Baryon Structure with PTIR and beyond

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INFN Roma1

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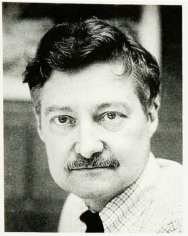
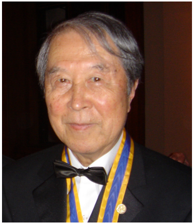
Chapter 1:
Perturbative Integral Representation

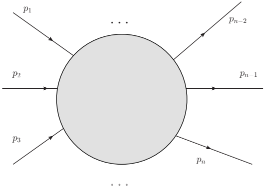



- 1954 Wick and Cutkosky introduced the first integral representation
- Goal at this time: solve the BSE



- 1955: Nambu derived a Integral representation for Scattering Amplitudes.
- 1st attempt to derive general Integral representation, which turn to be wrong.

Nakanishi Integral Representation

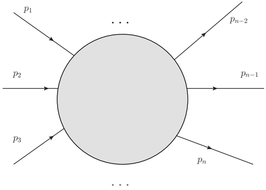


$$= \int_0^1 [\mathrm{d}z_i] \int_0^\infty \mathrm{d}\gamma \frac{\rho_j(\gamma, z_i)\delta\left(1 - \sum_i z_i\right)}{\left(\gamma - \sum_k z_k s_k\right)^j}$$

- Formula hold for any n-point function at any order of perturbation theory
- s_k are all the independent Poincaré invariant you can build from the p_i
- ρ_j is real and unique

N. Nakanishi, Graph Theory and Feynman Integrals, Gordon and Breach, 1971

Nakanishi Integral Representation



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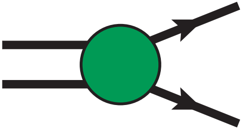
Caveat

Rigorous proof at all order of perturbation theory is not equivalent to a rigorous non-perturbative proof. **But it makes the procedure appealing for non-perturbative studies**.

Special cases

- Three-point Function (Vertex):



$$= \int_{-1}^{1} \mathrm{d}z \int_{0}^{\infty} \mathrm{d}\gamma \frac{\rho_V(\gamma, z)}{(q - \frac{z}{2}P)^2 - \gamma + i\epsilon}$$

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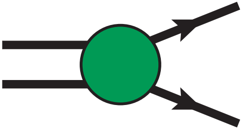

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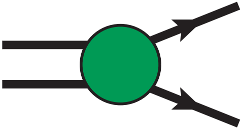

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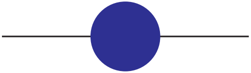

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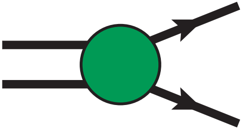

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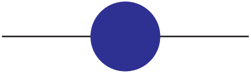

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This will be studied in the last part of the talk

Kallen-Lehmann representation:

$$S(p) = \int \mathrm{d}\omega \frac{\bar{\sigma}(\omega)}{p^2 - \omega^2 + i\epsilon}$$

- KL comes from insertion of a complete set of state and $\bar{\sigma}$ is **positive** $\neq$ NIR comes from perturbation theory and ρ is **real**.
- NIR allows for a bigger flexibility, and therefore might accomodate more theories.

Chapter 2:
Baryon Distribution Amplitudes

CM, J. Segovia, L. Chang, C.D. Roberts

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x, k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

Nucleon Distribution Amplitudes

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\Big[(\not{p}C)_{\alpha\beta}\,(\gamma_5 N^+)_\gamma\, V(z_i^-) + (\not{p}\gamma_5 C)_{\alpha\beta}\,(N^+)_\gamma\, A(z_i^-) - (ip^\mu \sigma_{\mu\nu} C)_{\alpha\beta}\,(\gamma^\nu \gamma_5 N^+)_\gamma\, T(z_i^-)\Big]$$

V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246, (1984)

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- 3 bodies Fock space interpretation (leading twist):

$$|P,\uparrow\rangle = \int \frac{[\mathrm{d}x]}{8\sqrt{6x_1x_2x_3}}|uud\rangle \otimes [\varphi(x_1,x_2,x_3)|\uparrow\downarrow\uparrow\rangle + \varphi(x_2,x_1,x_3)|\downarrow\uparrow\uparrow\rangle - 2T(x_1,x_2,x_3)|\uparrow\uparrow\downarrow\rangle]$$

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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

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Evolution and Asymptotic results

