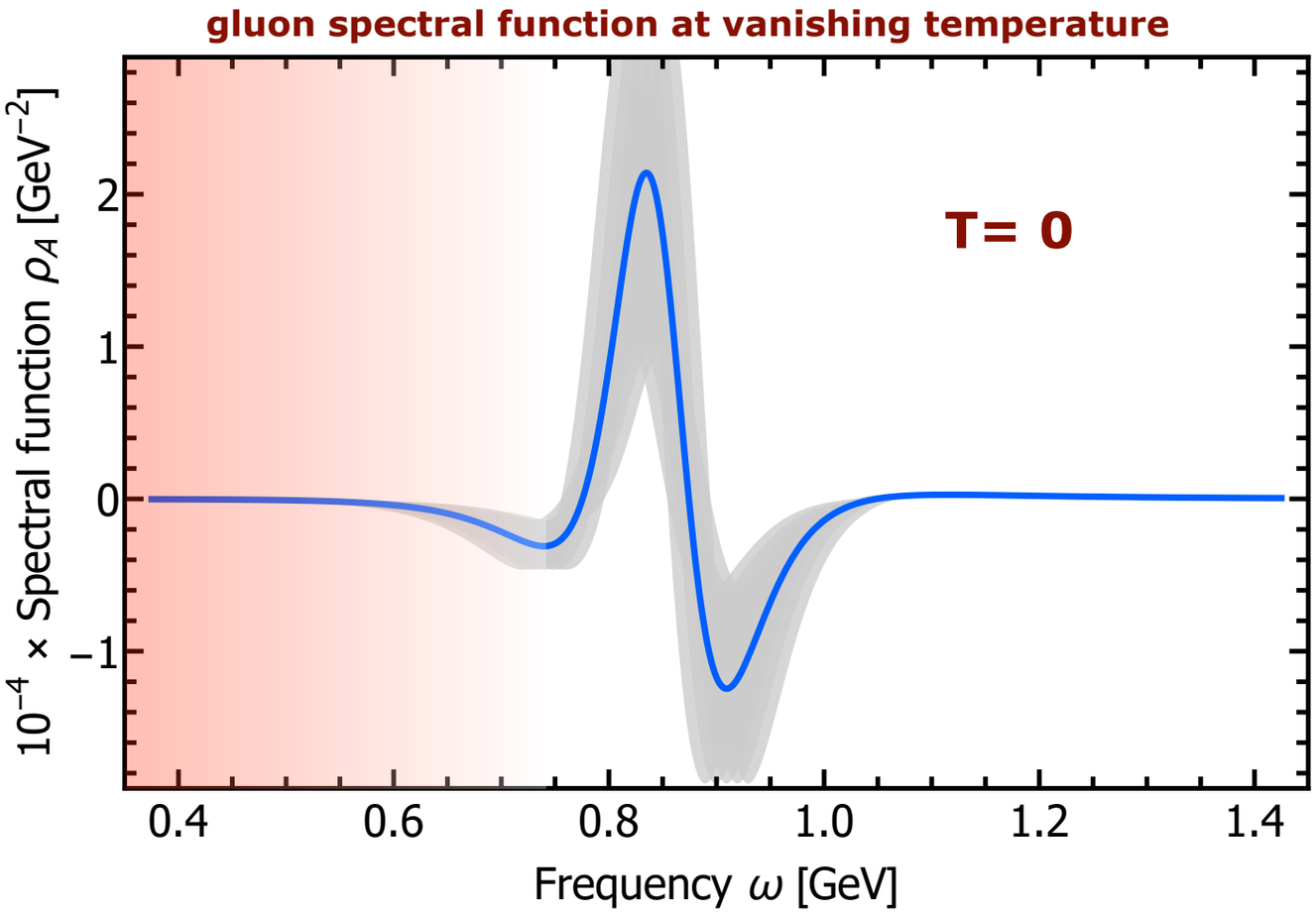
YM: Dudal, Oliveira, Silva, PoS LATTICE2013 (2014) 366
Tikhonov-continuation

QCD: Ilgenfritz, JMP, Rothkopf, Trunin; arXiv:1701.08610



Haas, Fister, JMP, PRD 90 (2014) 091501

Single particle spectral functions



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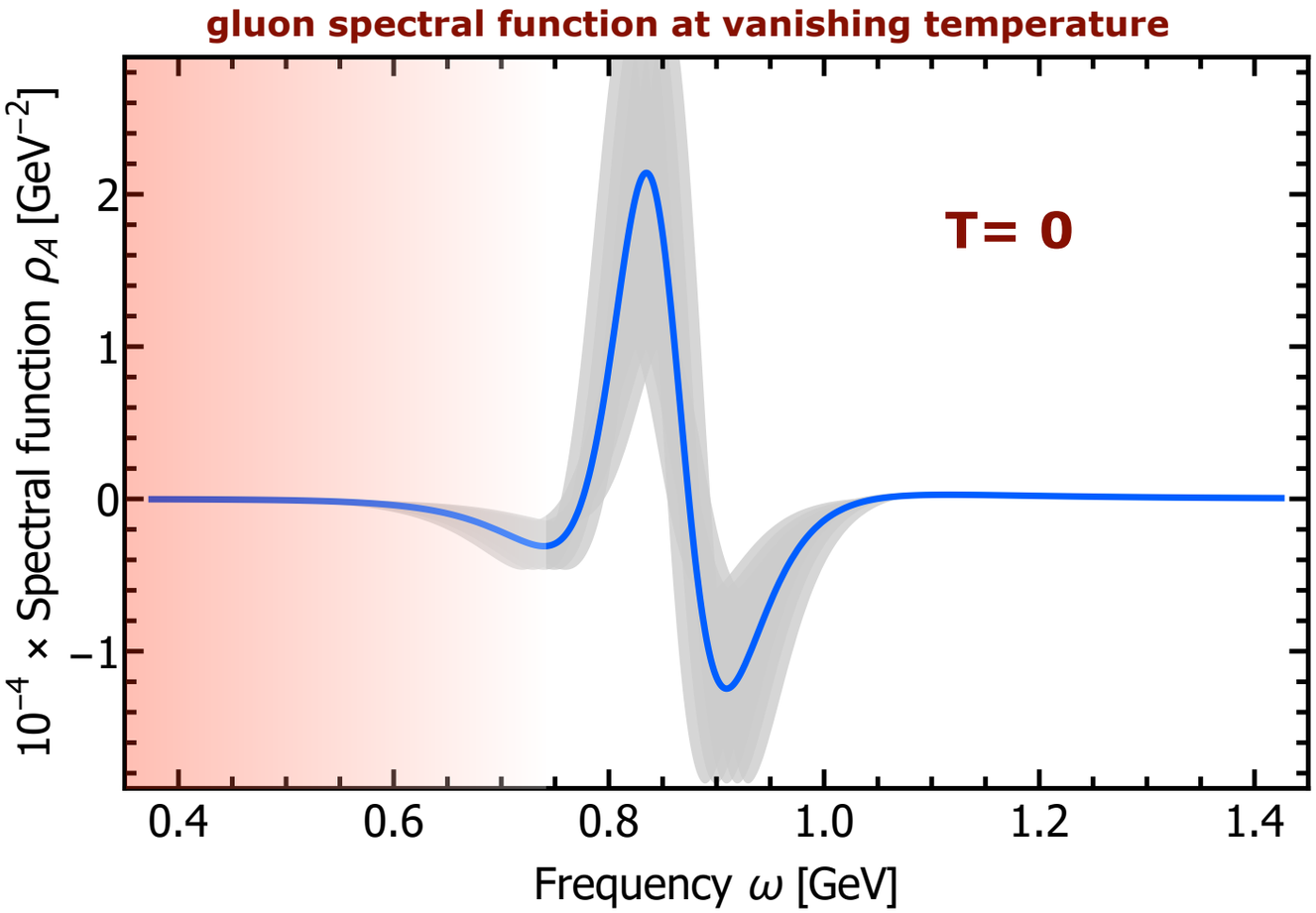
novel analytic IR (& UV) behaviour

qualitatively refined reconstruction

Cyrol, JMP, Rothkopf, Wink, arXiv:1804.00945

Single particle spectral functions



$$\rho(p) = 2\,\mathrm{Im}\langle A\,A\rangle_{\mathrm{ret}}(p)$$

novel analytic IR (& UV) behaviour

qualitatively refined reconstruction

'Those are my methods (principles), and if you don't like them...well, I have others'

Groucho Marx

direct computation

Real-time FRG: wait 5 minutes

Real time FRG: see also talk of Ralf-Arno Tripolt

Cyrol, JMP, Rothkopf, Wink, arXiv:1804.00945

Transport coefficients

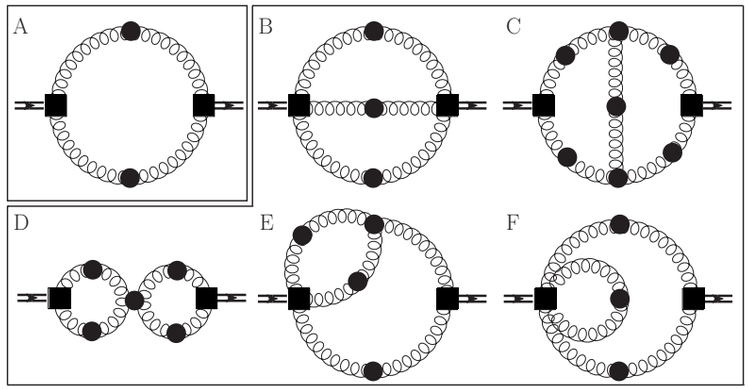
viscosity over entropy ratio in Yang-Mills theory

Kubo relation

$$\eta = \frac{1}{20} \left. \frac{d}{d\omega} \right|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

'3-loop' exact functional relation for $\rho_{\pi\pi}$

1 & 2-loop terms



Haas, Fister, JMP, PRD 90 (2014) 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

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Gluon spectral function

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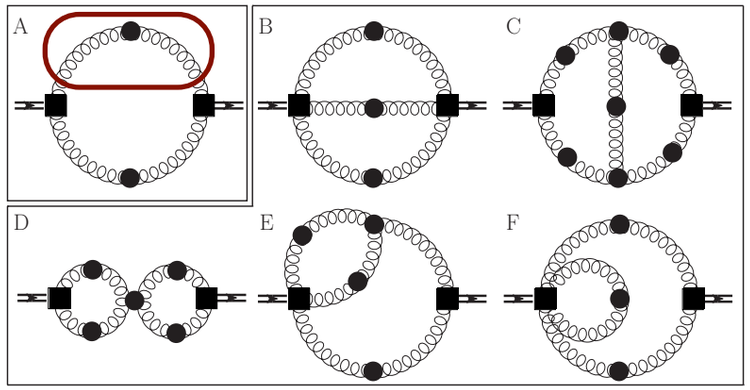

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Transport coefficients

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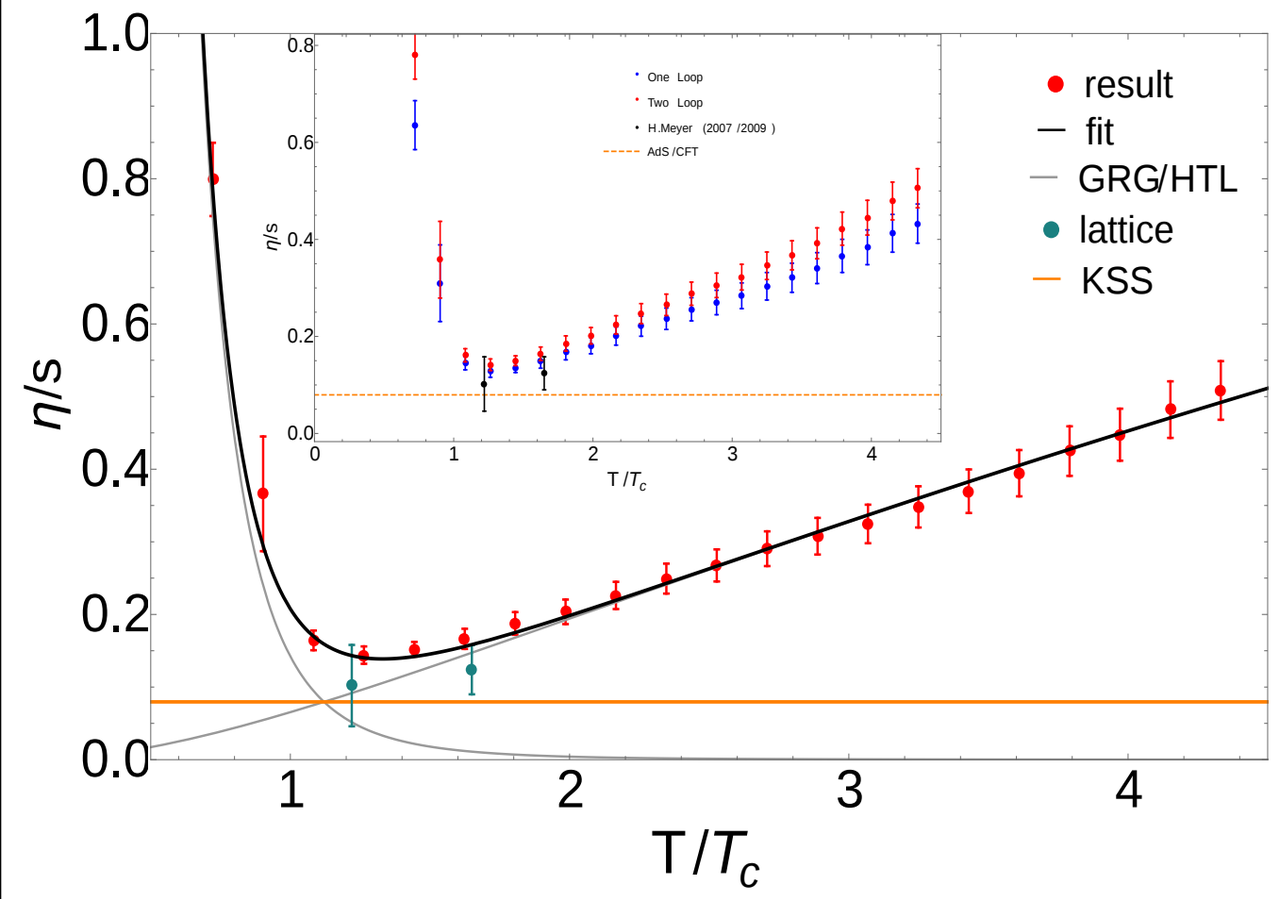
Yang-Mills viscosity over entropy ratio



recent lattice results: Astrakhantsev, Braguta, Kotov, JHEP 1704 (2017) 101
arXiv:1804.02382

Aiming at apparent convergence

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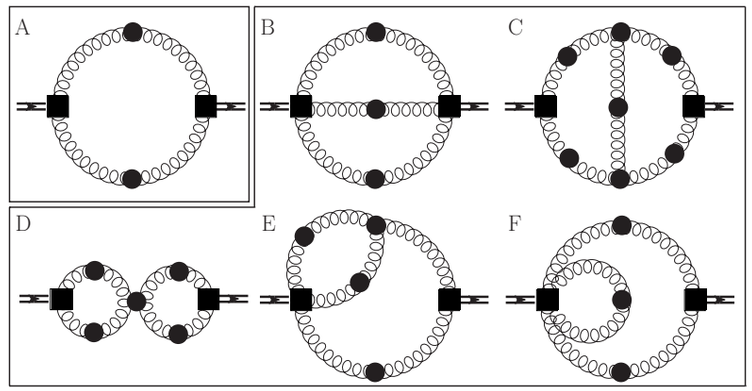


Haas, Fister, JMP, PRD 90 (2014) 091501

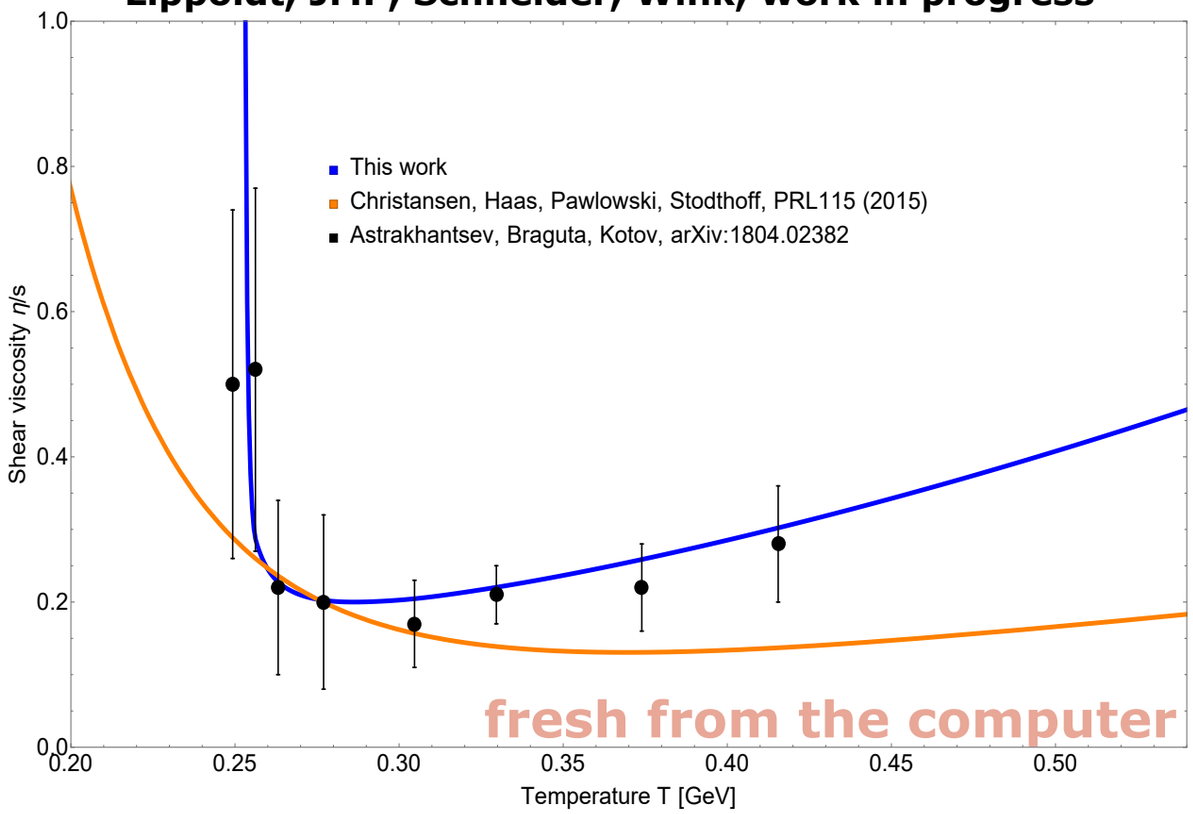
Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

