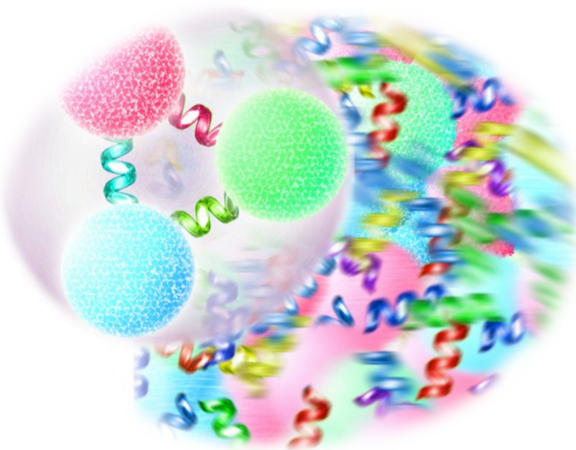

# Quark mass generation with non-abelian Ball-Chiu vertex

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Based on: ACA, J.C.Cardona, M.N.Ferreira and J. Papavassiliou, Phys.Rev. D98 (2018) no.1, 014002

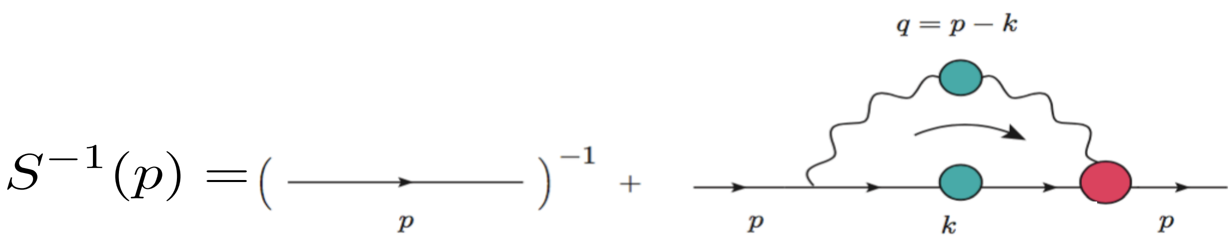
Supported by:
FAPESP
CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

**Emergent mass and its consequences in the Standard Model**
17 -21 September, 2018 – Trento, Italy

# Motivation

- The *dynamical mechanism that generates the quark masses should be included in any plausible description of the infrared QCD.*

- The study of *the chiral symmetry breaking in the continuum* involves almost invariably some version of *the Schwinger-Dyson for the quark propagator* (gap equation).




- The gap equation *displays "critical" behavior*: the *support of the kernel* throughout the entire range of integration *must exceed* a certain critical value in order to *generate non-trivial solutions*.

- Most of the support comes from the infrared region, i.e. around the QCD mass scale, the study of CSB furnishes stringent probes on approaches aiming towards a quantitative description of the non-perturbative sector of QCD.

**C.D.Roberts and A.G.Williams,** Prog. Part. Nucl.Phys.33, 477 (1994)

- The role of the quark-gluon vertex is a key ingredient for the gap equation.

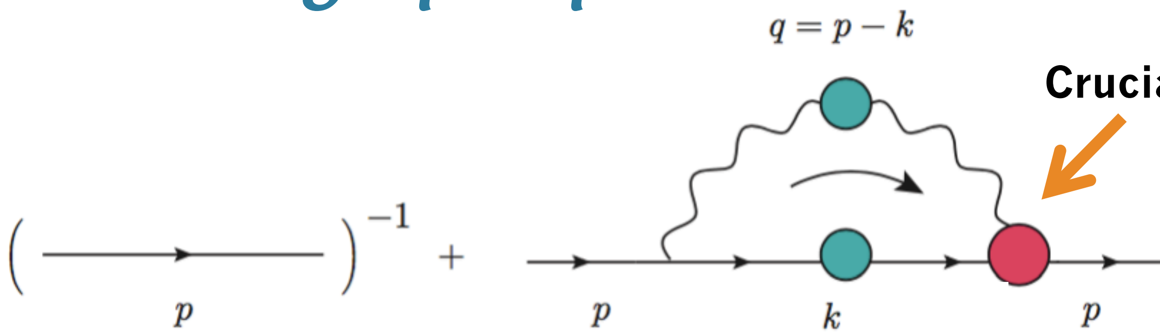
- Recently, the non-transverse form factors of the vertex were determined from the STI that it satisfies → gauge technique

**ACA, J. C. Cardona, M. N. Ferreira and J. Papavassiliou**, Phys.Rev.D96, no. 1, 014029 (2017)
Previous studies → kinematic special configurations:
**ACA and J. Papavassiliou**, Phys. Rev. D83, 014013 (2011)
**E. Rojas, J. P. B. C. de Melo, B. El-Bennich, O. Oliveira and T. Frederico**, JHEP 1310, 193 (2013)

- It is natural to study the CSB pattern that emerges if we couple the dynamical equation governing the quark propagator with the form factors of the non-transverse part of the quark-gluon vertex.

# *The gap equation*

$$S^{-1}(p) =$$




**Crucial ingredient**

$$S^{-1}(p) = \not{p} - m_0 - iC_{\mathrm{r}} g^2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S(k) \Gamma_\nu(k,p) \Delta^{\mu\nu}(p-k)$$

**Gap Equation**

Full quark propagator

$$S^{-1}(p) = A(p^2)\not{p} - B(p^2)$$

*Dynamical quark mass*

$$\mathcal{M}(p) = \frac{B(p)}{A(p)}$$

*Chiral Symmetry breaking occurs when B ≠ 0*

# Simple Ansatz for $\Gamma_\mu$

- The quark dynamical mass equation is given by

$$\mathcal{M}(p^2) = 4\int_k \mathcal{K}(p,k)\,\frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}$$

- The kernel $\mathcal{K}(p,k)$ depends on the approximation used for the quark-gluon vertex

- A simple Ansatz is the Abelian approximation for $\Gamma_\mu$ ( satisfies the QED Ward identity).

$$q^\mu \Gamma_\mu(p,k) = S^{-1}(p) - S^{-1}(k)$$

- In this case

$$\mathcal{K}(p,k) \propto g^2 \Delta(p-k)$$

- However, the kernel does not have enough strength for generating the quark mass

# Inflating the kernel

↓

## means better knowledge of the quark-gluon vertex

↓

- **Use an improved quark-gluon vertex (abelianization not good)**

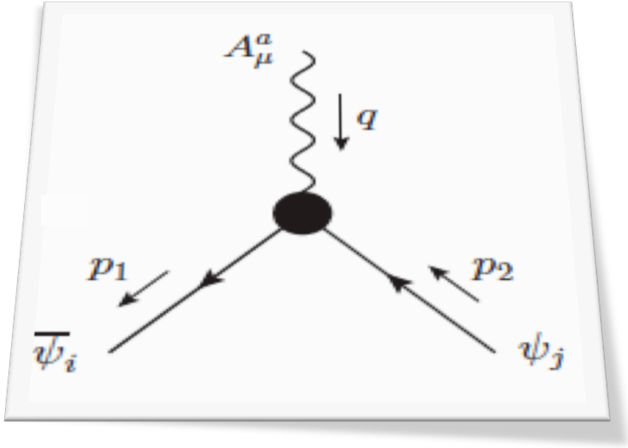
  ✓ **Slavnov-Taylor identity instead of Ward identity**

$$q^{\mu}\Gamma_{\mu}^{\rm STI}(q,p_2,-p_1)=F(q)[S^{-1}(p_1)H(q,p_2,-p_1)-\overline{H}(-q,p_1,-p_2)S^{-1}(p_2)]\,.$$

  ✓ **Include quark-ghost scattering kernel H is numerically crucial!**

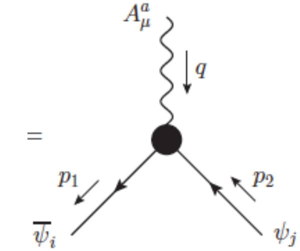
$$D(q^2)=\frac{iF(q^2)}{q^2}$$


**A. C. A. and J. Papavassiliou**, Phys. Rev. D83, 014013 (2011).
**A. C. A., J. C. Cardona, M. N. Ferreira and J. Papavassiliou,** Phys. Rev. D96, no. 1, 014029 (2017).




# The quark-gluon vertex

# The full quark-gluon vertex




$$q = p_1 - p_2$$

- The most general decomposition of the full quark-gluon vertex has 12 tensorial structures.
- It can be separated in a "non-transverse" and "transverse" parts

$$\Gamma_\mu(q,p_2,-p_1) = \Gamma^{(\mathrm{L})}_\mu(q,p_2,-p_1) + \Gamma^{(\mathrm{T})}_\mu(q,p_2,-p_1)\,,$$

- The transverse (8 tensorial structures) is automatically conserved

$$q^\mu \Gamma^{(\mathrm{T})}_\mu(q,p_2,-p_1) = 0\,.$$

- and the "non-transverse" (4 structures )

$$\Gamma^{(\mathrm{L})}_\mu(q,p_2,-p_1) = \sum_{i=1}^{4} L_i(q,p_2,-p_1)\lambda_{i,\mu}(p_1,p_2)\,,$$

$$\begin{aligned}
\lambda_{1,\mu} &= \gamma_\mu\,, \\
\lambda_{2,\mu} &= (\not{p}_1 + \not{p}_2)(p_1+p_2)_\mu\,, \\
\lambda_{3,\mu} &= (p_1+p_2)_\mu\,, \\
\lambda_{4,\mu} &= \widetilde{\sigma}_{\mu\nu}(p_1+p_2)^\nu\,,
\end{aligned}$$

**J. S. Ball and T.W. Chiu**, Phys.Rev. D 22, 2542 (1980).

- The longitudinal part saturates the non-Abelian Slavnov-Taylor identity:

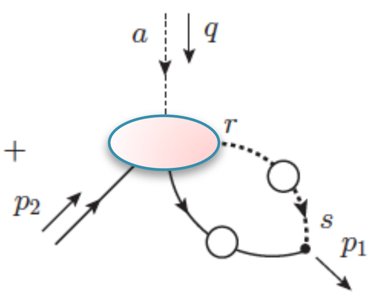
$$q^\mu \Gamma_\mu^{(\mathrm{L})}(q, p_2, -p_1) = F(q^2)\left[S^{-1}(p_1)H(q, p_2, -p_1) - \overline{H}(-q, p_1, -p_2)S^{-1}(p_2)\right],$$

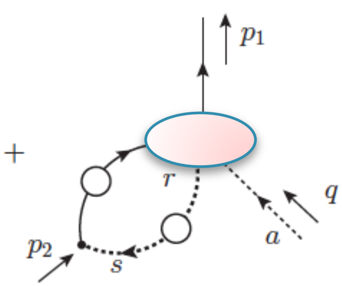
where:

$S^{-1}(p_1)$ → inverse of the quark propagator $S^{-1}(p) = A(p^2)\not{p} - B(p^2)\,,$

$F(q^2)$ → ghost dressing function $D(q^2) = \dfrac{iF(q^2)}{q^2}$

$H$ and $\overline{H} := \gamma^0 H^\dagger \gamma^0$ are the quark-ghost scattering kernel

$H^a(q, p_2, -p_1) = -gt^a +$ 


$\overline{H}^a(-q, p_1, -p_2) = gt^a +$ 

- The quark-ghost scattering kernel H has the following Lorentz decomposition

$$H = X_0 \mathbb{I} + X_1 \not{p}_1 + X_2 \not{p}_2 + X_3 \widetilde{\sigma}_{\mu\nu} p_1^\mu p_2^\nu$$

$$\widetilde{\sigma}_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$$

with $X_i(q^2, p_2^2, p_1^2)$ being the form factors (function of the momenta)

