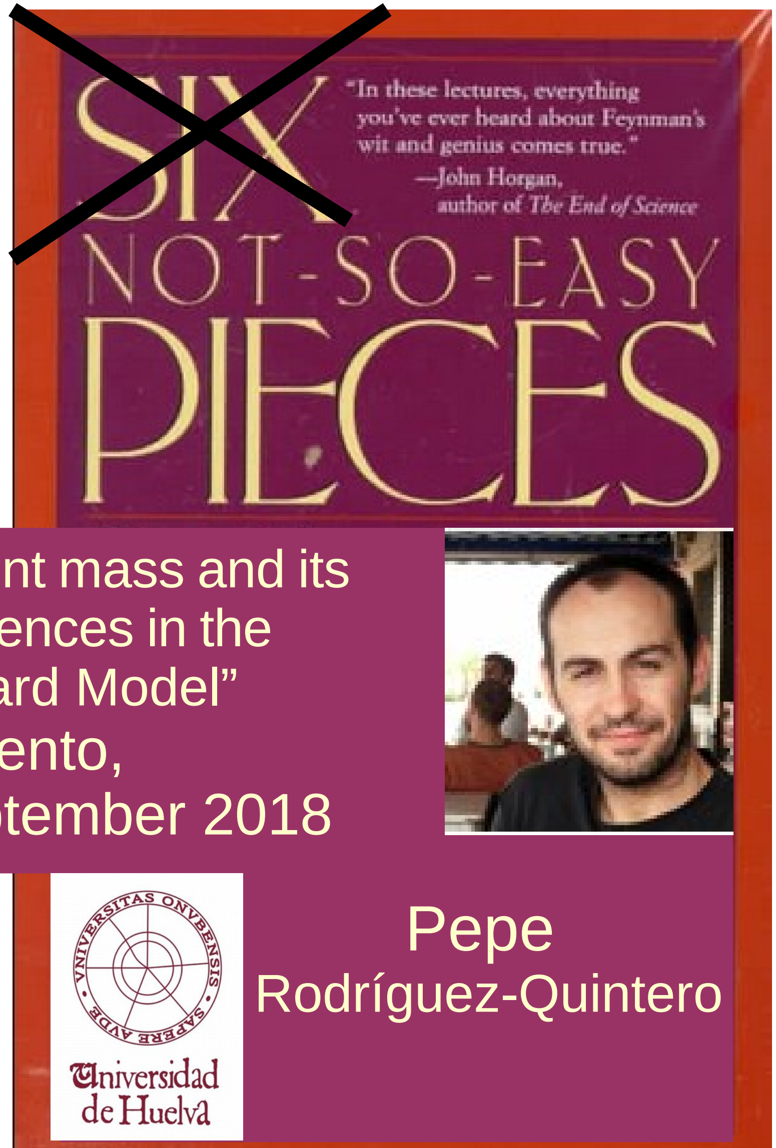
The gluon mass and the gauge sector of QCD

Two

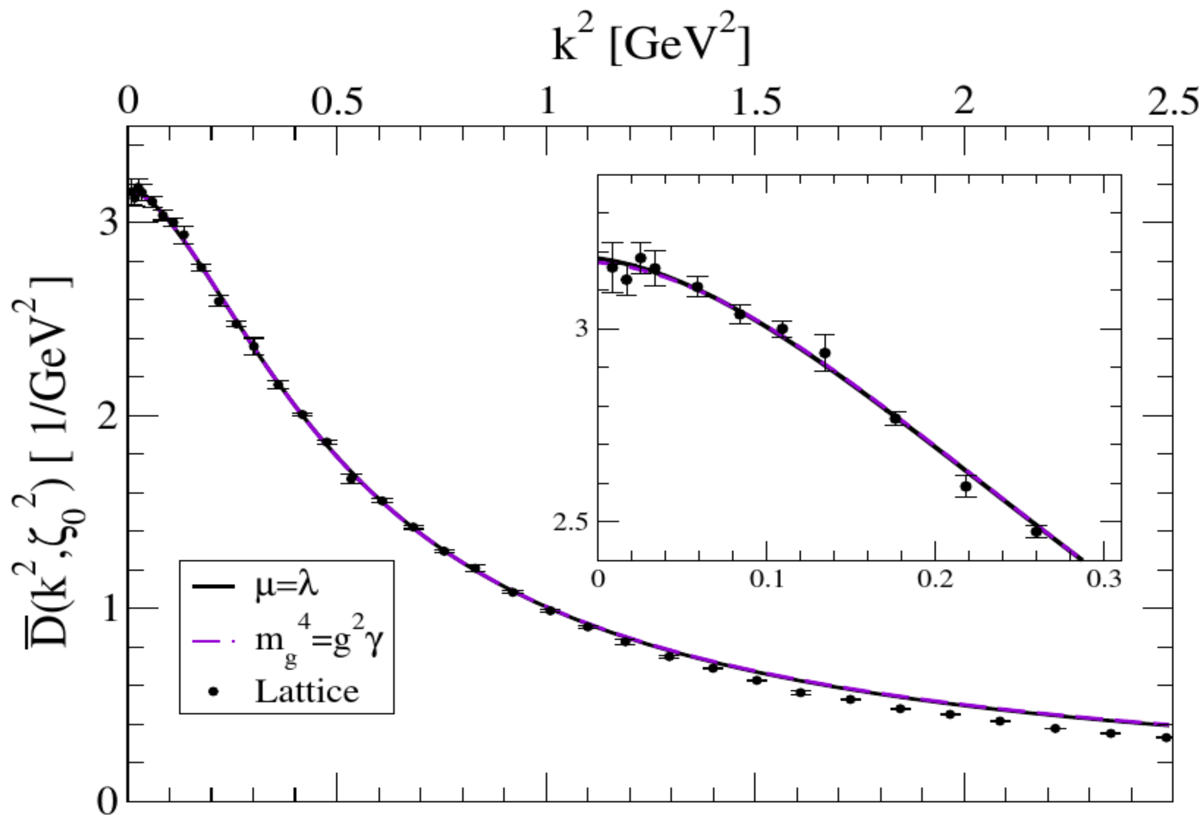


On "Emergent mass and its consequences in the Standard Model"
Trento,
17-21 September 2018

Pepe
Rodríguez-Quintero

Universidad de Huelva

Piece one: Gribov and gluon masses



In collaboration with: F. Gao, S. Qin and C.D. Roberts
[Phys.Rev. D97 (2018) no.3, 034010]

The Gribov problem

- The gauge-fixing condition, defined as the unambiguous selection of one unique element from each “gauge-field orbit” is nonperturbatively inadequate.

i.e., Landau gauge: $\partial_\mu A^\mu = 0$; Not enough!!!

◆ Gribov suggestion: minimization of $\boxed{F_A[U] = \frac{1}{2} \int d^4x \, [A_\mu^a(x)]_U \, [A_\mu^a(x)]_U}$

[V. N. Gribov, Nucl. Phys. B139 (1978),1]

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second derivative in Landau gauge

Fadeev-Popov operator: $\mathcal{M}^{ab}(x,y) = [-\partial^2 \delta^{ab} + \partial_\mu f^{abc} A^c_\mu(x)] \delta^4(x-y)$

- **In perturbation theory:** the ghost propagator reads

$$(\mathcal{M}^{-1})^{ab} = \frac{1}{k^2} \frac{1}{1-\sigma(k,A)} \longrightarrow \boxed{\sigma(0,A) < 1} \longrightarrow \left\{ \begin{array}{l} G_{Gribov}(k^2) = \dfrac{k^4}{k^4 + m_G^4} \\ F_{Gribov}(k^2) = \dfrac{128\pi^2 m_G^2}{N_c g^2 k^2} \end{array} \right.$$

First Gribov region

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- Zwanziger horizon condition: explicit modification of the action incorporating the horizon term: $\boxed{\gamma \int d^4x\, h(x)}$ [D. Zwanziger, Nucl.Phys.B321 (1989) 591]

$$h(x) = g^2 f^{abc} A^b_\mu(x) [\mathcal{M}^{-1}]^{ad}(x,x) f^{dec} A^e_\mu(x)$$

$$\mathcal{M}^{ab}(x,y) = [-\partial^2 \delta^{ab} + \partial_\mu f^{abc} A^c_\mu(x)] \delta^4(x-y)$$

$$\langle h[\gamma] \rangle = d(N^2-1)$$ Condition fixing the Gribov scale

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- **An equivalent local action** can be derived by incorporating auxiliary fields [D.. Zwanziger, Nucl.Phys.B399(1993)477]

Gluon dressing function: $\boxed{\mathcal{D}^\gamma(k^2) = \dfrac{k^2}{k^4 + 2Ng^2\gamma}}$

Gribov mass: m_γ^4

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Gribov-Zwanziger approach

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- The auxiliary fields localizing the horizon term take a non-zero dimension-two condensates such that [D. Dudal et al., Phys.Rev.D72(2005)014016] [D. Dudal et al., Phys.Rev.D77(2008)071501]

$$\boxed{\overline{\mathcal{D}}(k^2) = \frac{k^2 + M^2}{k^4 + k^2 m^2 + \lambda^4}}$$

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$$\left\{\begin{array}{l} \lambda^4 = 2N m_\gamma^4 - \mu^2 M^2 \\ m^2 = M^2 - \mu^2 \\ \text{where: } \mu^2 = \frac{3}{32}\langle A_\mu^a A_a^\mu \rangle \neq 0 \\ M^2 \neq 0 \end{array}\right.$$

The partonic constraints

- An illustrative example: $\mathcal{S}_f(k^2) = 1/(k^2 + \nu^2)$. a free-parton propagator!
  - 4-d dual in configuration space:

$$\chi = \sqrt{x^2}$$

$$s_f(\chi) = \int \frac{d^4k}{(2\pi)^4}\, \mathrm{e}^{ik\cdot x}\, \mathcal{S}_f(k^2)$$

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  - 1-d dual in configuration space:

$$\chi = \sqrt{\vec{x}^{\,2} + \tau^2}$$

$$\text{rest mass} = 2\frac{d^2}{d\tau^2}\sigma(\tau)\Big|_{\tau=0}$$

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- Asymptotic freedom implies that the two-points QCD Schwinger functions must be convex-down, both in momentum space (for $k^2 > k_P^2 > \Lambda_{QCD}^2$) and in configuration space (for $x^2 < x_P^2 < 1/\Lambda_{QCD}^2$). A real, positive effective mass implies the same for the 1-d Schwinger function in the vicinity of $\tau = 0$.

