Emergent mass and its consequences in the SM ECT*, Trento, 19.09.2018

On the order of the chiral phase transition at zero and small baryon density

Owe Philipsen

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Motivation
- Lattice results at zero density: 1st or 2nd order transition?
- Finite density

HIC for FAIR Helmholtz International Center

NIC

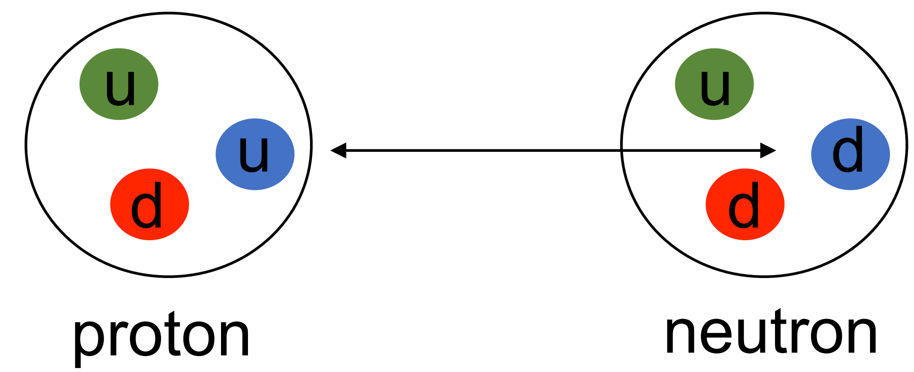
CRC-TR 211 Strong-interaction matter under extreme conditions

Chiral symmetry breaking and restoration

For $m_q = 0$ QCD symmetric under rotations in flavor space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

↑ anomalous



Confinement + spontaneous symmetry breaking: $\sigma = \langle \bar{\psi}\psi \rangle \neq 0$

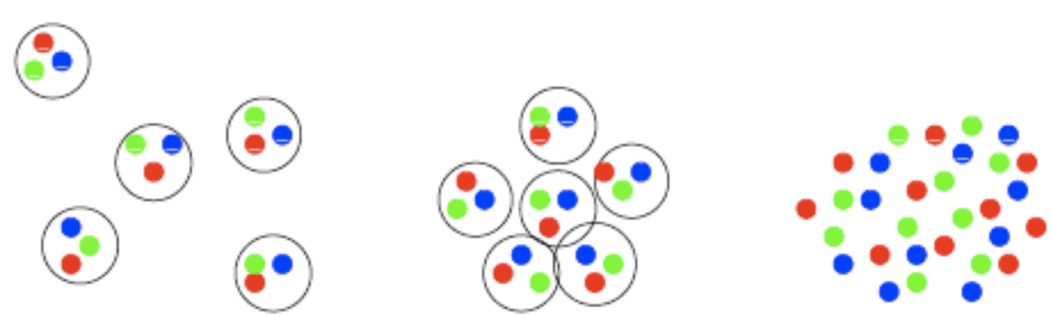
Responsible for visible mass in the Universe

Extreme conditions:

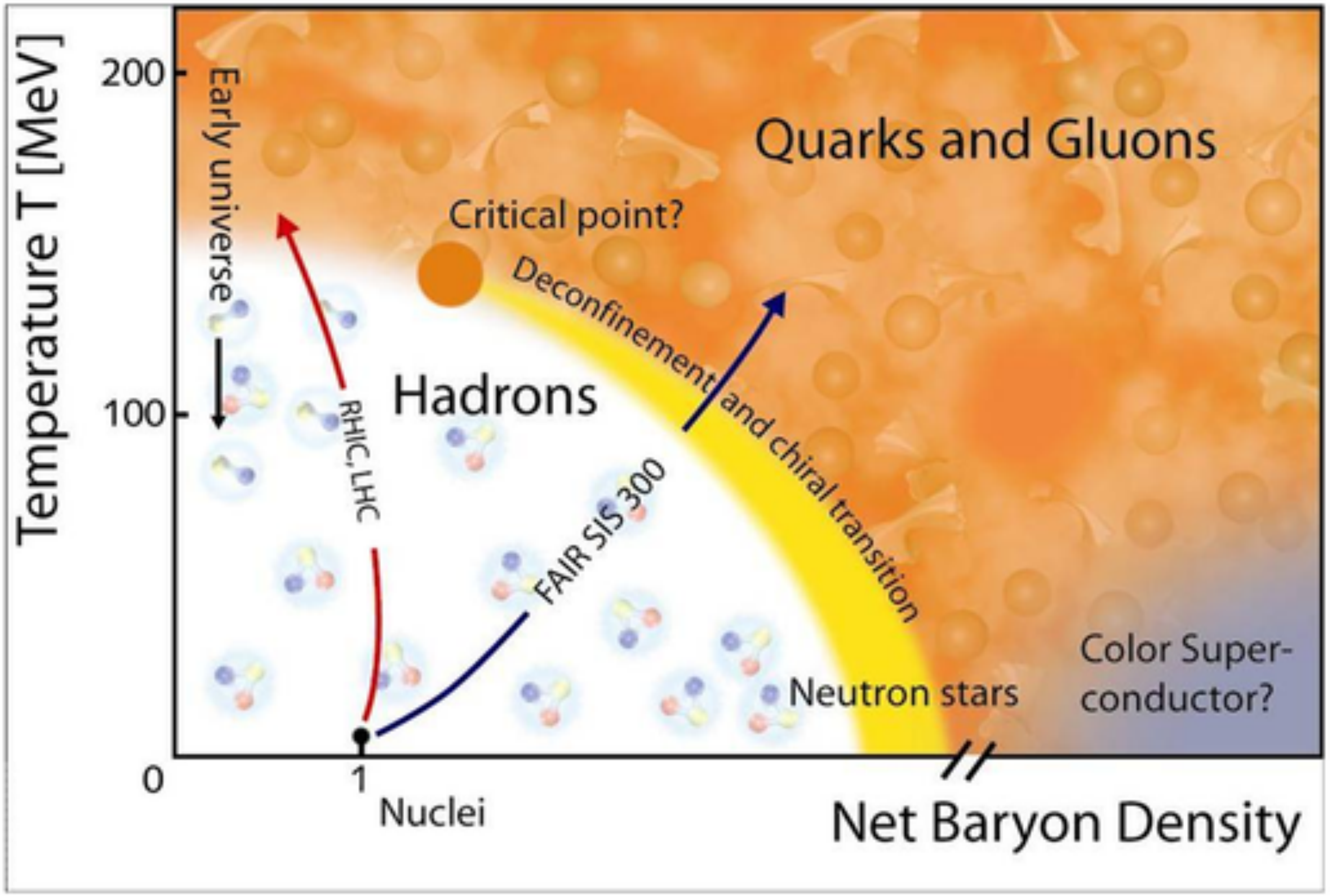
T, μ_B →

Phase transitions?

-Heavy-ion collisions
-Early Universe
-Compact stars



The QCD phase diagram



No Monte Carlo of Lattice QCD: sign problem!

Theory: how to calculate p.t., critical temperature

deconfinement/chiral phase transition $\longrightarrow$ quark gluon plasma

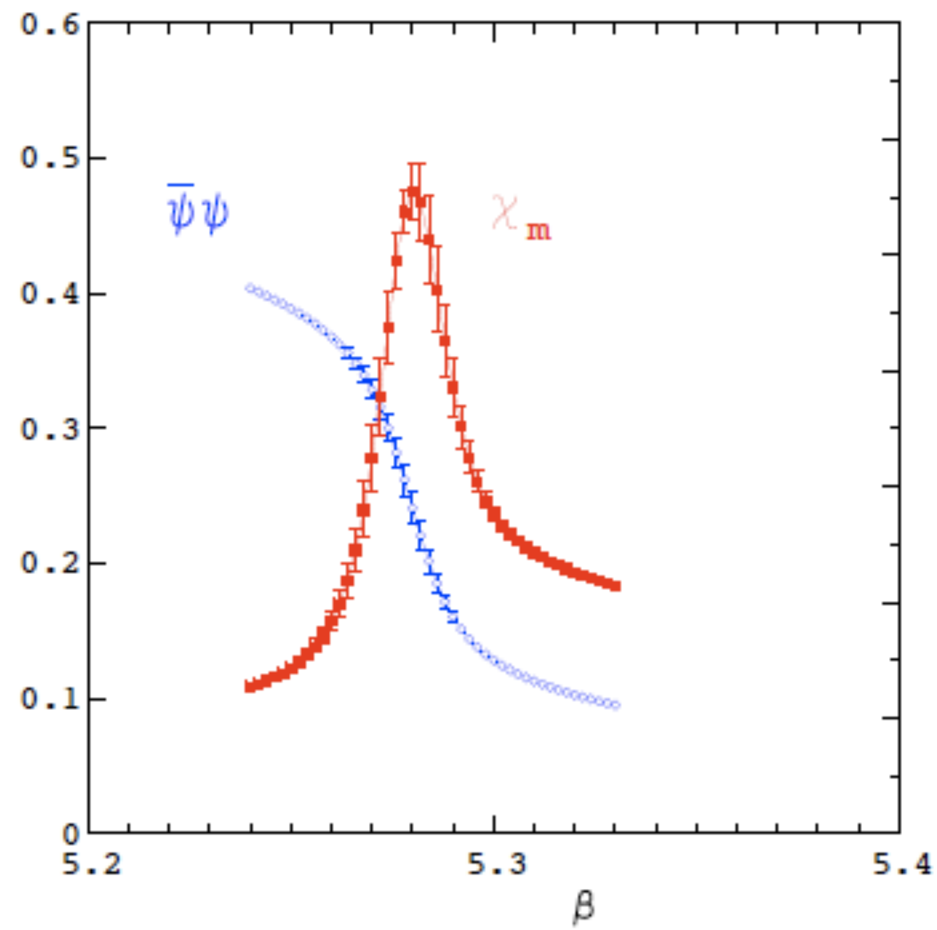
"order parameter":

chiral condensate $\langle\bar{\psi}\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$$\Rightarrow \chi_{max} = \chi(\beta_c) \quad \Rightarrow T_c$$



lattice coupling β, viz. T

only pseudo-critical on finite V!

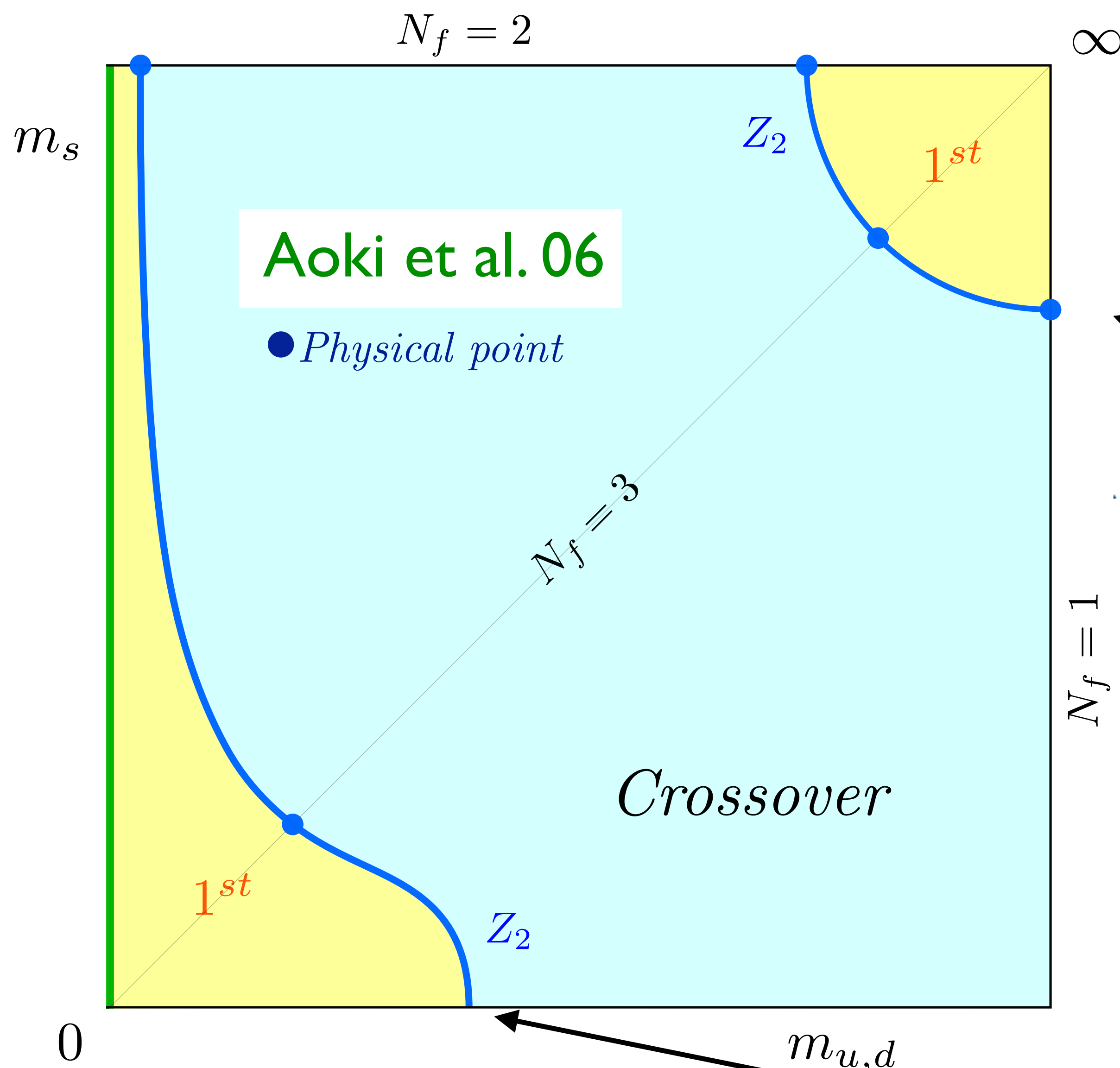
Order of transition:

finite volume scaling

$$\chi_{max} \sim V^\sigma$$

$\sigma = 1$	1st order
$\sigma =$ crit. exponent	2nd order
$\sigma = 0$	crossover

The nature of the QCD thermal transition



Pure gauge + static quarks:

deconfinement p.t.:
breaking of global $Z(3)$ symmetry

deconfinement critical line

chiral critical line

de Forcrand, O.P. 07

$N_t = 4, a \sim 0.3$ fm

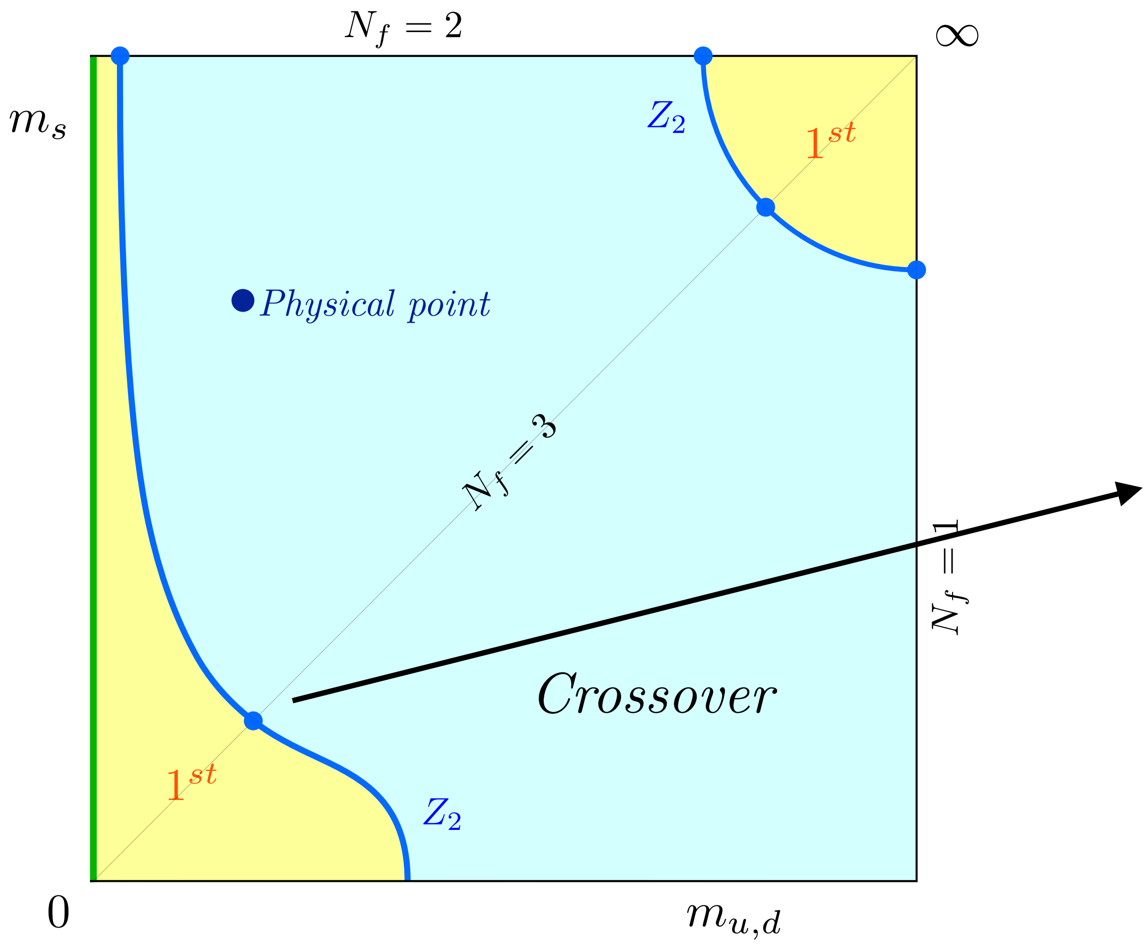
chiral p.t.
restoration of global symmetry in flavour space