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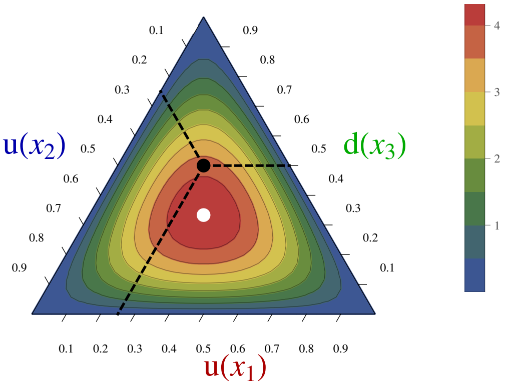
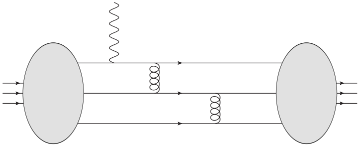


Form Factors: Nucleon case



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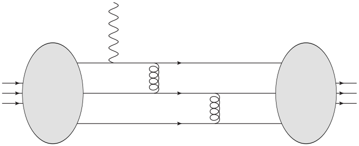

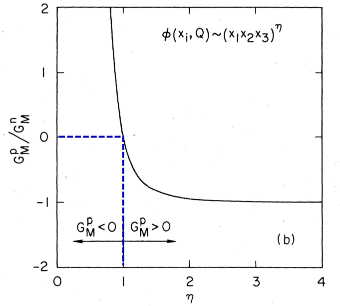

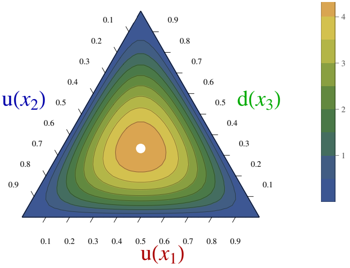

$\eta = 1$

S. Brodsky and G. Lepage, PRD 22, (1980)

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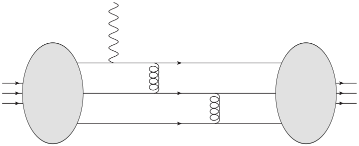
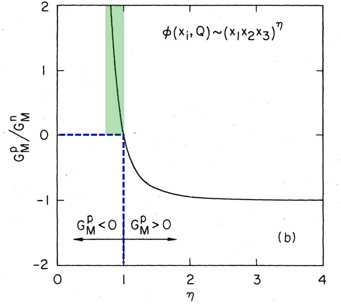
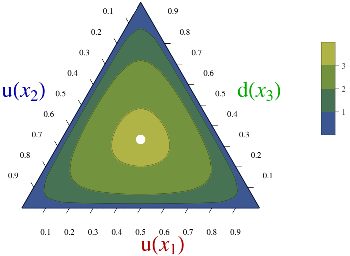
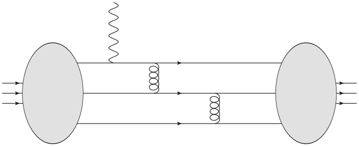


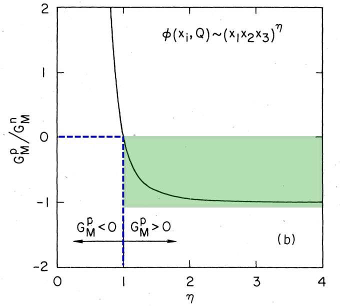


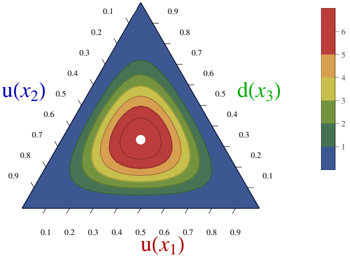


$\eta = 0.5$

S. Brodsky and G. Lepage, PRD 22, (1980)







$\eta = 2$

S. Brodsky and G. Lepage, PRD 22, (1980)

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, JHEP 2016 02

- Algebraic parametrisation inspired by the results obtained from DSEs and Faddeev equations.
- It is based on Nakanishi representation, which is completely general.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD one.

Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

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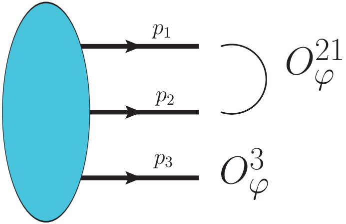

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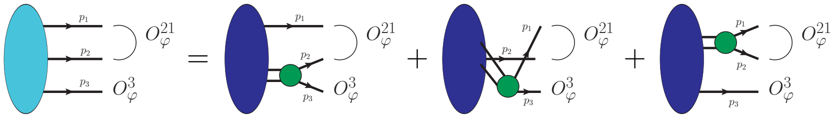

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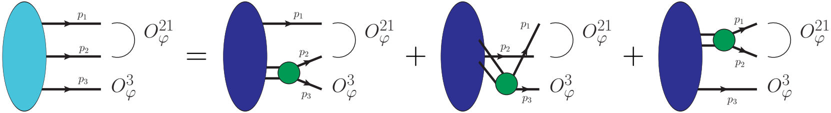

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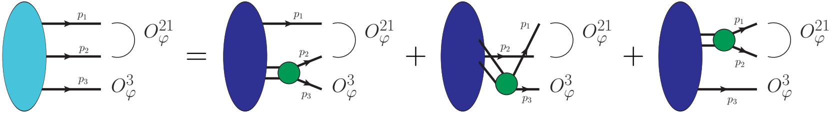
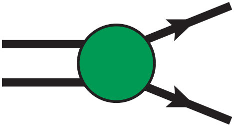


- The operator then selects the relevant component of the wave function.
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)



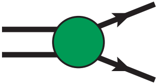
At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

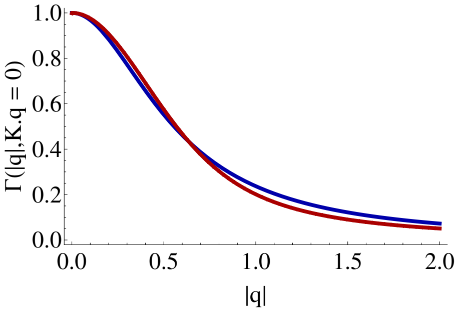
$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$

The model used:



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1 - z^2)}{(\Lambda^2 + (q + \frac{z}{2}P)^2)}$$

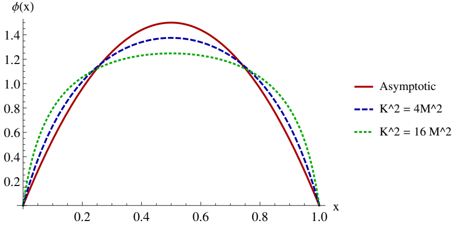
Comparable to scalar diquark amplitude previously used:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

$$\phi(x) \propto 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar diquark



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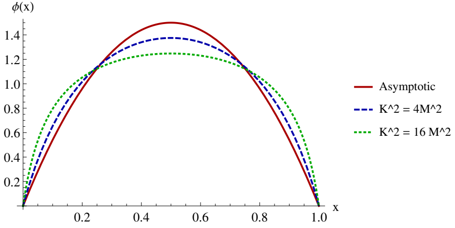
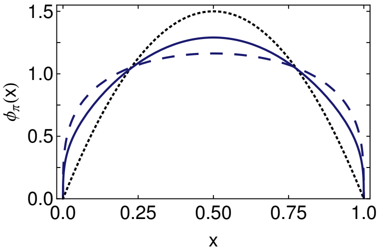


Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

Diquark DA

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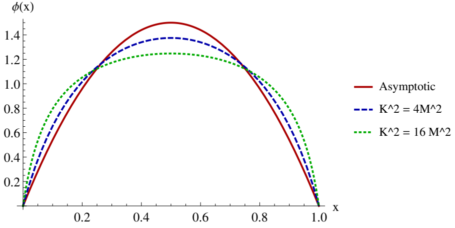

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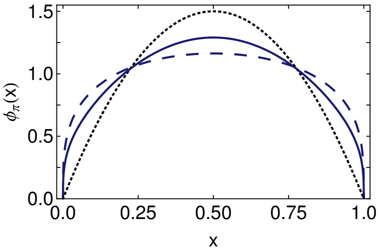
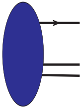


Pion figure from L. Chang *et al.*, PRL 110 (2013)

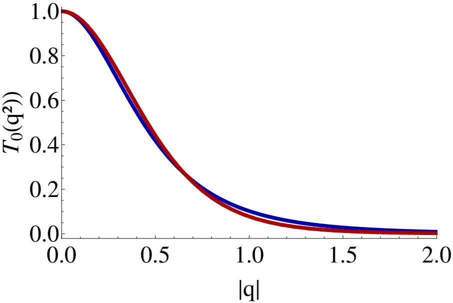
- This results provide a broad and concave meson DA parametrisation
- The endpoint behaviour remains linear

Nucleon Quark-Diquark Amplitude



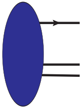
$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \, \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (q - \frac{1+3z}{6}P)^2)^3}$$

