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Three concepts of gauge boson mass

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Three different concepts of gauge boson mass appear in the front form of relativistic dynamics of field quanta. Identification of these concepts is presented on examples rooted in the standard model: massive QED, running gluon mass in QCD, and quark binding in mesons and baryons.

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Enumeration of mass concepts:

1. **Lagrangian** formal field theory - massive QED;
2. **Hamiltonian** renormalization - gluon mass running;
3. **Solution** quark binding in mesons and baryons,

hopefully helps in answering: Which type do I consider?

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1. Massive Abelian gauge field

T. Kibble, D. Soper

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_A + \mathcal{L}_{A\phi} - \mathcal{V}_\phi,$$

$$\begin{aligned}
\mathcal{L}_\psi &= \bar{\psi}\left[(i\partial_\mu - gA_\mu)\gamma^\mu - m\right]\psi \ , \\
\mathcal{L}_A &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
\mathcal{L}_{A\phi} &= \left[(i\partial^\mu - g'A^\mu)\phi\right]^\dagger (i\partial_\mu - g'A_\mu)\phi \\
\mathcal{V}_\phi &= -\mu^2\ \phi^\dagger\phi + \frac{\lambda^2}{2}\ (\phi^\dagger\phi)^2
\end{aligned}$$

gauge symmetry $\quad \psi \to e^{igf}\psi \quad A^\mu \to A^\mu - \partial^\mu f \quad \phi \to e^{ig'f}\phi$

Choice of field variables $\phi = \varphi \, e^{ig'\theta}/\sqrt{2}$

Gauge symmetry in terms of ψ, A^μ, φ and θ

$$\mathcal{L}_\psi = \bar{\psi}\left[(i\partial_\mu - gA_\mu)\,\gamma^\mu - m\right]\psi$$

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{A\phi} = \frac{1}{2}\,(\partial^\mu\varphi)^2 + \frac{1}{2}\,g'^2(A^\mu + \partial^\mu\theta)^2\varphi^2$$

$$\mathcal{V}_\phi = \mathcal{V}(\varphi/\sqrt{2})$$

$\psi \to e^{igf}\psi \qquad A^\mu \to A^\mu - \partial^\mu f \qquad \varphi \to \varphi \qquad \theta \to \theta + f$

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Gauge choice $f = -\theta$

$$\psi = e^{ig\theta}\tilde{\psi} \qquad A^\mu = \tilde{A} - \partial^\mu\theta \qquad \varphi = \tilde{\varphi} \qquad \theta = \tilde{\theta} + \theta$$

$$\begin{aligned}
\mathcal{L}_\psi &= \bar{\tilde{\psi}}\left[\left(i\partial_\mu - g\tilde{A}_\mu\right)\gamma^\mu - m\right]\tilde{\psi} \\
\mathcal{L}_A &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} \\
\mathcal{L}_{A\phi} &= \frac{1}{2}\left(\partial^\mu\tilde{\varphi}\right)^2 + \frac{1}{2}g'^2\tilde{A}^{\mu\,2}\tilde{\varphi}^2 \\
\mathcal{V}_\phi &= \mathcal{V}(\tilde{\varphi}/\sqrt{2})
\end{aligned}$$

massive limit

$$\tilde{\varphi} = \varphi = v + h \qquad v = \sqrt{2}\,\mu/\lambda \;\to\infty \qquad \lambda\to 0 \qquad g'v = \kappa \quad \text{constant}$$

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Massive limit

$$
\begin{aligned}
\mathcal{L} = & \; \bar{\tilde{\psi}} \left[\left(i\partial_\mu - g\tilde{A}_\mu \right) \gamma^\mu - m \right] \tilde{\psi} \\
& + -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\kappa^2 \tilde{A}^{\mu\,2} \\
& + \frac{1}{2}(\partial^\mu h)^2 - \frac{1}{2}(\sqrt{2}\,\mu)^2 h^2 \qquad\qquad - \frac{\mu^2}{2\lambda^2}
\end{aligned}
$$

current upper limit on the photon mass, or κ, is 10^{-18} eV/c^2 [PDG]

no earthly data on h

astrophysical data stimulate searches for the DM and DE

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Gauge choice $\tilde{A}^+ = 0$ Dirac's front form, IMF $\leftrightarrow$ CMS, vacuum

$$\partial^+ f = A^+$$

$$f(x) = \frac{1}{4}\left(\int_{-\infty}^{x^-} - \int_{x^-}^{\infty}\right) dy^- A^+(x^+, y^-, x^\perp)$$