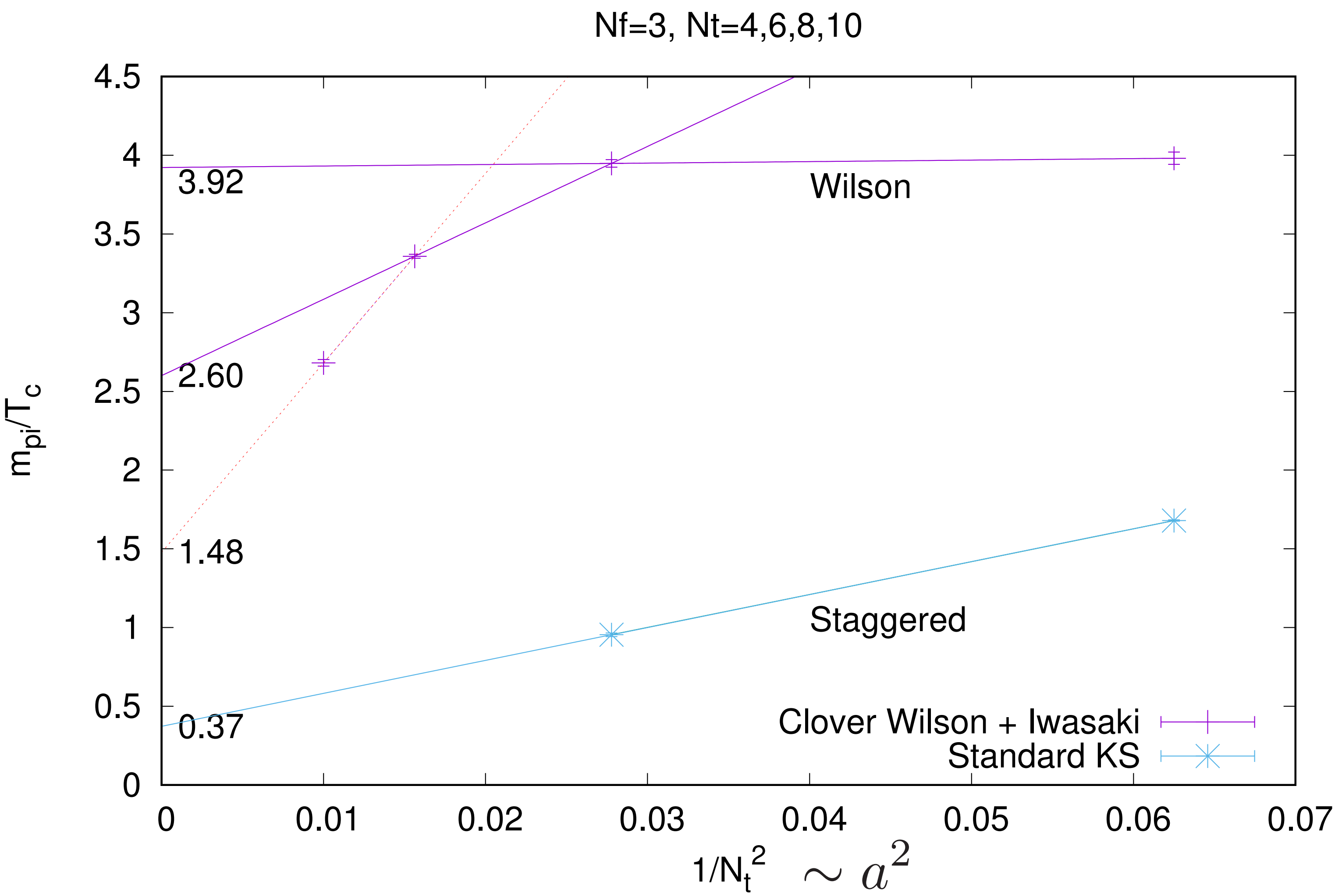
$SU(2)_L \times SU(2)_R \times U(1)_A$

anomalous

The nature of the QCD thermal transition

…has horribly large cut-off effects!

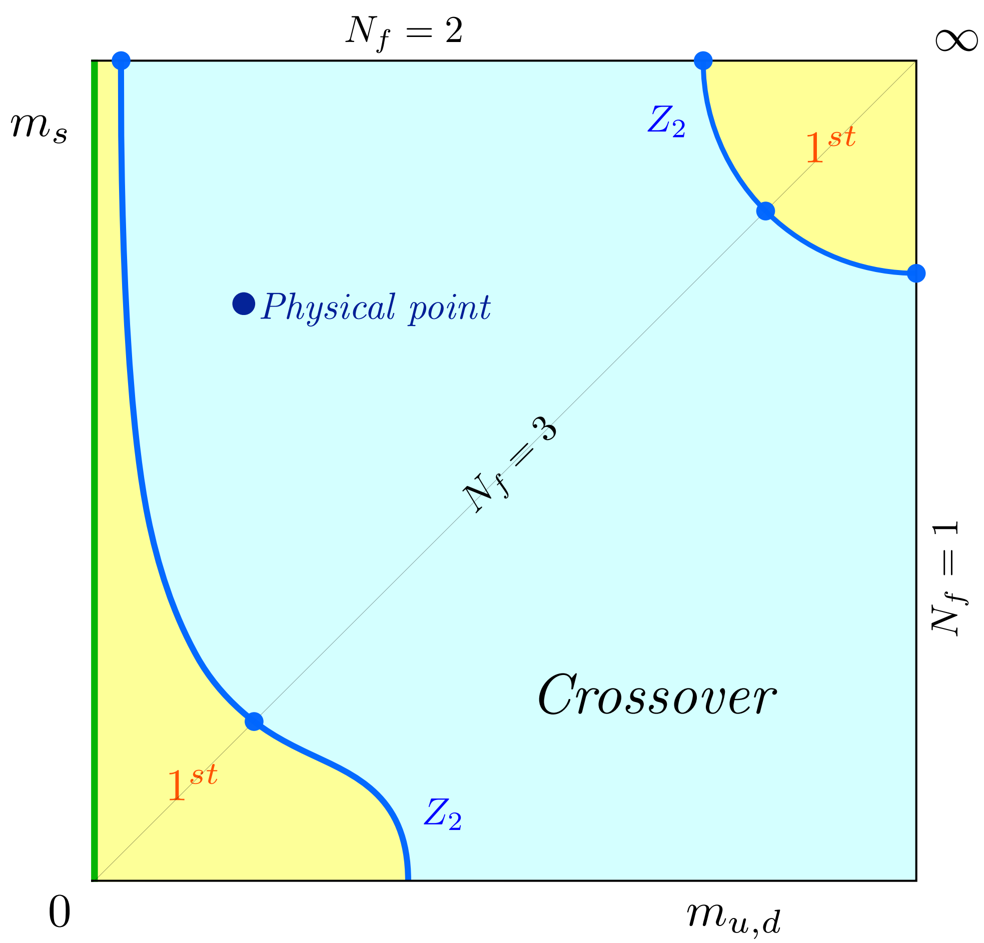




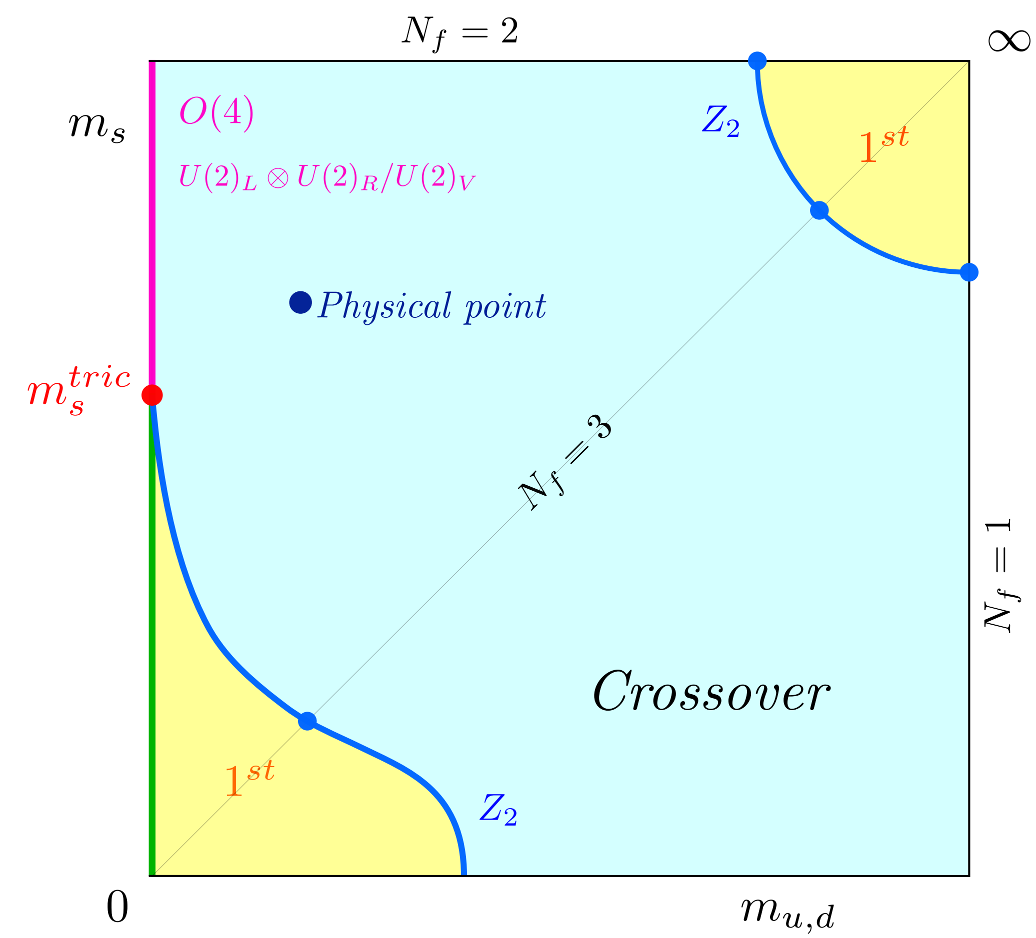
de Forcrand, talk @ LAT2017

The nature of the QCD thermal transition

…is still unknown in the continuum limit



a ⟶ 0?



Pisarski, Wilczek 84:
$U_A(1)$ anomaly?

1st order region seen on coarse lattices but shrinks with decreasing a
Only upper bounds with improved actions

Base for exploration of phase diagram at finite baryon density

The nature of the QCD thermal transition

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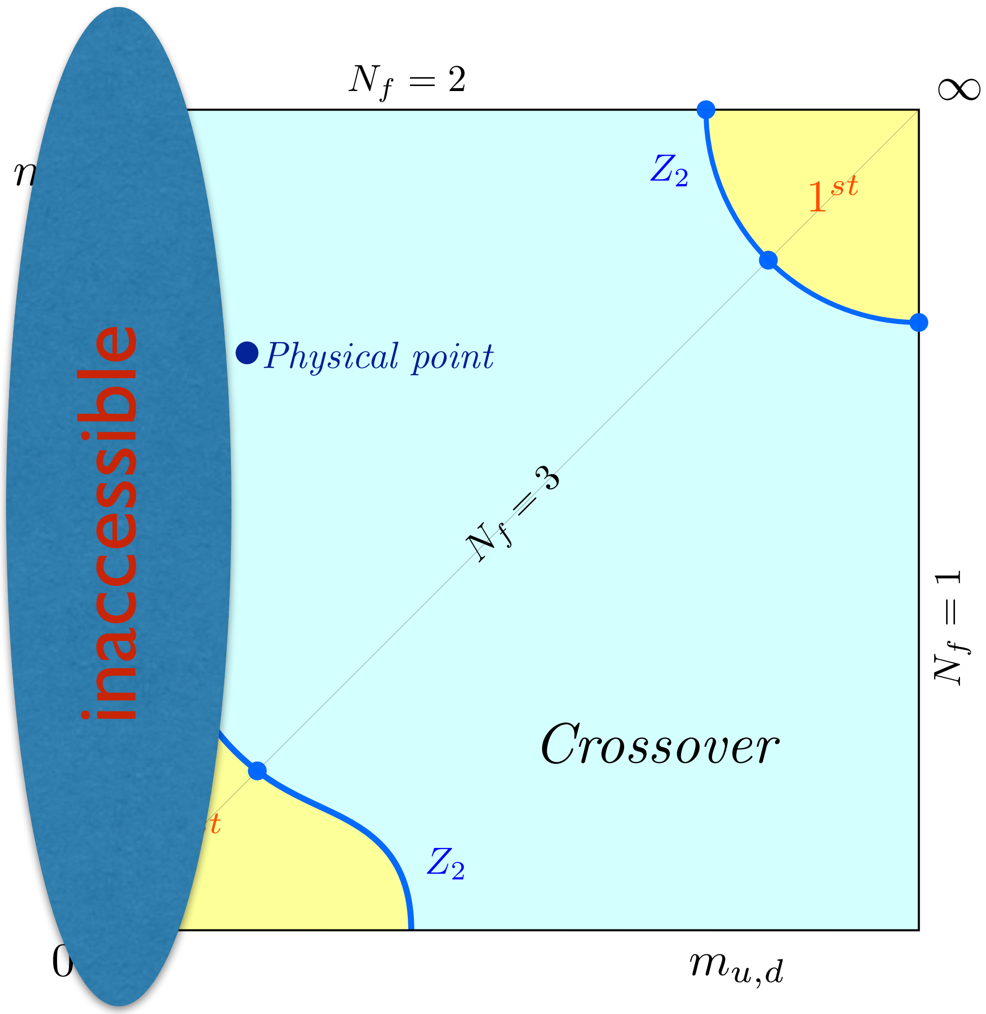

a → 0?

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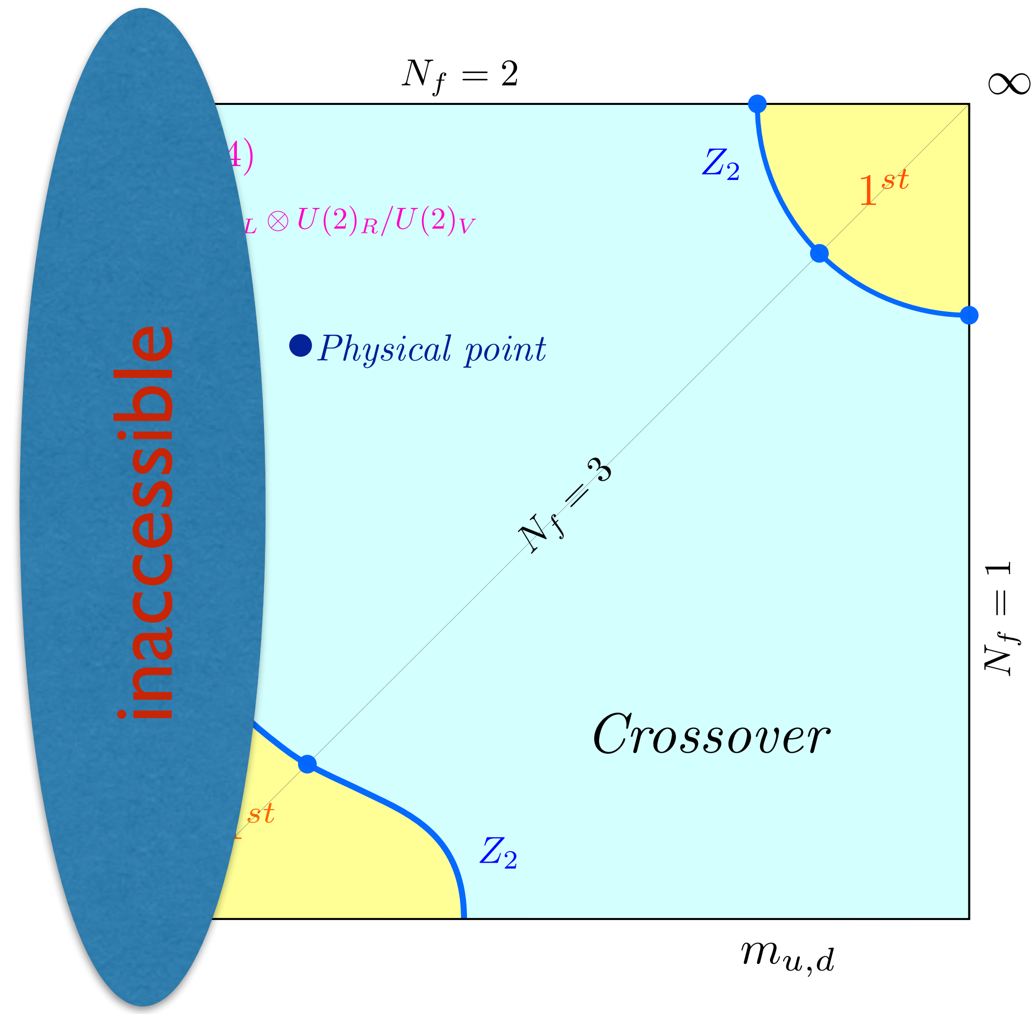


1st order region seen on coarse lattices but shrinks with decreasing a
Only upper bounds with improved actions