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- Applied to the RGZ gluon propagator: $\overline{\mathcal{D}}(k^2) = \dfrac{k^2 + M^2}{k^4 + k^2 m^2 + \lambda^4}$
  - 1-d dual in configuration space:

$$\Delta(\tau) = \frac{1}{\pi} \int_0^\infty dk\ \cos(\tau k)\, \overline{\mathcal{D}}(k^2)$$

$$= \frac{\exp(-\tau \lambda c_{\varphi/2})}{2 \lambda s_\varphi} \left[\left(1 + \frac{\lambda^2}{M^2}\right) s_{\varphi/2} \cos(\tau \lambda s_{\varphi/2}) + \left(1 - \frac{\lambda^2}{M^2}\right) c_{\varphi/2} \sin(\tau \lambda s_{\varphi/2}) \right]$$

$$s_\varphi = (1 - c_\varphi^2)^{1/2}$$

$$s_{\varphi/2} = (1 - c_{\varphi/2}^2)^{1/2}$$

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  - 1-d dual in configuration space: weak constraint: $\boxed{\lambda = M}$

$$\Delta(\tau) = \frac{1}{\pi} \int_0^\infty dk \, \cos(\tau k) \, \overline{\mathcal{D}}(k^2)$$

$$= \frac{\exp(-\tau \lambda c_{\varphi/2})}{\cancel{2} \lambda s_\varphi} \left[\cancel{\left(1 + \frac{\lambda^2}{M^2}\right)} s_{\varphi/2} \cos(\tau \lambda s_{\varphi/2}) + \cancel{\left(1 - \frac{\lambda^2}{M^2}\right)} c_{\varphi/2} \sin(\tau \lambda s_{\varphi/2}) \right]$$

0

$\cancel{2}$

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$$m_\gamma \geq 0: \quad \boxed{\lambda^2 \geq \frac{\mu^2}{3}}$$

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$$= \frac{\exp(-\tau \lambda c_{\varphi/2})}{\cancel{2} \lambda s_\varphi} \left[\cancel{\left(1 + \frac{\lambda^2}{M^2}\right)} s_{\varphi/2} \cos(\tau \lambda s_{\varphi/2}) + \cancel{\left(1 - \frac{\lambda^2}{M^2}\right)} c_{\varphi/2} \sin(\tau \lambda s_{\varphi/2}) \right]$$

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- Applied to the RGZ gluon propagator: $\overline{\mathcal{D}}(k^2) = \dfrac{k^2 + M^2}{k^4 + k^2 m^2 + \lambda^4}$
  - 1-d dual in configuration space: strong constraint: $\boxed{\lambda^2 \geq \mu^2}$

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$$= \frac{\exp(-\tau\lambda c_{\varphi/2})}{2\lambda s_\varphi}\left[\left(1 + \frac{\lambda^2}{M^2}\right) s_{\varphi/2} \cos(\tau\lambda s_{\varphi/2}) + \left(1 - \frac{\lambda^2}{M^2}\right) c_{\varphi/2} \sin(\tau\lambda s_{\varphi/2})\right]$$

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$$\boxed{\left.\frac{d^2}{d\tau^2}\Delta(\tau)\right|_{\tau=0} = \frac{\lambda}{4c_{\varphi/2}}[1 - \mu^2/\lambda^2] \;\; \geq \;\; 0}$$

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  - 4-d dual in configuration space:

$$\mathcal{d}(\chi) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x}\, \overline{\mathcal{D}}(k^2)$$

$$= \frac{1}{4\pi^2\chi} \sum_{i=\pm} a_i b_i K_1(\chi b_i)$$

$$\chi = \sqrt{x^2}$$

$$a_\pm = \frac{r \pm m^2 \mp 2M^2}{2r}$$

$$b_\pm = \tfrac{1}{\sqrt{2}}\left(m^2 \pm r\right)^{1/2}$$

$$r = \sqrt{(m^2-2\lambda^2)(m^2+2\lambda^2)}$$

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$$= \frac{1}{4\pi^2 \chi} \sum_{i=\pm} a_i b_i K_1(\chi b_i) \overset{\chi \simeq 0}{\approx} \frac{1}{4\pi^2 \chi^2}$$

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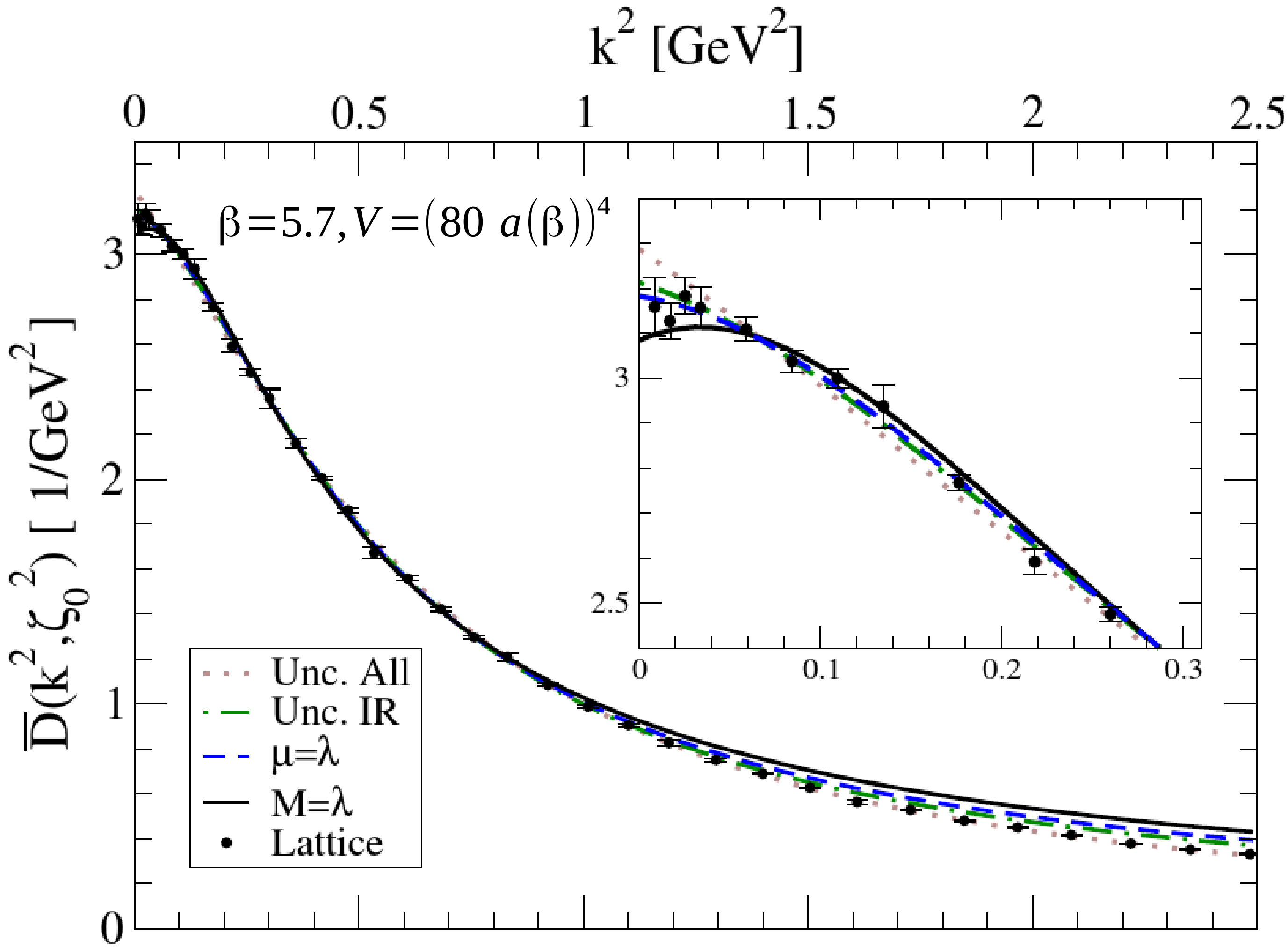
- It behaves pretty like the free-propagator 4-d dual at low-distance,
- but the partonic rest-mass structure is only exposed by the 1-d dual with the appropriate constraints!

Interpreting the lattice data



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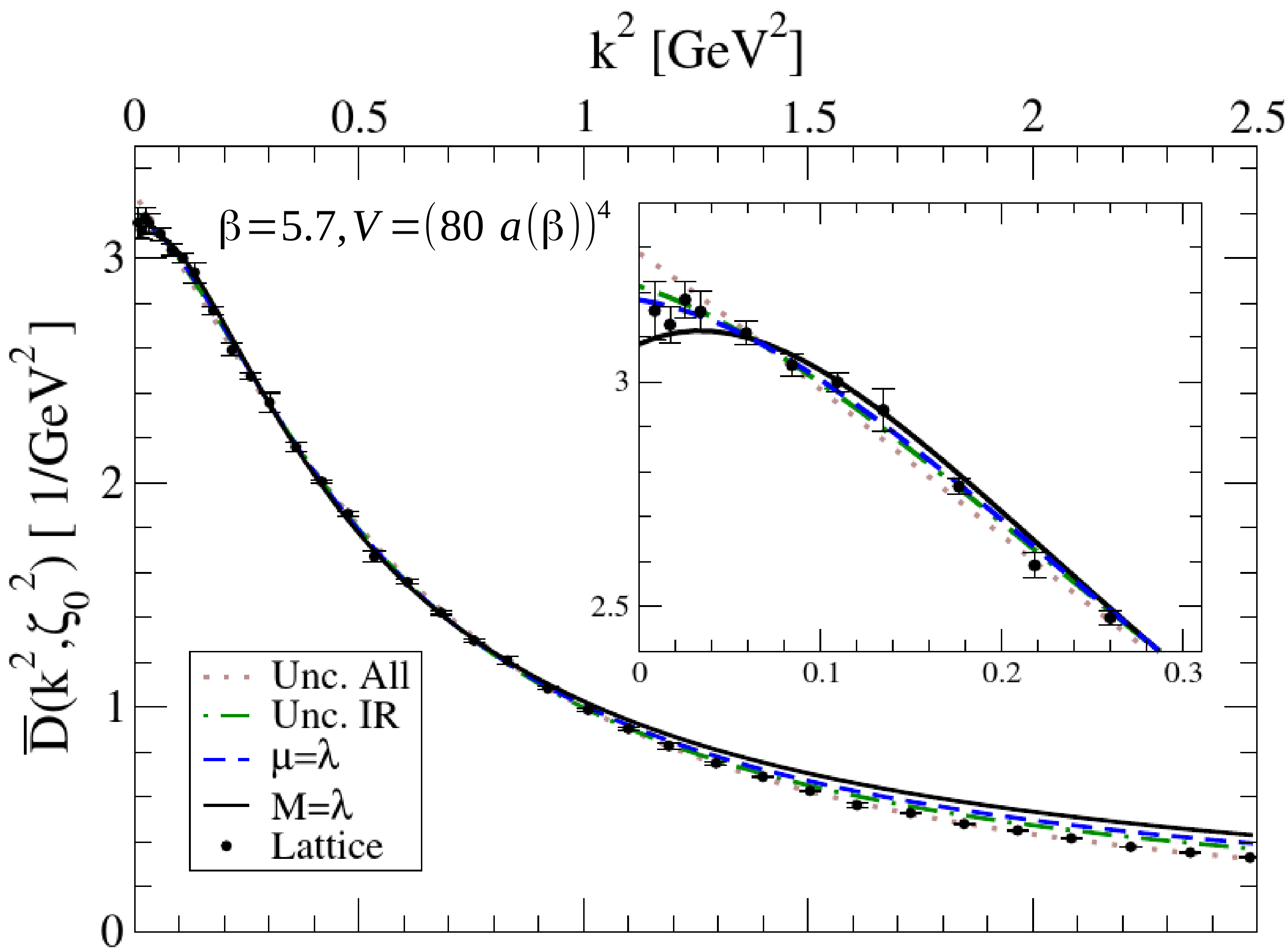
$$\overline{D}(k^2,\zeta_{GZ}^2) := \overline{D}(k^2)$$

$$\overline{D}(k^2,\zeta_0^2) = \frac{\overline{D}(k^2,\zeta_{GZ}^2)}{\zeta_0^2\,\overline{D}(\zeta_0,\zeta_{GZ})}$$

$$\overline{D}(k^2,\zeta_0^2) = \frac{D_{Lat}(k^2,a)}{\zeta_0^2 D_{Lat}(\zeta_0^2,a)}$$