- its conjugated counterpart

$$\overline{H} = \overline{X}_0 \mathbb{I} + \overline{X}_2 \not{p}_1 + \overline{X}_1 \not{p}_2 + \overline{X}_3 \widetilde{\sigma}_{\mu\nu} p_1^\mu p_2^\nu .$$

where $\overline{X}_i := X_i(q^2, p_1^2, p_2^2)$

- At tree level:

$$X_0^{(0)} = 1 \text{ and } X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 0,$$

- Substituting the decompositions in the STI

$$q^{\mu}\Gamma^{\rm STI}_{\mu}(q,p_2,-p_1)=F(q)[S^{-1}(p_1)H(q,p_2,-p_1)-\overline{H}(-q,p_1,-p_2)S^{-1}(p_2)]\,.$$

whose decompositions are given by

$$H(q,p_2,-p_1)=X_0\mathbb{I}+X_1\not{p}_1+X_2\not{p}_2+X_3\widetilde{\sigma}_{\mu\nu}p_1^{\mu}p_2^{\nu}\,,$$
$$\overline{H}(-q,p_1,-p_2)=\overline{X}_0\mathbb{I}+\overline{X}_2\not{p}_1+\overline{X}_1\not{p}_2+\overline{X}_3\widetilde{\sigma}_{\mu\nu}p_1^{\mu}p_2^{\nu}\,,$$

$$\Gamma^{\rm STI}_{\mu}(q,p_2,-p_1)=L_1\gamma_{\mu}+L_2(\not{p}_1-\not{p}_2)(p_1-p_2)_{\mu}+L_3(p_1-p_2)_{\mu}+L_4\tilde{\sigma}_{\mu\nu}(p_1-p_2)^{\nu}\,,$$

*We obtain for the form factors ….*

$$L_1 = \frac{F(q)}{2}\left\{A(p_1)[X_0 - (p_1^2 + p_1\cdot p_2)X_3] + A(p_2)[\overline{X}_0 - (p_2^2 + p_1\cdot p_2)\overline{X}_3]\right\}$$
$$+ \frac{F(q)}{2}\left\{B(p_1)(X_2 - X_1) + B(p_2)(\overline{X}_2 - \overline{X}_1)\right\};$$
$$L_2 = \frac{F(q)}{2(p_1^2 - p_2^2)}\left\{A(p_1)[X_0 + (p_1^2 - p_1\cdot p_2)X_3] - A(p_2)[\overline{X}_0 + (p_2^2 - p_1\cdot p_2)\overline{X}_3]\right\}$$
$$- \frac{F(q)}{2(p_1^2 - p_2^2)}\left\{B(p_1)(X_1 + X_2) - B(p_2)(\overline{X}_1 + \overline{X}_2)\right\};$$
$$L_3 = \frac{F(q)}{p_1^2 - p_2^2}\left\{A(p_1)\left(p_1^2 X_1 + p_1\cdot p_2 X_2\right) - A(p_2)\left(p_2^2\overline{X}_1 + p_1\cdot p_2\overline{X}_2\right) - B(p_1)X_0 + B(p_2)\overline{X}_0\right\};$$
$$L_4 = \frac{F(q)}{2}\left\{A(p_1)X_2 - A(p_2)\overline{X}_2 - B(p_1)X_3 + B(p_2)\overline{X}_3\right\}.$$

**ACA and J. Papavassiliou,** Phys. Rev. D83, 014013 (2011)

- Ball-Chiu vertex (Abelian) is recovered using the tree level of H and F

$$L_1^{\rm BC} = \frac{A(p_1) + A(p_2)}{2}, \qquad L_2^{\rm BC} = \frac{A(p_1) - A(p_2)}{2(p_1^2 - p_2^2)},$$
$$L_3^{\rm BC} = \frac{B(p_2) - B(p_1)}{p_1^2 - p_2^2}, \qquad L_4^{\rm BC} = 0.$$

$$X_0^{(0)} = 1 \text{ and } X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 0,$$

$$F^{[0]} = 1$$

**J. S. Ball and T.W. Chiu,** Phys.Rev. D 22, 2542 (1980).

- Notice that, we can do a hybrid assumptions: H is  tree level but not F

$$X_0^{(0)} = 1 \text{ and } X_1^{(0)} = X_2^{(0)} = X_3^{(0)} = 0,$$

- We obtain the **minimally "non-abelianized" Ball-Chiu vertex**

**H is turned off (tree level)**

$$L_1^{\rm FBC} = F(q)\frac{[A(p)+A(k)]}{2}\,, \qquad L_2^{\rm FBC} = F(q)\frac{[A(p)-A(k)]}{2(p^2-k^2)},$$

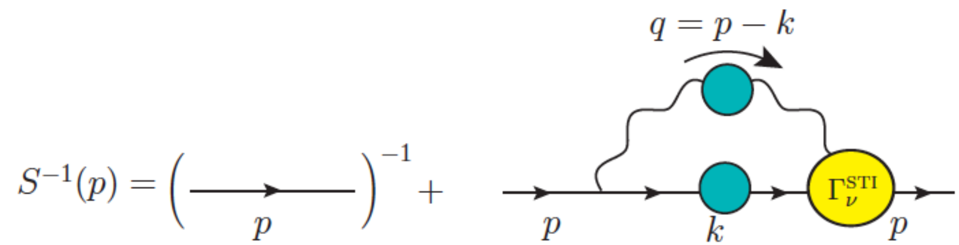
$$L_3^{\rm FBC} = -F(q)\frac{[B(p)-B(k)]}{p^2-k^2}\,, \qquad L_4^{\rm FBC} = 0\,.$$

$$\Gamma_\mu^{\rm FBC} = F(q)\Gamma_\mu^{\rm BC};$$

**C. S. Fischer and R. Alkofer**, Phys.Rev.D 67, 094020 (2003).

# Gap equation

- Plugging the complete non-transverse structure of the vertex in the gap equation




$$p^2 A(p) = Z_{\mathrm{F}} p^2 + Z_1 4\pi C_F \alpha_s \int_k \mathcal{K}_{\mathrm{A}}(k,p)\Delta(q)F(q)\,,$$

$$B(p) = Z_1 4\pi C_F \alpha_s \int_k \mathcal{K}_{\mathrm{B}}(k,p)\Delta(q)F(q)\,,$$

$$\mathcal{K}_{\mathrm{A}}(k,p) = \left\{\frac{3}{2}(k\cdot p)\overline{L}_1 - [\overline{L}_1 - (k^2+p^2)\overline{L}_2]h(p,k)\right\}\mathcal{Q}_{\mathrm{A}}(k)$$
$$- \left\{\frac{3}{2}p\cdot(k+p)\overline{L}_4 + (\overline{L}_3 - \overline{L}_4)h(p,k)\right\}\mathcal{Q}_{\mathrm{B}}(k)\,,$$
$$\mathcal{K}_{\mathrm{B}}(k,p) = \left\{\frac{3}{2}k\cdot(k+p)\overline{L}_4 - (\overline{L}_3 + \overline{L}_4)h(p,k)\right\}\mathcal{Q}_{\mathrm{A}}(k)$$
$$+ \left\{\frac{3}{2}\overline{L}_1 - 2h(p,k)\overline{L}_2\right\}\mathcal{Q}_{\mathrm{B}}(k)\,,$$

$$L_i = F(q)\overline{L}_i/2$$

$$h(p,k) := \frac{[k^2p^2 - (k\cdot p)^2]}{q^2}$$

$$\mathcal{Q}_{\mathrm{f}}(k) := \frac{f(k)}{[A^2(k)k^2 + B^2(k)]}$$

# Renormalization of the gap equation

Renormalization constants:
$Z_c \rightarrow$ ghost field
$Z_F \rightarrow$ quark field
$Z_H \rightarrow$ quark-ghost kernel
$Z_1 \rightarrow$ vertex

- The STI imposes the relation

$$Z_1 = Z_c^{-1} Z_F Z_H^{-1},$$

- In the Landau gauge, the quark self-energy and the quark-ghost kernel are finite at one-loop

$$Z_F = Z_H = 1, \quad \Rightarrow \quad Z_1 = Z_c^{-1}.$$

- We obtain the approximate version

$$p^2 A(p) = p^2 + \underline{Z_c^{-1}} 4\pi C_F \alpha_s \int_k \mathcal{K}_A(k,p)\Delta(q)F(q)\,,$$

$$B(p) = \underline{Z_c^{-1}} 4\pi C_F \alpha_s \int_k \mathcal{K}_B(k,p)\Delta(q)F(q)\,.$$

Presence of $Z_c^{-1}$

- **The presence of $Z_c^{-1}$ complicates the analysis**, especially in a **non-perturbative setting**.
- **Multiplicative renormalization constants** are instrumental for the systematic cancellation of **overlapping divergences**.
- The inclusion of the **contributions** stemming from the **transverse parts of the vertices** is also needed for the systematic **cancellation of overlapping divergences**.

For QED it was studied by
**A. Kizilersu and M. Pennington**, Phys. Rev. D79, 125020 (2009).

- Since **in this analysis the transverse part is completely undetermined** → the **cancellation of the overlapping divergences is excluded** from the outset.
- **A typical manifestation** of the mismatches induced if we impose $Z_c^{-1} = 1$ **is the failure of $\mathcal{M}(p)$ to display the correct anomalous dimension in the deep ultraviolet**

- The asymptotic behavior of $\mathcal{M}(p)$ at one-loop is given by

$$\mathcal{M}_{\rm UV}(p) = \frac{C}{p^2}\left[\ln\left(\frac{p^2}{\Lambda^2}\right)\right]^{\gamma_f - 1},$$

- With the approximation $Z_c^{-1} = 1$ we obtain

$\gamma_f = 48/(35C_{\rm A} - 8n_f)$. instead of $\gamma_f = 12/(11C_{\rm A} - 2n_f)$

- A simple way to remedy to this problem is to carry out the substitution

$$Z_c^{-1}\mathcal{K}_{\rm A,B}(p,k) \to \mathcal{K}_{\rm A,B}(p,k)\mathcal{C}(q),$$

**C. S. Fischer and R. Alkofer,** Phys. Rev. D67, 094020 (2003),
**ACA and J. Papavassiliou**, Phys. Rev. D83, 014013 (2011)

where $\mathcal{C}(q)$ should display the appropriate ultraviolet characteristics to convert the product

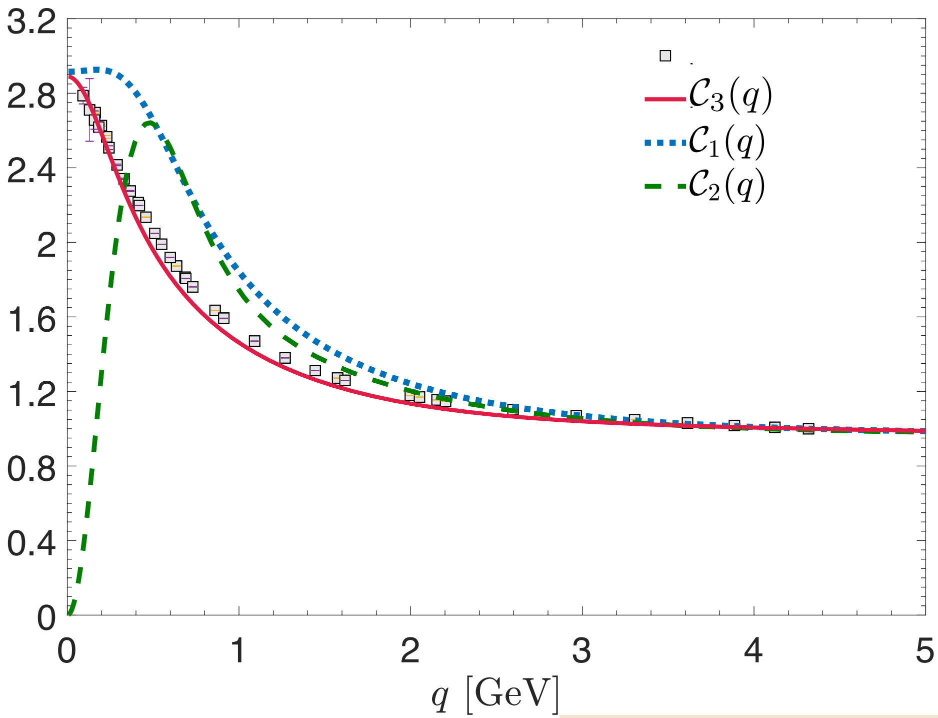
$$\mathcal{R}(q) = \alpha_s(\mu)\Delta(q,\mu)F(q,\mu)\mathcal{C}(q,\mu)\,,$$

into a renormalization-group invariant (RGI) ( $\mu$ -independent).