Preliminary estimations of the parameters through comparison to Chebychev moments:



red curve from Segovia *et al.*,

Nucleon Quark-Diquark Amplitude



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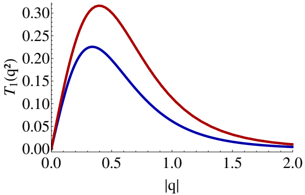

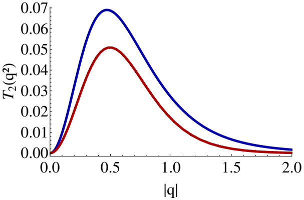

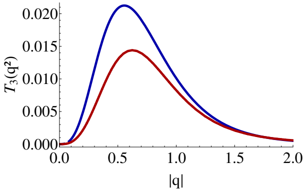

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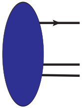

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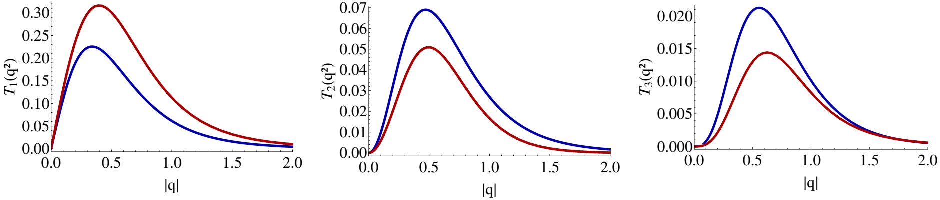


red curves from Segovia *et al.*,

There are still some works necessary to improve the comparison of higher Chebychev moments

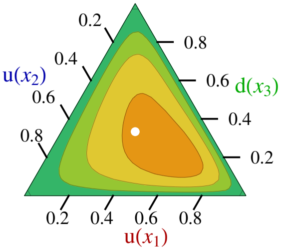
- We do not compute the PDA directly but Mellin moments of it:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \varphi(x_1, x_2, 1 - x_1 - x_2)$$

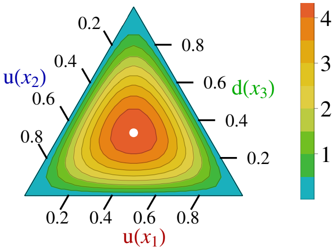
- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to right down our moments as:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta)$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and φ



Nucleon DA
(Evolved at 2GeV)

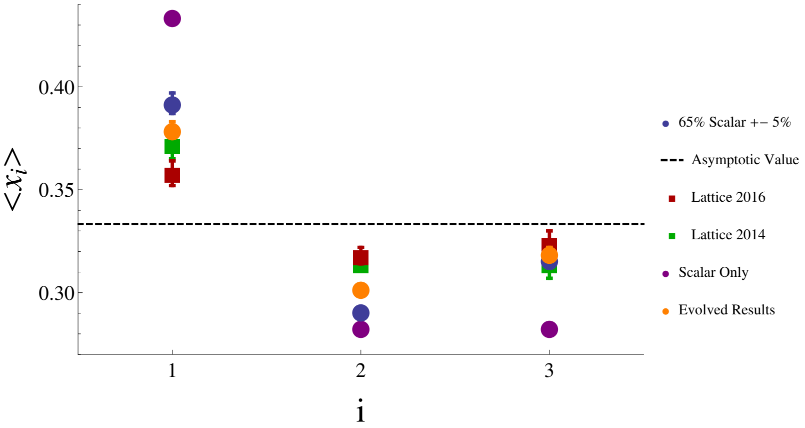


Asymptotic DA

- Nucleon DA is skewed compared to the asymptotic one
- These properties are consequences of our quark-diquark picture

Comparison with lattice

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Chapter 3:
Fermion Propagators and Spectral Representation

CM, G. Salmè

Computing the Nakanishi weight

- We have assume a specific (simplified) form for the Nakanishi weight

Can we do better?