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(A) quenched	k_0	ζ_0	λ	M	z_0	M/λ	μ/λ
unconstrained	4.5	1.1	0.84	2.10	0.43	2.49	2.33
	ζ_0	1.1	0.72	1.09	0.75	1.50	1.27
weak	ζ_0	1.0	0.59	0.59	1.04	1	0.45
strong	ζ_0	1.0	0.68	0.88	0.84	1.29	1

$$g^2\langle A^a_\mu A^a_\mu\rangle = \frac{32}{3}\mu^2 \approx \boxed{40\,GeV^2}$$

$$g^2\langle A^a_\mu A^a_\mu\rangle_{ph} = 1-7\,GeV^2$$

$$\overline{D}(k^{2,}\zeta^2_{GZ}) := \overline{D}(k^2)$$

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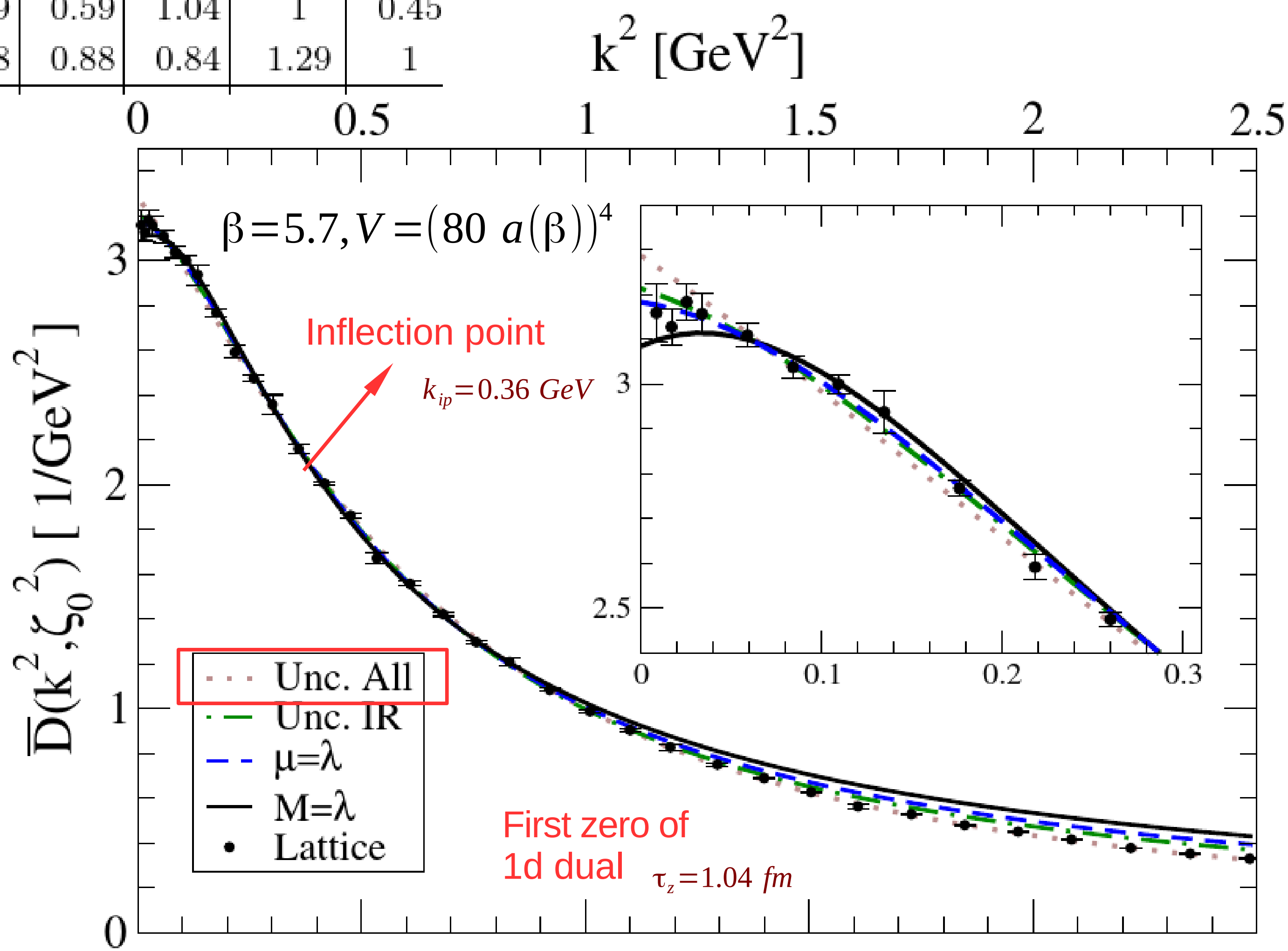

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$$g^2\langle A_\mu^a A_\mu^a\rangle = \frac{32}{3}\mu^2 \approx \boxed{9\,GeV^2}$$

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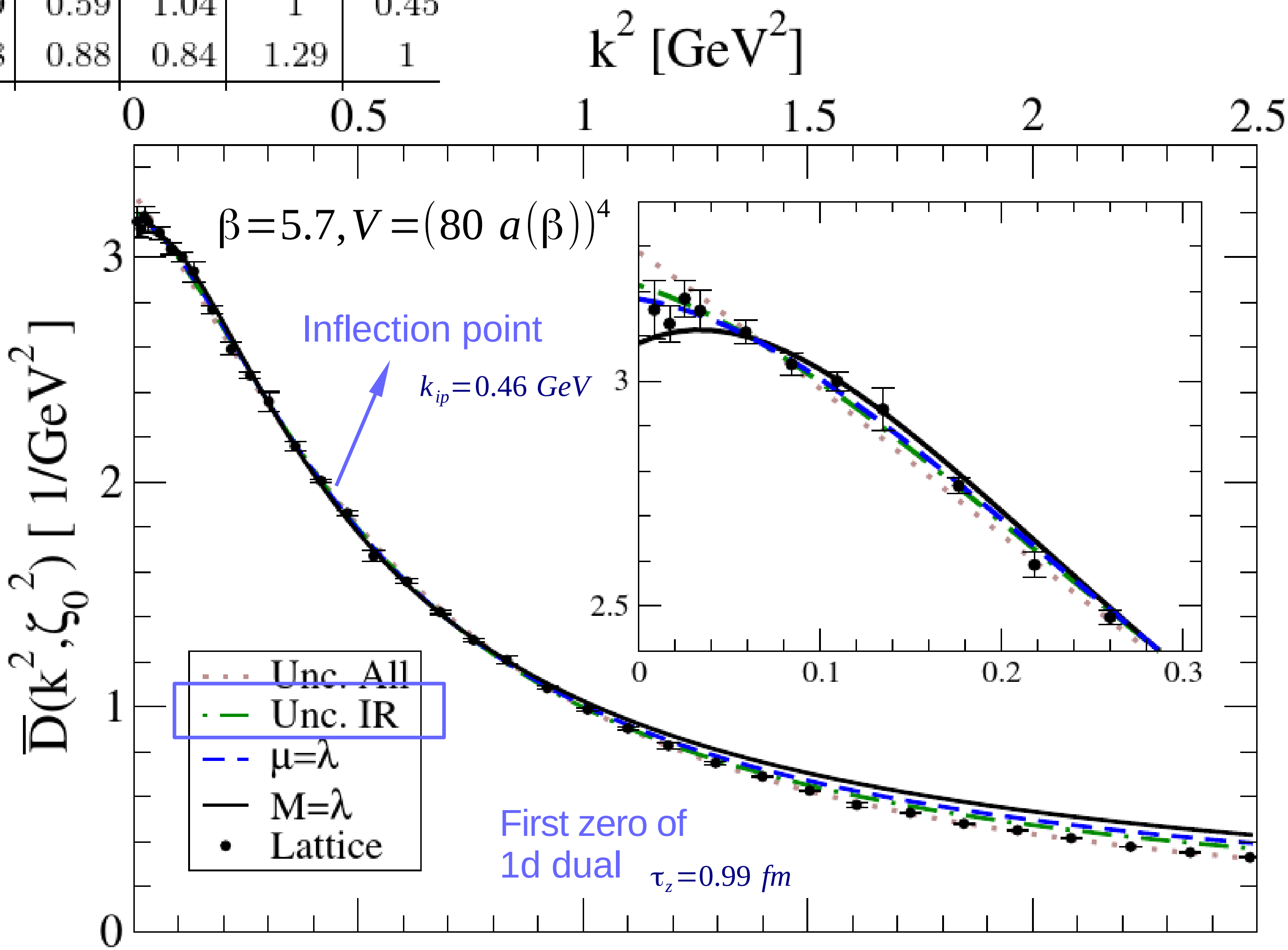

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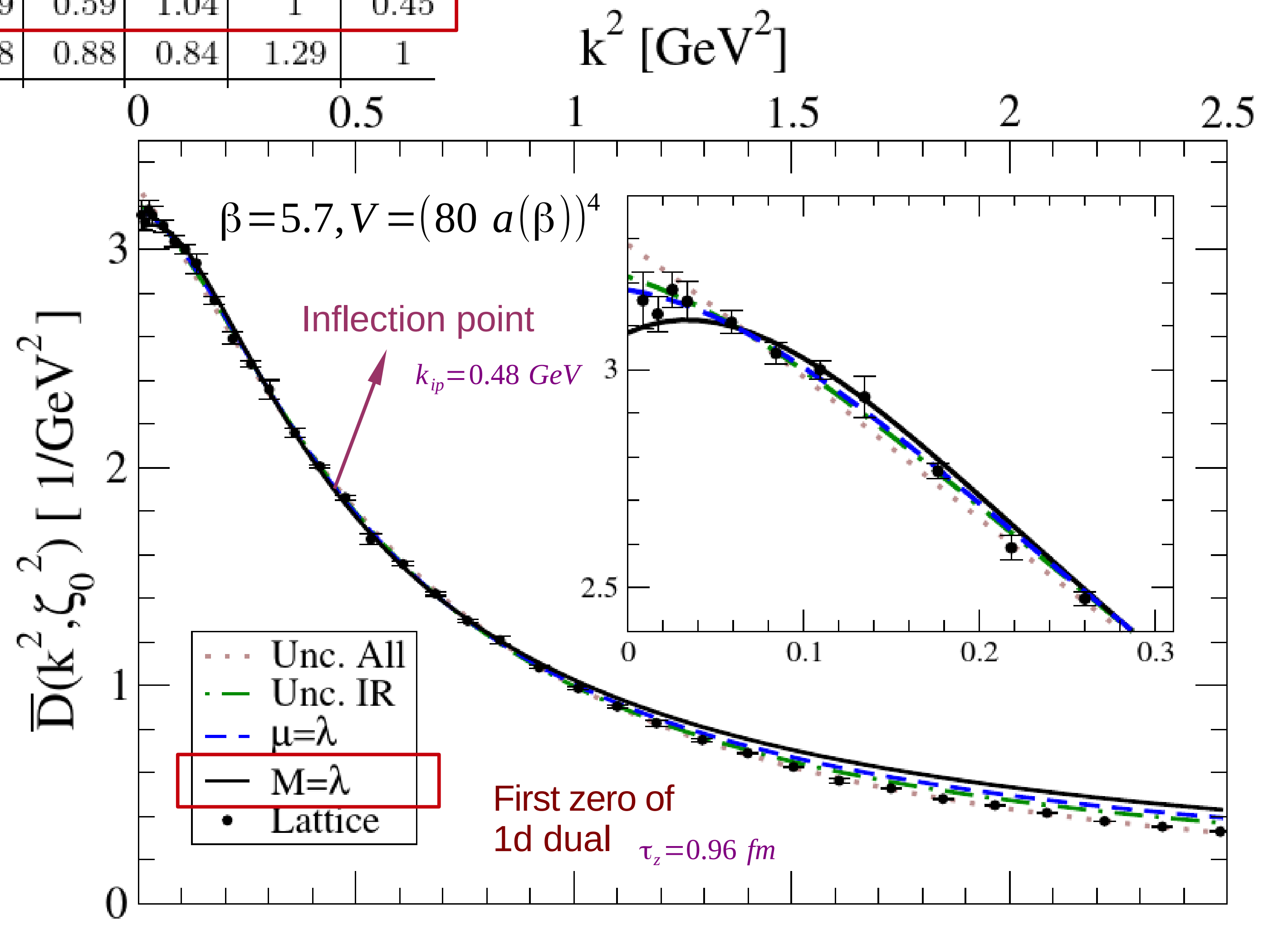

Interpreting the lattice data

(A) quenched	k_0	ζ_0	λ	M	z_0	M/λ	μ/λ
unconstrained	4.5	1.1	0.84	2.10	0.43	2.49	2.33
	ζ_0	1.1	0.72	1.09	0.75	1.50	1.27
weak	ζ_0	1.0	0.59	0.59	1.04	1	0.45
strong	ζ_0	1.0	0.68	0.88	0.84	1.29	1

$$g^2\langle A^a_\mu A^a_\mu\rangle = \frac{32}{3}\mu^2 \approx \boxed{5\ GeV^2}$$

$$g^2\langle A^a_\mu A^a_\mu\rangle_{ph} = 1-7\,GeV^2$$

$$\overline{D}(k^2,\zeta^2_{GZ}) := \overline{D}(k^2)$$

$$\overline{D}(k^2,\zeta^2_0) = \frac{\overline{D}(k^2,\zeta^2_{GZ})}{\zeta^2_0\ \overline{D}(\zeta_0,\zeta_{GZ})}$$