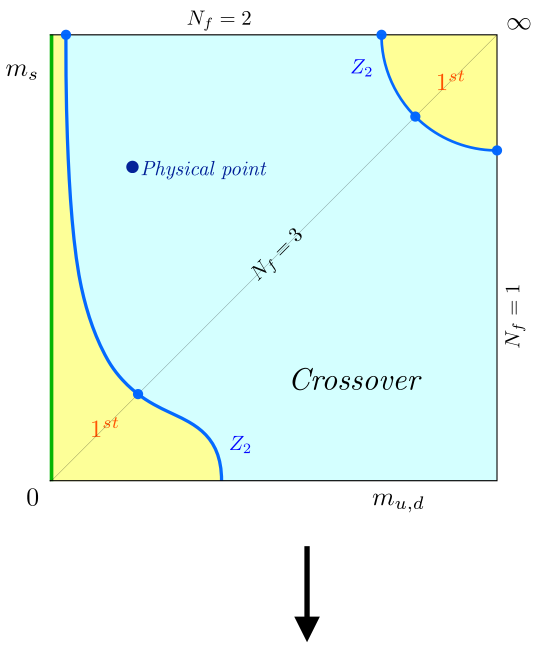
Base for exploration of phase diagram at finite baryon density

Lattice results on the anomaly…

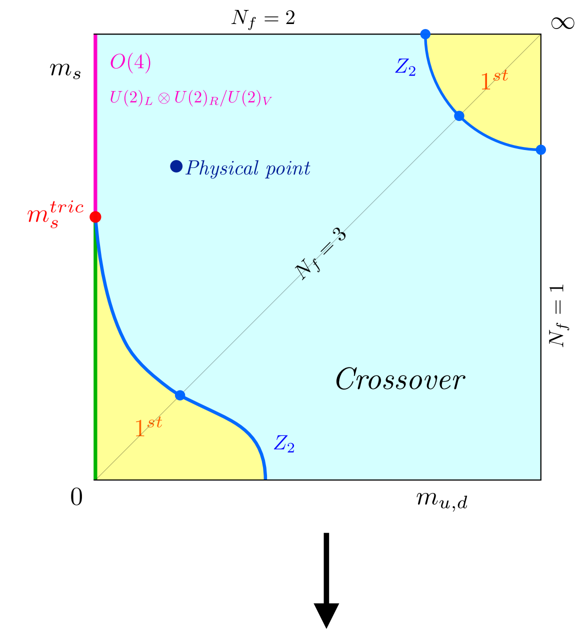
- fate of $U(1)_A$ lattice
  - HotQCD (DW, 2012) broken
  - JLQCD (topology fixed overlap, 2013) restores
  - TWQCD (optimal DW, 2013) restores ?
  - LLNL/RBC (DW, 2014) broken
  - HotQCD (DW, 2014) broken
  - Dick et al. (overlap on HISQ, 2015) broken
  - Brandt et al. (O(a) improved Wilson 2016) restores
  - JLQCD (reweighted overlap from DW, 2016) restores
  - JLQCD (current: see Suzuki et al Lattice 2017) restores
  - Ishikawa et al (Wilson, 2017) at least Z_4 restores

Y. Aoki, talk @ xQCD2018

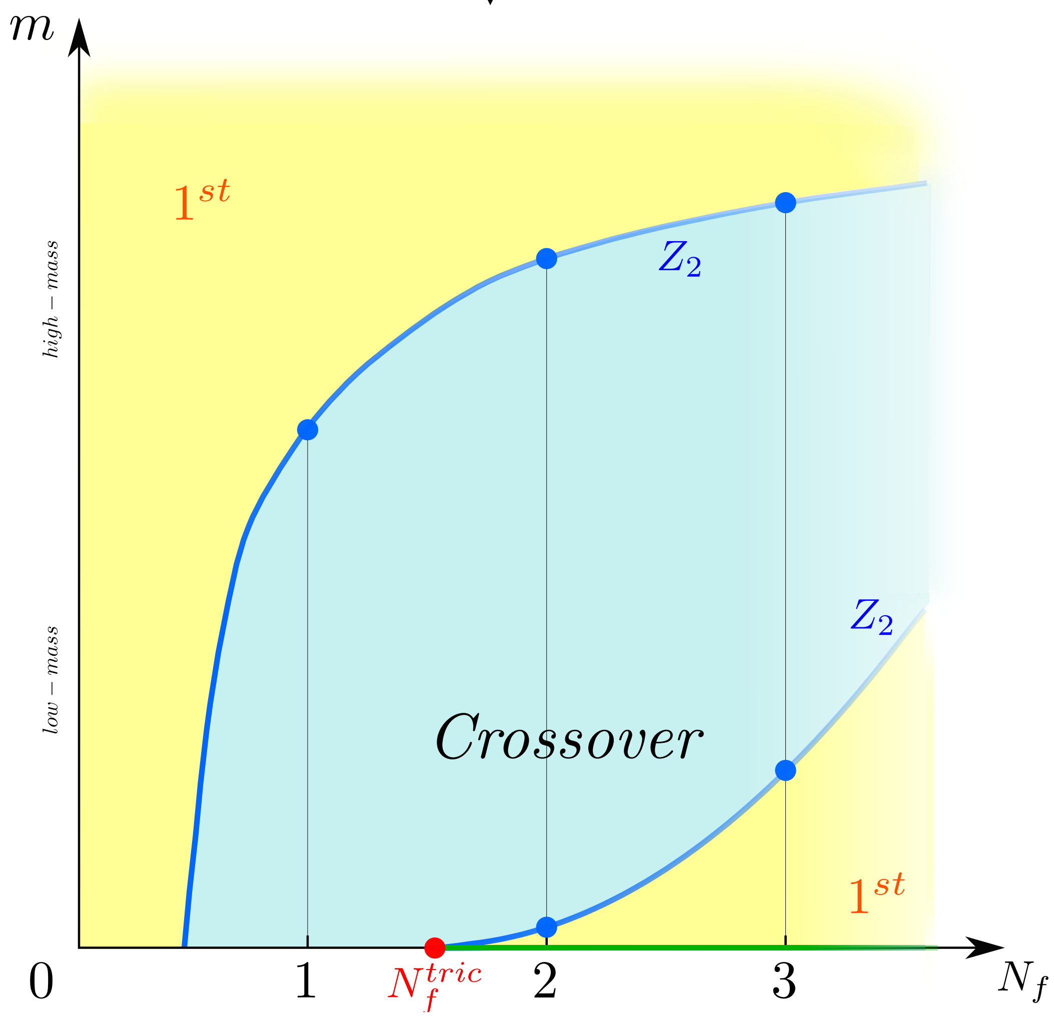
Statistical system with “continuous Nf”



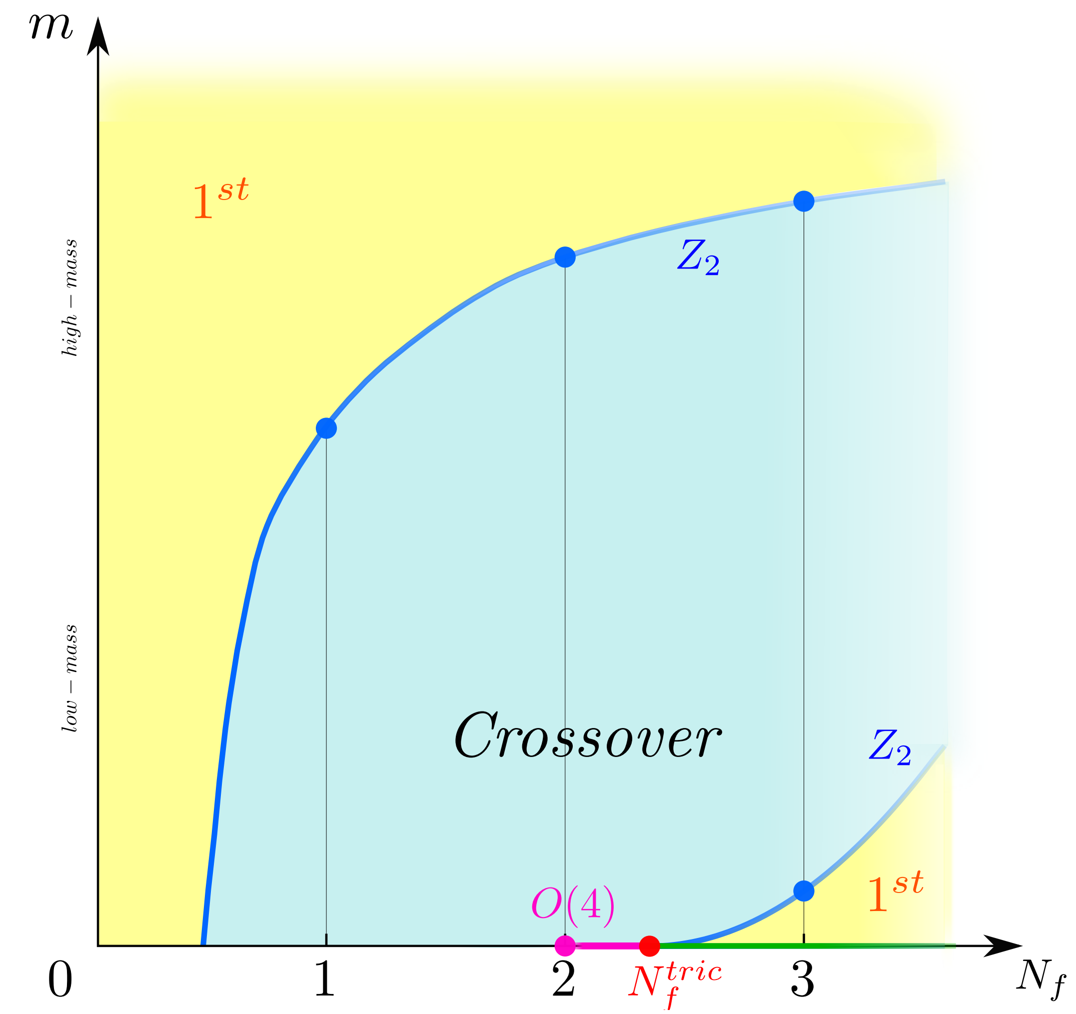
Cuteri, Sciarra, O.P. , 17



$$Z_{N_\mathrm{f}}(m) = \int \mathcal{D}U \, [\det M(U,m)]^{N_\mathrm{f}} \, e^{-\mathcal{S}_\mathrm{G}}$$

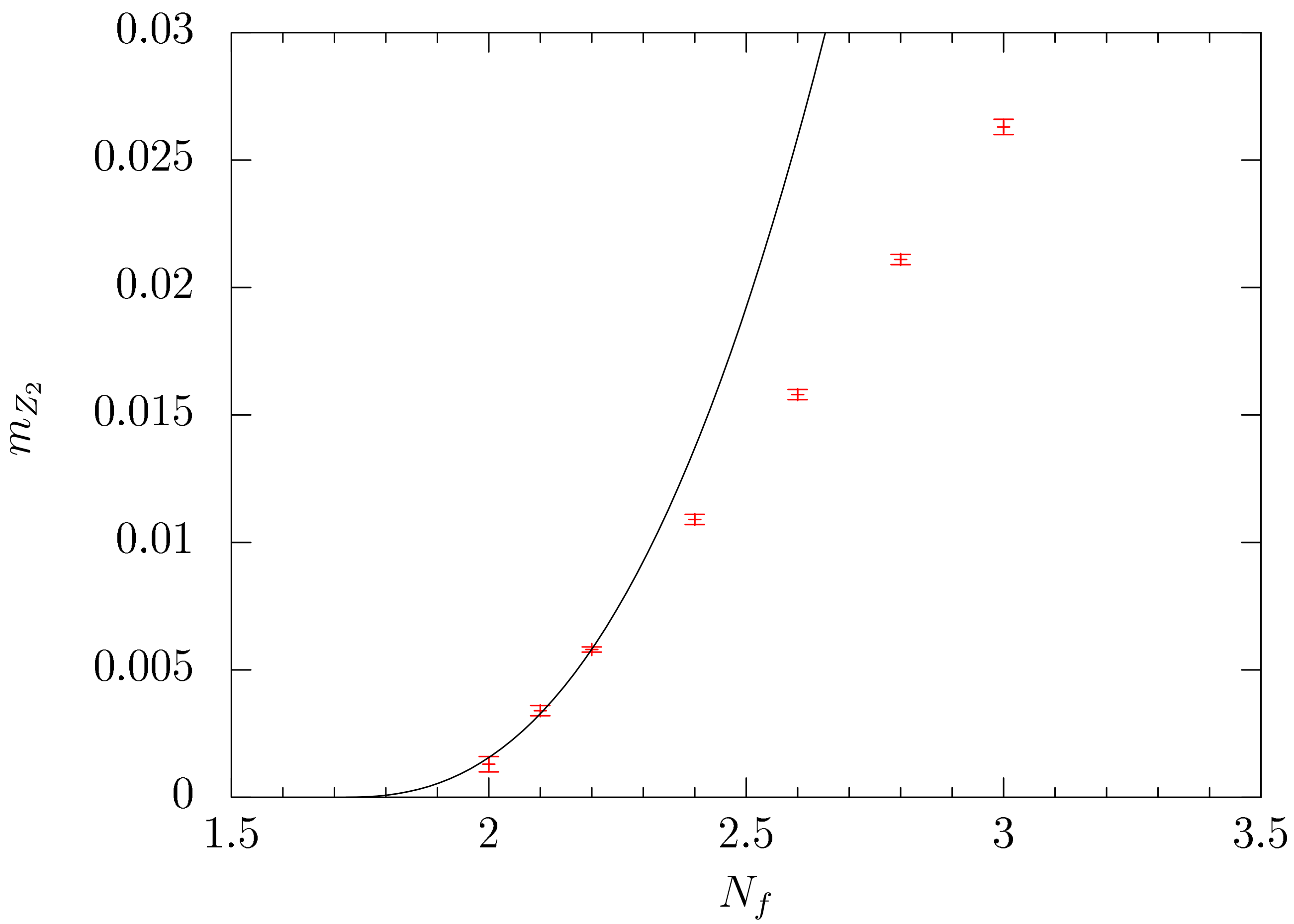


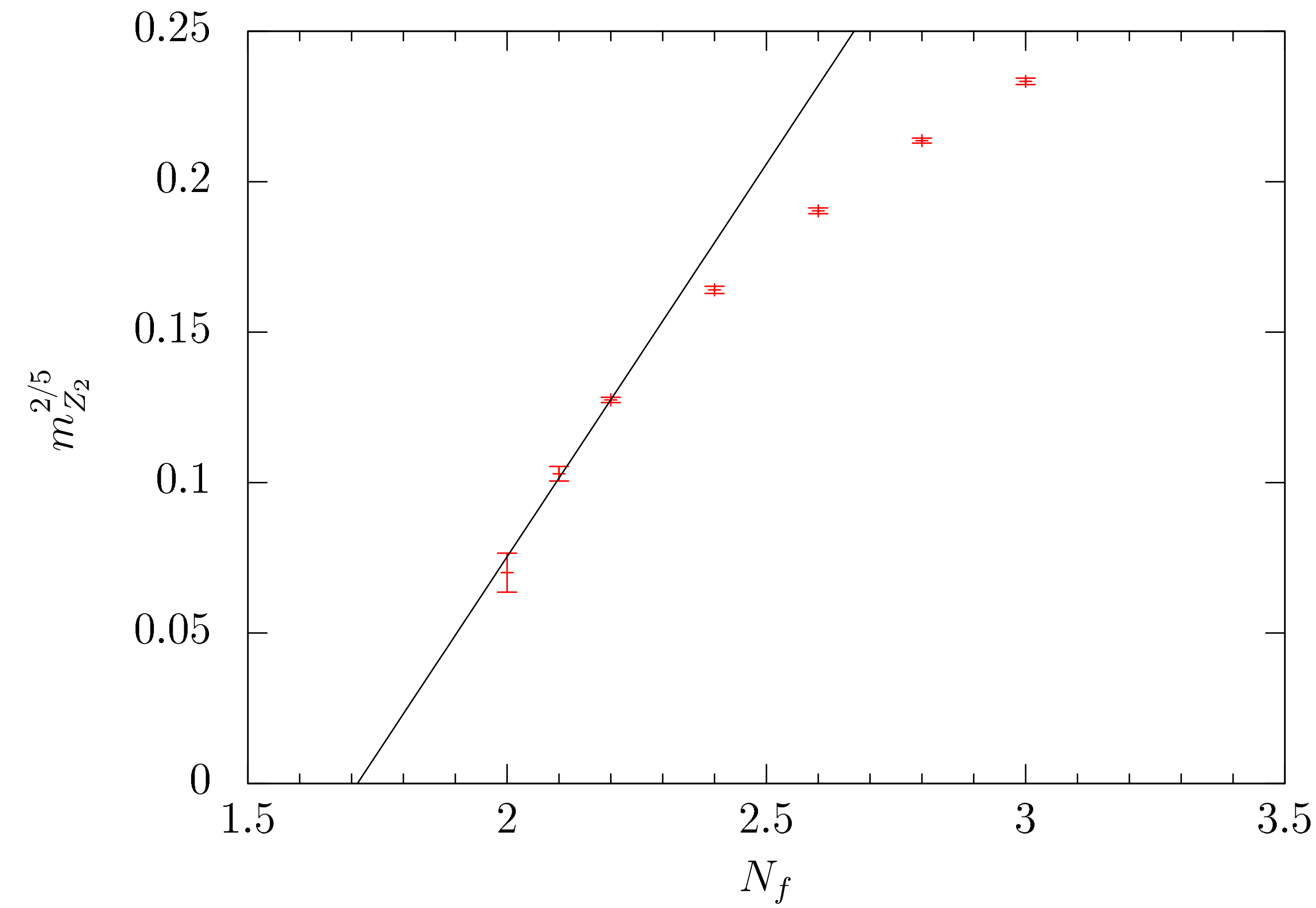
$$\langle \bar{\psi}\psi \rangle \begin{cases} > & 0 \\ = & 0 \\ < & 0 \end{cases}$$



Numerical results for varying Nf

$N_\tau = 4, a \approx 0.26\text{fm}$





Tricritical scaling $\sim m^{2/5}$ observed!