- The requirement that $\mathcal{R}(q)$ **be RGI fixes the ultraviolet behavior of** $\mathcal{C}(q)$

$$\mathcal{C}_{\rm UV}(q) = 1 + \frac{9C_{\rm A}\alpha_s}{48\pi}\ln\left(\frac{q^2}{\mu^2}\right)$$

- However, the low-energy completion of $\mathcal{C}(q)$ remains undetermined → necessity of **introducing specific Ansätze for it**




$$\mathcal{C}_1(q) = [1 + G(q)]^{-1} \,,$$

$$\mathcal{C}_2(q) = \frac{q^2}{q^2 + a_1} \left[ 1 + \exp\left( -\frac{a_2 q^2}{\mu^2} \right) \right] [1 + G(q)]^{-1} \,,$$

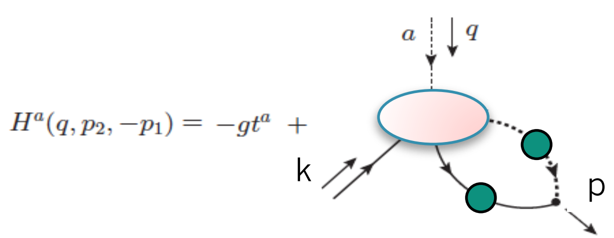
$$\mathcal{C}_3(q) = F(q) \,,$$

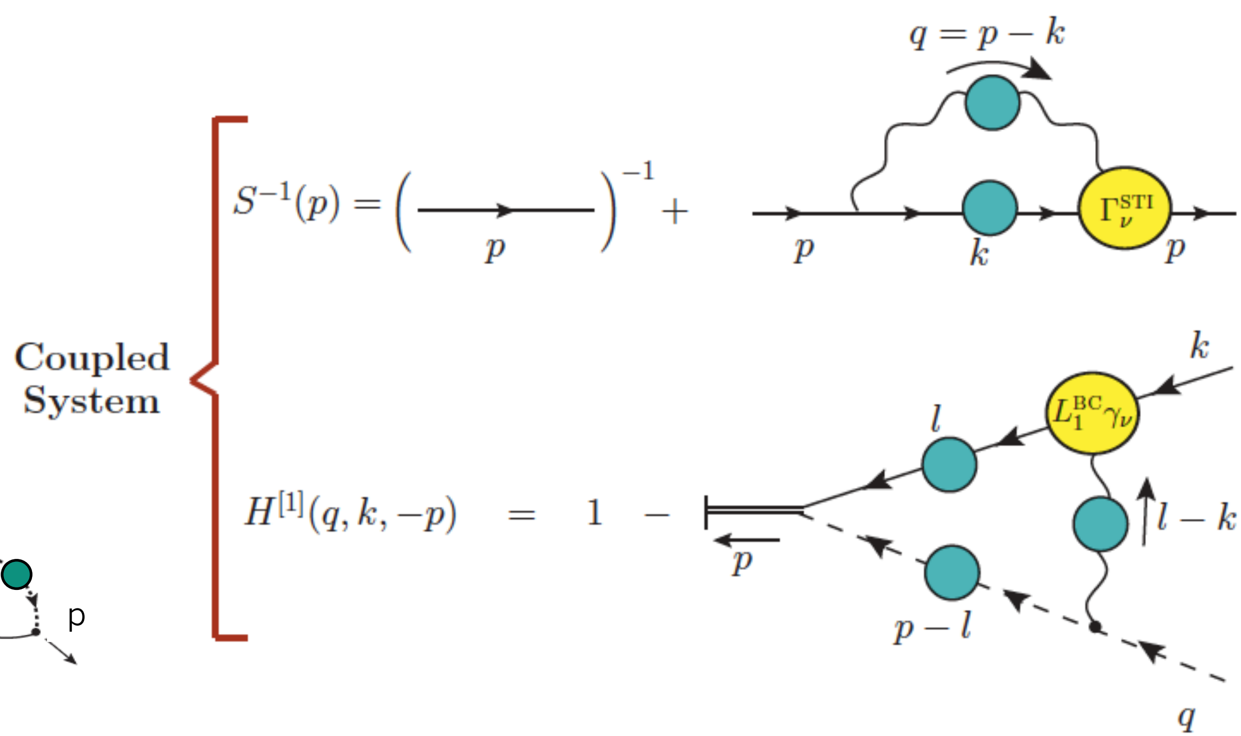
These three Ansätze are to be understood as representative cases of a wider range of qualitatively similar realizations

# Coupled system

- We solve numerically a coupled system of *six nonlinear integral equations for*

$$A(p), B(p), X_1, X_2, X_3 \text{ and } X_4$$

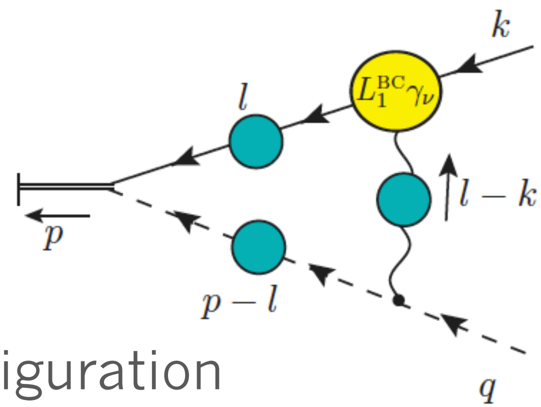


# *Scattering quark-ghost kernel*

**One-loop dressed approximation:**

$H^{[1]}(q, k, -p) \quad = \quad 1 \quad -$




- For a general kinematic configuration

$$H^{[1]} = 1 - \frac{1}{2} i C_{\mathrm{A}} g^2 \int_l \Delta^{\mu\nu}(l-k) G_\mu^{(0)}(p-l) D(l-p) S(l) L_1^{\mathrm{BC}}(l-k, k, -l)\gamma_\nu$$

Depends on:

✓ Gluon propagator: $\Delta(q)$

✓ Ghost propagator D(q) or F(q)

✓ Quark propagator: A(k), B(k)

$$H = X_0 \mathbb{I} + X_1 \not{p}_1 + X_2 \not{p}_2 + X_3 \widetilde{\sigma}_{\mu\nu} p_1^\mu p_2^\nu \,.$$

# Form factors of the scattering kernel

- Projecting out the form factors

$$X_0 = 1 + i\pi C_{\rm A}\alpha_s \int_l \mathcal{K}(p,k,l) A(l) \mathcal{G}(k,q,l)\,,$$
$$X_1 = i\pi C_{\rm A}\alpha_s \int_l \frac{\mathcal{K}(p,k,l)B(l)}{h(p,k)} \left[k^2\mathcal{G}(p,q,l) - (p\cdot k)\mathcal{G}(k,q,l)\right]\,,$$
$$X_2 = i\pi C_{\rm A}\alpha_s \int_l \frac{\mathcal{K}(p,k,l)B(l)}{h(p,k)} \left[p^2\mathcal{G}(k,q,l) - (p\cdot k)\mathcal{G}(p,q,l)\right]\,,$$
$$X_3 = -i\pi C_{\rm A}\alpha_s \int_l \frac{\mathcal{K}(p,k,l)A(l)}{h(p,k)} \left[k^2\mathcal{G}(p,q,l) - (p\cdot k)\mathcal{G}(k,q,l) - \mathcal{T}(p,k,l)\right]$$

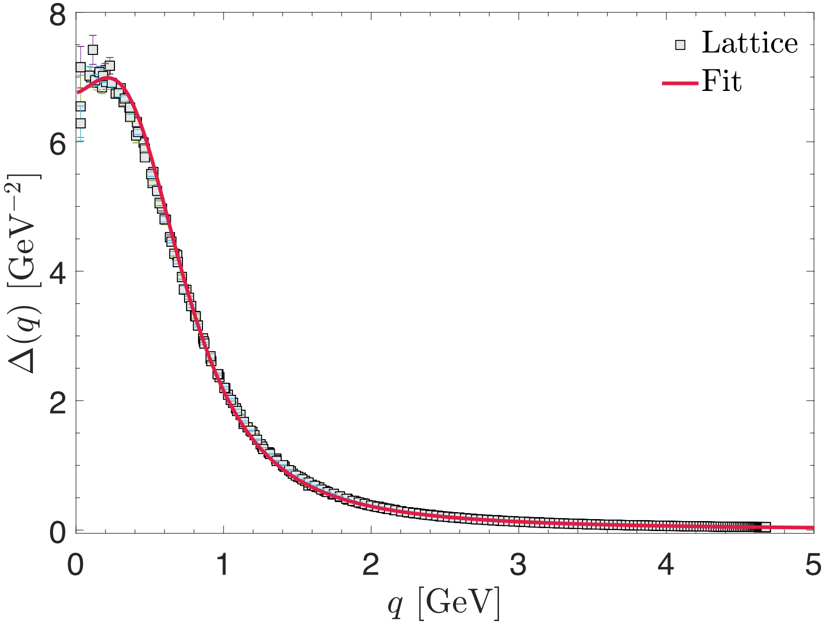
where

$$\mathcal{K}(p,k,l) = \frac{F(l-p)\Delta(l-k)[A(l)+A(k)]}{(l-p)^2[A^2(l)l^2 - B^2(l)]}$$

$$\mathcal{G}(r,q,l) = (r\cdot q) - \frac{[r\cdot(l-k)][q\cdot(l-k)]}{(l-k)^2}\,,$$
$$\mathcal{T}(p,k,l) = (k\cdot q)[(p\cdot l)-(p\cdot k)] - (p\cdot q)[(k\cdot l) - k^2]$$

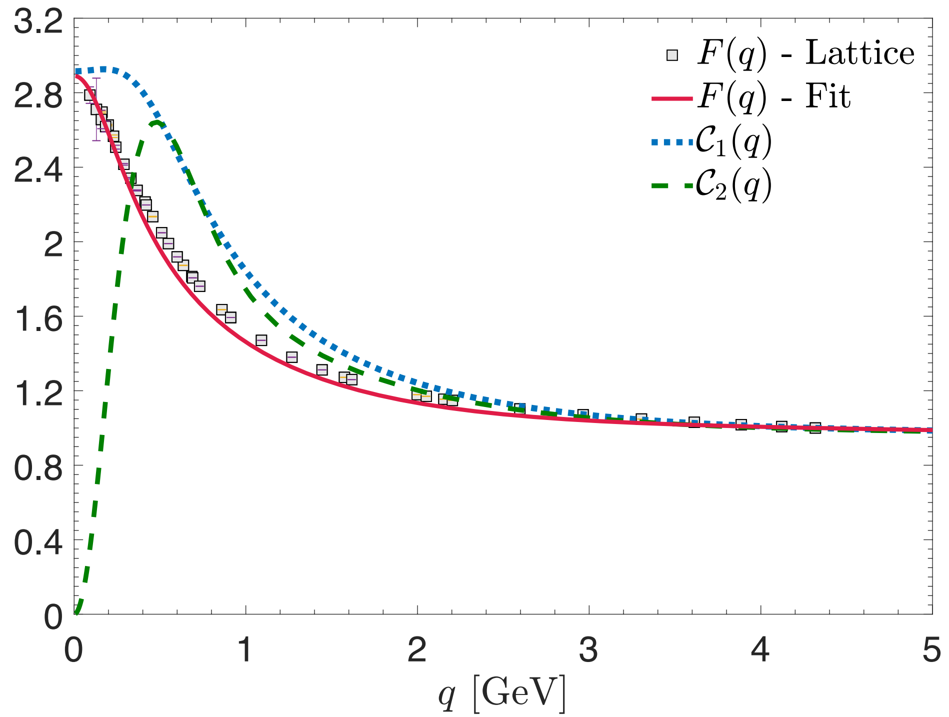
**ACA, J. C. Cardona, M. N. Ferreira and J. Papavassiliou**, Phys.Rev.D96, no. 1, 014029 (2017)