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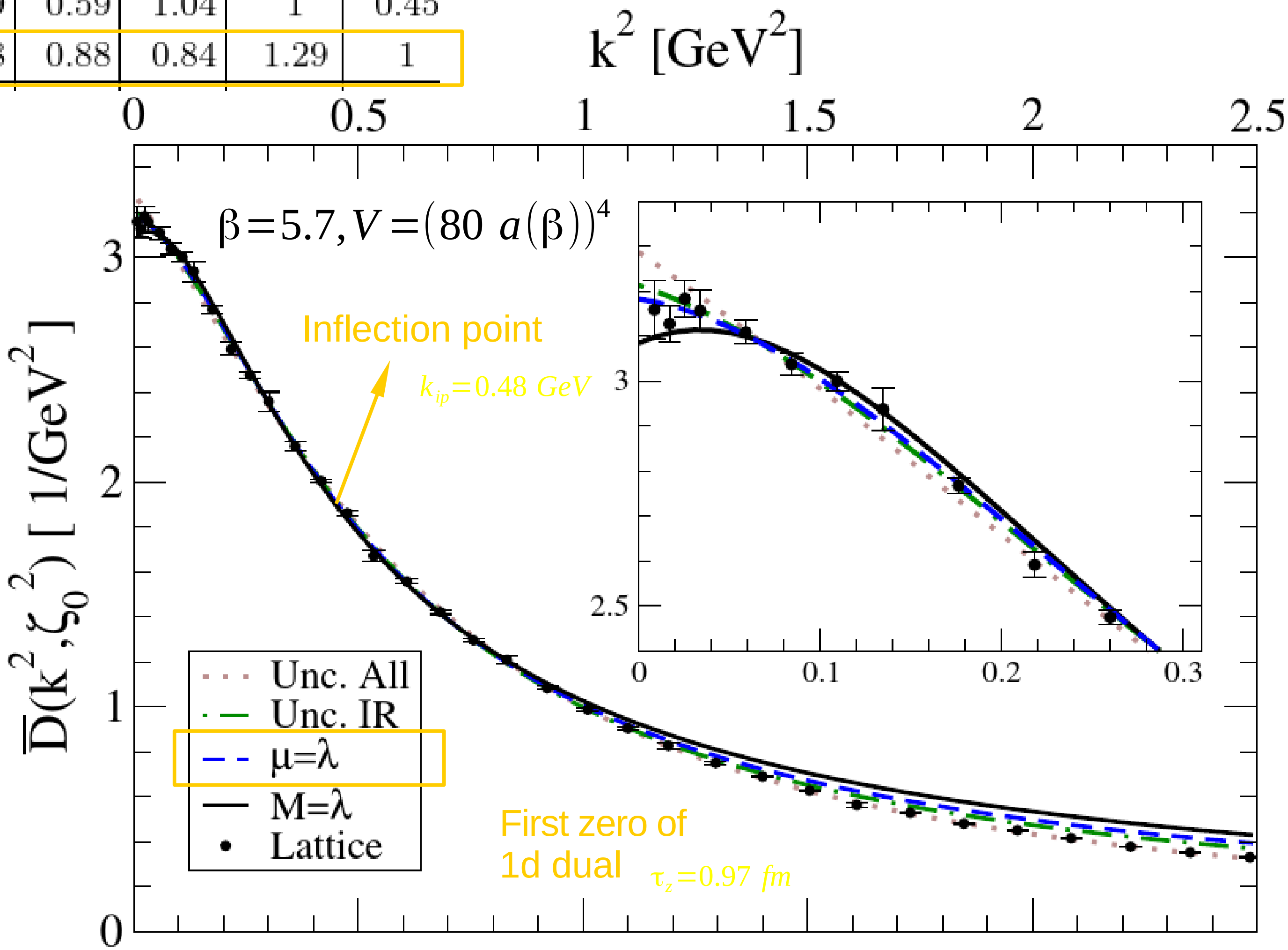

The Gribov and the gluon masses

- The RGZ scenario:

$$\overline{\mathcal{D}}(k^2) = \frac{k^2 + M^2}{k^4 + k^2 m^2 + \lambda^4}$$

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Gribov mass

$$\begin{cases} \lambda^4 = 2N m_\gamma^4 - \mu^2 M^2 \\ m^2 = M^2 - \mu^2 \end{cases}$$

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$$\left\{ m_g^2 = \frac{\lambda^4}{M^2} \right.$$

(a manifestation of more elaborated mechanisms as Schwinger's one!!!)

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- The RGZ + partonic scenario:
  - Weak constraint: $M^2 = \lambda^2 \, , \;\; \lambda^2 \geq \frac{\mu^2}{3}$
  - Strong constraint: $\lambda^2 \geq \mu^2$

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0.53 GeV 0.56 GeV

The Gribov and the gluon masses

$$g^2\langle A_\mu^a A_\mu^a\rangle = \frac{32}{3}\mu^2 \approx \boxed{4\ GeV^2}$$

$$g^2\langle A_\mu^a A_\mu^a\rangle_{ph} = 1-7\,GeV^2$$

(A) quenched	k_0	ζ_0	λ	M	z_0	M/λ	μ/λ
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str. $+m_\gamma = m_g$	ζ_0	1.0	0.67	0.84	0.87	1.26	0.94

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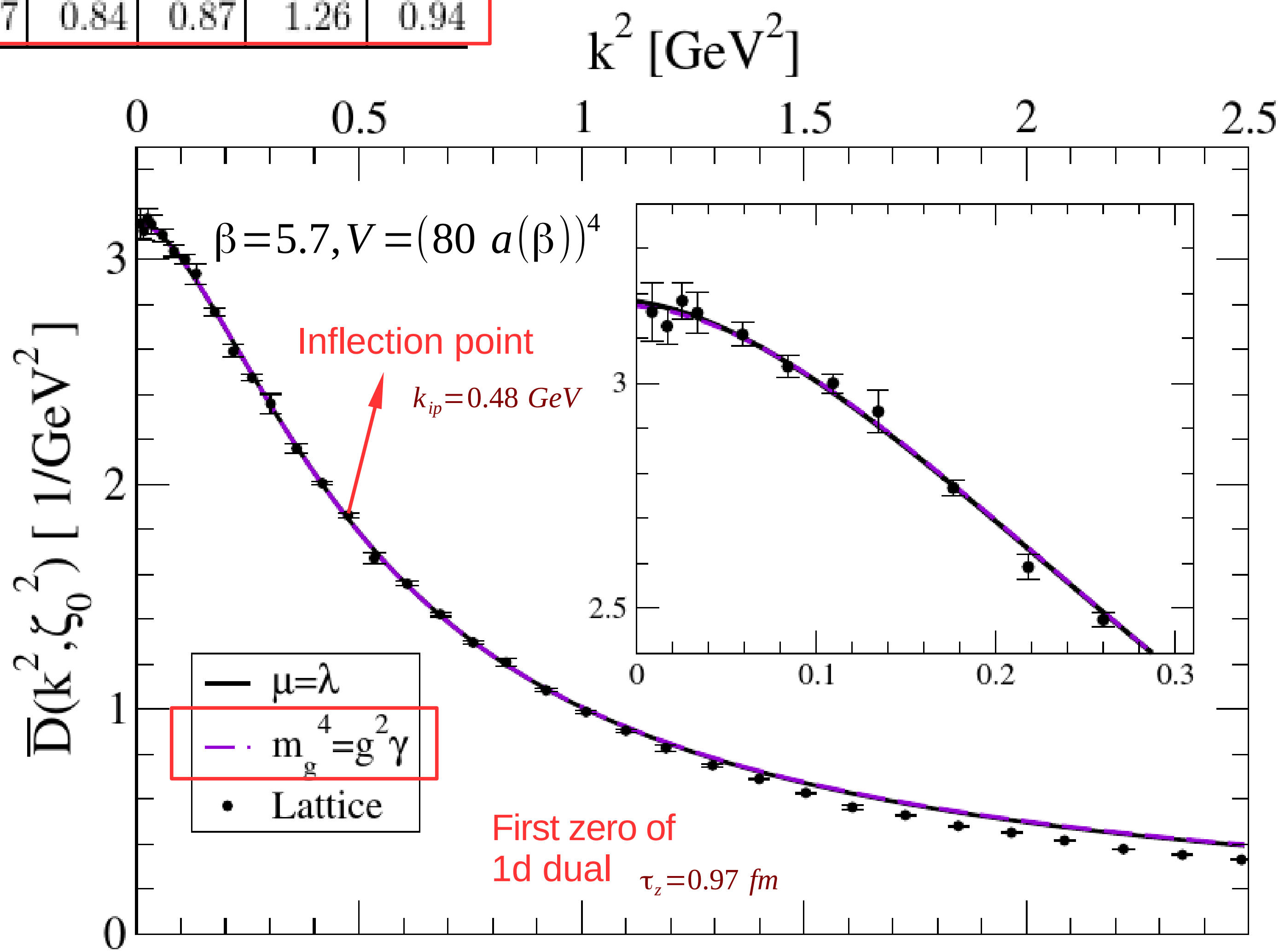

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  - Third synthetic and favoured case: $\boxed{\lambda^2 \geq \mu^2 \quad + \quad m_\gamma = m_g = 0.53\,GeV}$

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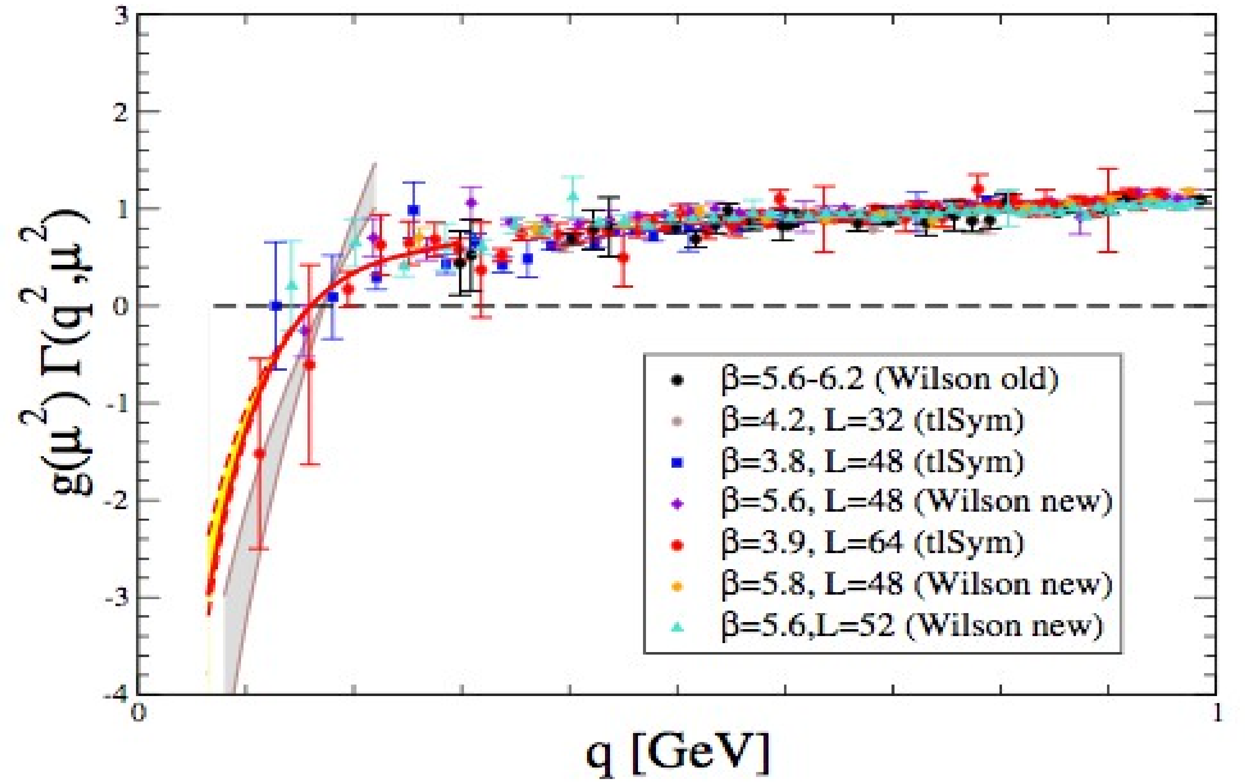
Conclusions:

- Both emergent phenomena, the appearance of a horizon scale and a gluon mass , play the same role in screening longwavelength gluon modes, thereby dynamically eliminating the Gribov ambiguities.
- Together, they set a confinement scale of the order of 1 GeV.
- We can thus sensibly conjecture that both emergent phenomena are equivalent!!!