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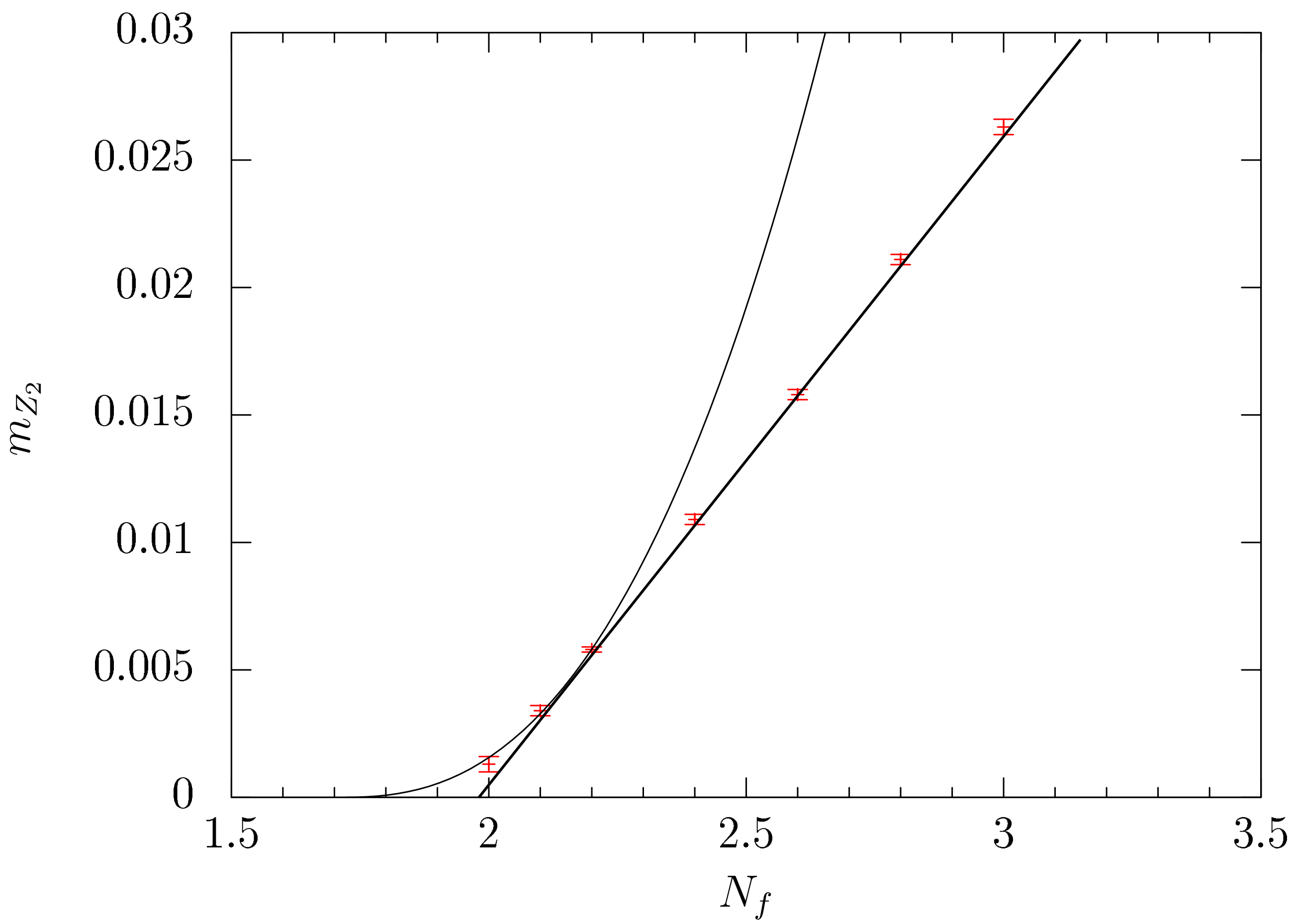
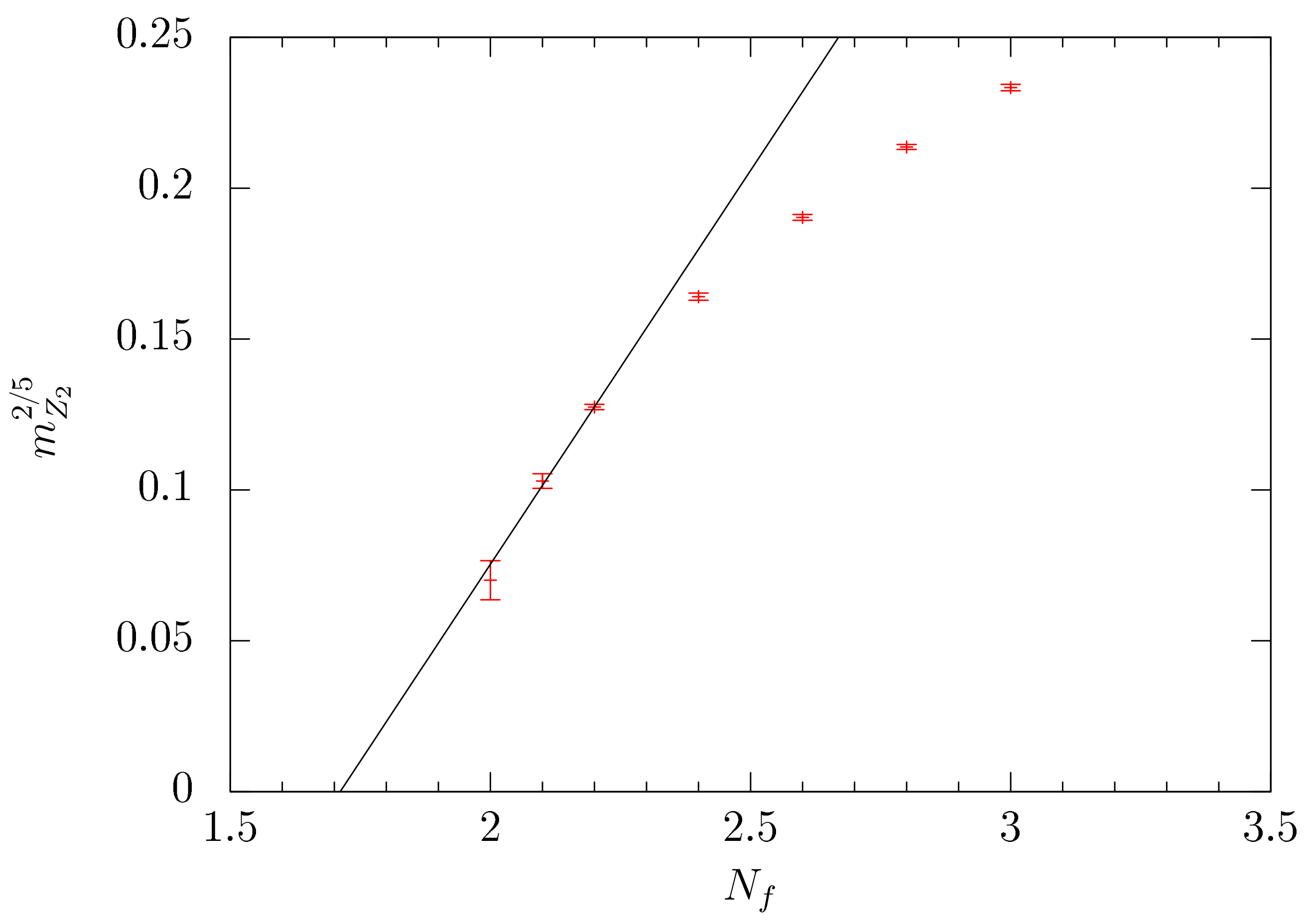
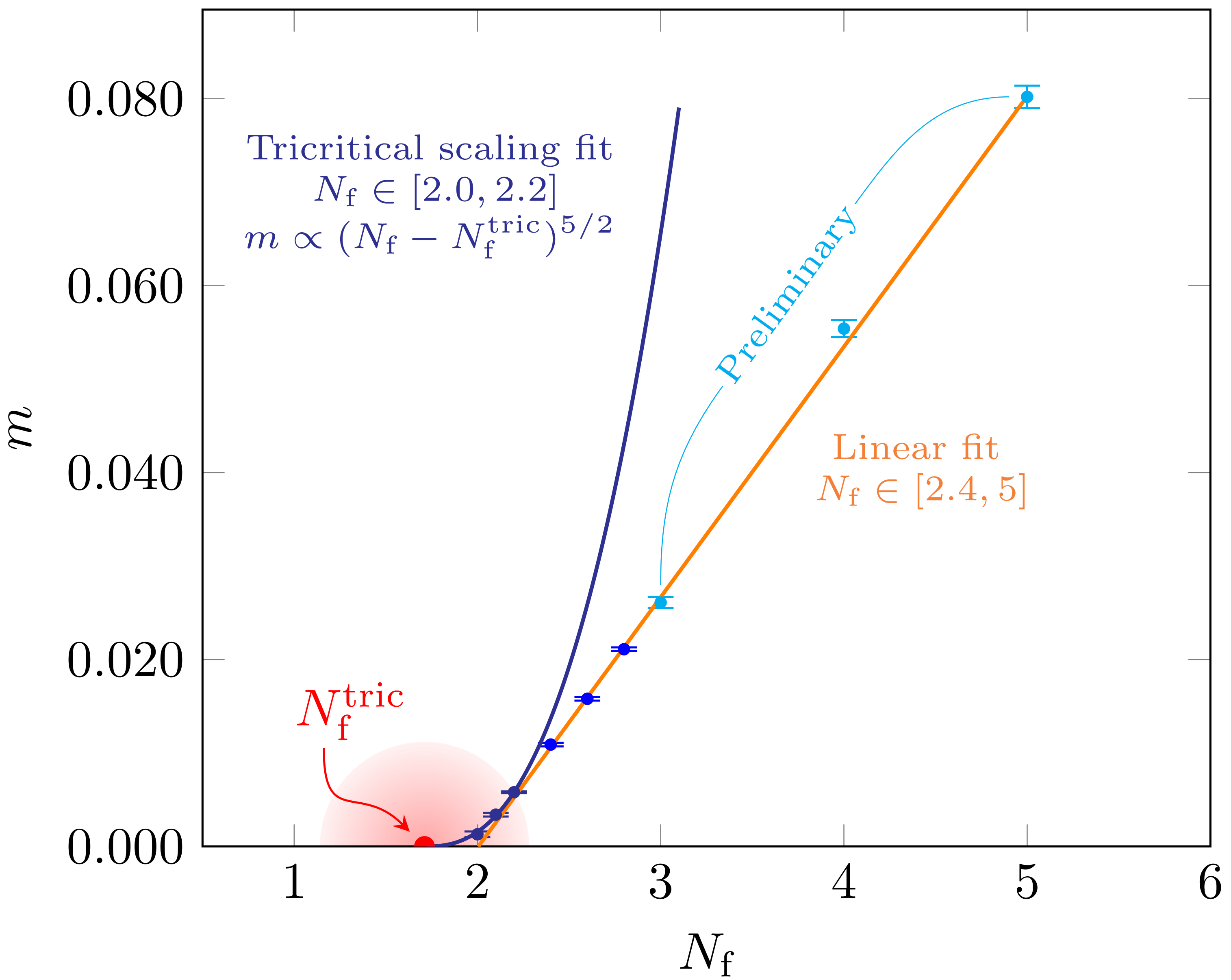




Tricritical scaling $\sim m^{2/5}$ observed!

Plus linear behaviour??

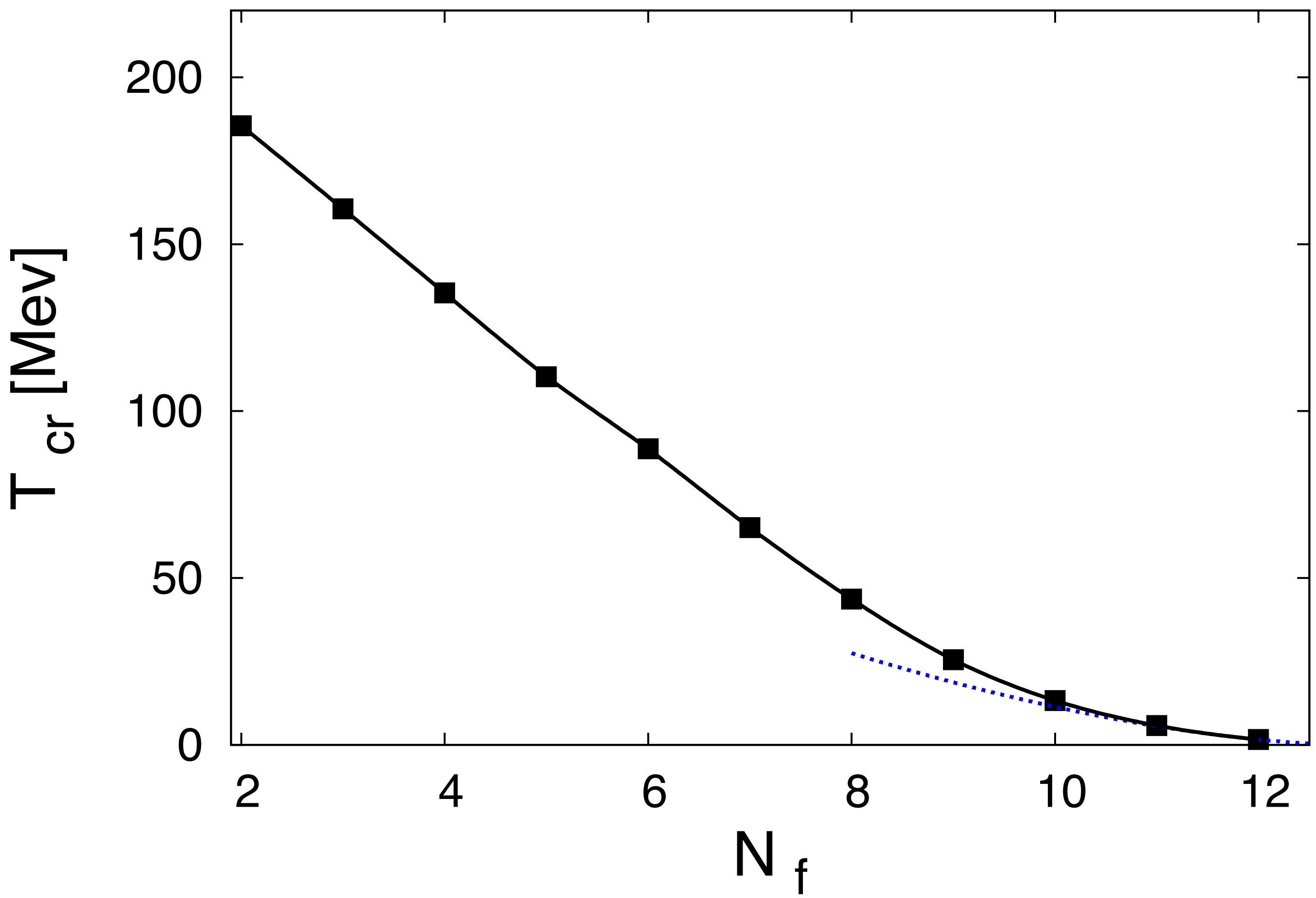
Linearity confirmed with larger Nf:



Scaling region plus linear region

Linear Nf-dependence:

Braun, Gies 09: chiral transition towards the conformal window…. RG treatment



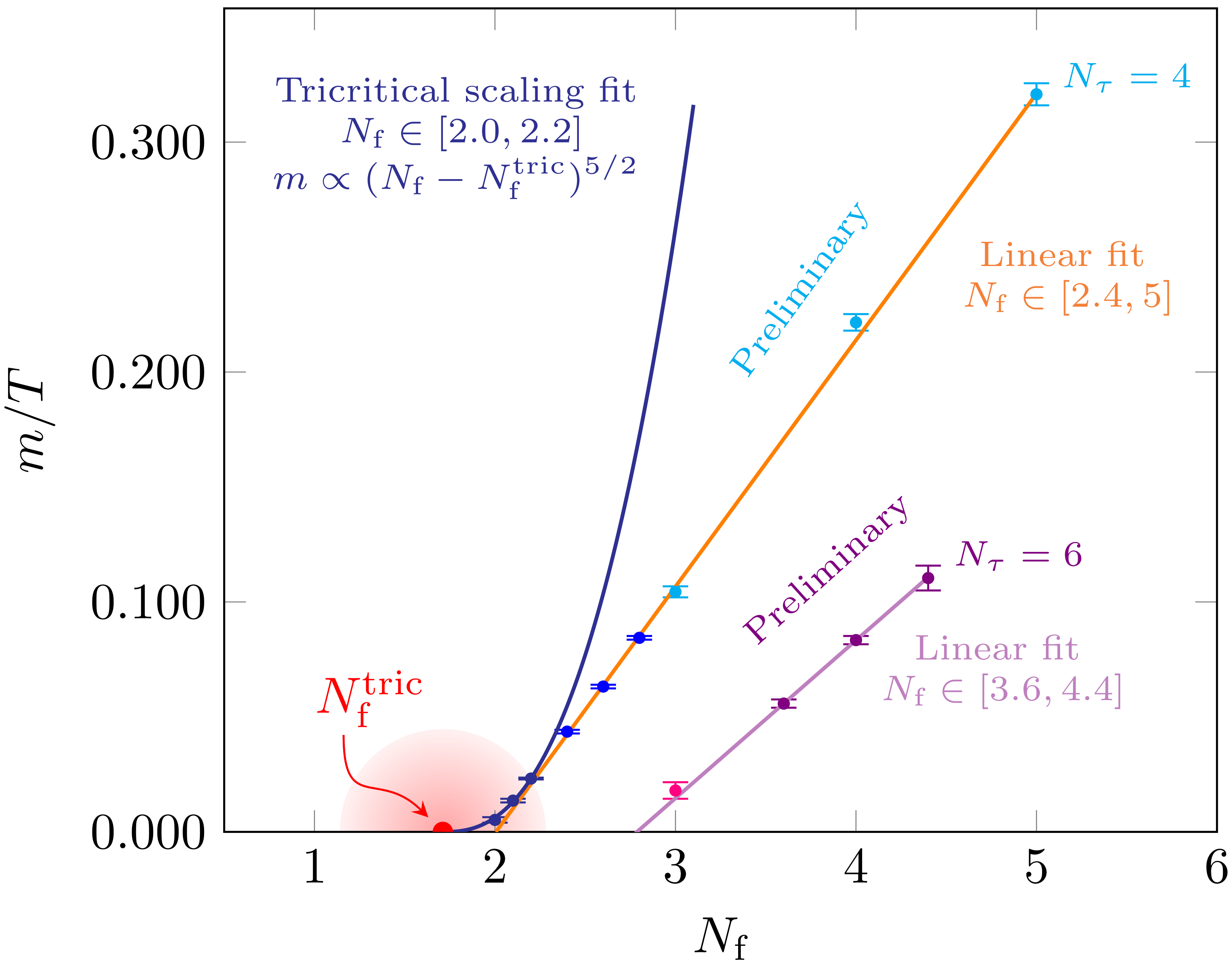
$$\Lambda_{\rm QCD} \simeq \mu_0\, {\rm e}^{-\frac{1}{4\pi b_0 \alpha(\mu_0)}}$$

$$\simeq \mu_0\, {\rm e}^{-\frac{6\pi}{11 N_{\rm c} \alpha(\mu_0)}} \left(1 - \epsilon N_{\rm f} + \mathcal{O}((\epsilon N_{\rm f})^2)\right) \qquad \epsilon = \frac{12\pi}{121 N_{\rm c}^2 \alpha(\mu_0)} \simeq 0.107$$

Inherited by all dimensionful quantities!