$$\mathcal{L}_\psi = \bar{\tilde{\psi}}\left[\left(i\partial_\mu - g\tilde{A}_\mu\right)\gamma^\mu - m\right]\tilde{\psi}$$

$$\mathcal{L}_A = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\mathcal{L}_{A\phi} = \frac{1}{2}(\partial^\mu\tilde{\varphi})^2 + \frac{1}{2}g'^2(\tilde{A}^\mu + \partial^\mu\tilde{\theta})^2\tilde{\varphi}^2$$

$$\mathcal{V}_\phi = \mathcal{V}[\tilde{\varphi}/\sqrt{2}]$$

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Massive limit in gauge $\tilde{A}^+ = 0$

$$\mathcal{L}_\psi = \bar{\tilde{\psi}} \left[\left(i\partial_\mu - g\tilde{A}_\mu \right) \gamma^\mu - m \right] \tilde{\psi}$$

$$\mathcal{L}_A = -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\mathcal{L}_{A\phi} = \frac{1}{2} (\partial^\mu \tilde{h})^2 + \frac{1}{2} \kappa^2 (\tilde{A}^\mu - \kappa^{-1} \partial^\mu B)^2$$

$$\mathcal{V}_\phi = -\frac{\mu^2}{2\lambda^2} + \frac{1}{2} (\sqrt{2}\, \mu)^2 \, \tilde{h}^2$$

$$\tilde{B} = -\kappa \, \tilde{\theta}$$

Field equations

$$\left[\left(i\partial_\mu - g\tilde{A}_\mu\right)\gamma^\mu - m\right]\tilde{\psi} = 0$$

$$\Box\tilde{A}^\beta - \partial^\beta\partial_\alpha\tilde{A}^\alpha = g\bar{\tilde{\psi}}\gamma^\beta\tilde{\psi} - \kappa^2\left(\tilde{A}^\beta - \kappa^{-1}\partial^\beta\tilde{B}\right)$$

$$\kappa^2\,\partial_\mu(\tilde{A}^\mu - \kappa^{-1}\partial^\mu\tilde{B}) = 0$$

$$(\Box + 2\mu^2)\tilde{h} = 0$$

the front form $\quad x^\pm = x^0 \pm x^3 \quad \partial^\pm = 2\frac{\partial}{\partial x^\mp} \quad x^\perp = (x^1, x^2)$

constraint on $\tilde{A}^-$

$$-\partial^+\partial_\alpha\tilde{A}^\alpha = -\partial^+\left(\frac{1}{2}\,\partial^+\tilde{A}^- - \partial^\perp\tilde{A}^\perp\right) = g\bar{\tilde{\psi}}\gamma^+\tilde{\psi} + \kappa\partial^+\tilde{B}$$

Three polarizations return to gauge $\hat{B} = 0$

$A^\mu = \tilde{A}^\mu - \kappa^{-1} \partial^\mu \tilde{B}$

massive limit with fermion charge $g = 0$ (free fields)

$$\tilde{A}^- = \frac{2}{\partial^+} \partial^\perp \tilde{A}^\perp - \frac{1}{\partial^+} 2\kappa \tilde{B}$$

$$\hat{A}^\perp_k = \frac{k^\perp}{\kappa} \, iB_k \ , \quad \hat{A}^+_k = \frac{k^+}{\kappa} \, iB_k \ , \quad \hat{A}^-_k = -\frac{2\kappa}{k^+} \, iB_k + \frac{k^{\perp\, 2} + \kappa^2}{\kappa k^+} \, iB_k$$

$$\varepsilon_k = \left(\frac{k^{\perp\, 2} - \kappa^2}{\kappa k^+}, \ \frac{k^+}{\kappa}, \ \frac{k^\perp}{\kappa} \right) = \frac{k}{\kappa} - \eta \frac{\kappa}{k^+}$$