Piece two: the zero-crossing of the three-gluon vertex



In collaboration with: A. Athenodorou, D. Binosi, Ph. Boucaud, F. De Soto, J. Papavassiliou and S. Zafeiropoulos

[Phys.Rev. D95 (2017) no.11, 114503]

[Phys.Lett. B761 (2016) 444-449]

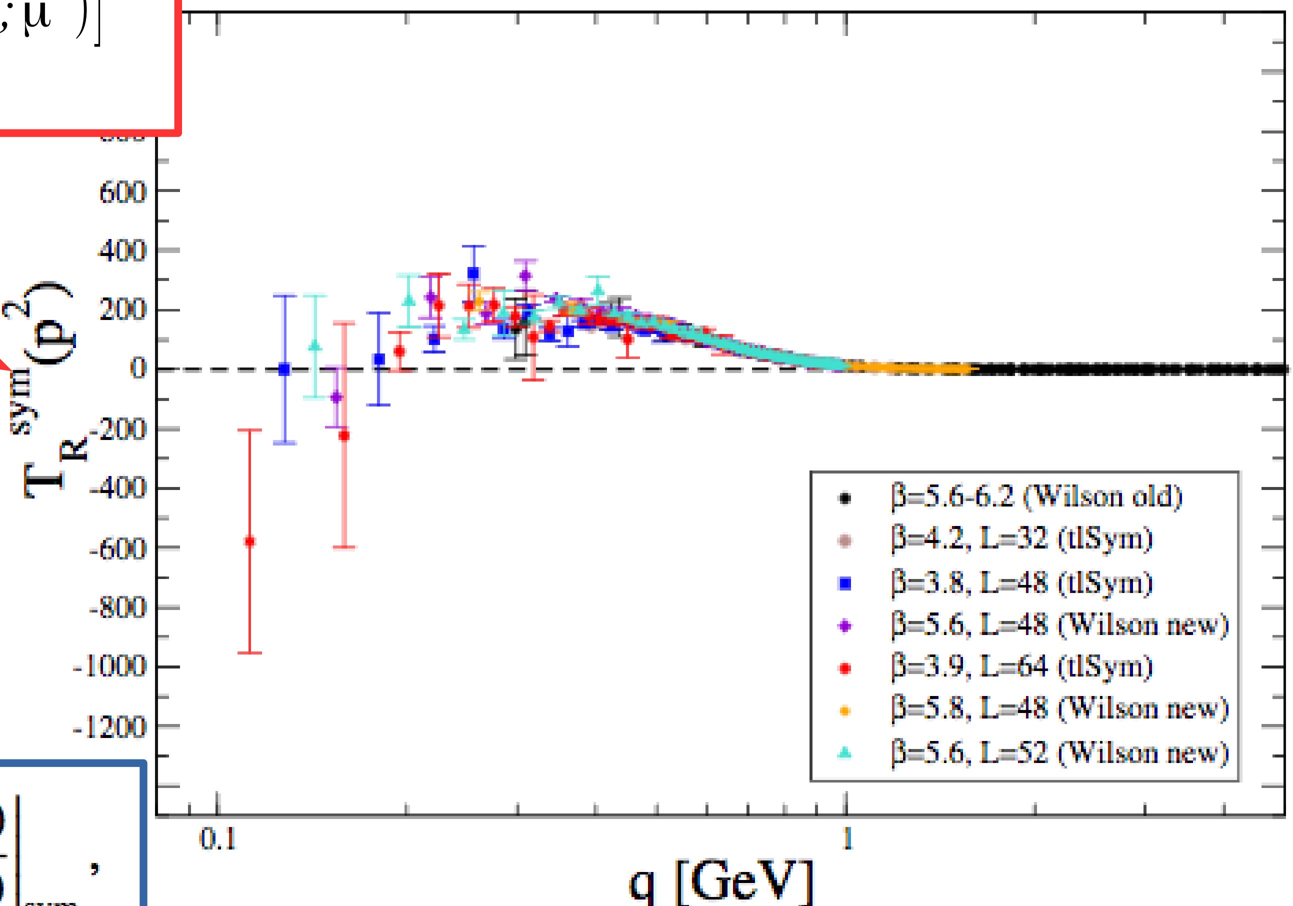
The zero-crossing of the three-gluon vertex

$$g^i(\mu^2)\,\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym, asym.$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3\frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$



$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\substack{\rm sym\\ \rm asym}},$$

Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This appears to happen below ~0.2 GeV.

The zero-crossing of the three-gluon vertex

M. Tissier, N. Wschebor, PRD84(2011)045018
A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
G. Eichmann et al.; PRD89(2014)105014
A.K. Cyrol et al.; arXiv:1605.01856[hep-ph]
A. Cucchieri, A. Maas, T. Mendes;
PRD74(2006)014503;PRD77(2008)094510

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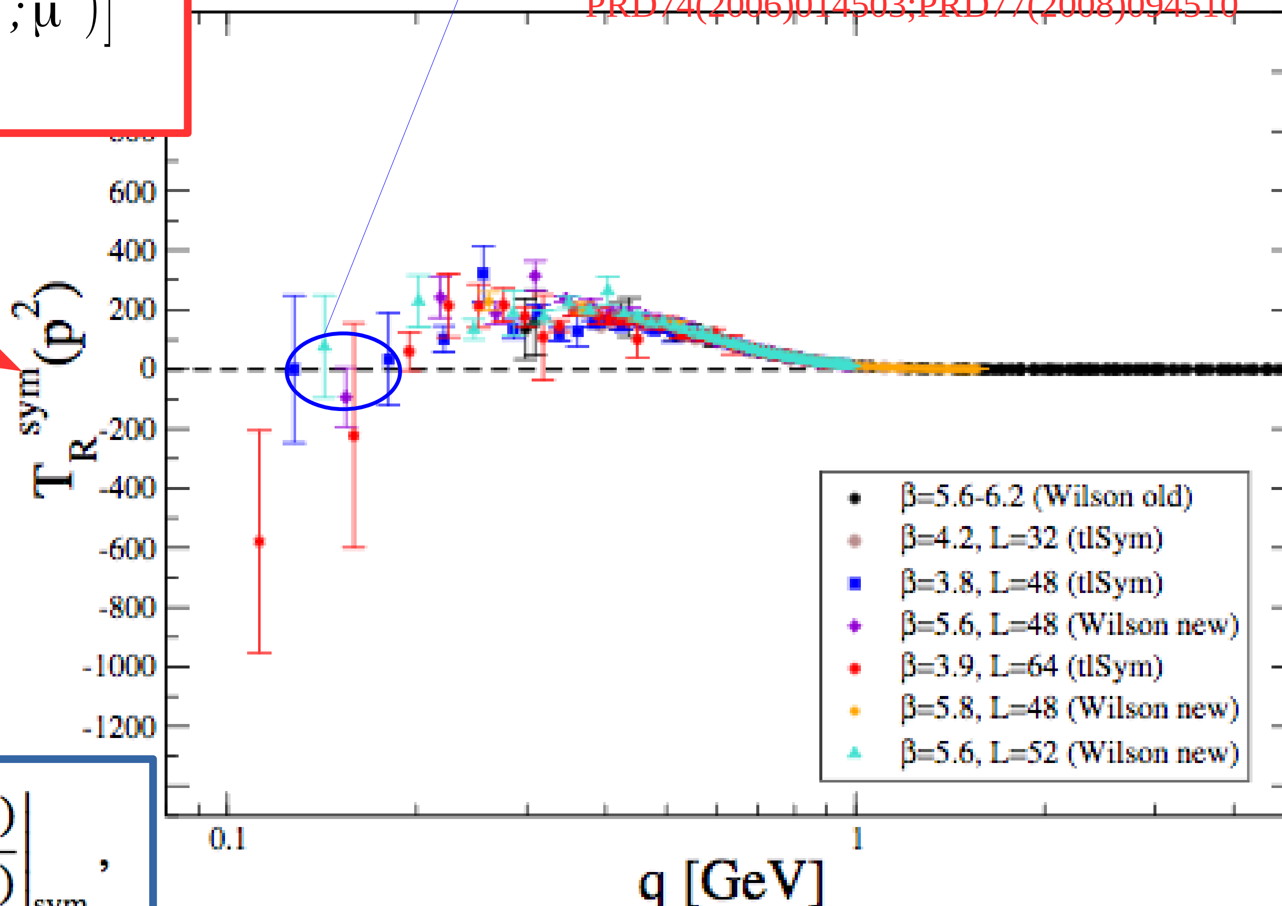
zero-crossing



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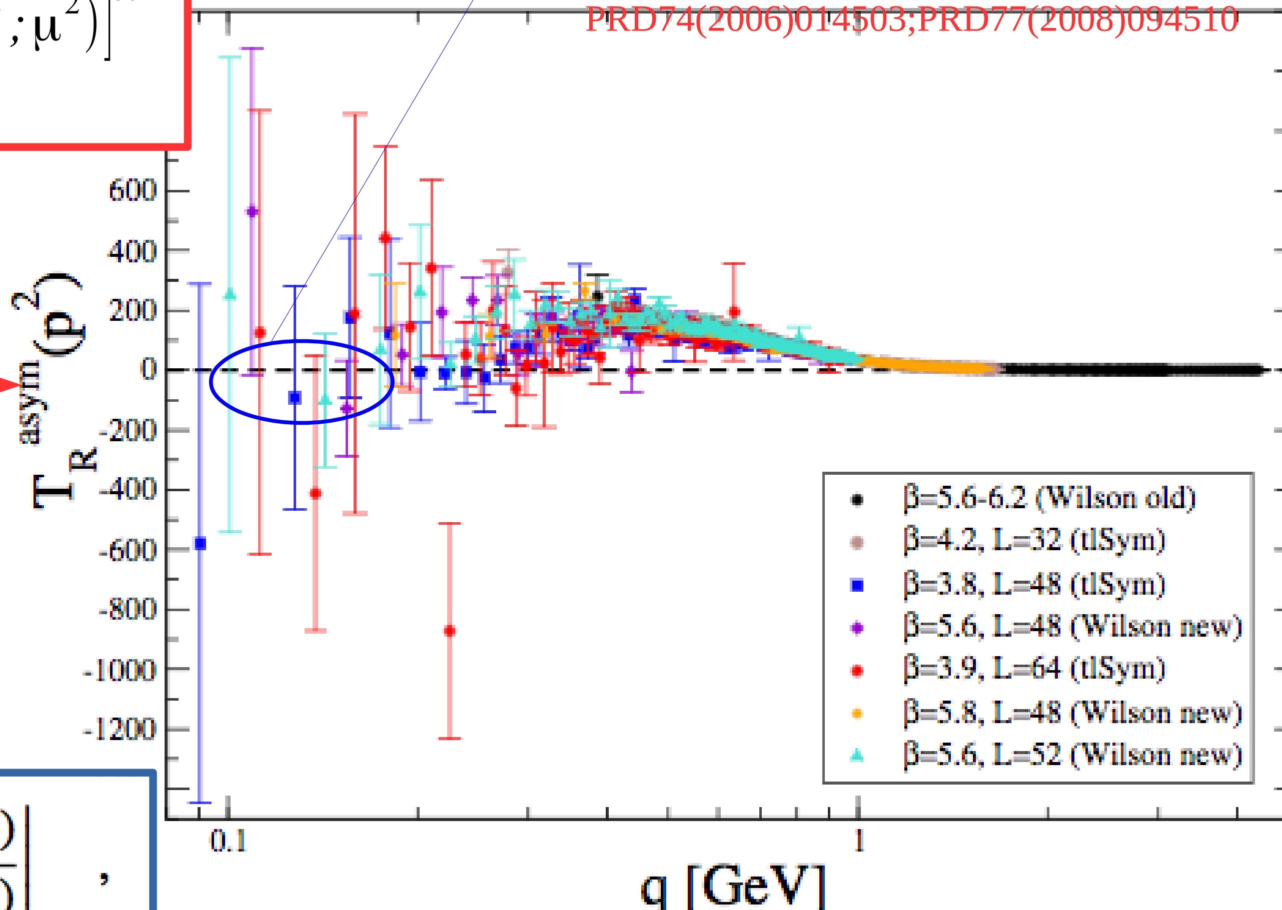
Zero-crossing?



