Graduated Visitor Future postdoc

ON THE STRUCTURE OF NEUTRAL PSEUDOSCALARS

Khépani Raya-Montaño

Emergent mass and its consequences in the SM
Sep 17 – Sep 21. ECT*, Trento, Italy

Motivation

- Understanding strong interactions is still being a challenge for physicists, even decades after the formulation of the fundamental theory of quarks and gluons, namely, **Quantum Chromodynamics** (**QCD**).

- QCD is characterized by two emergent phenomena: **confinement** and **dynamical chiral symmetry breaking** (**DCSB**), which have far reaching consequences in the hadron spectrum and their properties.

- Due to the non perturbative nature of QCD, unraveling the hadron structure, from the fundamental degrees of freedom, is an outstanding problem.

- **Dyson-Schwinger equations** (**DSEs**) combine the IR and UV behavior of the theory at once. Therefore, DSEs are an ideal platform to study quarks, gluons and hadrons.

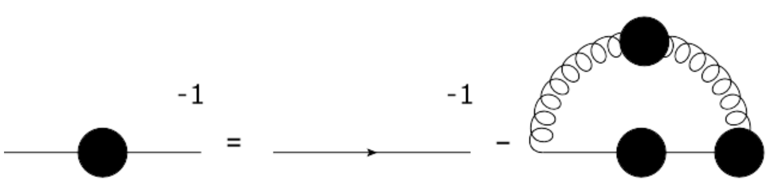
Outline

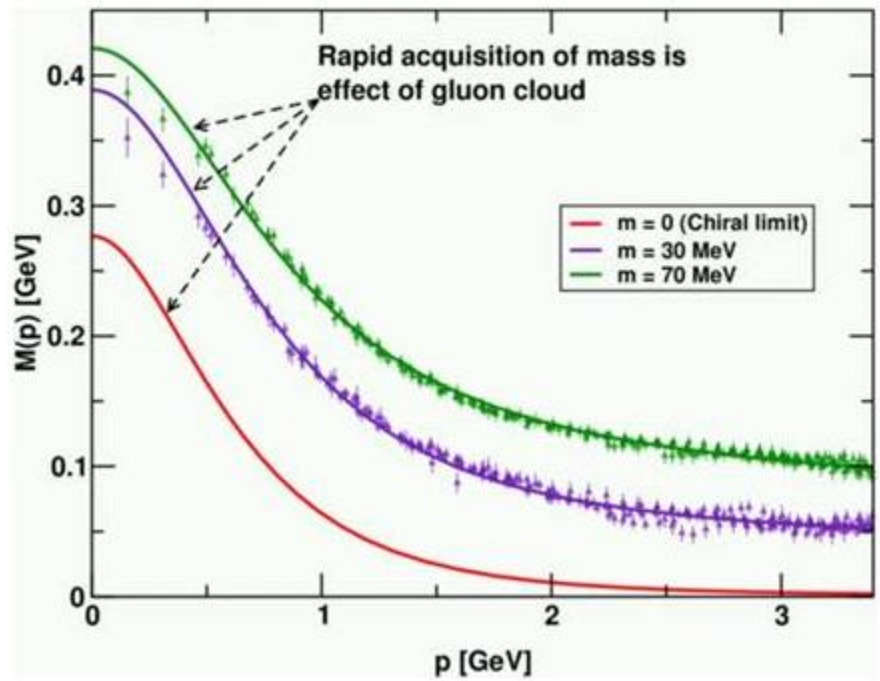
1. **The basics (DSEs)**
   - **Quark propagator and Bethe-Salpeter equation**
   - Perturbation theory integral representations
2. Parton distribution amplitudes
3. Two photon transition form factors
4. η-η' PDAs and TFFs
5. Conclusions and scope

The basics: Dyson-Schwinger equations

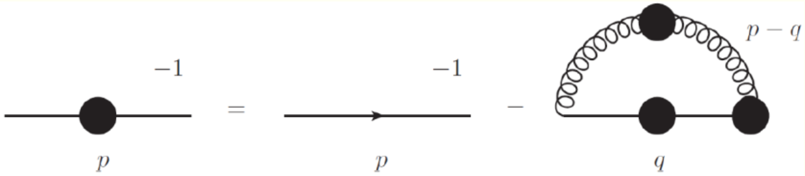
- DCSB is a critical emergent phenomenom in QCD. It is the most important mass generating mechanism for visible matter in the universe (~98% of proton mass).
- Quark propagator encodes this phenomenom in the mass function, M(p^2).

$$S(p;\zeta) = \frac{Z(p^2;\zeta)}{p^2 + M(p^2)}$$



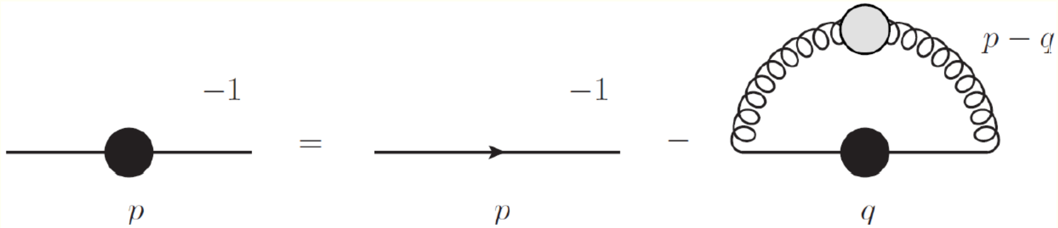


Quark propagator



- DSEs are an infinite tower of coupled equations. To extract the encoded physics, one must systematically truncate.

- The simplest symmetry preserving truncation, which gives a correct description of pseudoscalar and light vector meson phenomenology, is the **rainbow** truncation:



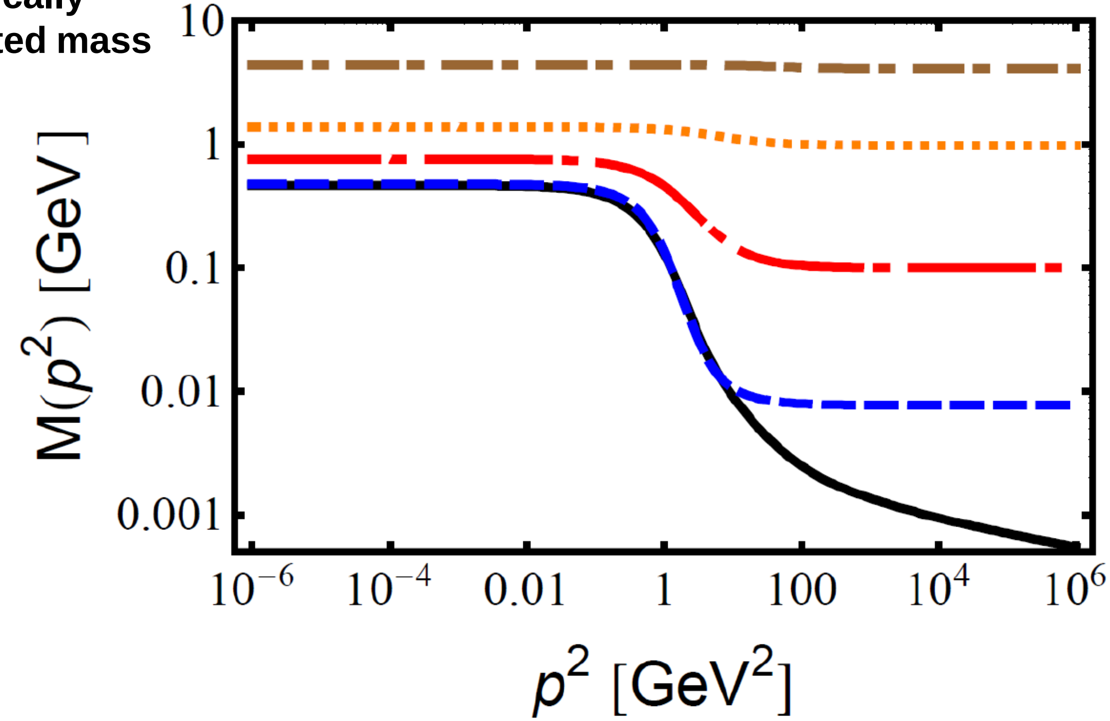
$$S^{-1}(p,\zeta) = [Z_{2F} S_0^{-1}(p)] + \int_q^{\Lambda} G((p-q)^2) D^0_{\mu\nu}(p-q,\zeta) \frac{\lambda^a}{2}\gamma_\mu S(q) \frac{\lambda^a}{2}\gamma_\nu,$$

$$S(p,\zeta) = Z(p^2;\zeta^2)(i\gamma\cdot p + M(p^2))^{-1} = (i\gamma\cdot p\, A(p^2;\zeta^2) + B(p^2;\zeta^2))^{-1}$$

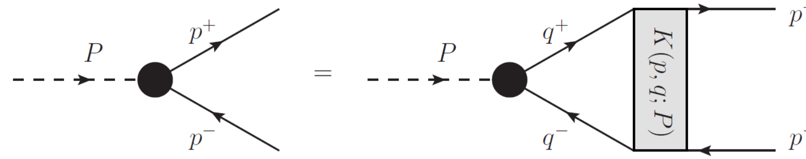
$G(k^2)$: Phys.Rev. C84, 042202(R) (2011)

Mass function

- **Mass function** for different current-quark masses: The **lighter** the **quark** is, the **stronger** the effect of **DCSB** is. Even when m=0, a dynamically generated mass appears (this is DCSB).



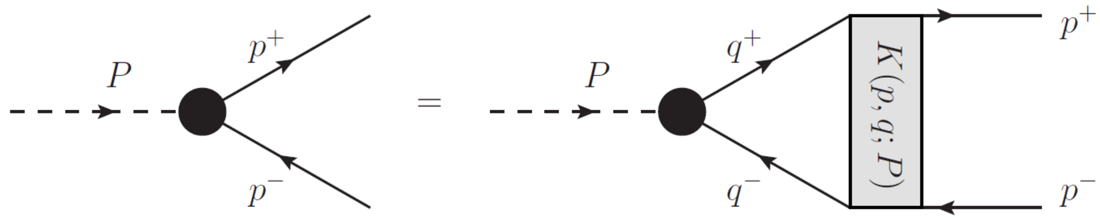
Bethe-Salpeter equation



- Quarks and gluons are not found free in nature, they form hadrons. Baryons are colorless bound states of three quarks and mesons are colorless bound states of quark-antiquark pair.

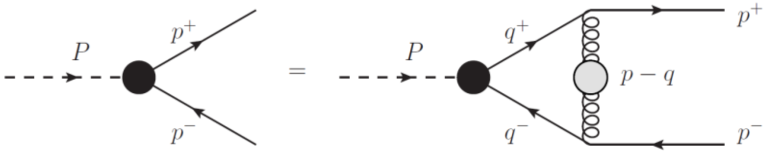
- The **Bethe-Salpeter equation** (**BSE**) is the equation in quantum field theory that describes meson-like systems:

$$\Gamma_M(p;P) = \int_q K(q,p;P)[S(q^+)\Gamma_M(q;P)S(q^-)] \ , \ q^\pm = q \pm P/2 \ .$$



- $\Gamma(p;P)$ is the **Bethe-Salpeter amplitude** (**BSA**). $\mathrm{K}(q,p;P)$ is the scattering kernel, which should be determined and is related to the truncation of the gap equation.

Bethe-Salpeter equation



- The Interaction kernel, K(q,p;P), is related to the truncation of the gap equation via the axial vector Ward-Takahashi identity (**Phys.Lett. B733 (2014) 202-208**, Qin et al.):

$$[\Sigma(p^+)\gamma_5 + \gamma_5\Sigma(p^-)] = \int_q K(p,q;P)[\gamma_5 S(q^-) + S(q^+)\gamma_5]\ .$$

- A kernel sufficient for many needs:

$$K(p,q;P) = -k^2 G(k^2) D^0_{\mu\nu}(k)\left[\frac{\lambda^c}{2}\gamma_\mu\right]\left[\frac{\lambda^c}{2}\gamma_\nu\right]\ ,\ k = p - q.$$

- Together with the rainbow approximation, it is called **rainbow-ladder** truncation (**RL**).

$$\Gamma_M(p;P) = -\int_q k^2 G(k^2) D^0_{\mu\nu}(k)\frac{\lambda^c}{2}\gamma_\mu[S(q^+)\Gamma_M(q;P)S(q^-)]\frac{\lambda^c}{2}\gamma_\nu\ .$$

Bethe-Salpeter equation

- For a pseudoscalar meson, the BSA is characterized by 4 tensor structures:

$$\Gamma_M(p;P) = \gamma_5(iE_M(p;P) + \gamma\cdot P F_M(p;P) + \gamma\cdot p\; p\cdot P G_M(p;P) + p_\alpha \sigma_{\alpha\beta} P_\beta H_M(q;P))$$

- In the case of the pion, the axial vector Ward-Takahashi identity relates the dominant BSA, E_π(p;P), with the quark propagator as follows:

$$f_\pi E_\pi(p^2) = B(p^2)$$

"Goldstone's theorem"

- The relationship above is **exact in the chiral limit**, and it implies that the **two-body** problem is **solved** (almost) completely, once solution of **one body** problem **is known**.

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Perturbation theory integral representations

- The quark propagator may be expressed as:

$$S(p;\zeta) = -i\gamma \cdot p\, \sigma_v(p^2;\zeta) + \sigma_s(p^2;\zeta)$$

- Numerical solutions are parametrized in terms of N pairs of complex conjugate poles:

$$\sigma_v(q) = \sum_{k=1}^{N} \left(\frac{z_k}{q^2 + m_k^2} + \frac{z_k^*}{q^2 + m_k^{*2}} \right) \;,\; \sigma_s(q) = \sum_{k=1}^{N} \left(\frac{z_k m_k}{q^2 + m_k^2} + \frac{z_k^* m_k^*}{q^2 + m_k^{*2}} \right) \;.$$

- These are constrained to the UV conditions of the free quark propagator form. For our computations, N=2 is adequate.

Phys.Rev. D67 (2003) 054019.

M. S. Bhagwat, M. A. Pichowsky, and P. C. Tandy

Perturbation theory integral representations

- BSAs are written in a Nakanishi-like representation (**Phys. Rev. 130, 1230–1235 (1963).** N. Nakanishi):

$$\mathcal{F}^k(p;P) = \int_{-1}^{1} dz\ \rho(z) \int_0^{\infty} d\Lambda\ \delta(\Lambda - \Lambda_c) \frac{1}{(p^2 + z\ p\cdot P + \Lambda^2)^n}\ .$$

- All BSAs can be written in such simple form.
- More than fifty years old. **Forgotten** for most of that time.
- **Rediscovered** 5 years ago: pion PDA and elastic form factor (Phys.Rev.Lett. 110 (2013) no.13, 132001; Phys.Rev.Lett. 111 (2013) no.14, 141802).
- **Now put to great use**: PDAs, form factors, PDFs, etc.; mesons and baryons; light and heavy.