# *Ingredients: Gluon and ghost propagators*




Renormalized at:

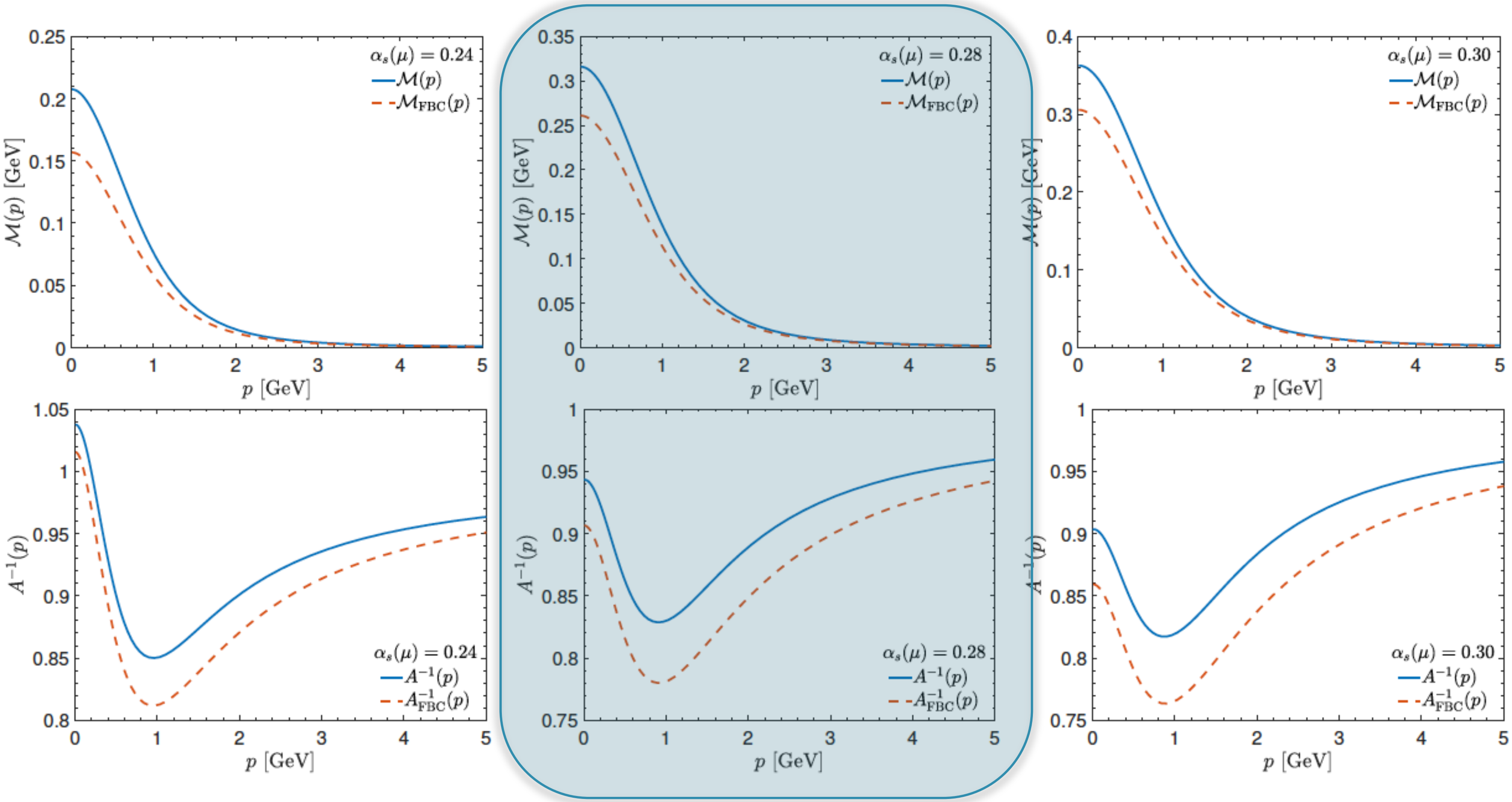
$$\mu = 4.3\,\mathrm{GeV}$$




**I. L. Bogolubsky, et al**. PoS LATTICE, 290 (2007).

# Numerical Results

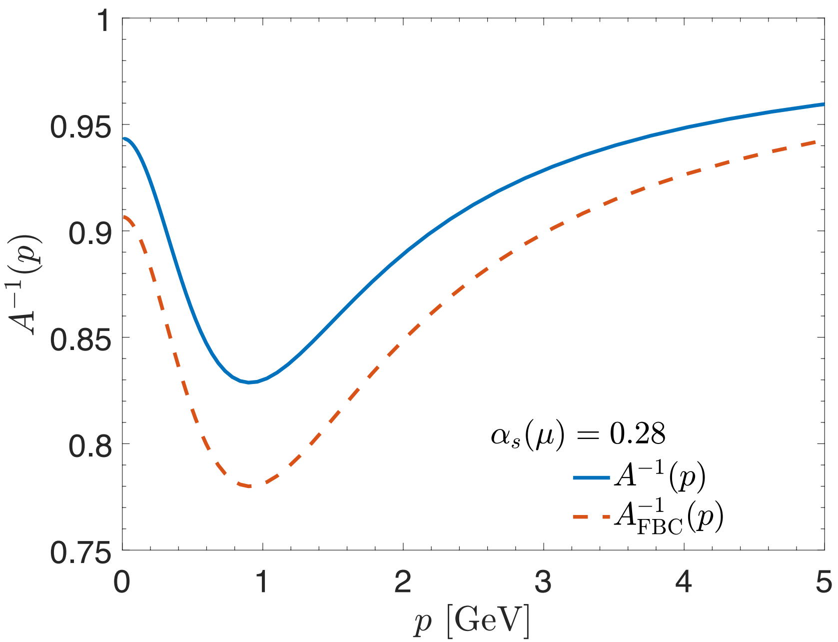
Dynamical quark mass and quark wave function

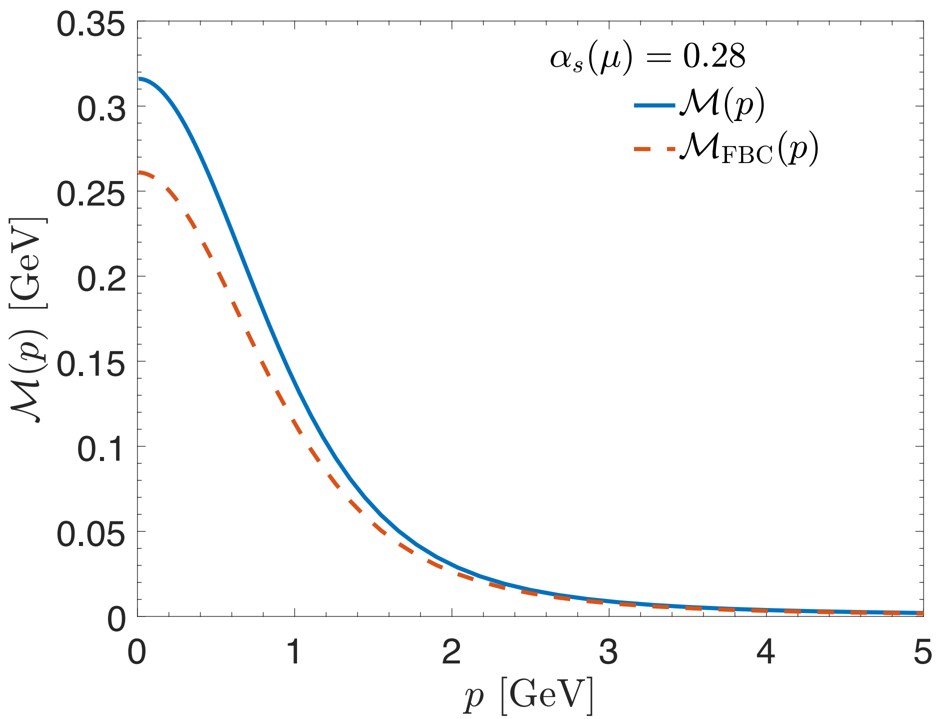



H is turned on - blue curves
H is turned off – orange curves

# Numerical Results

- The quark propagator results

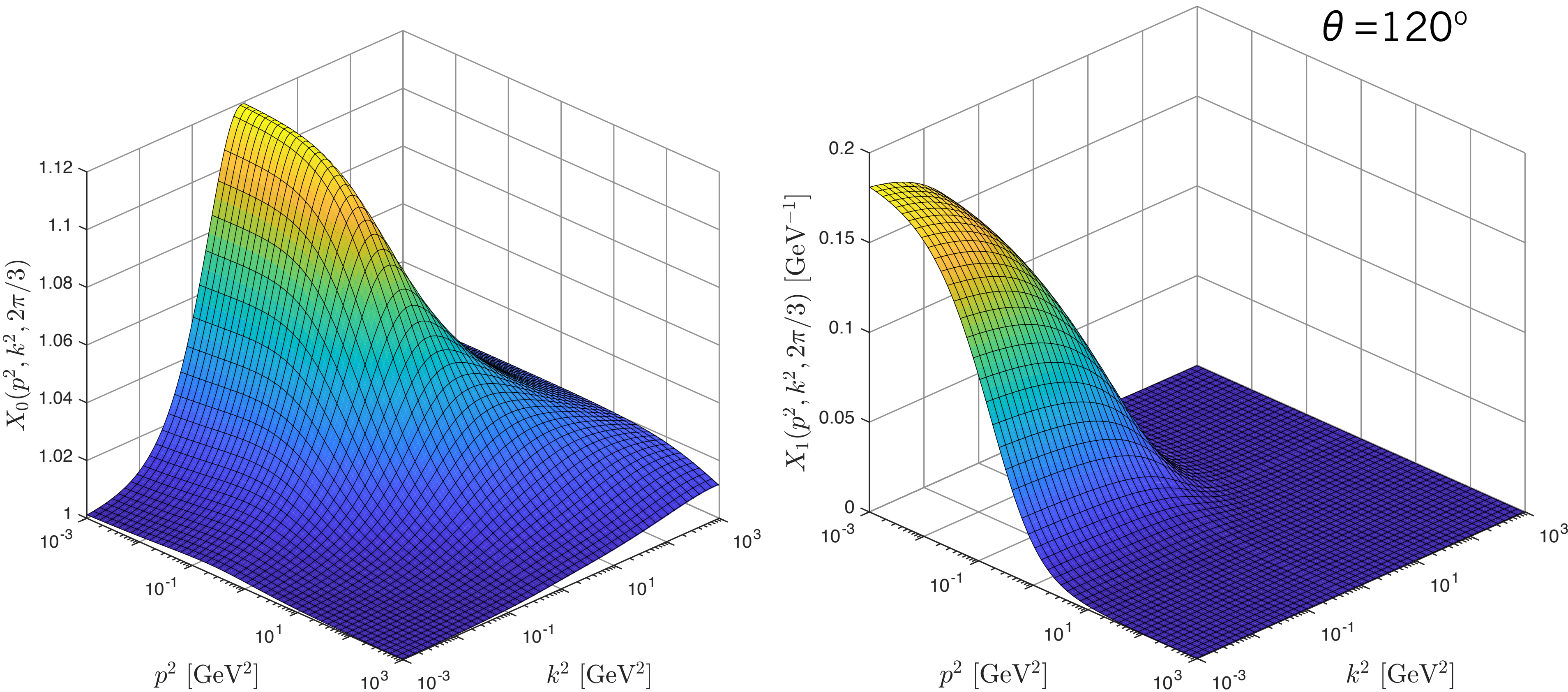



- Generates a dynamical mass of $\mathcal{M}(0) = 316\,\mathrm{MeV}$
- The effect of H increases ~20% of the value of the dynamical mass!