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FF Hamiltonian gauge $A^+ = 0$ $A^\perp$ and B

$$T^{\mu\nu} = \sum_i \frac{\partial \mathcal{L}}{\partial \partial_\mu f_i} \, \partial^\nu f_i - g^{\mu\nu} \mathcal{L}$$

$$P^- = \frac{1}{2} \int d^2x^\perp dx^- \, T^{+\,-} = \int d^2x^\perp dx^- \, \mathcal{H}$$

$$\begin{aligned} \mathcal{H} = & \frac{1}{2} \bar{\psi} \gamma^+ \frac{-\partial^{\perp\,2} + m^2}{i\partial^+} \psi + \frac{1}{2} A^\perp (-\partial^{\perp\,2} + \kappa^2) A^\perp + \frac{1}{2} B(-\partial^{\perp\,2} + \kappa^2) B \\ & + g \bar{\psi} \not{A} \psi + \frac{1}{2} g^2 \bar{\psi} \gamma^+ \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ \psi + g^2 \bar{\psi} \not{A} \frac{\gamma^+}{2i\partial^+} \not{A} \psi \\ & + g \bar{\psi} \gamma^+ \psi \frac{1}{i\partial^+} (-i\kappa B) + \frac{1}{2} \, h(-\partial^{\perp\,2} + 2\mu^2) h - \frac{\mu^4}{2\lambda^2} \end{aligned}$$

Massive gauge boson: third polarization plus scalar h **SINGULAR** $\mathcal{H}$

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2. Gluon mass term in $\mathcal{H}_{QCD}$ (long story made very short)

Wilsonian RG insufficient for Hamiltonians, Wegner's equation etc.

$\rightarrow$ renormalization group procedure for effective particles (RGPEP)

lowest-order solution (2nd) for the gluon mass term μ^2

μ^2 is a function of the gluon size s $\qquad t = s^4$

$$\mathcal{H} = \mathcal{H}_f + \mathcal{H}_I$$

$$a_t = \mathcal{U}_t \, a_0 \, \mathcal{U}_t^\dagger \qquad \rightarrow \qquad \frac{d}{dt}\mathcal{H} = \left[[\mathcal{H}_f, \tilde{\mathcal{H}}], \mathcal{H}\right]$$

initial condition at $t = 0$ is the FF canonical $\mathcal{H}_{QCD}$ with counterterms

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Initial condition = FF Hamiltonian for point-like gluons

$$\mathcal{L} = -\frac{1}{2} \mathrm{tr}\, F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{T}^{\mu\nu} = -F^{a\mu\alpha} \partial^\nu A^a_\alpha + g^{\mu\nu} F^{a\alpha\beta} F^a_{\alpha\beta}/4$$

$$A^+ = 0 \qquad\qquad A^- = \frac{1}{\partial^+} 2\, \partial^\perp A^\perp - \frac{2}{\partial^{+\,2}}\, ig\, [\partial^+ A^\perp, A^\perp]$$

$$\mathcal{H} = \frac{1}{2} \int dx^- d^2 x^\perp \; \mathcal{T}^{+\,-}|_{x^+=0}$$

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$$\mathcal{T}^{+-} = \mathcal{H}_{A^2} + \mathcal{H}_{A^3} + \mathcal{H}_{A^4} + \mathcal{H}_{[\partial AA]^2}$$

$$\mathcal{H}_{A^2} = -\frac{1}{2} A^{\perp a} (\partial^\perp)^2 A^{\perp a}$$

$$\mathcal{H}_{A^3} = g\, i\partial_\alpha A^a_\beta [A^\alpha, A^\beta]^a$$

$$\mathcal{H}_{A^4} = -\frac{1}{4} g^2 \, [A_\alpha, A_\beta]^a [A^\alpha, A^\beta]^a$$

$$\mathcal{H}_{[\partial AA]^2} = \frac{1}{2} g^2 \, [i\partial^+ A^\perp, A^\perp]^a \frac{1}{(i\partial^+)^2} [i\partial^+ A^\perp, A^\perp]^a$$

quantum theory

$$A^\mu \to \hat{A}^\mu = \sum_{\sigma c} \int [k] \left[t^c \varepsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \varepsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx} \right]_{x^+=0}$$

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Regularization set in $\mathcal{H} = \mathcal{H}_f + \mathcal{H}_I$ at $t = 0$

$$\mathcal{H}_f = \frac{1}{2} \int dx^- d^2x^\perp \; : \mathcal{H}_{A^2}(\hat{A}) :$$

$$\mathcal{H}_I = \frac{1}{2} \int dx^- d^2x^\perp \; : \mathcal{H}_{A^3}(\hat{A}) + \mathcal{H}_{A^4}(\hat{A}) + \mathcal{H}_{[\partial A A]^2}(\hat{A}) :$$

every $a^\dagger$ and a in every term of $\mathcal{H}_I$ is given a factor $r_{\Delta\delta}(\kappa^\perp, x)$