Allows for simulations on finer lattices

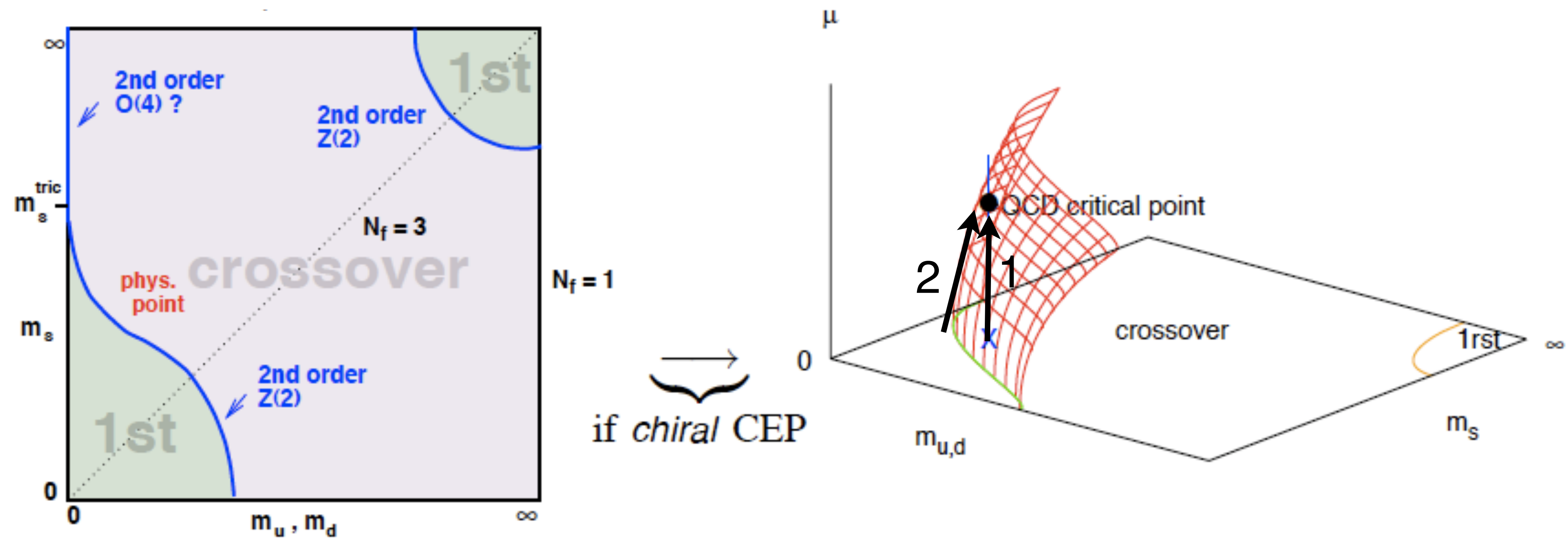
Cuteri, Sciarra, O.P. , in progress



Eventual continuum limit should be possible!

$N_f = 3,\ N_\tau = 6$ from de Forcrand, Kim, O.P. , 07

Extension to finite baryon density



Two strategies:

1 follow vertical line: $m = m_{\rm phys}$, turn on μ sign problem!

2 follow critical surface: $m = m_{\rm crit}(\mu)$

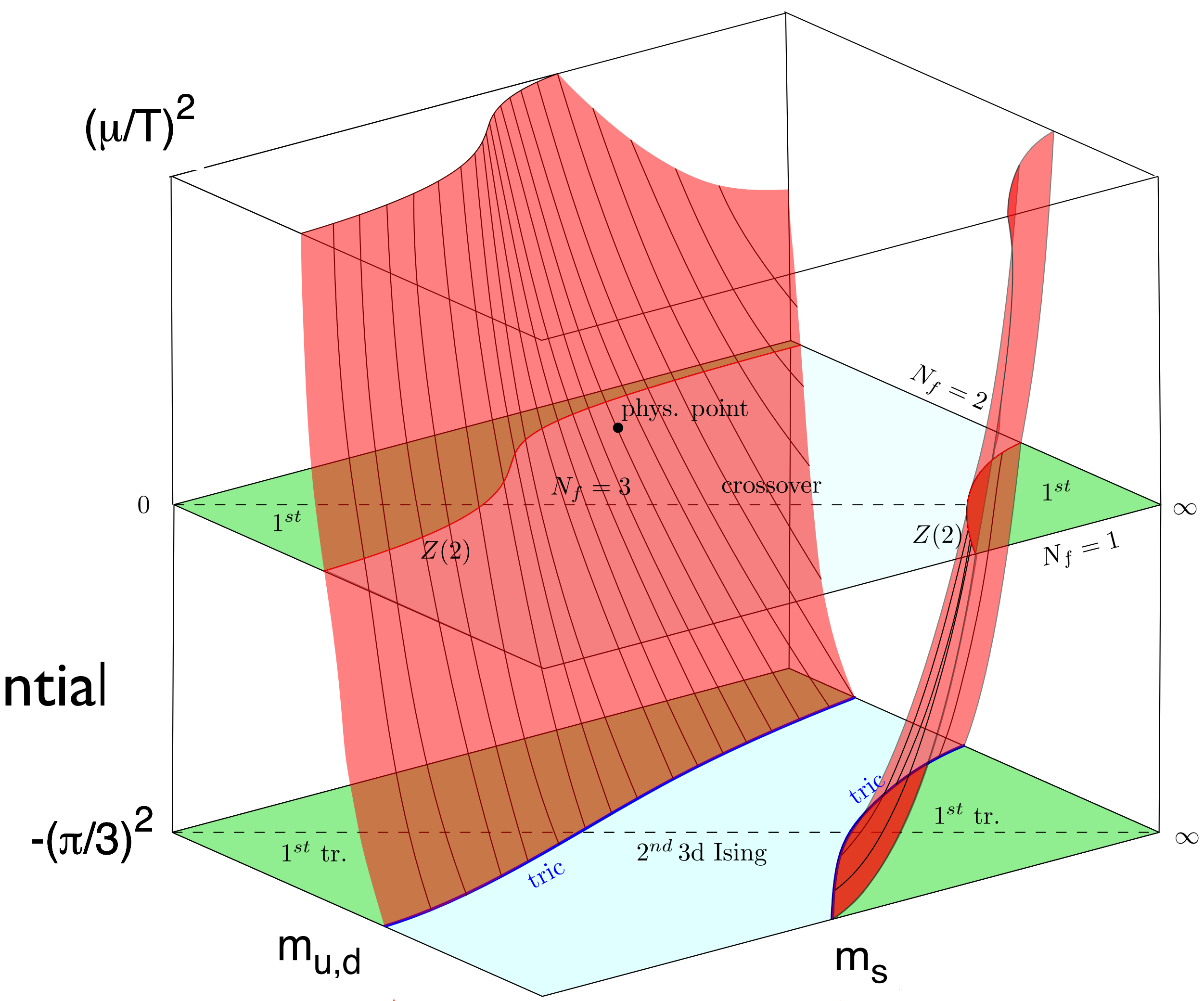
Some methods trying (1) give indications of critical point, but systematics not yet controlled

Extension to finite baryon density

Real and imaginary chemical potential, coarse Nt=4,6 lattices

Real chemical potential:
sign problem

Imaginary chemical potential
no sign problem



staggered,Nf=3:
de Forcrand, O.P. 10

staggered,Nf=2:
D'Elia, Sanfillippo 11

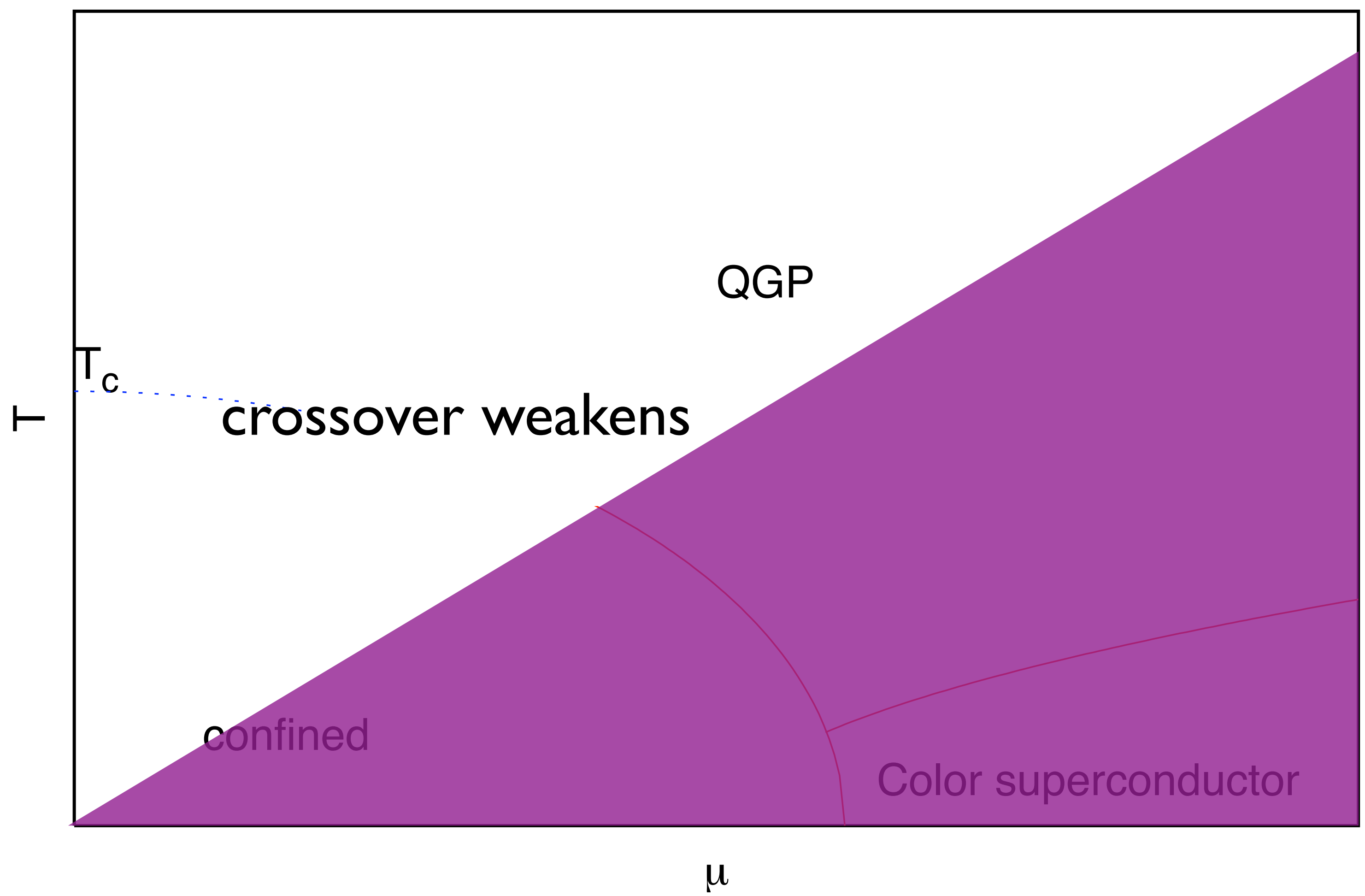
Wilson,Nf=2:
Pinke, O.P. 14

Non-trivial phase structure
Roberge-Weiss Z(3) symmetry!

shape, sign of curvatures determined by tricritical scaling!

transition weakens with real chemical potential!

The lattice-calculable region of the phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- No critical point in the controllable region, some signals beyond

Cluster expansion model (CEM) for baryon number

Vovchenko, Steinheimer, Stöcker, O.P.

QCD thermodynamics with relativistic fugacity/cluster expansion:

$$\frac{p(T,\mu_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cosh\left(\frac{k\,\mu_B}{T}\right)$$

Imaginary μ_B:

Lattice QCD is problematic at real μ but tractable at imaginary μ

$\mu_B \to \mathrm{i}\tilde{\mu}_B \quad \Rightarrow \quad$ QCD observables obtain trigonometric Fourier series form

Pressure: $$\frac{p(T,\mathrm{i}\tilde{\mu}_B)}{T^4} = \sum_{k=0}^{\infty} p_k(T) \cos\left(\frac{k\,\tilde{\mu}_B}{T}\right),$$

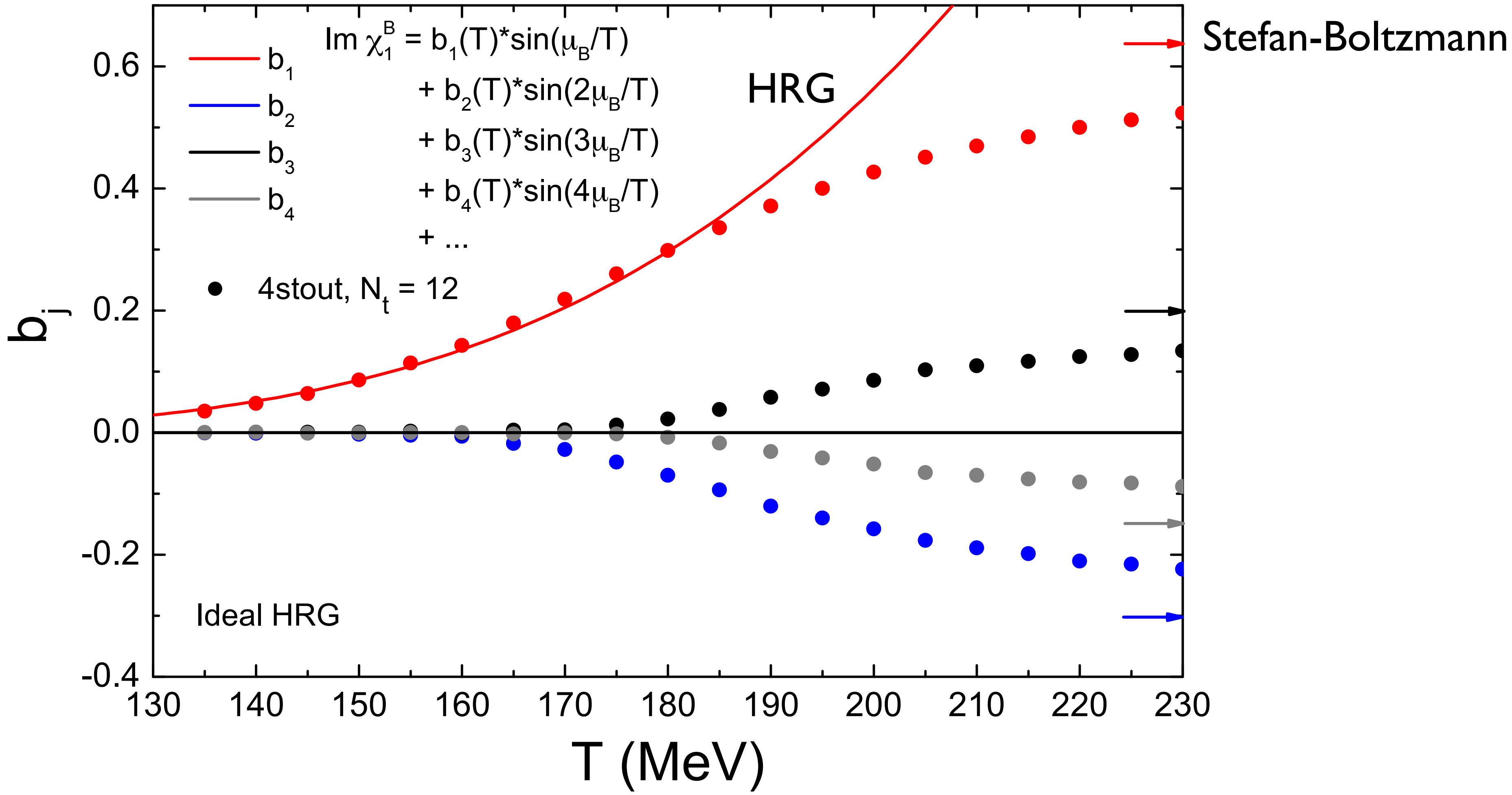
Net baryon density: $$\frac{\rho_B(T,\mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i} \sum_{k=1}^{\infty} b_k(T) \sin\left(\frac{k\,\tilde{\mu}_B}{T}\right), \qquad b_k(T) \equiv k\, p_k(T)$$

$$b_k(T) = \frac{2}{\pi\, T^3} \int_0^{\pi} d\tilde{\mu}_B \, [\mathrm{Im}\, \rho_B(T, i\tilde{\mu}_B)] \sin(k\,\tilde{\mu}_B/T)$$