Transport coefficients

viscosity over entropy ratio in Yang-Mills theory

Yang-Mills viscosity over entropy ratio

Lippoldt, JMP, Schneider, Wink, work in progress



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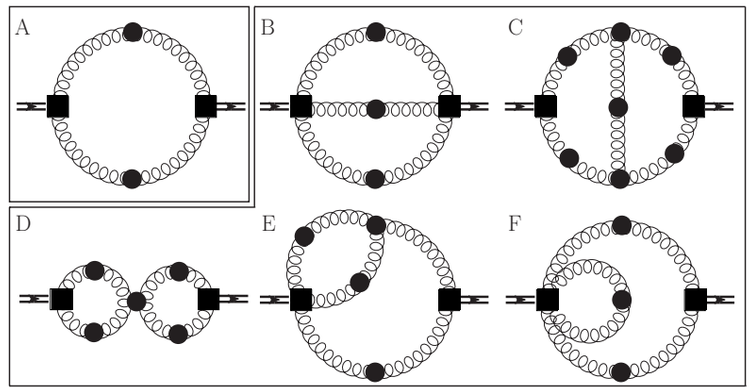


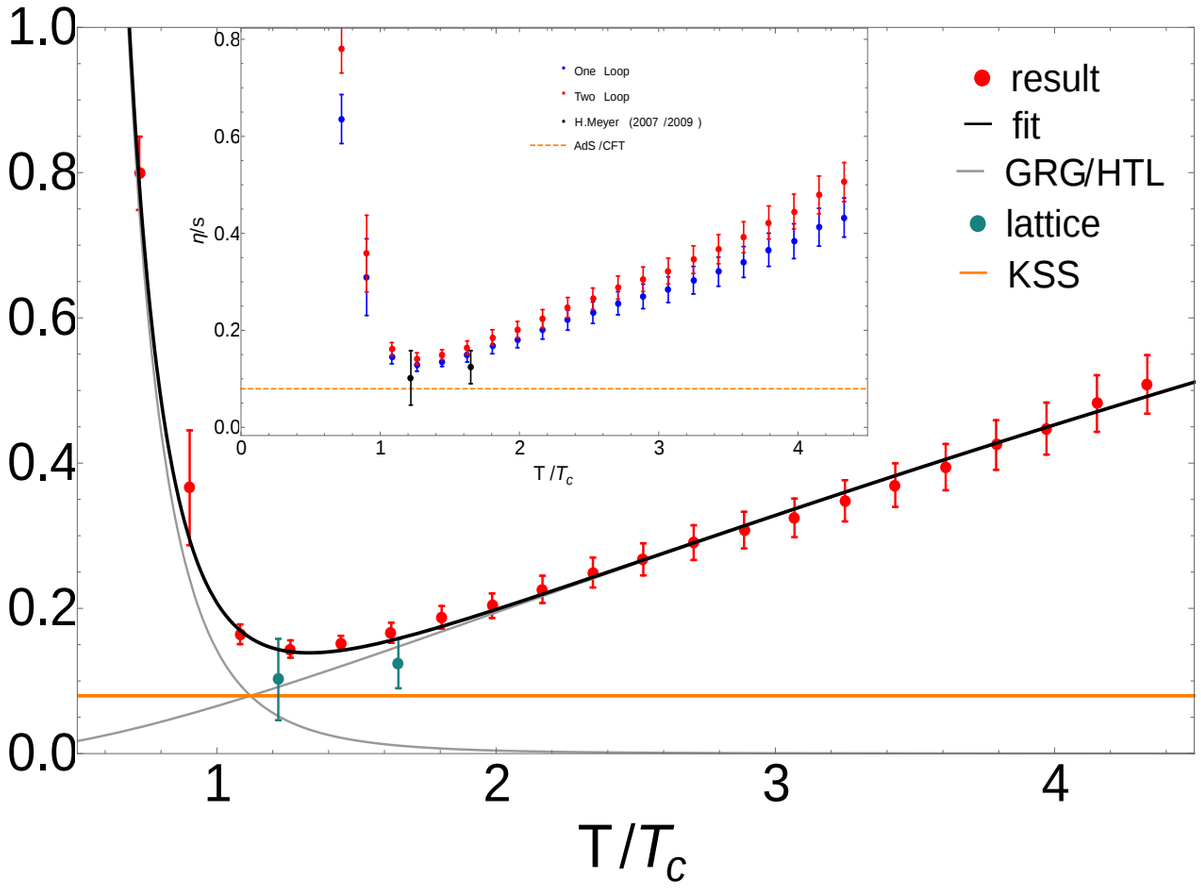
Haas, Fister, JMP, PRD 90 (2014) 091501

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

Transport coefficients

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$\frac{\eta}{s}(T) = \frac{a_{\rm qgp}}{\alpha_s^{\gamma_{\rm qgp}}(cT/T_c)} + \frac{a_{\rm grg}}{(T/T_c)^{\gamma_{\rm grg}}}$$

$$a_{\rm qgp} \approx 0.15$$

$$a_{\rm hrg} \approx 0.14$$

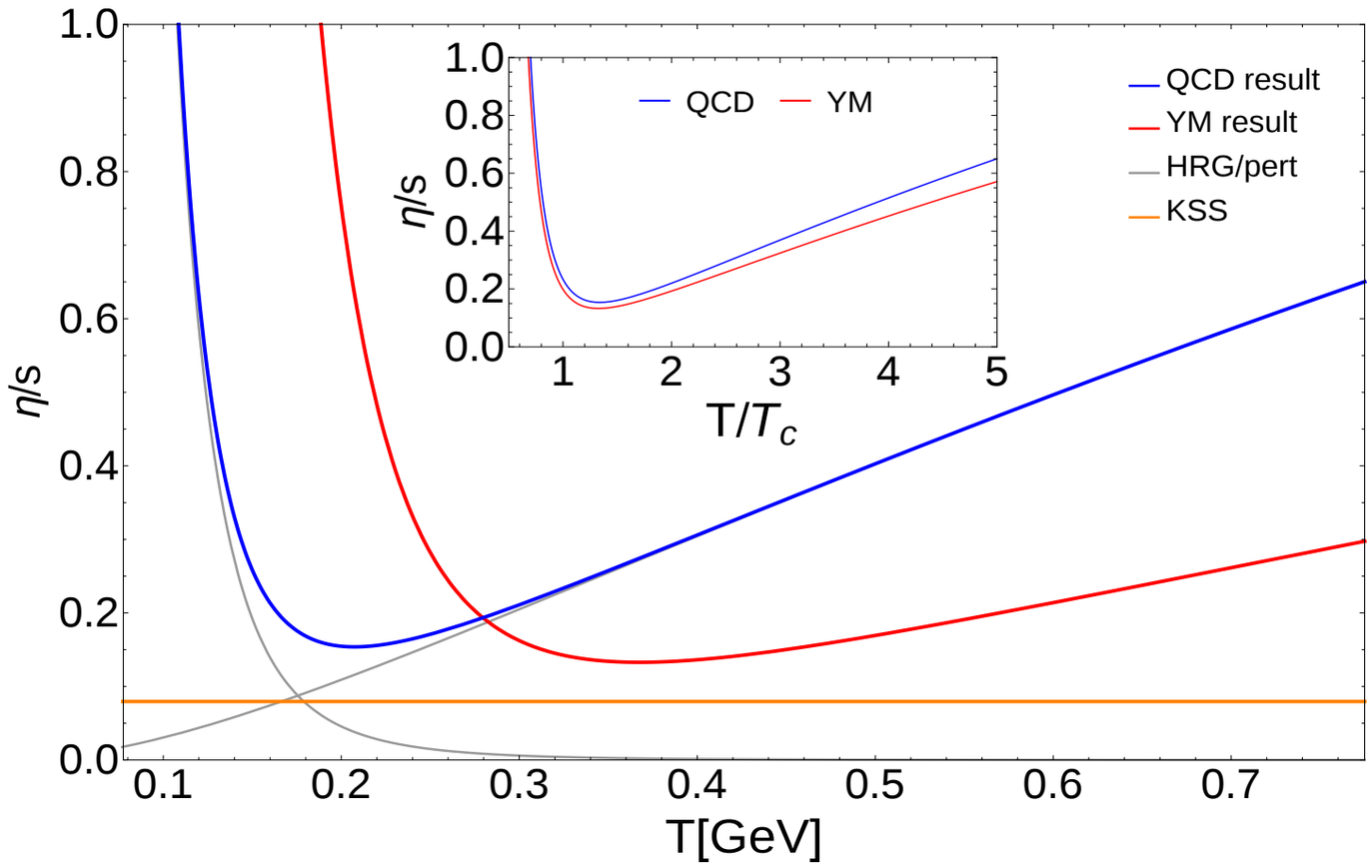
$$c \approx 0.66$$

Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

Transport coefficients

QCD - estimate for viscosity over entropy ratio

viscosity over entropy ratio



$$a_{\rm qgp} \approx 0.2$$

$$a_{\rm hrg} \approx 0.16$$

$$c \approx 0.79$$

QCD

$$\gamma_{\rm grg} \approx 5$$

$$\gamma_{\rm qgp} \approx 1.6$$

pure glue

$$a_{\rm qgp} \approx 0.15$$

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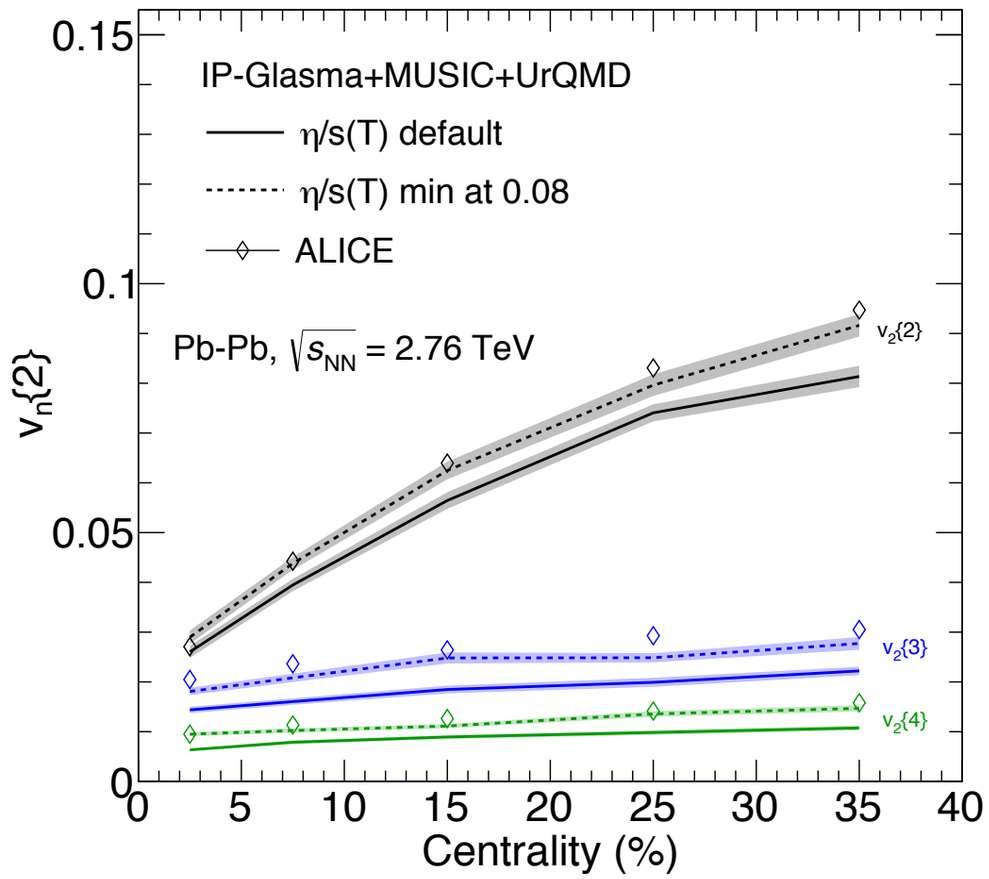
Christiansen, Haas, JMP, Strodthoff, PRL 115 (2015) 112002

QCD-assisted hydrodynamics

Dubla, Masciocchi, JMP, Schenke, Shen, Stachel, arXiv:1805.02985

IP-Glasma - MUSIC - UrQMD

v_n as function of centrality

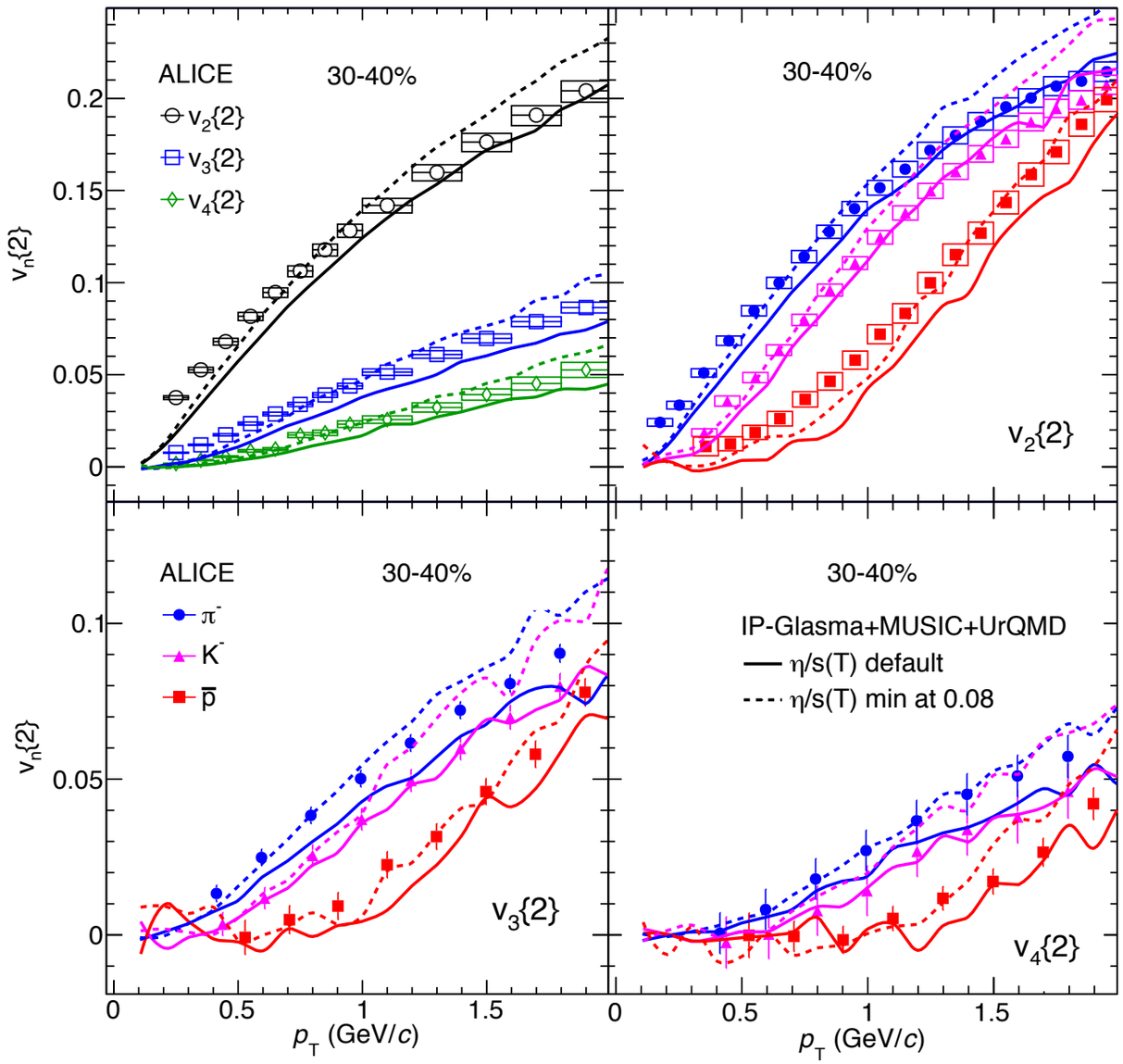


Test of systematic error

$$\eta/s(T) \rightarrow \eta/s(T) + d$$

$$d \in [-0.06, 0]$$

v_n as function of p_T



$$\frac{dN}{d(\varphi - \Psi_R)} = \frac{N_0}{2\pi}\left(1 + 2\sum_n v_n \cos[\, n\,(\varphi - \Psi_R)]\right)$$