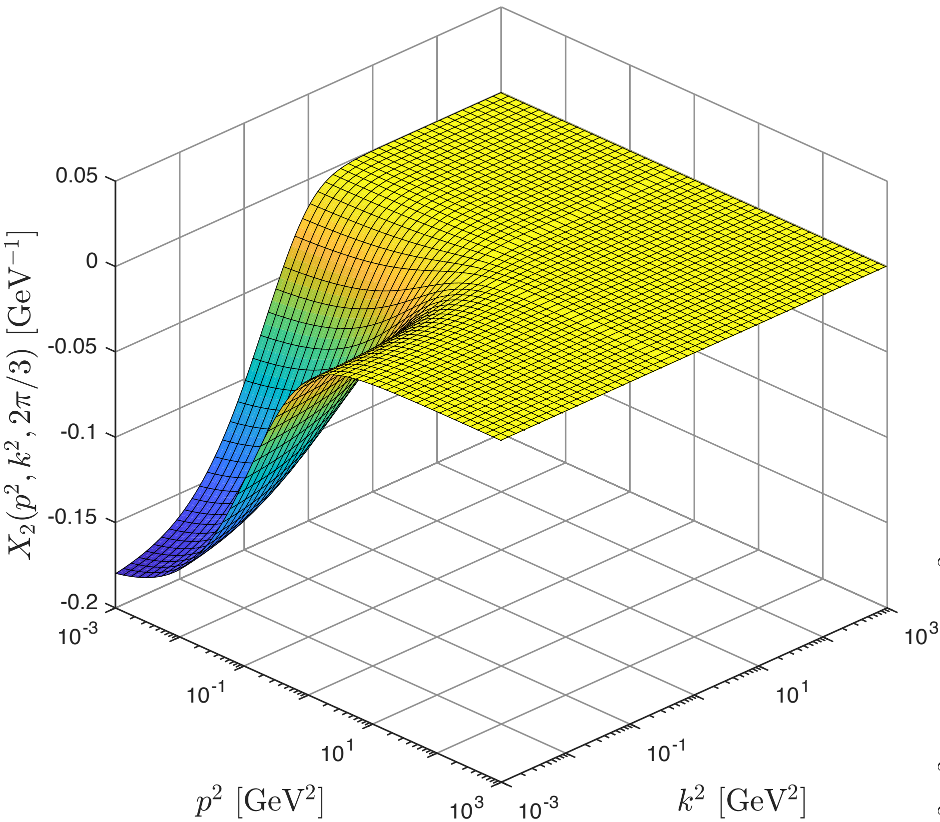
# Form factors of the scattering kernel

$$H(q,k,-p) = X_0\mathbb{I} + X_1\not{p} + X_2\not{k} + X_3\tilde{\sigma}_{\mu\nu}p^\mu k^\nu$$

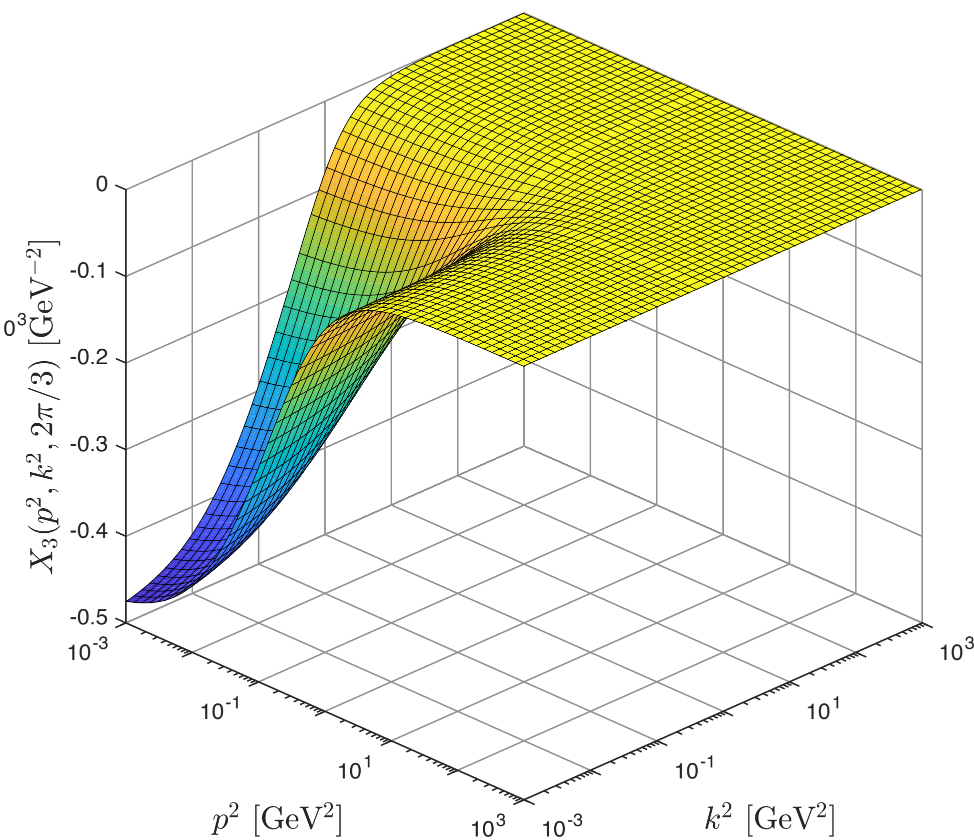



- ✓ Function of three variables $X_i(p, k, \theta)$;
- ✓ Perturbative behavior recovered for large momenta;
- ✓ Mild dependence on $\theta$.

$$H(q,k,-p) = X_0\mathbb{I} + X_1 \not{p} + X_2 \not{k} + X_3 \tilde{\sigma}_{\mu\nu} p^\mu k^\nu$$




$\theta$ =120°



# *Construction of* $L_1(p, k, \theta)$

$$q = p - k$$

- Substituting the $X_i$ in the $L_i(p,k,\theta)$

$$L_1 = \frac{F(q)}{2}\{A(p)[X_0 - (p^2 + p\cdot k)X_3] + A(k)[\overline{X}_0 - (k^2 + p\cdot k)\overline{X}_3]\} + \frac{F(q)}{2}\{B(p)(X_2 - X_1) + B(k)(\overline{X}_2 - \overline{X}_1)\};$$

- Functions of three variables: 2 momenta p and k and the angle between them.

- Similar procedure is performed to obtain self-consistently

$$L_2 = \cdots$$

$$L_3 = \cdots$$

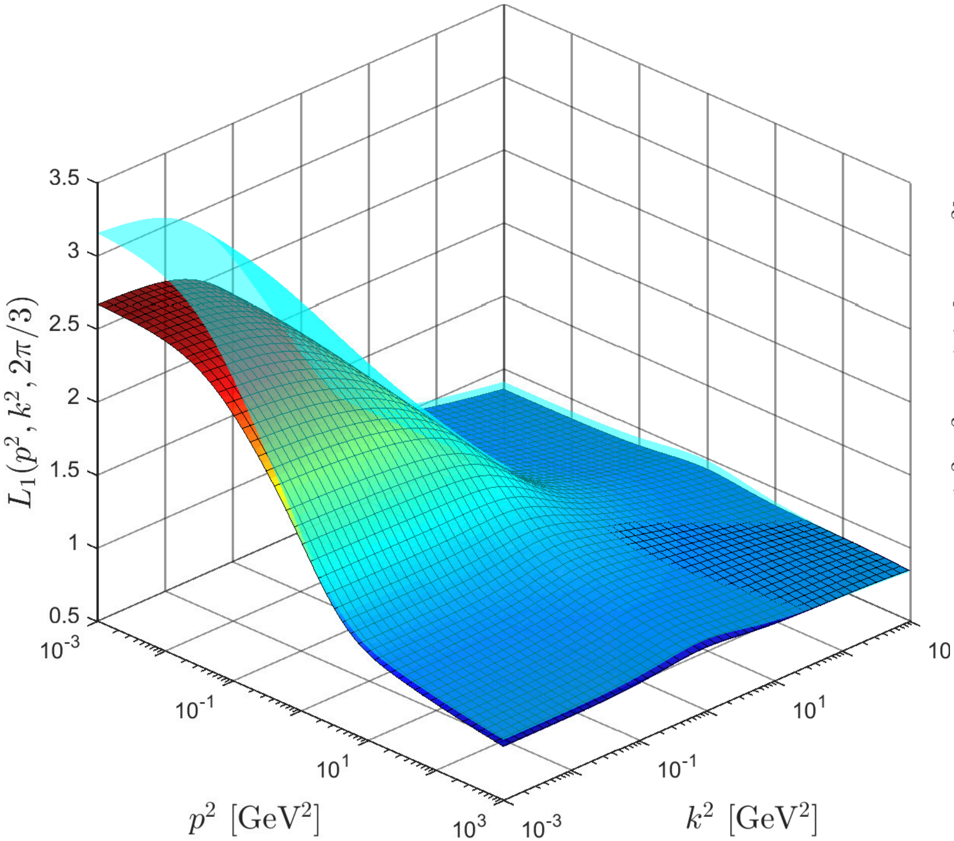
$$L_4 = \cdots$$

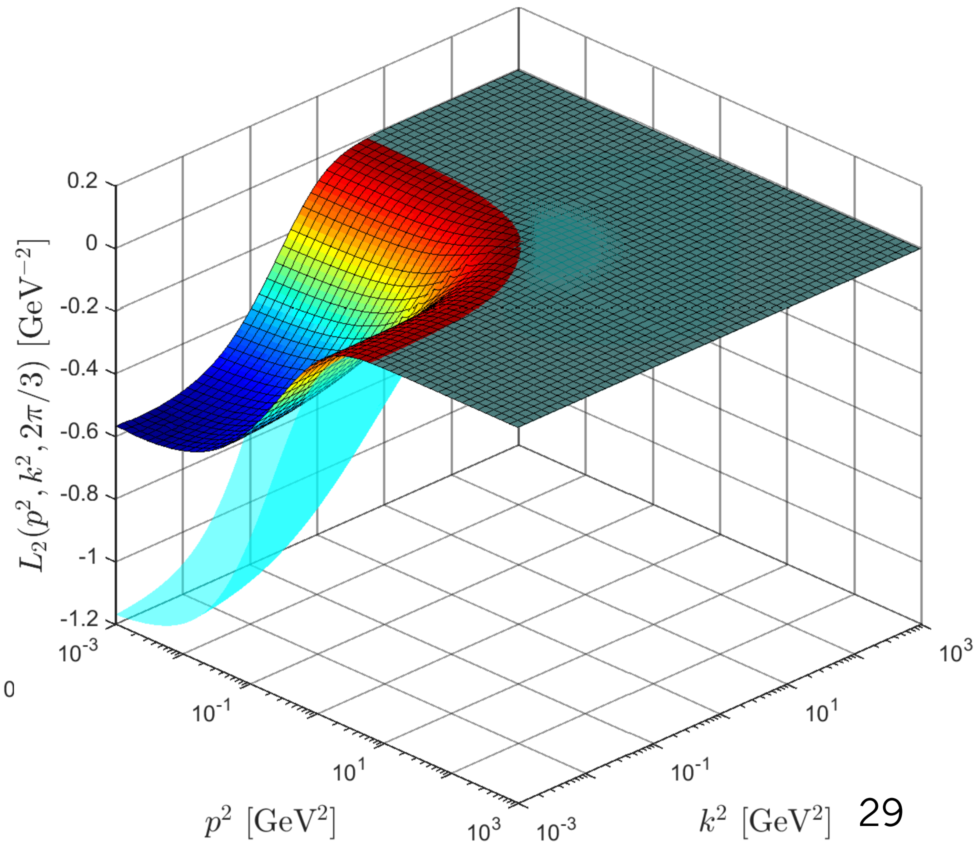
# Quark-gluon form factors

$$\Gamma^{\rm STI}_\mu(q,p_2,-p_1) = \underline{L_1}\gamma_\mu + \underline{L_2}(\not p_1 - \not p_2)(p_1-p_2)_\mu + L_3(p_1-p_2)_\mu + L_4\tilde\sigma_{\mu\nu}(p_1-p_2)^\nu\,,$$

- The $\mathbf{L_i}$ obtained indicate considerable deviations from the $\mathbf{L_i}^{\rm FBC}$ represented by the cyan surface.