Perturbation theory integral representations

- We separate IR and UV contributions as follows:

  - The **IR parts** of the non negligible amplitudes:

$$\mathcal{F}^i(p,P) = c^i_{\mathcal{F}} \int_{-1}^{1} dz\ \rho_{\nu^i_{\mathcal{F}}}(z)[a_{\mathcal{F}}\hat{\Delta}^4_{\Lambda^i_{\mathcal{F}}}(k_z^2) + \bar{a}_{\mathcal{F}}\hat{\Delta}^5_{\Lambda^i_{\mathcal{F}}}(k_z^2)]\ .$$

- The corresponding **UV terms**:

$$E^u(p,P) = c^u_E \int_{-1}^{1} dz\ \rho_{\nu^u_E}(z)\hat{\Delta}^{1+\alpha}_{\Lambda^u_E}(k_z^2)\ ,$$

$$F^u(p,P) = c^u_F \int_{-1}^{1} dz\ \rho_{\nu^u_F}(z)k^2\Lambda^u_F\Delta^{2+\alpha}_{\Lambda^u_F}(k_z^2)\ ,$$

$$G^u(p,P) = c^u_G \int_{-1}^{1} dz\ \rho_{\nu^u_G}(z)\Lambda^u_G\Delta^{2+\alpha}_{\Lambda^u_G}(k_z^2)\ .$$

- Some definitions

$$\hat{\Delta}_\Lambda(s) = \Lambda^2\Delta_\Lambda(s)\ ,\ \Delta_\Lambda(s) = (\Lambda^2+s)^{-1}\ ,$$

$$\rho_\nu(z) = \frac{1}{\sqrt{\pi}}\frac{\Gamma[\nu+3/2]}{\Gamma[\nu+1]}(1-z^2)^\nu\ ,$$

$$k_z^2 = k^2 + zk\cdot P\ .$$

Phys.Rev.Lett. 110 (2013) no.13, 132001

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Parton distribution amplitudes

- The PDA is a projection of the system's Bethe-Salpeter wave-function onto the light-front. It plays a crucial role in explaining and understanding a wide range of a given meson's properties.

- Given a pseudoscalar meson with total momentum$P$,a resolution scale ζ and a light-cone four-vector n ($n^2 = 0, n.P = -m_M$), the PDA reads as:

$$f_M \phi_M(x;\zeta) = Z_2 \int_q \delta(n \cdot q^+ - xn \cdot P) \gamma_5 \gamma \cdot n [S(q^+) \Gamma_M(q;P) S(q^-)] \ .$$

- The moments of the distribution are given by:

$$f_M (n \cdot P)^{m+1} < x^m >= \text{tr}_{CD} Z_2 \int_q (n \cdot q^+)^m \gamma_5 \gamma \cdot n [S(q^+) \Gamma_M(q;P) S(q^-)] \ ,$$

$$< x^m >= \int_0^1 dx \, x^m \phi_M(x;\zeta) \ .$$

Parton distribution amplitudes

- According to **Phys. Rev. D22, 2157 (1980**), in the neighborhood of the conformal limit, it is written in terms of 3/2-Gegenbauer polynomials.

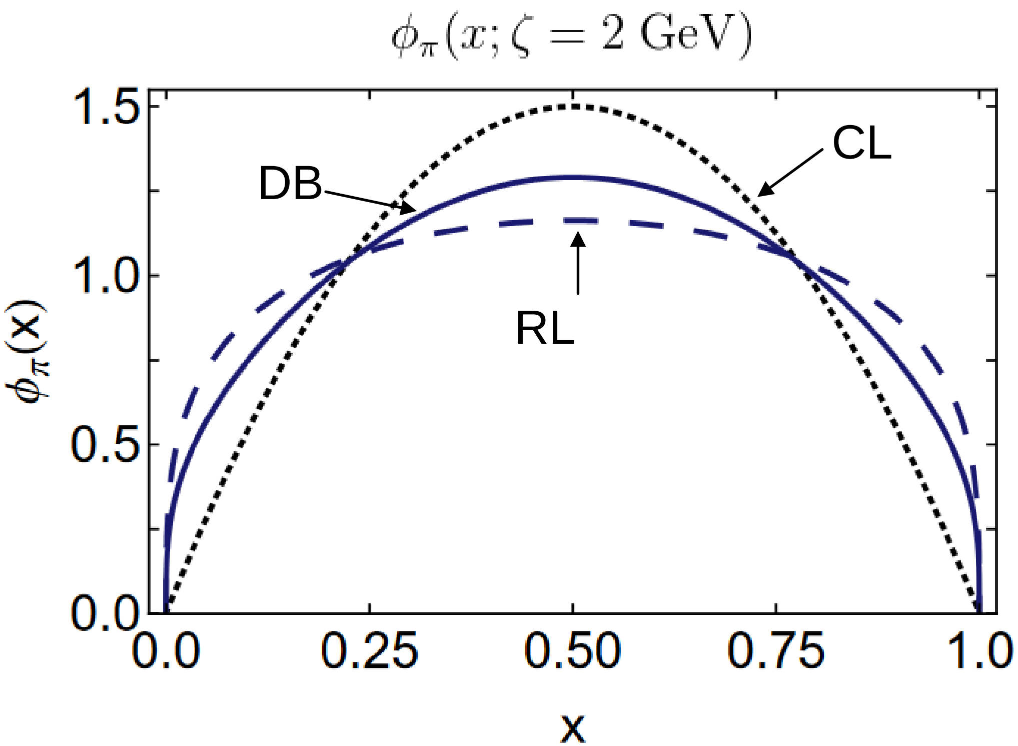
$$\phi_M(x;\zeta) = 6x(1-x)\left[1+\sum_{n=1}^{\infty} a_n^{(3/2)}(\zeta)\, C_n^{(3/2)}(2x-1)\right] .$$

- PDA should evolve with the resolution scale ζ^2 through the ERBL evolution equations (see **Phys. Lett. B87, 359 (1979)** and **Phys. Lett. B94, 245 (1980)**).

- Evolution enables the dressed quark/antiquark degrees of freedom to split into less well-dressed partons via the addition of gluons and sea quarks in the manner prescribed by QCD dynamics. The **asymptotic form** of the PDA is the well known result:

$$\phi^{CL}(x) = 6x(1-x) .$$

Pion PDA



- **Pion PDA** at any resolution scale is a **broad, concave** function of x.
- **Dilation** of the PDA is an **effect of DCSB**.
- It reaches its conformal form at multi-TeV scales.
- Chernyak-Zhitnitsky **double-hump** form was proven to be **erroneous**.

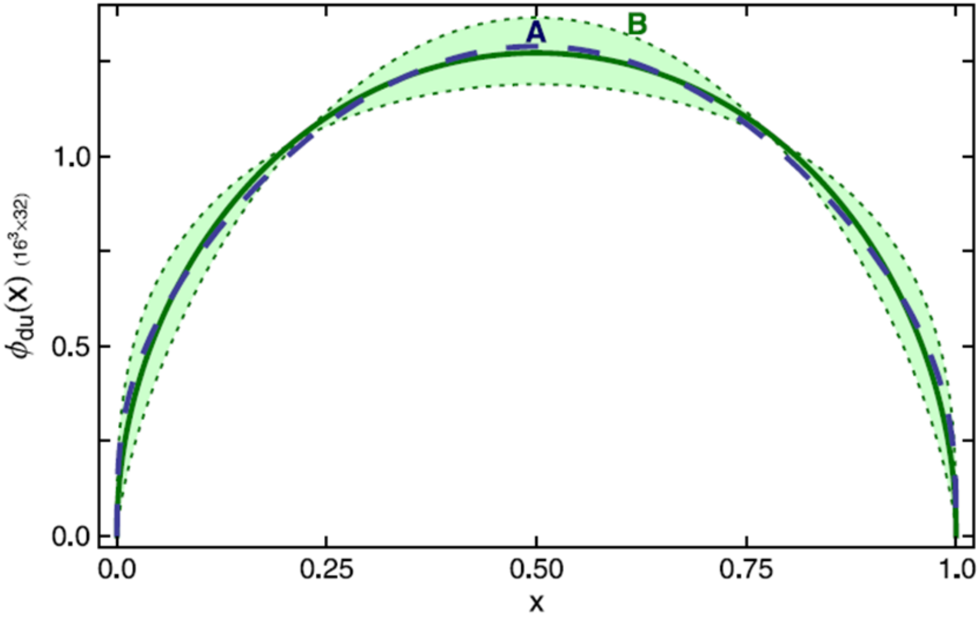
Phys.Rev.Lett. 110 (2013) no.13, 132001

"Imaging dynamical chiral symmetry breaking: pion wave function on the light front"

Lei Chang, Ian C. Cloët, J. Javier Cobos-Martinez, Craig D. Roberts, Sebastian. M. Schmidt, Peter. C. Tandy

Pion PDA

- Precise agreement of DSE with lQCD results (**Phys.Rev. D83 (2011) 074505**).
- Single humped form of PDA also confirmed by newer lQCD results (**Phys.Rev. D95 (2017) no.9, 094514**).



A: DSE prediction
Phys.Rev.Lett. 110 (2013) no.13, 132001.

$$< (2x-1)^2 >_{DSE} = 0.25$$

B: Infered PDA from lattice
Phys.Lett. B731 (2014) 13-18.

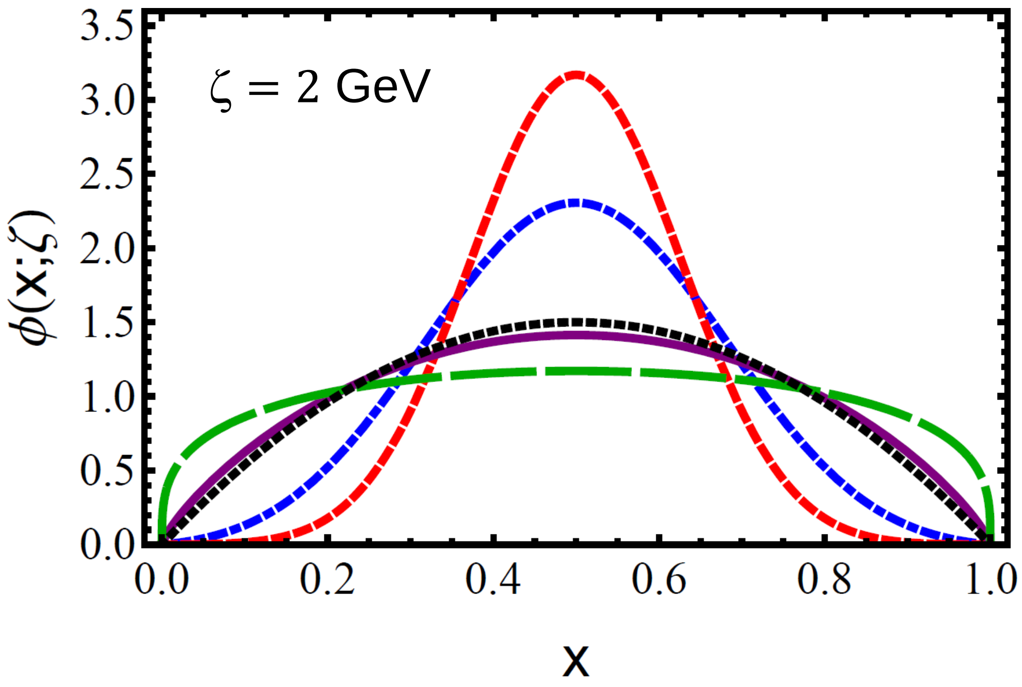
$$< (2x-1)^2 >_{lQCD} = 0.25(1)(2)$$

Phys.Lett. B731 (2014) 13-18.

"Distribution amplitudes of light-quark mesons from lattice QCD"

Jorge Segovia, Lei Chang, Ian C. Cloët, Craig D. Roberts, Sebastian M. Schmidt, Hong-shi Zong

PDAs: light and heavy mesons



❖[**Red**]: $\phi_{\eta_b}(x)$.

❖[**Blue**]: $\phi_{\eta_c}(x)$.

❖[**Purple**]: $\phi_{\eta_s}(x)$.

❖[**Green**]: $\phi_{\pi}(x)$.

❖[**Black**]: $\phi_{CL}(x)$.

➢ s-quark is the boundary between strong and weak mass generation being dominant.

➢ The last five years have shown us what the PDAs for mesons are, with **complete certainty**.

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Form factors

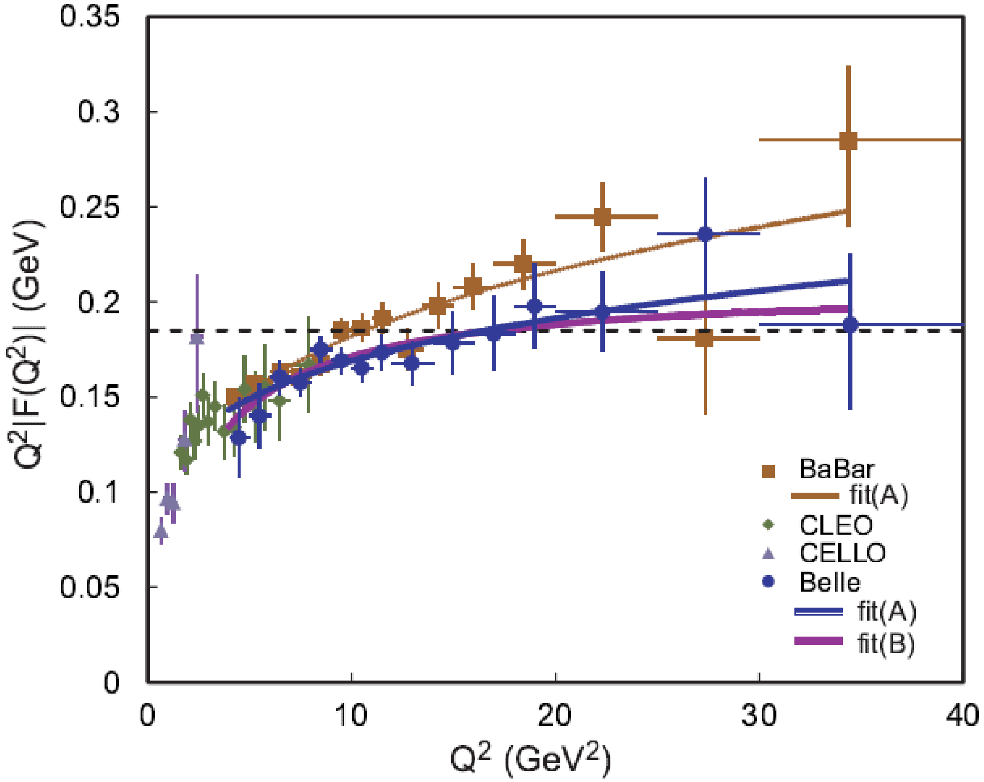
- Hadron form factors are intimately related to their internal structure, revealing many aspects of hadron structure, e.g. charge and current distributions.