QCD-assisted hydrodynamics

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IP-Glasma - MUSIC - UrQMD

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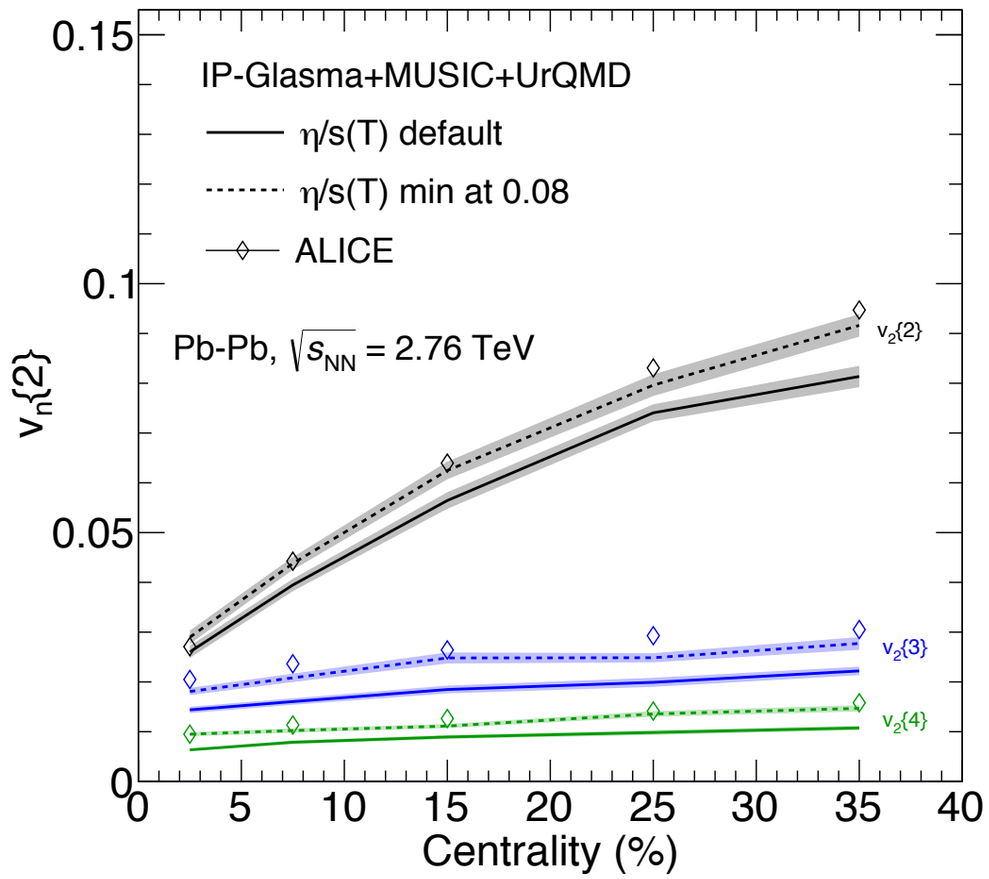

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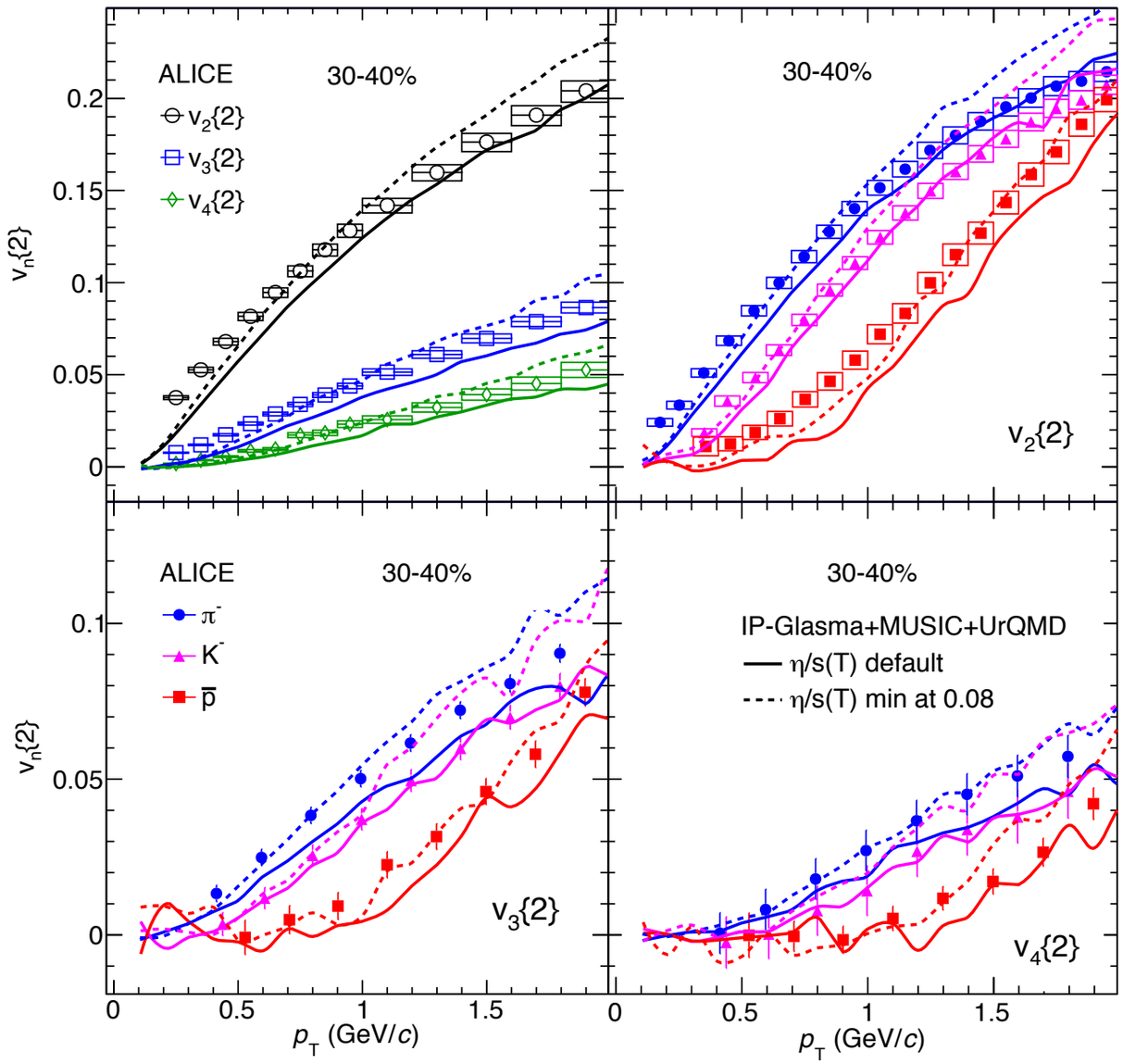

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IP-Glasma - MUSIC - UrQMD

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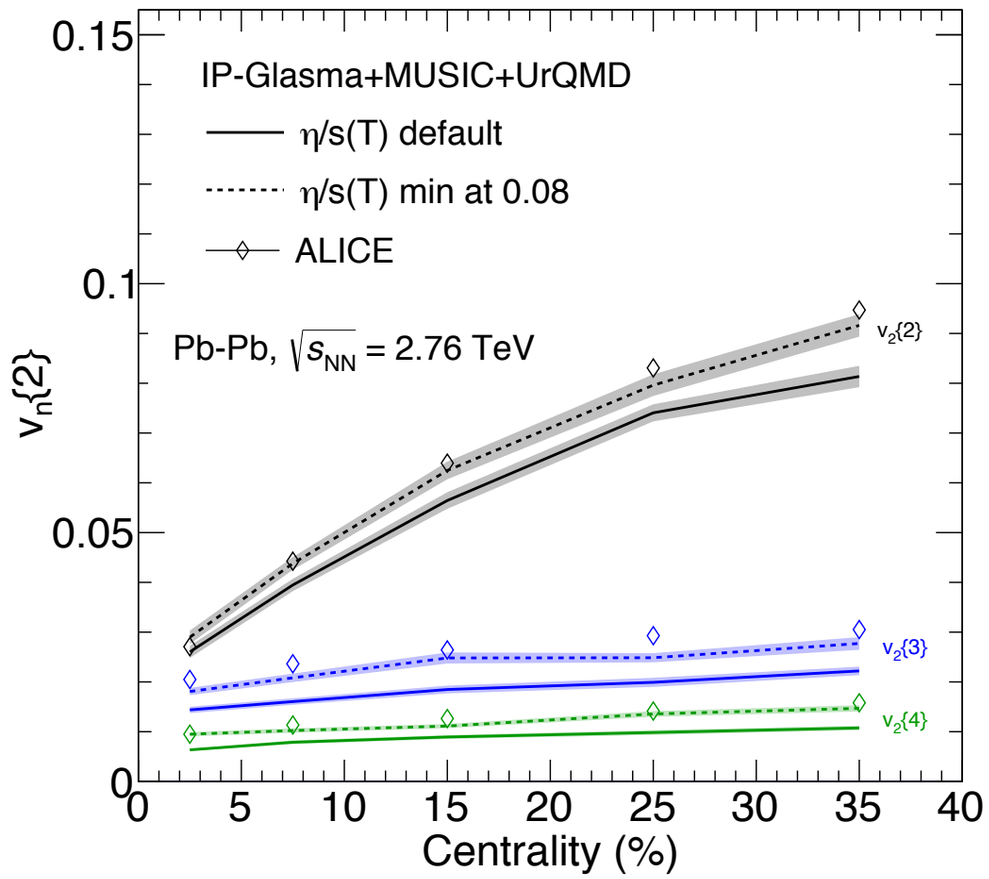

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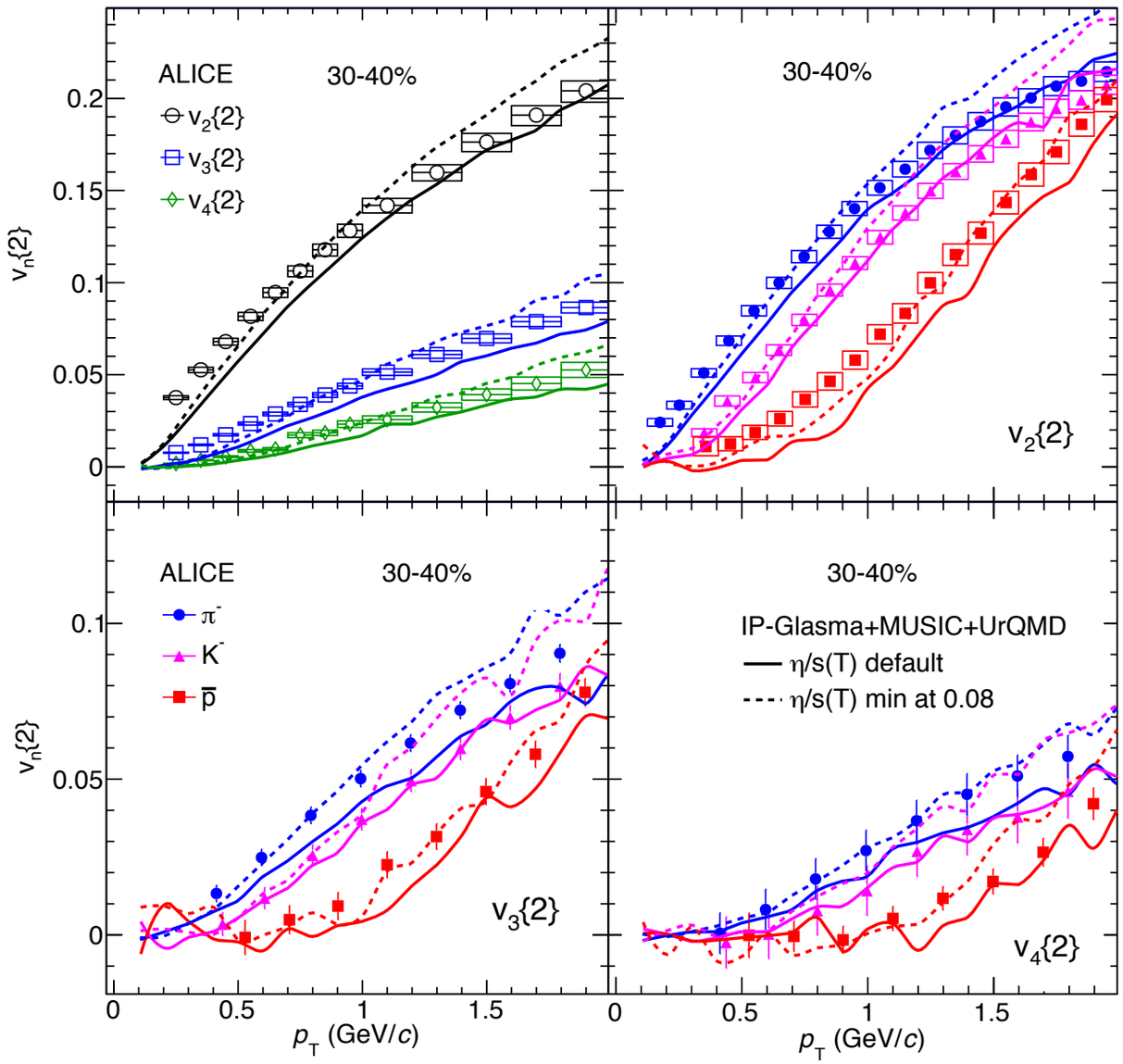

Initial state fluctuations?

Kinetic phase?

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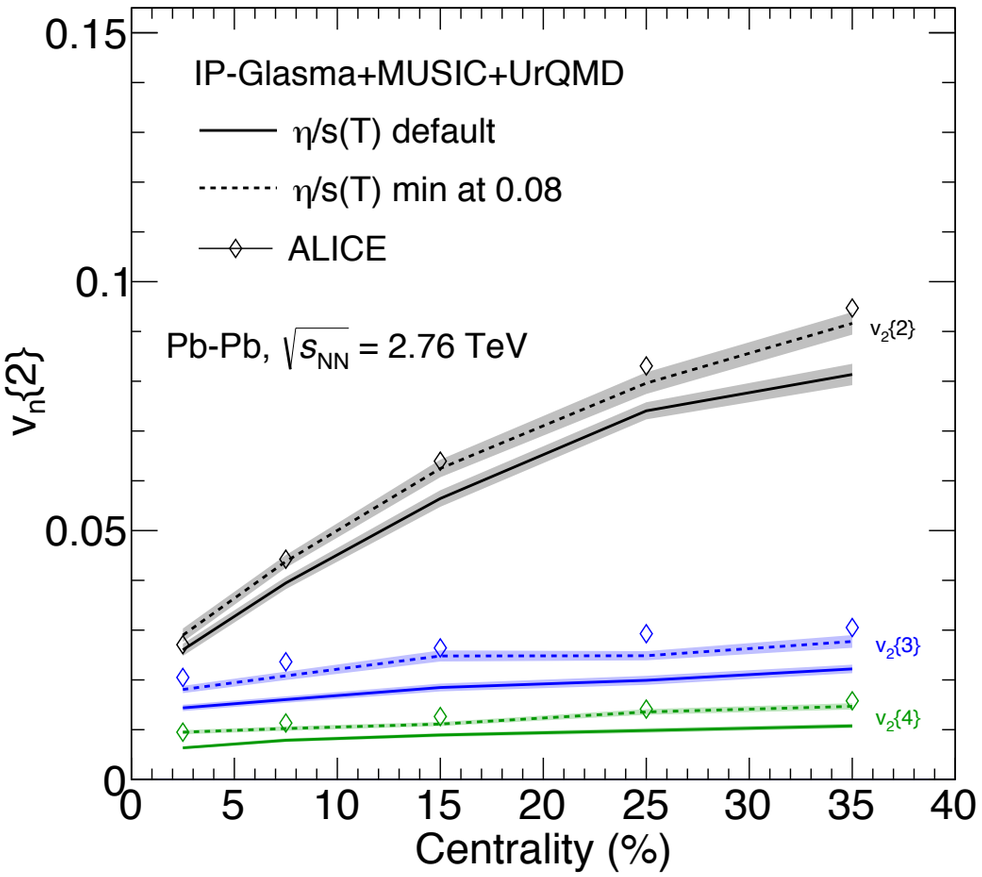
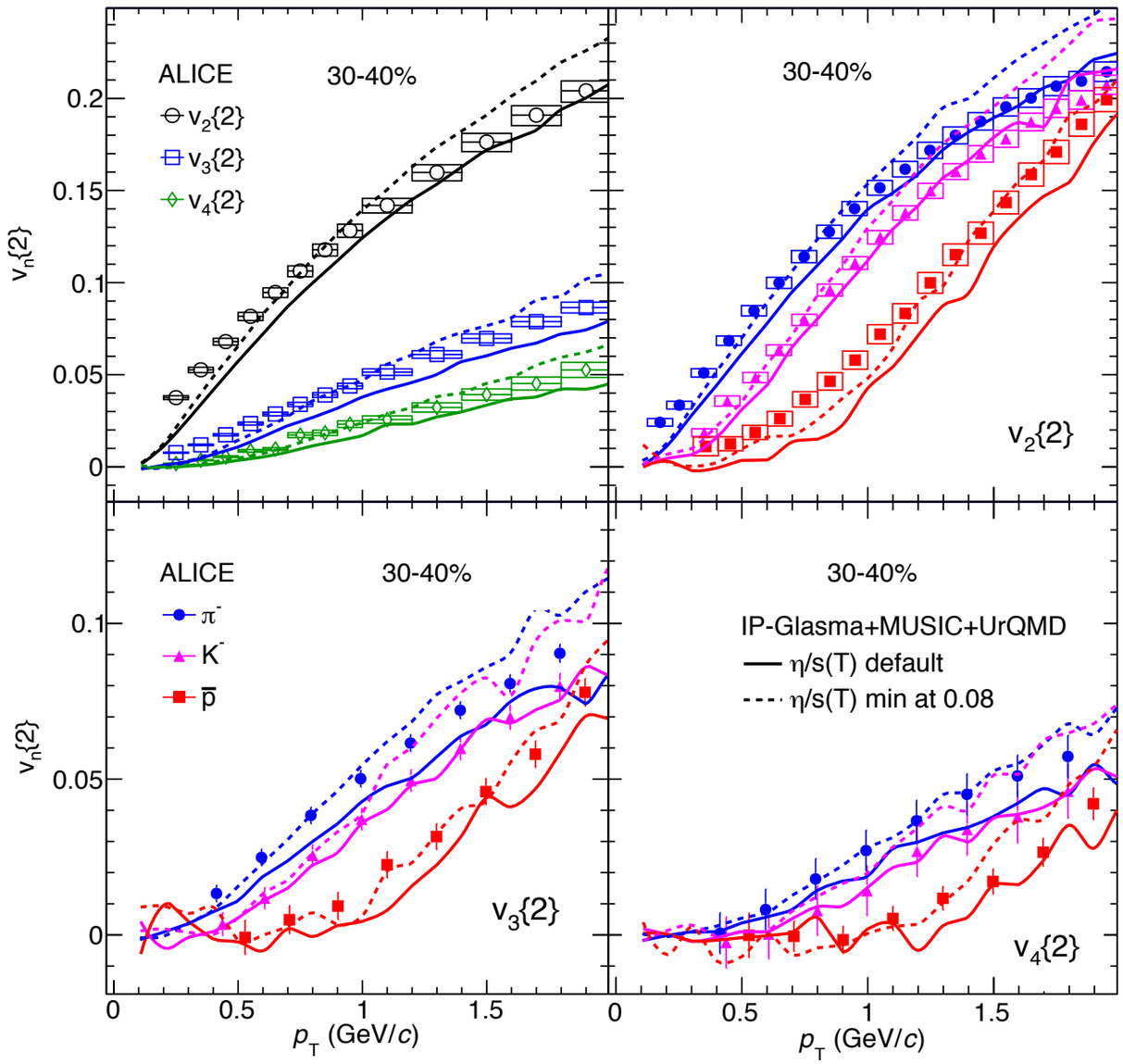
IP-Glasma - MUSIC - UrQMD