Coefficients $b_k(T)$ can and are now being calculated in LQCD

Coefficients calculated on the lattice:

$$\frac{\rho_B(T, \mathrm{i}\tilde{\mu}_B)}{T^3} = \mathrm{i} \sum_{j=1}^{\infty} b_j(T) \sin\left(j\, \tilde{\mu}_B / T\right)$$



V.V., A. Pásztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852; S. Borsányi, QM2017

Formulation of the CEM

- All observables are calculated from fugacity expansion for baryon density

$$\frac{\rho_B(T)}{T^3} = \chi_1^B(T) = \sum_{k=1}^{\infty} b_k(T)\,\sinh(k\,\mu_B/T)$$

- $b_1(T)$ and $b_2(T)$ are model input

- All higher order coefficients $\quad b_k(T) = \alpha_k\,\frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}} \qquad \alpha_k = \frac{[b_1^{\mathrm{SB}}]^{k-2}}{[b_2^{\mathrm{SB}}]^{k-1}}\,b_k^{\mathrm{SB}}$

Motivated by HRG with excluded volume;
Assumption: 2-particle interactions only (sufficiently dilute)

Baryon number susceptibilities at $\mu_B = 0$:

$$\chi_{2n}^B(T) \equiv \left.\frac{\partial^{2n}(p/T^4)}{\partial(\mu_B/T)^{2n}}\right|_{\mu_B=0} = \sum_{k=1}^{\infty} k^{2n-1}\,b_k(T) \simeq \sum_{k=1}^{k_{\max}} k^{2n-1}\,b_k(T)$$

Baryon number fluctuations:

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CEM-LQCD: $b_1(T)$ and $b_2(T)$ taken from LQCD simulations at imaginary μ_B

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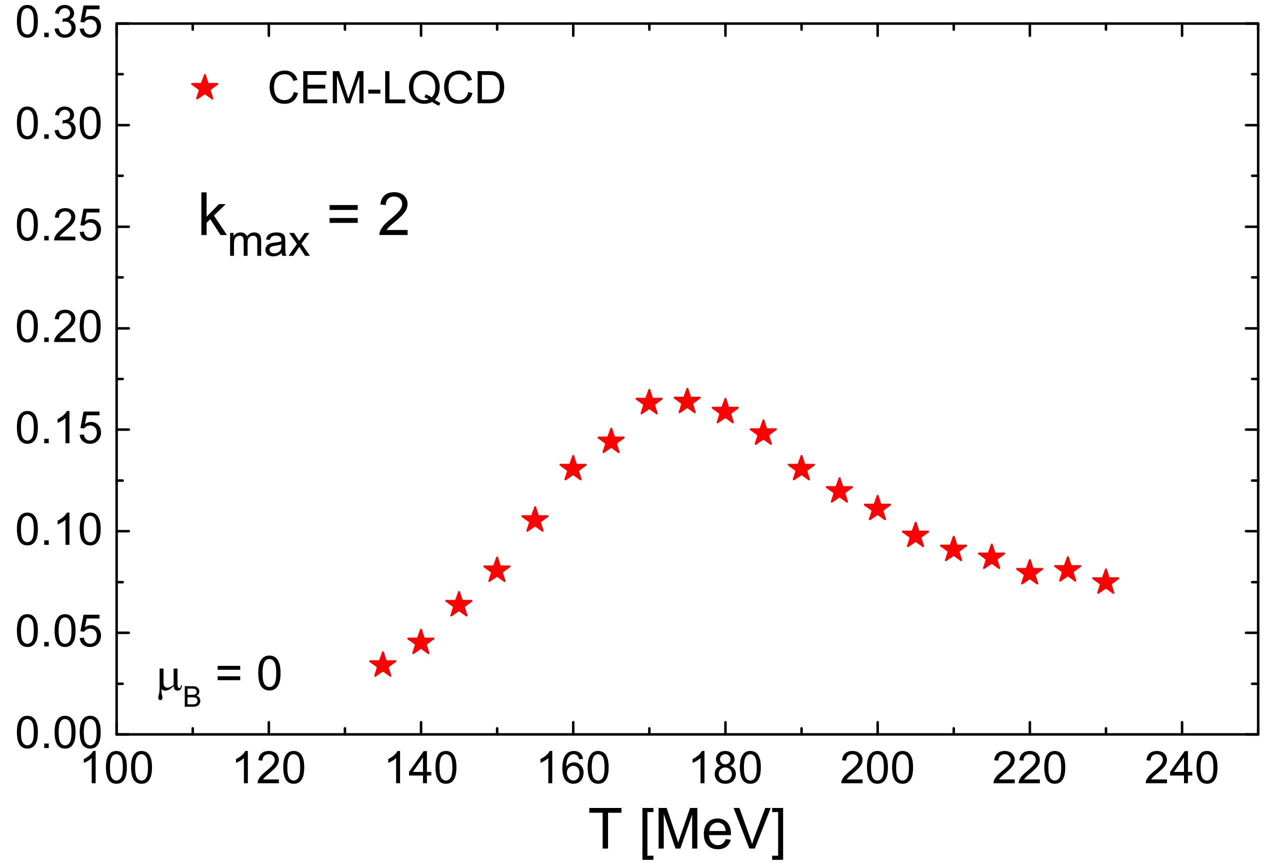
χ^B_2



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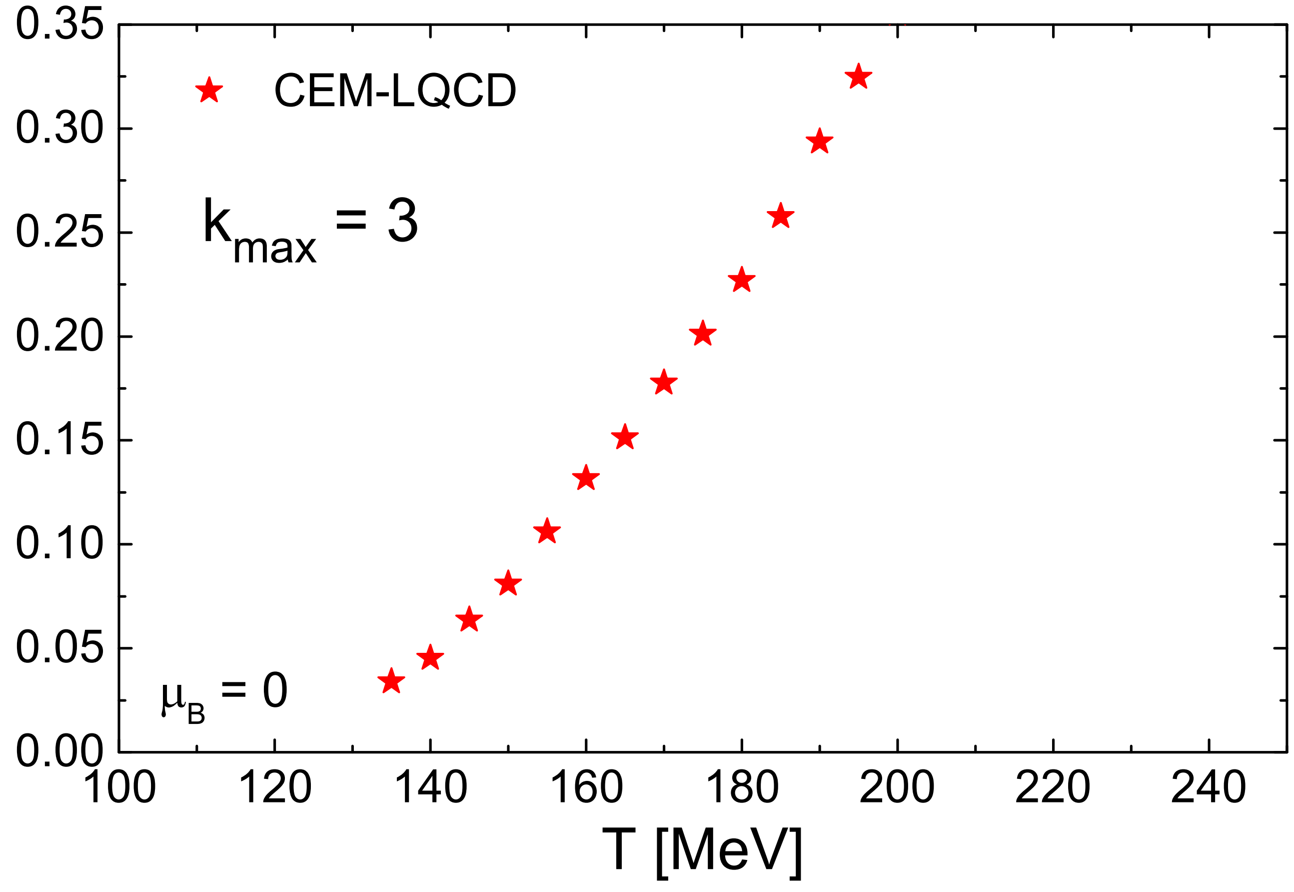
χ_2^B



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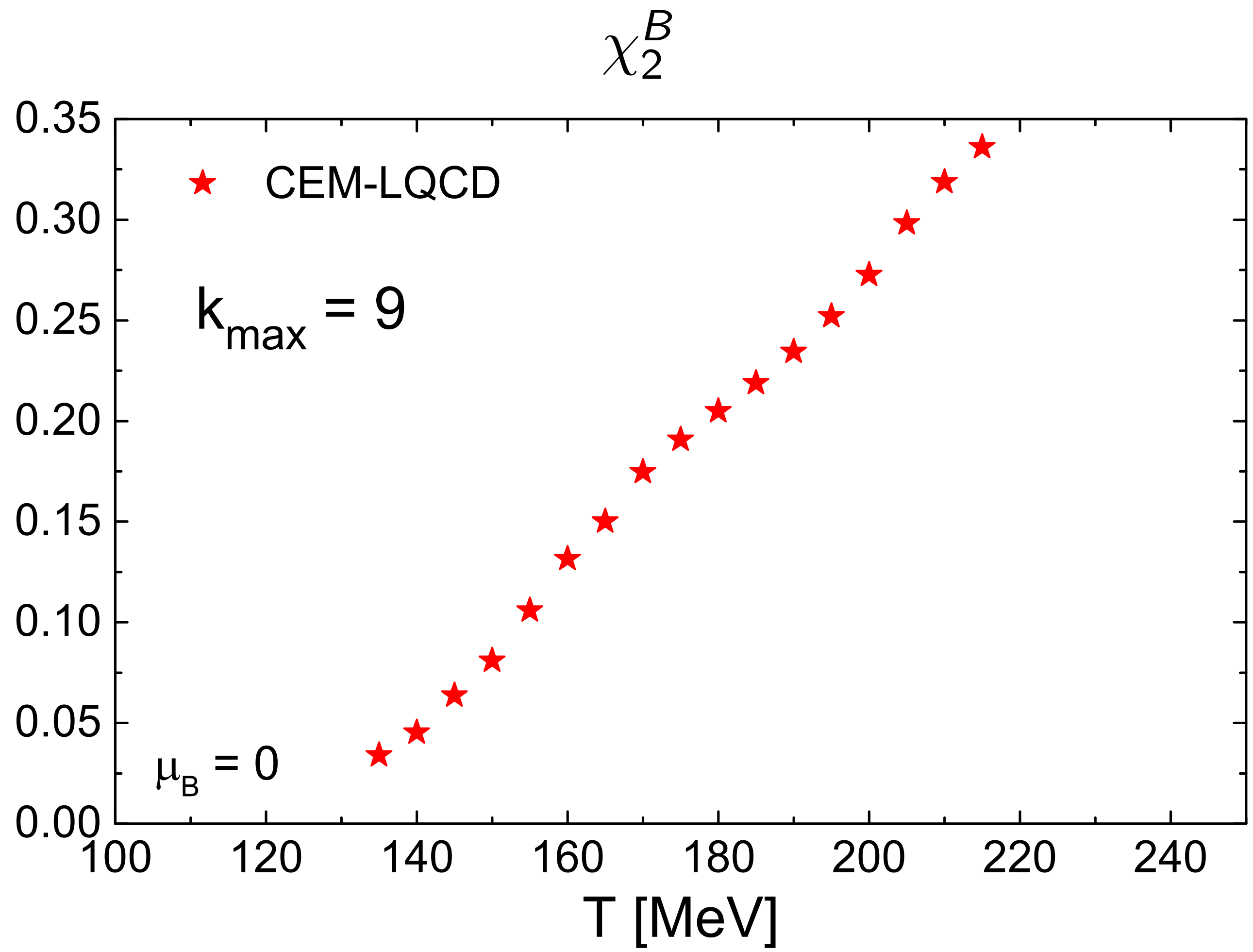

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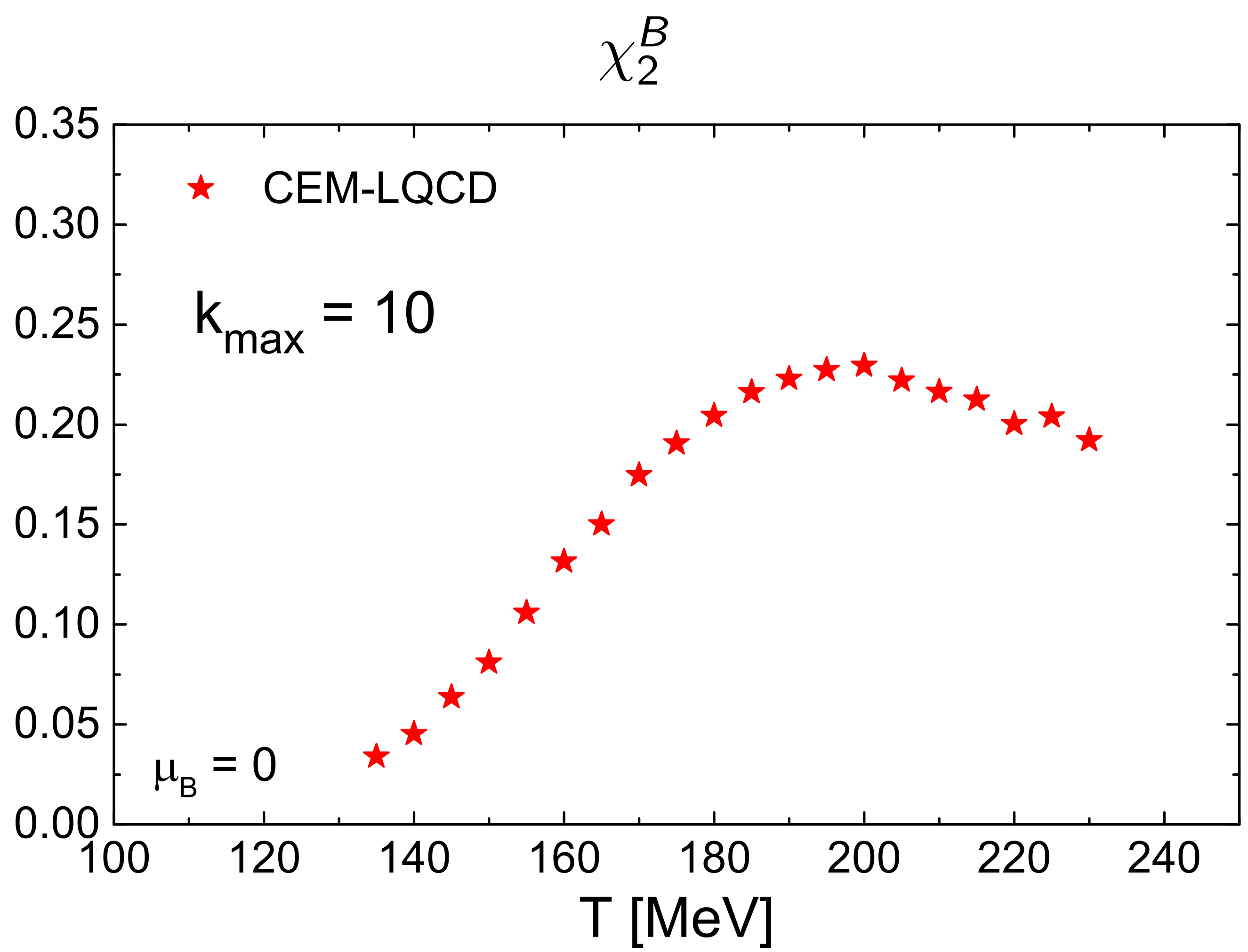