$k =$ gluon momentum $\qquad p =$ total momentum in the term

$x = k^+/p^+ \qquad \kappa^\perp = k^\perp - xp^\perp$

for example $\quad r_{\Delta\delta}(\kappa^\perp, x) = x^\delta \theta(x - \epsilon) e^{-(\kappa^\perp/\Delta)^2/x}$

Calculation of the effective gluon mass term $\quad s \ll 1/\Lambda_{QCD}$

$$\mathcal{H}_f = \sum_{\sigma c} \int [k] \, \frac{k^{\perp 2}}{k^+} \, a^\dagger_{k\sigma c} a_{k\sigma c}$$

$$\mathcal{H}_I = g(Y + Y^\dagger) + g^2 \mathcal{H}_\mu + O(g^2)$$

$$Y = \sum_{123} \int [123] \, \tilde{\delta}_{12.3} \, y_{123} \, a^\dagger_1 a^\dagger_2 a_3 \qquad\qquad Y \to \tilde{Y} \qquad y_{123} \to p_3^{+\,2} y_{123}$$

$$\mathcal{H}_\mu = \sum_{\sigma c} \int [k] \, \frac{\mu^2}{k^+} \, a^\dagger_{k\sigma c} a_{k\sigma c}$$

expand $\quad \dfrac{d}{dt}\mathcal{H} = \left[[\mathcal{H}_f, \tilde{\mathcal{H}}], \mathcal{H} \right] \quad$ in powers of g

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solution up to 2nd order in powers f g

$$\frac{d}{dt}\mathcal{H} = \left[[\mathcal{H}_f, \tilde{\mathcal{H}}], \mathcal{H}\right]$$

$$\frac{d}{dt}g(Y + Y^\dagger) + \frac{d}{dt}g^2\mathcal{H}_\mu + O(g^2)$$
$$= \left[[\mathcal{H}_f, g(\tilde{Y} + \tilde{Y}^\dagger) + O(g^2)], \mathcal{H}_f + g(Y + Y^\dagger) + g^2\mathcal{H}_\mu + O(g^2)\right]$$

first order term

$$\frac{d}{dt}(Y + Y^\dagger) = \left[[\mathcal{H}_f, \tilde{Y} + \tilde{Y}^\dagger], \mathcal{H}_f\right]$$
$$\frac{d}{dt}\, y_{123} = -\mathcal{M}_{12}^4 \, y_{123}$$

$\mathcal{M}_{12}$ is the invariant mass of two intermediate massless gluons

$$y_{123\,t} = e^{-t\ \mathcal{M}_{12}^4}\ y_{123\,0}$$

the initial condition from canonical $\mathcal{H}_{QCD}$

$$y_{123\,0} = \ i f^{c_1 c_2 c_3} \left(\varepsilon_1^* \varepsilon_2^* \cdot \varepsilon_3 \kappa - \varepsilon_1^* \varepsilon_3 \cdot \varepsilon_2^* \kappa \frac{1}{x_{2/3}} - \varepsilon_2^* \varepsilon_3 \cdot \varepsilon_1^* \kappa \frac{1}{x_{1/3}} \right)$$

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second order term - gluon mass squared

$$\frac{d}{dt}\mathcal{H}_\mu = \big[[\mathcal{H}_f, \tilde{Y} + \tilde{Y}^\dagger], Y + Y^\dagger\big]_{\mathcal{H}_f}$$

$$\frac{d}{dt}\mu^2 = 4N_c \int [x\kappa] \left[1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right] \kappa^2 \ \left(-\mathcal{M}_{12}^2\right) \ e^{-2 \ t \ \mathcal{M}_{12}^4}$$

This is negative - when the gluon size s grows, the gluon mass decreases.

including quarks

$\mathcal{M}^2_{g\,12} = \kappa^2/[x(1-x)]$ and $\mathcal{M}^2_{q\,12} = (\kappa^2 + m_q^2)/[x(1-x)]$

$$\mu_t^2 = \mu_0^2 + 2g_0^2 \int [x\kappa]\ N_c \left[x(1-x) + \frac{1-x}{x} + \frac{x}{1-x}\right] \left[e^{-2\ t\ \kappa^4/[x(1-x)]^2} - 1\right]$$

$$+ \sum_q 2g_0^2 \int [x\kappa]\ \frac{\left[x^2 + (1-x)^2\right]\kappa^2 + m_q^2}{\kappa^2 + m_q^2} \left[e^{-2\ t\ (\kappa^2+m_q^2)^2/[x(1-x)]^2} - 1\right]$$

the gluon part is divergent due to small x singularity

FF vacuum, recall $\varphi = v + h$ and $\kappa = g'v$

the quark part is finite no matter how large is m_q^2

μ^2 decreases when t increases because of the RGPEP factor

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3. Gluon mass in solving QCD