v_n as function of centrality



v_n as function of p_T



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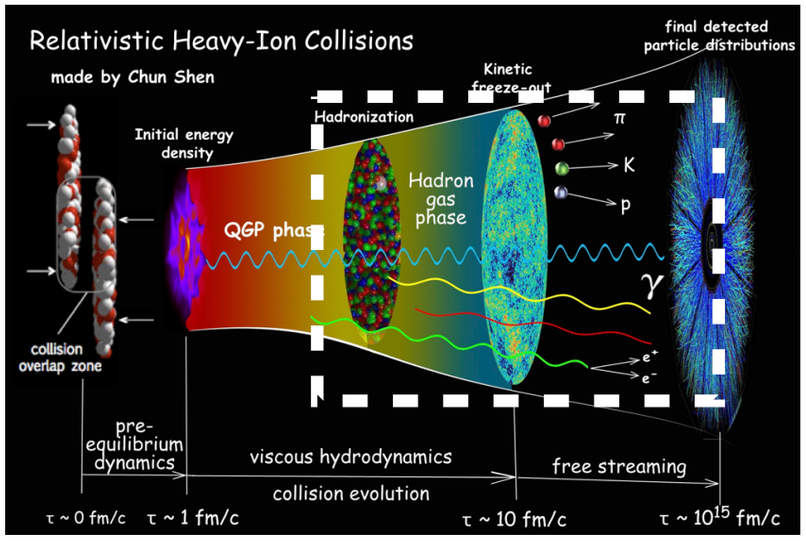
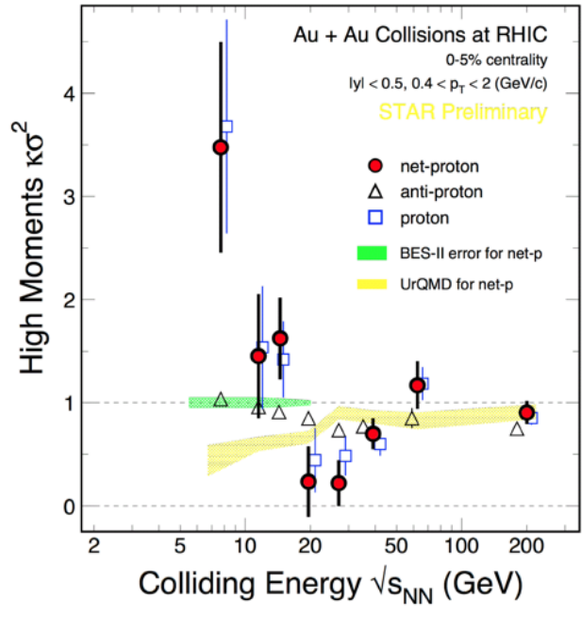
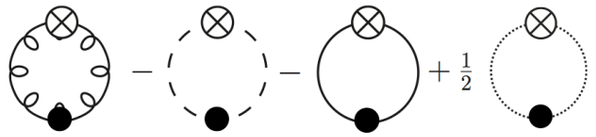
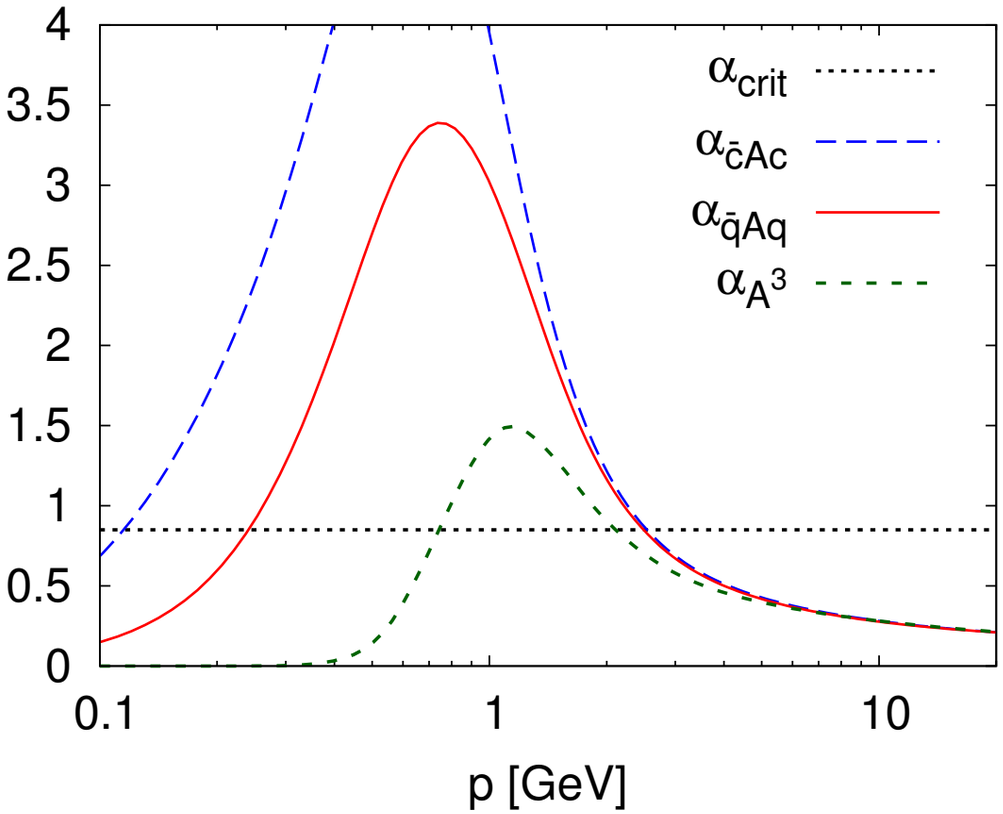




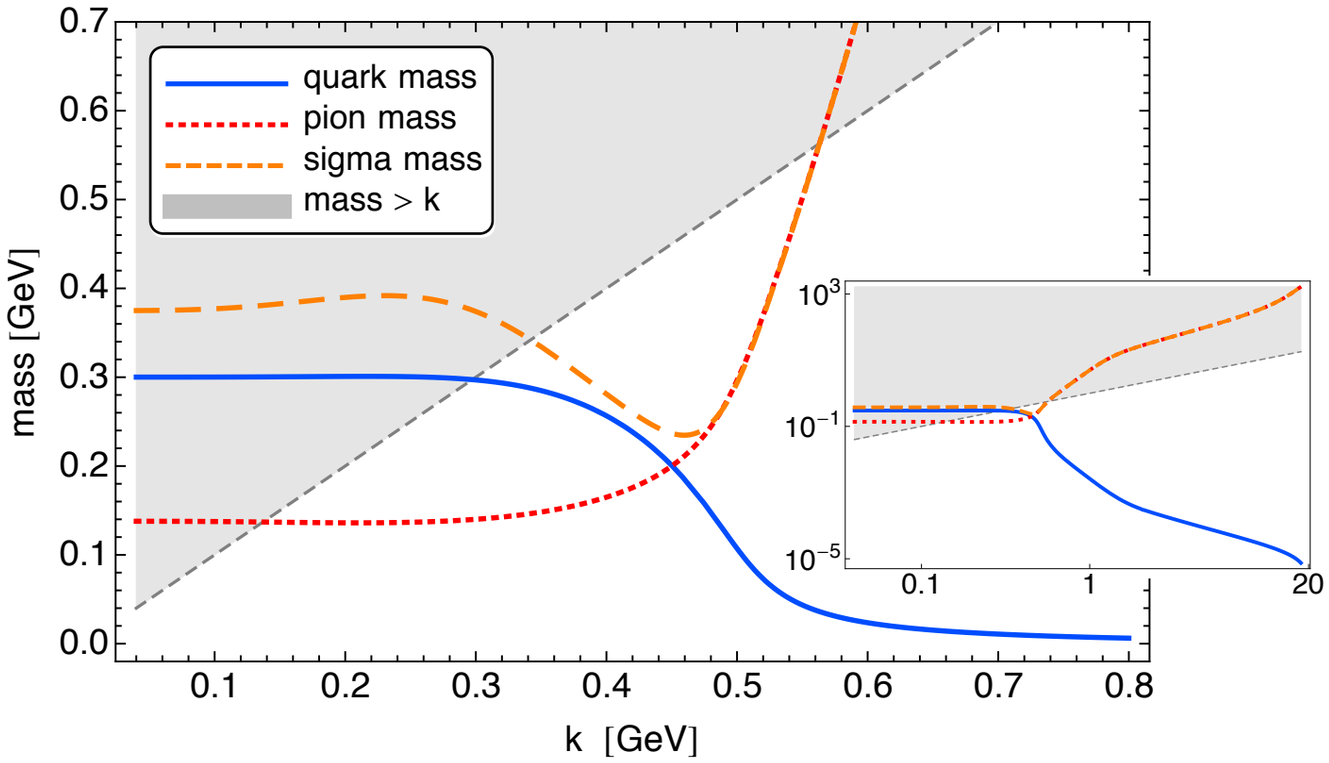
On the unreasonable effectiveness of low energy effective theories

$$\partial_t \Gamma_k[\phi] = \tfrac{1}{2}$$  $$+ \tfrac{1}{2}$$

Sequential decoupling of gluon, quark, sigma, pion fluctuations



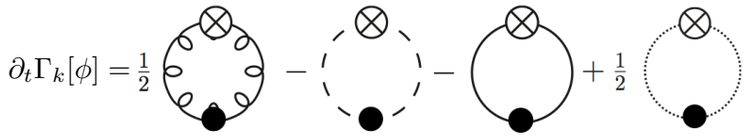
Mitter, JMP, Strodthoff, PRD 91 (2015) 054035



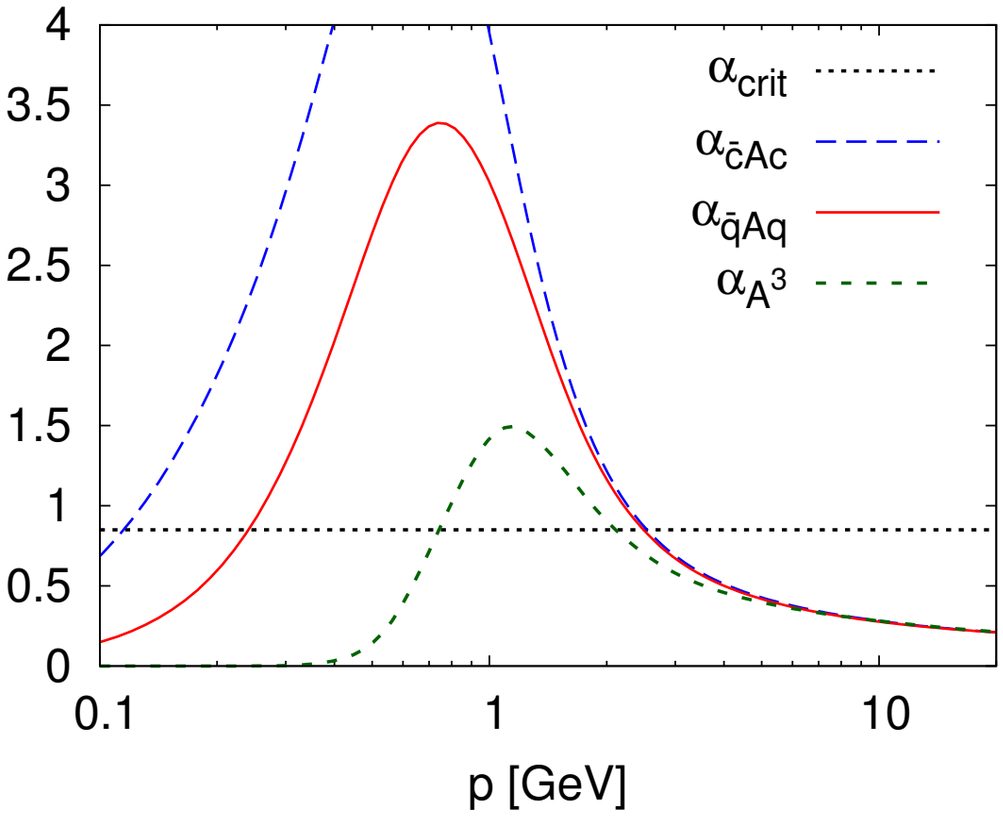
Braun, Fister, Haas, JMP, Rennecke, PRD 94 (2016) 034016