- Modern facilities around the world have studied form factors for many years, giving us deeper understanding about the structure of matter. For instance:

1. **JLab:** Nucleon elastic and transition form factors. Pion elastic form factor.
2. **Babar:** $\gamma\gamma^{*}\rightarrow\pi^{0}, \eta, \eta', \eta_{c}$, transition form factors.
3. **Belle:** $\gamma\gamma^{*}\rightarrow\pi^{0}$ transition form factor.
4. **BES III:** light-meson and charmonium decays.

Pion electromagnetic transition form factor

- The manner by which the conformal limit is approached is currently receiving keen scrutiny.
- Above $Q^2 > 10\,GeV^2$, **Babar** data is far **above** the asymptotic **QCD prediction***.
- Data subsequently published by Belle appears to be in general agreement with the conformal limit.
- Deeper understanding is needed.



[Babar]: **Phys. Rev. D80, 052002 (2009).**
[Belle]: **Phys. Rev. D86, 092007 (2012).**

*Phys. Rev. D 22, 2157 (1980)

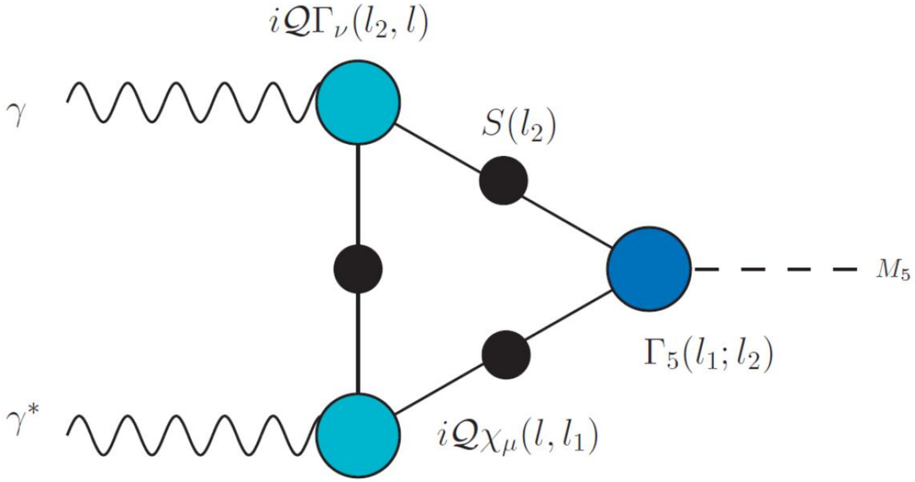
Pion transition form factor

- For a pseudoscalar meson M_5, the $\gamma\gamma^*\to M_5$ transition is written as:

$$T_{\mu\nu}(k_1,k_2) = \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G_{M_5}(k_1^2, k_1\cdot k_2, k_2^2)\ ,$$

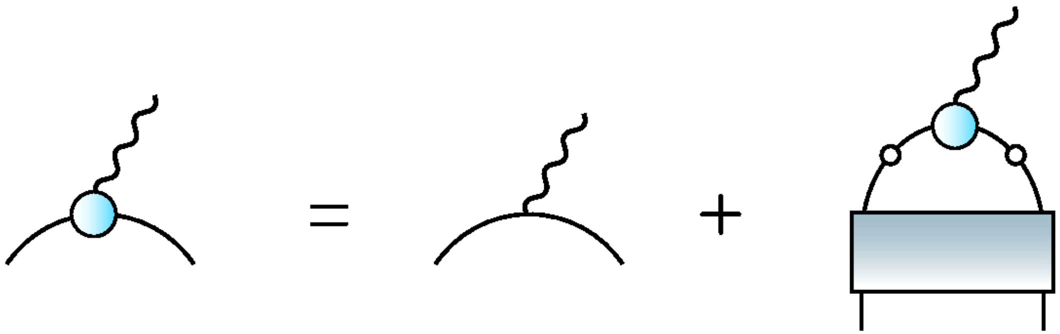
$$T_{\mu\nu}(k_1,k_2) = \mathrm{tr}\int \frac{d^4 l}{(2\pi)^4} i\mathcal{Q}\chi_\mu(l,l_1)\Gamma_{M_5}(l_1,l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2,l)$$

- We know quark propagators and BSA from their PTIRs, the remaining ingredient is **quark-photon vertex**.



Quark-photon vertex

- The DSE for the **quark-photon vertex** (**QPV**) is:



- I shall discuss its construction following our work on pion transition form factor:

PHYSICAL REVIEW D **93,** 074017 (2016)

Structure of the neutral pion and its electromagnetic transition form factor

Khépani Raya,[1,*] Lei Chang,[2,†] Adnan Bashir,[1,‡] J. Javier Cobos-Martinez,[1,§] L. Xiomara Gutiérrez-Guerrero,[3,∥] Craig D. Roberts,[4,¶] and Peter C. Tandy[5,**]

Quark-photon vertex

- When studying the elastic or transition form factors, it is the photon which probes its constituents, highlighting the importance of the quark-photon vertex.

- In principle, one could solve the DSE for the quark-photon vertex. However, from the **Ward-Green-Takahashi identities** (**WGTIs**) one can construct an Ansätz.

- In constructing the quark-photon vertex structure, several efforts have been done. Some of them: **Phys.Rev. D95 (2017) no.3 034041**, **Phys.Lett. B722 (2013) 384-388, Phys.Rev. C85 (2012) 045205, Phys.Rev. D79 (2009) 125020**.

- For computing form factors and other objects, quark-photon vertex should be mathematically **consistent** and **computationally useful**.

Quark-photon vertex

- Änsatz construction **demands** the following:
  - QPV should satisfy longitudinal WGTI and be free of kinematic singularities.
  - It should reduce to the bare vertex in the UV limit, and have the same transformation properties.
  - It must be capable of producing the abelian anomaly.
- Additionally, it **should expedite the computation** of form factors or other non perturbative objects.
- Our chosen änsatz is built through the gauge technique (**R. Delbourgo and P. C. West, J. Phys. A10, 1049 (1977)**).
- It has been employed succesfully when computing elastic and transition form factors: **Phys. Rev. Lett. 111 no.14, 141802 (2013)** (Chang et al.)**, Phys.Rev. D93 (2016) no.7, 074017 (KR et al.)** and **Phys.Rev. D95 (2017) no.7, 074014 (KR et al.) .**

Quark-photon vertex

- Unamputated vertex is written as:

$$S\Gamma_\mu S \to \chi(k_f, k_i) = \sum_{j=1}^{3} T_{j\mu}(k_f, k_i) X_j(k_f, k_i) \ .$$

- The tensor structures:

$$T_{1\mu}(k_f, k_i) = \gamma_\mu \ , \ \ T_{2\mu}(k_f, k_i) = \beta \ \gamma \cdot k_f \gamma_\mu \gamma \cdot k_i + \bar{\beta} \ \gamma \cdot k_i \gamma_\mu \gamma \cdot k_f \ ,$$

$$T_{3\mu}(k_f, k_i) = i\beta \ (\gamma \cdot k_f \gamma_\mu + \gamma_\mu \gamma \cdot k_i) + i\bar{\beta} \ (\gamma \cdot k_i \gamma_\mu + \gamma_\mu \gamma \cdot k_f) \ .$$

- With dressing functions:

$$X_1(k_f, k_i) = \Delta_{k^2\sigma_v}(k_f^2, k_i^2) \ , \ \ X_2(k_f, k_i) = \Delta_{\sigma_v}(k_f^2, k_i^2) \ ,$$

$$X_3(k_f, k_i) = \Delta_{\sigma_s}(k_f^2, k_i^2) \ .$$

Quark-photon vertex

- A proper choice of β ensures that the abelian anomaly is satisfied:

$$\beta \to \beta(Q^2) = 1 + s_0 \text{Exp}\left[-\mathcal{E}_5/M_f^E\right] \ ,$$

$$\mathcal{E}_5 = \sqrt{\frac{1}{4}Q^2 + m_5^2} - m_5 \ , \ M_f^E = \{p|p^2 = M(p^2) \ , \ p > 0\} \ .$$

- The value s_0 is fixed by the abelian anomaly or decay widths. Transverse pieces associated with s_0 are exponentially suppresed.

- Our ansätz is written in terms of quark propagator's dressing functions. We spared the need to solve the DSE for the QPV, and instead, we can employ the PTIRs of $S(p)$. This expedites the computation of the form factors.

Elastic and transition form factors

- The expressions of elastic and transition form factors:

$$K_\mu F_\pi(Q^2) = N_c \mathrm{tr}_{\mathrm{D}} \int \frac{d^4k}{(2\pi)^4} \chi_\mu(k+p_i, k+p_f) \Gamma_\pi(k_i; P_i) S(k) \Gamma_\pi(k_f; -P_f)$$

$$T_{\mu\nu}(k_1, k_2) = \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G_{M_5}(k_1^2, k_1 \cdot k_2, k_2^2) \; ,$$

$$T_{\mu\nu}(k_1, k_2) = \mathrm{tr} \int \frac{d^4l}{(2\pi)^4} i\mathcal{Q} \chi_\mu(l, l_1) \Gamma_{M_5}(l_1, l_2) S(l_2) i\mathcal{Q} \Gamma_\nu(l_2, l)$$

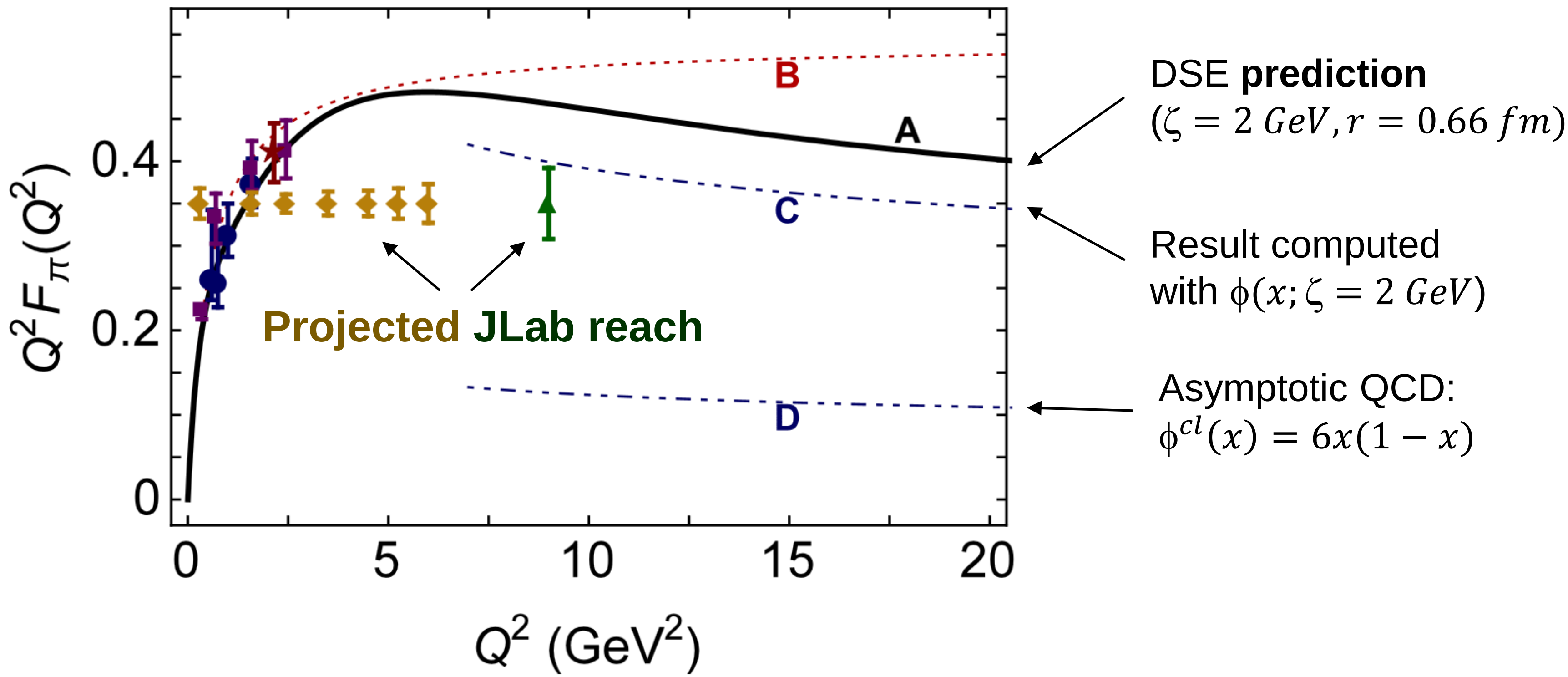
- Computation of the form factors reduces to the task of summing a series of terms, all of which involve a single four-momentum integral. The integrand **denominator** in every term is a **product of quadratic forms**.

Elastic and transition form factors

- Feynman parametrization enables **straightforward evaluation** of the **four momentum integration**.
- Then we numerically integrate over the Feynman parameters and the spectral integrals. The complete result follows after summing the series.
- This novel technique allowed to compute, **for the first time** in a framework with **direct connection to QCD**, pion elastic and $\gamma\gamma^*$ transition form factors on the **whole range** of space-like momentum.
- Many subsequent works follow this analysis: **Phys.Rev. D95 (2017) no.7, 074014** (K. Raya et al.), *"Valence quark distribution amplitudes of* η*,* η*' and their* $\gamma\gamma^*$ *transition form factors"* (M. Ding, K. Raya et al.) [in preparation], for example.