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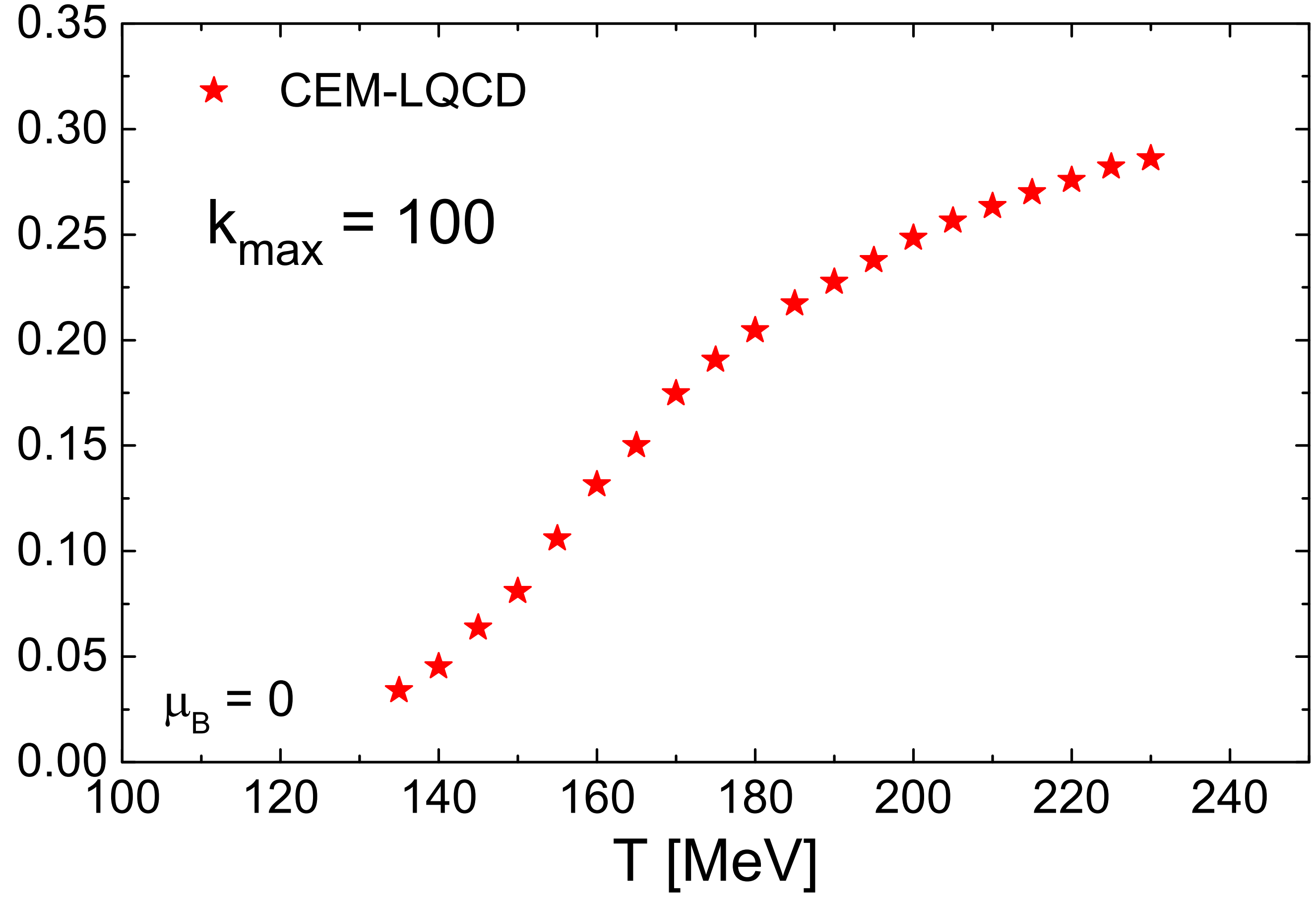
χ^B_2



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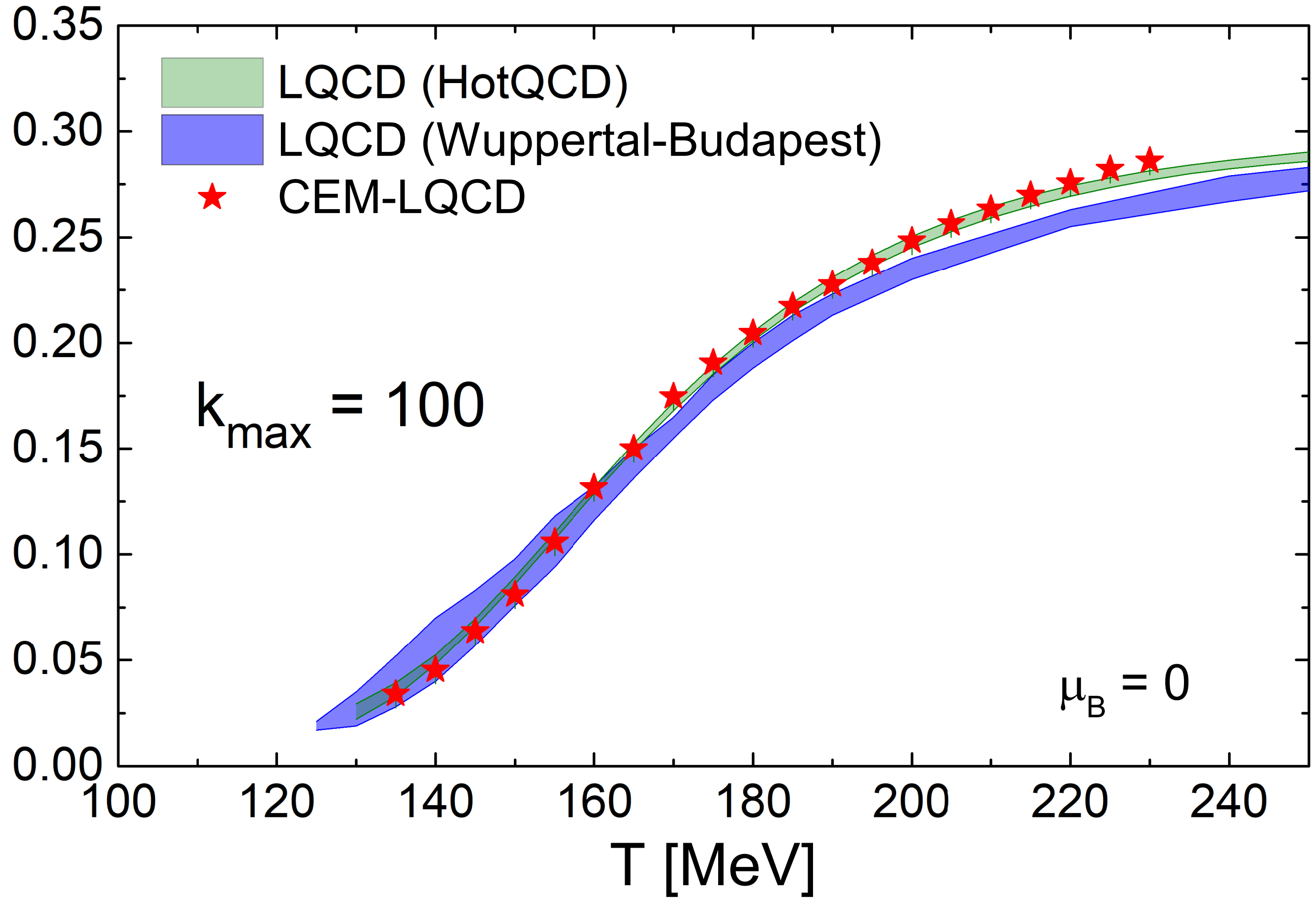
χ_2^B



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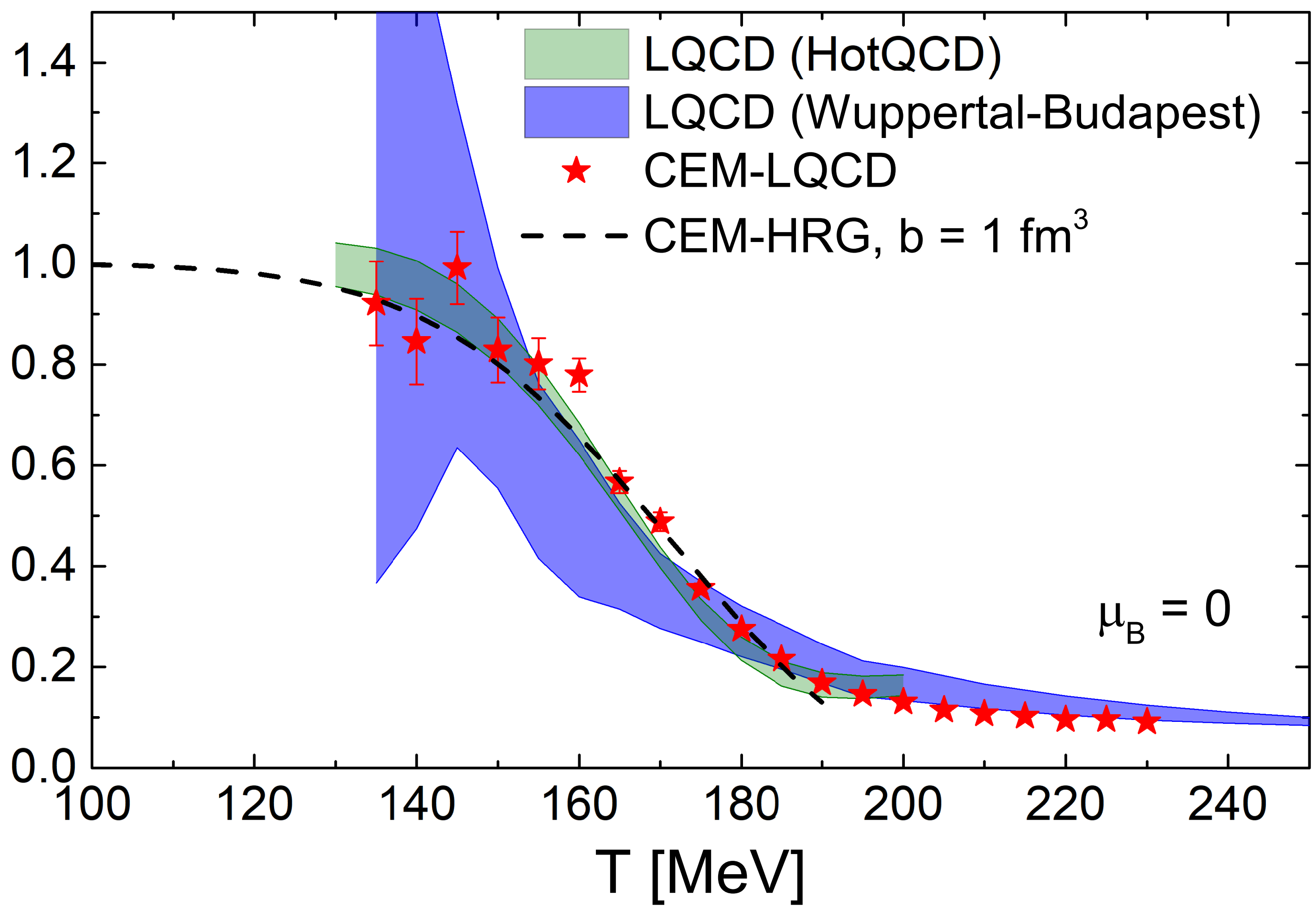
χ^B_4/χ^B_2



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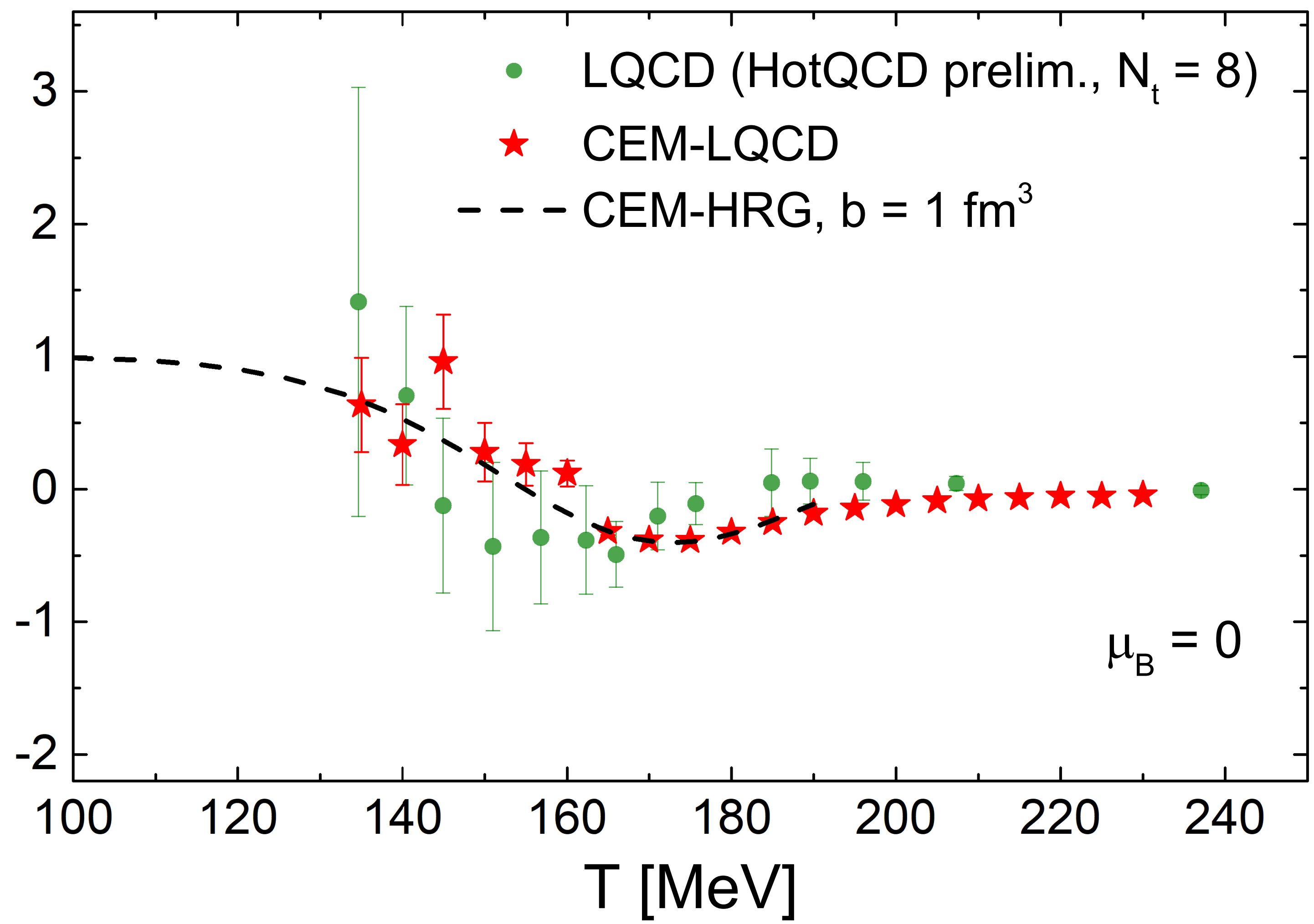
CEM-LQCD: $b_1(T)$ and $b_2(T)$ taken from LQCD simulations at imaginary μ_B