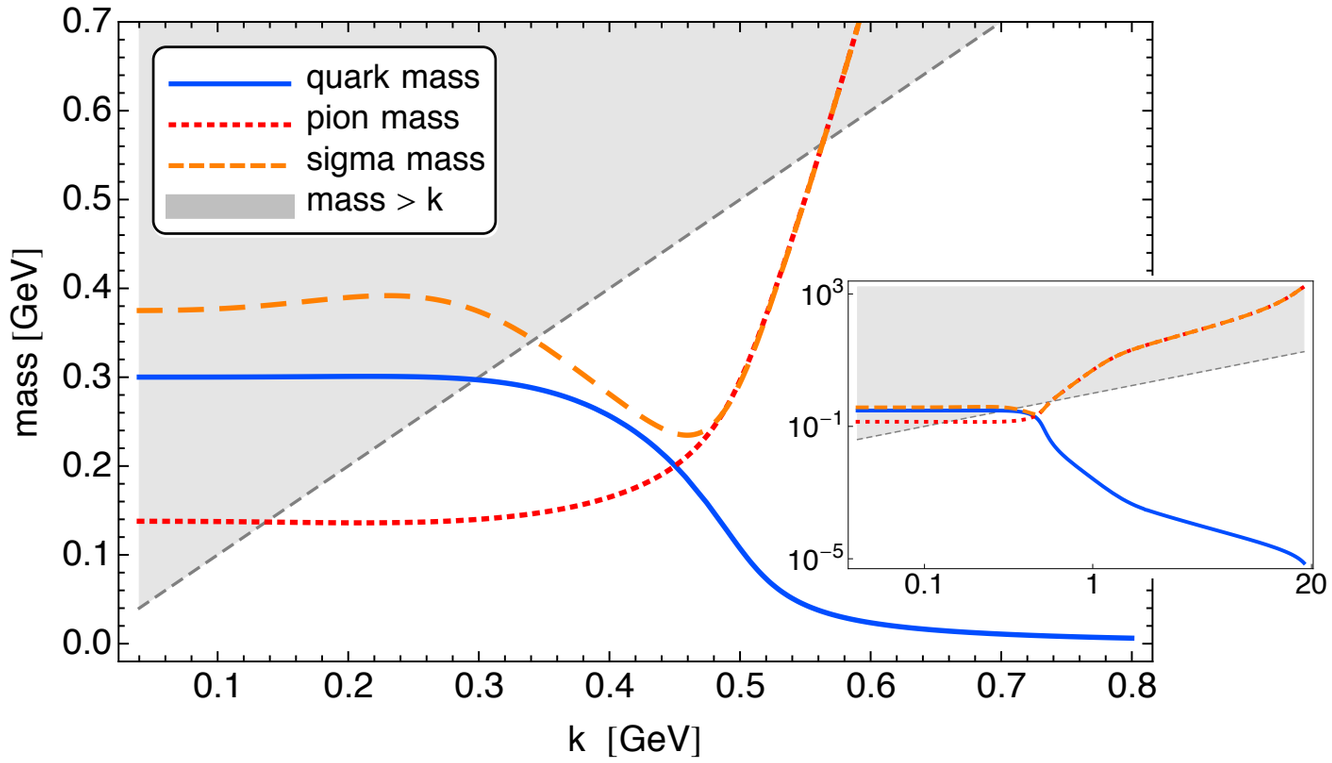
Rennecke, PRD 92 (2015) 076012

On the unreasonable effectiveness of low energy effective theories

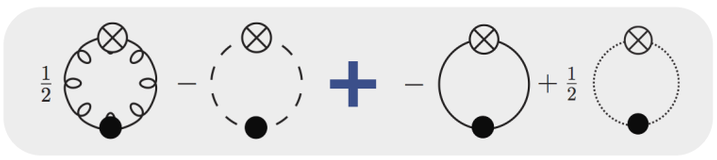


Sequential decoupling of gluon, quark, sigma, pion fluctuations

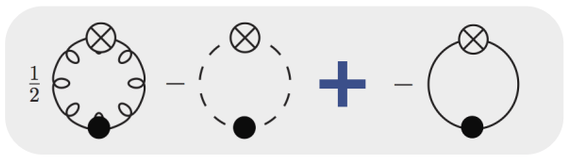




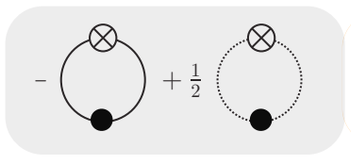
PQM-model



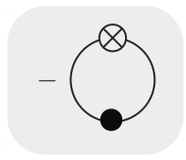
PNJL-model



QM-model

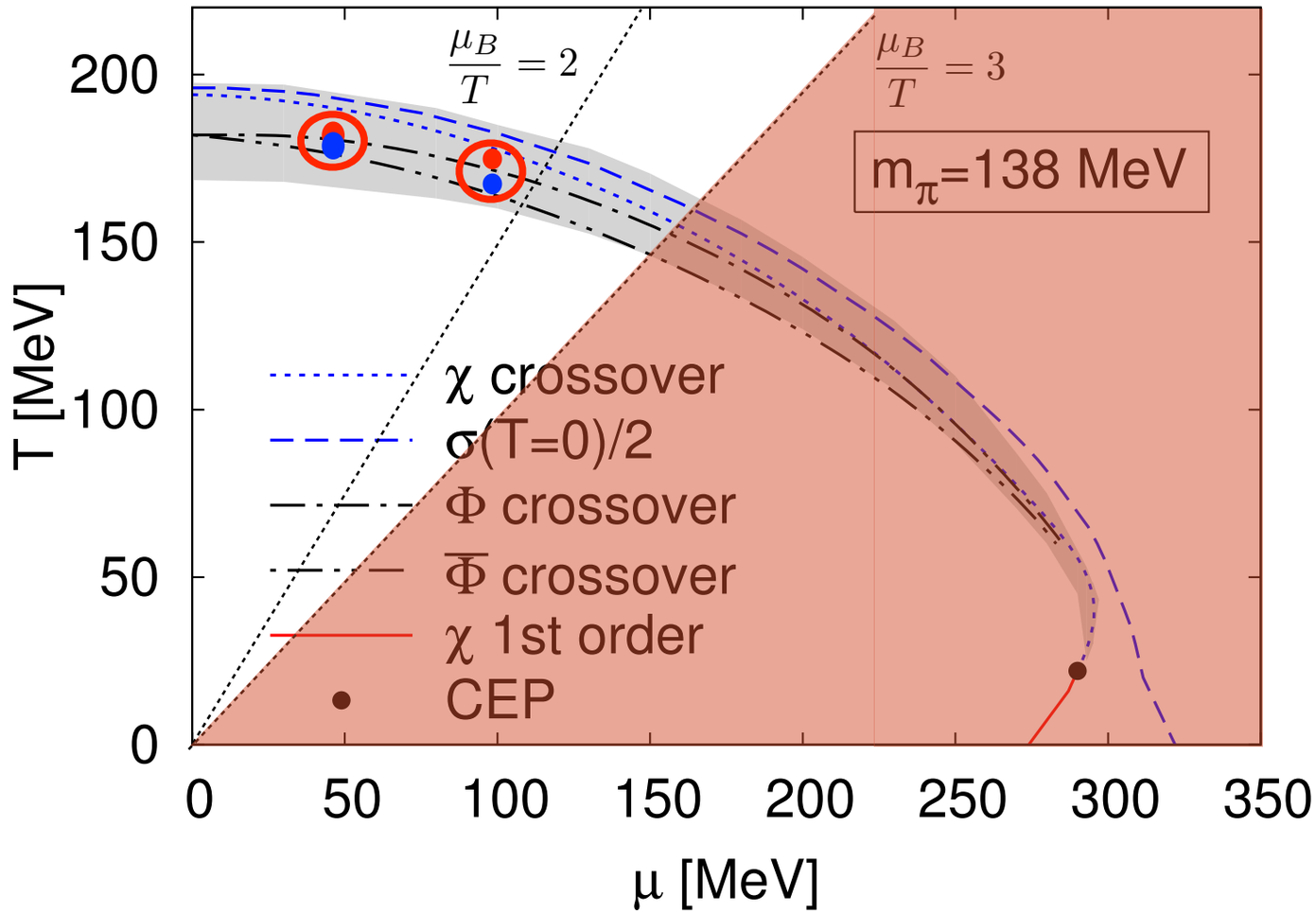


NJL-model



QCD at finite density

Phase diagram of the QCD-enhanced PQM



Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 1, 014007

FRG QCD results at finite density

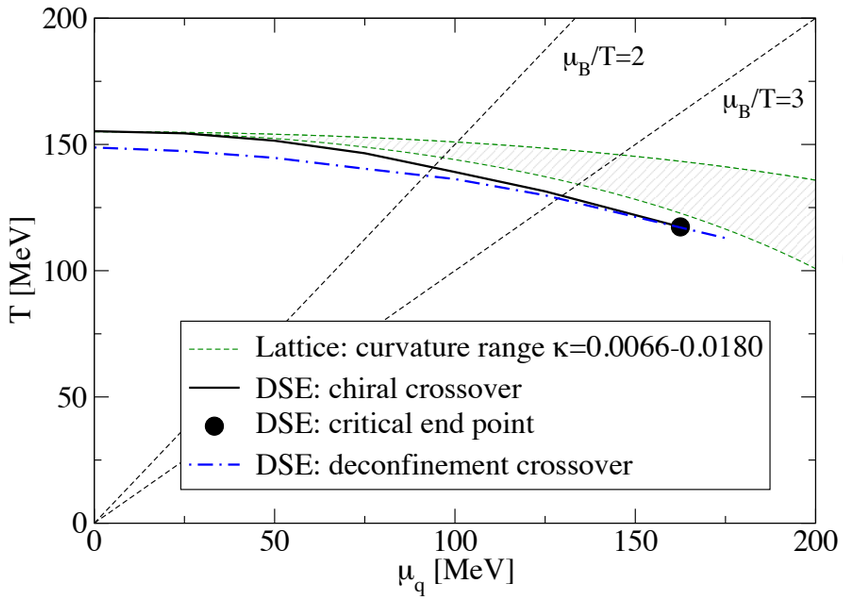
Haas, Braun, JMP '09, unpublished

Extension of FRG QCD results at imaginary chemical potential

Braun, Haas, Marhauser, JMP, PRL 106 (2011) 022002

Phase structure at finite density

Phase diagram of 2+1 flavor QCD



Kaczmarek at al. '11
Endrodi, Fodor, Katz, Szabo '11
Cea, Cosmai, Papa '14

Fischer, Fister, Luecker, JMP, PLB732 (2014)

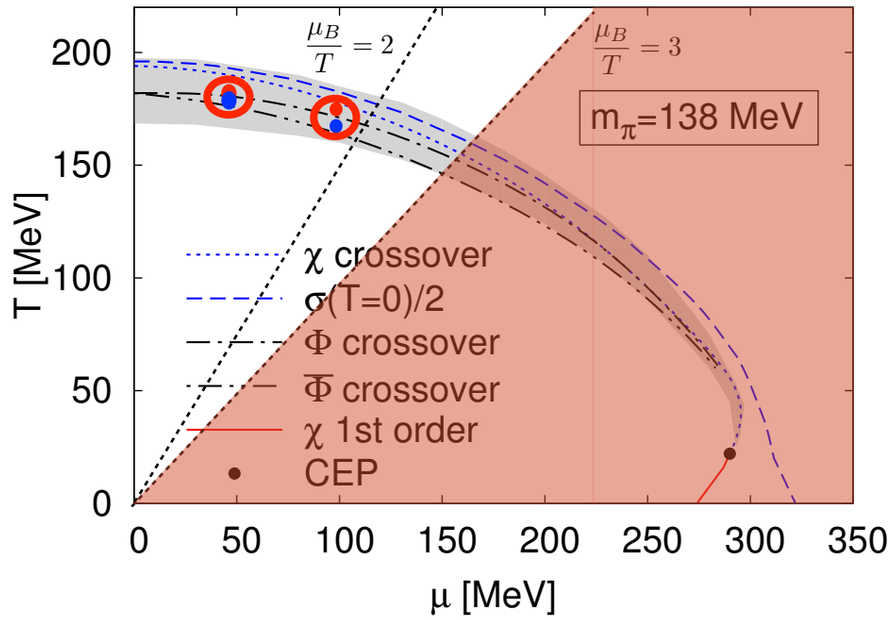
Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022

Eichmann, Fischer, Welzbacher, PRD 93 (2014) 034013

Chiral phase structure

Qin, Chang, Chen, Liu, Roberts, PRL 106 (2011) 172301

Phase diagram of QCD-enhanced 2-flavor PQM-model



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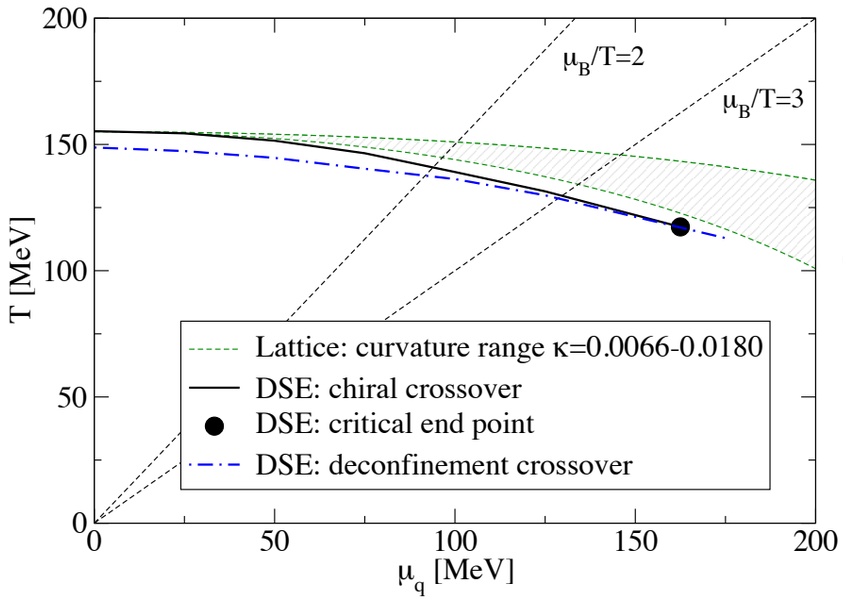

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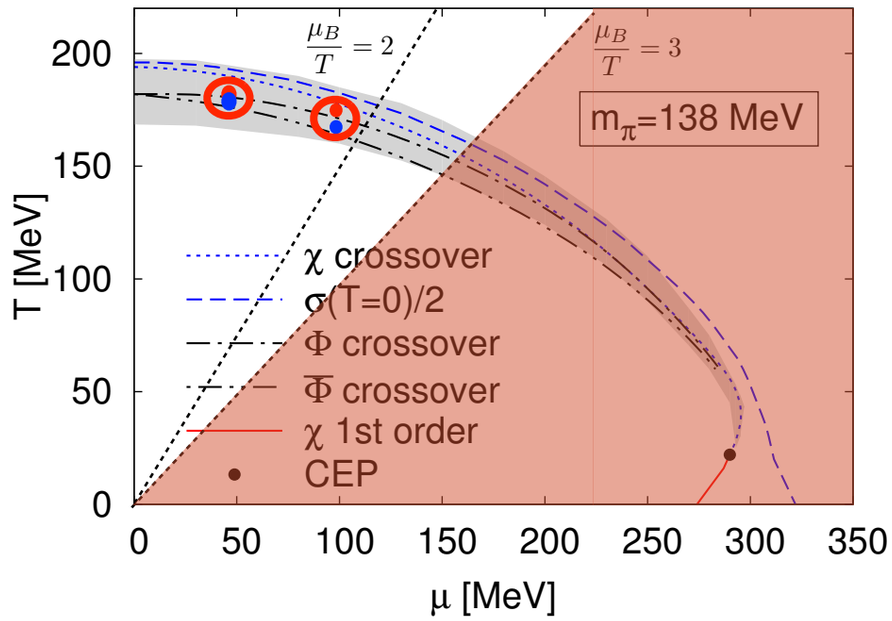

Herbst, JMP, Schaefer, PLB 696 (2011) 58-67
PRD 88 (2013) 014007

FRG QCD results at finite density

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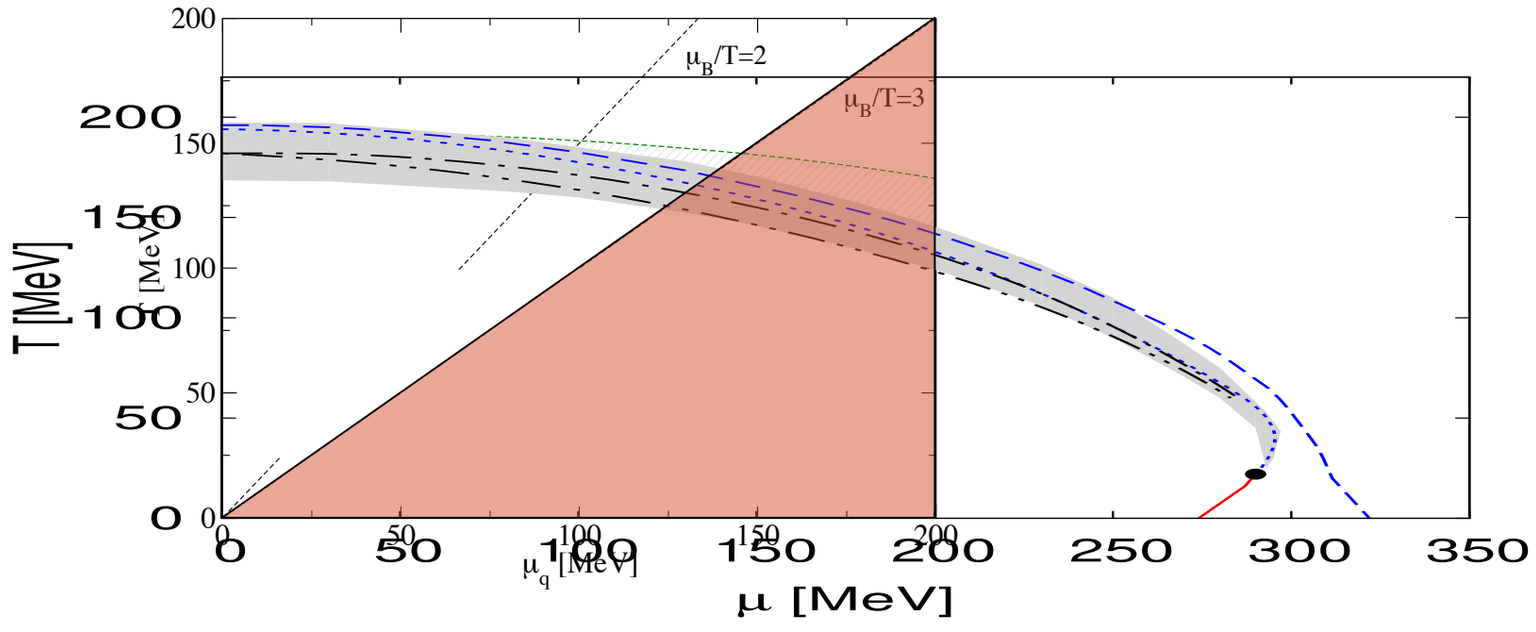
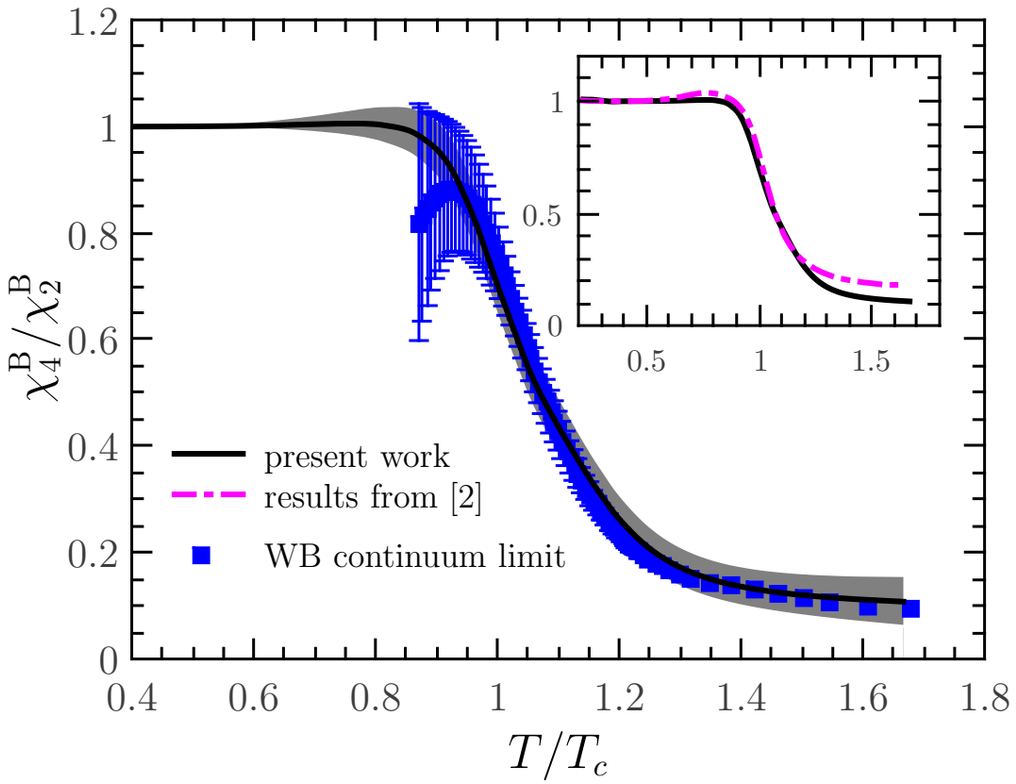
Comparison with 2 flavor vs 2+1 flavor scale matching of T_c