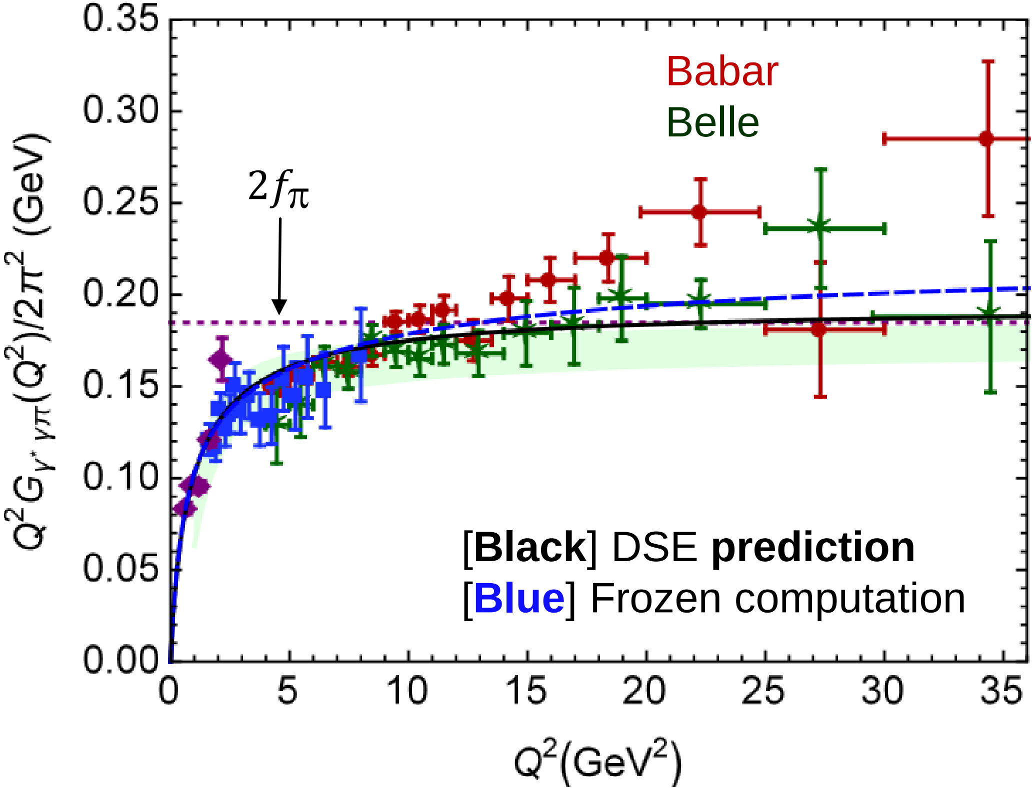
Pion elastic form factor

Phys.Rev.Lett. 111 (2013) no.14, 141802. Chang et al.



- **DSE prediction** of charged pion form factor **motivated a re-evaluation** of the reach **of JLab12 program**, with the aim of testing such prediction (so that green point was added).

Pion transition form factor

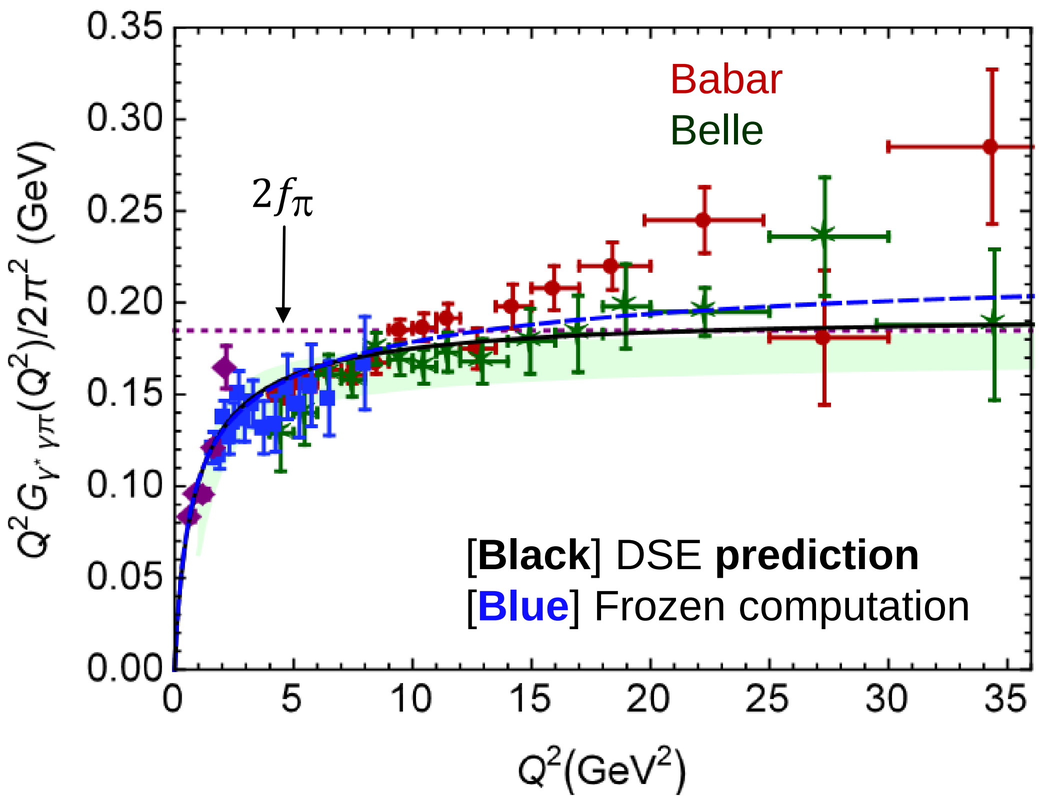


- Our result satisfies the Abelian anomaly, $2f_\pi G_\pi(Q^2=0)=1$, while also being in agreement with the conformal limit, $2f_\pi$.
- The interaction radius is similar to that of the charged pion ($r = 0.68\ fm$).
- DSE **prediction** agrees with all experimental data, except with Babar above $10GeV^2$.

Phys.Rev. D93 (2016) no.7, 074017.

KR, L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, P.C. Tandy

Pion transition form factor

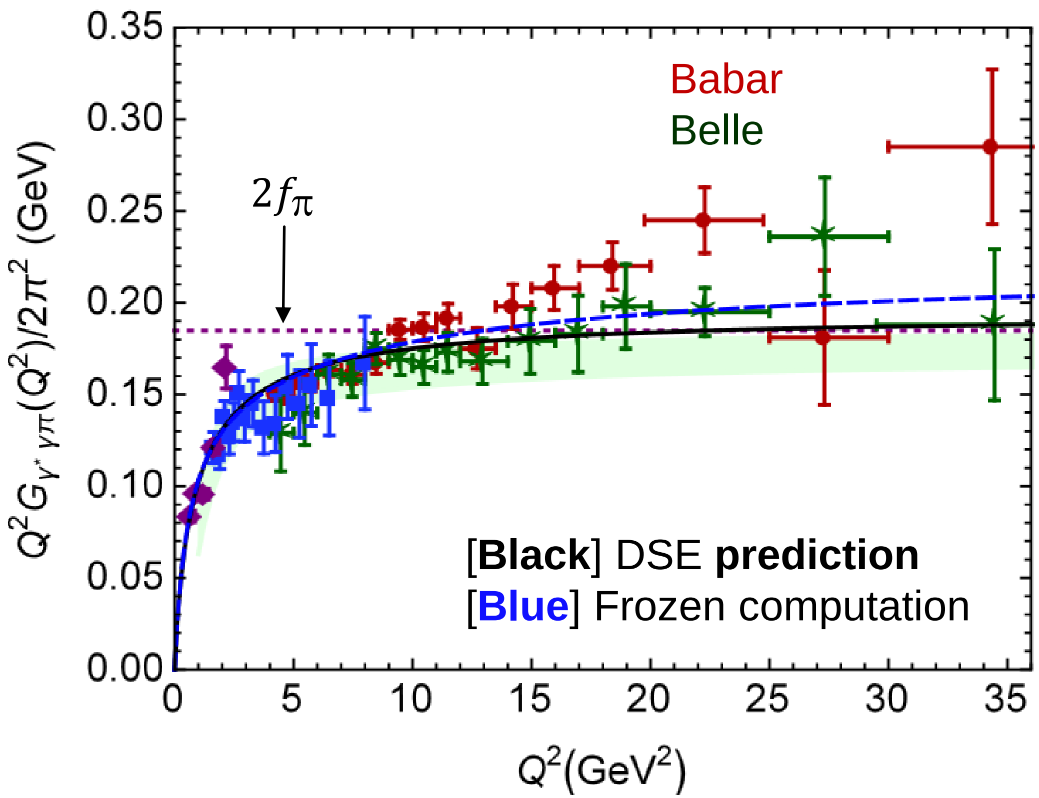


- $F_\pi(Q^2)$ evolves as $1/Q^2$ with scaling violations (~ ln Q^2).
- For the transition form factor, the **conformal limit is a constant**, independent of logarithms.
- But, **the way you reach that constant is completely determined by the logarithms**. We improved DSE methods to include this effect.

Phys.Rev. D93 (2016) no.7, 074017.

KR, L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, P.C. Tandy

Pion transition form factor



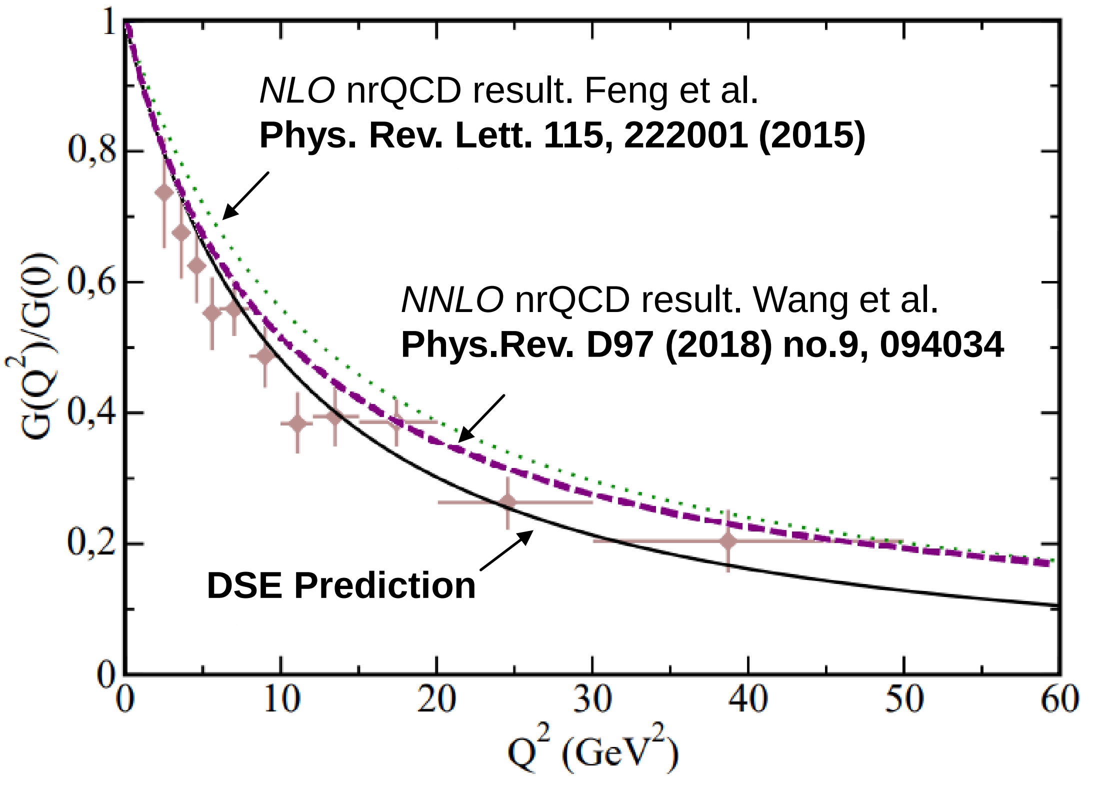
DSE prediction does not conciliate with Babar:

- A pion **PDA** that is a **broad, concave** function at the hadronic scale, **explains both** $F_\pi(Q^2)$, $G_\pi(Q^2)$ and their hard photon limits.
- Babar data, however favors a *flat-top* PDA, which yields a correct power-law, but produces an erroneous value of the anomalous dimension.
- Belle data strongly supports our conclusions.

Phys.Rev. D93 (2016) no.7, 074017.

KR, L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, P.C. Tandy

η_c, η_b transition form factors

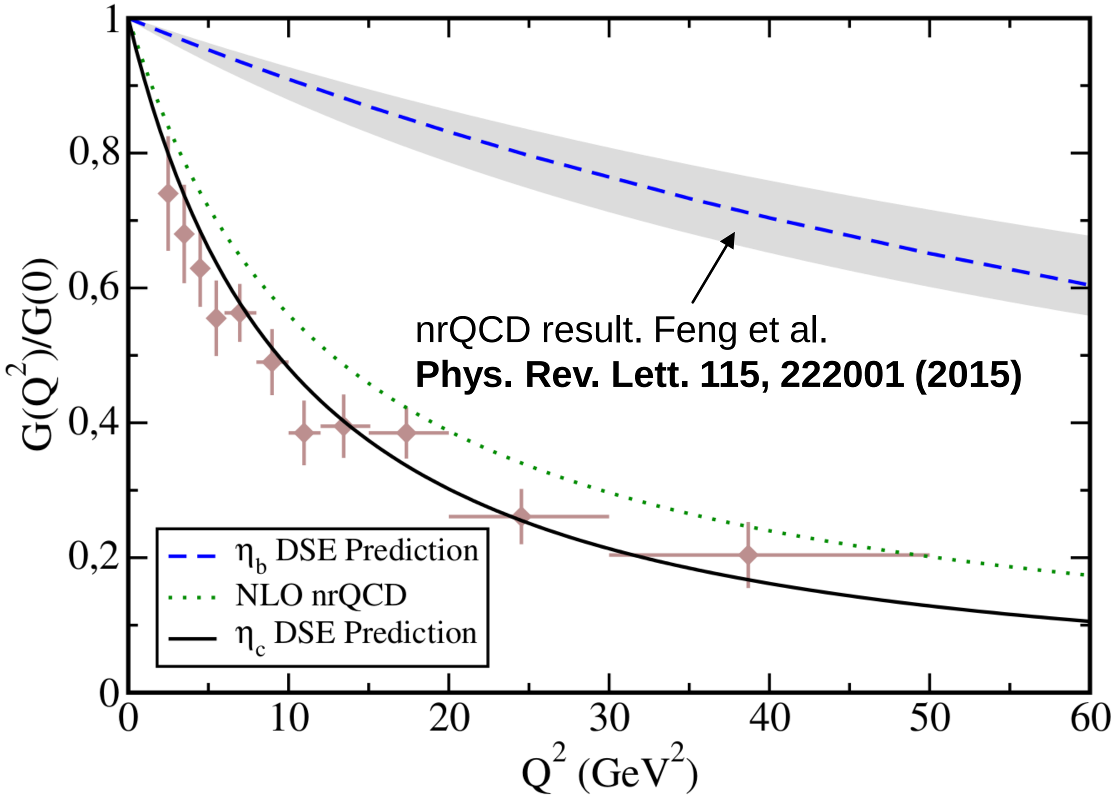


Phys.Rev. D95 (2017) no.7, 074014.

"Partonic structure of neutral pseudoscalars via two photon transition form factors"

KR, M. Ding, A. Bashir, L. Chang, C.D. Roberts

η_c, η_b transition form factors



Phys.Rev. D95 (2017) no.7, 074014.