$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\substack{\rm sym\\ \rm asym}},$$

Let's consider now the asymmetric case: the results are much noisier (surely because of the zero-momentum gluon field in the correlation function), although there appear to be strong indications for the happening of the zero-crossing.

The zero-crossing of the three-gluon vertex

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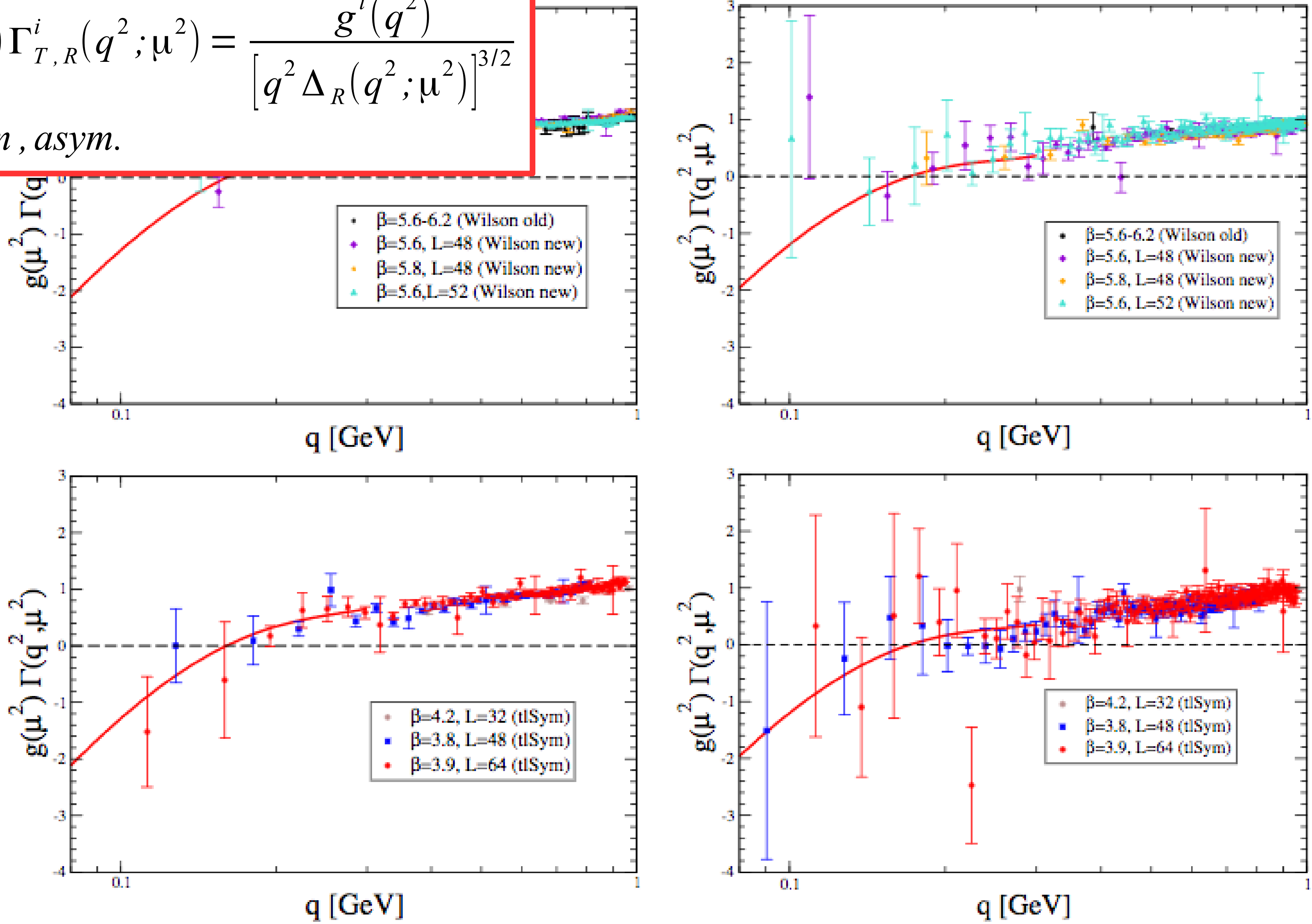
$i = sym, asym.$

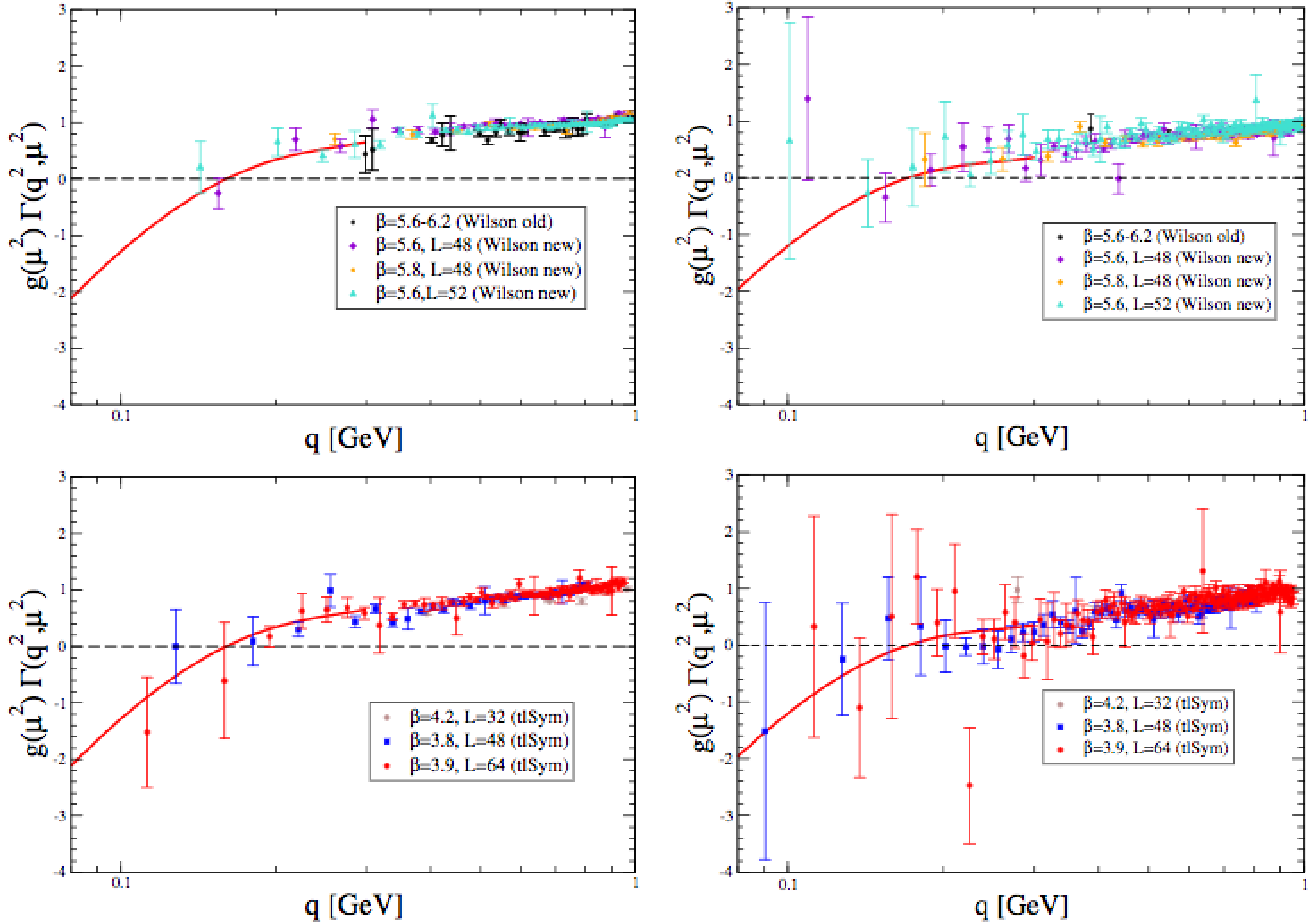


After le
trend is

The zero-crossing of the three-gluon vertex

$g^i(\mu^2$

$i = sym$



After le

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The zero-crossing of the three-gluon vertex

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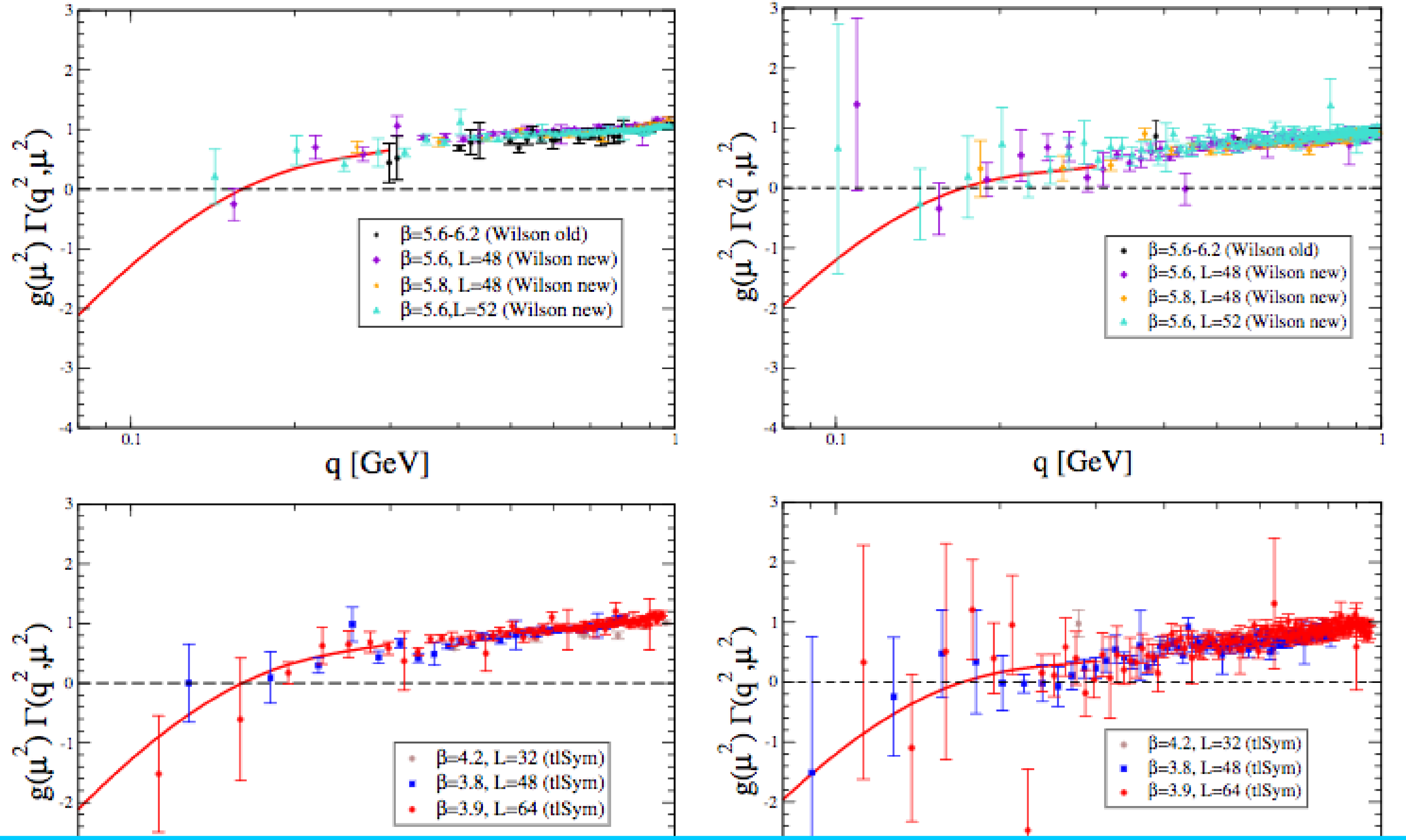


After leg amputation, the 1PI form factor for the tree-level tensor shows clearly the zero-crossing. The trend is the same for both Wilson and tlSym actions and symmetric and asymmetric configurations.