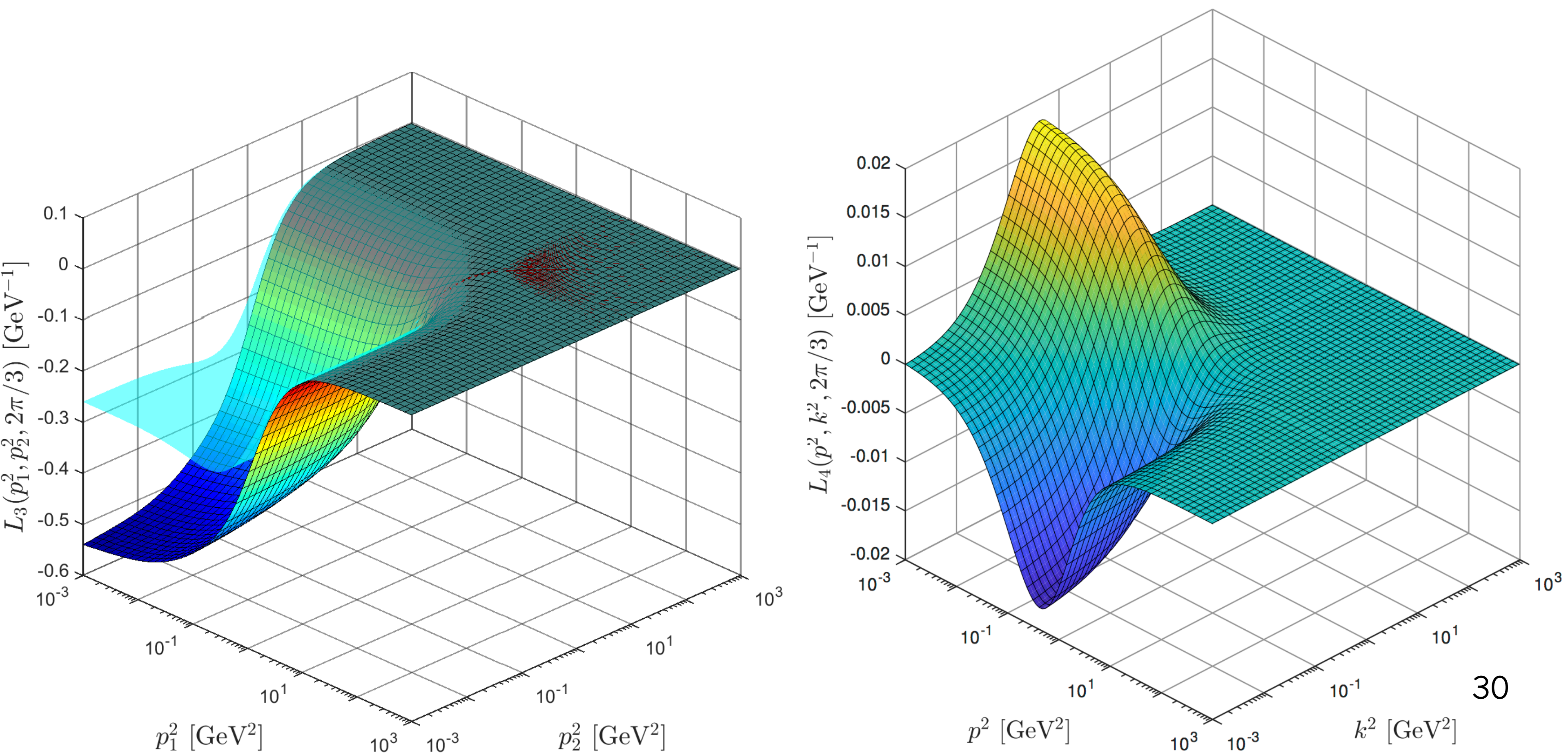
$$L_1^{\rm FBC} = F(q)\frac{[A(p)+A(k)]}{2}\,,\qquad L_2^{\rm FBC} = F(q)\frac{[A(p)-A(k)]}{2(p^2-k^2)}\,,$$
$$L_3^{\rm FBC} = -F(q)\frac{[B(p)-B(k)]}{p^2-k^2}\,,\qquad L_4^{\rm FBC} = 0\,.$$



$$\Gamma_\mu^{\rm STI}(q, p_2, -p_1) = L_1 \gamma_\mu + L_2(\not{p}_1 - \not{p}_2)(p_1 - p_2)_\mu + \underline{L_3}(p_1 - p_2)_\mu + \underline{L_4}\tilde{\sigma}_{\mu\nu}(p_1 - p_2)^\nu \,,$$

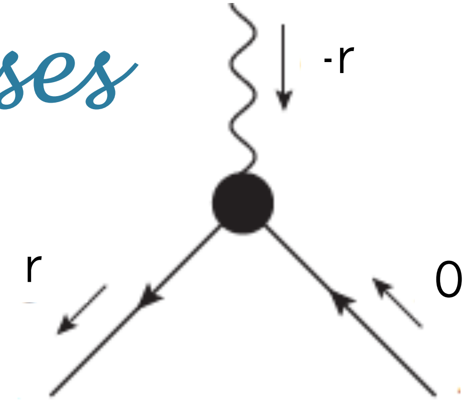
- $L_4$ has a suppressed structure but nonvanishing!
- When we neglected the contribution of scattering kernel H → $L_4$=0
- The four form factors are infrared finite in the entire range of momenta;

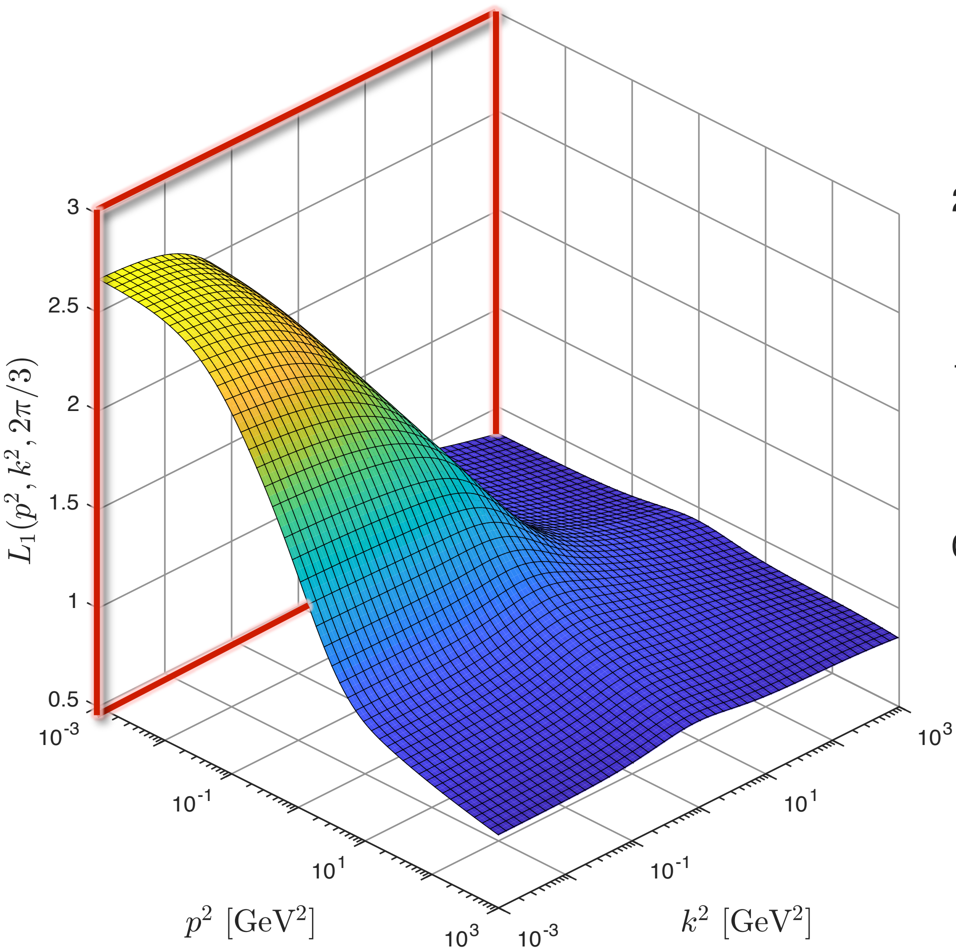


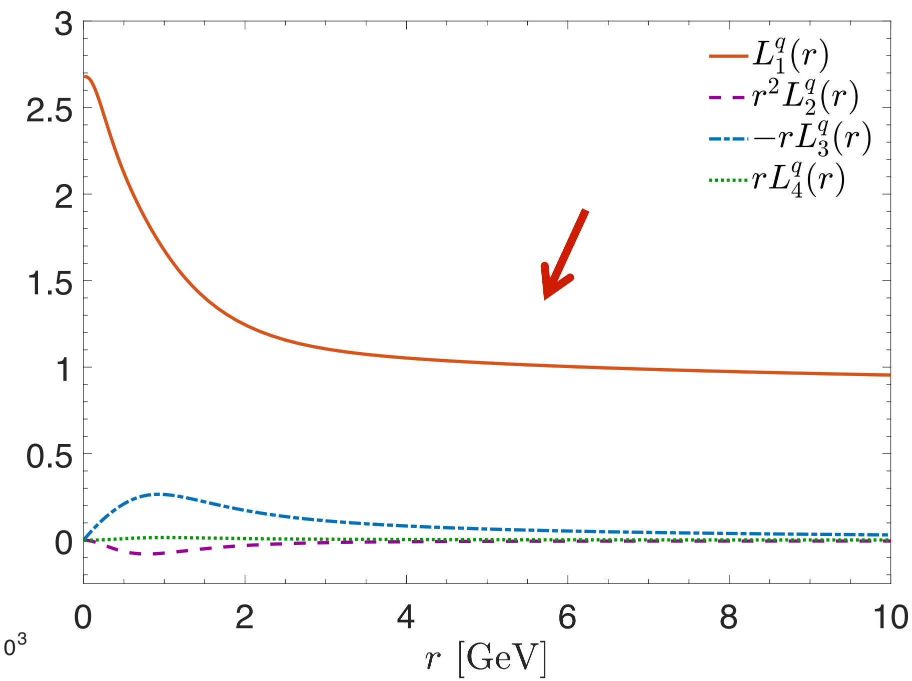
# Special kinematics cases

- *Soft quark limit* → momentum of the quark vanishes

$p \to 0$ and $k \to r$ (independent of $\theta$)