"Partonic structure of neutral pseudoscalars via two photon transition form factors"

KR, M. Ding, A. Bashir, L. Chang, C.D. Roberts

Outline

1. The basics (**DSEs**)
   - Quark propagator and Bethe-Salpeter equation
   - Perturbation theory integral representations
2. Parton distribution amplitudes
3. Two photon transition form factors
4. **η-η' PDAs and TFFs**
5. Conclusions and scope

η- η′ mesons

- It is convenient to separate quark flavors:

$$|\eta> = \cos_\eta \phi \, \frac{1}{\sqrt{2}}(|u\bar{u}> + |d\bar{d}>) - \sin_\eta \phi \, |s\bar{s}> \ ,$$

$$|\eta'> = \sin_\eta \phi \, \frac{1}{\sqrt{2}}(|u\bar{u}> + |d\bar{d}>) + \cos_\eta \phi \, |s\bar{s}> \ .$$

- The Bethe-Salpeter amplitude of the η-η′ system is:

$$\Gamma(p;P) = \lambda_q \Gamma^q(p;P) + \lambda_s \Gamma^s(p;P) \ ,$$

$$\lambda_q = (1/\sqrt{2})\mathrm{diag}(1,1,0) \ , \ \lambda_s = \mathrm{diag}(0,0,1) \ .$$

- **Lightest** (**heaviest**) solution corresponds to η (η′). Dirac structure is contained in $\Gamma^{q/s}$ ($m_u = m_d$).

η- η′ mesons

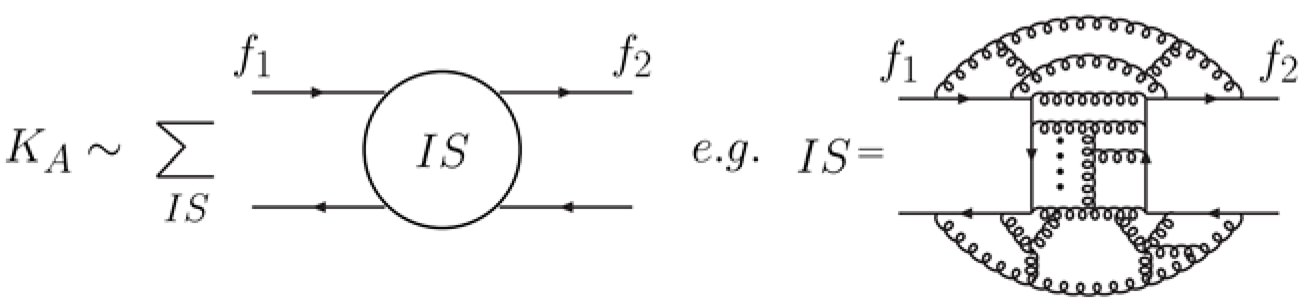
- Therefore, Bethe-Salpeter equation is written as:

$$\Gamma(p;P) = \int_q (K_L + K_A)^{rs}_{tu} \, (\mathcal{S}\,\Gamma(q;P)\,\mathcal{S})_{sr} \;, \quad \mathcal{S} = \mathrm{diag}(S^q, S^q, S^s) \;.$$

Ladder **Anomaly**

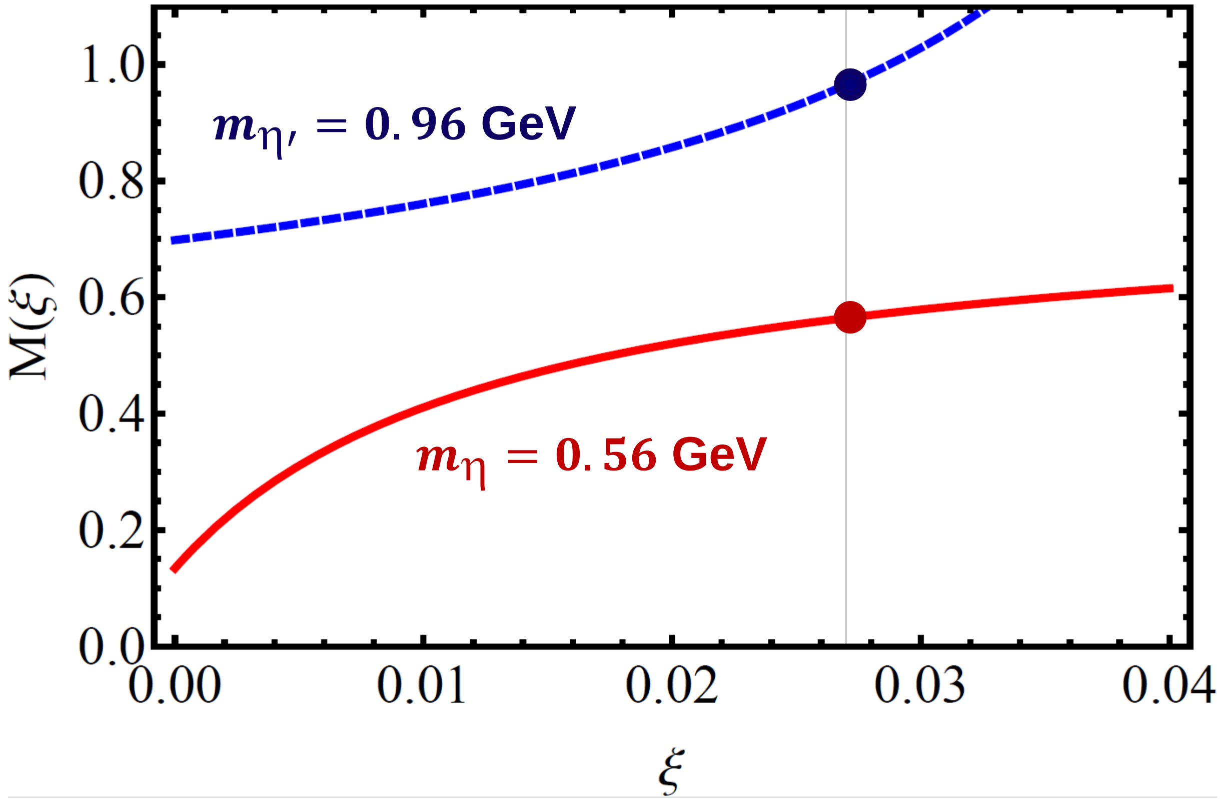
- We follow the discussion of Mandar et al. (**Phys. Rev. C76, 045203 (2007)**). The anomaly kernel is written as:

$$(K_A)^{rs}_{tu}(q,k,P) = -\xi(k-q)\{\cos\theta_\xi^2[\zeta\gamma_5]_{rs}[\zeta\gamma_5]_{tu} - \sin\theta_\xi^2[\zeta\gamma\cdot P\gamma_5]_{rs}[\zeta\gamma\cdot P\gamma_5]_{tu}$$



η- η′ mesons

- When increasing the strenght of the anomaly, the mixing between the u/d and s quark flavors is produced.



❖ Ideal mixing produces:

$m_{ss} = 0.7$ GeV

$m_{qq} = 0.134$ GeV

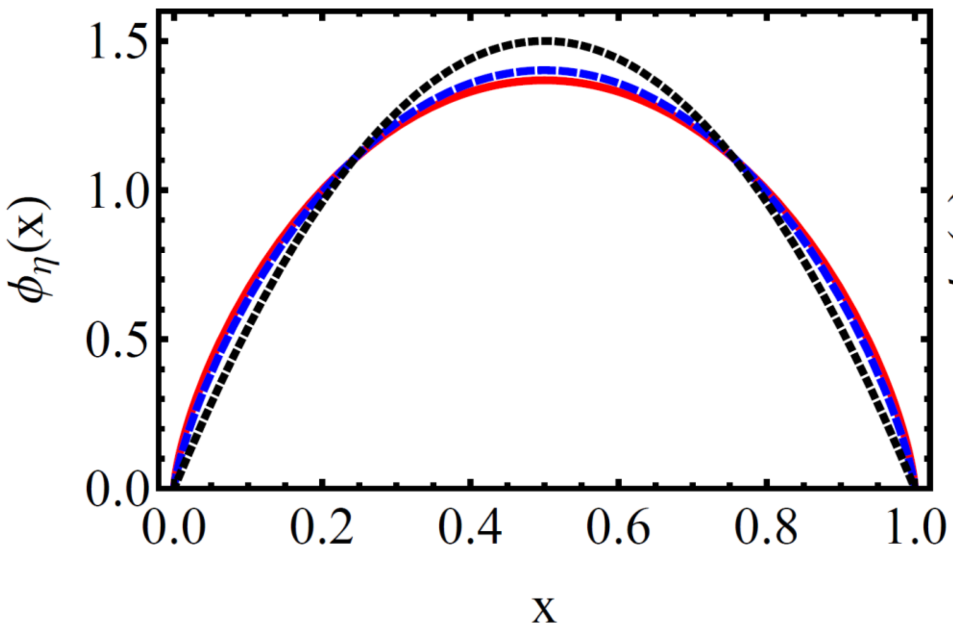
❖ Infered mixing angle:

$$\phi = 42.8^o$$

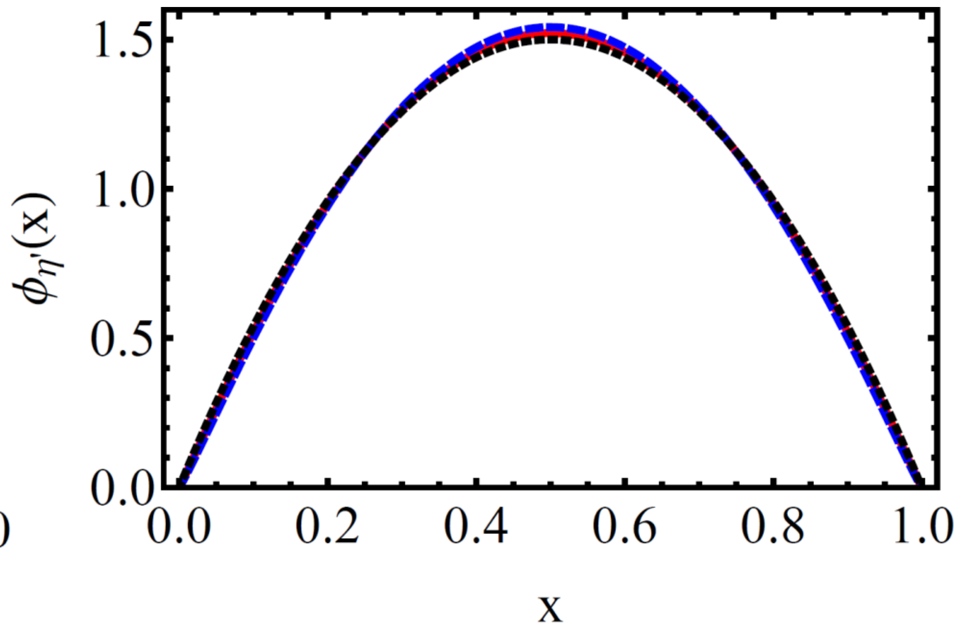
→ Similar flavor content

η- η′ PDAs

- η-η′ PDAs lie close to the conformal distribution.



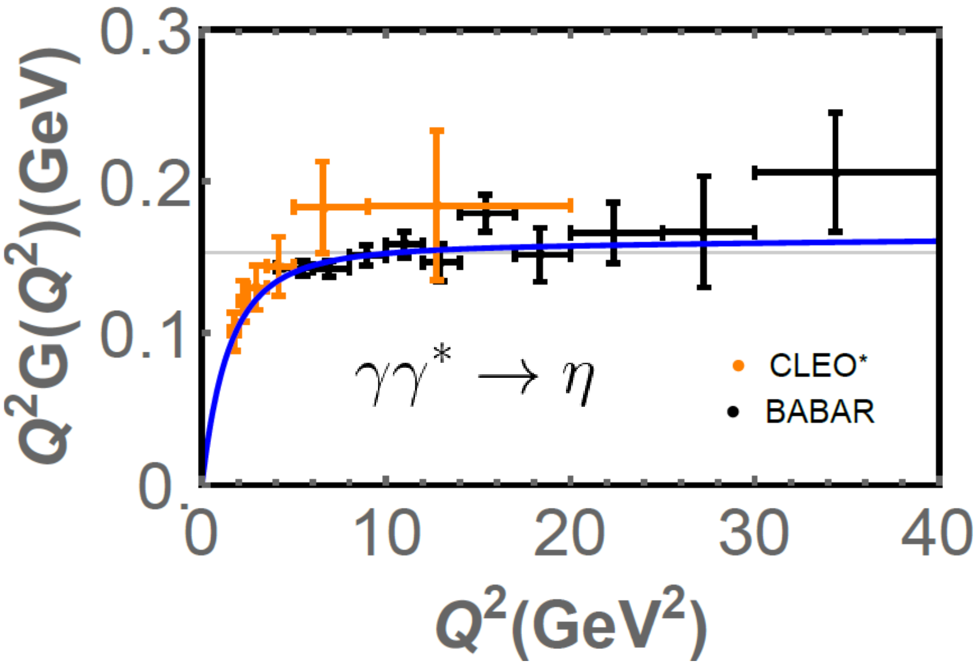
[Blue] $\phi_\eta^s(x;\zeta= 2\text{ GeV})$.
[Red] $\phi_\eta^q(x;\zeta = 2\text{ GeV})$.
[Black] $\phi(x) = 6x(1-x)$.

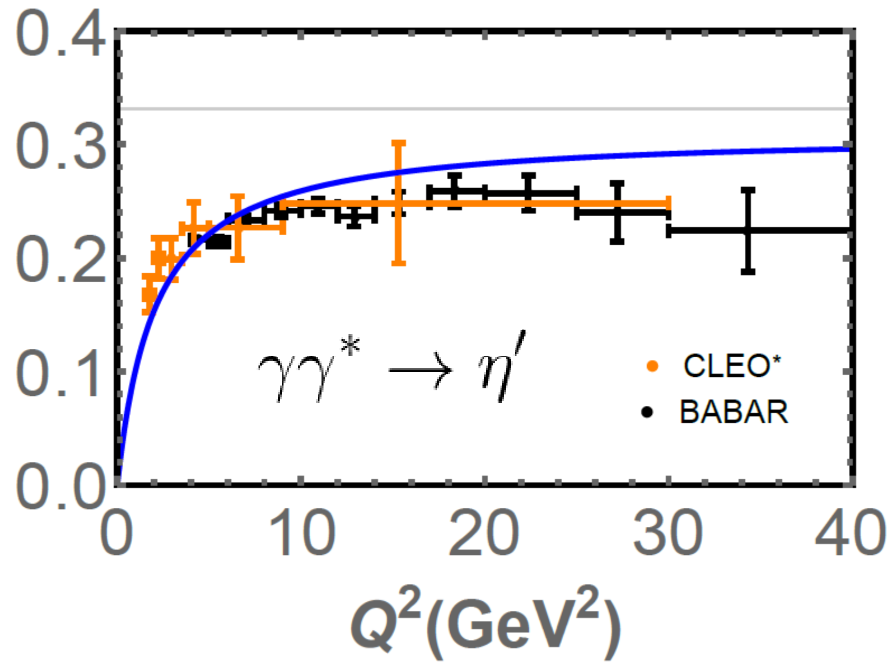


[Blue] $\phi_{\eta'}^s(x;\zeta= 2\text{ GeV})$.
[Red] $\phi_{\eta'}^q(x;\zeta = 2\text{ GeV})$.
[Black] $\phi(x) = 6x(1-x)$.