Fluctuations as a measure of confinement



$$\chi_n^{\rm B} = \frac{\partial^n}{\partial(\mu_{\rm B}/T)^n}\frac{p}{T^4}$$

Skewness, Kurtosis

$$\sigma^2 = VT^3\chi_2^{\rm B} \qquad S = \frac{\chi_3^{\rm B}}{\chi_2^{\rm B}\sigma} \qquad \kappa = \frac{\chi_4^{\rm B}}{\chi_2^{\rm B}\sigma^2}$$

[2] Fu, JMP, PRD 92 (2015) 116006

2+1: Wen, Huang, Fu, arXiv:1809.04233

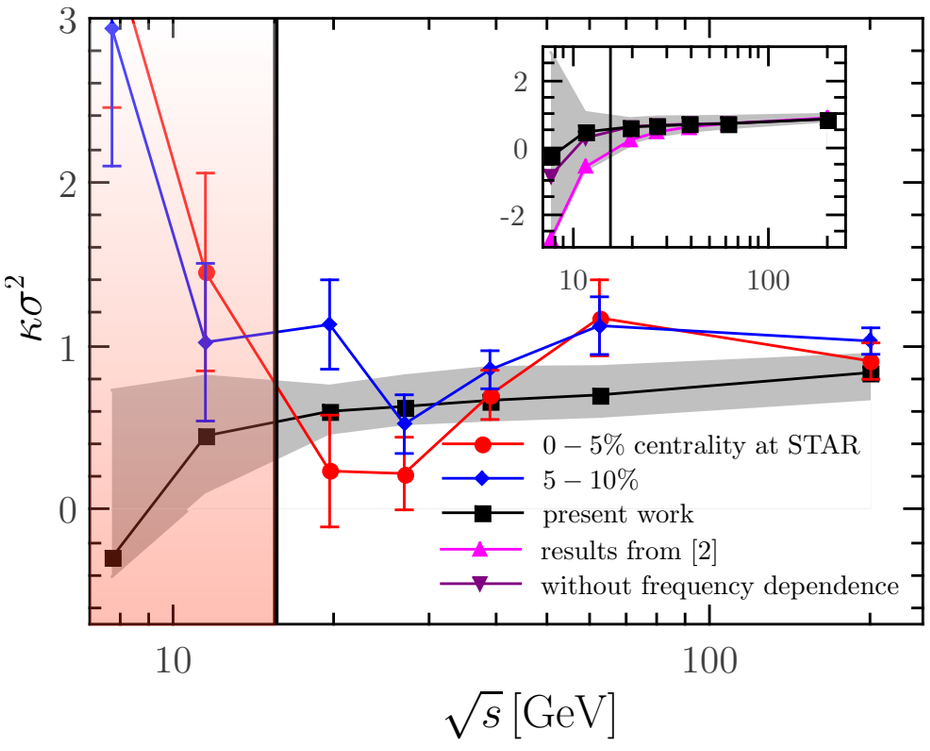
Karsch, Schaefer, Wagner, Wambach, PLB 698 (2011) 256

Friman, Karsch, Redlich, Skokov, EPJ C71 (2011) 1694

Schaefer, Wagner, PRD 85 (2012) 034027

Skokov, Friman, Redlich, PRC 88 (2013) 034911

Almasi, Friman, Redlich, Nucl.Phys. A956 (2016) 356-359



[2] Fu, JMP, PRD 93 (2016) 091501

Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

Transport approach to QCD

Blum, Jiang, Mitter, Nahrgang, JMP, Rennecke, Wink

Time evolution of the critical (scalar) σ mode

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

quantum equation of motion **noise field**

Extension of mean-field version

Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)

see also

Stephanov, Rajagopal, Shuryak PRL81 (1998)

Mukherjee, Venugopalan, Yin PRC92 (2015)

Herold, Nahrgang, Yan, Kobdaj PRC93 (2016)

Nahrgang, Bluhm, Schäfer, Bass arXiv:1804.05728

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Input from equilibrium low energy effective action of QCD

$\mathrm{Re}\,\Gamma^{(2)}_\sigma(\omega,\vec{p})$ $\mathrm{Im}\,\Gamma^{(2)}_\sigma(\omega,\vec{p})$ $U(\sigma)$

kinetic term **diffusion term** $\eta\,\partial_t\sigma$ **effective potential**

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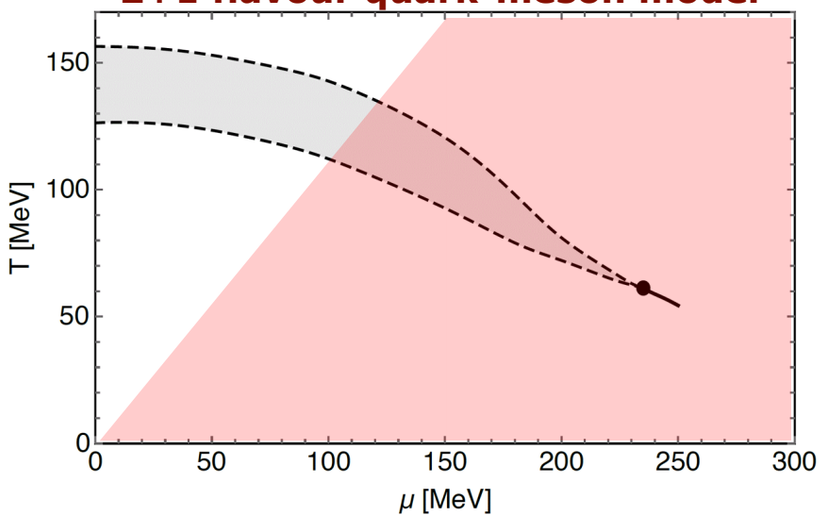
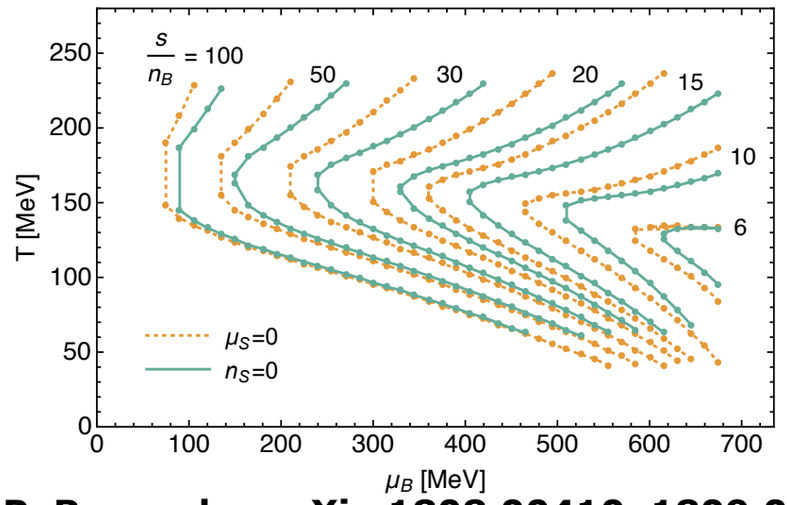
Phase structure of low energy QCD

2+1 flavour quark-meson model



Schaefer, Rennecke, PRD 96 (2017) 016009

strangeness neutrality & strangeness fluctuations



Fu, JMP, Rennecke, arXiv:1808.00410, 1809.01594

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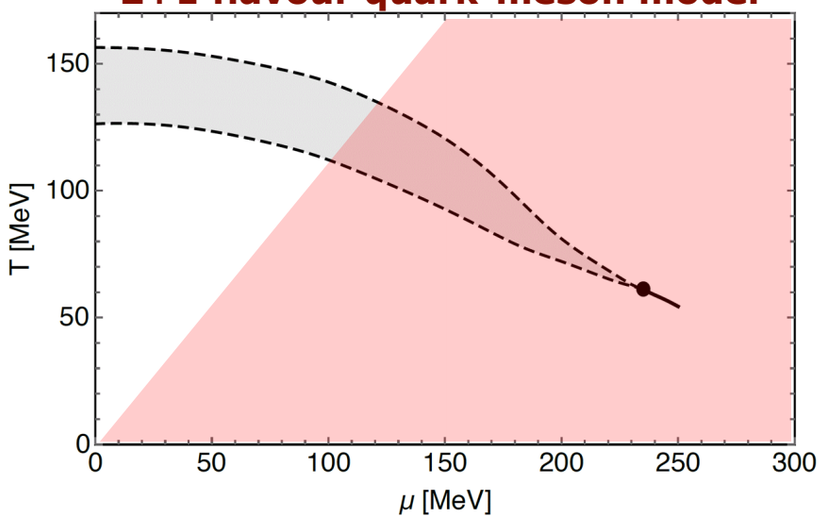
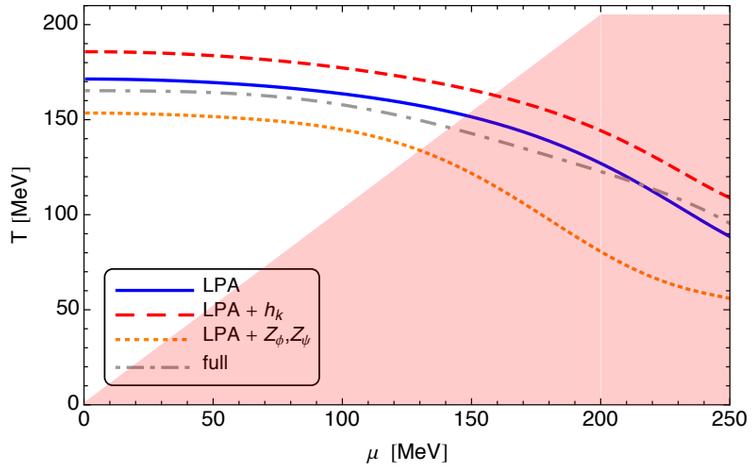


Schaefer, Rennecke, PRD 96 (2017) 016009

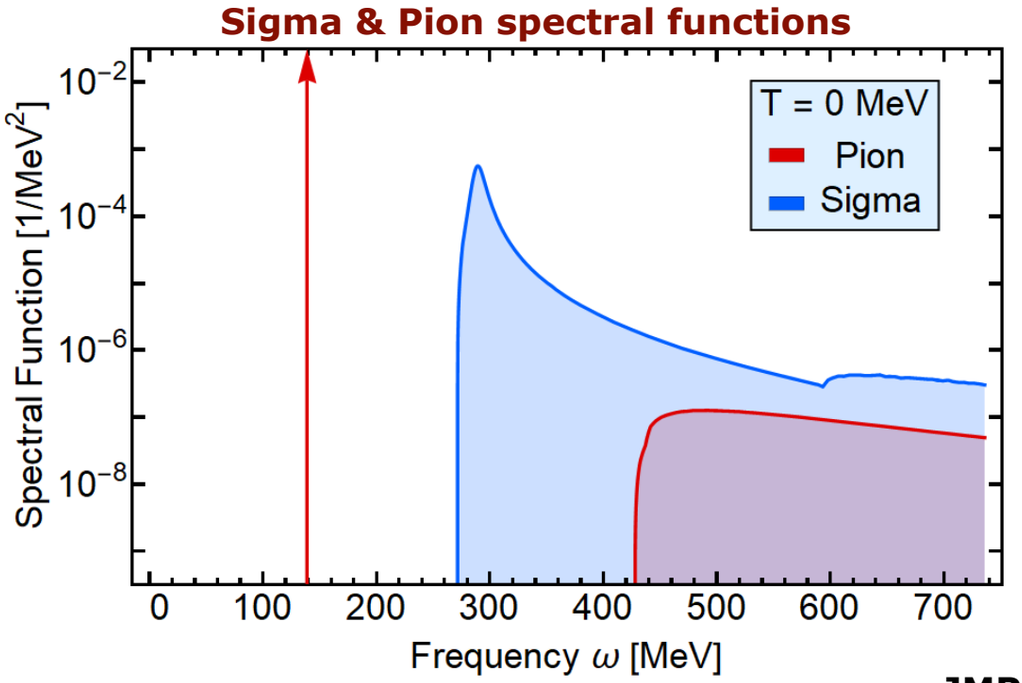
Comparison of truncations (2 flavours)



JMP, Rennecke, PRD 90 (2014) 076002

Pion & sigma spectral functions

Show case in linear sigma model



JMP, Strodthoff, Wink, arXiv:1711.07444

Real-time FRG computations, e.g.

Flörchinger JHEP 1205 (2012) 021

Kamikado, Strodthoff, von Smekal, Wambach, EPJC 74 (2014) 2806

JMP, Strodthoff, PRD 92 (2015) 094009

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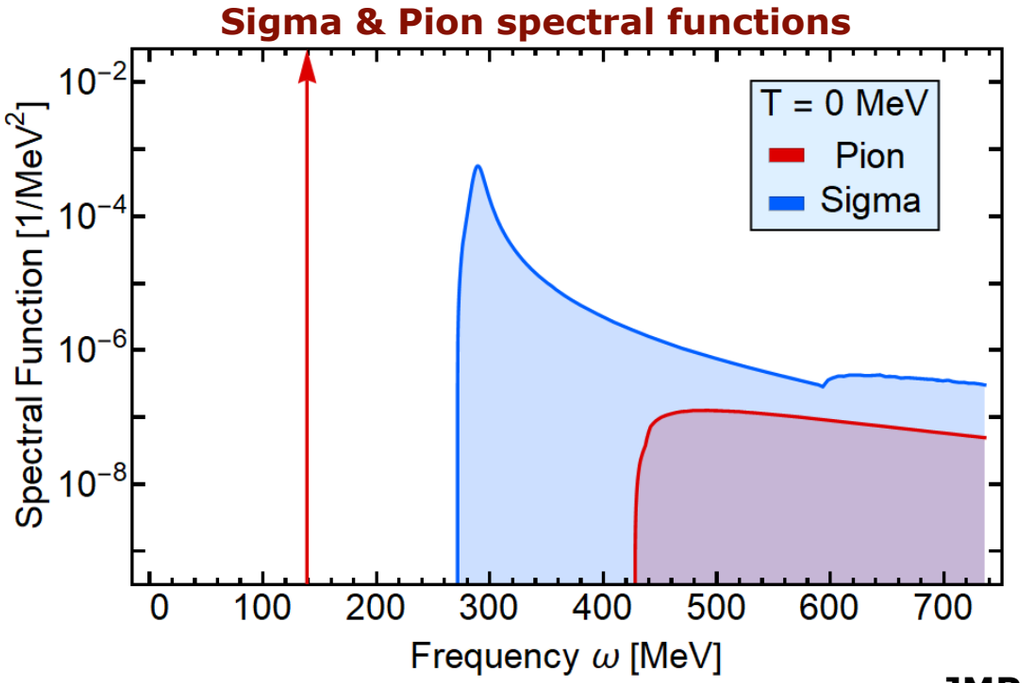