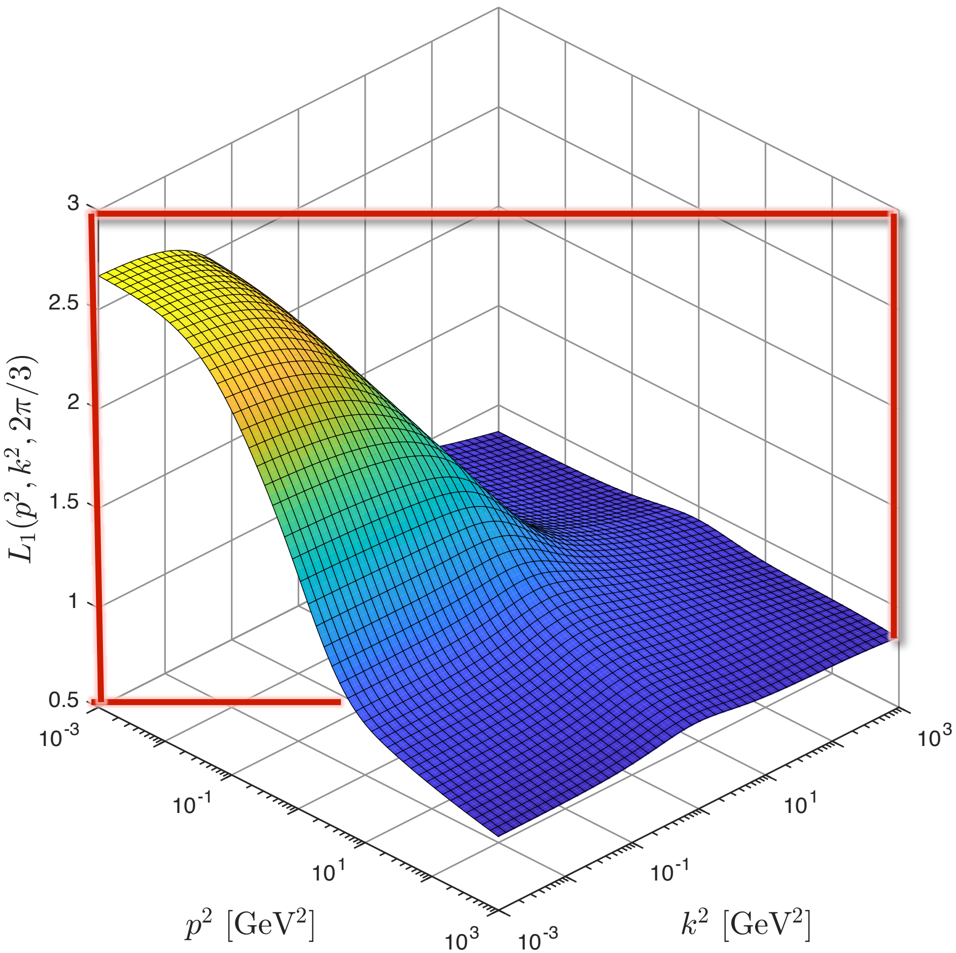


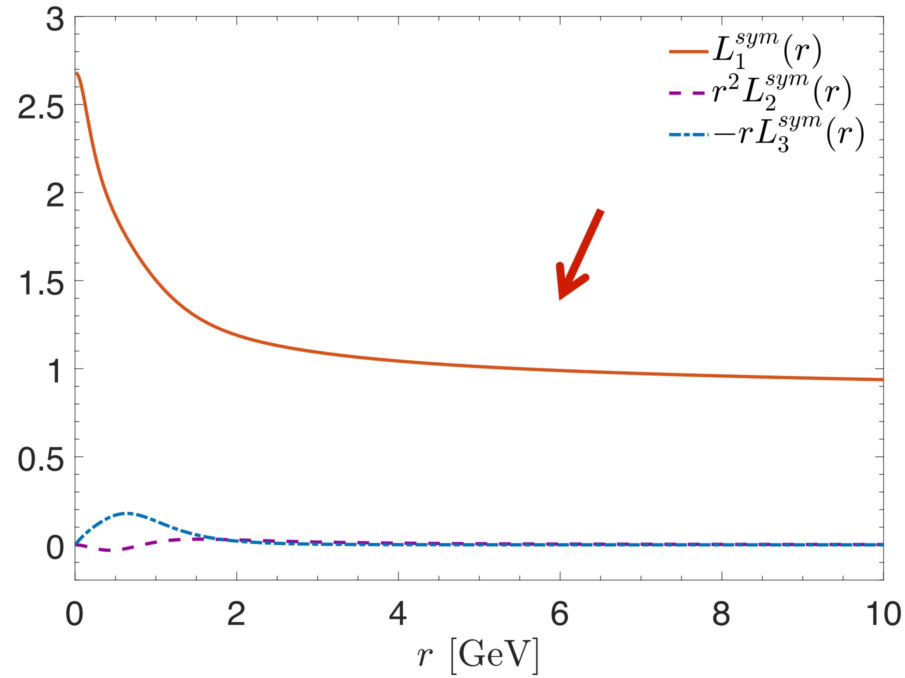



- *Totally symmetric configuration* → all squared momenta are equal and $\theta$ =120°

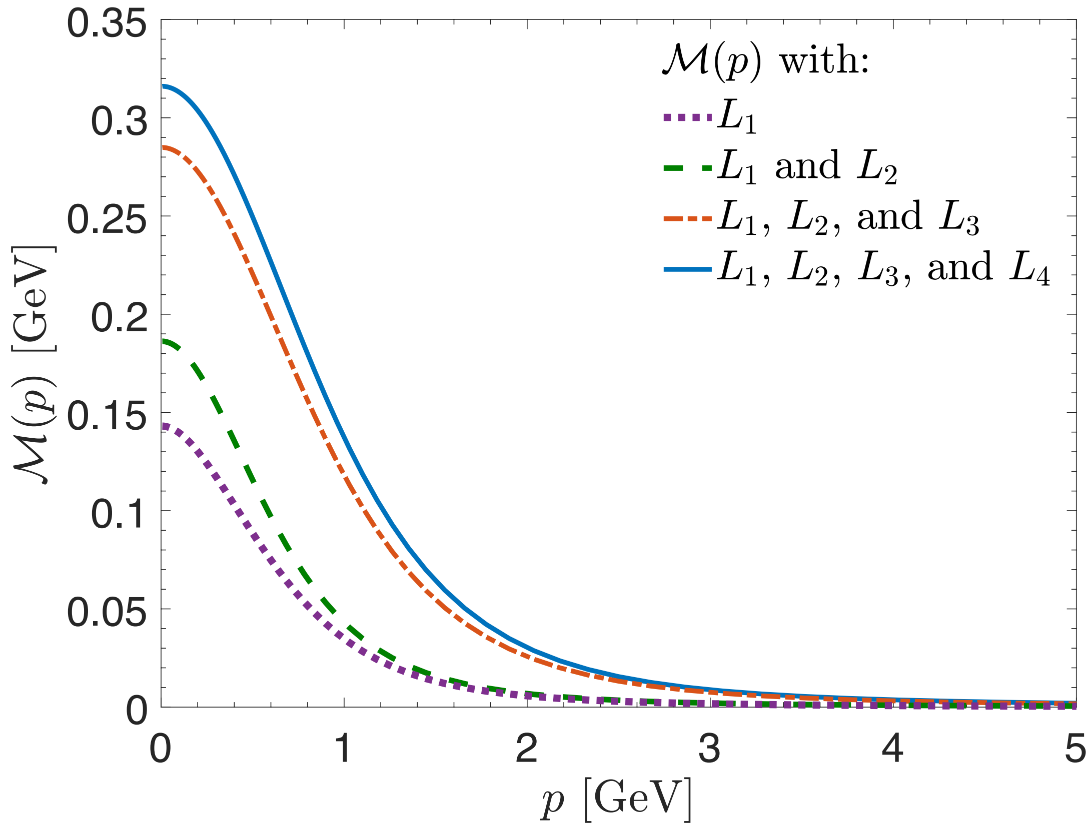
$$p^2 = k^2 = q^2 = r^2$$



# Impact of the individual form factors on the quark mass

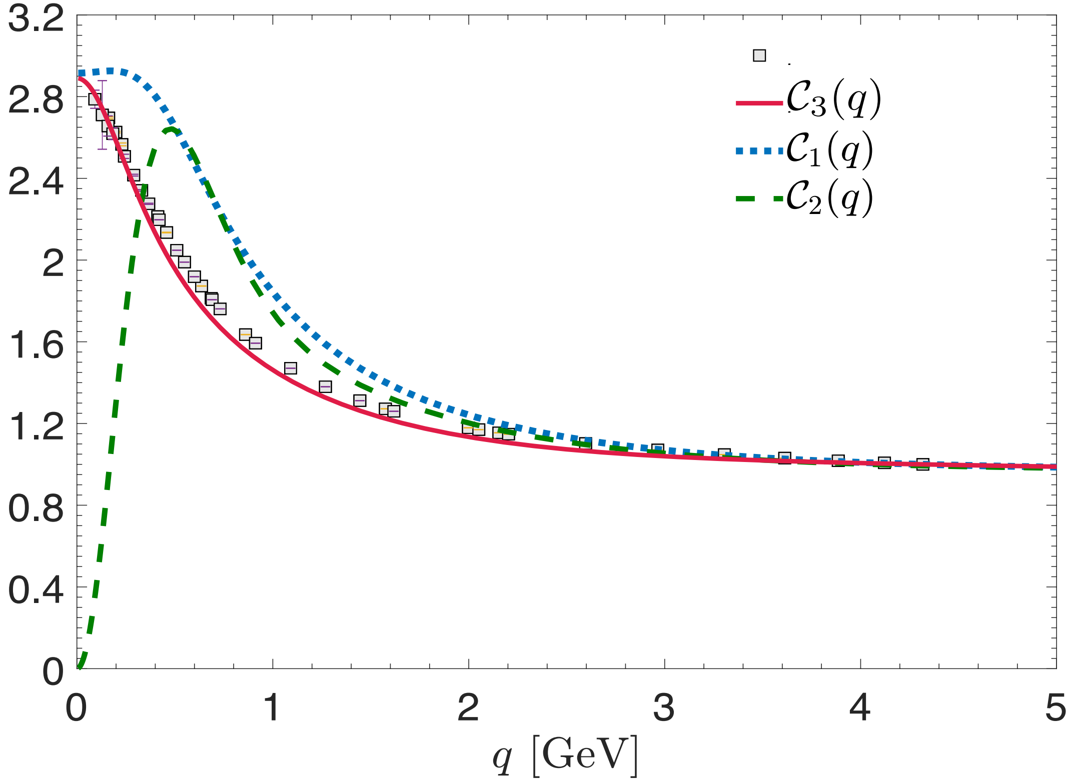
- When we turn on one by one the form factors




| Form Factor | % of the mass generated |
|---|---|
| $L_1$ | 54% |
| $L_2$ | 13% |
| $L_3$ | 23% |
| $L_4$ | 10% |