The zero-crossing of the three-gluon vertex

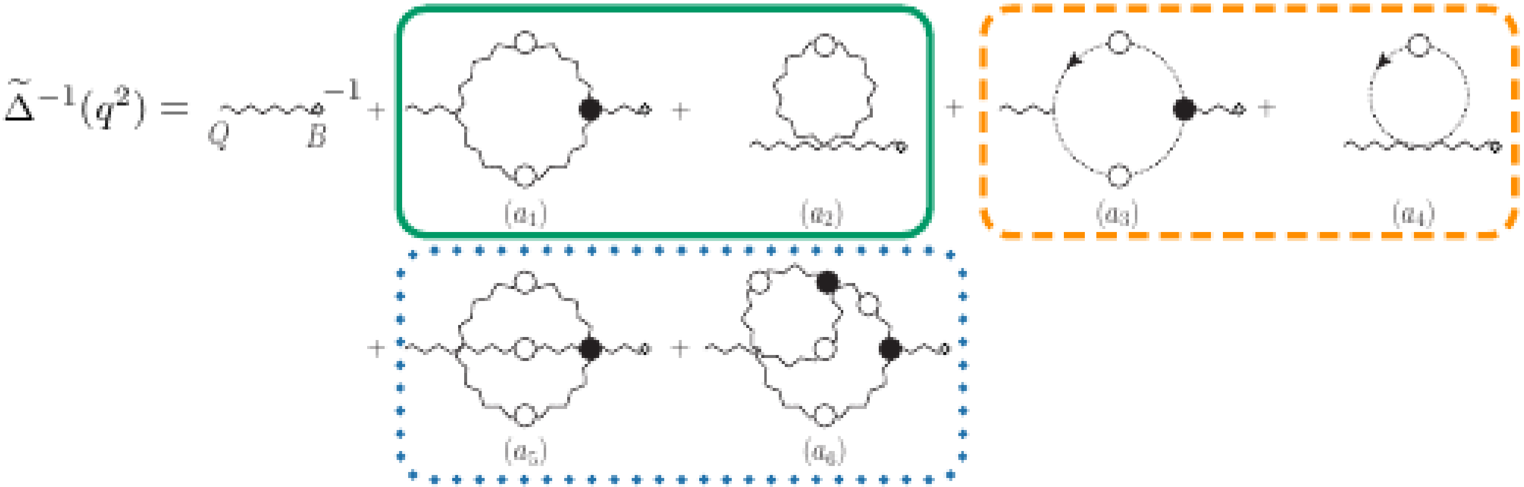
A.C Aguilar et al.; PRD89(2014)05008

DSE-based explanation:

In PT-BFM truncation

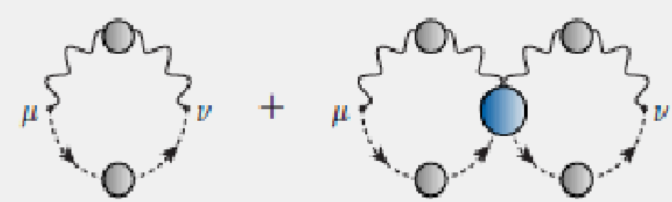
$$\boxed{\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) \;+\; \ldots}$$

cf. Daniele's, Joannis' or Cristina's talk!!!



$$[1 + G(q^2)]^2\Delta^{-1}(q^2) = \widehat{\Delta}^{-1}(q^2).$$

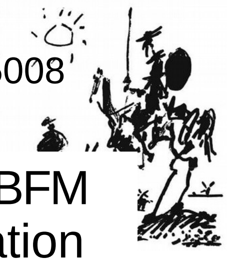
$\Lambda_{\mu\nu}(q) =$



$$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008



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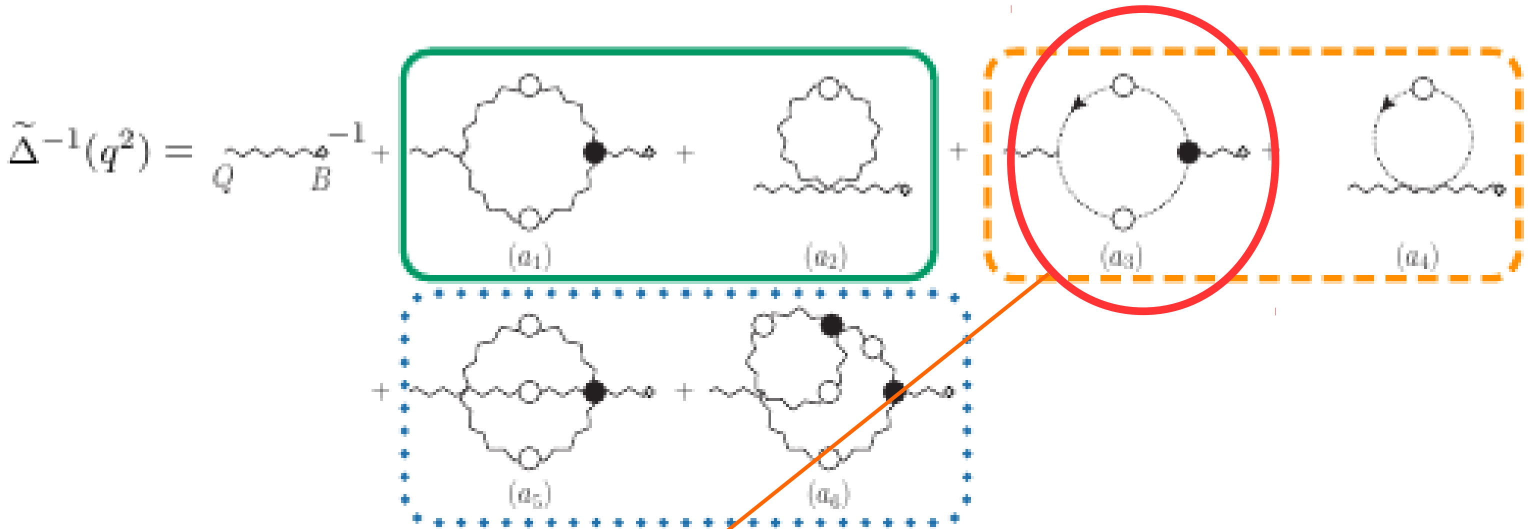

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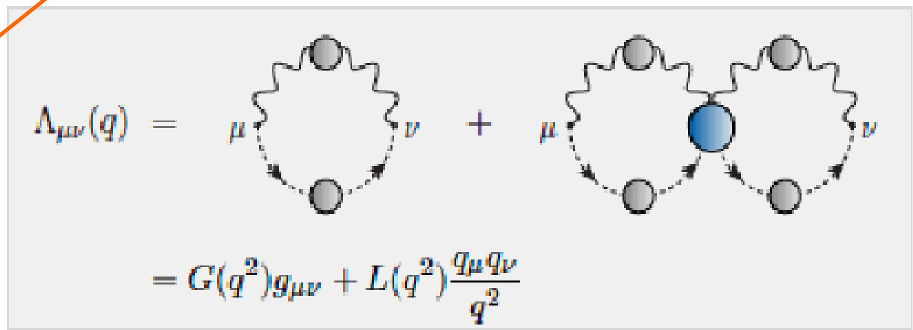
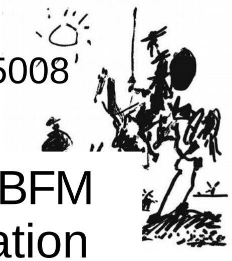


$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008



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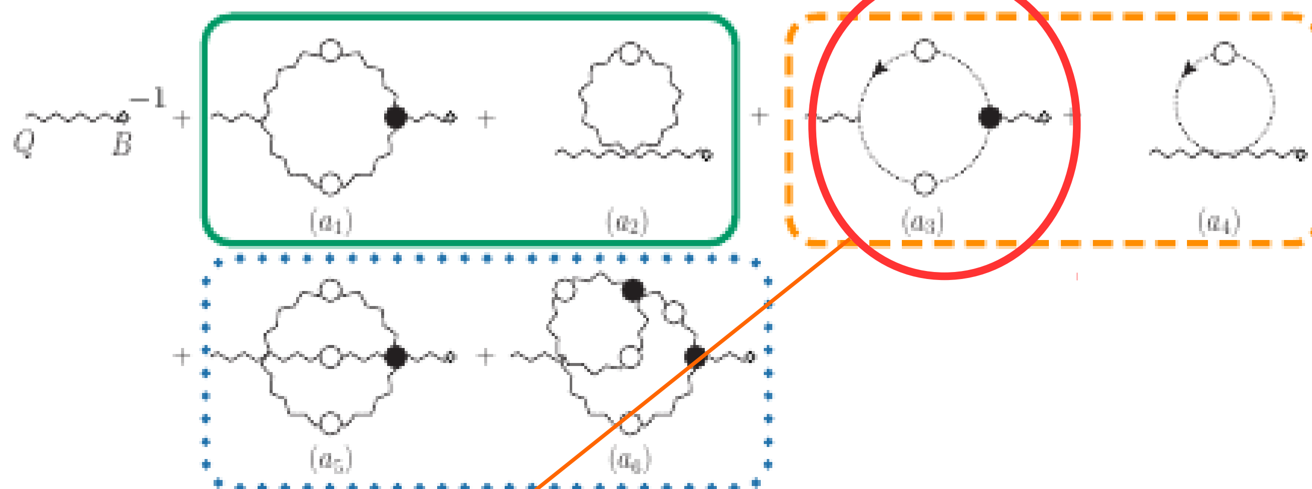

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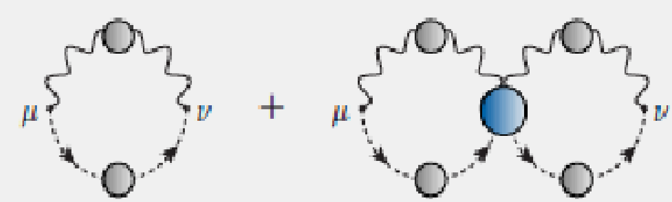

$$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

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d=4

$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

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aniele's,
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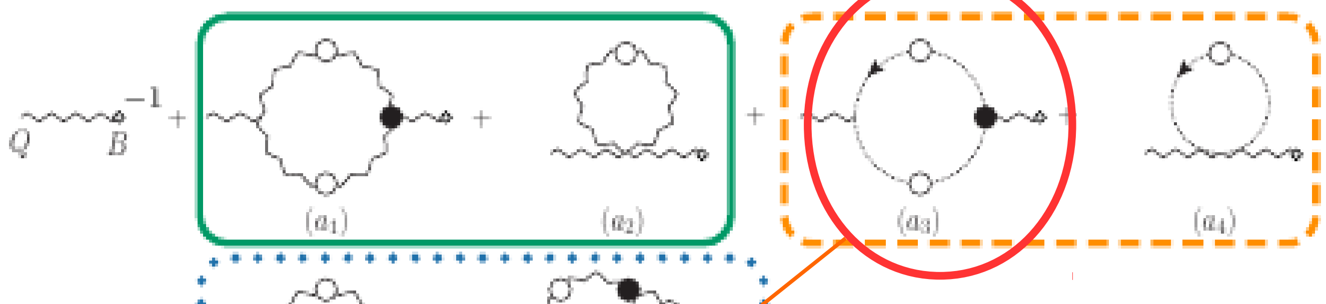
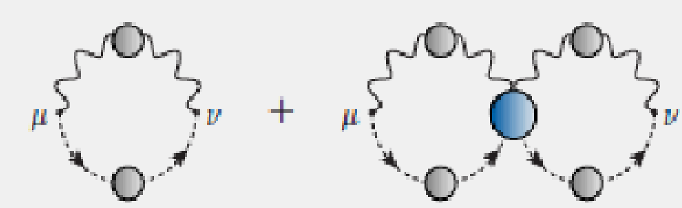


A logarithmic divergent contribution at vanishing momentum, pulling down the 1PI form factor and generating a zero crossing, can be understood within a DSE framework.