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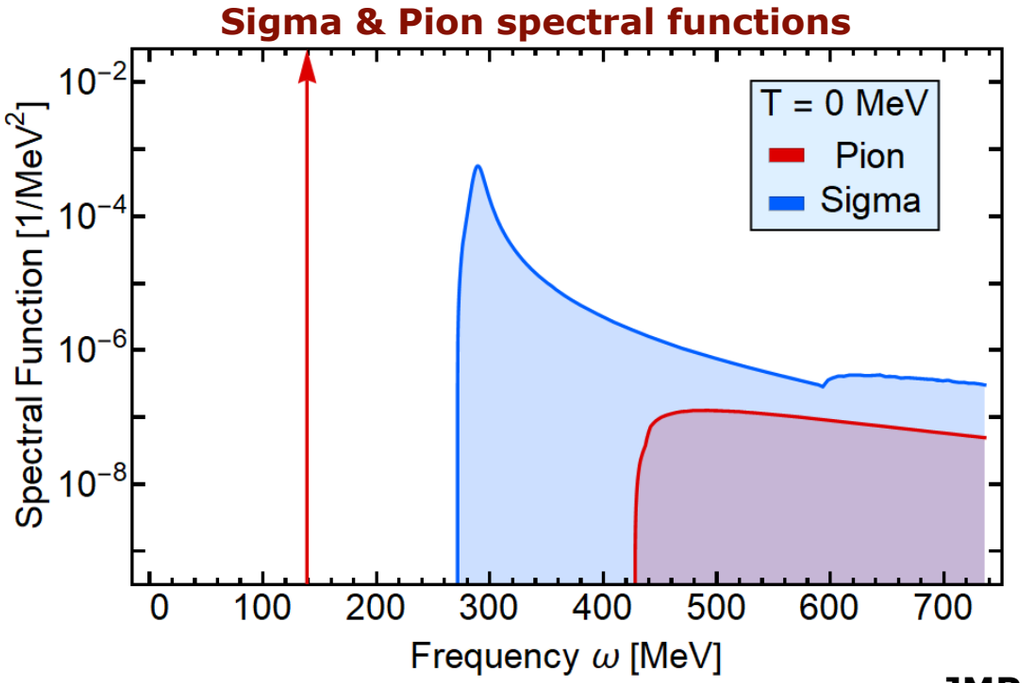
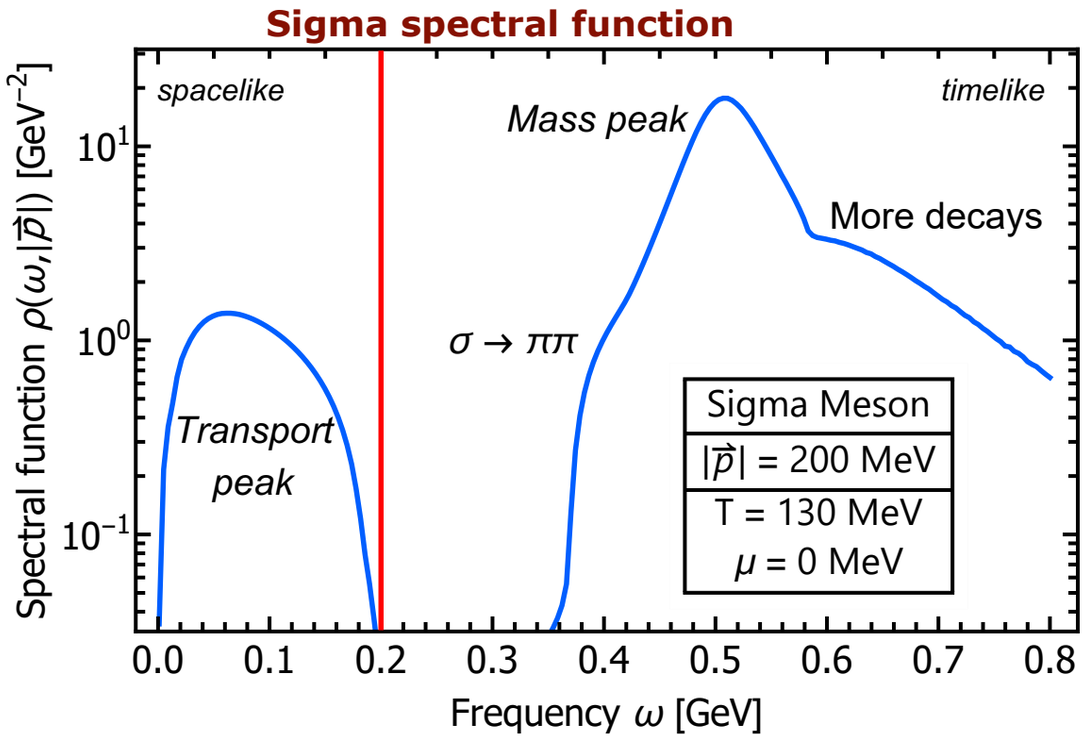


JMP, Strodthoff, Wink, arXiv:1711.07444

2+1 flavour quark-meson model sigma spectral function

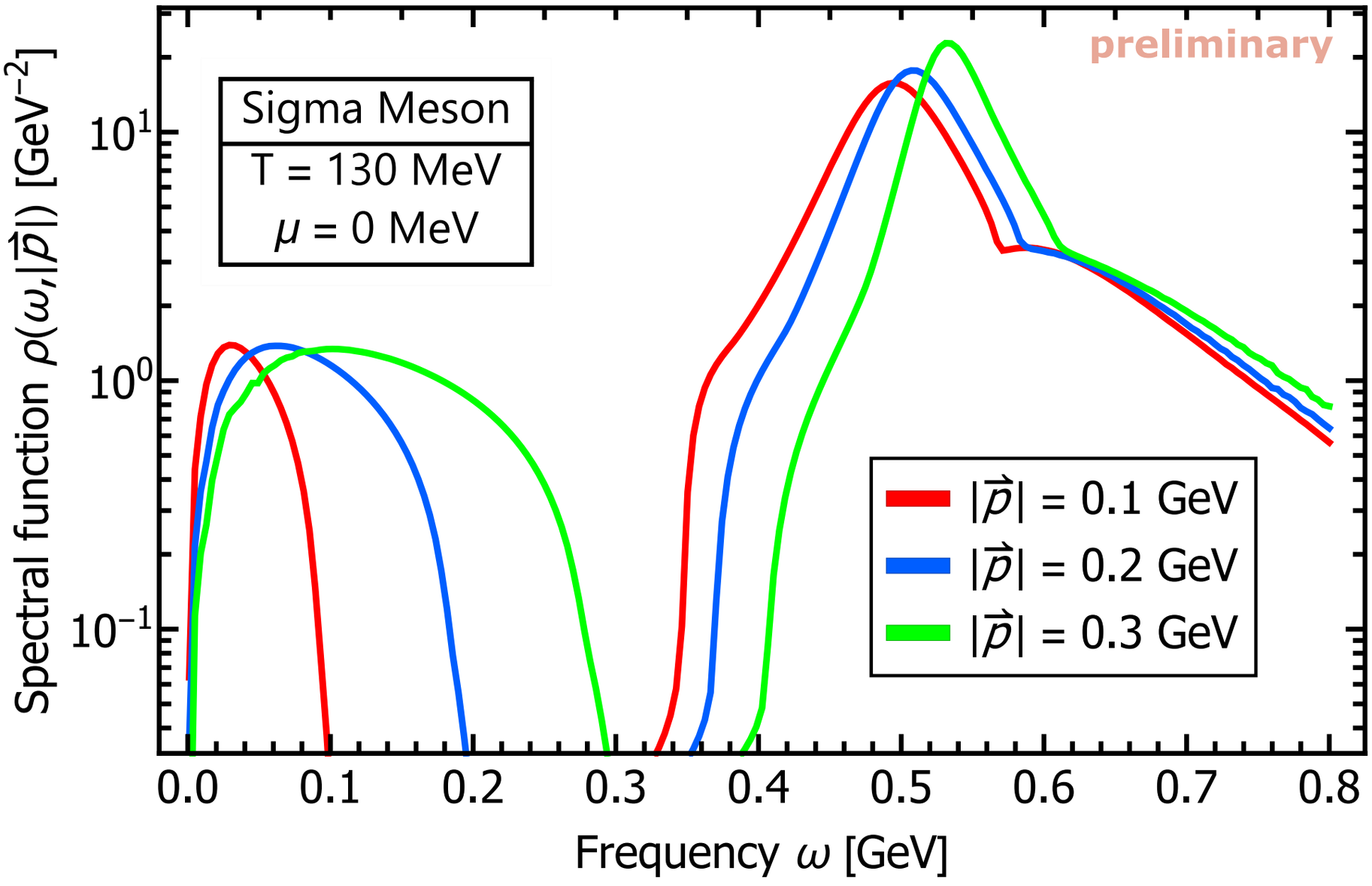


preliminary

JMP, Rennecke, Wink, in prep

Pion & sigma spectral functions

2+1 flavour quark-meson model sigma spectral function

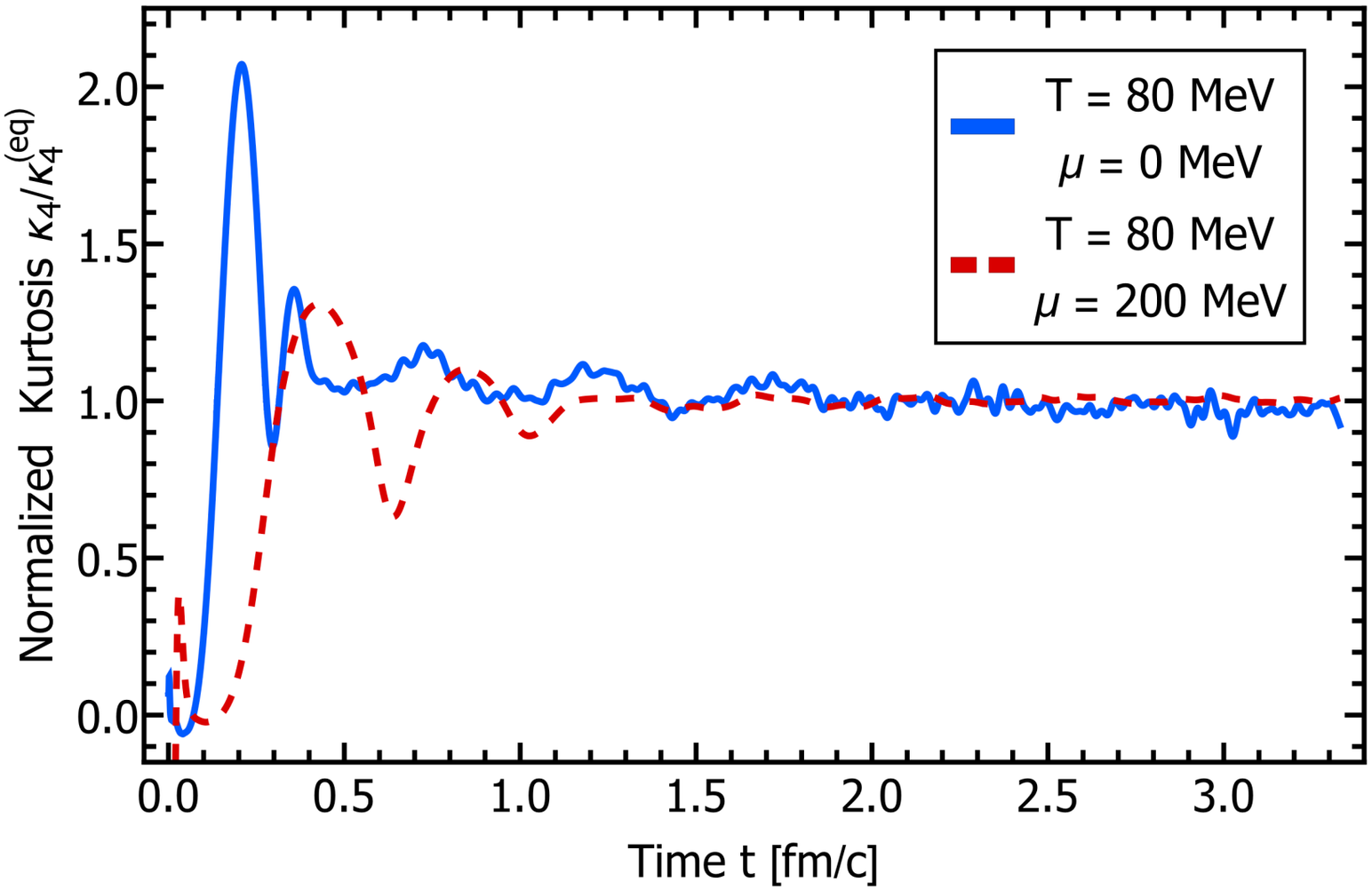


JMP, Rennecke, Wink, in prep

Time evolution of cumulants

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, arXiv:1808.01377

Time evolution of kurtosis



nth central moment of the sigma field: χ_n

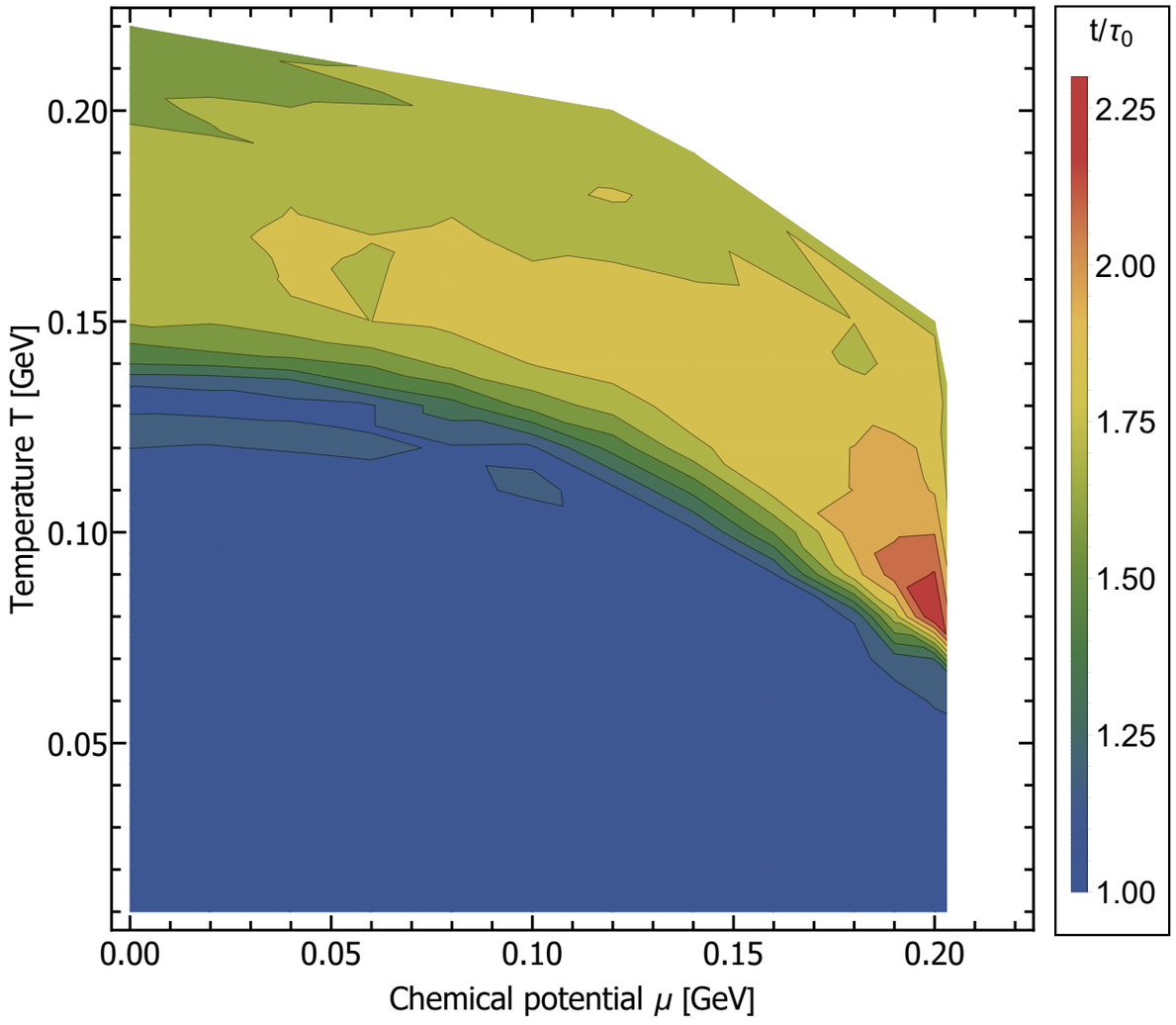
$$\chi_2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle$$

kurtosis: $\kappa = \dfrac{\chi_4}{\chi_2^2} - 3$

Equilibration time phase structure

Blum, Jiang, Nahrgang, JMP, Rennecke, Wink, arXiv:1808.01377

Equilibration time of sigma-kurtosis



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Equilibration time phase structure

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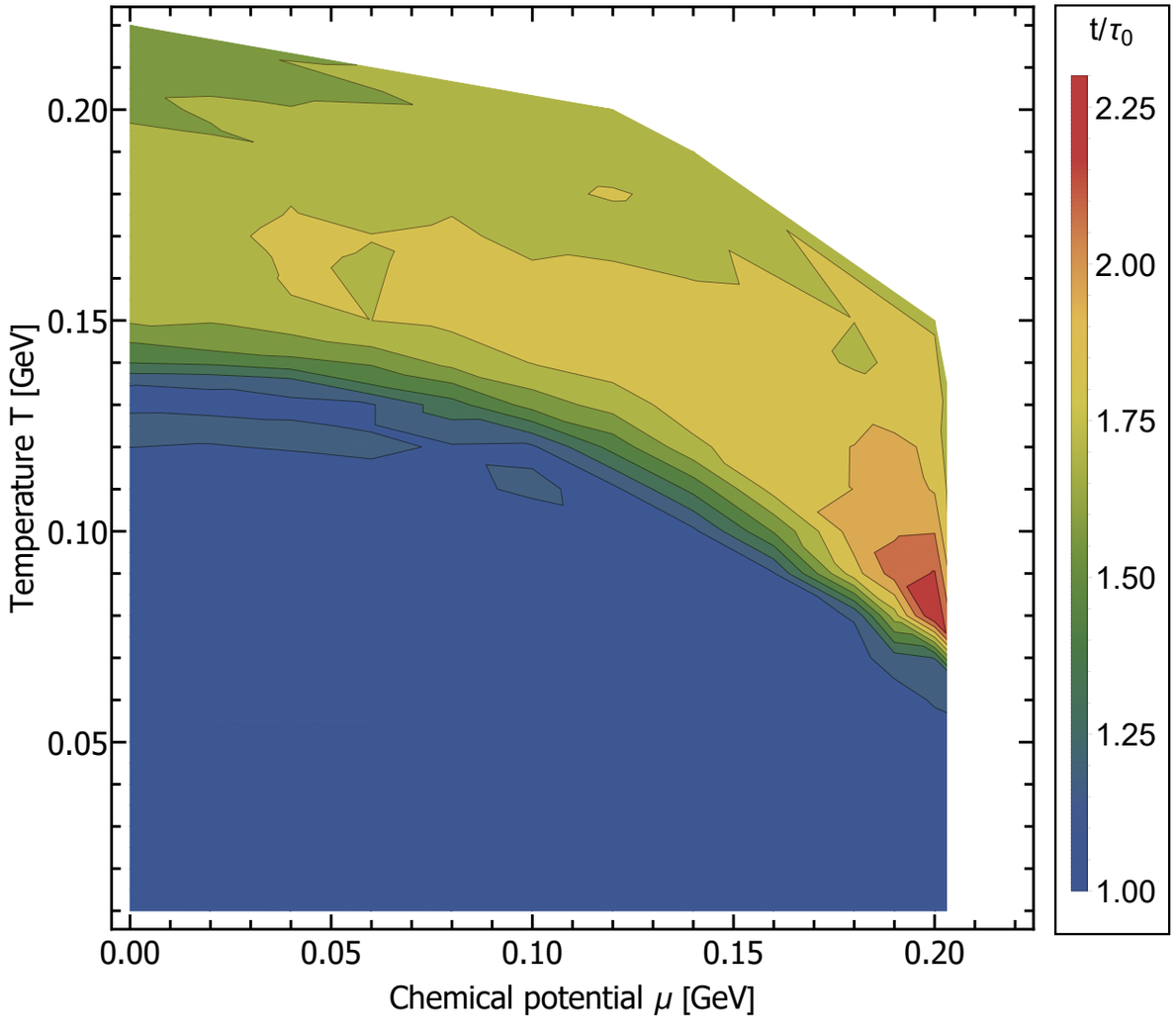