η- η′ TFFs

- The **good agreement** of our preliminary computation **with experimental data** is encouraging.

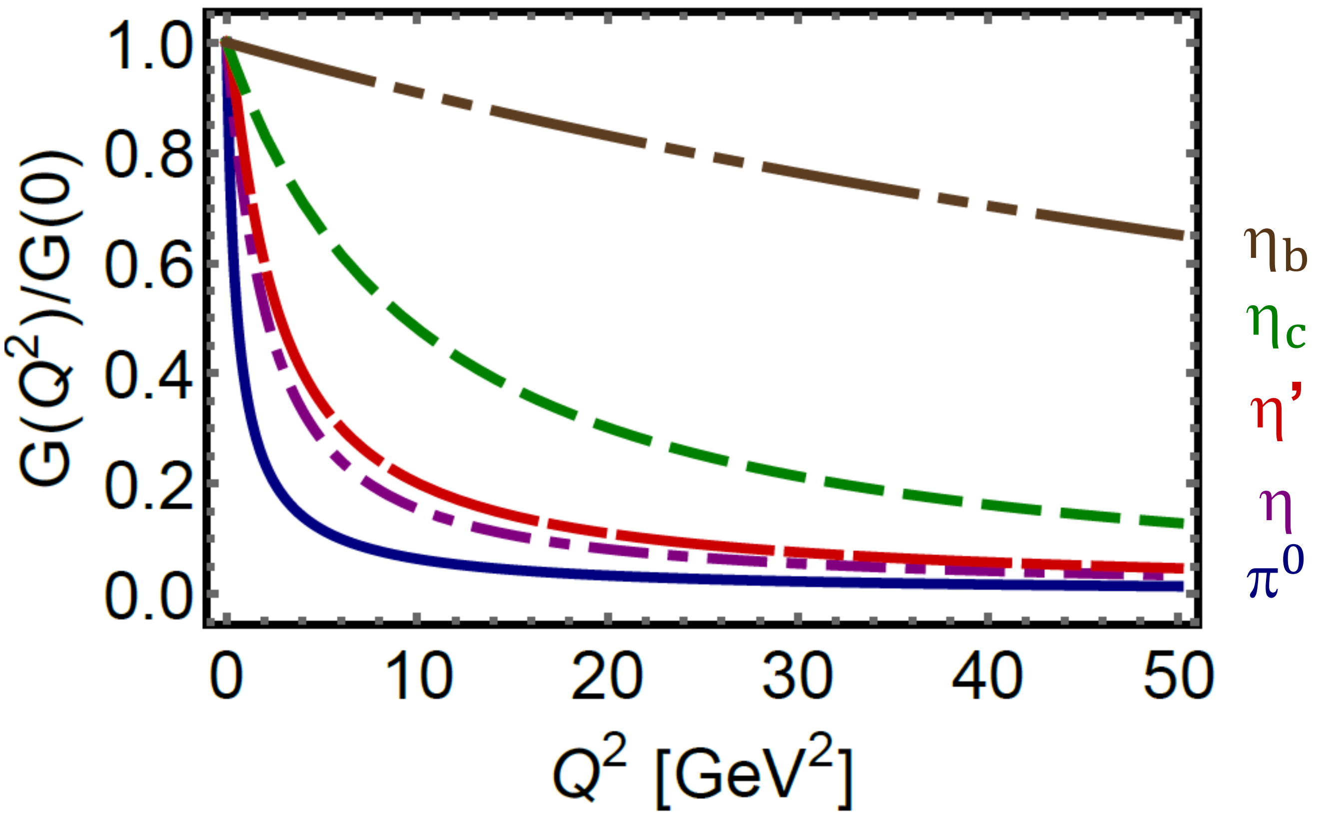




- Subtleties, however, must be taken into consideration: **scale dependence of** the flavor mixed **decay constants**, for example.

See **Phys.Rev. D90 (2014) no.7, 074019**

$\gamma\gamma^*$ transition form factors

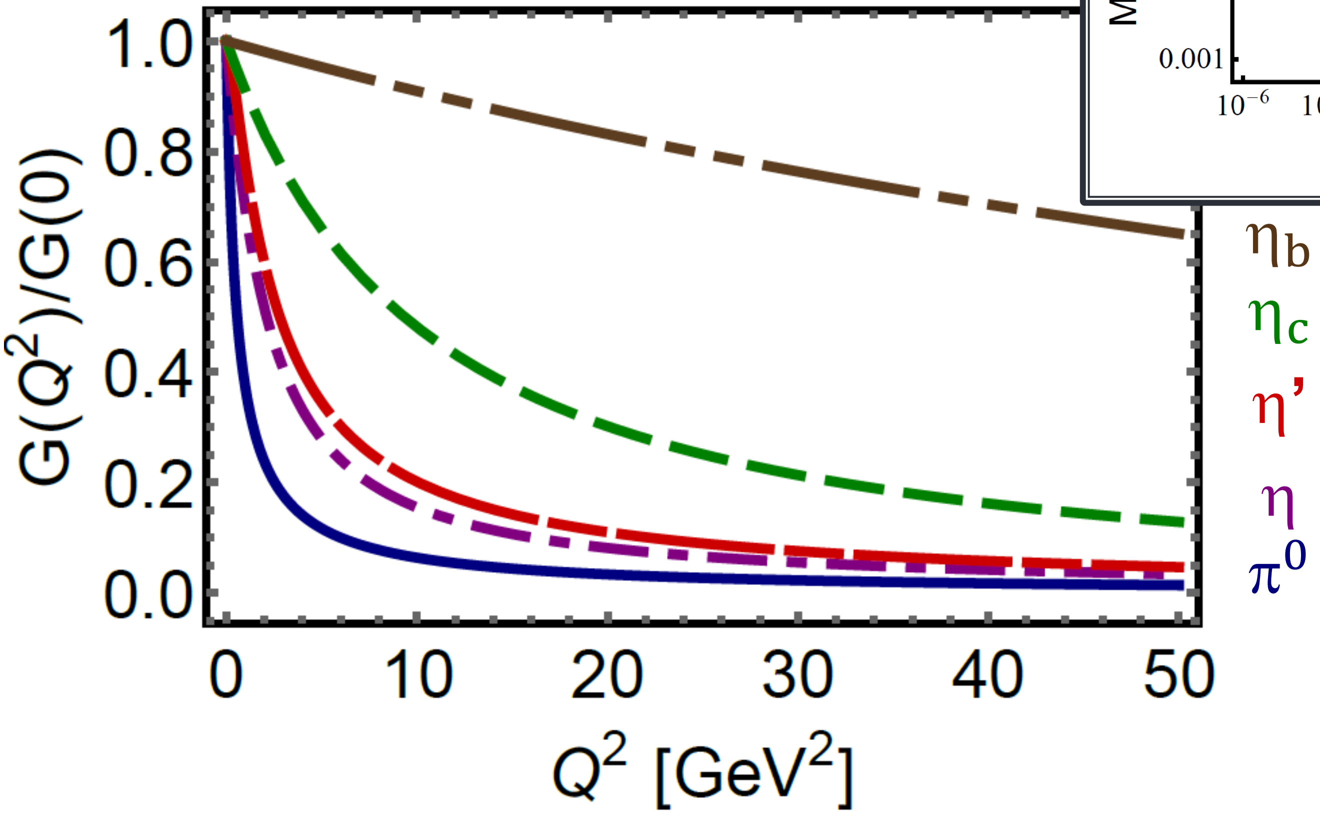


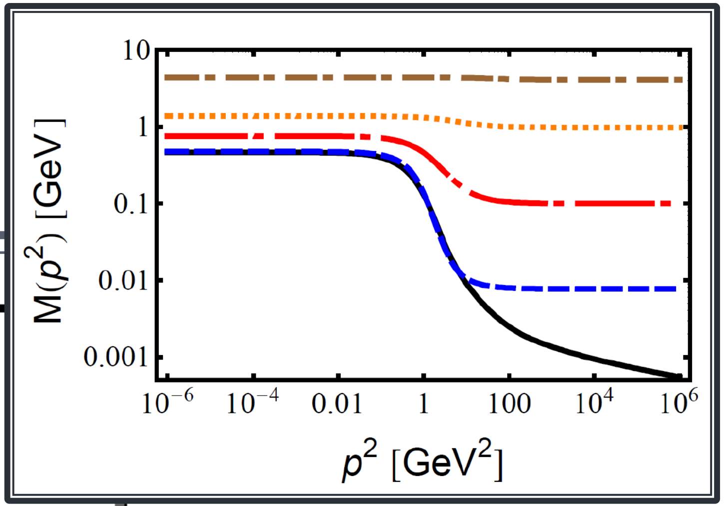
Phys.Rev. D93 (2016) no.7, 074017. K. Raya et al.

Phys.Rev. D95 (2017) no.7, 074014. K. Raya, M. Ding et al.

η- η' in preparation… M. Ding, K. Raya et al.

$\gamma\gamma^*$ transition form factors





Phys.Rev. D93 (2016) no.7, 074017. K. Raya et al.

Phys.Rev. D95 (2017) no.7, 074014. K. Raya, M. Ding et al.

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5. **Conclusions and scope**

Conclusions

Phys.Rev. D95 (2017) no.7, 074014. KR et al.
Phys.Rev. D93 (2016) no.7, 074017. KR et al.

- We described a computation of $\gamma\gamma^* \to$ neutral pseudoscalar transition form factors, in which all elements employed are determined by solutions of QCD's Dyson-Schwinger equations.
- The novel analysis techniques we employed made it possible to compute $F_\pi(Q^2), G(Q^2),$ on the entire domain of space-like momenta, for the first time in a framework with a **direct connection to QCD**.
- Our QCD based theoretical computation demonstrates that the results of asymptotic QCD are faithfully reproduced, while also successfully agreeing with *"all"* data. It strongly suggests that **Belle**, not Babar, **is correct** on neutral pion.
- Starting from the DSE of the quark propagator, the picture of $\gamma\gamma^* \to$neutral pseudoscalar **has been completed**.

Conclusions

Phys.Rev. D95 (2017) no.7, 074014. KR et al.
Phys.Rev. D93 (2016) no.7, 074017. KR et al.

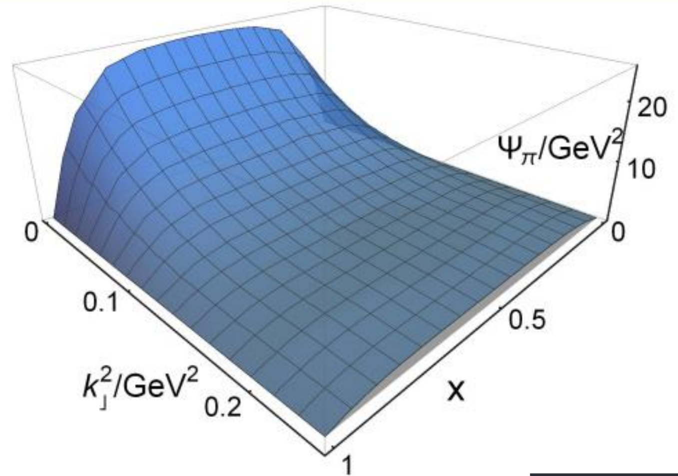
- Within a single systematic and consistent approach, starting from quark propagator, **we unified** the description of $\gamma\gamma^*$ **transition form factors** with:
  - ✓ Valence quark **distribution amplitudes**
  - ✓ Charged pion elastic form factor
  - ✓ Masses, decay constants, etc.
- A sound understanding of the distribution of valence-quarks within mesons has been reached.
  - ➢ Smooth connection of **Goldstone modes** with systems containing the **heaviest valence quarks** that can today be studied experimentally.

❖ **Predictions** for elastic and transition form factors and parton distributions of all types **are arising.**

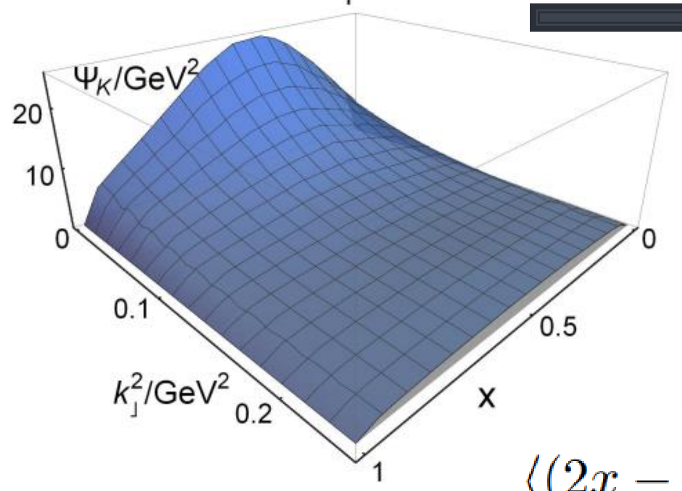
Scope

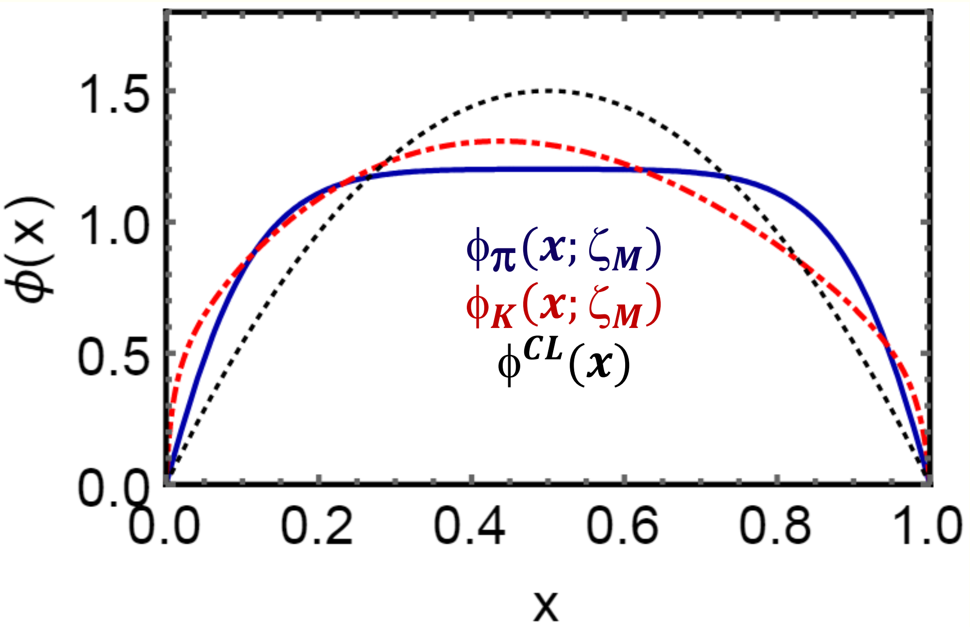
LFWF and PDA: pion and kaon

Phys.Rev. D97 (2018) no.9, 094014



$$\varphi_K(x) = \frac{1}{16\pi^3}\int d^2k_\perp \psi_K^{\uparrow\downarrow}(x, k_\perp^2)$$