$$[1+G(q^2)]^2\Delta^{-1}(q^2) = \widehat{\Delta}^{-1}(q^2).$$

$\Lambda_{\mu\nu}(q) =$



$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$

$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

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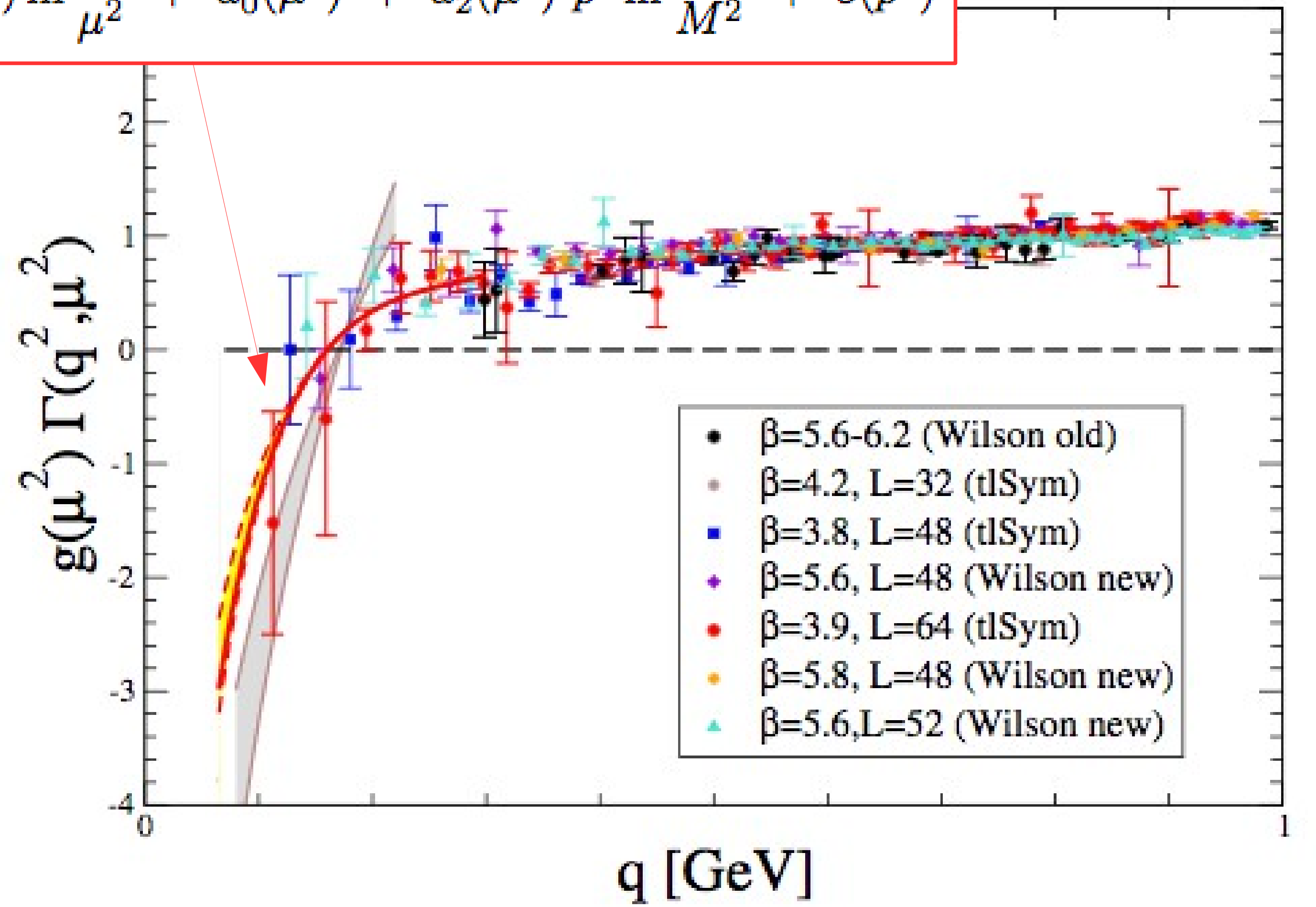
The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

Ph. Boucaud et al.; PRD95(2017)114503

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) \;=\; a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} \;+\; a_0^i(\mu^2) \;+\; a_2^i(\mu^2)\; p^2 \ln\frac{p^2}{M^2} \;+\; o(p^2)$$

i = symmetric



We can thus perform a fit, only over a deep IR domain, of our data to a DSE-grounded formula and describe the behaviour of the 1PI form factor.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

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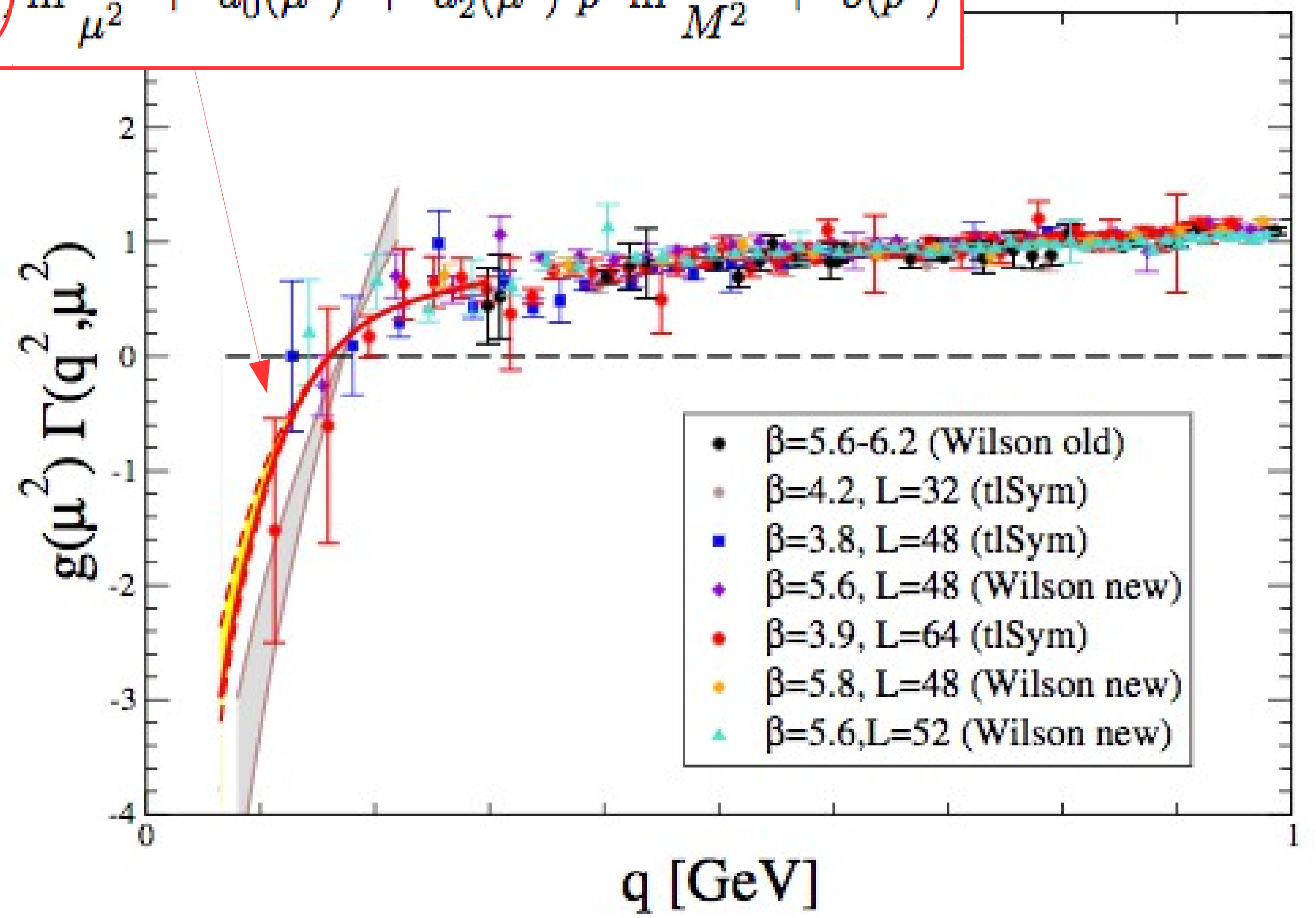
$$g_R^i(\mu^2)\,c\,F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



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The zero-crossing of the three-gluon vertex

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Ph. Boucaud et al.; PRD95(2017)114503

i = asymmetric

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) \;=\; a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} \;+\; a_0^i(\mu^2) \;+\; a_2^i(\mu^2)\,p^2\ln\frac{p^2}{M^2} \;+\; o(p^2)$$

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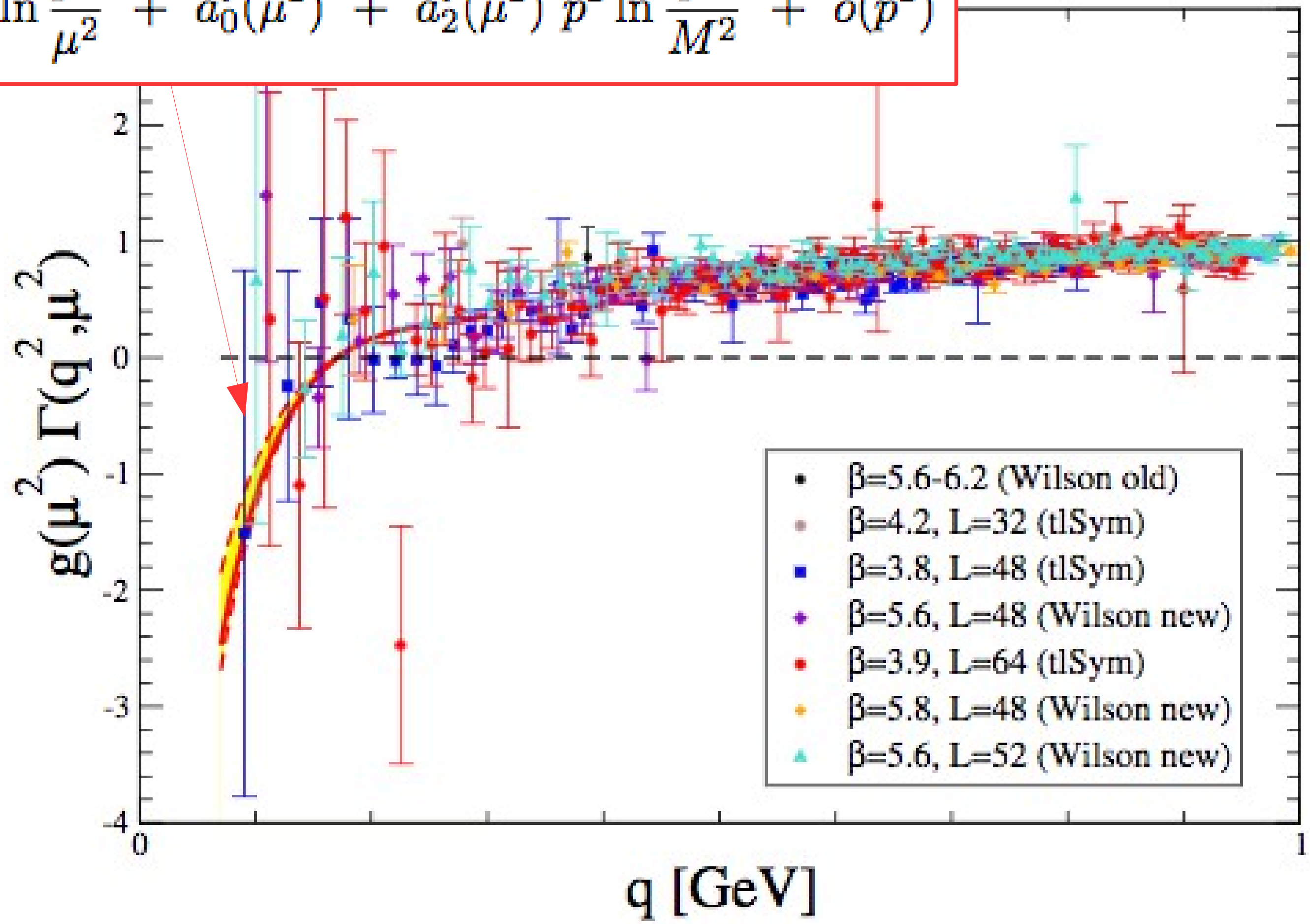

The low-momenta asymptotic 1PI form factor obtained from DSE within the PT-BFM is fully consistent with lattice data for both symmetric and asymmetric kinematic configurations.

Conclusions

- Lattice contemporary results for the three-gluon Green's functions provide, as a main feature, a zero-crossing at very low-momenta...

- … that can be understood as being driven by a negative logarithmic singularity for the 3-gluon 1-PI vertex.

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Thank you!!!