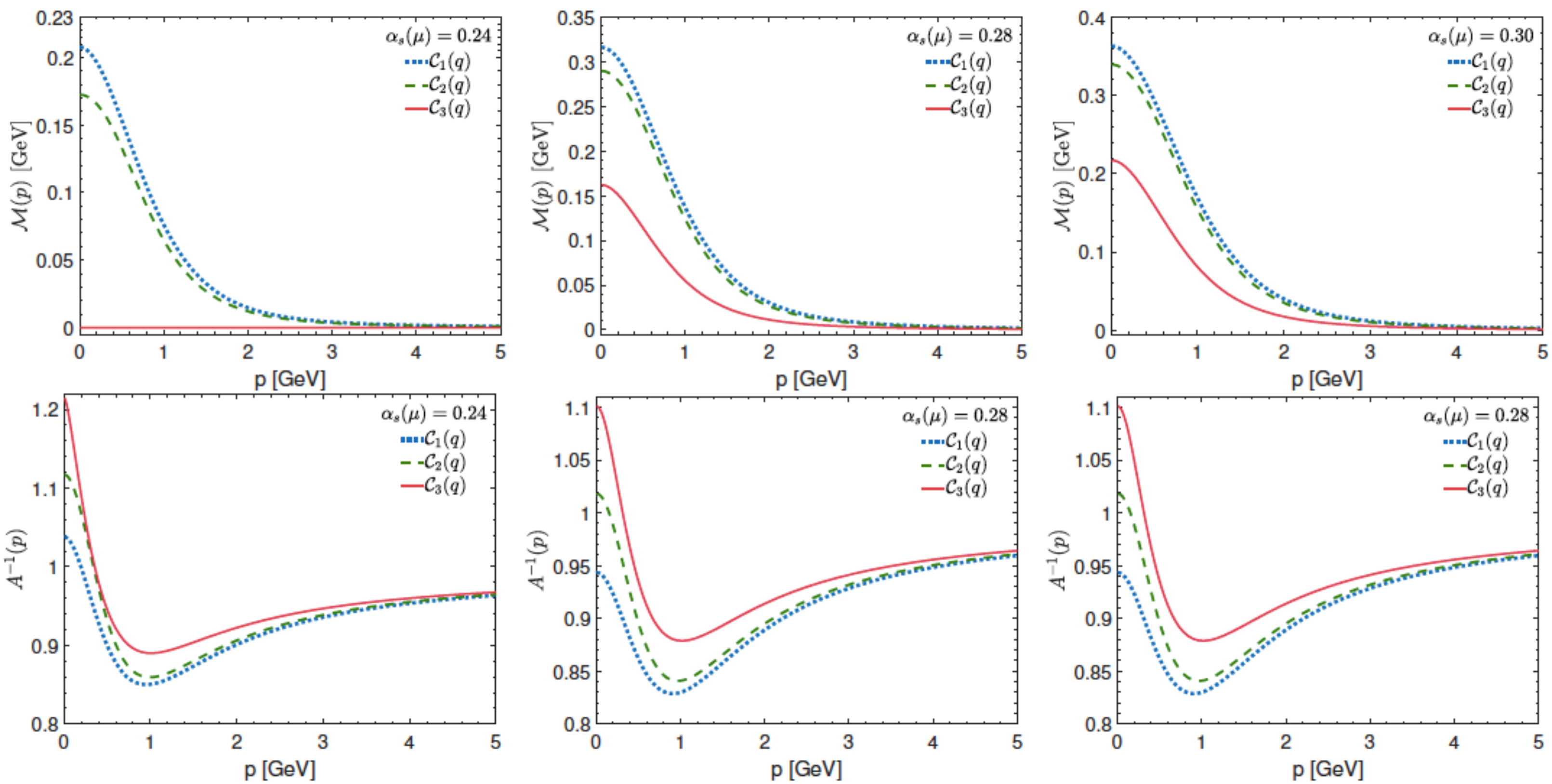
◎ $L_4$ is usually neglected, but it impact is of the order of the $L_2$

# The influence of $\mathcal{C}_i(q)$




$C_2(q)$ is more suppressed in the deep IR compared to $C_1(q)$ and $C_3(q)$.
$C_3(q)$ is more suppressed than $C_1(q)$ and $C_2(q)$ in range of 500 MeV -2 GeV

# The influence of $\mathcal{C}_i(q)$




- Either $C_3(q)$ does not provide sufficient strength to the kernel to trigger the onset of the dynamical mass generation or the values of masses are phenomenologically disfavored.

- $C_2(q)$ is more suppressed in the deep IR compared to $C_1(q)$ and $C_3(q)$, but the first two models generate quark masses of comparable size.

# *Fits for the dynamical quark mass*

- The running quark mass can be fitted by the physically motivated fit

$$\mathcal{M}(p) = \frac{\mathcal{M}_1^3}{\mathcal{M}_2^2 + p^2 \left[\ln(p^2 + \mathcal{M}_3^2)/\Lambda^2\right]^{1-\gamma_f}}$$

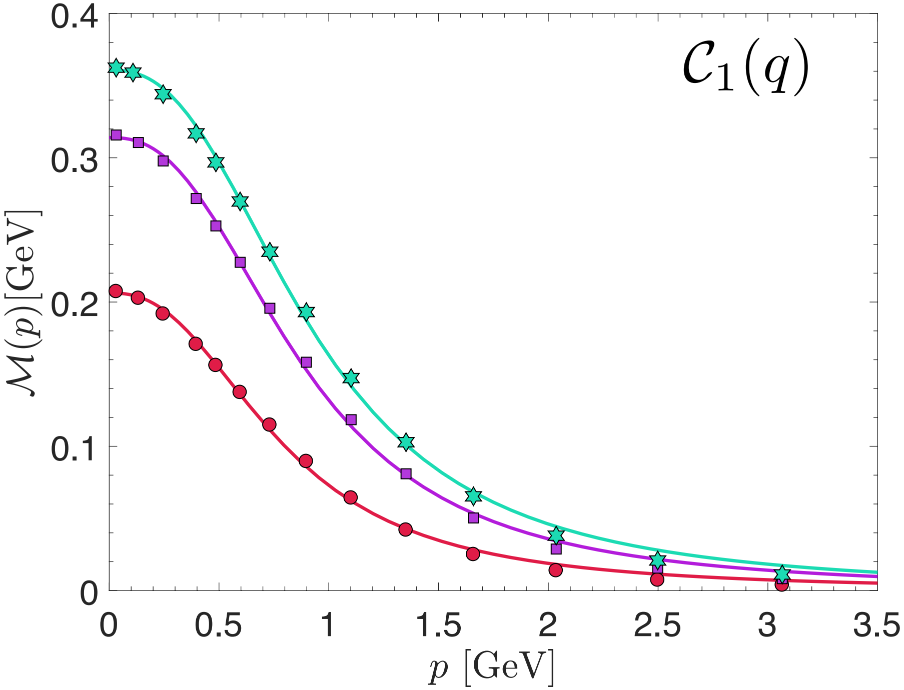
where $(\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3)$ are adjustable parameters.

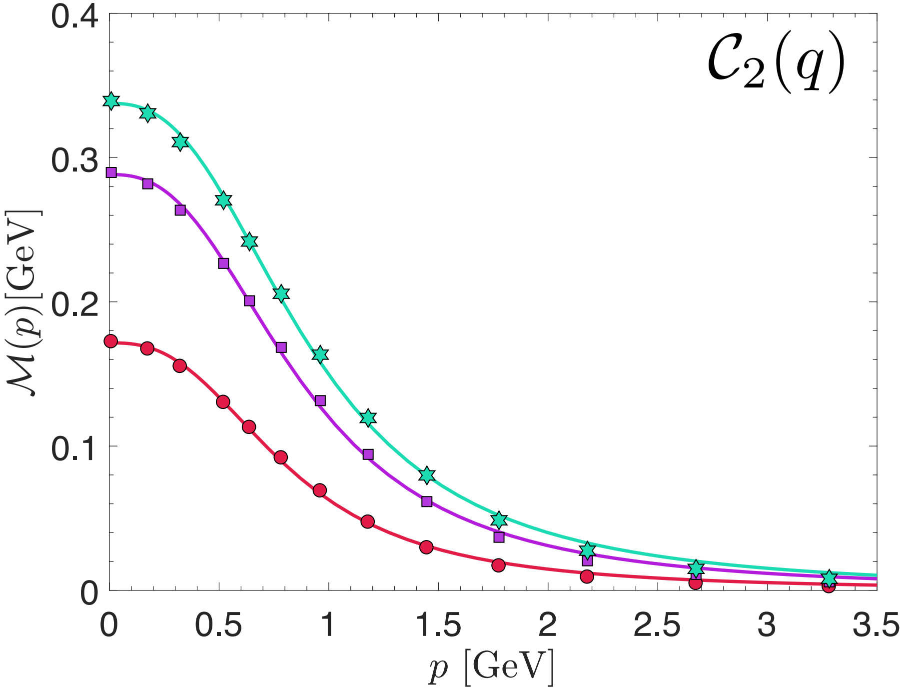
It is the IR completion of the UV power law behavior.

- Other possibility is

$$\mathcal{M}(p) = \frac{\mathcal{M}_0}{1 + (p^2/\lambda^2)^{1+d}} ,$$

# Fits for the dynamical quark mass



# Pion decay constant

- To appreciate the impact of H on a physical observable sensitive to the dynamical quark mass→ pion decay constant.
- Improved version of the Pagels-Stokar-Cornwall formula

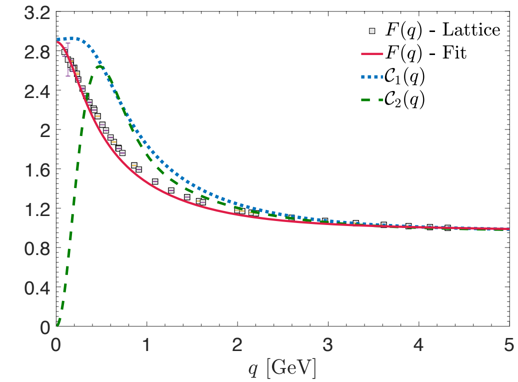
**H. Pagels and S. Stokar**, Phys. Rev. D20, 2947 (1979).
**J. M. Cornwall**, Phys. Rev. D22, 1452 (1980).
**C. D. Roberts**, Nucl. Phys. A605, 475 (1996).

$$f_\pi^2 = \frac{3}{8\pi^2}\int_0^\infty dy\, y B^2(y)\left\{\sigma_V^2 - 2\left[\sigma_S\sigma_S' + y\sigma_V\sigma_V'\right] - y\left[\sigma_S\sigma_S'' - (\sigma_S')^2\right] - y^2\left[\sigma_V\sigma_V'' - (\sigma_V')^2\right]\right\},$$

$$\sigma_V := \frac{A(y)}{yA^2(y)+B^2(y)},$$

$$\sigma_S := \frac{B(y)}{yA^2(y)+B^2(y)}$$

# Values for $f_\pi$




- It should be compared $f_\pi^{exp} = 93\,\mathrm{MeV}$

| | $f_\pi$ with $\mathcal{C}_1(q)$ | | $f_\pi$ with $\mathcal{C}_2(q)$ | | $f_\pi$ with $\mathcal{C}_3(q)$ | |
|---|---|---|---|---|---|---|
| $\alpha_s$ | $\Gamma_\mu^{\mathrm{FBC}}$ | $\Gamma_\mu^{\mathrm{STI}}$ | $\Gamma_\mu^{\mathrm{FBC}}$ | $\Gamma_\mu^{\mathrm{STI}}$ | $\Gamma_\mu^{\mathrm{FBC}}$ | $\Gamma_\mu^{\mathrm{STI}}$ |
| 0.24 | 62 | 73 | 52 | 67 | 0 | 0 |
| 0.28 | 87 | 97 | 83 | 93 | 40 | 61 |
| 0.30 | 97 | 107 | 93 | 103 | 57 | 75 |

- When phenomenological compatible quark masses are generated, the inclusion of H amounts to a 10% increase in the value of $f_\pi$.

# Conclusions

- ✓ **CSB with realistic results (masses of the order 300-350 MeV)** can be obtained from the study of the gap equation, supplemented by:
  - ✧ **The complete longitudinal non-Abelian quark-gluon vertex** ( with the quark-ghost scattering kernel).

- ✓ The **quark-ghost scattering kernel** is responsible for an increase of almost **20% of the dynamical quark mass**.

- ✓ The longitudinal **quark-gluon form factors are all finite and they display a sizable difference when compared to the case were H is turned off (tree-level H=1)**

- ✓ **$L_4$ contributes with 10% of the dynamical quark mass** and practically has the same impact as the form factor $L_2$.