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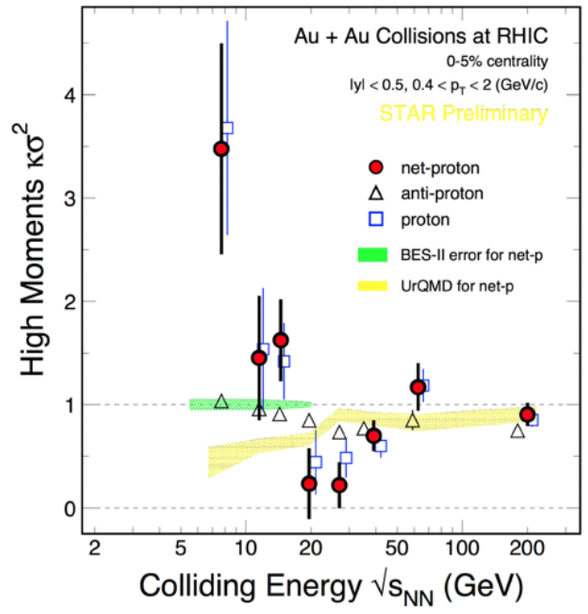
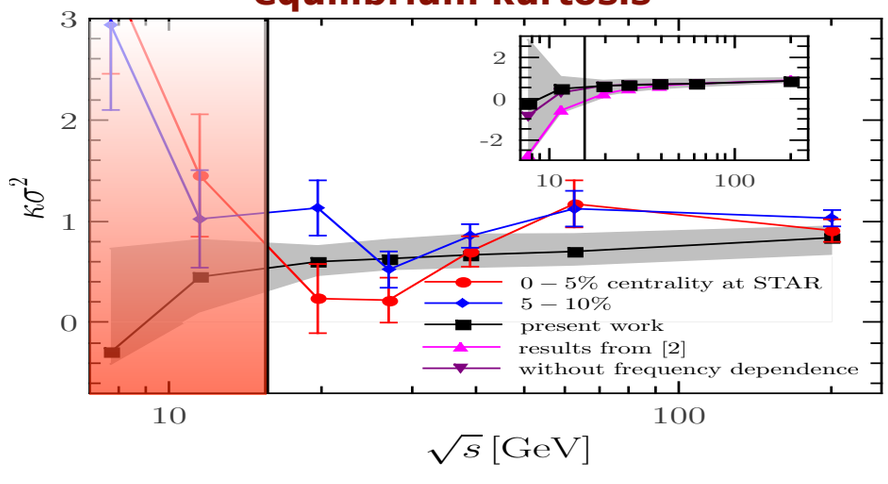
kurtosis of barvon number fluctuations



Luo, Cu, NST 28 (2017)

equilibrium kurtosis



Fu, JMP, Schaefer, Rennecke, PRD 94 (2016) 116020

kurtosis: $\kappa = \frac{\chi_4}{\chi_2^2} - 3$

Outline

- QCD from functional methods

Applications

- QCD-assisted hydrodynamics

- QCD-assisted transport

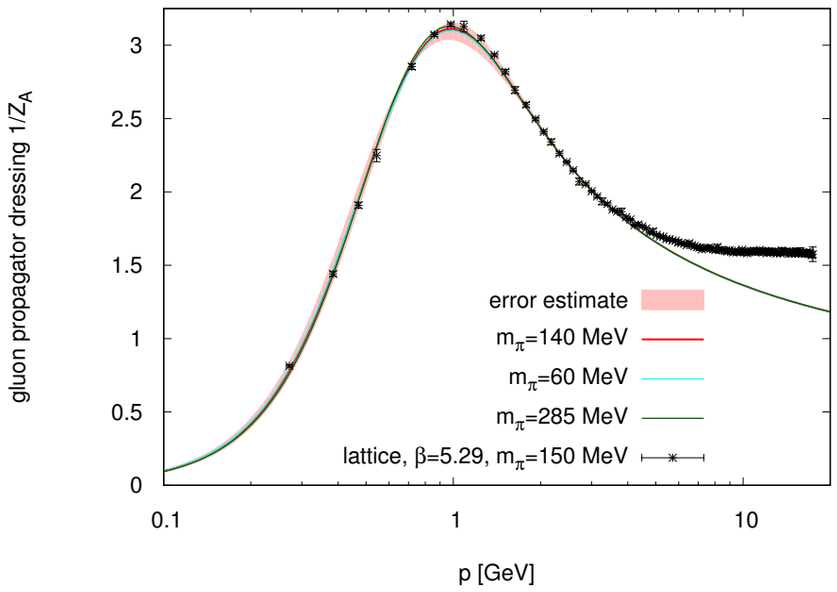
- Summary & outlook

Summary & Outlook

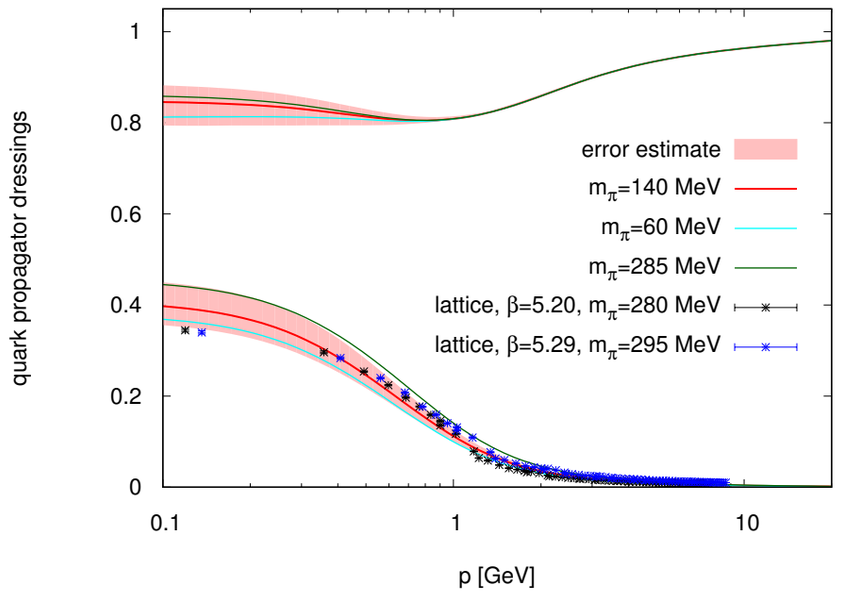
- **Phase structure of QCD**

$$\frac{f_{\pi,\mathrm{FRG}}}{f_{\pi,\mathrm{lattice}}} = 0.99$$

Gluon correlations

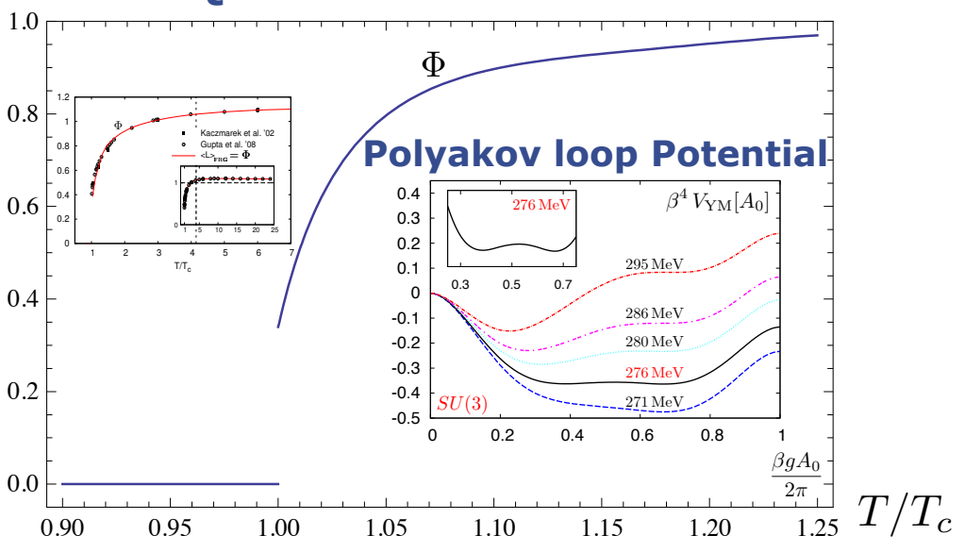


chiral symmetry breaking

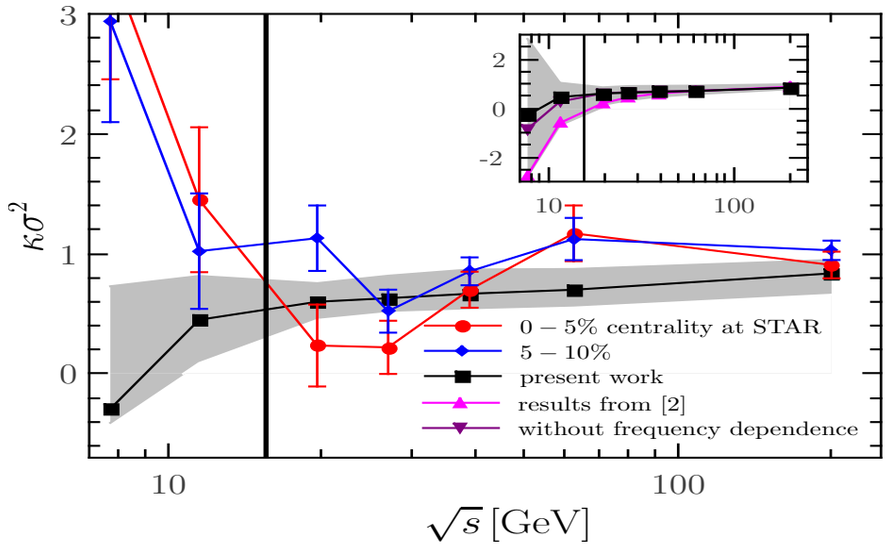


fQCD

Quark Confinement



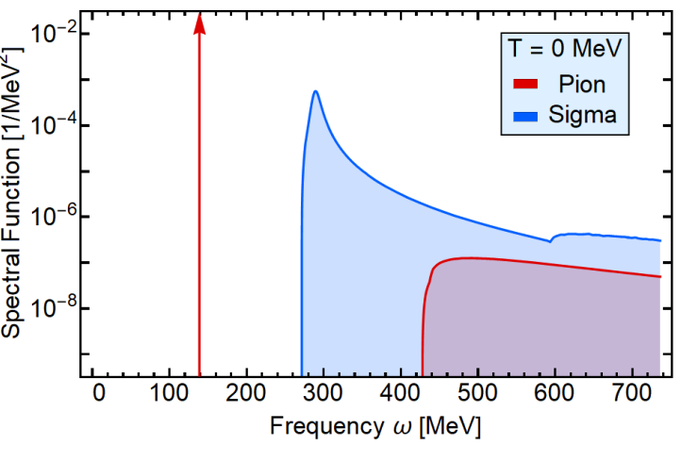
fluctuations at finite density

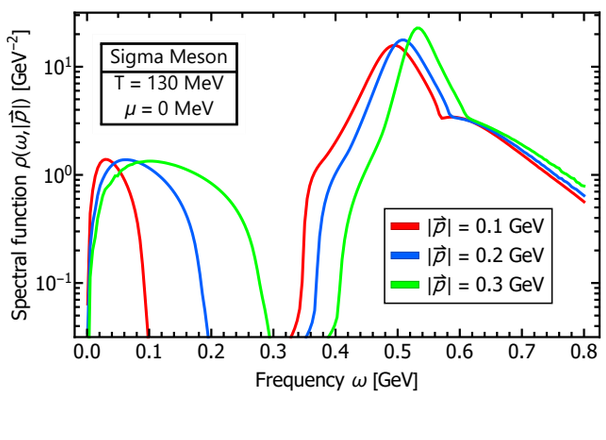


Summary & Outlook

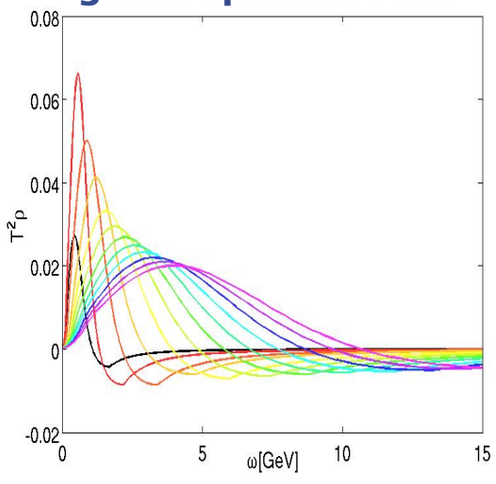
QCD Transport

meson spectral fcts.

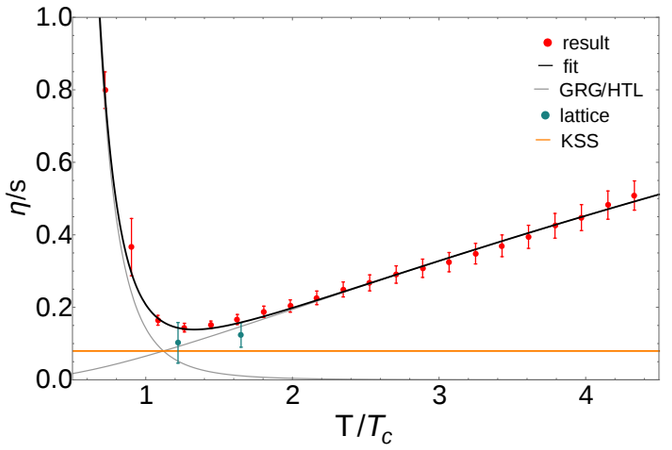




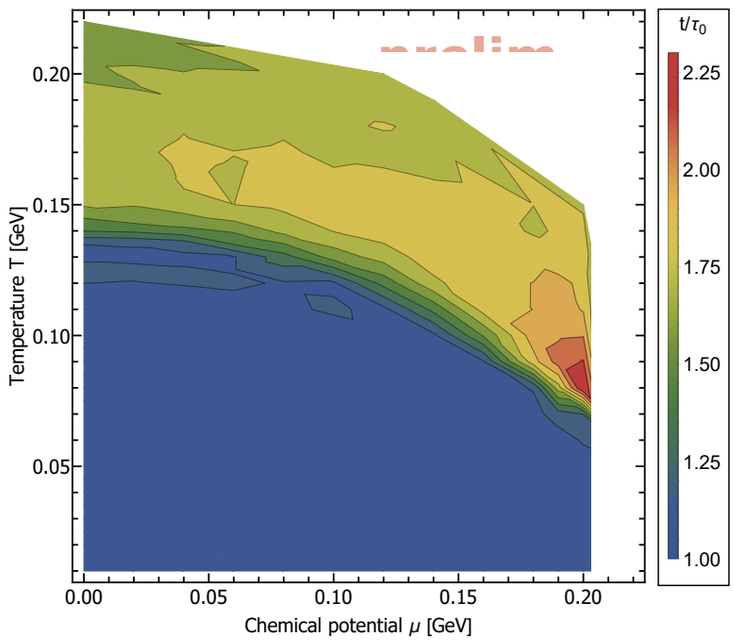
gluon spectral fcts.



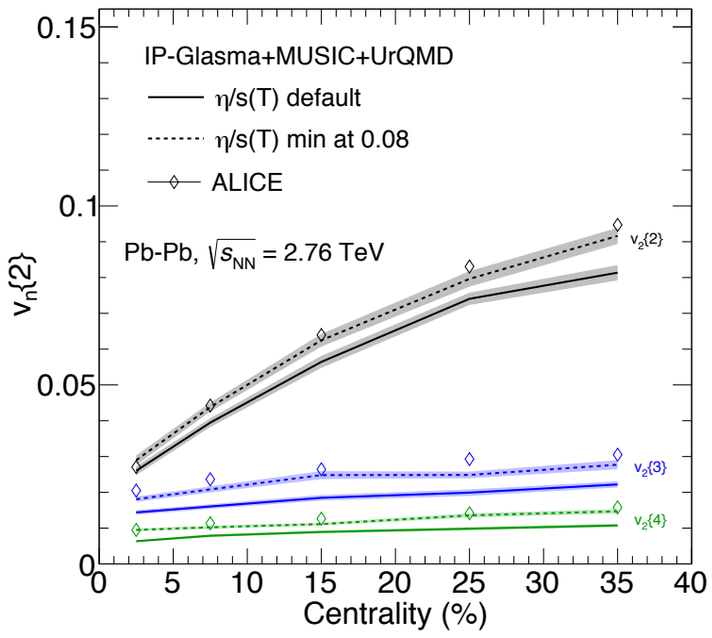
glue viscosity/entropy



Equilibration phase structure



V_n in QCD-assisted Hydro



Summary & Outlook

- QCD phase structure at low density

- Transport in pure glue & low energy EFTs

Summary & Outlook

- QCD phase structure at low density
- Transport in pure glue & low energy EFTs
- Towards quantitative precision
  - baryons, high density regime & CEP
- Threefold way to transport
  - imaginary time lattice simulations: Yang-Mills, QCD
  - real-time: Yang-Mills, QCD, finite density