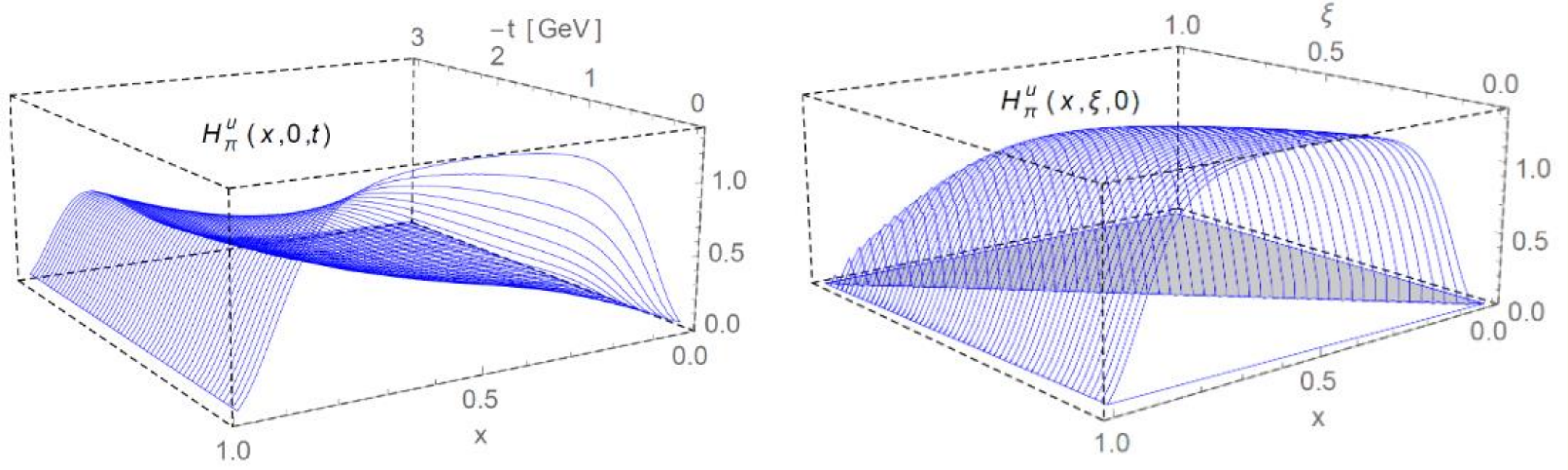
$$\langle (2x-1)^2\rangle_{\varphi_\pi} := \int_0^1 dx\,(2x-1)^2\varphi_\pi(x) \approx 0.25\,,$$

$$\langle 2x-1\rangle_{\varphi_K} \approx -0.04\,,\ \langle (2x-1)^2\rangle_{\varphi_K} \approx 0.25\,.$$

Overlap GPD representation: Pion

- A two-particle truncated expression for the pion and Kaon **GPDs**, in the **DGLAP** kinematic **domain**, is obtained from the **overlap of the LFWF**:

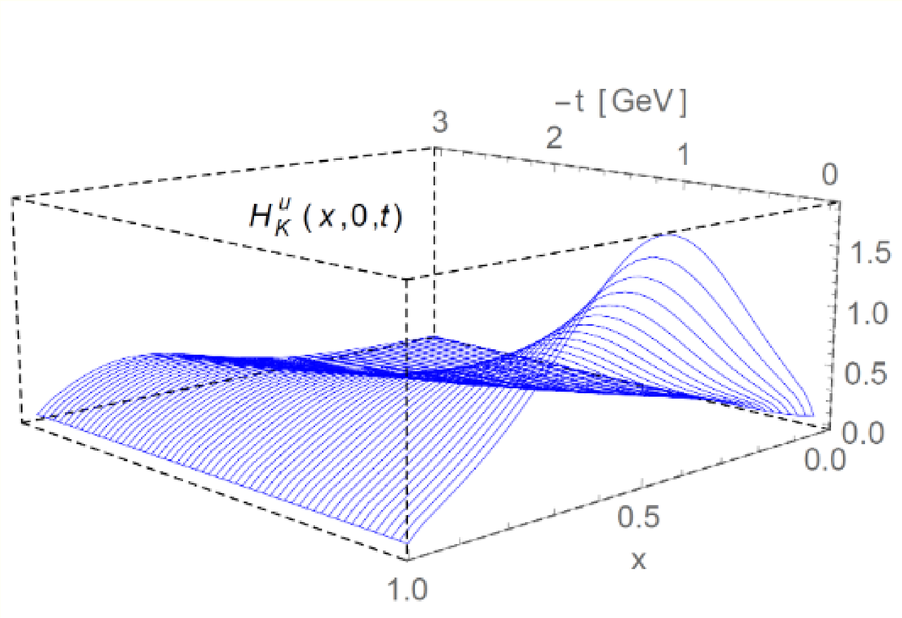
$$H_M^q(x,\xi,t) = \int \frac{d^2\mathbf{k}_\perp}{16\,\pi^3} \Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi}, \mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right) \Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi}, \mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right).$$

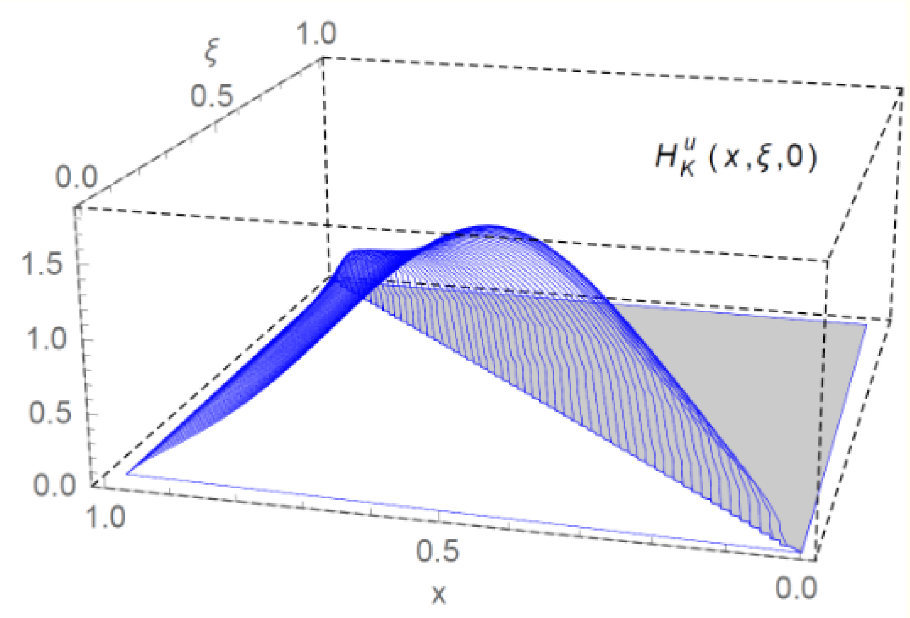


Overlap GPD representation: Kaon

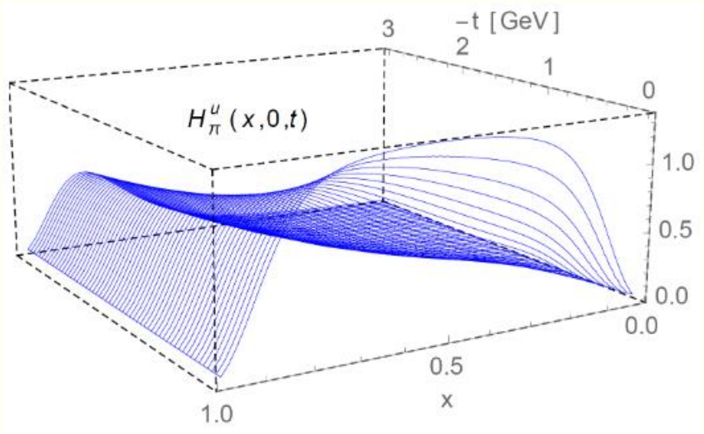
- A two-particle truncated expression for the pion and Kaon **GPDs**, in the **DGLAP** kinematic **domain**, is obtained from the **overlap of the LFWF**:

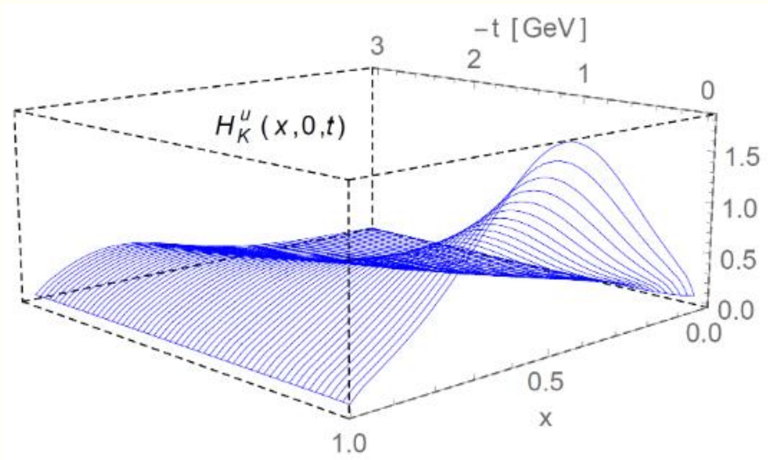
$$H_M^q(x,\xi,t) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\,\pi^3} \Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi}, \mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right) \Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi}, \mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right).$$