χ^B_6/χ^B_2



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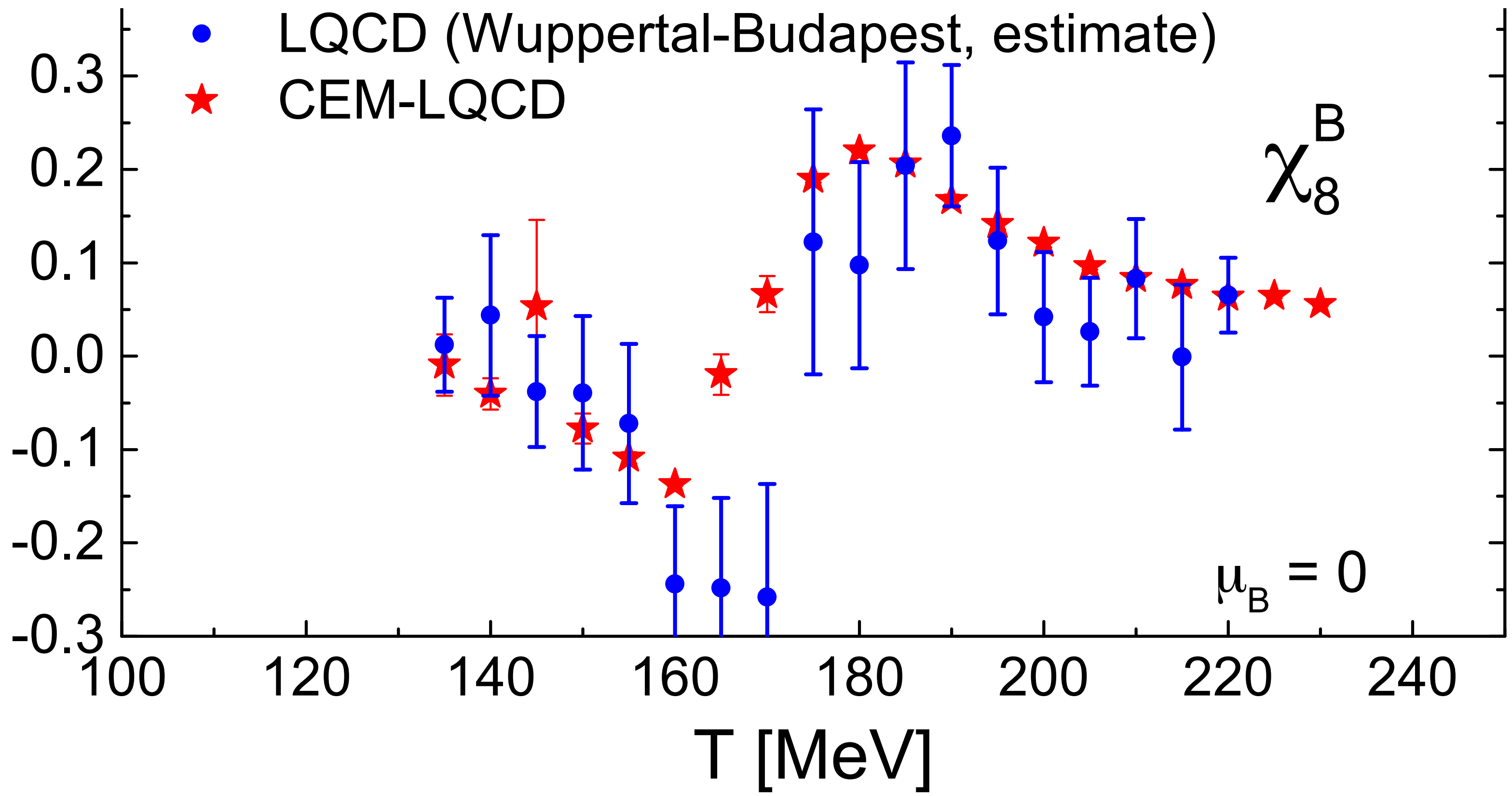
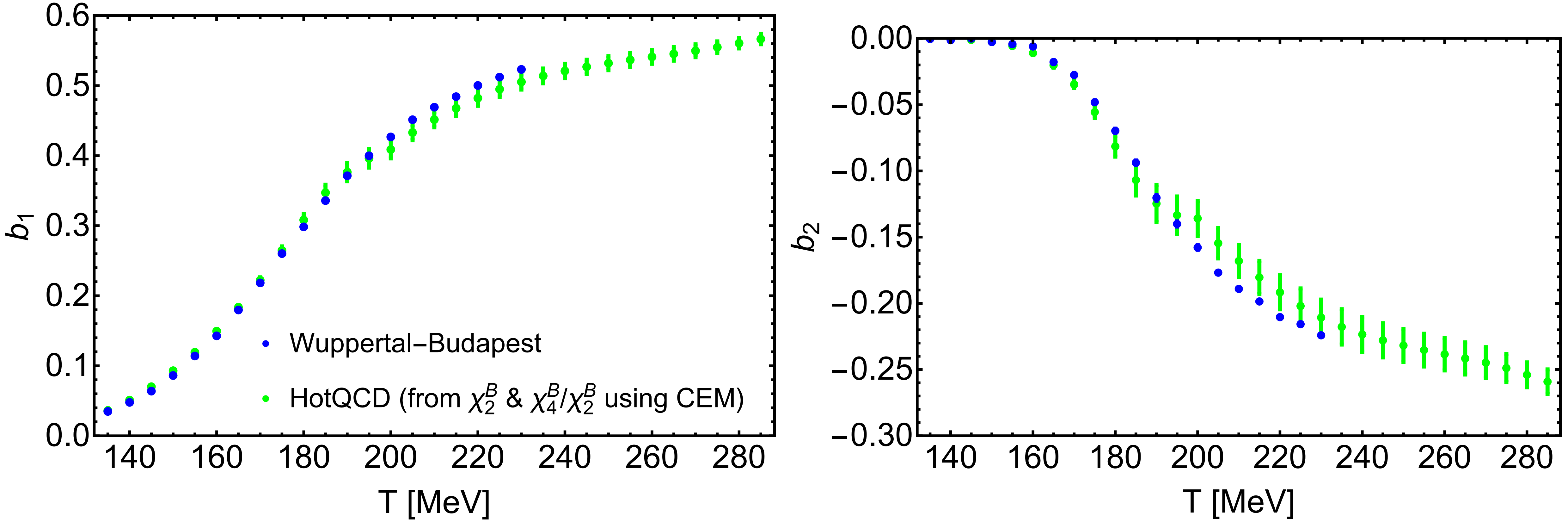


Yet another test of CEM



Note: this involves all coefficients!

Radius of convergence

Taylor expansion of QCD pressure:

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \frac{\chi_2^B(T)}{2!}(\mu_B/T)^2 + \frac{\chi_4^B(T)}{4!}(\mu_B/T)^4 + \ldots$$

Radius of convergence $r_{\mu/T}$ of the expansion is the distance to the nearest singularity of p/T^4 in the *complex* μ_B/T plane at a given temperature T

If the nearest singularity is at a real μ_B/T value, this could point to the QCD critical point

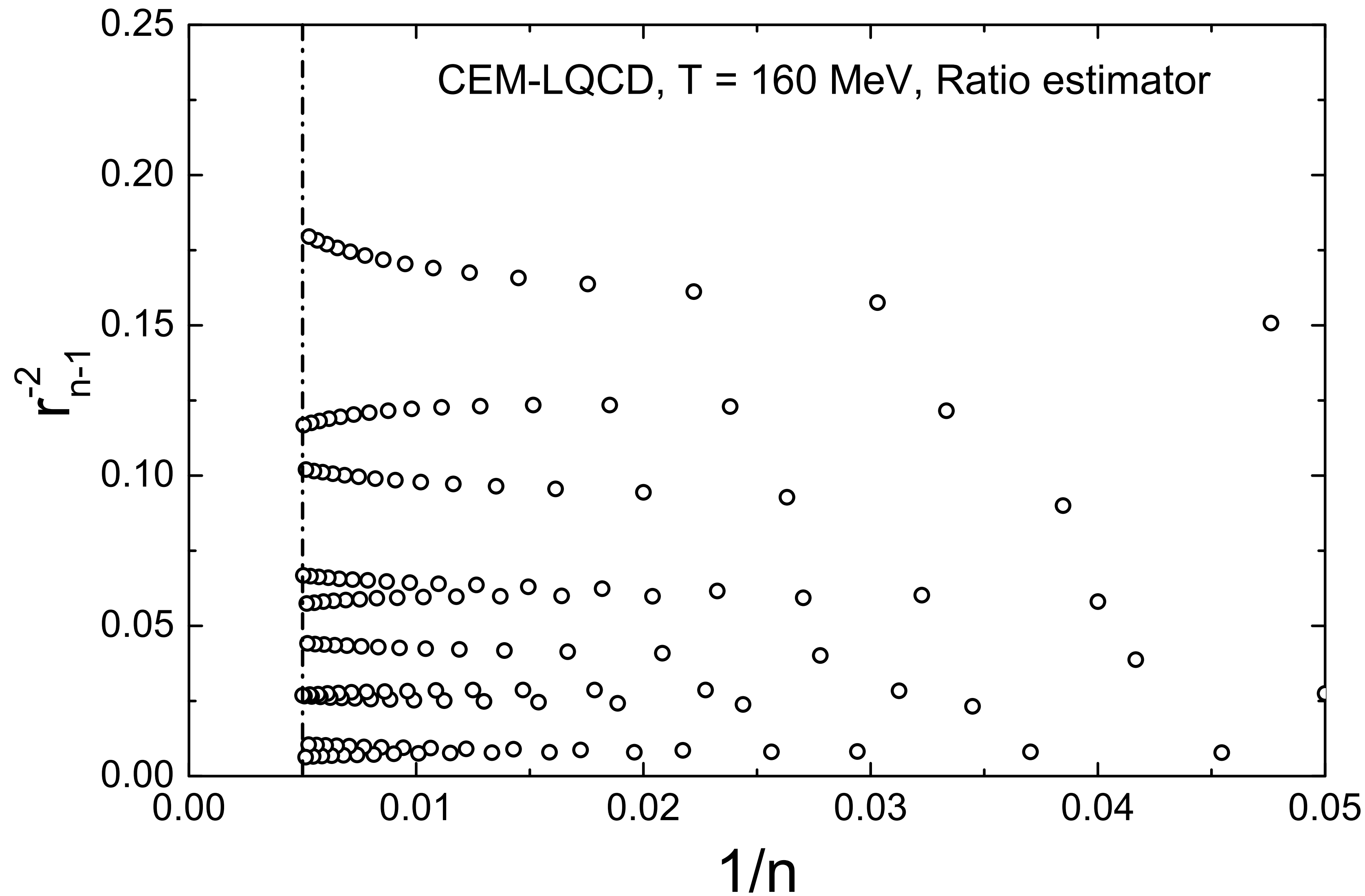
Lattice QCD strategy: Estimate $r_{\mu/T}$ from few leading terms
M. D'Elia et al., 1611.08285; S. Datta et al., 1612.06673; A. Bazavov et al., 1701.04325

Ratio estimator: $$r_n = \left|\frac{(2n+2)(2n+1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^{1/2}, \qquad r_{\mu/T} = \lim_{n\to\infty} r_n$$

CEM allows to analyze r_n to very high order

Domb-Sykes plot: $1/r_n^2$ vs $1/n$, linear extrapolation to $1/n = 0$ yields $r_{\mu/T}$

CEM-LQCD @ $T = 160$ MeV

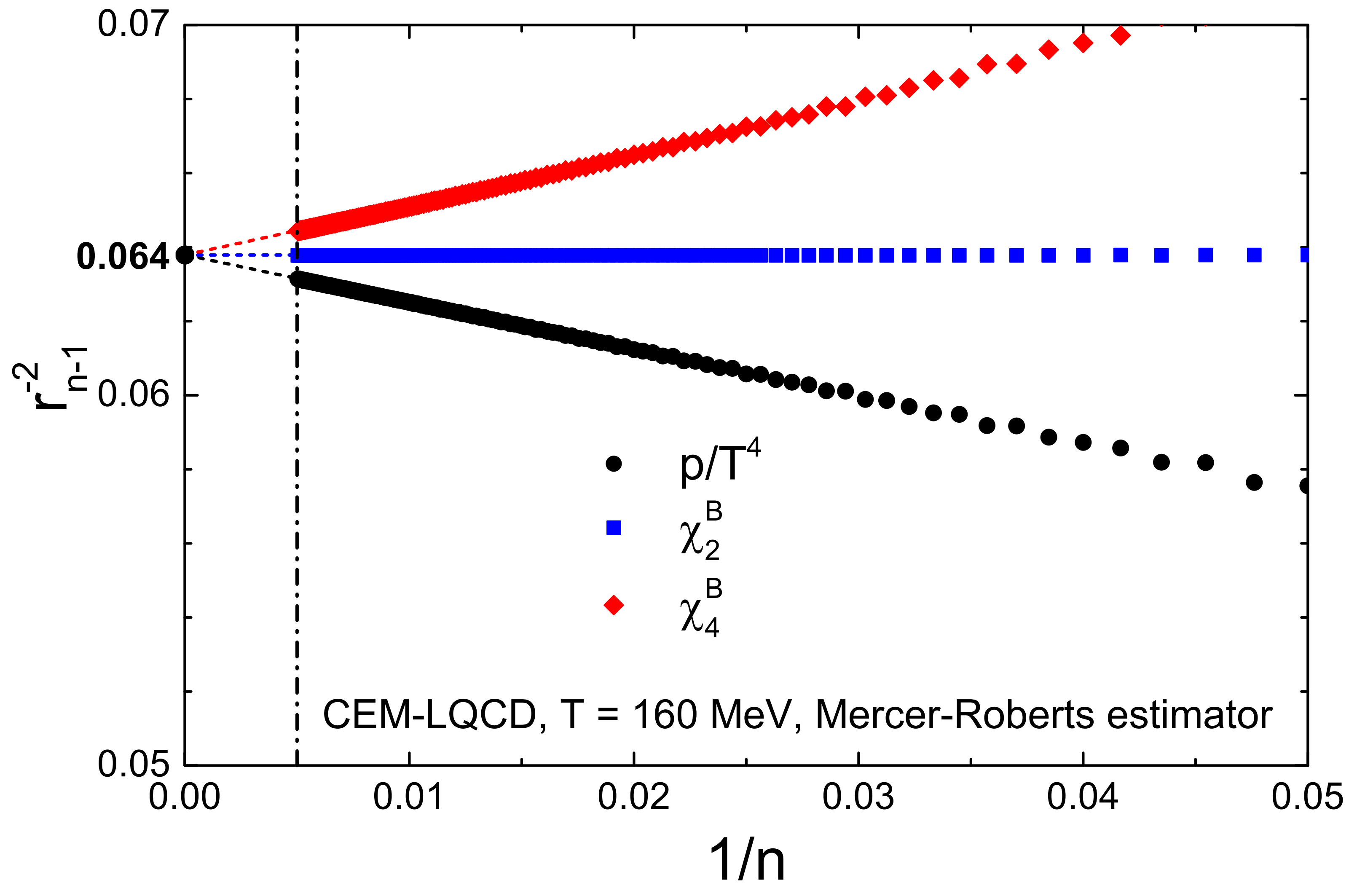


$$\lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)\chi^B_{2n}}{\chi^B_{2n+2}} \right|^{1/2}$$ DOES NOT EXIST!

Reason: coefficients have neither same nor alternating signs (required for ratio test)

Mercer-Roberts estimator

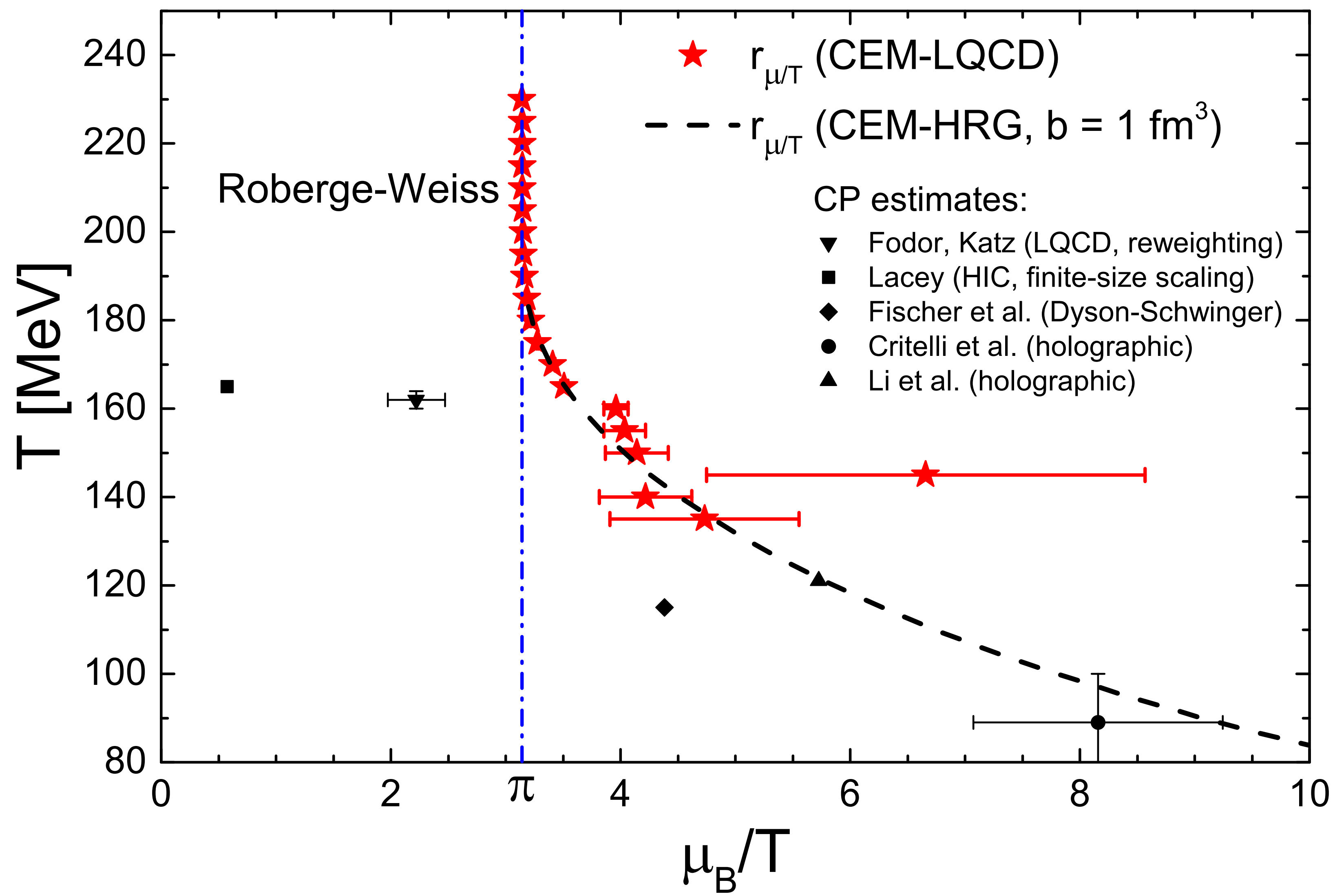
$$r_n = \left| \frac{c_{n+1}\, c_{n-1} - c_n^2}{c_{n+2}\, c_n - c_{n+1}^2} \right|^{1/4}, \qquad c_n = \frac{\chi_{2n}^B}{(2n)!}.$$



Three different observables show same radius of convergence!

Radius of convergence of CEM

Applying the same procedure at other temperatures



Radius of convergence of Taylor expansion sees Roberge-Weiss transition

R-W transition expected at $T > T_{RW}$ and $\mathrm{Im}[\mu_B/T] = \pi$ [Roberge, Weiss, NPB '86]

Lattice estimate: $T_{RW} \sim 200$ MeV [C. Bonati et al., 1602.01426]

Conclusions

- Order of chiral phase transition not yet settled in the continuum
- For physical quark masses it is a crossover
- For small baryon density the crossover weakens, no sign of criticality
- Cluster expansion model for baryon number:
  EoS for small densities to all orders in chemical potential
  no critical point below $\mu_B < \pi T$
- If there is a phase transition at larger density: is it chiral?