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- We can try to extract numerically the Nakanishi weight using various techniques dedicated to the inverse problem
- But the problem is intrinsically ill-posed in the sens of Hadamard, and sophisticated techniques would be required:
  - Tikhonov regularisation (J.Carbonell *et al.*, Phys.Lett. B769 (2017) 418-423)
  - Maximal Entropy method (F. Gao *et al.*,Phys.Lett. B770 (2017) 551-555)

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We try to answer it with an abelian theory first

Divide and Solve

We write the propagator and self energy following:

$$S(p,\zeta) = \int \mathrm{d}\omega \frac{\not{p}\bar{\sigma}_v(\omega,\zeta) - \bar{\sigma}_m(\omega,\zeta)}{p^2 - \omega + i\epsilon}, \quad \Sigma(p,\zeta) = \int \mathrm{d}s \frac{\not{p}\rho_A(s,\zeta) - \rho_B(s,\zeta)}{p^2 - s + i\epsilon}$$

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Nota Bene

The momenta can be handle entirely algebraically, allowing direct calculations in Minkowsky space.

Choosing the vertex

- Bare Vertex : $\gamma^\mu \rightarrow$ breaks WTI and upsets our main organiser

V. Sauli, JHEP 0302 (2003) 001

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- Ball-Chiu Vertex: $\Gamma^\mu_{BC}(p,q) = \gamma^\mu \frac{A(p^2)+A(q^2)}{2} + \frac{(p+q)^\mu}{p^2-q^2}\left((\not{p}+\not{q})\frac{A(p^2)-A(q^2)}{2} - (B(p^2)-B(q^2))\right)$
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  $\Gamma^\mu_Q = \Gamma^\mu_{BC}(p,q) + \Gamma^\mu_\perp[(A(p^2)-A(q^2), B(p^2)-B(q^2)]$
  - Fulfil the WTI longitudinal and transverse (in a simplified way)
  - It preserves the multiplicative renormalisability of the theory (we neglect τ_5)
  - It does not introduce new unknown, and is entirely fixed by the quark propagator

S. Qin *et al.*,Phys.Lett. B722 (2013) 384-388

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The Qin vertex seems well appropriate for our study

The New Gap System

From the previous considerations, we can derive a new Gap equation system (On-shell renormalisation, MOM scheme):

$$\begin{pmatrix}\sigma_v\\ \sigma_m\end{pmatrix} \propto \begin{pmatrix}\rho_A\\ \rho_B\end{pmatrix} + \begin{pmatrix}O_{11} & O_{12}\\ O_{21} & O_{22}\end{pmatrix}\begin{pmatrix}\sigma_v\\ \sigma_m\end{pmatrix}$$

$$\begin{pmatrix}\rho_A\\ \rho_B\end{pmatrix} \propto \begin{pmatrix}Q_{11} & Q_{12}\\ Q_{21} & Q_{22}\end{pmatrix}\begin{pmatrix}\rho_A\\ \rho_B\end{pmatrix}$$

where the O and the Q are operators depending resp. on the ρ and σ.

- The same type of equation can be derived for the photon
- No momentum dependence remains: Momenta are integrated out.
- These equations are derived used the Minkowski metric.

We have now started to think about numerical solution to this new problem

Summary

Summary

Baryon PDA with NIR

- **DSE compatible** framework for Baryon PDAs.
- Simple Nakanishi representation works for the nucleon PDA.
- Improved results for the scalar diquark
- We need to add the axial-vector diquark

Direct computation

- Derived a new set of equations for the ρ and σ
- No momentum dependence remains, everything is derived in Minkowski space
- Numerical part is still to be done

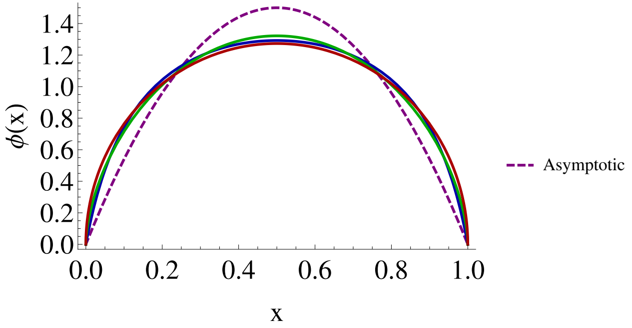
Thank you for your attention

Back up slides

$n = -1$ Mellin Moment

$$\langle x^{-1} \rangle = \int_0^1 \mathrm{d}x \frac{\varphi(x)}{1-x} \qquad \phi_{\ln}(x) \propto 1 - \frac{\ln\left[1 + \kappa x(1-x)\right]}{\kappa x(1-x)}$$

	$x(1-x)$	$\phi_{\ln}(x)$	$(x(1-x))^\nu$	$\sqrt{x(1-x)}$
$\langle x^{-1} \rangle$	3	3.41	3.66	4
$\frac{\langle x^{-1} \rangle}{\langle x^{-1} \rangle_{As}}$	1	1.14	1.22	1.33



Meson Form Factors