SDG, M. Gómez-Rocha, J. More, K. Serafin, Phys. Lett. B **773**, 172 (2017)
K. Serafin, M. Gómez-Rocha, J. More, SDG, arXiv:1805.03436

Kamil Serafin, Thursday 16:30

QCD of one heavy flavor of quarks

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{2}\mathrm{tr}F^{\mu\nu}F_{\mu\nu}$$

RGPEP yields H_t order by order in powers of g

$$m \gg s^{-1} \gg \Lambda_{\mathrm{QCD}}$$

$$H_t = H_{t0} + g_t H_{t1} + g_t^2 H_{t2} + \ldots$$

solution for H_t up to 2nd order

Bound-state eigenvalue problem

$$H_t|\Psi\rangle = E|\Psi\rangle$$

$$|\Psi\rangle = |M\rangle \quad \text{or} \quad |\Psi\rangle = |B\rangle$$

$$|M\rangle = \left|Q_t\bar{Q}_t\right\rangle + \left|Q_t\bar{Q}_t\,G_t\right\rangle + \left|Q_t\bar{Q}_t\,2G_t\right\rangle + \ldots$$

$$|B\rangle = |3Q_t\rangle + |3Q_t\,G_t\rangle + |3Q_t\,2G_t\rangle + \ldots$$

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Mesons:

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & H_{t\,0} + g^2 H_{t2} & g H_{t1} \\ \cdot & g H_{t1} & H_{t\,0} + g^2 H_{t2} \end{bmatrix} \begin{bmatrix} \cdot \\ |Q_t \bar{Q}_t \, G_t\rangle \\ |Q_t \bar{Q}_t\rangle \end{bmatrix} = E_M \begin{bmatrix} \cdot \\ |Q_t \bar{Q}_t \, G_t\rangle \\ |Q_t \bar{Q}_t\rangle \end{bmatrix}$$

Baryons:

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & H_{t\,0} + g^2 H_{t2} & g H_{t1} \\ \cdot & g H_{t1} & H_{t\,0} + g^2 H_{t2} \end{bmatrix} \begin{bmatrix} \cdot \\ |3Q_t \, G_t\rangle \\ |3Q_t\rangle \end{bmatrix} = E_B \begin{bmatrix} \cdot \\ |3Q_t \, G_t\rangle \\ |3Q_t\rangle \end{bmatrix}$$

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Gluon mass and $Q\bar{Q}$ and $3Q$ eigenvalue problems

$$\begin{bmatrix} H_{t\,0} + \mu^2_{M\,t} & gH_{t1} \\ gH_{t1} & H_{t\,0} + g^2 H_{t2} \end{bmatrix} \begin{bmatrix} |Q_t \bar{Q}_t\, G_t\rangle \\ |Q_t \bar{Q}_t\rangle \end{bmatrix} = E_M \begin{bmatrix} |Q_t \bar{Q}_t\, G_t\rangle \\ |Q_t \bar{Q}_t\rangle \end{bmatrix}$$

$$\begin{bmatrix} H_{t\,0} + \mu^2_{B\,t} & gH_{t1} \\ gH_{t1} & H_{t\,0} + g^2 H_{t2} \end{bmatrix} \begin{bmatrix} |3Q_t\, G_t\rangle \\ |3Q_t\rangle \end{bmatrix} = E_B \begin{bmatrix} |3Q_t\, G_t\rangle \\ |3Q_t\rangle \end{bmatrix}$$

.

$$\langle l|H_{\mathrm{eff\,t}}|r\rangle = \langle l| \Big[H_{t0} + g^2 H_{t2} + \frac{1}{2} g H_{t1} \left(\frac{1}{E_l - H_{t0} - \mu_t^2} + \frac{1}{E_r - H_{t0} - \mu_t^2} \right) g H_{t1} \Big] |r\rangle$$

In heavy-quark QCD, the assumption of a gluon mass leads to a harmonic oscillator potential among quarks in quarkonia and baryons, with frequencies that do not depend on the assumed value of the gluon mass.

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Of which type is your gauge boson mass ?

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