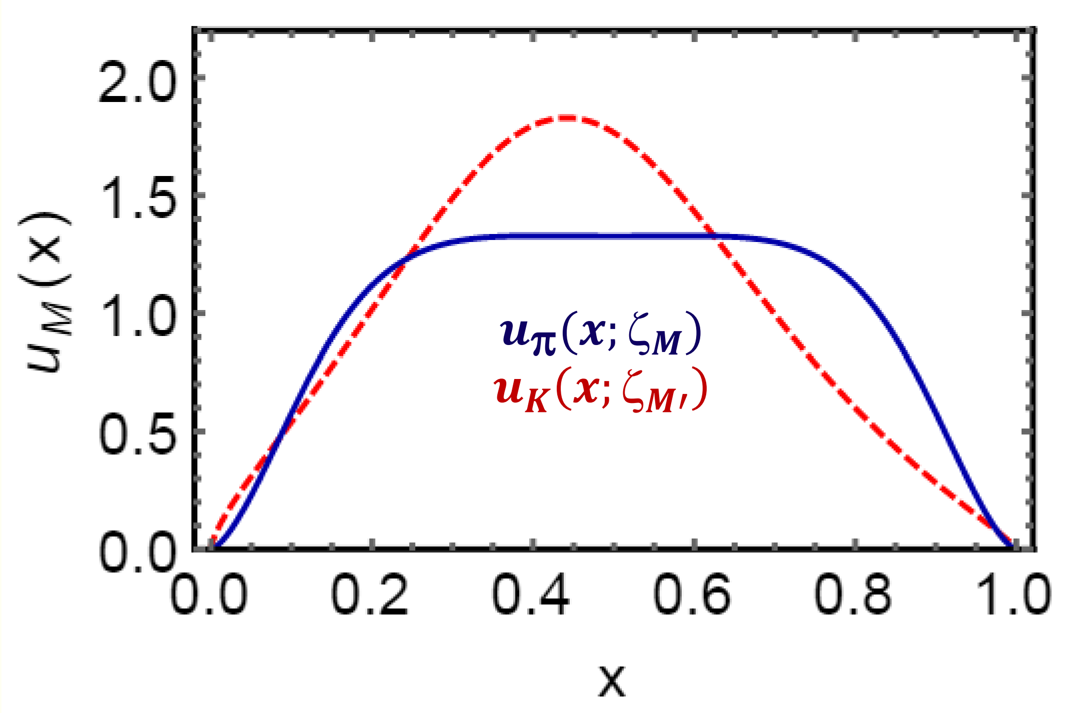


GPDs and PDFs: Pion and Kaon



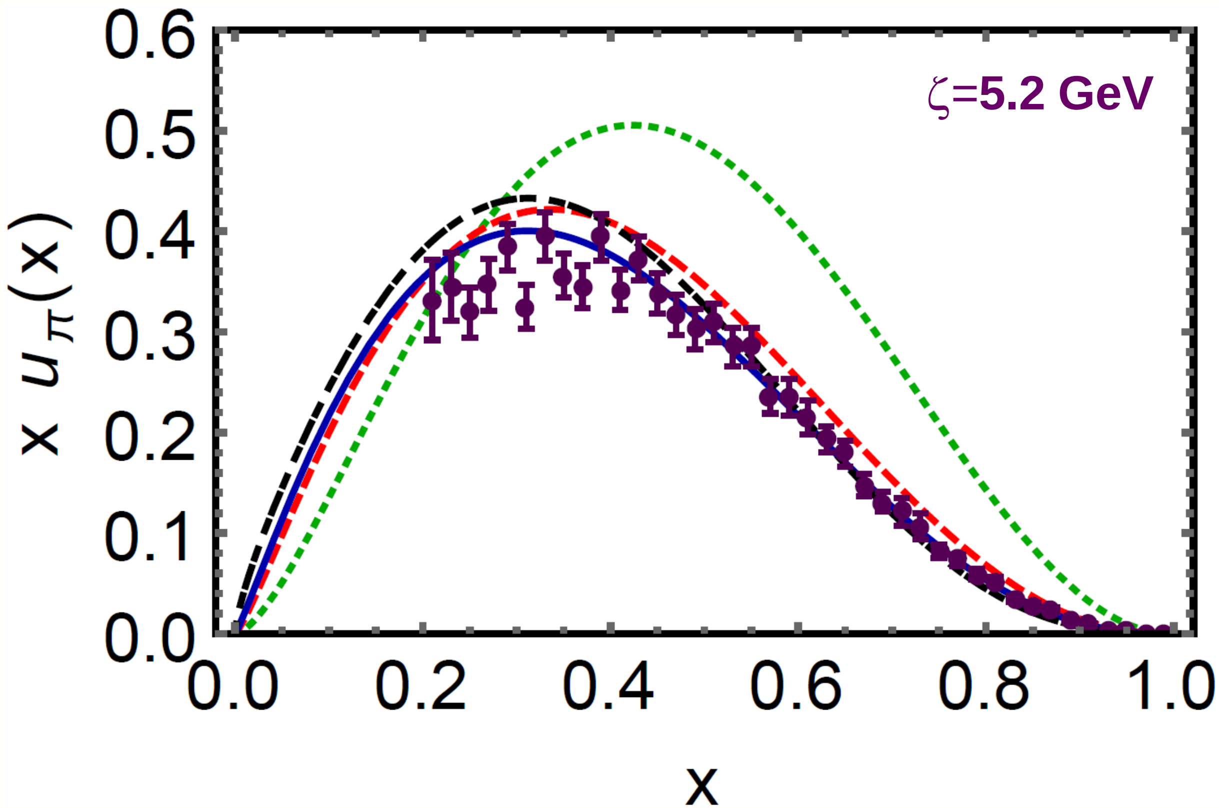


$$H^u_M(x,0,0) = u_M(x)$$



$\zeta_{M,M'}$: **Intrinsic** model **scales**

Pion PDF



$\langle x \rangle = \mathbf{0.24}$

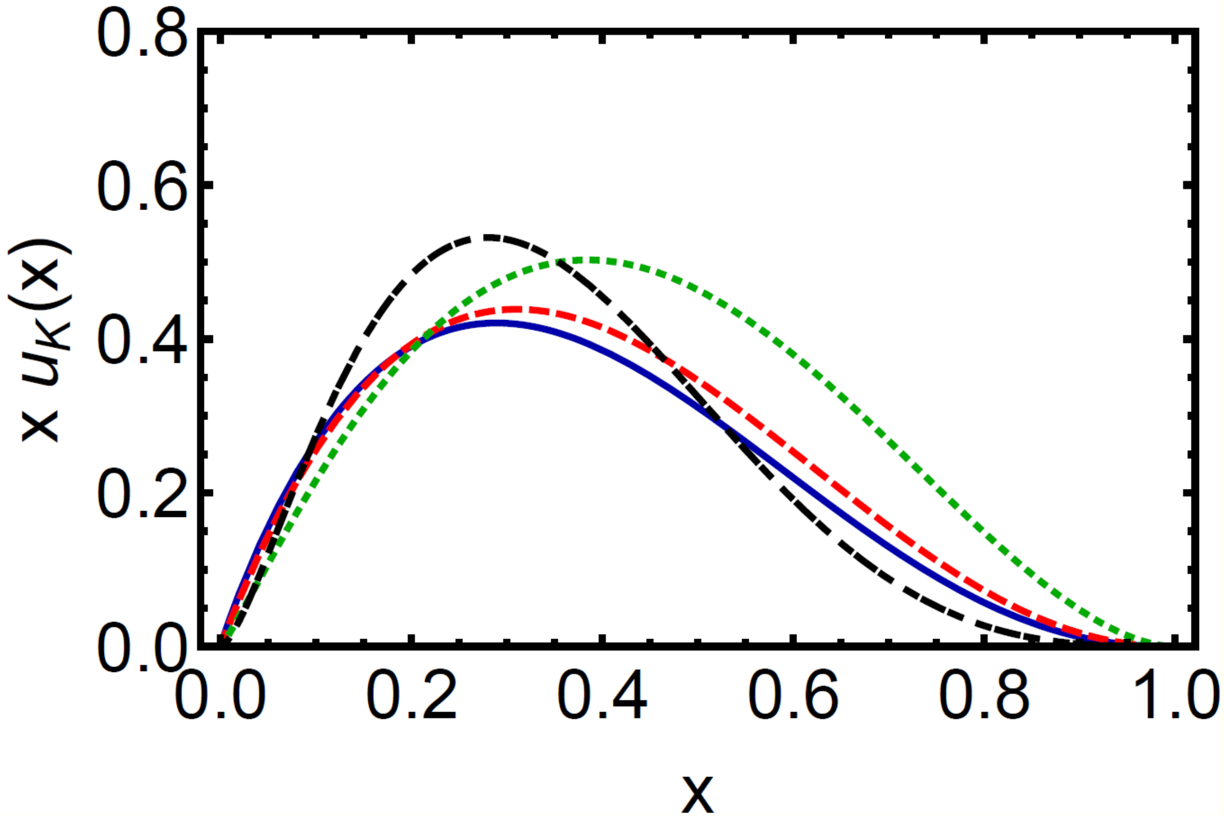
$\langle x \rangle = \mathbf{0.26}$

$\langle x \rangle = \mathbf{0.33}$

$\langle x \rangle = \mathbf{0.26}^{*}$

ζ_0 = 0.349 GeV: Obtained directly from the experimental data (π).
ζ_0 = 0.374 GeV: Obtained to best fit the lattice moments at 2 GeV (π).
ζ_0 = 0.510 GeV: Typical hadron scale. See for example: **Phys.Lett. B737 (2014) 23-29** and **Phys.Rev. D93 (2016) no.7, 074021***.

Kaon PDF



$< x > = 0.22$

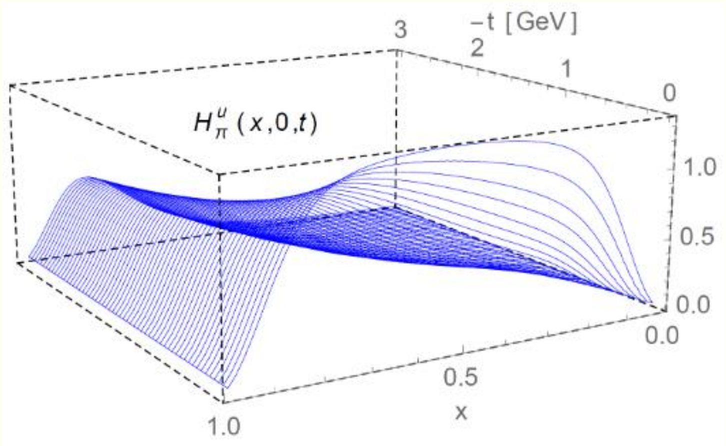
$< x > = 0.24$

$< x > = 0.29$

$< x > = 0.28$*

ζ_0 = 0.349 GeV: Obtained directly from the experimental data (π).
ζ_0 = 0.374 GeV: Obtained to best fit the lattice moments at 2 GeV (π).
ζ_0 = 0.510 GeV: Typical hadron scale. See for example: **Phys.Lett. B737 (2014) 23-29** and **Phys.Rev. D93 (2016) no.7, 074021***.

GPDs and EFF: Pion

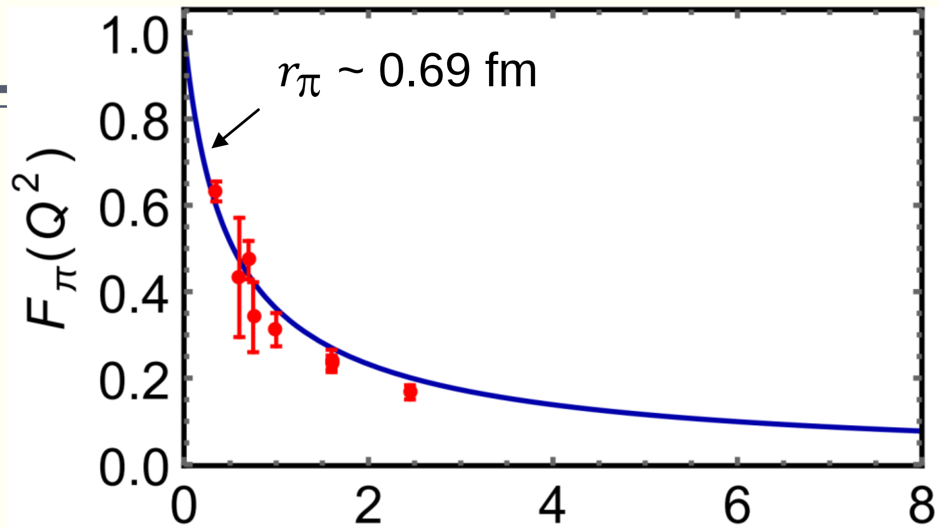


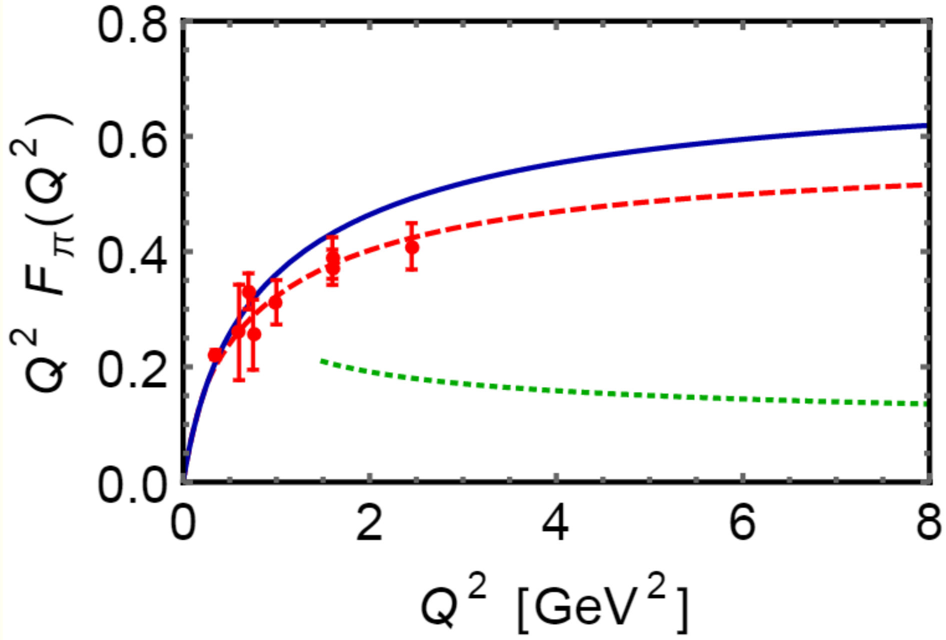
$$F^q_M(-t=\Delta^2) = \int_{-1}^{1} dx\, H^q_M(x,\xi,t)$$

Blue: Computed from GPD

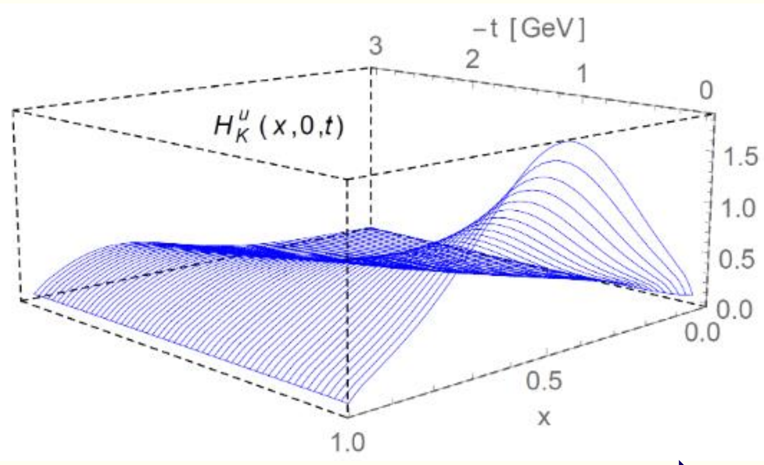
Green: Computed from HS formula

Red: '*Evolved'* form factor

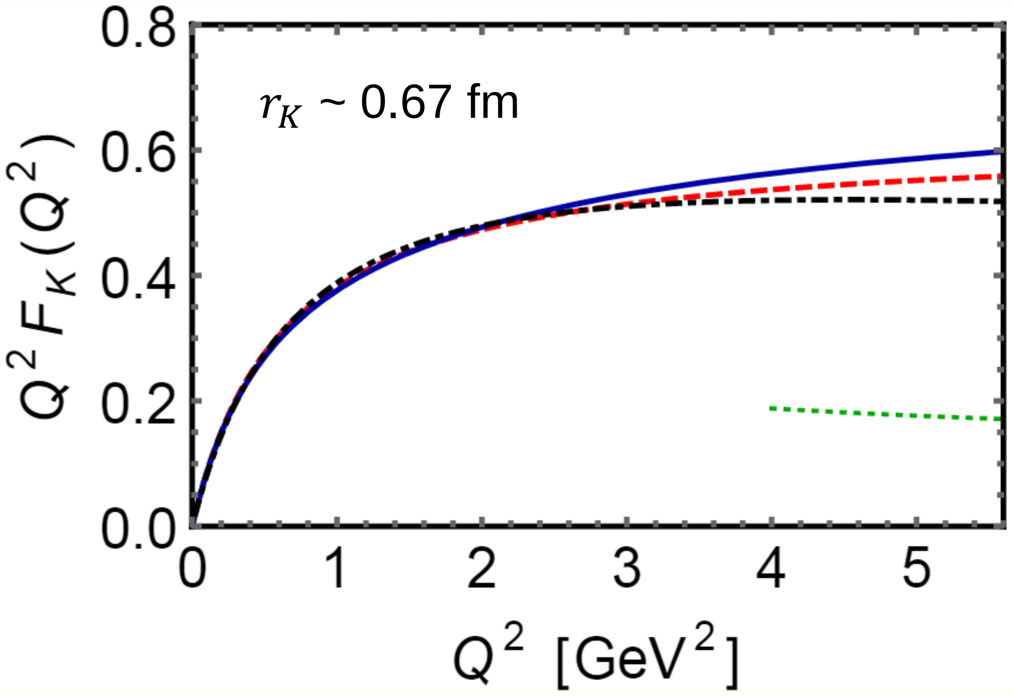




GPDs and PDFs: Kaon



$$F_M^q(-t=\Delta^2)=\int_{-1}^{1}dx\ H_M^q(x,\xi,t)$$



Blue: Computed from GPD

Green: Computed from HS formula

Black: DSE result, **Phys.Rev. D96 (2017) no.3, 034024**

Final remarks

Phys.Rev. D95 (2017) no.7, 074014. KR et al.
Phys.Rev. D93 (2016) no.7, 074017. KR et al.

- With several facilities at work all around the world, hadron physics is a very active field today: **it is the time to be interested in hadron physics**.

- Continuum QCD has evolved to the point where QCD connected predictions for elastic and transition form factors and parton distributions of **all types are within reach**:
  - **PDFs and GPDs:** Phys.Lett. B737 (2014) 23-29; Phys.Lett. B741 (2015) 190-196; Phys.Rev. D93 (2016) no.7, 074021
  - **PDAs and form factors:** Phys.Rev.Lett. 110 (2013) no.13, 132001; Phys.Rev.Lett. 111 (2013) no.14, 141802; Phys.Lett. B753 (2016) 330-335, Phys.Rev. D93 (2016) no.7, 074017; Phys.Lett. B783 (2018) 263-267.

- Lattice QCD and experiments provide crucial information to improve the theoretical predictions. Exist now an array of exciting predictions waiting for empirical validation.